SAMPLE QUESTION PAPER 4

A HIGHLY SIMULATED SAMPLE QUESTION PAPER FOR CBSE CLASS XII

MATHEMATICS

GENERAL INSTRUCTIONS

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
3. Both Part A and Part B have choices.

PART – A

1. It consists of two sections- I and II.
2. Section I comprises of 16 Very Short Answer Type Questions.
3. Section II contains two Case Studies. Each Case Study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

PART – B

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section -III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

TIME : 3 HOURS ***MAX. MARKS : 80***

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. Find the principal value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ and $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

2. Find the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^3+\sin\left(\frac{dy}{dx}\right)+1=0$.

Or Find the value of $\cos\left\{\frac{\pi}{2}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$.

3. Show that the relation R on the set $A=\{1, 2, 3\}$ given by $R=\{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

Or Show that the relation R on the set $A=\{1, 2, 3\}$ given by $R=\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric not transitive.

4. For what value of k, the matrix $\begin{bmatrix}2-k & 4\\ -5 & 1\end{bmatrix}$ is not invertible?

Or Find x, if $\begin{bmatrix}1 & 2 & x\\ 1 & 1 & 1\\ 2 & 1 & -1\end{bmatrix}$ is singular matrix.

5. If $\sin(x+y)=\log(x+y)$, then find the value of dy/dx.

Or Find the area of the parallelogram having adjacent sides $\vec{a}$ and $\vec{b}$ given by $2\hat{i}+\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}+4\hat{k}$, respectively.

6. Find the value of a, so that the sum of the squares of the roots of the equation $x^2-(a-2)x-a+1=0$ assure the least value.

Or Let A be a non-singular matrix of order n, where $n \geq 2$. Then, find adj (adj A).

7. Prove that the function $f(x)=\frac{x^3}{3}-x^2+x-9$ is increasing on R.

8. If $y=2x+|x|$, then find $\frac{dy}{dx}$ at $x=-1$.

9. If $\int \frac{2^{1/x}}{x^2}dx=\frac{2^{1/x}}{k}+C$, then find the value of k.

10. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared atleast once?

11. The probability of a discrete random variable X is given below

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

Then, find value of k.

12. Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

13. Find the angle between the vectors $5\hat{i}+3\hat{j}+4\hat{k}$ and $6\hat{i}-8\hat{j}-\hat{k}$.

14. Given, $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$, find $|\vec{a}\times\vec{b}|$.

15. If a line makes angle α, β, γ with the positive direction of X, Y and Z-axes respectively, find the value of $2+\cos 2\alpha+\cos 2\beta+\cos 2\gamma$.

16. Find the magnitude of $\vec{a}$ given by $\vec{a}=(\hat{i}+3\hat{j}-2\hat{k})\times(-\hat{i}+3\hat{k})$.

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each part carries 1 mark

17. If $f(x)$ is a continuous function defined on $[-a, a]$, then

$$\int_{-a}^{a} f(x)\,dx=\begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is a odd function} \end{cases}$$

A function $f(x)$ is even, when $f(-x)=f(x)$ and odd when $f(-x)=-f(x)$.

On the basis of above information, answer the following questions.

(i) If $f(x)$ is even function then the value of $\int_{-1}^{1}(f(x)-f(-x))\,dx$ is

(a) 1 (b) 2 (c) 0 (d) -2

(ii) If $g(x)$ is odd function then the value of $\int_{-a}^{a}(g(x)+g(-x))\,dx=$

(a) 1 (b) 2 (c) 0 (d) -2

(iii) $\int_{-\pi}^{\pi}\frac{x}{1+\cos^2 x}dx=$

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 0

(iv) $\int_{-\pi}^{\pi}|\sin x|\,dx=$

(a) 1 (b) 2 (c) 3 (d) 4

(v) $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx=$

(a) 0 (b) 1 (c) -2 (d) 2

18. Suppose a random variable X had the following probability distribution:

X	-2	-1	0	1	2	3
$P(X)$	0.1	K	0.2	$2K$	0.3	K

On the basis of above information, answer the following questions.

(i) The value of K is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

(ii) The most expected value of X is

(a) 0 (b) 1 (c) 2 (d) 3

(iii) $P(X>1)=$

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

(iv) $P(X<1)=$

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

(v) $P(X\geq -1)=$

(a) 0.9 (b) 0.8 (c) 0.7 (d) 0.6

PART B

Section III

All questions are compulsory. In case of internal choices attempt any one.

19. Show that a matrix which is both symmetric as well as skew-symmetric is a null matrix.

20. Two events A and B are such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{2}{3}$. Are the events A and B mutually independent?

21. Is the function $f(x)=\begin{cases}\dfrac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ continuous at $x=0$?

Or

If $xy=1$, prove that $\frac{dy}{dx}+y^2=0$.

22. If $x^y=e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.

23. Find the set of values of $\cot^{-1}(1)$ and $\cot^{-1}(-1)$.

24. Find the equation of a line passing through the point $(3,-1,6)$ and perpendicular to two lines

$$\vec{r}=(3\hat{i}-\hat{j}+4\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$$

and $\vec{r}=(12\hat{i}+10\hat{j}+\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

25. If $y=\frac{e^x+\log x}{\sin 3x}$, then find $\frac{dy}{dx}$.

Or If $y=\frac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx}=\frac{\cos^{-1}x}{(1-x^2)^{3/2}}$.

