Chapter 8

Mechanical Properties of Solids

Solutions

SECTION - A

Objective Type Questions (One option is correct)

1. The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is

(1) $\frac{l_1+l_2}{2}$ (2) $\sqrt{l_1 l_2}$ (3) $\frac{l_1T_2-l_2T_1}{T_2-T_1}$ (4) $\frac{l_1T_2+l_2T_1}{T_2+T_1}$

Sol. Answer (3)

$T_1 = k(l_1 - l_0)$ …(i)

$T_2 = k(l_2 - l_0)$ …(ii)

Solve (i) and (ii) to get value of l_0.

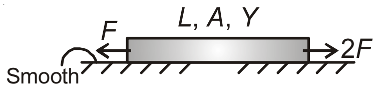
2. A uniform rod of mass m, length L, area of cross section A is rotated about an axis passing through one of its ends and perpendicular to its length with constant angular velocity ω in a horizontal plane. If Y is the Young's modulus of the material of the rod, the increase in its length due to rotation of rod is

(1) $\frac{m\omega^2L^2}{AY}$ (2) $\frac{m\omega^2L^2}{2AY}$ (3) $\frac{m\omega^2L^2}{3AY}$ (4) $\frac{2m\omega^2L^2}{AY}$

Sol. Answer (3)

Increase in length of rod will be due to tension in the rod.

3. A rod of mass M is placed over smooth horizontal surface. Forces $2F$, F are applied at the ends of the rod as shown. If length, cross-sectional area and Young's modulus of rod are L, A and Y respectively, then total elongation of rod will be ($F = Mg$)



(1) $\frac{MgL}{YA}$ (2) $\frac{4MgL}{YA}$ (3) $\frac{MgL}{2YA}$ (4) $\frac{3MgL}{2YA}$

Sol. Answer (4)

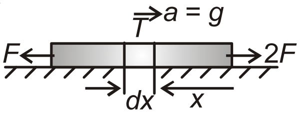
$$dl = \frac{T}{\left(\frac{YA}{dx}\right)}$$

$$dl = \frac{T}{YA}dx$$

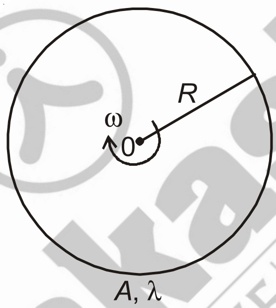
$$dl = \left(\frac{Mg}{YA}\right)\left(2-\frac{x}{L}\right)dx$$

$$\Delta l = \int_0^L \left(\frac{Mg}{YA}\right)\left(2-\frac{x}{L}\right)dx$$

$$\Delta l = \frac{3}{2}\frac{MgL}{YA}$$



4. A circular wire of radius R, placed on a horizontal surface is made to rotate about a fixed vertical axis passing through its center as shown. If breaking stress of the material of wire is S_0, then find maximum value of ω so that wire does not break (Cross-sectional area of wire is A, λ is the linear mass density of wire)



(1) $\frac{2}{R}\sqrt{\frac{S_0A}{\lambda}}$ (2) $\frac{1}{R}\sqrt{\frac{2S_0A}{\lambda}}$ (3) $\frac{1}{R}\sqrt{\frac{S_0A}{2\lambda}}$ (4) $\frac{1}{R}\sqrt{\frac{S_0A}{\lambda}}$

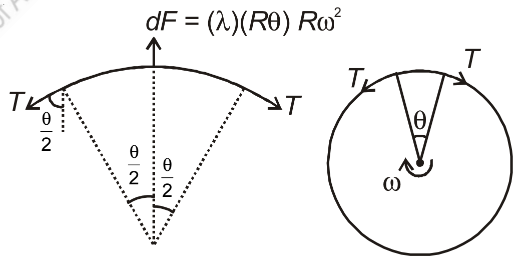
Sol. Answer (4)

$T = \lambda R^2\omega^2$

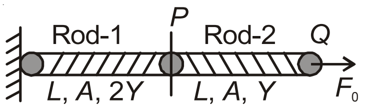
$S_0A = \lambda R^2\omega^2$

$\Rightarrow \quad \omega = \frac{1}{R}\sqrt{\frac{S_0A}{\lambda}}$

$T_{max} = S_0A$



5. Consider an arrangement shown in the diagram which is combination of two rods (1) and (2). Their length, cross-sectional area and Young's modulus are shown in diagram. If force F_0 is applied at end Q of rod (2), then total elongation of combined system is [P is the common end of rod (1) and (2)]



(1) $\frac{3F_0L}{YA}$ (2) $\frac{F_0L}{2YA}$ (3) $\frac{3F_0L}{2YA}$ (4) $\frac{F_0L}{YA}$

