Chapter 20

Trigonometric Functions-II

Solutions

SECTION - A

Objective Type Questions (One option is correct)

1. The number of solutions of the equation $\cos(\pi\sqrt{x-4})\cos(\pi\sqrt{x})=1$ is

   (1) Zero (2) 1 (3) 2 (4) Infinite

Sol. Answer (2)

$\cos\pi\sqrt{x-4}\cos\pi\sqrt{x}=1$

Only possible when

$\cos\pi\sqrt{x-4}=1$ and $\cos\pi\sqrt{x}=1$

$\pi\sqrt{x-4}=2n\pi\pm\frac{\pi}{2}$ $\quad$ $\pi\sqrt{x}=2m\pi\pm\frac{\pi}{2}$

$\sqrt{x-4}=\frac{(2n+1)}{2}$ $\quad$ $\sqrt{x}=\frac{(2m+1)}{2}$

$\sqrt{x-4}=\frac{1}{2},\frac{5}{2},\frac{7}{2},\frac{9}{2}$ $\quad$ $\sqrt{x}=\frac{1}{2},\frac{5}{2},\frac{7}{2}$........

$\therefore$ No solution exists. Hence only $x = 4$ is a valid solution.

2. The number of solutions of the equation $\sin x.\cos x(\cos x - \sin x)^2.(\sin x + \cos x) = \lambda$, where $\lambda > \frac{1}{2\sqrt{2}}$ in the interval $[0, 4\pi]$, is

   (1) 0 (2) 2 (3) 4 (4) 8

Sol. Answer (1)

$\sin 2x.(\cos x - \sin x).\cos 2x = 2\lambda$

$\sin 4x.\cos\left(x+\frac{\pi}{4}\right)=\frac{4}{\sqrt{2}}\lambda = 2\sqrt{2}\lambda$

L.H.S. can never exceed one while RHS = $2\sqrt{2}\lambda>1$

Since $\lambda > \frac{1}{2\sqrt{2}}$

$\therefore$ No solution

3. If $c^2 = a^2 + b^2$, $2s = a + b + c$, then $4s(s - a)(s - b)(s - c)$

(1) s^4 (2) b^2c^2 (3) c^2a^2 (4) a^2b^2

Sol. Answer (4)

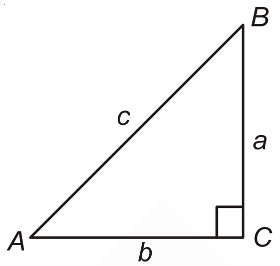
Given that $c^2 = a^2 + b^2$

$\Rightarrow \angle c = 90°$

Now,

$4s(s - a)(s - b)(s - c) = 4(\text{Area})^2$

$= 4 \times \frac{1}{4}a^2b^2 = a^2b^2$



4. A triangle has its sides in the ratio 4 : 5 : 6, then the ratio of circumradius to the inradius of the triangle is

(1) $\frac{15}{7}$ (2) $\frac{13}{6}$ (3) $\frac{16}{7}$ (4) $\frac{17}{6}$

Sol. Answer (3)

Let $a = 4\lambda$, $b = 5\lambda$, $c = 6\lambda$

We know that

$\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

$$\frac{b^2 + c^2 - a^2}{2bc} + \frac{a^2 + c^2 - b^2}{2ac} + \frac{a^2 + b^2 - c^2}{2ab} = 1 + \frac{r}{R}$$

$$1 + \frac{r}{R} = \frac{23}{16} \Rightarrow \frac{r}{R} = \frac{7}{16} \Rightarrow \frac{R}{r} = \frac{16}{7}$$

5. In ΔABC, if $a^2 + b^2 + c^2 = ac + ab\sqrt{3}$, then

(1) $\frac{r}{r_2} = \frac{r_3}{r_1}$ (2) $\frac{r}{r_1} = \frac{r_2}{r_3}$

(3) $r_1 + r_2 + r_3 = 2R + r$ (4) $rr_1r_2r_3 = 4^3$

Sol. Answer (1)

Given that,

$a^2 + b^2 + c^2 = ac + ab\sqrt{3}$

$\Rightarrow a^2 + b^2 + c^2 - ac - ab\sqrt{3} = 0$

$$\Rightarrow \left(\frac{a\sqrt{3}}{2} - b\right)^2 + \left(\frac{a}{2} - c\right)^2 = 0$$

Above equation is possible when

$\dfrac{a\sqrt{3}}{2} - b = 0 \;\&\; \dfrac{a}{2} - c = 0$

$\sqrt{3}a = 2b = 2\sqrt{3}c = k$ (let)

$\Rightarrow \quad a = \dfrac{k}{\sqrt{3}}, b = \dfrac{k}{2}, c = \dfrac{k}{2\sqrt{3}}$

$\Rightarrow$ $b^2 + c^2 = a^2 \Rightarrow$ triangle is right angled triangle now taking (1) option

$\Rightarrow \quad \dfrac{r}{r_2} = \dfrac{r_3}{r_1} \Rightarrow rr_1 = r_2r_3$

$\Rightarrow \quad \dfrac{\Delta^2}{s(s-a)} = \dfrac{\Delta^2}{(s-b)(s-c)}$

$\Rightarrow$ $s^2 - sa = s^2 - s(b + c) + bc$

$\Rightarrow$ $(b + c - a)(b + c + a) = 2bc$

$\Rightarrow$ $(b + c - a)s - bc$

$\Rightarrow$ $(b + c)^2 - a^2 = 2bc \Rightarrow b^2 + c^2 - a^2 = 0$

$\Rightarrow$ $\boxed{b^2 + c^2 = a^2}$ proved

So option (1) is correct

6. In a triangle ABC, a, b and c are the sides of the triangle satisfying the relation $r_1 + r_2 = r_3 - r$ then the perimeter of the triangle

(1) $\dfrac{2ab}{a+b-c}$ (2) $\dfrac{ab}{a+c-b}$ (3) $\dfrac{ab}{a-b-c}$ (4) $\dfrac{bc}{b+c-a}$

Sol. Answer (1)

Given that $r_1 + r_2 = r_3 - r$

$\Rightarrow \quad \dfrac{\Delta}{s-a} + \dfrac{\Delta}{s-b} = \dfrac{\Delta}{s-c} - \dfrac{\Delta}{s} \quad \Rightarrow \dfrac{2s-a-b}{(s-a)(s-b)} = \dfrac{s-s+c}{s(s-c)}$

$\Rightarrow$ $(s - a)(s - b) = s(s - c) \quad \Rightarrow \boxed{2s = \dfrac{2ab}{a+b-c}}$

Hence, option (1) is correct

7. In ΔABC, which is not right angled, if $p = \sin A \sin B \sin C$ and $q = \cos A \cos B \cos C$. Then the equation having roots $\tan A$, $\tan B$ and $\tan C$ is

(1) $qx^3 - px^2 + (1 + q)x - p = 0$ (2) $qx^3 + 2px^2 + qx - p = 0$

(3) $qx^3 - px^2 + (2 + q)x + pq = 0$ (4) $qx^3 - px^2 + qx + p = 0$

Sol. Answer (1)

Given that

$p = \sin A.\sin B.\sin C$

$q = \cos A.\cos B.\cos C$

Now, $\tan A.\tan B.\tan C = \dfrac{p}{q}$

And also tan$A$ + tan$B$ + tan$C$ = tan$A$.tan$B$.tan$C$ = $\frac{p}{q}$

Now, tan$A$.tan$B$ + tan$B$.tan$C$. + tan$C$.tan$A$

$= \frac{\sin A.\sin B}{\cos A.\cos B} + \frac{\sin B.\sin C}{\cos B.\cos C} + \frac{\sin C.\sin A}{\cos C.\cos A}$

= sin$A$.sin$B$.cos$C$ + sin$B$sin$C$.cos$A$ + sin$A$sin$C$.cos$B$/cos$A$cos$B$cos$C$

= sin$B$(sin$A$cos$C$ + cos$A$sin$C$) + sin$A$.sin$C$.cos$B$/cos$A$cos$B$cos$C$

= sin$B$ sin(A + C) + sin$A$. sin$C$. cos$B$ /cos$A$cos$B$cos$C$

$= \frac{\sin^2 B + \sin A \sin C.\cos B}{\cos A.\cos B.\cos C}$

$= \frac{1-\cos^2 B + \sin A.\sin C.\cos B}{\cos A.\cos B.\cos C}$

$= \frac{1-\cos B(\cos B - \sin A \sin C)}{\cos A.\cos B.\cos C}$

$= \frac{1-\cos B\{-\cos(A+C) - \sin A \sin C\}}{\cos A.\cos B.\cos C}$

$= \frac{1+\cos B.\cos A \cos C}{\cos A.\cos B.\cos C} = \frac{1+q}{q}$

So required equation is

$$x^3 - \frac{p}{q}x^2 + \left(\frac{1+q}{q}\right)x - \frac{p}{q} = 0$$

$$\Rightarrow \boxed{qx^3 - px^2 + (1+q)x - p = 0}$$

Hence option (1) is correct

8. In ΔABC, if $\frac{R}{r} \le 2$, and a = 2, then Δ is equal to (Δ = $ar(\Delta ABC)$)

(1) 3 (2) 9 (3) $\sqrt{3}$ (4) $\frac{\sqrt{3}}{4}$

Sol. Answer (3)

Given that

$\frac{R}{r} \le 2 \Rightarrow \frac{r}{R} \ge \frac{1}{2}$

$\Rightarrow 1 + \frac{r}{R} \ge \frac{3}{2} \Rightarrow \cos A + \cos B + \cos C \ge \frac{3}{2}$ …(i)

But we know that

cos$A$ + cos$B$ + cos$C$ $\le \frac{3}{2}$ …(ii)

So from (i) & (ii) we get

$\Rightarrow$ $\cos A + \cos B + \cos C = \dfrac{3}{2}$

$\Rightarrow$ $A = B = C$

Hence triangle is equilateral

So area of the triangle is = $\dfrac{\sqrt{3}}{4}(\text{sinde})^2 = \dfrac{\sqrt{3}}{4} \times 4 = \sqrt{3}$

$\Rightarrow$ Option (3) is correct

9. In ΔABC, if r_1, r_2, r_3 are exradii opposites to angles A, B and C respectively. Then $\left(\dfrac{1}{r_1}+\dfrac{1}{r_2}\right)\left(\dfrac{1}{r_2}+\dfrac{1}{r_3}\right)\left(\dfrac{1}{r_3}+\dfrac{1}{r_1}\right)$ is equal to

(1) $\dfrac{64R^3}{abc}$ (2) $\dfrac{16R^3}{a^2b^2c^2}$ (3) $\dfrac{64R^3}{a^2b^2c^2}$ (4) $\dfrac{R^3}{abc}$

Sol. Answer (3)

$$\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{1}{r_2}+\frac{1}{r_3}\right)\left(\frac{1}{r_3}+\frac{1}{r_1}\right)$$

$$= \frac{abc}{\Delta^3}$$

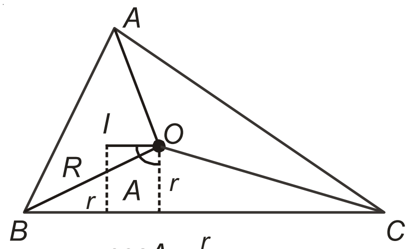
$$= \frac{64R^3}{a^2b^2c^2} \left(\text{using} \frac{abc}{4\Delta} = R\right)$$

So option (3) is correct.