26. Find the point of local maxima or local minima for the function, $f(x)=\sin x+\cos x$, where $0<x<\frac{\pi}{2}$.

Also find the local maximum or local minimum value.

27. Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$.

Or Prove that $\int_0^{\pi}\frac{x}{(1+\sin x)}dx=\pi$.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

28. Find the value of c for which the vectors $\vec{a}=(c\log_2 x)\hat{i}-6\hat{j}+3\hat{k}$ and $\vec{b}=(\log_2 x)\hat{i}+2\hat{j}+(2c\log_2 x)\hat{k}$ make an obtuse angle for any $x\in(0,\infty)$.

29. Show that the relation R in the S at $A=\{x: x\in W, 0\le x\le 12\}$ given by $R=\{(a,b): |a-b| \text{ is multiple of } 4\}$ is an equivalence relation. Also, find the set of all elements related to 2.

Or Show that the function $f:R\to\{x\in R: -1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}$, $x\in R$ is one-one and onto function.

30. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and $y(0)=-1$, then show that $y(1)=-\frac{1}{2}$.

Or It is given that the rate of which some bacteria multiply is proportional to the instantaneous number present. If the original number of bacteria doubles in two hours, in how many hours will it be five times?

31. Evaluate $\int_0^{\pi}\log(1+\cos x)\,dx$.

32. A man is moving away from a tower 41.6 m high at the rate of 2 m/sec. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.

33. Find the area of region bounded by the curve $y^2=4x$ and the line $x=4$.

34. Evaluate $\int \frac{x^3}{x^{16}+4}dx$.

35. If $y=(\sin x)^x+\sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. A school wants to award its students for the values of honesty, regularity and hardwork with a total cash award of ₹ 6000. Three times the award money for hardwork added to that given for honesty amounts to ₹ 11000. The award money given for honesty and hardwork together is double the one given for regularity. Represent the above situation algebraically and find the award money for each value, by using matrix method.

Or The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

37. Solve the given LPP graphically.

Maximise $(Z)=22x+18y$

Subject to constraints

$x+y\le 20,\ 3x+2y\le 48,\ x\ge 0,\ y\ge 0$

Or Solve the following LPP graphically:

Minimise $Z=5x+10y$

subject to the constraints $x+2y\le 120$

$x+y\ge 60,\ x-2y\ge 0$ and $x,\ y\ge 0$

38. If straight lines having direction cosines given by $al+bm+cn=0$ and $fmn+gnl+hlm=0$ are perpendicular, then show that $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0.$

Or Find the vector and cartesian equation of a line through the point $(1,-1,1)$ and perpendicular to the lines joining the points $(4,3,2),(1,-1,0)$ and $(1,2,-1),(2,1,1)$.

ANSWERS

1. $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$ **2.** Degree is not defined *Or* $-\frac{1}{2}$ **4.** $k=22$ *Or* $x=4$

5. $\frac{dy}{dx}=-1$ *Or* $\sqrt{35}$ sq unit **6.** $a=1$ *Or* $|A|^{n-2}A$ **8.** $\frac{dy}{dx}=1$ **9.** $k=-\log 2$

10. $\frac{2}{5}$ **11.** $k=32$ **12.** 0 **13.** $\theta=\cos^{-1}\left(\frac{\sqrt{2}}{5\sqrt{101}}\right)$ *Or* $\theta=\frac{\pi}{4}$

14. $|\vec{a}\times\vec{b}|=16$ **15.** 1 **16.** $\sqrt{91}$ **17.** (i) (c) (ii) (c) (iii) (d) (iv) (d) (v) (a)

18. (i) (a) (ii) (c) (iii) (d) (iv) (d) (v) (a) **20.** Mutually independent

21. Function is discontinuous at $x=0$ **23.** $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ **24.** $\vec{r}=(3\hat{i}-\hat{j}+6\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

25. $\frac{dy}{dx}=\frac{\left(e^x+\frac{1}{x}\right)\sin 3x-3(e^x+\log x)\cos 3x}{\sin^2 3x}$ **26.** Point of local maximum is $\frac{\pi}{4}$ and local maximum value is $\sqrt{2}$.

27. $\log\left|\frac{2-\sin x}{1-\sin x}\right|+C$ **28.** $C\in\left(\frac{-4}{3},0\right)$ **29.** {2, 6, 10} **30.** *Or* $\frac{2\log 5}{\log 2}$ h **31.** $-\pi\log e 2$

32. The angle of elevation of the top of tower is decreasing at the rate of $\frac{4}{125}$ radians/sec. **33.** $\frac{64}{3}$ sq units

34. $\frac{1}{32}\tan^{-1}\left(\frac{x^8-2}{2x^4}\right)-\frac{1}{64}\log\left|\frac{x^8-2x^4+2}{x^8+2x^4+2}\right|+C$ **35.** $\frac{dy}{dx}=(\sin x)^x[x\cot x+\log\sin x]+\frac{1}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}}$

36. ₹ 500, ₹ 2000, ₹ 3500 *Or* $x=1, y=2$ and $z=3$

37. Maximum value of $Z=392$ at point $B(8,12)$ or minimum value of $Z=300$ at point (60,0)

38. $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(10\hat{i}-4\hat{j}-7\hat{k})$ and $\frac{x-1}{10}=\frac{y+1}{-4}=\frac{z-1}{-7}$