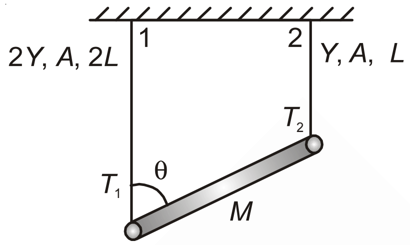
Sol. Answer (3)

$$k_1 = \frac{2YA}{L}, k_2 = \frac{YA}{L}$$

$$k_{eq} = \left(\frac{2YA}{3L}\right)$$

$$\Delta x_t = \left(\frac{F_0}{k_r}\right) = \left(\frac{3F_0L}{2YA}\right)$$

6. If a rod of mass M is attached at end of two wire (1) and (2) as shown in figure. If Δl_1 and Δl_2 are the elongation of wire 1 and 2 respectively due to weight of rod, then (Symbol have their usual meanings)



(1) $\Delta l_2 = 4\Delta l_1$ (2) $\Delta l_1 = 4\Delta l_2$ (3) $\Delta l_1 = \Delta l_2$ (4) $\Delta l_1 = 2\Delta l_2$

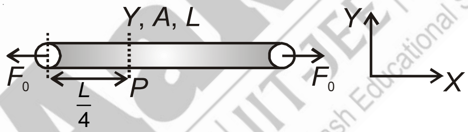
Sol. Answer (3)

Since $T_1 = T_2$

$k_1 = k_2$

$\Rightarrow \Delta l_1 = \Delta l_2$

7. Consider a metal rod shown in figure. Two forces are applied at the ends of the rod. Find displacement of point P



(1) $\frac{F_0L}{4YA}(-\hat{i})$ (2) $\frac{F_0L}{4YA}(\hat{i})$ (3) $\frac{F_0L}{2YA}(-\hat{i})$ (4) $\frac{F_0L}{2YA}(\hat{i})$

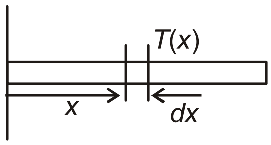
Sol. Answer (1)

Calculate elongation in both the rod to calculate displacement of point P.

8. A rod of length l, cross-sectional area A and mass M is rotated in a horizontal circle with constant angular velocity ω_0 about one of its end. Find elongation produced in the rod due to rotation if Young's modulus of the material of the rod is Y.

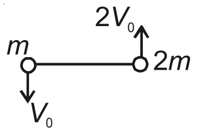
(1) $\frac{Ml^2\omega_0^2}{AY}$ (2) $\frac{2Ml^2\omega_0^2}{AY}$ (3) $\frac{Ml^2\omega_0^2}{3AY}$ (4) $\frac{2Ml^2\omega_0^2}{3AY}$

Sol. Answer (3)



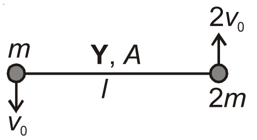
$$\Delta l = \int \frac{T(x)dx}{AY}$$

9. Two point masses of mass m and $2m$ placed on smooth horizontal plane are connected by a light rod of length l. Masses are given velocities in horizontal plane perpendicular to rod as shown. If area of cross section of rod is A and Young's modulus is Y, then elongation in the rod, will be



(1) $\dfrac{mv_0^2}{AY}$ (2) $\dfrac{2mv_0^2}{AY}$ (3) $\dfrac{6mv_0^2}{AY}$ (4) $\dfrac{3mv_0^2}{AY}$

Sol. Answer (3)



$v_{CM} = v_0$

$$T = \frac{m(2v_0)^2}{2\frac{l}{3}} = \frac{6mv_0^2}{l}$$

$$\Delta l = \frac{Tl}{AY} = \frac{6mv_0^2}{AY}$$

10. The length of a wire is l_1 when the tension in it is 3 N and is l_2 when the tensions is 4 N. What will be its length when tension in it is 7 N?

(1) $4l_2 - 3l_1$ (2) $\dfrac{4l_2 - 3l_1}{7}$ (3) $4l_1 - l_2$ (4) $4l_1 - 3l_2$

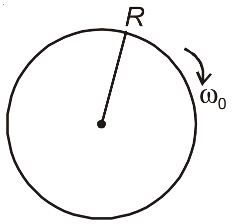
Sol. Answer (1)

$3 = k(l_1 - l_0)$

$4 = k(l_2 - l_0)$

$7 = k(l_1 + l_2 - 2l_0)$

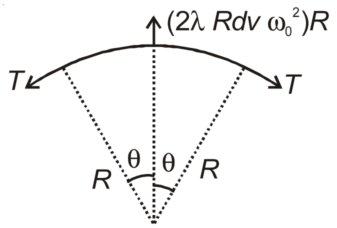
11. A ring of mass m and radius R made of a material having Young's modulus Y is rotating with constant angular speed ω_0 in a horizontal x-y plane as shown in figure. If cross-sectional area of wire of ring is A, then increase in the radius of ring due to rotation is



(1) $\dfrac{2m\omega_0^2R^2}{YA}$ (2) $\dfrac{m\omega_0^2R^2}{YA}$ (3) $\dfrac{m\omega_0^2R^2}{2\pi YA}$ (4) $\dfrac{2\pi m\omega_0^2R^2}{YA}$