10. In triangle ABC if the line joining incentre to the circumcentre is parallel to the base BC, then the value of $(\cos B + \cos C - 1)$ is equal to

(1) 1 (2) 2 (3) 3 (4) 0

Sol. Answer (4)



$\cos A = \dfrac{r}{R}$

In ΔABC $\cos A + \cos B + \cos C = 1 + \dfrac{r}{R}$

$\Rightarrow$ $\dfrac{r}{R} + \cos B + \cos C = 1 + \dfrac{r}{R}$

$\Rightarrow \cos B + \cos C - 1 = 0$

So, option (4) is correct

11. Consider a regular polygon of 12 sides each of length one unit, then which of the following is not true?

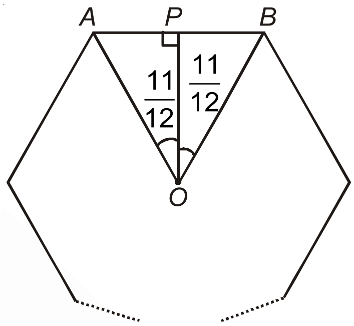
(1) The area of polygon is $\left(6+3\sqrt{3}\right)$

(2) The circum-radius of polygon is $\frac{1}{2}\left(\sqrt{6}+\sqrt{2}\right)$

(3) The radius of incircle is $\frac{2+\sqrt{3}}{2}$

(4) Each internal angle is of 135°

Sol. Answer (4)



$\angle AOP = \frac{\pi}{12}$

Area of polygon $= 12 \times ArAOB$

$= 12 \times \frac{1}{2} \times AB \times OP$

$= 6 \times 1 \times OP$ $\qquad \tan \pi/12 = \frac{1}{2OP}$

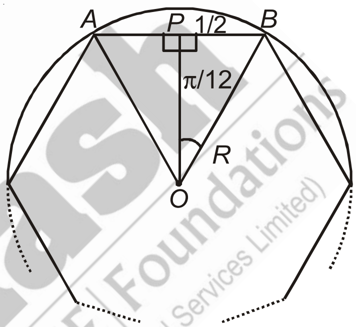
$= \frac{6}{2\tan \pi/12}$

$= \frac{3}{\tan \pi/12}$

$= \frac{3}{\sqrt{3}-1}(\sqrt{3}+1)$

$= 6+3\sqrt{3}$

So option (1) is true.

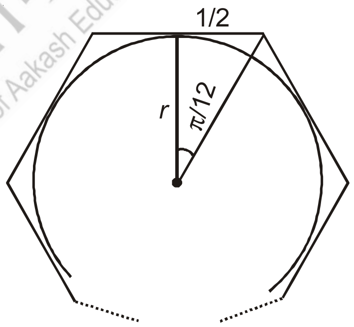


(ii) $\sin\frac{\pi}{12} = \frac{1}{2R}$

$\therefore\ R = \frac{1}{2\sin\frac{\pi}{12}}$

$\therefore\ R = \frac{\sqrt{6}+\sqrt{2}}{2}$

So option (2) is true.



$\tan\frac{\pi}{12} = \frac{1}{2r}$

$\therefore\ r = \frac{2+\sqrt{3}}{2}$

So option (3) is true.

(iv) Each internal angle $= \frac{(2n-4)90}{n}$

$= \frac{20 \times 90}{12}$

$= 150°$

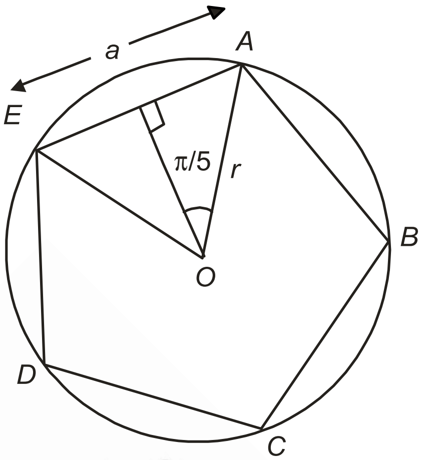
Hence, option (4) is answer not true.

12. A regular pentagon is inscribed in a circle. If A_1 and A_2 represents the area of circle and that of regular pentagon respectively, then $A_1 : A_2$ is

(1) $\frac{\pi}{5}\cos\frac{2\pi}{5}$ (2) $\frac{2\pi}{5}\sin\frac{2\pi}{5}$ (3) $\frac{2\pi}{5}\text{cosec}\frac{2\pi}{5}$ (4) $\frac{2\pi}{5}\cos\frac{2\pi}{5}$

Sol. Answer (3)

$$\frac{A_1}{A_2} = \frac{\pi r^2}{5\times\frac{1}{2}\times a\times h}$$



$$= \frac{\pi\times r\times r}{\frac{5}{2}\times a\times h}$$

$$= \frac{2\pi}{5}\times\frac{1}{2\sin\frac{\pi}{5}}\times\frac{1}{\cos\frac{\pi}{5}}$$

$$= \frac{2\pi}{5}\times\frac{1}{\sin 2\frac{\pi}{5}}$$

∴ Option (3) is true.

13. In ΔABC, if O is the circumcentre of ΔABC and R_1, R_2, R_3 are the radii of circumcircles of triangles OBC, OCA and OAB respectively. Then the value of $\frac{a}{R_1}+\frac{b}{R_2}+\frac{c}{R_3}$ is equal to

(1) $\frac{abc}{R}$ (2) $\frac{abc}{2R}$ (3) $\frac{abc}{4R^3}$ (4) $\frac{abc}{R^3}$

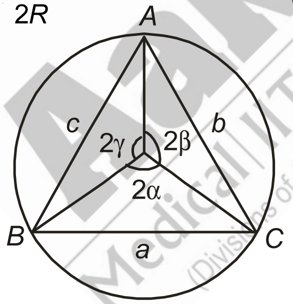
Sol. Answer (4)

Let $\angle AOB = 2\gamma$

$\angle BOC = 2\alpha$

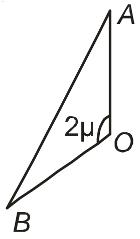
$\angle AOC = 2\beta$

In ΔAOB



$$\frac{c}{\sin 2\gamma} = 2R_3$$

$$\frac{C}{R_3} = 2\sin 2\gamma$$



Similarly $\frac{a}{R_1} = 2\sin 2\alpha$

$$\frac{b}{R_2} = 2\sin 2\beta$$

$$\therefore \quad \frac{a}{R_1}+\frac{b}{R_2}+\frac{c}{R_3}$$

$= 2\sin 2\alpha + 2\sin 2\beta + 2\sin 2\gamma$

$= 2[\sin 2\alpha + \sin 2\beta + \sin 2\gamma]$

$= 2[4\sin\alpha \sin\beta \sin\gamma]$

$= 8\sin\alpha \sin\beta \sin\gamma$

$= 8\dfrac{a}{2R}\cdot\dfrac{b}{2R}\cdot\dfrac{c}{2R}$

$= \dfrac{abc}{R^3}$

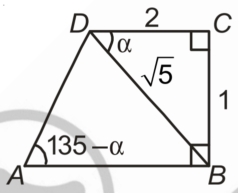
Hence, option (4) is correct.

14. *ABCD* is a trapezium such that *AB* and *DC* are parallel and *BC* is perpendicular to them. If *BC* = 1 cm, *CD* = 2 cm and $\angle ADB$ = 45°, then the length of *AB* is (in cm)

(1) $\dfrac{12}{5}$ (2) $\dfrac{8}{5}$ (3) $\dfrac{5}{3}$ (4) $\dfrac{16}{3}$

Sol. Answer (3)

Using cosine formula in ΔDCB



$$\cos\alpha = \frac{5+4-1}{4\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Apply law of sin in ΔADB

$$\frac{\sin 45°}{AB} = \frac{\sin(135-\alpha)}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{2}AB} = \frac{\sin 135 \cos\alpha - \cos 135 \sin\alpha}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{2}AB} = \left(\frac{2}{\sqrt{10}} + \frac{1}{\sqrt{10}}\right)\frac{1}{\sqrt{5}}$$

$$\therefore AB = \frac{5}{3}$$

Here, option (3) is correct.

SECTION - B

Objective Type Questions (More than one options are correct)

1. The angles *A*, *B*, *C* of a triangle *ABC* satisfy 4cos*A*cos*B* + sin2*A* + sin2*B* + sin2*C* = 4. Then which of the following statements is/are correct?

(1) The triangle *ABC* is right angled

(2) The triangle *ABC* is isosceles

(3) The triangle *ABC* is neither isosceles nor right angled

(4) The triangle *ABC* is equilateral

Sol. Answer (1, 2)

4cos*A*.cos*B* + sin2*A* + sin2*B* + sin2*C* = 4

$$\Rightarrow 4.\frac{1}{2}.\frac{1}{2} + \frac{\sqrt{3}}{2}\times 3$$

$2[\cos(A + B) + \cos(A - B)] + 2\cos(A + B)\cos(A - B) + 2\sin C\cos C = 2$

$\Rightarrow\ -\cos C + \cos(A - B) - \cos C\cos(A - B) + \sin C\cos C = 2$

$\Rightarrow\ -\text{coc}C + \cos(A - B) + \cos C$

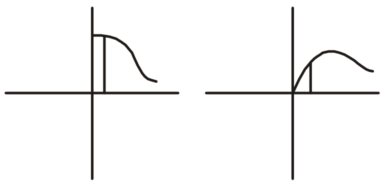
2. Which of the following statement(s) is/are correct?

(1) $\cos(\sin 1) > \sin(\cos 1)$ (2) $\cos(\sin 1.5) > \sin(\cos 1.5)$

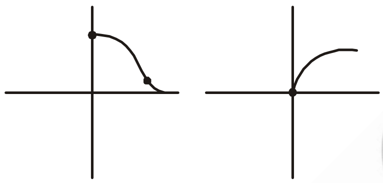
(3) $\cos\left(\sin\frac{7\pi}{18}\right) > \sin\left(\cos\frac{7\pi}{18}\right)$ (4) $\cos\left(\sin\frac{5\pi}{18}\right) > \sin\left(\cos\frac{5\pi}{18}\right)$

Sol. Answer (1, 2, 3, 4)

1. cos(sin1) sin(cos1)



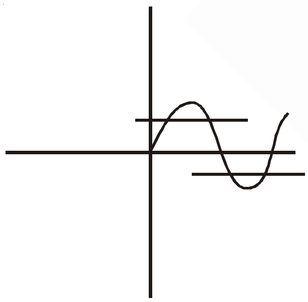
2.



3. If $\sin\theta = K$, $-1 \le K \le 1$, then number of values of θ, for same value of K in $[0, 2\pi]$ may be

(1) 1 (2) 2 (3) 3 (4) 4

Sol. Answer (1, 2, 3)



4. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)\}$, then the length of hypoteneous is

(1) $2[1 + \cos(\alpha - \beta)]$ (2) $2[1 - \cos(\alpha + \beta)]$ (3) $4\cos^2\left(\frac{\alpha-\beta}{2}\right)$ (4) $4\sin^2\left(\frac{\alpha+\beta}{2}\right)$

Sol. Answer (1, 3)

$a = \cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)$

$= 2\cos(\alpha + \beta)\cos(\alpha + \beta) + 2\cos(\alpha + \beta)$

$= 2\cos(\alpha + \beta) \times 2\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$= 4\cos(\alpha + \beta)\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$b = \sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)$

$= 2\sin(\alpha + \beta)\cos(\alpha - \beta) + 2\sin(\alpha + \beta)$

$= 2\sin(\alpha + \beta).2\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$= 4\sin(\alpha + \beta)\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$\therefore\ a^2 + b^2\left(\frac{\alpha-\beta}{2}\right) = 16\cos^2(\alpha + \beta)\cos^4 + 16\sin^2(\alpha + \beta).\cos^4\left(\frac{\alpha-\beta}{2}\right) = 16\cos^4\left(\frac{\alpha-\beta}{2}\right)$

$A = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$

5. If $\sin\theta + \sqrt{3}\cos\theta = 6y - y^2 - 11$, $\theta \in [0, 4\pi]$, $y \in R$ holds for

(1) No value of y and two values of θ

(2) One value of y and two values of θ

(3) Two values of y and one value of θ

(4) Two pairs of (y, θ)

Sol. Answer (2, 4)

$\sin\theta + \sqrt{3}\cos\theta = 6y - y^2 - 11$

$2\left(\sin\left(\theta + \frac{\pi}{3}\right)\right) = -(y^2 - 6y + 9) + 9 - 11$

$2\sin\left(\theta + \frac{\pi}{3}\right) = -(y-3)^2 - 2$

LHS $\in [-2, 2]$,

RHS ≥ -2

$\therefore$ only solution exists if LHS = RHS = –2

$\sin\left(\theta + \frac{\pi}{3}\right) = -1$ and $(y - 3) = 0$

$\because\ \theta \in [0, 4\pi]$, $y = 3$

$\Rightarrow\ \theta + \frac{\pi}{3} = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore$ Two values of θ and one value of y.

