5

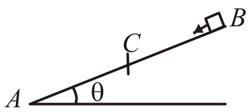
Work, Energy and Power

TOPIC 1 Work

1. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box ? **[4 Sep. 2020 (II)]**

(a) 3280 J (b) 2780 J
(c) 5690 J (d) 5250 J

2.

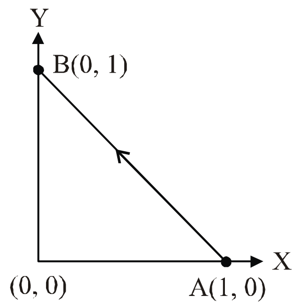


A small block starts slipping down from a point *B* on an inclined plane *AB*, which is making an angle θ with the horizontal section *BC* is smooth and the remaining section *CA* is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point *A*) of the inclined plane. If *BC* = 2*AC*, the coefficient of friction is given by μ = *k* tanθ. The value of *k* is _________.

[NA 2 Sep. 2020 (I)]

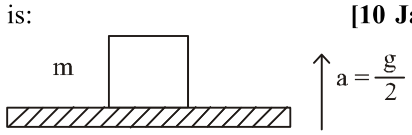
3. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units)

[9 Jan. 2020 I]



(a) 2J (b) $\frac{1}{2}$J (c) 1J (d) $\frac{3}{2}$J

4. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. work done by normal reaction on block in time t is: **[10 Jan. 2019 I]**



(a) $-\frac{m g^2 t^2}{8}$ (b) $\frac{m g^2 t^2}{8}$

(c) 0 (d) $\frac{3m g^2 t^2}{8}$

5. A body of mass starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant s and is the distance covered by the body. The total work done by all the forces acting on the body in the first second after the start of the motion is: **[Online April 16, 2018]**

(a) $\frac{1}{8}ma^4t^2$ (b) $4ma^4t^2$

(c) $8ma^4t^2$ (d) $\frac{1}{4}ma^4t^2$

6. When a rubber-band is stretched by a distance x, it exerts restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is: **[2014]**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}\left(aL^2 + bL^3\right)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

7. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ? **[2004]**

(a) 12 J (b) 3.6 J (c) 7.2 J (d) 1200 J

8. A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\vec{i} - \vec{j})m$. The work done on the particle in joules is **[2004]**

(a) +10 (b) +7 (c) –7 (d) +13

9. A spring of spring constant 5×10^3 N/m is stretched initially by 5cm from the unstretched position. Then the work required to stretch it further by another 5 cm is **[2003]**

(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

10. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is **[2002]**

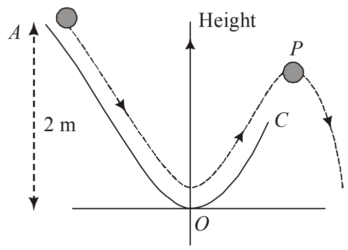
(a) 16 J (b) 8 J (c) 32 J (d) 24 J

TOPIC 2 Energy

11. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is (g = 10 ms^{-2}) __________.

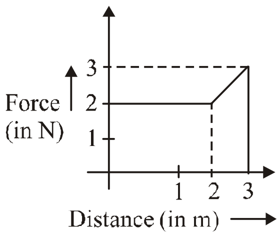
[NA 3 Sep. 2020 (I)]

12. A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is: (Figure drawn is schematic and not to scale; take g = 10 ms^{-2}) ______ . **[NA 7 Jan. 2020 I]**



13. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is :

[7 Jan. 2020 II]



(a) 4 J (b) 2.5 J
(c) 6.5 J (d) 5 J

14. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be: **[12 April 2019 II]**

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

15. A body of mass 1 kg falls freely from a height of 100m, on a platform of mass 3 kg which is mounted on a spring having spring constant k = 1.25×10^6 N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that g = 10 ms^{-2}, the value of x will be close to : **[11 April 2019 I]**

(a) 40 cm (b) 4 cm (c) 80 cm (d) 8 cm

16. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

[9 April 2019 I]

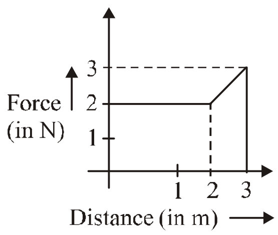
(a) $\frac{MgL}{2n^2}$ (b) $\frac{MgL}{n^2}$ (c) $\frac{2MgL}{n^2}$ (d) nMgL

17. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by: **[9 April 2019 II]**

(a) $\frac{v^2}{g}$ (b) $\frac{2v^2}{7g}$
(c) $\frac{2v^2}{5g}$ (d) $\frac{v^2}{2g}$

18. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is :

[8 April 2019 I]

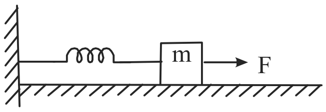


(a) 4 J (b) 2.5 J
(c) 6.5 J (d) 5 J

19. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? **[10 Jan. 2019 II]**

(a) 9 J (b) 12 J
(c) 10 J (d) 15 J

20. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is: **[9 Jan. 2019 I]**



(a) $\frac{2F}{\sqrt{mk}}$ (b) $\frac{F}{\pi\sqrt{mk}}$ (c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{F}{\sqrt{mk}}$

21. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? **[9 Jan. 2019 II]**

(a) 850 J (b) 950 J (c) 875 J (d) 900 J

22. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is: **[2018]**

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$

(c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

23. Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$

The first particle is at a distance r = 1, and the second, at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to **[Online April 16, 2018]**

(a) 10^{-1} (b) 6×10^{-2} (c) 6×10^2 (d) 3×10^{-3}

24. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be: **[2017]**

(a) 10^{-4} kg m^{-1} (b) 10^{-1} kg m^{-1} s^{-1}

(c) 10^{-3} kg m^{-1} (d) 10^{-3} kg s^{-1}

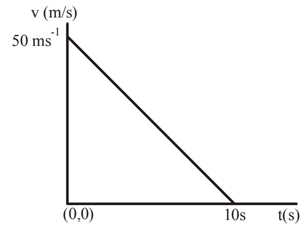
25. An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered as $t \to \infty$ is **[Online April 8, 2017]**

(a) 3h (b) ∞ (c) $\frac{5}{3}h$ (d) $\frac{8}{3}h$

26. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 second will be **[2017]**

(a) 9 J (b) 18 J (c) 4.5 J (d) 22 J

27. Velocity–time graph for a body of mass 10 kg is shown in figure. Work–done on the body in first two seconds of the motion is : **[Online April 10, 2016]**

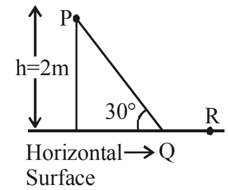


(a) – 9300 J (b) 12000 J

(c) –4500 J (d) –12000 J

28. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The value of the coefficient of friction μ and the distance x (= QR), are, respectively close to : **[2016]**



(a) 0.29 and 3.5 m (b) 0.29 and 6.5 m

(c) 0.2 and 6.5 m (d) 0.2 and 3.5 m

29. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} : **[2016]**

(a) 9.89×10^{-3} kg (b) 12.89×10^{-3} kg

(c) 2.45×10^{-3} kg (d) 6.45×10^{-3} kg

30. A particle is moving in a circle of radius r under the action of a force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for r = 0) : **[Online April 11, 2015]**

(a) $\frac{1}{2}\alpha r^3$ (b) $\frac{5}{6}\alpha r^3$ (c) $\frac{4}{3}\alpha r^3$ (d) αr^3

31. A block of mass $m = 0.1$ kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3 ms^{-1}.

The total initial energy of the spring is : **[Online April 10, 2015]**

(a) 0.3 J (b) 0.6 J
(c) 0.8 J (d) 1.5 J

32. A bullet looses $\left(\frac{1}{n}\right)^{th}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be: **[Online April 19, 2014]**

(a) $\frac{n^2}{2n-1}$ (b) $\frac{2n^2}{n-1}$ (c) infinite (d) n

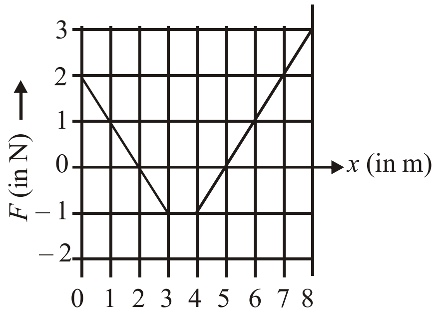
33. A spring of unstretched length l has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is: **[Online April 12, 2014]**

(a) $\frac{1}{2}mv^2$ (b) mv^2 (c) $\frac{1}{3}mv^2$ (d) $\frac{1}{6}mv^2$

34. Two springs of force constants 300 N/m (Spring A) and 400 N/m (Spring B) are joined together in series. The combination is compressed by 8.75 cm. The ratio of energy stored in A and B is $\frac{E_A}{E_B}$. Then $\frac{E_A}{E_B}$ is equal to : **[Online April 9, 2013]**

(a) $\frac{4}{3}$ (b) $\frac{16}{9}$ (c) $\frac{3}{4}$ (d) $\frac{9}{16}$

35. The force $\vec{F} = F\hat{i}$ on a particle of mass 2 kg, moving along the x-axis is given in the figure as a function of its position x. The particle is moving with a velocity of 5 m/s along the x-axis at $x = 0$. What is the kinetic energy of the particle at $x = 8$ m? **[Online May 26, 2012]**



(a) 34 J (b) 34.5 J (c) 4.5 J (d) 29.4 J

36. A particle gets displaced by $\Delta\bar{r} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ m under the action of a force $\vec{F} = \left(7\hat{i} + 4\hat{j} + 3\hat{k}\right)$. The change in its kinetic energy is **[Online May 7, 2012]**

(a) 38 J (b) 70 J (c) 52.5 J (d) 126 J

37. At time $t = 0$ a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to **[2011 RS]**

(a) constant (b) t
(c) $\frac{1}{\sqrt{t}}$ (d) $\sqrt{t}$

38. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is

$D = \left[U(x = \infty) - U_{\text{at equilibrium}}\right]$, D is **[2010]**

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

39. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[2008]**

(a) 200 J - 500 J (b) 2×10^5 J - 3×10^5 J
(c) 20, 000 J - 50,000 J (d) 2,000 J - 5, 000 J

40. A 2 kg block slides on a horizontal floor with a speed of 4m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15N and spring constant is 10,000 N/m. The spring compresses by **[2007]**

(a) 8.5 cm (b) 5.5 cm
(c) 2.5 cm (d) 11.0 cm

41. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[2007]**

(a) $K/2$ (b) K
(c) Zero (d) $K/4$

42. A particle of mass 100g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is **[2006]**

(a) –0.5 J (b) –1.25 J
(c) 1.25 J (d) 0.5 J

43. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) J$.