Sol. Answer (3)

$$T = \left(\frac{m\omega_0^2 R}{2\pi}\right)$$

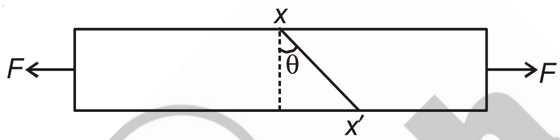


$$\Delta l = \frac{(T)}{YA} 2\pi R$$

$$\Delta l = \frac{m\omega_0^2 R^2}{YA}$$

$$\Delta R = \frac{m\omega_0^2 R^2}{2\pi YA}$$

12. A metallic rod of cross-sectional area A is subjected to equal and opposite tensile forces F at its ends as shown in figure. Consider a plane xx' making an angle θ as indicated in figure. For what value of θ is the shearing stress on plane xx' is maximum?



(1) 0º (2) 90º (3) 45º (4) 60º

Sol. Answer (3)

$$\text{Stress}_{(xx')} = \frac{F\cos(90-\theta)}{\left(\dfrac{A}{\cos\theta}\right)} = \frac{F}{A}\sin\theta\ \cos\theta$$

13. A metallic solid sphere of radius R is subjected to an excess pressure of ΔP. If bulk modulus of material of sphere is B, then relative change in density of material can be given as

(1) Zero (2) $\dfrac{2\Delta P}{B-\Delta P}$ (3) $\dfrac{\Delta P}{B}$ (4) $\dfrac{B}{B-\Delta P}$

Sol. Answer (3)

$$B = \frac{\Delta P}{\left(\dfrac{\Delta V}{V_0}\right)} \Rightarrow \Delta V = \frac{V_0 \Delta P}{B}$$

$$\therefore \quad \frac{\Delta\rho}{\rho_0} = \frac{\rho' - \rho_0}{\rho_0} = \frac{\rho'}{\rho_0} - 1$$

$$\text{Now, } \frac{\rho'}{\rho_0} = \frac{V_0}{V'} = \frac{V_0}{V_0 - \Delta V} = \frac{V_0}{V_0 - V_0\dfrac{\Delta P}{B}}$$

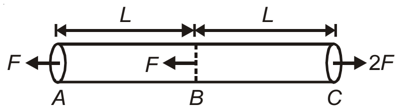
$$\Rightarrow \quad \frac{\Delta\rho}{\rho_0} = \frac{\Delta P}{B - \Delta P}$$

$$\therefore \quad \frac{\Delta P}{B} << .1 \Rightarrow \frac{\Delta\rho}{\rho_0} = \frac{\Delta P}{B}$$

SECTION - B

Objective Type Questions (More than one options are correct)

1. A uniform rod of length $2L$, area of cross-section A and Young's modulus Y is lying at rest under the action of three forces as shown.

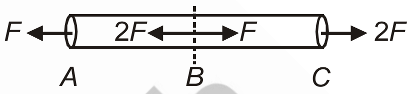


Select the correct alternatives.

(1) The stress at any cross-section is $\dfrac{2F}{A}$
(2) The stress at any cross-section in section AB is $\dfrac{F}{A}$
(3) The total extension is $\dfrac{3FL}{AY}$
(4) The total extension is $\dfrac{6FL}{AY}$

Sol. Answer (2, 3)

Stress in rod $AB = \dfrac{F}{A}$



$$\Delta L = \frac{F_1L_1 + F_2L_2}{YA} = \frac{2F \times L + F \times L}{YA} = \frac{3FL}{YA}$$

2. If a wire is stretched to triple its length, then

(1) Stress is double of Young's modulus
(2) Strain is unity
(3) Young's modulus is equal to the half of the elastic energy per unit volume
(4) Its radius is halved

Sol. Answer (1, 3)

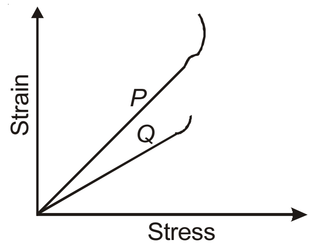
$$\frac{\Delta l}{l} = \text{strain} = \frac{2l}{l} = 2$$

$\Rightarrow$ stress = Y × strain = $2Y$

and elastic potential energy = $\dfrac{1}{2}$ × stress × strain = $2Y$

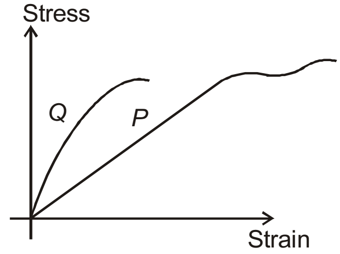
Δr can be calculated only if Poisson's ratio is known.