6. The possible value of $\theta \in [-\pi, \pi]$ satisfying the equation $2(\cos\theta + \cos 2\theta) + (1 + 2\cos\theta)\sin 2\theta = 2\sin\theta$ are

(1) $-\frac{\pi}{2}$

(2) $-\frac{\pi}{3}$

(3) $\frac{\pi}{3}$

(4) π

Sol. Answer (1, 2, 3, 4)

$\theta \in [-\pi, \pi]$

$2(\cos\theta + \cos 2\theta) + (1 + 2\cos)\sin 2\theta = 2\sin\theta$

or, $2(\cos\theta + \cos 2\theta) + 2(\cos\theta + 2\cos^2\theta)\sin\theta = 2\sin\theta$

or, $2(\cos\theta + \cos 2\theta) + 2\sin[\cos\theta + 2\cos^2\theta - 1] = 0$

or, $2(\cos\theta + \cos 2\theta) + 2\sin\theta(\cos\theta + \cos 2\theta) = 0$

or, $2(\cos\theta + \cos 2\theta)(1 + \sin\theta) = 0$

7. The number of all possible triplets (p, q, r) such that $p + q\cos 2\theta + r\sin^2\theta = 0$ for all θ, is

(1) $(k, -k, -2k)$ (2) $\left(\frac{k}{2}, -\frac{k}{2}, -k\right)$ (3) $\left(-\frac{k}{2}, \frac{k}{2}, k\right)$ (4) $(-k, k, 2k)$

Sol. Answer (1, 2, 3, 4)

$p + q\cos 2\theta + r\sin^2\theta = 0 \quad \forall\theta \in R$

or, $p + q(1 - 2\sin^2\theta) + r\sin^2\theta = 0 \quad \forall\theta \in R$

or, $(r - 2q)\sin^2\theta + (q + p) = 0 \quad \forall\theta \in R$

$\Rightarrow r - 2q = q + p = 0$

$\Rightarrow r = 2q,\ q = -p$

$\Rightarrow \frac{r}{2} = \frac{q}{1} = \frac{-p}{1}$

Or, $\frac{p}{-1} = \frac{q}{1} = \frac{r}{2}$

8. In a triangle ABC, $\frac{a}{b} = 2 + \sqrt{3}, \angle C = 60°$ then in the triangle

(1) One angle is 105°

(2) One angle is four times another angle

(3) One angle is 25°

(4) One angle is five times another angle

Sol. Answer (1, 2)

In ΔABC

Using $\sin\theta$ formula $\frac{\sin A}{\sin B} = \frac{a}{b} = \frac{2+\sqrt{3}}{1}$

Apply components and dividends $\frac{a+b}{a-b} = \frac{3+\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{3+\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}} = \frac{3+\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \cot\frac{A-B}{2} = 1$

$\therefore A - B = 90°$

and $A + B = 120°$

$\angle A = 105°$

$\therefore \angle B = 15°$

$\therefore$ Option (1) and (2) are correct but option (3) & (4) are not correct.

9. In a triangle ABC

(1) $\sin A.\sin B.\sin C = \dfrac{\Delta}{2R^2}$

(2) $\sin A.\sin B.\sin C = \dfrac{r}{2R}(\sin A + \sin B + \sin C)$

(3) $a\cos A + b\cos B + c\cos C = \dfrac{abc}{2R^2}$

(4) $\sin A.\sin B.\sin C = \dfrac{R}{2r}(\sin A + \sin B + \sin C)$

Sol. Answer (1, 2)

(1) $\sin A.\ \sin B.\ \sin C$

$= \dfrac{2\Delta}{bc}\cdot\dfrac{2\Delta}{ac}\cdot\dfrac{2\Delta}{ab}$ $\quad \Delta = \dfrac{1}{2}ab\sin C$

$= \dfrac{8\Delta^3}{a^2b^2c^2}$ $\quad \Delta = \dfrac{abc}{4R}$

$= \dfrac{8\Delta^3}{16\Delta^2R^2}$

$= \dfrac{\Delta}{2R^2}$ So option (1) is correct

(2) RHS $\dfrac{r}{2R}(\sin A + \sin B + \sin C)$

$= \dfrac{r}{2R}\left(\dfrac{2\Delta}{bc}+\dfrac{2\Delta}{ac}+\dfrac{2\Delta}{ab}\right)$

$= \dfrac{r}{2R}.2\Delta\left(\dfrac{a+b+c}{abc}\right)$

$= \dfrac{r\Delta}{R}\cdot\dfrac{2s}{abc}$

$= \dfrac{2\Delta^2}{R.abc}$

$= \dfrac{2\Delta^2}{R.4\Delta R}$

$= \dfrac{\Delta}{2R^2} = \text{LHS}$

So option 2 is correct

(3) Since option (2) is correct $\therefore$ option 4 is not correct

Hence correct answers are (1, 2)

10. If $\cos A + \cos B = 4\sin^2\left(\dfrac{C}{2}\right)$, then

(1) $2\sin B = \sin A + \sin C$

(2) $\cos C = 1 - \dfrac{r}{R}$

(3) $\cos A + \cos B = \dfrac{2r}{R}$

(4) a, c, b are in G.P.

Sol. Answer (2, 3)

$$\cos A + \cos B = 4\sin^2 \frac{C}{2}$$

$$\Rightarrow \quad 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = 4\sin^2\frac{C}{2}$$

$$\Rightarrow \quad \cos\frac{A-B}{2} = 2\sin\frac{C}{2}$$ Multiply by $2\cos\frac{C}{2}$ both sides

$$\Rightarrow \quad 2\cos\frac{C}{2}\cos\frac{A-B}{2} = 2\sin C$$

$\Rightarrow$ sin$A$ + sin$B$ = 2sin$C$

$\therefore$ *a, b, c* are in A.P.

$\therefore$ Option (1) is not correct

So we know that cos$A$ + cos$B$ + cos$C$ = $1+\frac{r}{R}$

$$\therefore \; 4\sin^2\frac{C}{2} + \cos C = 1 + \frac{r}{R}$$

$$(1-\cos C) + \cos C = 1 + \frac{r}{R}$$

$$1 - \frac{r}{R} = \cos C$$

$\therefore$ Option (2) is correct

Again cos$A$ + cos$B$ + cos$C$ = $1+\frac{r}{R}$

Put cos$C$ = $1+\frac{r}{R}$

$$\therefore \; \cos A + \cos B + 1 - \frac{r}{R} = 1 + \frac{r}{R}$$

$$\therefore \; \cos A + \cos B = \frac{2r}{R}$$

$\therefore$ Option (3) is correct

As *a*, *c*, *b* are in A.P. and all three are not equal

$\therefore$ *a*, *c*, *b* are not in G.P

11. In a triangle *ABC*, point D and E are taken on side BC such that *BD* = *DE* = *EC*. If angle *ADE* = angle *AED* = θ, then

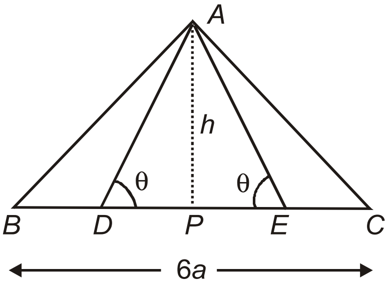
(1) tanθ = 3tan$B$ (2) 3tanθ = tan$C$ (3) $\frac{6\tan\theta}{\tan^2\theta - 9} = \tan A$ (4) $\angle B = \angle C$

Sol. Answer (1, 3, 4)

Let *BC* = 6*a*

$\therefore$ $BD = DE = EC = 2a$

And $DP = PE = a$



$$\tan B = \frac{h}{3a}$$

$$\tan\theta = \frac{h}{a}$$

$\therefore$ $\tan\theta = 3\tan B$

$\therefore$ Option (1) is correct

By figure it can be seen that $\angle B = \angle C$

$\therefore$ Option (4) is correct

And option (2) is incorrect

Now option (3); LHS $\Rightarrow$ $\dfrac{6\tan\theta}{\tan^2\theta - 9} = \dfrac{6\dfrac{h}{a}}{\left(\dfrac{h}{a}\right)^2 - 9} = \dfrac{6ah}{h^2 - 9a^2}$

L.H.S. $\tan A = -\tan(B + C)$

$$= \frac{[\tan B + \tan C]}{1 - \tan B \tan C}$$

$$= \frac{\tan B + \tan C}{\tan B \tan C - 1} = \frac{\dfrac{h}{3a} + \dfrac{h}{3a}}{\dfrac{h^2}{9a^2} - 1} = \frac{6ah}{h^2 - 9a^2}$$

$\therefore$ Option (3) is correct

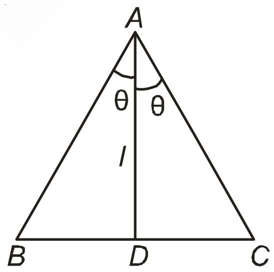
12. In a triangle, with usual notations, the length of the bisector of angle A is

(1) $\dfrac{2bc\cos\dfrac{A}{2}}{b+c}$ (2) $\dfrac{2bc\sin\dfrac{A}{2}}{b+c}$ (3) $\dfrac{abc\ \text{cosec}\dfrac{A}{2}}{2R(b+c)}$ (4) $\dfrac{2\Delta}{b+c}\text{cosec}\dfrac{A}{2}$

Sol. Answer (1, 3, 4)

Let $AD = l$

Ar ΔABD + Ar ΔADC = Ar ΔABC



$$\frac{1}{2}cl\sin\theta + \frac{1}{2}bl\sin\theta = \frac{1}{2}bc\sin 2\theta$$

$$\frac{1}{2}l\sin\theta[b+c] = \frac{1}{2}bc2\sin\theta\cos\theta$$

$$l = \frac{2bc\cos\theta}{b+c} = \frac{2bc\cos A/2}{b+c}$$

So option (1) is correct and option (2) is incorrect

For option (3)

$$=\frac{a}{a}\frac{2bc\cos A/2}{b+c}=\frac{2abc\cos A/2}{(b+c).2R\sin A}=\frac{2abc\cos A/2}{(b+c).2R.2\sin A/2\cos A/2}$$

$$=\frac{abc\,\text{cosec}\;A/2}{2R(b+c)}$$

For option (4)

$$\frac{abc\text{ cosec }A/2}{2R(b+c)}=\frac{4R\;\Delta\text{cosec }A/2}{2R(b+c)}=\frac{2\Delta\text{cosec}A/2}{b+c}$$

13. An ordered triplet solution (x, y, z) with x, y, z $\in$ (0, 1) and satisfying $x^2 + y^2 + z^2 + 2xyz = 1$ is

(1) $\left(\cos\frac{\pi}{6}, \cos\frac{7\pi}{18}, \cos\frac{4\pi}{9}\right)$

(2) $\left(\cos\frac{2\pi}{5}, \cos\frac{\pi}{3}, \cos\frac{\pi}{10}\right)$

(3) $\left(\cos\frac{7\pi}{12}, \cos\frac{\pi}{4}, \cos\frac{\pi}{6}\right)$

(4) $\left(\cos\frac{\pi}{12}, \cos\frac{4\pi}{9}, \cos\frac{17\pi}{36}\right)$

Sol. Answer (1, 4)

Let $A=\cos^{-1}x, B=\cos^{-1}y, C=\cos^{-1}z,$ where $A + B + C = \pi$

$A + B + C = \pi$

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$

$$\cos^{-1}[xy-\sqrt{1-x^2}\sqrt{1-y^2}]=\cos^{-1}(-z)$$

$$xy+z=\sqrt{1-x^2}\sqrt{1-y^2}$$

$(xy + z^2) = (1 - x^2)(1 - y^2)$

$x^2y^2 + z^2 + 2xyz = 1 - x^2 - y^2 + x^2y^2$

$x^2y^2 + z^2 + 2xyz = 1$ which is the given identity

$\therefore\;\; A + B + C = \pi$

which is satisfied by options (1) and (4) and A, B, C should be acute.

