THREE DIMENSIONAL GEOMETRY

EXERCISE – 1: Basic Subjective Questions

Section–A (1 Mark Questions)

1. Find the distance of the plane $\vec{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin.
2. If the plane $2x-3y+6z-11=0$ makes an angle $\sin^{-1}\alpha$ with x-axis, then find the value of α.
3. If a plane passes through the points $(2, 0, 0), (0, 3, 0)$ and $(0, 0, 4)$, then find the equation of plane.
4. Find the vector equation of the line through the points $(3, 4, -7)$ and $(1, -1, 6)$.
5. Find the cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$.

Section–B (2 Marks Questions)

6. If the direction cosines of a line are k, k, and k, then find the value of k.
7. Find the sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$.
8. Find the unit vector normal to the plane $x+2y+3z-6=0$.
9. Find the intercepts made by the plane $2x-3y+5z+4=0$ on the coordinate axis.
10. Find the angle between the line $\vec{r}=(5\hat{i}-\hat{j}-4\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}-\hat{k})+5=0$.
11. Find the angle between the planes $\vec{r}.(2\hat{i}-3\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-\hat{j})=4$.
12. Find the equation of a line, which is parallel to $2\hat{i}+\hat{j}+3\hat{k}$ and which passes through the point $(5, -2, 4)$.
13. If the foot of perpendicular drawn from the origin to a plane is $(5, -3, -2)$, then find the equation of plane.

Section–C (3 Marks Questions)

14. Check whether the line $\vec{r}=2\hat{i}-3\hat{j}-\hat{k}+\lambda(\hat{i}-\hat{j}+2\hat{k})$ lies in the plane $\vec{r}.(3\hat{i}+\hat{j}-\hat{k})+2=0$.
15. Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at $60°$ *to OX* and at $45°$ *to OY* and $|\overrightarrow{OA}|=10$ units.
16. Find the Cartesian equation of the line which is parallel to the vector $3\hat{i}-2\hat{j}+6\hat{k}$ and which passes through the point (1, -2, 3).
17. Find the angle between the lines $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda(2\hat{i}+\hat{j}+2\hat{k})$ and $\vec{r}=(2\hat{j}-5\hat{k})+\mu(6\hat{i}+3\hat{j}+2\hat{k})$
18. Prove that the lines $x=py+q,\ z=ry+s$ and $x=p'y+q',\ z=r'y+s'$ are perpendicular, if $pp'+rr'+1=0$.
19. Find the equation of a plane which bisects the line joining the points $A(2,3,4)$ and $B(4,5,8)$ at right angles.
20. If the line drawn from the point $(-2,-1,-3)$ meets a plane at right angle at the point $(1,-3,3)$, then find the equation of the plane.
21. Find the equation of the plane passing through the points (2, 1, 0), (3, -2, -2) and (3, 1, 7)
22. Find the distance of a point (2, 4, -1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.
23. Find the equation of the line passing through the point (3, 0, 1) and parallel to the planes $x+2y=0$ and $3y-z=0$.
24. Find the distance between the point P(6, 5, 9) and the plane determined by the points A(3, – 1, 2), B(5, 2, 4) and C(– 1, – 1, 6).

Section–D (5 Marks Questions)

25. Find the angle between the lines whose direction cosines are given by the equation $l+m+n=0$ and $l^2+m^2-n^2=0$.
26. Find the foot of perpendicular from the point (2, 3, -8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also find the perpendicular distance from the given point to the line.
27. Find the length and the foot of perpendicular from the point $\left(1,\frac{3}{2},2\right)$ to the plane $2x-2y+4z+5=0$.
28. Find the equation of the plane through the points (2, 1, -1), (-1, 3, 4) and perpendicular to the plane $x-2y+4z=10$
29. Find the shortest distance between the lines given by $\vec{r}=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$ and $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$
30. Show that the straight lines whose direction cosines are given by $2l+2m-n=0$ and $mn+nl+lm=0$ are at right angles.