3. In plotting stress versus strain curves for two materials P and Q,a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are)



(1) P has more tensile strength than Q
(2) P is more ductile than Q
(3) P is more brittle than Q
(4) The Young's modulus of P is more than that of Q

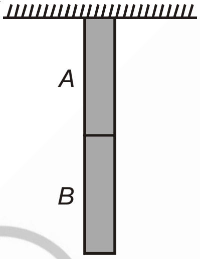
Sol. Answer (1, 2)



(1) Also, P breaks at a greater stress.

(2) P is more ductile

4. Two thin wires A and B of equal length L and equal cross-sectional area A and masses M_1 and M_2 respectively are joined end to end and suspended as shown in the figure. Wires A and B have Young's modulus Y_1 and Y_2 respectively. The total elongation in the combined wire is



(1) $\dfrac{[M_1Y_2 + M_2(Y_1 + 2Y_2)]gL}{2AY_1Y_2}$ (2) $\dfrac{(M_1Y_1 + M_2Y_2)gL}{AY_1Y_2}$

(3) $\dfrac{(2M_1Y_2 + M_2Y_1)gL}{AY_1Y_2}$ (4) $\dfrac{[(M_1 + M_2)Y_1 + M_2Y_2]gL}{2AY_1Y_2}$

Sol. Answer (1)

Elongation in B = $\dfrac{(M_2g)/2}{\left(\dfrac{Y_2A}{L}\right)}$ = ΔL_1 (say)

Elongation in A = $\dfrac{M_2g}{\left(\dfrac{Y_1A}{L}\right)} + \dfrac{(M_1g)/2}{\left(\dfrac{Y_1A}{L}\right)}$ = ΔL_2 (say)

$\therefore$ Total elongation = $\Delta L_1 + \Delta L_2$

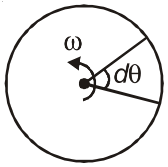
5. A uniform ring of mass M and radius R is made of thin wire of cross-sectional area A and Young's modulus Y. The ring is rotated about its axis with constant angular speed ω. The elongation produced in the wire is

(1) $\dfrac{M\omega^2R^2}{2YA}$ (2) $\dfrac{M\omega^2(2\pi R)}{YA}$ (3) $\dfrac{M\omega^2R^2}{YA}$ (4) $\dfrac{2M\omega^2R^2}{YA}$

Sol. Answer (3)

$$2T\sin\left(\frac{d\theta}{2}\right) = (dm)\omega^2R$$

$$2T\left(\frac{d\theta}{2}\right) = \left(\frac{M}{2\pi R}Rd\theta\right)\omega^2R$$



$$\Rightarrow T = \frac{M\omega^2R}{2\pi} \quad \because \Delta L = \frac{T\times 2\pi R}{YA} = \frac{M\omega^2R^2}{YA}$$

6. For a perfectly rigid body

(1) Young's modulus is infinity and bulk modulus is zero

(2) Young's modulus is zero and bulk modulus is infinity

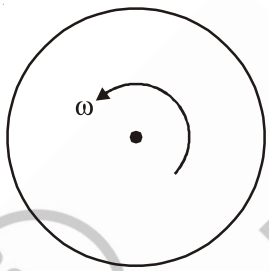
(3) Both Young's modulus and bulk modulus are infinity

(4) Both Young's modulus and bulk modulus are zero

Sol. Answer (3)

For a perfectly rigid body both Young's modulus and Bulk modulus are infinite.

7. A wire of mass *M*, length *L* and cross-section *A* is shaped into a ring. It is placed on smooth horizontal surface and is rotated about its axis with constant angular speed ω as shown. If Young's modulus of material of wire is *Y*, then



(1) Tension developed in the wire is $\frac{M\omega^2 L}{\pi^2}$

(2) Elongation produced in the wire is $\frac{M\omega^2 L^2}{4\pi^2 YA}$

(3) Stress developed in the wire is $\frac{M\omega^2 L}{4\pi^2 A}$

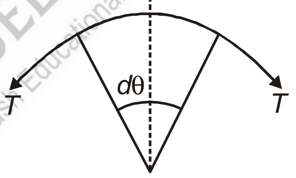
(4) Strain developed in the wire is $\frac{M\omega^2 L}{\pi^2 YA}$

Sol. Answer (2, 3)

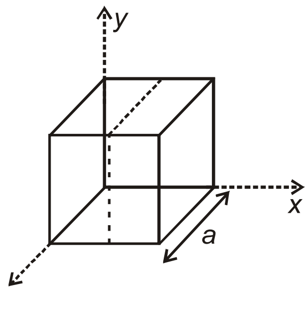
$T d\theta = \frac{M}{L} R d\theta \omega^2 L$

$T = M\omega^2 R$

$2\pi R = L$



8. A solid cubical block of side length *a* and density ρ is made to move with constant velocity $v_0\hat{i}$, over a rough horizontal surface (coefficient of friction between block and ground is μ). The magnitude of longitudinal and shear stress S_1 and S_2 respectively, at the transverse cross-section parallel to *y*-*z* plane through centre of cube, is



(1) $S_1 = \frac{\mu\rho a g}{2}$

(2) $S_2 = 0$

(3) $S_1 = \mu\rho a g$

(4) $S_2 = \frac{\mu\rho a g}{4}$

Sol. Answer (1, 2)

$f = \mu mg$

$f = (\mu)(\rho)a^3g$

Longitudinal stress $= \dfrac{\mu\rho a^3 g}{(2)a^2} = \left(\dfrac{\mu\rho ag}{2}\right)$

9. Two identical rods P and Q each of cross-sectional area A, length L and Young's modulus Y are placed on smooth horizontal surface. They are acted by horizontal forces as shown below.