14. In a ΔABC, $a\cos B + b\cos C + c\cos A = \frac{a+b+c}{2}$, then

(1) Triangle in isosceles

(2) Triangle may be equilateral

(3) $\sin(A - B) + \sin(B - C) + \sin(C - A) = \frac{3}{2}$

(4) $4\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{B-C}{2}\right)\sin\left(\frac{C-A}{2}\right)=1$

Sol. Answer (1, 2)

In ΔABC

$$a\cos B + b\cos C + c\cos A = \frac{a+b+c}{2}$$

$$2R\sin A\cos B + 2R\sin B\cos C + 2R\sin C\cos A = \frac{2R\sin A+2R\sin B+2R\sin C}{2}$$

$\sin(A + B) + \sin(A - B) + \sin(B + C) + \sin(B - C) + \sin(C + A) + \sin(C - A) = \sin A + \sin B + \sin C$

$\Rightarrow \sin(B + C) + \sin(C - A) + \sin(A + B) = 0$

$\Rightarrow 4\sin\frac{B-A}{2}\sin\frac{A-C}{2}\sin\frac{C-B}{2} = 0$

$\therefore \sin\frac{B-A}{2} = 0$ or $\sin\frac{A-C}{2} = 0$ or $\sin\frac{C-B}{2} = 0$

$\therefore B = A$ or $A = C$ or $C = B$

Option (1) is correct

Option (2) is correct (If $A = B = C$)

Option (3) is not correct

Option (4) is not correct

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

There are trigonometrical equations that are nonstandard in aspect in that the equation may contain dissimilar terms, say algebraic as well as trigonometric expressions, or there may be only one equation in more than one unknowns. In such cases we make use of the extreme values of trigonometric functions or algebraic functions. In particular $|\sin x| \le 1$, $|\cos x| \le 1$, and maxima/minima of quadratic expressions are widely made use of.

1. The equation $\sin\left(\frac{\pi x}{6}\right) = x^2 - 6x + 10$ holds for

(1) Infinitely many values of x
(2) Finitely many values of x
(3) Just one value of x
(4) No value of x

Sol. Answer (3)

$\sin\left(\frac{x\pi}{6}\right) = (x - 3)^2 + 1 \ge 1, \ \forall x \in R$

We know $\sin\left(\frac{\pi x}{6}\right) \le 1$

Equality holds at $x = 3$.

Hence the given equation has only one solution.

2. The equation $8\cos^4\frac{x}{2}\sin^2\frac{x}{2} = x^2 + \frac{1}{x^2}, x \in (0, 4\pi]$ holds for

(1) No value of x
(2) Exactly two values of x, both greater than π
(3) Exactly two values of x, one smaller than π and the other greater than π
(4) Just one value of x

Sol. Answer (1)

$2\cos^2\frac{x}{2}\sin^2 x = x^2 + \frac{1}{x^2} \ge 2 \ \ \forall x \in R$, except $x = 0$

$\Rightarrow \cos^2\dfrac{x}{2}\sin^2 x \geq 1$ (which is not possible)

Because $0 \leq \cos^2\dfrac{x}{2} \leq 1$ and $0 \leq \sin^2 x \leq 1$

Hence $\cos^2\dfrac{x}{2}\sin^2 x \ngtr 1$

But if $\cos^2\dfrac{x}{2}\sin^2 x = 1$

Then $\cos^2\dfrac{x}{2} = 1$ and $\sin^2 x = 1$

$\Rightarrow \dfrac{x}{2} = n\pi$ and $x = k\pi \pm \dfrac{\pi}{2}n,\ k \in Z$, which is not possible simultaneously. Hence no solution exits.

3. The equation $\text{cosec}\dfrac{x}{2} + \text{cosec}\dfrac{y}{2} + \text{cosec}\dfrac{z}{2} = 6$, where $0 < x, y, z < \dfrac{\pi}{2}$ and $x + y + z = \pi$, have

(1) Three ordered triplet (x, y, z) solutions

(2) Two ordered triplet (x, y, z) solutions

(3) Just one ordered triplet (x, y, z) solution

(4) No ordered triplet (x, y, z) solution

Sol. Answer (3)

$\text{cosec}\dfrac{x}{2} + \text{cosec}\dfrac{y}{2} + \text{cosec}\dfrac{z}{2} = 2 + 2 + 2$

$\Rightarrow x = y = z = \dfrac{\pi}{3}$

Comprehension-II

Recall that $\sin x + \cos x = u$ (say)

and $\sin x\cos x = v$ (say) are connected by

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$\Rightarrow u^2 = 1 + 2v$

$\Rightarrow v = \dfrac{u^2 - 1}{2}$

It follows that any rational integral function of $\sin x + \cos x$, and $\sin x\cos x$ *i.e.*, $R(\sin x + \cos x, \sin x\cos x)$, or in our notation $R(u, v)$ can be transformed to $R\left(u, \dfrac{u^2-1}{2}\right)$. Thus to solve an equation of the form $R(u, v) = 0$, we form a polynomial equation in u and then look for solutions.

1. The solution set of $\sin x + \cos x - 2\sqrt{2}\sin x\cos x = 0$ is completely described by

(1) $x = 2n\pi + \dfrac{\pi}{4}, 2n\pi - \dfrac{5\pi}{12}, 2n\pi + \dfrac{11\pi}{12}, n \in Z$

(2) $x = 2n\pi - \dfrac{\pi}{4}, 2n\pi + \dfrac{\pi}{12}, 2n\pi + \dfrac{7\pi}{12}, n \in Z$

(3) $x = 2n\pi + \dfrac{\pi}{4}, 2n\pi - \dfrac{\pi}{12}, 2n\pi - \dfrac{7\pi}{12}, n \in Z$

(4) $x = 2n\pi - \dfrac{\pi}{4}, 2n\pi + \dfrac{\pi}{12}, 2n\pi - \dfrac{7\pi}{12}, n \in Z$

Sol. Answer (1)

sin$x$ + cos$x$ = $2\sqrt{2}$ sin$x$ cos$x$

$\Rightarrow u = 2\sqrt{2}v = 2\sqrt{2}\left(\frac{u^2-1}{2}\right) \quad \Rightarrow \sqrt{2}u^2 - u - \sqrt{2} = 0$

$\Rightarrow u = \frac{1 \pm \sqrt{1+8}}{2\sqrt{2}} = \sqrt{2}, -\frac{1}{\sqrt{2}} \quad \Rightarrow$ sin x + cos x = $\sqrt{2}$, $-\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(x+\frac{\pi}{4}\right) = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow x + \sin\frac{\pi}{4} = n\pi + (-1)^{n-1}.\frac{\pi}{6}$

and $\sin\left(x+\frac{\pi}{4}\right) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{2}$

$\Rightarrow x = n\pi - \frac{\pi}{4} + (-1)^n \frac{\pi}{2} = n\pi + \frac{\pi}{4}$ for n is even or odd.

2. The complete solution of the equation
sin2x – 12(sin$x$ – cos$x$) + 12 = 0 is given by

(1) $x = 2n\pi + \frac{\pi}{2}, (2n-1)\frac{\pi}{4}, n \in Z$

(2) $x = n\pi + \frac{\pi}{2}, (2n+1)\pi, n \in Z$

(3) $x = 2n\pi + \frac{\pi}{2}, (2n+1)\pi, n \in Z$

(4) $x = n\pi + \frac{\pi}{2}, (2n-1)\pi, n \in Z$

Sol. Answer (3)

Let u = cosx – sinx, v = sinx cos$x$ = $\frac{1-u^2}{2}$

Thus the given equation reduces to 2v + 12u + 12 =0

$\Rightarrow$ 1 – u^2 + 12u + 12 = 0

$\Rightarrow$ u^2 – 12u – 13 = 0

$\Rightarrow$ (u + 1) (u – 13) = 0

$\Rightarrow$ cos$x$ – sin$x$ = –1 as $u \neq 13$

$\Rightarrow \cos\left(x+\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

$\Rightarrow x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}, (2n-1)\pi, n \in Z$ (by taking positive and negative respectively)

3. The number of solutions of the equation $\sin\theta + \cos\theta = 1 + \sin\theta\cos\theta$ in the interval $[0, 4\pi]$ is

(1) Four (2) Six (3) Eight (4) Five

Sol. Answer (4)

Let $y = \sin x + \cos x$

Then the given equation can be reduced to

$$y = 1 + \frac{y^2 - 1}{2}$$

$$\Rightarrow y^2 - 2y + 1 = 0$$

$$\Rightarrow y = 1$$

$$\Rightarrow \sin x + \cos x = 1 \qquad \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

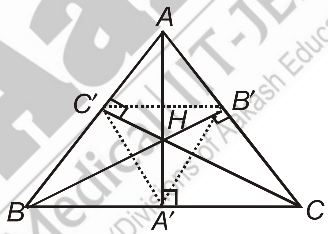
$$\Rightarrow x + \frac{\pi}{4} = \frac{\pi}{4}, \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}, 3\pi - \frac{\pi}{4}, 4\pi + \frac{\pi}{4}, 5\pi - \frac{\pi}{4}$$

$$\Rightarrow x = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}, 4\pi, \frac{9\pi}{2}$$

Hence in the given interval $x = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}, 4\pi$ five solution exists.

Comprehension-III

Consider a triangle ABC, and that AA', BB', CC' be the perpendicular from A, B and C upon the sides opposite to them. These three perpendiculars meet in H, called the orthocentre of the triangle. The triangle $A'B'C'$ formed by the feet of the perpendicular is called the pedal triangle of ABC. (Assume $\angle A, B, C \neq 90°$)



1. Suppose the triangle ABC have angles 60°, 70° and 50°. Then the pedal triangle $A'B'C'$ have angles given by

(1) 80°, 60°, 40° (2) 120°, 40°, 20° (3) 30°, 65°, 85° (4) 45°, 55°, 80°

Sol. Answer (1)

$\angle A' = \pi - 2A = \pi - 120 = 60°$

$\angle B' = \pi - 2B = \pi - 140 = 40°$

$\angle C' = \pi - 2C = \pi - 100 = 80°$

2. Suppose a triangle ABC has its sides 13, 14 and 15 cm. Then the circumradius of the pedal triangle is (in cm)

(1) $\frac{65}{24}$ (2) $\frac{65}{8}$ (3) $\frac{26}{3}$ (4) $\frac{65}{16}$

Sol. Answer (4)

Given $a = 13$, $b = 14$, $c = 15$

Let R is circumradius of ΔABC, then

$$R = \frac{abc}{4\Delta}$$

Now, $2s = 13 + 14 + 15 = 42$

$s = 21$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84$$

$$R = \frac{13 \cdot 14 \cdot 15}{4 \cdot 21 \cdot 4} = \frac{65}{8}$$

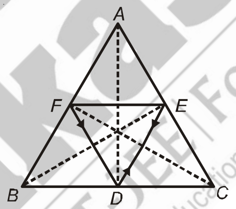
Now, circumradius of pedal triangle = $\frac{R}{2} = \frac{65}{16}$

3. Suppose ABC is an acute angled triangle, then the area of the pedal triangle is (R being the circum radius of triangle ABC)

(1) $\frac{R^2}{2}\sin 2A \sin 2B \sin 2C$ (2) $R^2 \sin 2A \sin 2B \sin 2C$

(3) $\frac{R^2}{2}\sin A \sin B \sin C$ (4) $R^2 \sin A \sin B \sin C$

Sol. Answer (1)



Area of $\Delta DEF = \frac{DE \cdot EF \cdot FD}{4 \cdot R'}$

where R' is the circumradius of ΔDEF

$$= \frac{(a\cos A)(b\cos B)(c\cos C)}{4\cdot\left(\frac{R}{2}\right)}$$

$$= \frac{(2R\sin A\cos A)(2R\sin B\cos B)(2R\sin C \cdot \cos C)}{2R}$$

$$= \frac{R^3}{2R}\sin 2A \cdot \sin 2B \cdot \sin 2C$$

$$= \frac{1}{2}R^2 \sin 2A \cdot \sin 2B \cdot \sin 2C$$

Comprehension-IV

Let us consider a triangle ABC having $BC = 5$ cm, $CA = 4$ cm, $AB = 3$ cm D, E are points on BC such that $BD = DE = EC$, $\angle CAE = \theta$, then

1. AE^2 is equal to

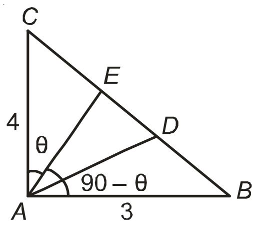
(1) $\frac{73}{3}$ (2) $\frac{73}{5}$ (3) $\frac{73}{7}$ (4) $\frac{73}{9}$

Sol. Answer (4)

Given $BD = DE = EC = x$(let)