EXERCISE – 2: Basic Objective Questions

Section–A (Single Choice Questions)

1. For every point $P(x, y, z)$ on the xy -plane,

(a) $x=0$ (b) $y=0$

(c) $z=0$ (d) $x=y=z=0$

2. For every point $P(x, y, z)$ on the x-axis (except the origin),

(a) $x=0, y=0, z \neq 0$ (b) $x=0, z=0, y \neq 0$

(c) $y=0, z=0, x \neq 0$ (d) $x=y, z=0$

3. A rectangular parallelepiped is formed by planes drawn through the points (5, 7, 9) and (2, 3, 7) parallel to the coordinate planes. The length of an edge of this rectangular parallelepiped is

(a) 2 (b) 3

(c) 4 (d) All of these

4. A parallelepiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelepiped is

(a) 7 (b) $\sqrt{38}$

(c) $\sqrt{155}$ (d) None of these

5. The xy -plane divides the line segment joining the points $(-1,3,4)$ and $(2,-5,6)$.

(a) internally in the ratio 2 : 3

(b) externally in the ratio 2 : 3

(c) internally in the ratio 3 : 2

(d) externally in the ratio 3 : 2

6. If the x-coordinate of a point P on the join of $Q(2,2,1)$ and $R(5,1,-2)$ is 4, then its z- coordinate is

(a) 2 (b) 1

(c) –1 (d) –2

7. The distance of the point $P(a, b, c)$ from x-axis is

(a) $\sqrt{b^2+c^2}$ (b) $\sqrt{a^2+c^2}$

(c) $\sqrt{a^2+b^2}$ (d) None of these

8. Ratio in which the xy -plane divides the join of (1, 2, 3) and (4, 2, 1) is

(a) 3 : 4 internally (b) 3 : 1 externally

(c) 1 : 2 internally (d) 2 : 1 internally

9. If $P(3,2,-4), Q(5,4,-6)$ and $R(9,8,-10)$ are collinear, then R divides PQ in the ratio.

(a) 3 : 2 internally (b) 3 : 2 externally

(c) 2 : 1 internally (d) 2 : 1 externally

10. $A(3,2,0), B(5,3,2)$ and $C(-9,6,-3)$ are the vertices of a triangle ABC. If the bisector of $\angle BAC$ meets BC at D, then coordinates of D is

(a) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$ (b) $\left(-\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

(c) $\left(-\frac{19}{8}, -\frac{57}{16}, \frac{17}{16}\right)$ (d) none of these

11. If O is the origin and $OP=3$ with direction ratios proportional to $-1,2,-2,$ then the coordinates of P are

(a) $(-1,2-2)$ (b) $(1,2,2)$

(c) $\left(-\frac{1}{9}, \frac{2}{9}, -\frac{2}{9}\right)$ (d) $(3,6,-9)$

12. The angle between the two diagonals of a cube is

(a) 30° (b) 45°

(c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\cos^{-1}\left(\frac{1}{3}\right)$

13. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$ is equal to.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{8}{3}$

14. The shortest distance between the lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4} \text{ is}$$

(a) $\sqrt{30}$ (b) $2\sqrt{30}$

(c) $5\sqrt{30}$ (d) $3\sqrt{30}$

15. The plane $2x-(1+\lambda)y+3\lambda z=0$ passes through the intersection of the planes

(a) $2x-y=0$ and $y-3z=0$

(b) $2x+3z=0$ and $y=0$

(c) $2x-y+3z=0$ and $y-3z=0$

(d) None of these

16. The acute angle between the planes $2x-y+z=6$ and $x+y+2z=3$ is

(a) $45°$ (b) $60°$

(c) $30°$ (d) $75°$

17. The equation of the plane through the intersection of the planes $x+2y+3z=4$ and $2x+y-z=-5$ and perpendicular to the plane $5x+3y+6z+8=0$ is

(a) $7x-2y+3z+81=0$

(b) $23x+14y-9z+48=0$

(c) $51x+15y-50z+173=0$

(d) None of these

18. The distance between the planes

$2x+2y-z+2=0$ and $4x+4y-2z+5=0$ is.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) None of these

19. The image of the point $(1,3,4)$ in the plane $2x-y+z+3=0$ is.

(a) $(3,5,2)$ (b) $(-3,5,2)$

(c) $(3,5,-2)$ (d) $(3,-5,2)$

20. The equation of the plane containing the two lines

$$\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{3} \text{ and } \frac{x}{2}=\frac{y-2}{-1}=\frac{z+1}{3} \text{ is}$$

(a) $8x+y-5z-7=0$ (b) $8x+y+5z-7=0$

(c) $8x-y-5z-7=0$ (d) None of these

21. The equation of the plane $\vec{r}=\hat{i}-\hat{j}+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$ in scalar product form is

(a) $\vec{r}.(5\hat{i}-2\hat{j}-3\hat{k})=7$ (b) $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=7$

(c) $\vec{r}.(5\hat{i}-2\hat{j}+3\hat{k})=7$ (d) None of these

22. The distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$ is

(a) 9 (b) 13

(c) 17 (d) None of these

Section–B (Case Study Questions)

Case Study–1

23. A cricket match is organized between two clubs *A* and *B* for which a team from each club is chosen. Remaining players of club *A* and club *B* are respectively sitting on the plane represented by the equation $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})=3$ and $\vec{r}.(\hat{i}+3\hat{j}+2\hat{k})=8$, to cheer the team of their own clubs.