(1) The extension in rod P is zero

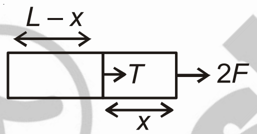
(2) The extension in rod Q is $\dfrac{FL}{AY}$

(3) The extension in rod P is $\dfrac{FL}{AY}$

(4) The elastic potential energy stored in rod P is $\dfrac{2F^2L}{3YA}$

Sol. Answer (2, 3, 4)

$\Delta L_P = \dfrac{2FL}{2AY} = \dfrac{FL}{AY}$



$\Delta L_Q = \dfrac{FL}{AY}$

Elastic potential energy $= \dfrac{1}{2}\dfrac{\text{stress}^2}{Y}A.dx$

$$= \frac{1}{2Y}\cdot\frac{4F^2}{A^2L^2}\int_0^L (L-x)^2 .dx .A$$

$$= \frac{2F^2}{YAL^2}\left[L^2L + \frac{L^3}{3} - 2L.\frac{L^2}{2}\right]$$

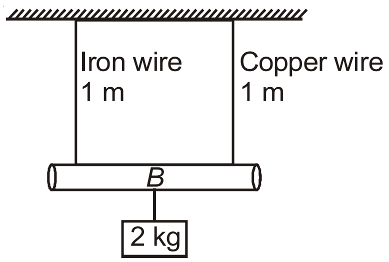
$$= \frac{2F^2}{3YAL^2}\left[L^3 + \frac{L^3}{3} - L^3\right] = \frac{2F^2L}{3yA}$$

SECTION - C

Linked Comprehension Type Questions

Comprehension

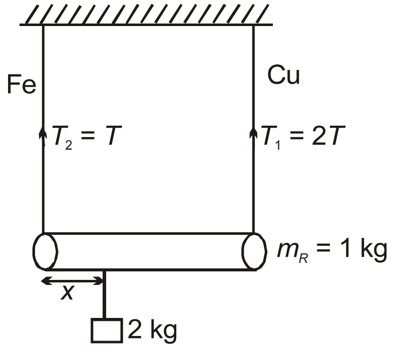
An iron rod B of length 1 m and mass 1 kg is suspended with help of two wires as shown in figure. The area of cross-section of iron wire is 0.3 cm^2 and of copper wire is 0.6 cm^2.



1. At what distance from iron wire a weight can be hung to produce equal stress in wires?

(1) 0.82 m (2) 0.38 m (3) 0.67 m (4) 0.47 m

Sol. Answer (3)



$$\frac{T_1}{0.6} = \frac{T_2}{0.3} \Rightarrow T_1 = 2T_2 \quad \text{...(i)}$$

For rotational equilibrium about centre of mass of rod,

$T_2 \times x = T_1 (1 - x)$ ...(ii)

From (i) and (ii),

$x = 0.67$ m

2. In the above question, if $Y_{iron} : Y_{Cu} = 2 : 1$, then the ratio of extension produced in copper wire to iron wire is

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Sol. Answer (2)

$$\frac{Y_{iron}}{Y_{Cu}} = \frac{\Delta l_{Cu}}{\Delta l_{iron}} \Rightarrow \frac{\Delta l_{Cu}}{\Delta l_{iron}} = \frac{2}{1}$$

3. The ratio of elastic energy per unit volume stored in iron wire to copper wire when $Y_{iron} : Y_{Cu} = 2 : 1$

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Sol. Answer (3)

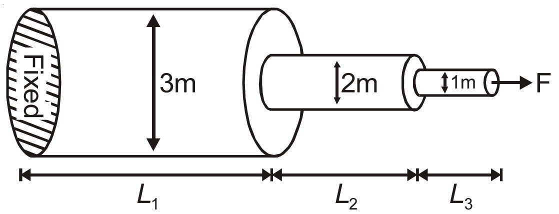
Energy density $= \frac{1}{2} \times$ stress $\times$ strain

$$\Rightarrow \frac{E_{iron}}{E_{Cu}} = \frac{1}{2}$$

SECTION - D

Matrix-Match Type Questions

1. Three wires of lengths L_1, L_2, L_3 and Young's moduli Y_1, Y_2 and Y_3 respectively are pulled by a force F as shown in figure. The extensions produced in wires are ΔL_1, ΔL_2, ΔL_3