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is **[2006]**

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

44. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is **[2006]**

(a) $Mg(\sqrt{2}+1)$ (b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2}-1)$

45. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is **[2005]**

(a) 20 m/s (b) 40 m/s
(c) $10\sqrt{30}$ m/s (d) 10 m/s

46. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to **[2004]**

(a) x (b) e^x (c) x^2 (d) $\log_e x$

47. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particles takes place in a plane. It follows that **[2004]**

(a) its kinetic energy is constant
(b) its acceleration is constant
(c) its velocity is constant
(d) it moves in a straight line

48. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is **[2003]**

(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J

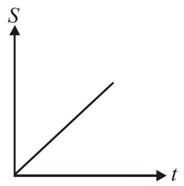
49. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be **[2002]**

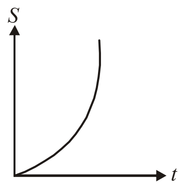
(a) E (b) $E/\sqrt{2}$ (c) $E/2$ (d) zero

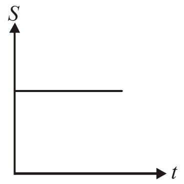
TOPIC 3 Power

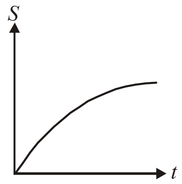
50. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) ________. **[5 Sep. 2020 (II)]**

51. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) : **[3 Sep. 2020 (II)]**

(a)


(b)


(c)


(d)


52. A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms^{-2}) **[7 Jan. 2020 I]**

(a) 1.7 ms^{-1} (b) 1.9 ms^{-1}
(c) 1.5 ms^{-1} (d) 2.0 ms^{-1}

53. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by $n^2R\,t^2$ where n is a constant. The power delivered to the particle by the force acting on it, is : **[Online April 10, 2016]**

(a) $\frac{1}{2}Mn^2R^2t^2$ (b) Mn^2R^2t
(c) MnR^2t^2 (d) MnR^2t

54. A car of weight W is on an inclined road that rises by 100 m over a distance of 1 Km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 ms^{-1}, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is: **[Online April 9, 2016]**

(a) 20 ms^{-1} (b) 5 ms^{-1} (c) 15 ms^{-1} (d) 10 ms^{-1}

55. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to **[Online April 25, 2013]**

(a) v^4 (b) v^2 (c) v (d) v

56. A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The center of gravity rises by 0.5 m before he leaps. After the leap the c.g. rises by another 1 m. The maximum power delivered by the muscles is : (Take g = 10 ms^{-2}) **[Online April 23, 2013]**

(a) 6.26×10^3 Watts at the start
(b) 6.26×10^3 Watts at take off
(c) 6.26×10^4 Watts at the start
(d) 6.26×10^4 Watts at take off

57. A body of mass 'm', accelerates uniformly from rest to 'v_1' in time 't_1'. The instantaneous power delivered to the body as a function of time 't' is **[2004]**

(a) $\frac{mv_1t^2}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$
(c) $\frac{mv_1t}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$

58. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to **[2003]**

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$

TOPIC 4 Collisions

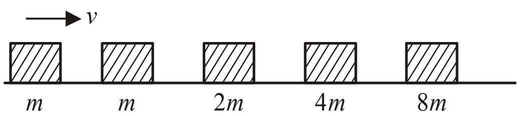
59. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is ______. **[NA 6 Sep. 2020 (I)]**

60. Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i}+\hat{j})\ ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particles A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = (\hat{i}+\sqrt{3}\hat{j})\ ms^{-1}$, the angle between $\vec{V}_1$ and $\vec{V}_2$ is : **[6 Sep. 2020 (II)]**

(a) 15º (b) 60º

(c) –45º (d) 105º

61. Blocks of masses *m*, 2*m*, 4*m* and 8*m* are arranged in a line on a frictionless floor. Another block of mass *m*, moving with speed *v* along the same line (see figure) collides with mass *m* in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8*m* starts moving the total energy loss is *p*% of the original energy. Value of '*p*' is close to : **[4 Sep. 2020 (I)]**



(a) 77 (b) 94

(c) 37 (d) 87

62. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s^2. Assume there is no rotational motion and losss of energy after the collision is negligiable.] **[3 Sep. 2020 (II)]**

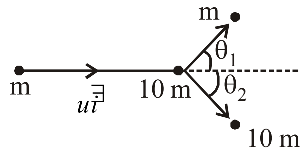
(a) 20 J (b) 21 J (c) 19 J (d) 23 J

63. A particle of mass *m* with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, *v* is given by : **[2 Sep. 2020 (I)]**

(a) $v=\sqrt{\frac{2}{3}}u$ (b) $v=\frac{u}{\sqrt{3}}$

(c) $v=\frac{u}{\sqrt{2}}$ (d) $v=\frac{1}{\sqrt{6}}u$

64. A particle of mass *m* is moving along the *x*-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of *n* is ___________. **[NA 2 Sep. 2020 (II)]**



65. Two particles of equal mass *m* have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is: **[9 Jan. 2020 I]**

(a) $\frac{1}{3}mu^2$ (b) $\frac{1}{8}mu^2$

(c) $\frac{3}{4}mu^2$ (d) $\sqrt{\frac{2}{3}}mu^2$

66. A body *A*, of mass *m* = 0.1 kg has an initial velocity of $3\hat{i}\ ms^{-1}$. It collides elastically with another body, *B* of the same mass which has an initial velocity of $5\hat{j}\ ms^{-1}$. After collision, *A* moves with a velocity $\vec{v}=4(\hat{i}+\hat{j})$. The energy of *B* after collision is written as $\frac{x}{10}$ *J*. The value of *x* is _______. **[NA 8 Jan. 2020 I]**

67. A particle of mass *m* is dropped from a height *h* above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is: **[8 Jan. 2020 II]**

(a) $\sqrt{\frac{1}{2}}$ (b) $\sqrt{\frac{3}{4}}$

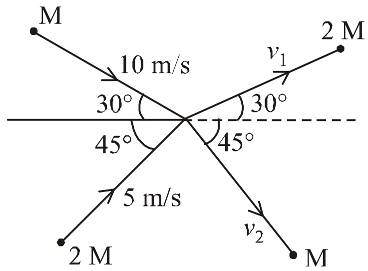
(c) $\frac{1}{2}$ (d) $\sqrt{\frac{3}{2}}$

68. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is : **[12 April 2019 I]**

(a) 0.28 ms^{-1} (b) 0.20 ms^{-1}

(c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

69. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly : **[10 April 2019 I]**



(a) 6.5 m/s and 6.3 m/s (b) 3.2 m/s and 6.3 m/s
(c) 6.5 m/s and 3.2 m/s (d) 3.2 m/s and 12.6 m/s

70. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body? **[9 April 2019 I]**
(a) 1.0 kg (b) 1.5 kg (c) 1.8 kg (d) 1.2 kg

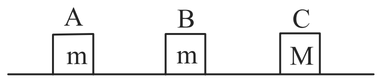
71. A particle of mass 'm' is moving with speed '$2v$' and collides with a mass '$2m$' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the original direction. **[9 April 2019 II]**
The speed of each of the moving particle will be:
(a) $\sqrt{2}\,v$ (b) $2\sqrt{2}\,v$
(c) $v/(2\sqrt{2})$ (d) $v/\sqrt{2}$

72. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.
If $m_2 = 0.5\,m_1$ and $v_3 = 0.5\,v_1$, then v_1 is : **[8 April 2019 II]**
(a) $v_4 - \frac{v_2}{2}$ (b) $v_4 - v_2$ (c) $v_4 - \frac{v_2}{4}$ (d) $v_4 + v_2$

73. An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is : **[12 Jan. 2019 II]**
(a) 2m (b) 3.5m (c) 1.5m (d) 4m

74. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is: ($g = 10\ ms^{-2}$) **[10 Jan. 2019 I]**
(a) 20m (b) 30m (c) 40m (d) 10m

75. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m? **[9 Jan. 2019 I]**



(a) 5 (b) 2 (c) 4 (d) 3

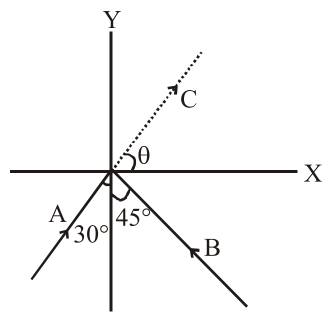
76. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is: **[2018]**
(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

77. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly: **[2018]**
(a) $2.35\times10^3\ N/m^2$ (b) $4.70\times10^3\ N/m^2$
(c) $2.35\times10^2\ N/m^2$ (d) $4.70\times10^2\ N/m^2$

78. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively: **[2018]**
(a) $(\cdot89, \cdot28)$ (b) $(\cdot28, \cdot89)$ (c) $(0, 0)$ (d) $(0, 1)$

79. A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass of unknown particle is: **[Online April 15, 2018]**
(a) $\frac{m}{\sqrt{3}}$ (b) $\frac{m}{2}$ (c) 2m (d) m

80. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X-axis is given by: **[Online April 9, 2017]**



(a) $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

(b) $\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$

(c) $\tan\theta = \frac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$

(d) $\tan\theta = \frac{1-\sqrt{3}}{1+\sqrt{2}}$

81. A neutron moving with a speed 'v' makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is : **[Online April 10, 2016]**

(a) 20.4 eV (b) 10.2 eV (c) 12.1 eV (d) 16.8 eV

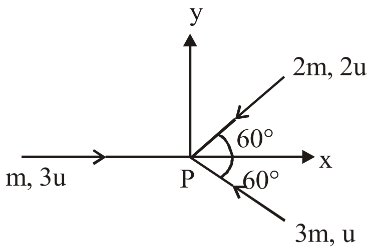
82. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed **v**. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : **[2015]**

(a) 56% (b) 62% (c) 44% (d) 50%

83. A bullet of mass 4 g is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately? **[Online April 12, 2014]**

(a) 0.19 m (b) 0.379 m (c) 0.569 m (d) 0.758 m

84. Three masses m, 2m and 3m are moving in x-y plane with speed 3u, 2u and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity of resulting mass will be: **[Online April 12, 2014]**



(a) $\frac{u}{12}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ (b) $\frac{u}{12}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

(c) $\frac{u}{12}\left(-\hat{i}+\sqrt{3}\hat{j}\right)$ (d) $\frac{u}{12}\left(-\hat{i}-\sqrt{3}\hat{j}\right)$

85. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement - I: Apoint particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[2013]**

(a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.