And $\angle ACE = \theta$

In ΔACE, by using cosine rule



$$\cos\theta = \frac{AE^2 + 16 - CE^2}{2AE \cdot 4} \text{ and } CE = \frac{5}{3}$$

$$\cos\theta = \frac{AE^2 + 16 - \frac{25}{9}}{2 \cdot AE \cdot 4} \quad \ldots\text{(i)}$$

In ΔAEB, by using sine rule

$$\frac{\sin(90-\theta)}{BE} = \frac{\sin B}{AE}$$

$$\cos\theta = \frac{BE}{AE}\sin B = \frac{10}{3.AE} \cdot \frac{4}{5} \quad \ldots\text{(ii)}$$

From equation (i) and (ii)

$$\frac{10}{3 \cdot AE} \cdot \frac{4}{5} = \frac{AE^2 + 16 - \frac{25}{9}}{2 \cdot AE \cdot 4}$$

$$\frac{64}{3} = AE^2 + \frac{119}{9}$$

$$\therefore\ AE^2 = \frac{73}{9}$$

2. $\tan\theta$ is equal to

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{3}{8}$ (4) $\frac{5}{8}$

Sol. Answer (3)

From equation (ii)

$$\cos\theta = \frac{8}{3 \cdot AE}$$

$$\cos\theta = \frac{8}{3\left(\frac{\sqrt{73}}{3}\right)} = \frac{8}{\sqrt{73}}$$

$$\therefore\ \tan\theta = \frac{3}{8}$$

3. AD^2 is equal to

(1) $\frac{52}{3}$ (2) $\frac{52}{9}$ (3) $\frac{52}{7}$ (4) 52

Sol. Answer (2)

In ΔABD, by using cosine rule

$$\cos B = \frac{BA^2 + BD^2 - AD^2}{2BA \cdot BD}$$

$$\frac{3}{5} = \frac{9 + \frac{25}{9} - AD^2}{2 \cdot 3 \cdot \frac{5}{3}}$$

$$6 = 9 + \frac{25}{9} - AD^2$$

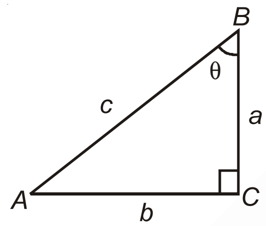
$$\therefore\ AD^2 = \frac{25}{9} + 3$$

$$AD^2 = \frac{52}{9}$$

Comprehension-V

Given a right angled triangle *ABC* with right angled at *C*. $\angle C = 90°$ and *a*, *b*, *c* are length of corresponding sides. $(b > a)$.

Solution of Comprehension-V



$a^2 + b^2 = c^2$

$\angle(A + B) = 90°$

1. cos(*A* – *B*) will be equal to

(1) cos2*B* (2) sin2*A* (3) cos2*A* (4) sin*B*

Sol. Answer (2)

$\therefore \cos(90° - B - B) = \cos(90° - 2B)$

$= \sin 2B$

Also $\cos(A - (90° - A)) = \cos(2A - 90°) = \sin(2A)$

2. sin2*A* will be equal to

(1) $\frac{2bc}{a^2}$ (2) $\frac{b^2 - a^2}{c^2}$ (3) $\frac{2ab}{c^2}$ (4) $\frac{b^2 - c^2}{2bc}$

Sol. Answer (3)

$\therefore\ \sin 2A = 2\sin A \cos A$

$$= 2 \cdot \frac{a}{c} \times \frac{b}{c}$$

$$= \frac{2ab}{c^2}$$

$c^2 - b^2 = a^2$

3. $\tan\frac{A}{2}$ will be equal to

(1) $\frac{2ab}{a^2-b^2}$ (2) $\frac{2ab}{b^2-a^2}$ (3) $\frac{c-b}{a}$ (4) $\frac{a}{c-b}$

Sol. Answer (3)

$$\tan^2\frac{A}{2}=\frac{1-\cos A}{1+\cos A}=\frac{1-\frac{b}{c}}{1+\frac{b}{c}}=\frac{c-b}{c+b}$$

$$=\frac{(c-b)^2}{c^2-b^2}=\frac{(c-b)^2}{a^2}$$

$$\Rightarrow \tan\frac{A}{2}=\frac{c-b}{a}$$

SECTION - D

Matrix-Match Type Questions

1. Match the value of the expression given in column-I with value greater than or equal to the number given in column-II.

Column-I	Column-II
(A) $2^{\sin x}+2^{\cos x}$	(p) 2
(B) $2^{\sin^2 x}+2^{\cos^2 x}$	(q) $2^{3/2}$
(C) $2^{\sin^6 x}+2^{\cos^6 x}$	(r) $2^{1-1\sqrt{2}}$
(D) $2^{\sin^4 x}+2^{\cos^4 x}$	(s) $2^{5/4}$
	(t) $2^{9/8}$

Sol. Answer A(r), B(p, q, r, s, t), C(p, r, t), D(p, r, s, t)

(A) $\frac{2^{\sin x}+2^{\cos x}}{2}\geq 2^{\frac{\sin x+\cos x}{2}}$

$$\Rightarrow \quad 2^{\sin x}+2^{\cos x}\geq 2^{\frac{\sin x+\cos x+2}{2}}$$

$$=2^{\frac{2-\sqrt{2}}{2}}$$

$$=2^{1-\frac{1}{\sqrt{2}}}$$

(B) $\frac{2^{\sin^2 x}+2^{\cos^2 x}}{2}\geq 2^{\sin x\cos x}$

$$\Rightarrow \quad 2^{\sin^2 x}+2^{\cos^2 x}\geq 2^{\sin x\cos x+1}$$

$$=2^{\frac{1}{2}\sin 2x+1}$$

$$=2^{-\frac{1}{2}+1}=2^{\frac{3}{2}}$$

(C) $2^{\sin^6 x} + 2^{\cos^6 x} \geq 2^{\sin^3 x \cos^3 x + 1}$

$= 2^{\frac{1}{8}(\sin 2x)^3 + 1} = 2^{\frac{9}{8}}$

(D) $2^{\sin^4 x} + 2^{\cos^4 x} \geq 2^{\sin^2 x \cos^2 x + 1}$

$= 2^{\frac{1}{4}(\sin 2x)^2 + 1} = 2^{\frac{5}{4}}$

2. Match the following.

Column-I	**Column-II**
(A) If $\frac{\sin\theta}{\sin\phi} = \frac{1}{2}$ and $\frac{\cos\theta}{\cos\phi} = \frac{3}{2}$; θ and φ are acute angles, then tanθ is equal to	(p) 12
(B) If $\frac{\sin\theta}{\sin\phi} = \frac{1}{2}$ and $\frac{\cos\theta}{\cos\phi} = \frac{3}{2}$; θ and φ are acute angles, then tanφ is equal to	(q) $\frac{1}{3}\sqrt{\frac{5}{3}}$
(C) The numerical value of $4\sin 50° - \sqrt{3}\tan 50°$ is equal to	(r) $\sqrt{\frac{5}{3}}$
(D) The minimum value of $9\tan^2\theta + 4\cot^2\theta$ is greater than or equal to	(s) 1
	(t) 2

Sol. Answer A(q), B(r), C(s), D(p)

(A) $\frac{\sin\theta}{\sin\phi} = \frac{1}{2}, \frac{\cos\theta}{\cos\phi} = \frac{3}{2}$

sinφ = 2sinθ

$\cos\phi = \frac{2}{3}\cos\theta$

$1 \pm 4\sin\theta + \frac{4}{9}\cos^2\theta$

$\Rightarrow \sec^2\theta = 4\sin^2\theta + \frac{4}{9}$

$\Rightarrow 1 + \sin^2\theta = 4\sin^2\theta + \frac{4}{9}$

$\Rightarrow 3\sin^2\theta = 1 - \frac{4}{9} = \frac{5}{9}$

$\Rightarrow \sin^2\theta = \frac{5}{27}$

$\Rightarrow \tan\theta = \frac{\sqrt{5}}{3\sqrt{3}} = \frac{1}{3}\sqrt{\frac{5}{3}}$

(B) $\dfrac{\sin\theta}{\sin\phi}=\dfrac{1}{2}, \dfrac{\cos\theta}{\cos\phi}=\dfrac{3}{2}$

$\Rightarrow \quad \sin\theta = \dfrac{1}{2}\sin\phi, \ \cos\theta = \dfrac{3}{2}\cos\phi$

$1 = \dfrac{1}{4}\sin^2\theta, + \dfrac{9}{4}\cos^2\phi$

$\Rightarrow \quad \sec^2\phi = \dfrac{1}{4}\sin^2\phi + \dfrac{9}{4}$

$\Rightarrow \quad 1+\sin^2\phi = \dfrac{1}{4}\sin^2\phi + \dfrac{9}{4}$

$\Rightarrow \dfrac{3}{4}\sin^2\phi = \dfrac{5}{4}$

$\Rightarrow \quad \sin^2\phi = \dfrac{5}{3}$

$\Rightarrow \quad \sin\phi = \sqrt{\dfrac{5}{3}}$

(C) $4\sin 50 - \sqrt{3}.\sin 50^\circ$

$= 4\sin 50^\circ - \sqrt{3}\,\dfrac{\sin 50^\circ}{\cos 50^\circ}$

$= \dfrac{4\sin 50^\circ \cos 50^\circ - \sqrt{3}\sin 50^\circ}{\cos 50^\circ}$

$= \dfrac{2\sin 100^\circ - \sqrt{3}\sin 50^\circ}{\cos 50^\circ}$

$= \dfrac{2\sin 100^\circ - 2\sin 50^\circ}{\cos 50^\circ}$

$= \dfrac{2\sin 100^\circ - \sin 80^\circ - \sin 20^\circ}{\cos 50^\circ}$

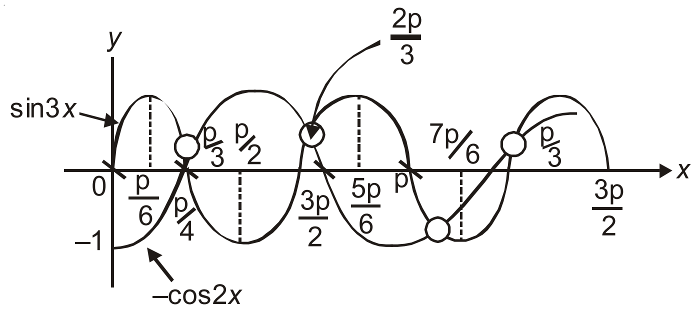
$= \dfrac{\sin 100^\circ - \sin 20^\circ}{\cos 50^\circ} = \dfrac{2\cos 60^\circ.\sin 40^\circ}{\cos 50^\circ}$

= 1

3. Match the following

Column-I	**Column-II**
(A) The number of solutions of $\sin 3x + \cos 2x = 0$ in $\left[0, \dfrac{3\pi}{2}\right]$ is	(p) 1
(B) The number of solutions of the equation $(1 - \tan x)(1 + \sin 2x) = 1 + \tan x$ in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ is	(q) 4
(C) The number of solutions of the equation $\sin(\pi^x) = e^x + e^{-x}$ is	(r) 2
(D) The number of solutions of the equation $\sin^3 x - 3\sin x\cos^2 x + 2\cos^3 x = 0$ in $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$ is	(s) 0

Sol. Answer A(q), B(r), C(s), D(p)

(A) 

It is clear from the graph that 4 points satisfy the given equation

(B) $(\cos x - \sin x)(1 + \sin 2x) = \cos x + \sin x$

$\Rightarrow -\sin x + \cos x - \sin x \sin 2x + \cos x \sin 2x = \sin x + \cos x$

$\Rightarrow -1 + \sin 2x + 2\cos^2 x = 1$ or $\sin x = 0$

$\Rightarrow -\sin x \cos x = 1 - \cos^2 x$

$\Rightarrow \sin x [\cos x + \sin x] = 0$

$\Rightarrow x = 0$ or $-\dfrac{\pi}{4}$

(C) $-1 \le \sin(\pi^x) \le 1$

But $e^x + e^{-x} \ge 2$

Hence no solution exists.