Match the Column-I with Column-II

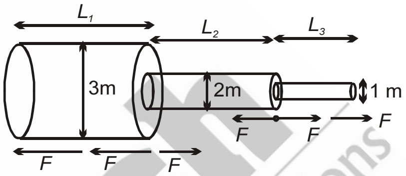
Column-I	Column-II
(A) If $9L_2 = 4L_1$ and $\Delta L_1 = \Delta L_2$, then	(p) $Y_2 = Y_1$
(B) If $L_2 < 4L_3$ and $\Delta L_2 = \Delta L_3$, then	(q) $Y_2 > Y_1$
(C) If $\Delta L_1 = \Delta L_2$ and $L_1 = L_2$, then	(r) $Y_2 < Y_3$
(D) If $L_2 = L_3$ and $\Delta L_2 = \Delta L_3$, then	(s) $4Y_2 = Y_3$

Sol. Answer A(p), B(r), C(q), D(r, s)

$$Y_1 = \frac{F.l}{A.\Delta l} = \frac{F \times L_1}{\frac{9}{4}\pi.\Delta L_1} = \frac{4FL_1}{9\pi\Delta L_1}$$

$$Y_2 = \frac{F.L_2}{\pi(1)^2.\Delta L_2} = \frac{F.L_2}{\pi.\Delta L_2}$$

$$Y_3 = \frac{F.L_3}{\pi\left(\frac{1}{4}\right).\Delta L_3} = \frac{4.F.L_3}{\pi\Delta L_3}$$



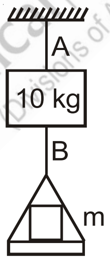
If $9L_2 = 4L_1$ and $\Delta L_1 = \Delta L_2 \Rightarrow Y_1 = Y_2$

If $L_2 < 4L_3$ and $\Delta L_2 = \Delta L_3 \Rightarrow Y_3 > Y_2$

If $\Delta L_1 = \Delta L_2$ and $L_1 = L_2 \Rightarrow Y_2 > Y_1$

If $L_2 = L_3$ and $\Delta L_2 = \Delta L_3 \Rightarrow Y_3 = 4Y_2 \Rightarrow Y_3 > Y_2$

2. In the arrangement shown, the breaking stress of wires *A* and *B* is 10^9 N/m^2. The Young's modulus are 1×10^{14} N/m^2 and 2×10^{14} N/m^2 and area of cross-section of the wires are 0.002 mm^2 and 0.004 mm^2 either of *A* or *B*. Match the columns.



Column-I	Column-II
(A) The possible values of *m* at which wire *A* breaks.	(p) 350 ± 10 J/m^3
(B) The possible values of *m* at which wire *B* breaks.	(q) 30 kg
(C) Possible values of elastic potential energy density of *A* if *m* = 5 kg.	(r) 20 kg
(D) Possible values of elastic potential energy density of *B* if *m* = 7 kg.	(s) 15 kg
	(t) 700 ± 100 J/m^3

Sol. Answer A(q, r, s), B(q, r), C(p, t), D(p)

For A, breaking stress = $\dfrac{(10+m)g}{A}$

If $A = 2\times10^{-7}\,\text{m}^2$ then $10^9 = \dfrac{(10+m)\times10}{2\times10^{-7}}$ $\therefore m = 10$ kg

If $A = 4\times10^{-7}\,\text{m}^2$ then $m = 30$ kg

If $m = 5$ kg then $u_A = \dfrac{1}{2}\times\dfrac{(\text{stress})^2}{y}$ $\therefore u_A = \dfrac{(150)^2}{2yA^2}$

If $y = 10^{14}\,\text{N/m}^2$ and $A = 2\times10^{-7}\,\text{m}^2$ $u_A = \dfrac{150\times150}{2\times10^{14}\times4\times10^{-14}} = 2812\ \text{J/m}^3$

And $A = 4\times10^{-7}\,\text{m}^2$, $u_A = 703\ \text{J/m}^2$

If $y = 2\times10^{14}$, $A = 2\times10^{-7}\,\text{m}^2$, $u_A = 1406\,\text{J/m}^3$

And $A = 4\times10^{-7}\,\text{m}^2$, $u_A = 352\,\text{J/m}^3$

For B, breaking stress = $\dfrac{mg}{A}$

If $A = 2\times10^{-7}\,\text{m}^2$ then $10^9 = \dfrac{m\times10}{2\times10^{-7}}$ $\therefore m = 20$ kg

If $A = 4\times10^{-7}\,\text{m}^2$ then $m = 40$ kg

If $m = 7$ kg then $u_B = \dfrac{(7g)^2}{2yA^2}$

If $y = 10^{14}\,\text{N/m}^2$, $A = 2\times10^{-7}\,\text{m}^2$, $u_B = \dfrac{70\times70}{2\times10^{14}\times4\times10^{-14}} = 612.5\ \dfrac{\text{J}}{\text{m}^3}$

$A = 4\times10^{-7}\,\text{m}^2$, $u_B = 153\ \text{J/m}^3$

If $y = 2\times10^{14}\,\text{N/m}^2$, $A = 2\times10^{-7}\,\text{m}^2$, $u_B = 306\ \text{J/m}^3$

$A = 4\times10^{-7}\,\text{m}^2$, $u_B = 76.5\ \text{J/m}^3$

SECTION - E

Assertion-Reason Type Questions

1. STATEMENT-1 : When a rod lying freely is heated, no thermal stress is developed in it.

and

STATEMENT-2 : On heating, the length of the rod increases.