(b) Statement-I is true, Statment - II is true, Statement - II is not the correct explanation of Statement - II.

(c) Statement - I is true, Statment - II is false.

(d) Statement - I is false, Statment - II is true.

86. A projectile of mass M is fired so that the horizontal range is 4 km. At the highest point the projectile explodes in two parts of masses $M/4$ and $3M/4$ respectively and the heavier part starts falling down vertically with zero initial speed. The horizontal range (distance from point of firing) of the lighter part is : **[Online April 23, 2013]**

(a) 16 km (b) 1 km (c) 10 km (d) 2 km

87. A moving particle of mass m, makes a head on elastic collision with another particle of mass $2m$, which is initially at rest. The percentage loss in energy of the colliding particle on collision, is close to **[Online May 19, 2012]**

(a) 33% (b) 67% (c) 90% (d) 10%

88. Two bodies A and B of mass m and $2m$ respectively are placed on a smooth floor. They are connected by a spring of negligible mass. A third body C of mass m is placed on the floor. The body C moves with a velocity v_0 along the line joining A and B and collides elastically with A. At a certain time after the collision it is found that the instantaneous velocities of A and B are same and the compression of the spring is x_0. The spring constant k will be **[Online May 12, 2012]**

(a) $m\frac{v_0^2}{x_0^2}$ (b) $m\frac{v_0}{2x_0}$

(c) $2m\frac{v_0}{x_0}$ (d) $\frac{2}{3}m\left(\frac{v_0}{x_0}\right)^2$

89. A projectile moving vertically upwards with a velocity of 200 ms^{-1} breaks into two equal parts at a height of 490 m. One part starts moving vertically upwards with a velocity of 400 ms^{-1}. How much time it will take, after the break up with the other part to hit the ground? **[Online May 12, 2012]**

(a) $2\sqrt{10}$ s (b) 5 s

(c) 10 s (d) $\sqrt{10}$ s

90. **Statement -1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement -2 : Principle of conservation of momentum holds true for all kinds of collisions. **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is the correct explanation of Statement -1.

(b) Statement -1 is true, Statement -2 is true; Statement -2 is **not** the correct explanation of Statement -1

(c) Statement -1 is false, Statement -2 is true.

(d) Statement -1 is true, Statement -2 is false.

91. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[2008]**

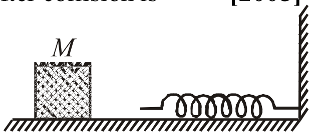
(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

92. A bomb of mass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velolcity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is **[2006]**

(a) 144 J (b) 288 J (c) 192 J (d) 96 J

93. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is **[2005]**



(a) $\frac{kL^2}{2M}$ (b) $\sqrt{Mk}\ L$ (c) $\frac{ML^2}{k}$ (d) zero

94. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision. **[2005]**

(a) $\sqrt{3}v$ (b) v (c) $\frac{v}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}v$

95. Consider the following two statements : **[2003]**

A. Linear momentum of a system of particles is zero

B. Kinetic energy of a system of particles is zero.

Then

(a) A does not imply B and B does not imply A

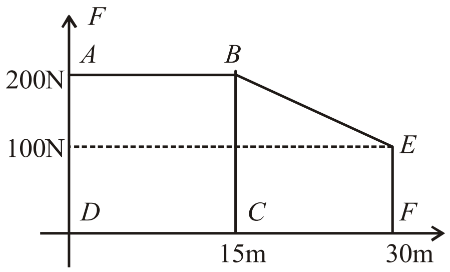
(b) A implies B but B does not imply A

(c) A does not imply B but B implies A

(d) A implies B and B implies A

Hints & Solutions

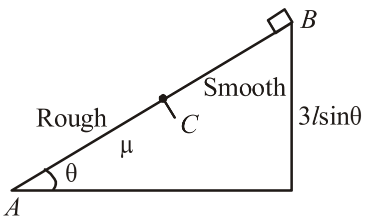
1. **(d)** The given situation can be drawn graphically as shown in figure.
Work done = Area under F-x graph
= Area of rectangle $ABCD$ + Area of trapezium $BCFE$



$$W = (200 \times 15) + \frac{1}{2}(100 + 200) \times 15 = 3000 + 2250$$

$$\Rightarrow W = 5250 \text{ J}$$

2. **(3)** If $AC = l$ then according to question, $BC = 2l$ and $AB = 3l$.



Here, work done by all the forces is zero.

$$W_{\text{friction}} + W_{mg} = 0$$

$$mg(3l)\sin\theta - \mu mg\cos\theta(l) = 0$$

$$\Rightarrow \mu mg \cos\theta l = 3mgl\sin\theta$$

$$\Rightarrow \mu = 3\tan\theta = k\tan\theta$$

$$\therefore k = 3$$

3. **(c)** Work done, $W = \int \vec{F} \cdot \vec{ds}$

$$= \left(-x\hat{i} + y\hat{j}\right) \cdot \left(d \times \hat{i} + dy\hat{j}\right)$$

$$\Rightarrow W = -\int_1^0 x\,dx + \int_0^1 y\,dy$$

$$= \left(0 + \frac{1}{2}\right) + \frac{1}{2} = 1J$$

4. **(d)** Here, $N - mg = ma = \frac{mg}{2} \Rightarrow N = \frac{3mg}{2}$
N = normal reaction
Now, work done by normal reaction 'N' on block in time t, $W = \vec{N}\vec{S} = \left(\frac{3mg}{2}\right)\left(\frac{1}{2}g/_2 t^2\right)$

or, $W = \frac{3mg^2t^2}{8}$

5. **(a)** From question, $v = a\sqrt{s} = \frac{ds}{dt}$

or, $2\sqrt{s} = at \Rightarrow S = \frac{a^2t^2}{4}$

$F = m \times \frac{a^2}{2}$

Work done $= \frac{ma^2}{2} \times \frac{a^2t^2}{4} = \frac{1}{8}ma^4t^2$

6. **(c)** Work done in stretching the rubber-band by a distance dx is
$dW = F\,dx = (ax + bx^2)dx$
Integrating both sides,

$$W = \int_0^L ax\,dx + \int_0^L bx^2\,dx = \frac{aL^2}{2} + \frac{bL^3}{3}$$

7. **(b)** Mass of over hanging part of the chain

$$m' = \frac{4}{2} \times (0.6)\text{kg} = 1.2 \text{ kg}$$

Weight of hanging part of the chain
= 1.2 × 10 = 12 N
C.M. of hanging part = 0.3 m below the table
Workdone in putting the entire cha in on the table = 12 × 0.30 = 3.6 J.

8. **(b)** Given, Force, $\vec{F} = \left(5\hat{i} + 3\hat{j} + 2\hat{k}\right)$

Displacement, $x = \left(2\hat{i} - \hat{j}\right)$
Work done,

$$W = \vec{F} \cdot \vec{x} = (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$$

$= 10 - 3 = 7$ joules

9. **(b)** Spring constant, $k = 5 \times 10^3$ N/m
Let x_1 and x_2 be the initial and final stretched position of the spring, then

Work done, $W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$

$$= \frac{1}{2} \times 5 \times 10^3 \left[(0.1)^2 - (0.05)^2\right]$$

$$= \frac{5000}{2} \times 0.15 \times 0.05 = 18.75 \text{ Nm}$$

10. **(b)** Small amount of work done in extending the spring by dx is
$dW = k\,x\,dx$

$$\therefore W = k\int_{0.05}^{0.15} x\,dx$$

$= \frac{800}{2}\left[(0.15)^2 - (0.05)^2\right]$
$= 400\,[(0.15+0.05)(0.15-0.05)]$
$= 400 \times 0.2 \times 0.1 = 8\text{ J}$

11. **(150.00)** From work energy theorem,

$W = F \cdot s = \Delta KE = \frac{1}{2}mv^2$

Here $V^2 = 2gh$

$\therefore F \cdot s = F \times \frac{2}{10} = \frac{1}{2} \times \frac{15}{100} \times 2 \times 10 \times 20$

$\therefore F = 150\text{ N}.$

12. **(10.00)** Kinetic energy = change in potential energy of the particle,

KE = mgΔh

Given, $m = 1$ kg,

$\Delta h = h_2 - h_1 = 2 - 1 = 1m$

$\therefore KE = 1 \times 10 \times 1 = 10\,J$

13. **(c)** We know area under F-x graph gives the work done by the body

$\therefore\ W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2 = 2.5 + 4 = 6.5\text{ J}$

Using work energy theorem,
Δ K.E = work done
∴ ΔK.E = 6.5 J

14. **(c)** $l_1 + l_2 = l$ and $l_1 = nl_2$

$\therefore\ l_1 = \frac{nl}{n+1}$ and $l_2 = \frac{l}{n+1}$

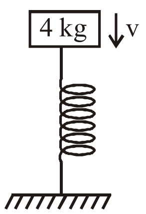
As $k \propto \frac{1}{l}$,

$\therefore\ \frac{k_1}{k_2} = \frac{l/(n+1)}{(nl)/(n+1)} = \frac{1}{n}$

15. **(b)** Velocity of 1 kg block just before it collides with 3 kg block $= \sqrt{2gh} = \sqrt{2000}$ m/s