(D) $\sin^3 x - 3\sin x \cos^2 x + 2\cos^3 x = 0$

$\Rightarrow \sin^2 x (\sin x - \cos x) + \sin x \cos x (\sin x - \cos x) - 2\cos^2 x (\sin x - \cos x) = 0.$

$\Rightarrow (\sin x - \cos x)(\sin x - \cos x)(\sin x + 2\cos x) = 0$

$\Rightarrow (\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$

$\Rightarrow \; x = \dfrac{\pi}{4}$ or $\tan x = -2 \;\Rightarrow\; x < -\dfrac{\pi}{4}$

$\Rightarrow \; x = \dfrac{\pi}{4} \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

4. Match the following

Column-I	**Column-II**
(A) The range of k such that the equation $\sin^8 x + \cos^8 x = k$ admits of a solution is given by	(p) $k \in \left[\dfrac{1}{2}, 1\right]$
(B) The range of k such that the equation $k\sin x - 4\cos x = k + 2$ admits of a solution is given by	(q) $k \in \left[\dfrac{1}{4}, 1\right]$
(C) The range of k such that the equation $\sin^4 x + \cos^4 x = k$ admits of a solution is given by	(r) $k \in (-\infty, 3]$
(D) The range of k such that the equation $\sin^6 x + \cos^6 x = k$ admits of a solution is given by	(s) $k \in \left[\dfrac{1}{8}, 1\right]$

Sol. Answer A(s), B(r), C(p), D(q)

(A) $\dfrac{\sin^8 x + \cos^8 x}{2} \ge \left(\sin^8 x \cos^8 x\right)^{1/2}$

$\Rightarrow \dfrac{k}{2} \ge \dfrac{\left(\sin^4 2x\right)}{16} = \dfrac{1}{16}$

$\Rightarrow k \ge \dfrac{1}{8}$

$\Rightarrow k \in \left[\dfrac{1}{8}, 1\right]$

(B) In order that the given equation admits of a solution we must have

$(k + 2)^2 \le k^2 + 16$

$\Rightarrow \; k^2 + 4k + 4 \le k^2 + 16$

$\Rightarrow \; k \le 3 \Rightarrow k \in (-\infty, 3]$

(C) Do as 'a' part (A.M. ≥ G.M)

(D) Do as 'a' part (A.M. ≥ G.M)

5. Match the following

Column-I	Column-II
(A) $\cos x \sin^3 x < \sin x \cos^3 x$, $0 \le x \le 2\pi$ is satisfied for x lying in	(p) $\left[\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right]$
(B) The value of x for which $4\sin^2 x - 8\sin x + 3 \le 0$, where $x \in [0, 2\pi]$ lies in	(q) $\left[-\pi, -\dfrac{3\pi}{4}\right] \cup \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$
(C) $\lvert\tan x\rvert \le 1$ and $x \in [-\pi, \pi]$ is	(r) $\left[\dfrac{3\pi}{2}, 2\pi\right] \cup \{0\}$
(D) $\cos x - \sin x \ge 1$ and $0 \le x \le 2\pi$ is	(s) $\left[0, \dfrac{\pi}{4}\right]$
	(t) $\left[\dfrac{3\pi}{4}, \pi\right]$

Sol. Answer A(s), B(p), C(q, s, t), D(r)

(A) $\cos x \sin^3 x - \sin x \cos^3 x \le 0$

$\Rightarrow \sin x \cos x(\sin^2 x - \cos^2 x) \le 0$

$\Rightarrow \sin x \cdot \cos x(\cos^2 x - \sin^2 x) \ge 0$

$\Rightarrow \sin x \cdot \cos x(\cos 2x) \ge 0$

$\Rightarrow 2\sin x \cdot \cos x(\cos 2x) \ge 0$

$\Rightarrow \sin 2x \cdot \cos 2x \ge 0$

$\sin 4x \ge 0$

$\Rightarrow 4x \in [0, \pi]$

$x \in \left[0, \dfrac{\pi}{4}\right]$

(B) $4\sin^2 x - 6\sin x - 2\sin x + 3 \le 0$

$\Rightarrow 2\sin x(2\sin x - 3) - 1(2\sin x - 3) \le 0$

$(2\sin x - 3)(2\sin x - 1) \le 0$

But $2\sin x - 3 \le 0$

$\Rightarrow 2\sin x - 1 \le 0$

$\sin x \ge \frac{1}{2}$

$\Rightarrow x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

(C) $|\tan x| \le -1 \Rightarrow -1 \le \tan x\ 2 \le 1$

$x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[-\pi, -\frac{3\pi}{4}\right]$

(D) $\cos x - \sin x \ge 1$

$\Rightarrow \sqrt{2}\left(\cos x \sin\frac{\pi}{4} - \sin x \cos\frac{\pi}{4}\right) \ge 1$

$\sin\left(\frac{\pi}{4} - x\right) \ge \frac{1}{\sqrt{2}} \Rightarrow x \in \left[\frac{3\pi}{2}, 2\pi\right] \cup [0]$

6. Based on the relation between the variables in column I match the type of the triangle in column II.

Column-I	**Column-II**
(A) $r_1 = r_2 + r_3 + r$	(p) Isosceles or right angled
(B) $(a^2 + b^2)\sin(A - B) = (a^2 - b^2)\sin(A + B)$	(q) Obtuse but not necessarily isosceles
(C) $\tan A \tan B < 1$	(r) Right angled but not necessary isosceles
(D) $R = 2r$	(s) Equilateral

Sol. Answer A(r), B(p), C(q), D(s)

(A) $r_1 = r_2 + r_3 + r$

$\Rightarrow r_1 - r = r_2 + r_3$

$\frac{\Delta}{s-a} - \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{(s-c)}$

$\frac{a}{s(s-a)} = \frac{2s-(s-c)}{(s-b)(s-c)}$

$\Rightarrow s(s-a) = (s-b)(s-c)$

$\Rightarrow \frac{(s-b)(s-c)}{s(s-a)} = 1$

$\tan^2\frac{A}{2} = 1$

$\frac{A}{2} = \frac{\pi}{4} \Rightarrow \angle A = 90°$

Then triangle is right angled

(B) $(a^2 + b^2)\sin(A - B) = (a^2 - b^2)\sin(A + B)$

$$\Rightarrow \frac{\sin(A-B)}{\sin(A+B)} = \frac{a^2 - b^2}{a^2 + b^2}$$

By using componendo and dividendo

$$\Rightarrow \frac{\sin(A-B)+\sin(A+B)}{\sin(A-B)-\sin(A+B)} = \frac{2a^2}{-2b^2}$$

$$\Rightarrow \frac{2\sin A\cos B}{-2\cos A\sin B} = \frac{a^2}{+b^2}$$

$$\Rightarrow \frac{\sin A}{\cos A} - \frac{\cos B}{\sin B} = \frac{\sin^2 A}{\sin^2 B}$$

$$\Rightarrow \frac{\cos B}{\cos A} = \frac{\sin A}{\sin B}$$

$\Rightarrow \sin^2 A = \sin^2 B$

$\Rightarrow \boxed{A = B}$ or $2A = \pi - 2B$

$$A + B = \frac{\pi}{2}$$

Then ΔABC is isosceles or right angled triangle

(C) $\tan A.\ \tan B < 1$

Let B is acute angle then

$\Rightarrow \tan A < \cot B$

$\Rightarrow \tan A < \cot B\ (\pi/2 - B)$

$$A + B < \frac{\pi}{2}$$

$$\therefore\ \angle C > \frac{\pi}{2}$$

(D) $R = 2r$

$$\because\ \cos A + \cos B + \cos C = 1 + \frac{r}{R} = \frac{3}{2}$$

Then it is possible

$\angle A = \angle B = \angle C$

7. Match the following

Column-I	**Column-II**
(A) In triangle ABC, the angle $A = 60°$ and the sides $b = 3$ cm, $c = 5$ cm, then the length of median through A is (in cm)	(p) $\frac{12}{5}$

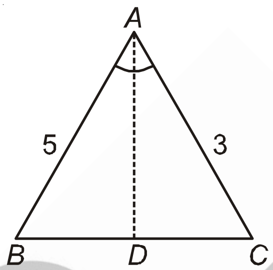
(B) In triangle ABC, the angle $A = 120°$ and the sides $b = 4$ cm, $c = 6$ cm, then the length of angle bisector through A is (in cm) — (q) $\frac{7}{2}$

(C) In triangle ABC, the medians through A and B are perpendicular, then the value of $\frac{a^2 + b^2}{c^2}$ — (r) 8

(D) If the lengths of median of a triangle are 3, 4, 5 cm, then the area of the triangle is (in cm^2) — (s) 5

Sol. Answer A(q), B(p), C(s), D(r)

(A) $\angle A = 60°$, $b = 3$ and $c = 5$



$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2}$$

$b^2 + c^2 - a^2 = bc$

$25 + 9 - a^2 = 15$

$a^2 = 19$

Length of median $AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

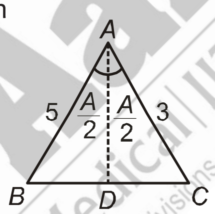
$= \frac{1}{2}\sqrt{16 + 50 - 19}$

$= \frac{7}{2}$

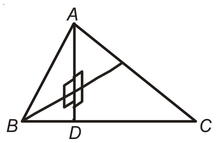
(B) $\angle A = 120°$, $b = 4$ and $c = 6$ cm

$$AD = \left(\frac{2bc}{b+c}\right)\cos\frac{A}{2}$$

$$\frac{48}{10} \cdot \frac{1}{2} = \frac{12}{5}$$

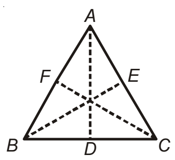


(C)



(D) Let AD, BE and CF are the median of triangle ABC

Let



$\therefore AD = 3$

$BE = 4$

$CF = 5$

8. Match the items of Column I with Column II.

Column-I	Column-II
(A) If in ΔABC, $\sin^2A + \sin^2B = \sin^2(A + B)$, then the triangle must be	(p) Right angled triangle
(B) In a ΔABC, $\frac{bc}{2\cos A} = b^2 + c^2 - 2bc\cos A$, then the ΔABC must be	(q) Equilateral
(C) In ΔABC, $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} = \sqrt{3}$, then Δ must be	(r) Isosceles
(D) In a ΔABC, the sides and the altitudes are in A.P. then Δ must be	(s) Obtuse angled
	(t) Acute angled

Sol. Answer A(p), B(r), C(q, r, t), D(q, r, t)

(A) In ΔABC

$\sin^2A + \sin^2B = \sin^2(A + B)$

$\Rightarrow \sin^2A + \sin^2B = \sin^2C$

$\Rightarrow$ By using Sine Rule

$a^2 + b^2 = c^2$

Triangle must be right angled triangle

(B) $\frac{bc}{2\cos A} = b^2 + c^2 - 2bc\cos A$

By using Cosine Rule

$$\frac{bc(2bc)}{2(b^2 + c^2 - a^2)} = b^2 + c^2 - 2bc\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\Rightarrow \frac{b^2c^2}{b^2 + c^2 - a^2} = a^2$$

$\Rightarrow b^2c^2 = a^2b^2 + a^2c^2 - a^4$

$\Rightarrow (b^2c^2 - a^2b^2) + a^4 - a^2c^2 = 0$

$\Rightarrow b^2(c^2 - a^2) + a^2(a^2 - c^2) = 0$

$\Rightarrow (a^2 - b^2)(a^2 - c^2) = 0$

$a^2 - b^2 = 0$ or $a^2 - c^2 = 0$

$a = b$ or $a = c$

(C) $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} = \sqrt{3}$ …(i)

$\because \tan\frac{A}{2}\cdot\tan\frac{B}{2} + \tan\frac{B}{2}\cdot\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2} = 1$

From equation (i)

By squaring both side

$$\Rightarrow \tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}+2\left(\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}-\tan\frac{C}{2}+\tan\frac{C}{2}\cdot\tan\frac{A}{2}\right)=3$$

$$\Rightarrow \tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}=1$$

Now, $\left(\tan\frac{A}{2}-\tan\frac{B}{2}\right)^2+\left(\tan\frac{B}{2}-\tan\frac{C}{2}\right)^2+\left(\tan\frac{C}{2}-\tan\frac{A}{2}\right)^2$

$$=2\left[\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}-\left(\tan\frac{A}{2}\cdot\tan\frac{B}{2}+\tan\frac{B}{2}\cdot\tan\frac{C}{2}+\tan\frac{C}{2}\cdot\tan\frac{A}{2}\right)\right]$$