Sol. Answer (2)

A rod lying freely will attain its natural length on heating

$\Rightarrow$ No thermal stress will be developed

Statement-1 is correct, Statement-2 is also correct, as rod expands on heating.

2. STATEMENT-1 : The value of modulus of elasticity depends on the magnitude of the stress and strain.

and

STATEMENT-2 : A given material have different moduli of elasticity depending on the type of stress applied.

Sol. Answer (4)

Modulus of elasticity = $\dfrac{\text{Stress}}{\text{Strain}}$

is property of a material but, depending on nature of deforming force (longitudinal, shear or volumetric) three different moduli are defined. So statement-2 is correct.

3. STATEMENT-1 : Steel is more elastic than rubber.

and

STATEMENT-2 : Under given deforming force, steel is deformed less than rubber.

Sol. Answer (1)

Both Statement-1 and Statement-2 are true and reason is the correct explanation of assertion.

Elasticity is a measure of tendency of the body to regain its original configuration. As steel is deformed less than rubber therefore steel is more elastic than rubber.

4. STATEMENT-1 : Bulk modulus of elasticity (K) represents incompressibility of the material.

and

STATEMENT-2 : Bulk modulus of elasticity is proportional to change in pressure.

Sol. Answer (1)

Refer theory

5. STATEMENT-1 : Strain is a unitless quantity.

and

STATEMENT-2 : Strain is equivalent to force

Sol. Answer (3)

Refer theory

SECTION - F

Integer Answer Type Questions

1. Two identical rods are pulled as shown in figure (1) and (2). The surfaces are smooth. If Δl_1 and Δl_2 are the elongations in two cases respectively, then find $\dfrac{\Delta l_2}{\Delta l_1}$.

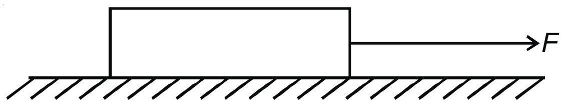


Fig (1)

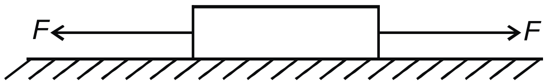


Fig (2)

Sol. Answer (2)

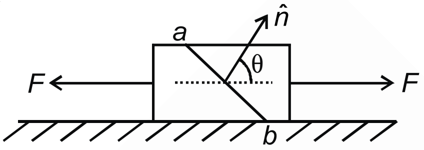
In 1st case, $T_{(x)} = \frac{m}{l} \cdot x \times \frac{F}{m} = \frac{F}{l}x$

$$e_1 = \int de = \int \frac{T_{(2)} \cdot dx}{AY} = \frac{Fl}{2AY}$$

In 2nd case, $e_2 = \frac{Fl}{AY}$

$$\frac{e_2}{e_1} = 2$$

2. A rod of area of cross-section A is being acted upon by two forces $F - F$ as shown in figure. Consider a section ab of rod whose normal makes angle θ with horizontal. The shear stress upon this section is maximum when $\theta = \frac{180°}{n}$. Find n.



Sol. Answer (4)

Shear force = $F \sin\theta$

$$\text{Shear stress} = \frac{\text{Shear Force}}{\text{Surface Area}} = \frac{F\sin\theta}{A/\cos\theta} = \frac{F}{2}\sin 2\theta$$

Shear stress will be max for $\theta = 45°$

3. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

Sol. Answer (4)

$$Y = \frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}} \qquad \Delta\ell = 25\times10^{-50} \text{ m}$$

$$Y = \frac{F}{A}\cdot\frac{\ell}{\Delta\ell}$$

$$\frac{\Delta Y}{Y}\times100 = \frac{10^{-5}}{25\times10^{-5}}\times100 = 4\%$$

4. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} Nm^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is ________. (Take $g = 10$ ms^{-2} and $\pi = 3.2$).