Using principle of conservation of linear momentum just before and just after collision, we get

$1 \times \sqrt{2000} = 4v \Rightarrow v = \frac{\sqrt{2000}}{4}$ m/s



Initial compression of spring
$1.25 \times 10^6 x_0 = 30 \Rightarrow x_0 \approx 0$
using work energy theorem,
$W_g + W_{sp} = \Delta KE$

$\Rightarrow\ 40 \times x + \frac{1}{2} \times 1.25 \times 10^6 (0^2 - x^2)$

$= 0 - \frac{1}{2} \times 4 \times v^2$

solving x ≈ 4 cm

16. **(a)** $W = u_f - u_i$

$= 0 - \left(-\frac{mg}{n} \times \frac{L}{2n}\right) = \frac{MgL}{2n^2}.$

17. **(c)** $mv = (m+M)\,V'$

or $v = \frac{mv}{m+M} = \frac{mv}{m+4m} = \frac{v}{5}$

Using conservation of ME, we have

$\frac{1}{2}mv^2 = \frac{1}{2}(m+4m)\left(\frac{v}{5}\right)^2 + mgh$

or $h = \frac{2}{5}\frac{v^2}{g}$

18. **(c)** We know area under F-x graph gives the work done by the body

$\therefore\ W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2$
$= 2.5 + 4$
$= 6.5\text{ J}$

Using work energy theorem,
Δ K.E = work done
∴ Δ K.E = 6.5 J

19. **(d)** Work done = $\vec{F}.\vec{d} = (3\vec{i} - 12\vec{j}).(4\vec{i}) = 12\text{J}$

From work energy theorem,
$w_{net} = \Delta K.E. = k_f - k_i$
$\Rightarrow 12 = k_f - 3$
$\therefore K_f = 15\text{J}$

20. **(d)** Maximum speed is at mean position or equilibrium

At equilibrium Position

$F = kx \Rightarrow x = \frac{F}{k}$

From work-energy theorem,

$W_F + W_{sp} = \Delta KE$

$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$

$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2$

or, $v_{max} = \frac{F}{\sqrt{mk}}$

21. **(d)** Position, $x = 3t^2 + 5$

$\therefore$ Velocity, $v = \frac{dx}{dt} \Rightarrow v = \frac{d(3t^2+5)}{dt}$

$\Rightarrow v = 6t + 0$

At t = 0 v = 0
And, at t = 5 sec v = 30 m/s
According to work-energy theorem, w = ΔKE

or, $W=\frac{1}{2}mv^2-0=\frac{1}{2}(2)(30)^2=900J$

22. (c) $F=-\frac{\partial u}{\partial r}\hat{r}=\frac{K}{r^3}\hat{r}$

Since particle is moving in circular path

$$F=\frac{mv^2}{r}=\frac{K}{r^3}\Rightarrow mv^2=\frac{K}{r^2}$$

$$\therefore\ K.E.=\frac{1}{2}mv^2=\frac{K}{2r^2}$$

Total energy = P.E. + K.E.

$$=-\frac{K}{2r^2}+\frac{K}{2r^2}=\text{Zero} \qquad (\because P.E.=-\frac{K}{2r^2}\text{ given})$$

23. (b) As the particles moving in circular orbits, So

$$\frac{mv^2}{r}=\frac{16}{r}+r^2$$

Kinetic energy, $KE_0=\frac{1}{2}mv^2=\frac{1}{2}[16+r^4]$

For first particle, r = 1, $K_1=\frac{1}{2}m(16+1)$

Similarly, for second particle, r = 4, $K_2=\frac{1}{2}m(16+256)$

$$\therefore\ \frac{K_1}{K_2}=\frac{\frac{16+1}{2}}{\frac{16+256}{2}}=\frac{17}{272}\simeq 6\times10^{-2}$$

24. (a) Let V_f is the final speed of the body.
From questions,

$$\frac{1}{2}mV_f^2=\frac{1}{8}mV_0^2 \Rightarrow V_f=\frac{V_0}{2}=5\,m/s$$

$$F=m\left(\frac{dV}{dt}\right)=-kV^2 \quad \therefore \quad (10^{-2})\frac{dV}{dt}=-kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2}=-100K\int_0^{10}dt$$

$$\frac{1}{5}-\frac{1}{10}=100K(10) \qquad \text{or,}\quad K=10^{-4}\,kgm^{-1}$$

25. (a) (K.E.)' = 50% of K.E. after hit i.e.,

$$\frac{1}{2}mv'^2=\frac{50}{100}\times\frac{1}{2}mv^2\Rightarrow v'=\frac{v}{\sqrt{2}}$$

Coefficient of restitution $=\frac{1}{\sqrt{2}}$

Now, total distance travelled by object is

$$H=h\left(\frac{1+e^2}{1-e^2}\right)=h\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)=3h$$

26. (c) Using, $F=ma=m\frac{dV}{dt}$

$$6t=1.\frac{dV}{dt} \qquad [\because\ m=1\text{ kg given}]$$

$$\int_0^v dV=\int 6t\,dt \quad V=6\left[\frac{t^2}{2}\right]_0^1=3\,ms^{-1}$$

[∵ t = 1 sec given]

From work-energy theorem,

$$W=\Delta KE=\frac{1}{2}m\left(V^2-u^2\right)=\frac{1}{2}\times1\times9=4.5\text{ J}$$

27. (c) Acceleration (a) $=\frac{v-4}{t}=\frac{(0-50)}{(10-0)}=-5\text{ m/s}^2$

u = 50 m/s
∴ v = u + at = 50 – 5t
Veocity in first two seconds t = 2

$v_{(at\ t=2)}=40\text{ m/s}$

From work-energy theorem,

$$\Delta K.E.=W=\frac{1}{2}(40^2-50^2)\times10=-4500\text{ J}$$

28. (a) Work done by friction at QR = μmg*x*

In triangle, $\sin 30°=\frac{1}{2}=\frac{2}{PQ}$

⇒ *PQ* = 4*m*
Work done by friction at PQ = μmg × Cos 30° × 4

$=\mu mg\times\frac{\sqrt{3}}{2}\times4=2\sqrt{3}\ \mu mg$

Since work done by friction on parts *PQ* and *QR* are equal,
μmg*x* = $2\sqrt{3}$ umg
⇒ *x* = $2\sqrt{3}\cong3.5m$
Using work energy theorem mg sin 30° × 4 = $2\sqrt{3}$ μmg + μmg*x*
⇒ 2 = $4\sqrt{3}$ μ
⇒ μ = 0.29

29. (b) $n=\frac{W}{\text{input}}=\frac{mgh\times1000}{\text{input}}=\frac{10\times9.8\times1\times1000}{\text{input}}$

Input $=\frac{98000}{0.2}=49\times10^4$J

Fat used $=\frac{49\times10^4}{3.8\times10^7}=12.89\times10^{-3}$kg.

30. (b) As we know, dU = F.dr

$$U=\int_0^r\alpha r^2dr=\frac{ar^3}{3} \qquad ...(i)$$

As, $\frac{mv^2}{r}=\alpha r^2$

$m^2v^2=m\alpha r^3$

or, $2m(KE)=\frac{1}{2}\alpha r^3$ ...(ii)

Total energy = Potential energy + kinetic energy
Now, from eqn (i) and (ii)
Total energy = K.E. + P.E.

$$= \frac{\alpha r^3}{3} + \frac{\alpha r^3}{2} = \frac{5}{6}\alpha r^3$$

31. **(b)** Applying momentum conservation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.1u + m(0) = 0.1(0) + m(3)$

$0.1u = 3m$

$$\frac{1}{2}0.1u^2 = \frac{1}{2}m(3)^2$$

Solving we get, $u = 3$

$$\frac{1}{2}kx^2 = \frac{1}{2}K\left(\frac{x}{2}\right)^2 + \frac{1}{2}(0.1)3^2$$

$$\Rightarrow \frac{3}{4}kx^2 = 0.9$$

$$\Rightarrow \frac{3}{2}\times\frac{1}{2}kx^2 = 0.9$$

$\therefore \frac{1}{2}Kx^2 = 0.6$ J (total initial energy of the spring)

32. **(a)** Let u be the initial velocity of the bullet of mass m. After passing through a plank of width x, its velocity decreases to v.

$$\therefore \quad u - v = \frac{4}{n} \text{ or, } v = u - \frac{4}{n} = \frac{u(n-1)}{n}$$

If F be the retarding force applied by each plank, then using work – energy theorem,

$$Fx = \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}mu^2\frac{(n-1)^2}{n^2}$$

$$= \frac{1}{2}mu^2\left[\frac{1-(n-1)^2}{n^2}\right]$$

$$Fx = \frac{1}{2}mu^2\left(\frac{2n-1}{n^2}\right)$$

Let P be the number of planks required to stop the bullet.
Total distance travelled by the bullet before coming to rest = Px
Using work-energy theorem again,

$$F(Px) = \frac{1}{2}mu^2 - 0$$

$$\text{or, } P(Fx) = P\left[\frac{1}{2}mu^2\frac{(2n-1)}{n^2}\right] = \frac{1}{2}mu^2$$

$$\therefore P = \frac{n^2}{2n-1}$$

33. **(d)**

34. **(a)** Given : $k_A = 300\,N/m,\ k_B = 400\,N/m$

Let when the combination of springs is compressed by force F. Spring A is compressed by x. Therefore compression in spring B

$x_B = (8.75 - x)$ cm

$F = 300 \times x = 400(8.75 - x)$

Solving we get, x = 5 cm

$x_B = 8.75 - 5 = 3.75$ cm

$$\frac{E_A}{E_B} = \frac{\frac{1}{2}k_A(x_A)^2}{\frac{1}{2}k_B(x_B)^2} = \frac{300\times(5)^2}{400\times(3.75)^2} = \frac{4}{3}$$