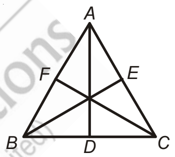
= 2[1 – 1] = 0

It is possible if

$$\tan\frac{A}{2}=\tan\frac{B}{2}=\tan\frac{C}{2}$$

$\angle A = \angle B = \angle C$

(D) Let a, b, c are the sides with usual rotations given a, b, c are in *A.P.*



$2b = a + c$ …(i)

Let P_1, P_2, P_3 are the length of altitudes *AP*, *BE*, *CE* respectively

Now, $\Delta=\frac{1}{2}aP_1=\frac{1}{2}bP_2=\frac{1}{2}cP_3$

Or $P_1=\frac{2\Delta}{a}, P_2=\frac{2\Delta}{b}, P_3=\frac{2\Delta}{c}$

And given P_1, P_2, P_3 are in *A.P.*

Then $2P_2 = P_1 + P_3$

$$2\left(\frac{2\Delta}{b}\right)=\frac{2\Delta}{a}+\frac{2\Delta}{b}$$

$$\frac{2}{b}=\frac{1}{a}+\frac{1}{b} \quad \text{…(ii)}$$

From (i) and (ii) a, b, c are in *A.P.* as well as in *H.P.* then it is possible if $a = b = c$

SECTION - E

Assertion-Reason Type Questions

1. STATEMENT-1 : If $\cos\theta=\frac{1}{7}$ and $\cos\phi=\frac{13}{14}$ where θ and ϕ both are acute angles, then the value of $\theta-\phi$ is $\frac{\pi}{3}$.

and

STATEMENT-2 : $\cos\frac{\pi}{3}=\frac{1}{2}$

Sol. Answer (4)

$\cos(\theta - \phi) = \cos\theta \cdot \cos\phi + \sin\theta \cdot \sin\phi$

$$= \frac{1}{7} \cdot \frac{13}{14} + \frac{\sqrt{48}}{7} \cdot \frac{\sqrt{27}}{14}$$

$$= \frac{13+36}{91} = \frac{49}{91}$$

2. STATEMENT-1 : The value of k for which $(\cos\theta - \sin\theta)^2 + k\sin\theta\cos\theta - 1 = 0$ is an identity is –2.

and

STATEMENT-2 : An identity in θ is satisfied by all real values of θ.

Sol. Answer (4)

$\sin^2\theta + \cos^2\theta - 2\sin\theta \cdot \cos\theta + k\sin\theta \cdot \cos\theta - 1$

$(k - 2)\sin\theta \cdot \cos\theta = 0 \Rightarrow k = 2$

Statement-1 is false, Statement-2 is true.

3. STATEMENT-1 : If θ is acute and $1 + \cos\theta = k$, then $\sin\frac{\theta}{2}$ is $\sqrt{\frac{2-k}{2}}$.

and

STATEMENT-2 : $2\sin^2\frac{x}{2} = 1 - \cos x$.

Sol. Answer (1)

$1 + \cos\theta = k$

$$1 + 1 - 2\sin^2\frac{\theta}{2} = k$$

$$\frac{2-k}{2} = \sin^2\frac{\theta}{2}$$

$$\Rightarrow \sin\frac{\theta}{2} = \sqrt{\frac{2-k}{2}}$$

Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation of (1).

4. STATEMENT-1 : The general solution of $2^{\sin x} + 2^{\cos x} = 2^{1-\frac{1}{\sqrt{2}}}$ is $n\pi + \frac{\pi}{4}$.

and

STATEMENT-2 : A.M. $\geq$ G.M.

Sol. Answer (1)

Statement-1

$$2^{\sin x} + 2^{\cos x} = 2^{1-\frac{1}{\sqrt{2}}}$$

$$\text{L.H.S.} = 2^{\sin x} + 2^{\cos x} \geq 2\sqrt{(2^{\sin x} 2^{\cos x})}$$

$$\geq 2\sqrt{2^{\sin x + \cos x}}$$

$\therefore$ LHS $\geq 2^{1-\frac{1}{\sqrt{2}}}$

RHS = $2^{1-\frac{1}{\sqrt{2}}}$

$\therefore$ LHS = RHS

$\therefore$ Equality holds if $2^{\sin x} = 2^{\cos x}$

$\Rightarrow \sin x = \cos x$

$\therefore \quad x = n\pi + \frac{\pi}{4}$

5. STATEMENT-1 : The equation $3\cos x + 4\sin x = 6$ has no solution.

and

STATEMENT-2 : Due to periodic nature of sine and cosine functions, the equation $3\cos x + 4\sin x = 6$ has infinitely many solutions.

Sol. Answer (3)

$-5 \leq 3\cos x + 4\sin x \leq 5$

$\therefore$ Given equation has no solution & statement 2 is false.

6. STATEMENT-1 : The number of real solution of the equation $\sin x = 4^x + 4^{-x}$ is zero.

and

STATEMENT-2 : $|\sin x| \leq 1 \; \forall x \in R$.

Sol. Answer (1)

Statement (2) : $|\sin x| \leq | \; \forall x \in R$ is true.

Statement (1) : $\sin x + 4^x + 4^{-x}$.

L.H.S. $\in$ [–1, 1] (as from statement-2)

R.H.S. $\Rightarrow$ No solution

$\therefore$ Both statements are correct and statement -2 is the correct explanation of statement -1.

7. STATEMENT-1 : $f(x) = \log_{\cos x}\sin x$ is well defined in $\left(0, \frac{\pi}{2}\right)$.

and

STATEMENT-2 : $\sin x$ and $\cos x$ are positive in $\left(0, \frac{\pi}{2}\right)$.

Sol. Answer (2)

Statement (2) is correct as both $\sin x$ and $\cos x$ are positive in I quadrant

Statement (1)

$f(x) = \log_{\cos x}(\sin x)$

$= \frac{\ln \sin x}{\ln \cos x}$

Is defined if $\cos x > 0$ and $\cos x \neq 1$

$\therefore$ It is defined in $\left(0, \frac{\pi}{2}\right)$ (as given)

$\therefore$ Statement (1) is true but (2) is not correct explanation for (1)

8. STATEMENT-1 : If in ΔABC, $3bc = (a - b + c)(a + b - c)$ then $A = 120°$.

and

STATEMENT-2 : $\cos 120° = -\frac{1}{2}$

Sol. Answer (2)

$3bc = (a - b + c)(a + b - c)$

$= [a - (b - c)][a + (b - c)]$

$\Rightarrow 3bc = a^2 - (b - c)^2$

$\Rightarrow 3bc = a^2 - b^2 - c^2 + 2bc$

$\Rightarrow b^2 + c^2 - a^2 = -bc$

$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = -\frac{1}{2}$

$\Rightarrow \cos A = -\frac{1}{2}$

$A = 120°$

9. $ABCD$ is a quadrilateral in which a circle is inscribed.

STATEMENT-1 : The length of the sides of the quadrilateral can be A.P.

and

STATEMENT-2 : The length of tangents from an external point to a circle are equal.

Sol. Answer (1)

If sum of opposite sides of a quadrilateral is equal, then and only then a circle can be inscribed in a quadrilateral

Statement-1 : Length of sides of quadrilateral can be in A.P. if common difference is zero.

Statement-2 : True.

10. STATEMENT-1 : In ΔABC, if $a < b \sin A$, then the triangle is possible.

and

STATEMENT-2 : In ΔABC $\frac{a}{\sin A} = \frac{b}{\sin B}$

Sol. Answer (4)

In ΔABC

$\frac{a}{\sin A} = \frac{b}{\sin A}$

$\sin A > \frac{a}{b}$

$\sin A > \frac{\sin A}{\sin B}$

$\boxed{\sin B > 1}$

11. Let *ABCD* be a cyclic quadrilateral then

STATEMENT-1 : $\sin A + \sin B + \sin C + \sin D = 0$

and

STATEMENT-2 : $\cos A + \cos B + \cos C + \cos D = 0$

Sol. Answer (4)

In cyclic quadrilateral *ABCD*

$A + C = 180$ and $B + D = 180$

$A = 180 - C$ and $B = 180 - D$

Then, $\sin A = \sin C$ and $\sin B = \sin D$

Or $\cos A = -\cos C$ and $\cos B = -\cos D$

Hence $\cos A + \cos B + \cos C + \cos D = 0$

12. STATEMENT-1 : In a triangle *ABC* if $\tan A : \tan B : \tan C = 1 : 2 : 3$, then $A = 45°$.

and

STATEMENT-2 : If $p : q : r = 1 : 2 : 3$, then $p = 1$

Sol. Answer (3)

Given,

$\tan A : \tan B : \tan C = 1 : 2 : 3$

$\tan A = k$

$\tan B = 2k$

$\tan C = 3k$

In ΔABC

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$6K = 6K^3$

$K \neq 0 \Rightarrow K = \pm 1\ K \neq -1$

$\Rightarrow K = 1$

$\tan A = 1 \Rightarrow A = 45°$

13. STATEMENT-1 : In an acute angled triangle minimum value of $\tan\alpha + \tan\beta + \tan\gamma$ is $3\sqrt{3}$.

and

STATEMENT-2 : If *a*, *b*, *c* are three positive real numbers then $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ also in a ΔABC, $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$.

Sol. Answer (1)

$\tan\alpha + \tan\beta + \tan\gamma \geq (\tan\alpha \cdot \tan\beta \cdot \tan\gamma)^{1/3}$

Equality will hold when $\tan\alpha = \tan\beta = \tan\gamma = \sqrt{5}$

SECTION - F

Integer Answer Type Questions

1. $x^2 - 2x + 2\cos^2\theta + \sin^2\theta = 0$, then maximum number of ordered pair (x, θ) such that $x \in R$, $\theta \in [0, 2\pi]$.

Sol. Answer (2)

$x^2 - 2x + 2\cos^2\theta + \sin^2\theta = 0,\ x,\ \theta \in R$

$x^2 - 2x + \cos^2\theta + 1 = 0$

$D = 4 - 4(\cos^2\theta + 1)$

$= -4\cos^2\theta$

$\therefore$ x is real $\Rightarrow \Delta \geq 0$

$\therefore$ $\cos\theta = 0$

In $[0, 2\pi]$, $\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$\therefore$ $x^2 - 2x + 1 = 0 \Rightarrow x = 1$

$\therefore$ $(x, 0) = \left(1, \dfrac{\pi}{2}\right), \left(1, \dfrac{3\pi}{2}\right)$

2. The smallest positive value of $\dfrac{8\sqrt{2}}{\pi}p$ for which $\sin(p\cos\theta) = \cos(p\sin\theta)$ has a solution in $[0, 2\pi]$ is

Sol. Answer (4)

$\sin(p\cos\theta) = \sin\left(\dfrac{\pi}{2} - p\sin\theta\right)$

$\Rightarrow$ $p\cos\theta = \dfrac{\pi}{2} - p\sin\theta$

$\Rightarrow$ $p(\sin\theta + \cos\theta) = \dfrac{\pi}{2}$

$\Rightarrow$ $\sin\theta + \cos\theta = \dfrac{\pi}{2p}$

$\Rightarrow$ $\sin\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{\pi}{2\sqrt{2}p}$

$\left|\dfrac{\pi}{2\sqrt{2}p}\right| \leq 1$

$\Rightarrow$ $p \geq \dfrac{\pi}{2\sqrt{2}}$

$\therefore$ $\dfrac{8\sqrt{2}}{\pi} \times \dfrac{\pi}{2\sqrt{2}} = 4$

$\Rightarrow$ Smallest value of p is 4.

3. If sec(θ – φ), secθ and sec(θ + φ) are in A.P. then $\cos^2\theta\sec^2\frac{\phi}{2}+3$, then is equal to

Sol. Answer (5)

sec(θ – φ), secθ, sec(θ + φ) ∈ A.P.