Sol. Answer (3)

$$\Delta L = \frac{W}{\left(\frac{YA}{L}\right)} = \frac{1.2\times10\times4\times1}{2\times10^{11}\times\pi\times(0.5)^2\times(10^{-6})} = 0.3\text{ mm}$$

L.C. of vernier $= \left(1-\frac{9}{10}\right)$mm = 0.1 mm

$\therefore$ Vernier reading = 3

5. The Poisson's ratio of a material is 0.25. If a force is applied to a wire of this material, and there is a decrease in the cross-section area by 4%. The percentage increase in the length is

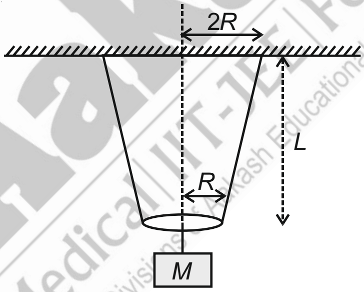
Sol. Answer (8)

$$\frac{1}{4} = \frac{\Delta r/r}{\Delta L/L} \qquad \frac{2\Delta r}{r} = 4\%$$

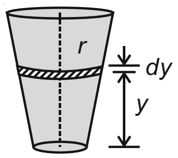
$$\frac{\Delta L}{L} = 4\frac{\Delta r}{r}$$

$$\frac{\Delta L}{L}\% = 4(2\%) = 8\%$$

6. A light conical wire is made from a material of Young's modulus Y and has a normal, unextended length L as shown. The radii, at the upper and lower ends of this conical wire, are $2R$ and R respectively. The upper end of wire is fixed to a rigid support and a mass M is suspended from its lower end. If length of this wire at equilibrium, is $L\left[1+\frac{2Mg}{N\pi R^2 Y}\right]$, then value of N?



Sol. Answer (4)



$$\int dl = \int \frac{Mg\cdot dy}{\pi r^2\cdot Y}$$

$$\tan\theta = \frac{2R-R}{L}$$

$$\frac{r-R}{y} = \frac{R}{L} \qquad r = y\frac{R}{L}+R$$

$$\int dl = \int \frac{M\cdot g\cdot dy}{\pi\cdot r^2\cdot Y} = \int_0^L \frac{Mg\cdot dy}{\pi\left(R+\frac{y}{L}R\right)^2 .Y}$$

$$\Delta l = \frac{Mg}{\pi Y}\left[\frac{\left(R+\frac{y}{L}R\right)^{-1}}{-1}\right]_0^L \frac{L}{R}$$

$$= \frac{MgL}{\pi RY}\left[-\frac{1}{R+\frac{y}{L}R}\right]_0^L$$

$$= \frac{MgL}{\pi RY}\left[\frac{1}{R}-\frac{1}{2R}\right]$$

$$\Delta L = \frac{MgL}{\pi RY}\left[\frac{1}{2R}\right] = \frac{MgL}{2\pi R^2 Y}$$

$$L' = L\left[1+\frac{Mg}{2\pi R^2 Y}\right]$$

7. A uniform iron rod of length L and radius r is suspended from the ceiling by one of its end. What will be the elastic potential energy stored in the rod due to its own weight, given $\rho^2 g^2 L^3 A = 12Y$ where ρ and Y are the density and Young's modulus of the iron rod respectively? [$A = \pi r^2$]

Sol. Answer (2)

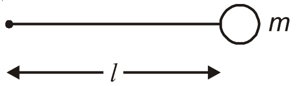
$$E = \frac{M^2 g^2 L}{6AY}$$

$$= \frac{(\rho \cdot AL)^2 \cdot g^2 \cdot L}{6AY}$$

$$= \frac{\rho^2 A^2 L^2 \cdot g^2 L}{6AY}$$

$$= \frac{1}{6}\frac{\rho^2 g^2 L^3 A}{Y}$$

8. A particle of mass m is attached to a metallic wire of length l, area of cross-section A and Young's modulus of elasticity Y. The other end of the wire is fixed. The particle is released from rest under gravity when the wire is in horizontal position and the particle moves along a vertical circle. Extension in the wire at an instant the string becomes vertical is $\frac{nmgl}{AY}$. Find the value of n.



Sol. Answer (3)

When string is vertical

$$T - mg = \frac{mv^2}{l} \quad \Rightarrow \quad T = 3mg$$

9. A wire of length 4 m and cross-sectional area 2×10^{-6} m^2 is stretched by 2 mm. If Young's modulus of material is 2×10^{11} N/m^2, then elastic energy stored per unit volume of wire is $\frac{K}{2} \times 10^4$ J/m^3, find K.

Sol. Answer (5)

$$U = \frac{1}{2} \times Y \times (\text{strain})^2$$

$$= \frac{1}{2} \times 2 \times 10^{11} \times \left(\frac{2 \times 10^{-3}}{4}\right)^2$$

$$= 2.5 \times 10^4 \text{ J/m}^3$$

10. A uniform rod of mass M, length L and cross-sectional area A is pulled by two forces on a smooth surface as shown. Young's modulus of the material of rod is Y. If elongation in the rod is $\frac{KFL}{2\gamma A}$, then find K.

Sol. Answer (4)

$$F_{av} = \frac{F + 3F}{2} = 2F$$

$$\therefore \quad \Delta L = \frac{F_{av}}{\left(\frac{YA}{L}\right)} = \frac{2FL}{YA}$$

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