35. **(d)**

36. **(a)** According to work-energy theorem,
Change in kinetic energy = work done

$$= \vec{F}.\Delta\vec{r} = \left(7\hat{i} + 4\hat{j} + 3\hat{k}\right).\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$= 14 + 12 + 12 = 38$ J

37. **(c)** K.E. $\propto t$

K.E. $= ct$ [Here, c = constant]

$$\Rightarrow \frac{1}{2}mv^2 = ct$$

$$\Rightarrow \frac{(mv)^2}{2m} = ct$$

$$\Rightarrow \frac{p^2}{2m} = ct \quad (\because p = mv)$$

$$\Rightarrow p = \sqrt{2ctm}$$

$$\Rightarrow F = \frac{dp}{dt} = \frac{d\left(\sqrt{2ctm}\right)}{dt}$$

$$\Rightarrow F = \sqrt{2\,cm}\times\frac{1}{2\sqrt{t}}$$

$$\Rightarrow F \propto \frac{1}{\sqrt{t}}$$

38. **(d)** At equilibrium : $F = \frac{-dU(x)}{dx}$

$$\Rightarrow F = \frac{-d}{dx}\left[\frac{a}{x^{12}} - \frac{b}{x^6}\right]$$

$$\Rightarrow F = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

$$\Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7} \quad \Rightarrow \quad x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a} \text{ and } U_{(x=\infty)} = 0$$

$$\therefore D = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$$

39. **(d)** The average speed of the athelete

$$v = \frac{5}{t} = \frac{100}{10} = 10\,m/s$$

$$\therefore \text{ K.E.} = \frac{1}{2}mv^2$$

Assuming the mass of athelet to 40 kg his average K.E would be

$$K.E = \frac{1}{2} \times 40 \times (10)^2 = 2000\,J$$

Assuming mass to 100 kg average kinetic energy

$$K.E. = \frac{1}{2} \times 100 \times (10)^2 = 5000\,J$$

40. (b) Suppose the spring gets compressed by x before stopping.
kinetic energy of the block = P.E. stored in the spring + work done against friction.

$$\Rightarrow \frac{1}{2} \times 2 \times (4)^2 = \frac{1}{2} \times 10{,}000 \times x^2 + 15 \times x$$

$$\Rightarrow 10{,}000\,x^2 + 30x - 32 = 0$$

$$\Rightarrow 5000x^2 + 15x - 16 = 0$$

$$\therefore \; x = \frac{-15 \pm \sqrt{(15)^2 - 4 \times (5000)(-16)}}{2 \times 5000}$$

$$= 0.055\text{m} = 5.5\text{cm}.$$

41. (d) Let u be the velocity with which the particle is thrown and m be the mass of the particle. Then

$$K = \frac{1}{2} mu^2. \qquad ...(1)$$

At the highest point the velocity is u cos 60° (only the horizontal component remains, the vertical component being zero at the top-most point). Therefore kinetic energy at the highest point.

$$K' = \frac{1}{2} m(u\cos 60°)^2 = \frac{1}{2} mu^2 \cos^2 60° = \frac{K}{4} \quad \text{[From 1]}$$

42. (b) Given, Mass of the particle, $m = 100$g
Initial speed of the particle, $\mu = 5$ m/s
Final speed of the particle, $v = 0$
Work done by the force of gravity
= Loss in kinetic energy of the body.

$$= \frac{1}{2} m\,(v^2 - u^2) = \frac{1}{2} \times \frac{100}{1000}\;(0^2 - 5^2)$$

$= -1.25$ J

43. (a) Potential energy

$$V(x) = \frac{x^4}{4} \frac{-x^2}{2} \text{ joule}$$

For maxima of minima

$$\frac{dV}{dx} = 0 \Rightarrow x^3 - x = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow \text{Min. P.E.} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}\,J$$

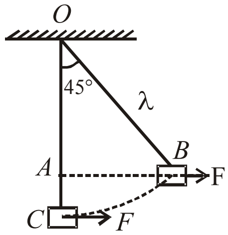
$K.E._{(max.)} + P.E._{(min.)} = 2$ (Given)

$$\therefore K.E._{(max.)} = 2 + \frac{1}{4} = \frac{9}{4}$$

$$K.E._{max.} = \frac{1}{2} m v^2_{max.}$$

$$\Rightarrow \frac{1}{2} \times 1 \times v^2_{max.} = \frac{9}{4} \Rightarrow v_{max.} = \frac{3}{\sqrt{2}}$$

44. (d) Work done by tension + Work done by force (applied) + Work done by gravitational force = change in kinetic energy
Work done by tension is zero



$$\Rightarrow 0 + F \times AB - Mg \times AC = 0$$

$$\Rightarrow F = Mg\left(\frac{AC}{AB}\right) = Mg\left[\frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right]$$

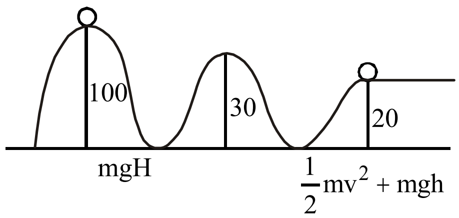
[$\because AB = \ell \sin 45° = \frac{\ell}{\sqrt{2}}$ and

$$AC = OC - OA = \ell - \ell\cos 45° = \ell\left(1 - \frac{1}{\sqrt{2}}\right)$$

where ℓ = length of the string.]

$$\Rightarrow F = Mg(\sqrt{2} - 1)$$

45. (b)



Using conservation of energy,
Total energy at 100 m height
= Total energy at 20m height

$$\text{m}\;(10 \times 100) = m\left(\frac{1}{2}v^2 + 10 \times 20\right)$$

or $\frac{1}{2}v^2 = 800$ or $v = \sqrt{1600} = 40$ m/s

Note :
Loss in potential energy = gain in kinetic energy

$$m \times g \times 80 = \frac{1}{2} mv^2$$

$$10 \times 80 = \frac{1}{2} v^2$$

$v^2 = 1600$ or $v = 40$ m/s

46. (c) Given : retardation $\propto$ displacement
i.e., $a = -kx$ [Here, k = constant]

But $a = v\frac{dv}{dx}$

$$\therefore \frac{vdv}{dx} = -kx \Rightarrow \int_{v_1}^{v_2} v\;dv = -\int_0^x kxdx$$

$$\Rightarrow \left[\frac{v^2}{2}\right]_{v_1}^{v_2} = -k\left[\frac{x^2}{2}\right]_0^x$$

$$\Rightarrow \left(v_2^2 - v_1^2\right) = -\frac{kx^2}{2}$$

$$\Rightarrow \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}m\left(\frac{-kx^2}{2}\right)$$

$\therefore$ Loss in kinetic energy, $\therefore \Delta K \propto x^2$

47. **(a)** Work done by such force is always zero since force is acting in a direction perpendicular to velocity.
$\therefore$ From work-energy theorem = ΔK = 0
K remains constant.

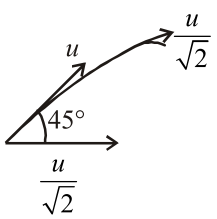
48. **(d)** The elastic potential energy

$$= \frac{1}{2} \times \text{Force} \times \text{extension}$$

$$= \frac{1}{2} \times F \times x$$

$$= \frac{1}{2} \times 200 \times 0.001 = 0.1 \text{ J}$$

49. **(c)** Let u be the speed with which the ball of mass m is projected. Then the kinetic energy (E) at the point of projection is



$$E = \frac{1}{2}mu^2 \qquad ...(i)$$

When the ball is at the highest point of its flight, the speed of the ball is $\frac{u}{\sqrt{2}}$ (Remember that the horizontal component of velocity does not change during a projectile motion).
$\therefore$ The kinetic energy at the highest point

$$= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\frac{mu^2}{2} = \frac{E}{2} \qquad \text{[From (i)]}$$

50. **(18)** Given, Mass of the body, $m = 2$ kg
Power delivered by engine, $P = 1$ J/s
Time, $t = 9$ seconds
Power, $P = Fv$

$$\Rightarrow P = mav \qquad [\because F = ma]$$

$$\Rightarrow m\frac{dv}{dt}v = P \qquad \left(\because a = \frac{dv}{dt}\right)$$

$$\Rightarrow v\ dv = \frac{P}{m}dt$$

Integrating both sides we get

$$\Rightarrow \int_0^v v\,dv = \frac{P}{m}\int_0^t dt$$

$$\Rightarrow \frac{v^2}{2} = \frac{Pt}{m} \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2} \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt$$

$$\therefore \text{ Distance, } x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \sqrt{\frac{2P}{m}} \times \frac{2}{3}t^{3/2}$$

$$\Rightarrow x = \sqrt{\frac{2\times 1}{2}} \times \frac{2}{3} \times 9^{3/2} = \frac{2}{3} \times 27 = 18.$$

51. **(b)** We know that
Power, $P = Fv$

But $F = mav = m\frac{dv}{dt}v$

$$\therefore P = mv\frac{dv}{dt} \Rightarrow P\,dt = mv\,dv$$

Integrating both sides $\int_0^t P\ dt = m\int_0^v v\ dv$

$$\text{P. t} = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$$

$$\text{Distance, } s = \int_0^t v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt = \sqrt{\frac{2P}{m}}\cdot\frac{t^{3/2}}{3/2}$$

$$\Rightarrow s = \sqrt{\frac{8P}{9m}}\cdot t^{3/2} \Rightarrow s \propto t^{3/2}$$

So, graph (b) is correct.

52. **(b)** Total force required to lift maximum load capacity against frictional force = 400 N

$F_{total} = Mg$ + friction
= 2000 × 10 + 4000
= 20,000 + 4000 = 24000 N

Using power, $P = F \times v$
60 × 746 = 24000 × v
$\Rightarrow v = 1.86$ m/s ≈ 1.9 m/s
Hence speed of the elevator at full load is close to 1.9 ms^{-1}

53. **(b)** Centripetel acceleration $a_c = n^2Rt^2$

$$a_c = \frac{v^2}{R} = n^2Rt^2$$

$$v^2 = n^2R^2t^2$$

v = nRt

$$a_c = \frac{dv}{dt} = nR$$

Power = ma_tv = m nR nRt = Mn^2R^2t.