$\Rightarrow$ 2secθ = sec(θ – φ) + sec(θ + φ)

$$\Rightarrow \frac{2}{\cos\theta}=\frac{1}{\cos(\theta-\phi)}+\frac{1}{\cos(\theta+\phi)}$$

$$\Rightarrow \frac{2}{\cos\theta}=\frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta-\phi)\cos(\theta+\phi)}$$

$$\Rightarrow \frac{2}{\cos\theta}=\frac{2\sin\theta\cos\phi}{\cos^2\theta-\sin^2\phi}$$

$\Rightarrow$ $\cos^2\theta - \sin^2\phi = \cos^2\theta\sin^2\phi$

$\Rightarrow$ $\cos^2\theta(1 - \cos\phi) = \sin^2\phi$

$$\Rightarrow \sin^2\theta=\frac{\sin^2\phi}{1-\cos\phi}$$

$$\Rightarrow \cos^2\theta=\frac{4\sin^2\frac{\phi}{2}\cos^2\frac{\phi}{2}}{2\sin^2\frac{\phi}{2}}$$

$$\Rightarrow \cos^2\theta.\sec^2\frac{\phi}{2}=2$$

$$\Rightarrow \cos^2\theta.\sec^2\frac{\phi}{2}+3=2+3=5$$

4. The sum of maximum and minimum values of the expression $5\cos x+3\sin\left(\frac{\pi}{6}-x\right)+4$ is

Sol. Answer (8)

$$5\cos x+3\sin\left(\frac{\pi}{6}-x\right)+4$$

$$=5\cos x+3\left(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right)+4$$

$$=\frac{13}{2}\cos x-\frac{3\sqrt{3}}{2}\sin x+4$$

Maximum value $=\sqrt{\frac{169}{4}+\frac{27}{4}}+4=\sqrt{\frac{196}{4}}+4$ = 7 + 4 = 11

Minimum value = – 7 + 4 = – 3

Sum of maximum and minimum value = 11 – 3 =8

5. Let PQ and RS be two parallel chords of a given circle of radius 6 cm lying on the same side of the centre. If the chords subtends angles of 72° and 144° at the centre and the distance between the chords is d, then d^2 is equal to

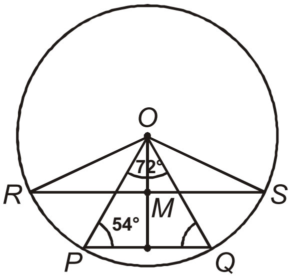
Sol. Answer (9)

$OM = 6\sin 18°$, $ON = 6\sin 54°$

$d = 6\sin 54° - 6\sin 18°$

$= 6(\cos 36° - \sin 18°)$

$= 6\left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right) = 6\left(\frac{1}{2}\right) = 3$



$\therefore\ d^2 = 9$

6. If $4\tan\theta\tan\phi = 3$, then $\frac{\cos(\theta-\phi)}{\cos(\theta+\phi)}$ is equal to

Sol. Answer (7)

$$\frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = k$$

$$\Rightarrow \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)} = \frac{k-1}{K+1}$$

$$\Rightarrow \frac{2\sin\theta\sin\phi}{2\cos\theta\cos\phi} = \frac{k-1}{k+1}$$

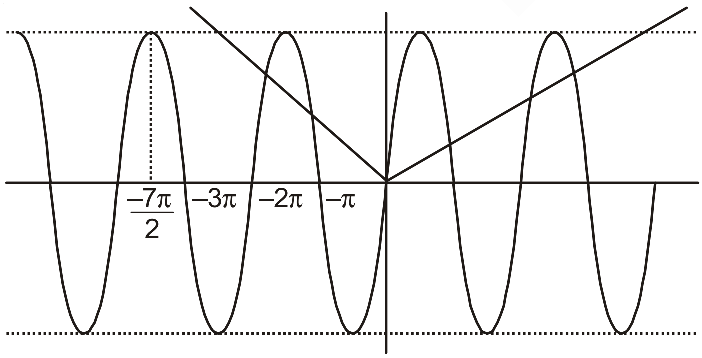
$$\Rightarrow \tan\theta\tan\phi = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{3}{4} = \frac{k-1}{k+1}$$

$\Rightarrow k = 7$

7. The number of solution of $10\sin x = |x|$ is ________.

Sol. Answer (6)



8. If $a, b \in [0, 2\pi]$ and equation $x^2 - 2x + 4 = 3\sin(ax + b)$ has at least one solution and the least positive value of $a + b$ is k, then $\frac{2k}{\pi}$ is equal to

Sol. Answer (1)

$x^2 - 2x + 4 = 3\sin(ax + b)$

$(x - 1)^2 + 3 = 3\sin(ax + b)$

LHS ≥ 3

RHS $\in [-3, 3]$

$n = 1$ is only solution.

$3 = 3\sin(a + b)$

$\Rightarrow a + b = \frac{\pi}{2}$

9. Let $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = \frac{-3}{2}$, then the value of $\cos\alpha + \cos\beta + \cos\gamma$ is ....

Sol. Answer (0)

$$\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$$

$2\cos(\alpha - \beta) + 2\cos(\beta - \gamma) + 2\cos(\gamma - \alpha) + 3 = 0$

$2\cos(\alpha - \beta) + 2\cos(\beta - \gamma) + 2\cos(\gamma - \alpha) + \cos^2\alpha + \sin^2\alpha + \cos^2\beta + \sin^2\beta + \cos^2\gamma + \sin^2\gamma = 0$

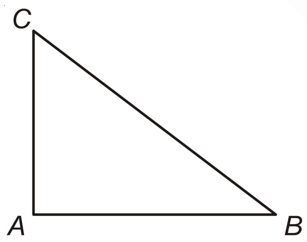
$(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

10. In right angled ΔABC, if $AB = AC$, then value of $\left[\frac{R}{r}\right]$ (value $[x]$ denote the greatest integer of x)

Sol. Answer (2)

In right angled ΔABC



$AB = AC$

$\therefore \angle A = 90°, \angle B = 45°, \angle C = 45°$

And $r = 4R\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

$$r = 4R\sin 45°\sin\frac{45°}{2}\cdot\sin\frac{45°}{2}$$

$$= \frac{4R}{\sqrt{2}}\sin^2\frac{45°}{2}$$

$$= \frac{4R}{\sqrt{2}}\left[\frac{1-\cos 45^\circ}{2}\right]$$

$$= \frac{2R}{\sqrt{2}}\left[1-\frac{1}{\sqrt{2}}\right]$$

$$\frac{r}{R} = \frac{2}{\sqrt{2}} - 1 = (\sqrt{2}-1)$$

$$\frac{R}{r} = \frac{1}{\sqrt{2}-1} = 2.414$$

$$\therefore \left[\frac{R}{r}\right] = 2$$

11. In a ΔABC, $bc = a$, $a = 2$, the value of $2\Delta R$ is equal to

Sol. Answer (2)

In a ΔABC

$bc = a$ and $a = 2$

$$\therefore \quad \Delta = \frac{abc}{4R}$$

$$\Rightarrow \Delta R = \frac{abc}{4}$$

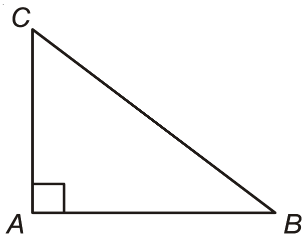
$$\Rightarrow \Delta R = \frac{a(a)}{4} = \frac{4}{4}$$

$\Rightarrow \Delta R = 1$

$\therefore 2\Delta R = 2$

12. In a triangle ABC with usual notation $b\text{cosec}B = a$, then value of $\left(\frac{b+c}{r+R}\right)$ is

Sol. Answer (2)



Given b cosec$B$ = a

$b = a \sin B$

$$\therefore \frac{a}{1} = \frac{b}{\sin B}$$

$\therefore \angle A = 90°$

$\therefore \Delta ABC$ is a right angled triangle then

$\therefore \ R = \frac{a}{2}$ and $r = (s-a)\tan\frac{A}{2}$

Now, $\frac{b+c}{r+R} = \frac{b+c}{(s-a)+\frac{a}{2}} = \frac{b+c}{s-\frac{a}{2}} = \frac{2(b+c)}{a+b+c-a}$

$\frac{b+c}{r+R} = 2$

13. In a triangle ABC, $2B = A + C$ and $b^2 = ac$, then $\frac{a(a+b+c)}{3bc}$ is

Sol. Answer (1)

Clearly $\angle B = 60°$

$\frac{1}{2} = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \ c^2 + a^2 - b^2 = ac$

But a, b, c are in G.P.

$\Rightarrow \ b^2 = ac$

$\Rightarrow \ c^2 + a^2 - 2ac = 0$

$\Rightarrow \ c = a = b$

Hence $\frac{a(a+b+c)}{3bc} = \frac{3a^2}{3a^2} = 1$

14. In an equilateral triangle with usual notations the value of $\frac{27r^2R}{r_1r_2r_3}$ is equal to

Sol. Answer (4)

In an equilateral triangle

$r = \frac{\Delta}{s}$

$\therefore$ Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$s = \frac{3a}{2}$

$r = \frac{\sqrt{3}a^2(2)}{4(3a)} = \frac{1}{2\sqrt{3}}a$

And $r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}a^2}{4\left(\frac{3a}{2}-a\right)} = \frac{\sqrt{3}\ a^2}{4\left(\frac{a}{2}\right)} = \frac{\sqrt{3}}{2}a = r_2 = r_3$

$$R = \frac{a}{\sin A} = \frac{2a}{\sqrt{3}}$$

Now $\frac{27r^2R}{r_1r_2r_3} = \frac{27a^2}{4\cdot 3}\cdot\frac{2a}{\sqrt{3}}\cdot\frac{8}{3\sqrt{3}a^3} = 4$

15. In a triangle *ABC*, $\frac{c\cos(A-\theta)+a\cos(C+\theta)}{b\cos\theta}$ is equal to

Sol. Answer (1)

In ΔABC

$\because$ $c\cos(A - \theta) + a\cos(C + \theta)$

$= c[\cos A.\cos\theta + \sin A.\sin\theta] + a[\cos C.\cos\theta - \sin C.\sin\theta]$

$= \cos\theta[c\cdot\cos A + a\cos C] + \sin\theta\,[c.\sin A - a.\sin C]$

By using projection formula and sine rule

$= \cos\theta\,(b) + 0$

$\therefore\ \frac{c\cos(A-\theta)+a\cos(C+\theta)}{b\cos\theta} = \frac{b\cos\theta}{b\cos\theta} = 1$

16. In a ΔABC if $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^n}$, then *n* is equal to ______.

Sol. Answer (2)

Given that

$$\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^n}$$

Taking L.H.S.

$$\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}$$

$$= \frac{s^2+(s-a)^2+(s-b)^2+(s-c)^2}{\Delta^2}$$

$$= \frac{(a+b+c)^2+(b+c-a)^2+(a-b+c)^2+(a+b-c)^2}{4\Delta^2}$$

$$= \frac{4(a^2+b^2+c^2)}{4\Delta^2} = \frac{a^2+b^2+c^2}{\Delta^2}$$

So $n = 2$

17. In ΔABC, medians *AD*, *CE* are drawn such that $AD = 5$, $\angle DAC = \frac{\pi}{8}$, $\angle ACE = \frac{\pi}{4}$. If area of ΔABC is *P* then

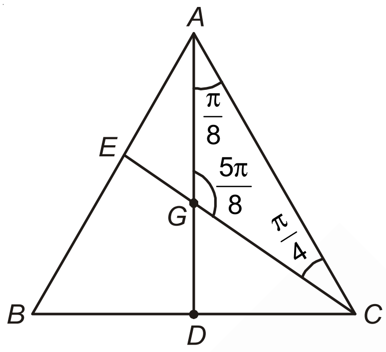
$\frac{3P}{5} =$

Sol. Answer (5)

From sine rule

$$\frac{AG}{\sin\frac{\pi}{4}} = \frac{CG}{\sin\frac{\pi}{8}}$$

$$AG = \frac{10}{3}$$



$$CG = \frac{10}{3}\sqrt{2} \times \sin\frac{\pi}{8}$$

Area of $\Delta ABC(P)$

= 3 Area Δ AGC

$$= 3 \times \frac{1}{2} \times AG \cdot CG \sin\left(\frac{5\pi}{8}\right)$$

$$= \frac{3}{2} \times \frac{10}{3} \times \frac{10\sqrt{2}}{3} \sin\frac{\pi}{8} \cdot \sin\left(\frac{5\pi}{8}\right)$$

$$= \frac{25}{3}\sqrt{2}\left(\cos\frac{3\pi}{4}\right)$$

$$P = \frac{25}{3}$$

Then, $\frac{3P}{5} = 5$

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