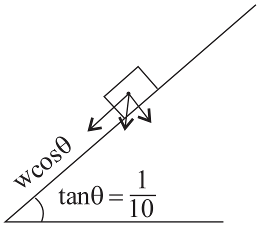
54. **(c)** While moving downhill power

$$P = \left(w \sin\theta + \frac{w}{20}\right)10$$

$$P = \left(\frac{w}{10} + \frac{w}{20}\right)10 = \frac{3w}{2}$$

$$\frac{P}{2} = \frac{3w}{4} = \left(\frac{w}{10} - \frac{w}{20}\right)V$$

$$\frac{3}{4} = \frac{v}{20} \Rightarrow v = 15 \text{ m/s}$$



$\therefore$ Speed of car while moving downhill v = 15 m/s.

55. **(d)** **56.** **(b)**

57. **(b)** Let a be the acceleration of body

Using, $v = u + at$

$$v_1 = 0 + at_1 \Rightarrow a = \frac{v_1}{t_1}$$

Velocity of the body at instant t,

$v = at$

$$\Rightarrow v = \frac{v_1 t}{t_1}$$

Instantaneous powr, P = $\vec{F} \cdot \vec{v} = (m\vec{a}) \cdot \vec{v}$

$$= \left(\frac{mv_1}{t_1}\right)\left(\frac{v_1 t}{t_1}\right) = m\left(\frac{v_1}{t_1}\right)^2 t$$

58. **(b)** Power, $P = Fv = ma.v$

$$\Rightarrow \quad P = \frac{mdv}{dt} v = c = \text{contant}$$

$$\left(\because F = ma = \frac{mdv}{dt}\right)$$

$mv_0 v = cdt$

Integrating both sides, we get

$$m\int_0^v v dv = c\int_0^t dt$$

$$\Rightarrow \frac{1}{2}mv^2 = ct$$

$$\Rightarrow \frac{v^2}{2} = \frac{c.t}{m}$$

$$\Rightarrow v_2 = \frac{2c.t}{m}$$

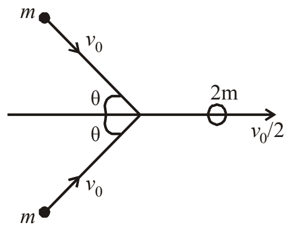
$$\Rightarrow v = \sqrt{\frac{2c}{m}} \times t^{1/2}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2c}{m}} \times t^{1/2} \quad \text{where } v = \frac{dx}{dt}$$

$$\Rightarrow \int_e^x dx = \sqrt{\frac{2c}{m}} \times \int_0^t t^{1/2} dt$$

$$\Rightarrow x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3} \qquad \Rightarrow \quad x \propto t^{3/2}$$

59. **(120)**



Momentum conservation along x direction,

$$2mv_0 \cos\theta = 2m\frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60°$$

Hence angle between the initial velocities of the two bodies

$= \theta + \theta = 60° + 60° = 120°$.

60. **(d)** Before collision,

Velocity of particle A, $u_1 = (\sqrt{3}\hat{i} + \hat{j})$ m/s

Velocity of particle B, $u_2 = 0$

After collision,

Velocity of particle A, $v_1 = (\hat{i} + \sqrt{3}\hat{j})$

Velocity of particle B, $v_2 = 0$

Using principal of conservation of angular momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow 2m_2(\sqrt{3}\hat{i} + \hat{j}) + m_2 \times 0 = 2m_2(\hat{i} + \sqrt{3}\hat{j}) + m_2 \times \vec{v}_2$$

$$\Rightarrow 2\sqrt{3}\hat{i} + 2\hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j} + \vec{v}_2$$

$$\Rightarrow \vec{v}_2 = (\sqrt{3} - 1)\hat{i} - (\sqrt{3} - 1)\hat{j}$$

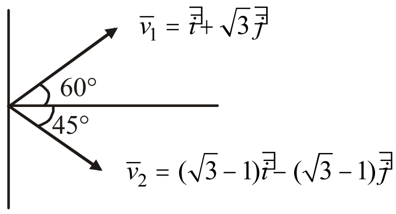
$$\Rightarrow \vec{v}_1 = \hat{i} + \sqrt{3}\hat{j}$$

For angle between $\vec{v}_1$ and $\vec{v}_2$,

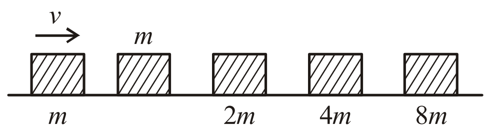
$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{\vec{v}_1\vec{v}_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3}-1)} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \theta = 105°$$

Angle between $\vec{v}_1$ and $\vec{v}_2$ is 105°



61. **(b)** According to the question, all collisions are perfectly inelastic, so after the final collision, all blocks are moving together.



Let the final velocity be v', using momentum conservation

$$mv = 16mv' \Rightarrow v' = \frac{v}{16}$$

Now initial energy $E_i = \frac{1}{2}mv^2$

Final energy : $E_f = \frac{1}{2}\times 16m \times \left(\frac{v}{16}\right)^2 = \frac{1}{2}\frac{mv^2}{16}$

Energy loss : $E_i - E_f = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{16}$

$\Rightarrow \frac{1}{2}mv^2\left[1-\frac{1}{16}\right] \Rightarrow \frac{1}{2}mv^2\left[\frac{15}{16}\right]$

The total energy loss is P% of the original energy.

$\therefore \ \%P = \frac{\text{Energy loss}}{\text{Original energy}}\times 100$

$= \frac{\frac{1}{2}mv^2\left[\frac{15}{16}\right]}{\frac{1}{2}mv^2}\times 100 = 93.75\%$

Hence, value of P is close to 94.

62. **(b)** Given,

Mass of block, m_1 = 1.9 kg

Mass of bullet, m_2 = 0.1 kg

Velocity of bullet, v_2 = 20 m/s

Let v be the velocity of the combined system. It is an inelastic collision.

Using conservation of linear momentum

$m_1 \times 0 + m_2 \times v_2 = (m_1 + m_2)v$

$\Rightarrow 0.1\times 20 = (0.1 + 1.9)\times v$

$\Rightarrow v = 1$ m/s

Using work energy theorem

Work done = Change in Kinetic energy

Let K be the Kinetic energy of combined system.

$(m_1 + m_2)gh$

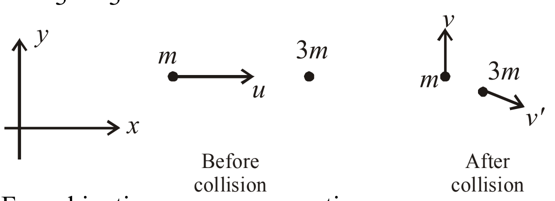
$= K - \frac{1}{2}(m_1 + m_2)V^2$

$\Rightarrow 2\times g \times 1 = K - \frac{1}{2}\times 2\times 1^2 \Rightarrow K = 21$ J

63. **(c)** From conservation of linear momentum

$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v}'$

$\vec{v}' = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$



From kinetic energy conservation,

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$

or, $mu^2 = mv^2 + \frac{mu^2}{3} + \frac{mv^2}{3}$

$\therefore v = \frac{u}{\sqrt{2}}$

64. **(10.00)**

From momentum conservation in perpendicular direction of initial motion.

$mu_1 \sin\theta_1 = 10mv_1 \sin\theta_2$ ...(i)

It is given that energy of m reduced by half. If u_1 be velocity of m after collision, then

$\left(\frac{1}{2}mu^2\right)\frac{1}{2} = \frac{1}{2}mu_1^2$

$\Rightarrow u_1 = \frac{u}{\sqrt{2}}$

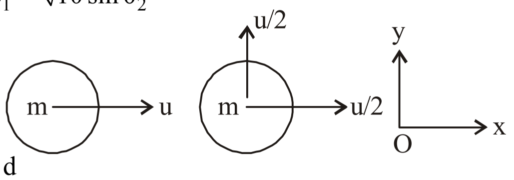
If v_1 be the velocity of mass 10 m after collision, then

$\frac{1}{2}\times 10m \times v_1^2 = \frac{1}{2^m}\frac{u^2}{2} \Rightarrow v_1 = \frac{u}{\sqrt{20}}$

From equation (i), we have

$\sin\theta_1 = \sqrt{10}\sin\theta_2$

65. **(b)**



x-direction

$mu + \frac{mu}{2} = 2mv_x' \Rightarrow V_x' = \frac{3u}{4}$

y-direction $0 + \frac{mu}{2} = 2mv_y' \Rightarrow v_y' = \frac{u}{4}$

$K.E._i = \frac{1}{2}m\,u^2 + \frac{1}{2}m\left[\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]$

$= \frac{1}{2}mu^2 + \frac{mu^2}{4} = \frac{3mu^2}{4}$

$K.E._f = \frac{1}{2}(2m)\left(v_x'\right)^2 + \frac{1}{2}(2m)\left(v_y'\right)^2$

$= \frac{1}{2}2m\left[\left(\frac{3u}{4}\right)^2 + \left(\frac{u}{4}\right)^2\right] = \frac{5}{8}mu^2$

$\therefore$ Loss in $KE = KE_f - KE_i$

$= mu^2\left(\frac{6}{8} - \frac{5}{8}\right) = \frac{mu^2}{8}$

66. **(a)** For elastic collision $KE_i = KE_f$

$\frac{1}{2}m\times 25 + \frac{1}{2}\times m \times 9 = \frac{1}{2}m\times 32 + \frac{1}{2}mv_B^2$

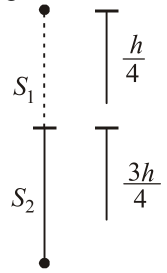
$34 = 32 + V_B^2 \Rightarrow V_B = \sqrt{2}$

$KE_B = \frac{1}{2}mv_B^2 = \frac{1}{2}\times 0.1\times 2 = 0.1J = \frac{1}{10}J$

$\therefore x = 1$

67. (d) Let t be the time taken by the particle dropped from height h to collide with particle thrown upward.

Using,



$v^2 - u^2 = 2gh$

$\Rightarrow v^2 - 0^2 = 2gh$

$\Rightarrow v = \sqrt{2gh}$

Downward distance travelled

$$S_1 = \frac{1}{2}gt^2 = \frac{1}{2}g.\frac{h}{2g} = \frac{h}{4}$$

Distance of collision point from ground

$$s_2 = h - \frac{h}{4} = \frac{3h}{4}$$

Speed of (A) just before collision

$$v_1 = gt = \sqrt{\frac{gh}{2}}$$

And speed of(B) just before collision

$$v_2 = \sqrt{2gh} - \sqrt{\frac{gh}{2}}$$

Using principle of conservation of linear momentum

$mv_1 + mv_2 = 2mv_f$

$$\Rightarrow v_f = \frac{m\left(\sqrt{2gh} - \sqrt{\frac{gh}{2}}\right) - m\sqrt{\frac{gh}{2}} = 0}{2m}$$

After collision, time taken (t_1) for combined mass to reach the ground is

$$\Rightarrow \frac{3h}{4} = \frac{1}{2}gt_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{3h}{2g}}$$

68. (b) $P_i = P_f$

or $0 = 20(0.7 - v) = 50v$

or $v = 0.2$ m/s

69. (a) Apply concervation of linear momentum in X and Y direction for the system then

$$M(10\cos30°) + 2M(5\cos45°) = 2M(v_1\cos30°) + M(v_2\cos45°)$$

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}\,v_1 + \frac{v_2}{\sqrt{2}} \quad ....(1)$$

Also

$$2M(5\sin45°) - M(10\sin30°) = 2Mv_1\sin30° - Mv_2\sin45°$$

$$5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \quad ....(2)$$

Solving equation (1) and (2)

$$\left(\sqrt{3}+1\right)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5 \Rightarrow v_1 = 6.5\text{m/s}$$

$v_2 = 6.3$ m/s

70. (b) $2u + 0 = 2\left(\frac{u}{4}\right) + mv_2$

and $\frac{1}{2} \times 2 \times u^2 + 0 = \frac{1}{2} \times 2 \times \left(\frac{u}{4}\right)^2 + \frac{1}{2}mv_2^2$

On solving, we get

m = 1.5 kg

71. (b) $m(2v) + 2mv = 0 + 2mv'\cos 45°$

or $v' = 2\sqrt{2}v$

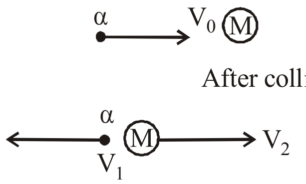
72. (b) $m_1v_1 + m_2v_2 = m_1v_2 + m_2v_1$

or $m_1v_1 + (0.5m_1)v_2 = m_1 0.5v_1 + (0.5m_1)v_4$

On solving, $v_1 = v_4 - v_2$

73. (d) Using conservation of momentum,

$mv_0 = mv_2 - mv_1$



$$\frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv_0^2$$

$\Rightarrow v_1 = 0.6v_0$

The collision is elastic. So,

$$\frac{1}{2}MV_2^2 = 0.64 \times \frac{1}{2}mv_0^2 \ [\because M = \text{mass of nucleus}]$$

$$\Rightarrow V_2 = \sqrt{\frac{m}{M}} \times 0.8V_0$$

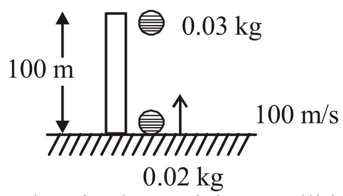
$$mV_0 = \sqrt{mM} \times 0.8V_0 - m \times 0.6V_0$$

$\Rightarrow 1.6m = 0.8\sqrt{mM}$

$\Rightarrow 4m^2 = mM$

$\therefore M = 4m$

74. (c)



Time taken for the particles to collide,

$$t = \frac{d}{V_{rel}} = \frac{100}{100} = 1\,\text{sec}$$

Speed of wood just before collision = gt = 10 m/s and speed of bullet just before collision = v – gt

$= 100 - 10 = 90$ m/s

$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95$ m

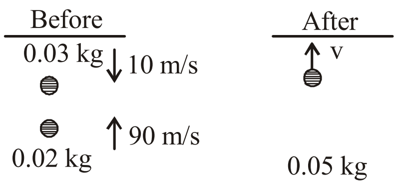
Now, using conservation of linear momentum just before and after the collision

$-(0.03)(10) + (0.02)(90) = (0.05)v$

$\Rightarrow 150 = 5v \qquad \therefore v = 30$ m/s

Max. height reached by body

$$h = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$$



$\therefore$ Height above tower = 40 m

75. (c) Kinetic energy of block A

$$k_1 = \frac{1}{2} m v_0^2$$

$\therefore$ From principle of linear momentum conservation

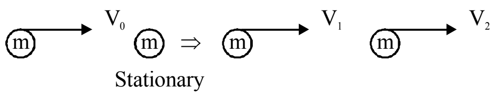
$$mv_0 = (2m + M)v_f \Rightarrow v_f = \frac{mv_0}{2m + M}$$

According to question, of $\frac{5}{6}$th the initial kinetic energy is lost in whole process.

$$\therefore \frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2} m v_0^2}{\frac{1}{2}(2m + M)\left(\frac{mv_0}{2m + M}\right)^2} = 6$$

$$\Rightarrow \frac{2m + M}{m} = 6 \therefore \frac{M}{m} = 4$$

76. (b)



$$\frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 = \frac{3}{2}\left(\frac{1}{2} m v_0^2\right)$$

$$\Rightarrow v_1^2 + v_2^2 = \frac{3}{2} v_0^2 \qquad \text{....(i)}$$

From momentum conservation

$mv_0 = m(v_1 + v_2)$ ....(ii)

Squarring both sides,

$(v_1 + v_2)^2 = v_0^2$

$\Rightarrow v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

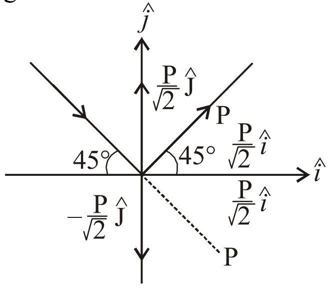
$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$$

Solving we get relative velocity between the two particles

$$v_1 - v_2 = \sqrt{2} v_0$$

77. (a) Change in momentum



$$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$$

$$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H \text{ molecule}$$

$$\Rightarrow I_{wall} = -\frac{2P}{\sqrt{2}}\hat{j}$$

Pressure, P

$$= \frac{F}{A} = \frac{\sqrt{2}P}{A} n \quad (\because n = \text{no. of particles})$$

$$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23}}{2 \times 10^{-4}} = 2.35 \times 10^3 \text{ N/m}^2$$

78. (a) For collision of neutron with deuterium:

Applying conservation of momentum :

$mv + 0 = mv_1 + 2mv_2$ .....(i)

$v_2 - v_1 = v$ .....(ii)

$\because$ Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\frac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

Applying Conservation of momentum

$mv + 0 = mv_1 + 12mv_2$ ....(iii)

$v = v_2 - v_1$ ....(iv)

From eqn (iii) and eqn (iv)

$$v_1 = -\frac{11}{13} v$$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

79. (d) Apply principle of conservation of momentum along x-direction,

$mu = mv_1 \cos 45° + Mv_2 \cos 45°$

$$mu = \frac{1}{\sqrt{2}}(mv_1 + Mv_2) \qquad \text{..... (i)}$$

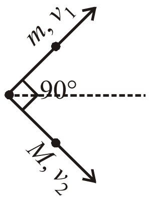
Along y-direction,

$o = mv_1 \sin 45° - Mv_2 \sin 45°$

$o = (mv_1 - Mv_2)\frac{1}{\sqrt{2}}$ ..... (ii)

$m, u_1 = u$ — Proton; $M, u_2 = 0$ — Unknown mass

Before collision



After collision

Coefficient of restution $e = 1 = \frac{v_2 - v_1 \cos 90}{u \cos 45}$

($\because$ Collision is elastic)

$\Rightarrow \frac{v_2}{\frac{u}{\sqrt{2}}} = 1$

$\Rightarrow u = \sqrt{2}v_2$ ..... (iii)

Solving eqs (i), (ii), & (iii), we get mass of unknown particle, $M = m$

80. **(a)** For particle C,

According to law of conservation of linear momentum, verticle component,

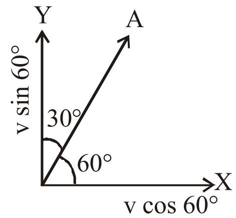
$2\text{ mv}' \sin\theta = \text{mv} \sin 60° + \text{mv} \sin 45°$

$2\text{mv}'\sin\theta = \frac{\text{mv}}{\sqrt{2}} + \frac{\text{mv}\sqrt{3}}{2}$ ...... (i)

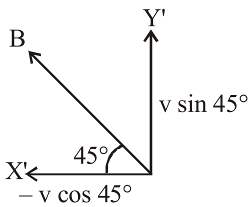
Horizontal component,

$2\text{ mv}' \cos\theta = \text{mv} \sin 60° - \text{mv} \cos 45°$

$2\text{mv}'\cos\theta = \frac{\text{mv}}{2} + \frac{\text{mv}}{\sqrt{2}}$ ...... (ii)



For particle A



For particle B

Dividing eq^n (i) by eq^n (ii),

$$\tan\theta = \frac{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}+\sqrt{3}}{1-\sqrt{2}}$$

81. **(a)** For inelastic collision $v' = \frac{m_1}{(m_1 + m_2)} v$

$= \frac{1}{(1+1)} v = \frac{v}{2}$

$n \rightarrow v(H)$ Before

$(n)(H) \rightarrow \frac{v}{2}$ After

Loss in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$

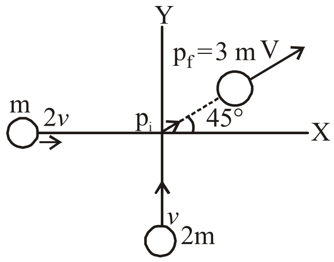
K.E. lost is used to jump from 1st orbit to 2nd orbit

ΔK.E. = 10.2ev

Minimum K.E. of neutron for inelastic collision

$\frac{1}{2}mv^2 = 2 \times 10.2 = 20.4 \text{ eV}$

82. **(a)**



Initial momentum of the system

$p_i = \sqrt{[m(2V)^2 \times 2m(2V)^2]}$

$= \sqrt{2}m \times 2V$

Final momentum of the system = 3mV

By the law of conservation of momentum

$2\sqrt{2}mv = 3mV \Rightarrow \frac{2\sqrt{2}v}{3} = V_{combined}$

Loss in energy

$\Delta E = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 - \frac{1}{2}(m_1 + m_2)V_{combined}^2$

$DE = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2 = 55.55\%$

Percentage loss in energy during the collision $\simeq$ 56%

83. **(b)** Given, $m_1 = 4g, u_1 = 300$ m/s

$m_2 = 0.8$ kg $= 800$ g, $u_2 = 0$ m/s

From law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Let the velocity of combined system = v m/s then,

$4 \times 300 + 800 \times 0 = (800 + 4) \times v$

$v = \frac{1200}{804} = 1.49 \text{ m/s}$

Now, $\mu = 0.3$ (given)

$a = \mu g$

$a = 0.3 \times 10$ (take $g = 10 \text{ m/s}^2$)

$= 3 \text{ m/s}^2$

then, from $v^2 = u^2 + 2as$

$(1.49)^2 = 0 + 2 \times 3 \times s$

$s = \frac{(1.49)^2}{6}$

$s = \frac{2.22}{6}$

$= 0.379$ m

84. **(d)** From the law of conservation of momentum we know that,

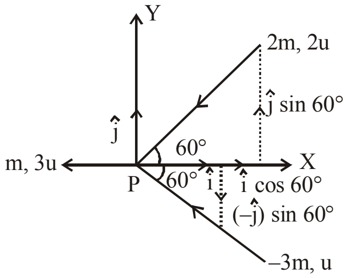
$m_1u_1 + m_2u_2 + ... = m_1v_1 + m_2v_2 + ...$

Given $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$

and $u_1 = 3u$, $u_2 = 2u$ and $u_3 = u$

Let the velocity when they stick $= \vec{v}$

Then, according to question,



$$m \times 3u\left(\hat{i}\right) + 2m \times 2u\left(-\hat{i}\cos 60° - \hat{j}\sin 60°\right)$$

$$+3\,m \times u\left(-\hat{i}\cos 60° + \hat{j}\sin 60°\right) = (m+2m+3m)\,\vec{v}$$

$$\Rightarrow\ 3mu\hat{i} - 4mu\frac{\hat{i}}{2} - 4mu\left(\frac{\sqrt{3}}{2}\hat{j}\right) - 3mu\frac{\hat{i}}{2}$$

$$+3mu\left(\frac{\sqrt{3}}{2}\hat{j}\right) = 6m\,\vec{v}$$

$$\Rightarrow\ mu\hat{i} - \frac{3}{2}mu\hat{i} - \frac{\sqrt{3}}{2}mu\hat{j} = 6m\,\vec{v}$$

$$\Rightarrow\ -\frac{1}{2}mu\hat{i} - \frac{\sqrt{3}}{2}mu\hat{j} = 6m\,\vec{v}$$

$$\Rightarrow\ \vec{v} = \frac{u}{12}\left(-\hat{i} - \sqrt{3}\hat{j}\right)$$

85. **(d)** Maximum energy loss $= \frac{P^2}{2m} - \frac{P^2}{2(m+M)}$

$$\left[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case of perfectly inelastic collision.

By comparing the equation given in statement I with above equation, we get\

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

86. **(c)**

87. **(c)** Fractional decrease in kinetic energy of mass m

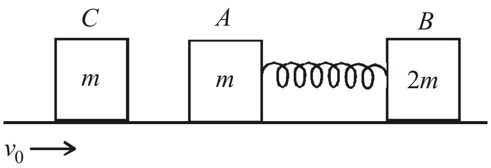
$$= 1 - \left(\frac{m_2 - m_1}{m_2 + m_1}\right)^2 = 1 - \left(\frac{2-1}{2+1}\right)^2$$

$$= 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

Percentage loss in energy

$$= \frac{8}{9} \times 100 \simeq 90\%$$

88. **(d)**



Initial momentum of the system block (C) $= mv_0$. After striking with A, the block C comes to rest and now both block A and B moves with velocity v when compression in spring is x_0.

By the law of conservation of linear momentum

$$mv_0 = (m + 2m)\,v \Rightarrow v = \frac{v_0}{3}$$

By the law of conservation of energy

K.E. of block C = K.E. of system + P.E. of system

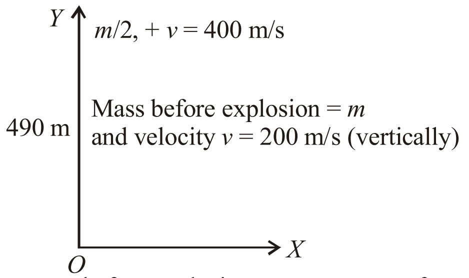
$$\frac{1}{2}mv_0^2 = \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{6}mv_0^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2 - \frac{1}{6}mv_0^2 = \frac{mv_0^2}{3}$$

$$\therefore\ k = \frac{2}{3}m\left(\frac{v_0}{x_0}\right)^2$$

89. **(c)**



Momentum before explosion = Momentum after explosion

$$m \times 200\hat{j} = \frac{m}{2} \times 400\hat{j} + \frac{m}{2}v$$

$$= \frac{m}{2}\left(400\hat{j} + v\right)$$

$$\Rightarrow\ 400\hat{j} - 400\hat{j} = v$$

$$\therefore\ v = 0$$

i.e., the velocity of the other part of the mass, $v = 0$

Let time taken to reach the earth by this part be t

Applying formula, $h = ut + \frac{1}{2}gt^2$

$$490 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow\ t^2 = \frac{980}{9.8} = 100$$

$$\therefore\ t = \sqrt{100} = 10\,\text{sec}$$

90. **(a)** In completely inelastic collision, all initial kinetic energy is not lost but loss in kinetic energy 15 as large as it can be. Linear momentum remain conserved in all types of collision. Statement -2 explains statement -1 correctly because applying the principle of conservation of momentum, we can get the common velocity and hence the kinetic energy of the combined body.

91. **(c)** Initial kinetic energy of the system

$$\text{K.E}_i = \frac{1}{2}mu^2 + \frac{1}{2}M(0)^2$$

$$= \frac{1}{2} \times 0.5 \times 2 \times 2 + 0 = 1\text{J}$$

Momentum before collision
= Momentum after collision

$m_1 u_1 + m_2 u_2 = (m + M) \times v$

$$\therefore\ 0.5 \times 2 + 1 \times 0 = (0.5 + 1) \times v \Rightarrow v = \frac{2}{3}\text{m/s}$$

Final kinetic energy of the system is

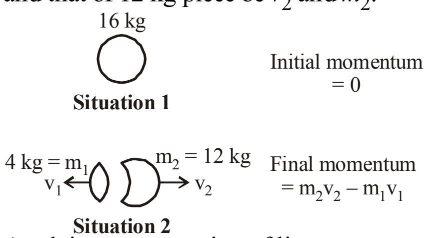
$$\text{K.E}_f = \frac{1}{2}(m + M)v^2$$

$$= \frac{1}{2}(0.5 + 1) \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3}\text{J}$$

$\therefore$ Energy loss during collision

$$= \left(1 - \frac{1}{3}\right)\text{J} = 0.67\text{J}$$

92. **(b)** Let the velocity and mass of 4 kg piece be v_1 and m_1 and that of 12 kg piece be v_2 and m_2.



Applying conservation of linear momentum

$16 \times 0 = 4 \times v_1 + 12 \times 4$

$$\Rightarrow\ v_1 = -\frac{12 \times 4}{4} = -12\ ms^{-1}$$

Kinetic energy of 4 kg mass

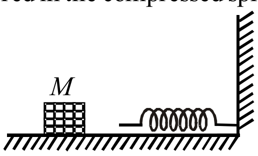
$$\therefore K.E. = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \times 4 \times 144 = 288\ J$$

93. **(b)** When the spring gets compressed by length L.

K.E. lost by mass M = P.E. stored in the compressed spring.

$$\frac{1}{2} M v^2 = \frac{1}{2} k L^2$$

$$\Rightarrow\ v = \sqrt{\frac{k}{M}} \cdot L$$



Momentum of the block, $= M \times v$

$$= M \times \sqrt{\frac{k}{M}} \cdot L = \sqrt{kM} \cdot L$$

94. **(d)** Considering conservation of momentum along x-direction,

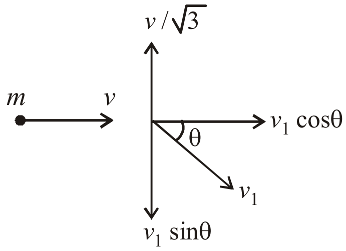
$mv = mv_1 \cos\theta$ ...(1)

where v_1 is the velocity of second mass

In y-direction,

$$0 = \frac{mv}{\sqrt{3}} - mv_1 \sin\theta$$

or $m_1 v_1 \sin\theta = \frac{mv}{\sqrt{3}}$ ...(2)



Squaring and adding eqns. (1) and (2) we get

$$v_1^2 = v^2 + \frac{v^2}{\sqrt{3}} \Rightarrow v_1 = \frac{2}{\sqrt{3}} v$$

95. **(c)** Kinetic energy of a system of particle is zero only when the speed of each particles is zero. This implies momentum of each particle is zero, thus linear momentum of the system of particle has to be zero.

Also if linear momentum of the system is zero it does not mean linear momentum of each particle is zero. This is because linear momentum is a vector quantity. In this case the kinetic energy of the system of particles will not be zero.

$\therefore$ A does not imply B but B implies A.

Given, force, $F = 200$ N extension of wire, $x = 1$mm.