Based on the above, answer the following:

(i) The Cartesian equation of the plane on which players of club *A* are seated is.

(a) $2x-y+z=3$ (b) $2x-y+2z=3$

(c) $2x-y+z=-3$ (d) $x-y+z=3$

(ii) The magnitude of the normal to the plane on which players of club B are seated, is.

(a) $\sqrt{15}$ (b) $\sqrt{14}$

(c) $\sqrt{17}$ (d) $\sqrt{20}$

(iii) The intercept form of the equation of the plane on which players of club B are seated is.

(a) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{8/3}=1$ (b) $\frac{x}{5}+\frac{y}{8/3}+\frac{z}{8/3}=1$

(c) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{4}=1$ (d) $\frac{x}{8}+\frac{y}{7}+\frac{z}{2}=1$

(iv) The distance of the plane, on which players of club B are seated, from the origin is.

(a) $\frac{8}{\sqrt{14}}$ units (b) $\frac{6}{\sqrt{14}}$ units

(c) $\frac{7}{\sqrt{14}}$ units (d) $\frac{9}{\sqrt{14}}$ units

Case Study–2

24. The Indian coast guard, while patrolling saw a suspicious boat with people. They were nowhere looking like fishermen. The coast guard were closely observing the movement of the boat for an opportunity to seize the boat. They observed that the boat is moving along a planar surface. At and instant of time, the coordinates of the position of the coast guard helicopter and the boat is (1, 3, 5) and (2, 5, 3) respectively.

Based on the above, answer the following questions:

(i) If the line joining of the helicopter and the boat is perpendicular to the plane in which the boat moves, then the equation of the plane is.

(a) $-x+2y-2z=6$ (b) $x+2y+2z=6$

(c) $x+2y-2z=6$ (d) $x-2y-2z=6$

(ii) If the coast guard decide to shoot the boat at that given instant of time, then what is the distance (in meters) that the bullet has to travel.

(a) 5m (b) 3m

(c) 6m (d) 4m

(iii) If the coast guard decides to shoot the boat at that given instant of time, when the speed of bullet is 36 m/sec, then what is the time taken for the bullet to travel and hit the boat.

(a) $\frac{1}{8}$ seconds (b) $\frac{1}{14}$ seconds

(c) $\frac{1}{10}$ seconds (d) $\frac{1}{12}$ seconds

(iv) At that given instant of time, the equation of line passing through the positions of the helicopter and boat is.

(a) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-5}{-2}$

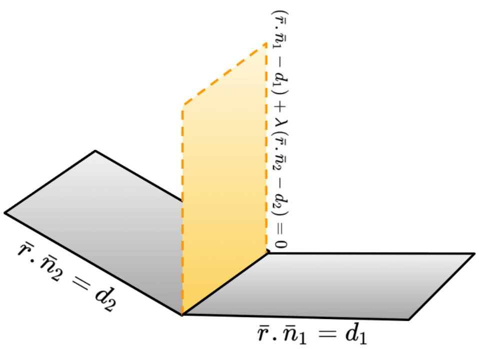
(b) $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-5}{-2}$

(c) $\frac{x+1}{-2}=\frac{y-3}{-1}=\frac{z-5}{-2}$

(d) $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+5}{2}$

Case Study–3

25. $P_1: x+3y-z=0$ and $P_2: y+2z=0$ are two intersecting planes. P_3 is a plane passing through the point $(2,1,-1)$ and through the line of intersection of P_1 and P_2.



Based on the above information, answer the following questions.

(i) The angle between P_1 and P_2 is.

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{55}}\right)$

(c) $\cos^{-1}\left(\frac{2}{\sqrt{11}}\right)$ (d) $\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)$

(ii) Equation of P_3 is __________.

(a) $4x+y-2z=10$ (b) $x+y-2z=3$

(c) $x+9y+11z=0$ (d) $4x-y+z=0$

(iii) Equation of plane parallel to P_3 and passing through (1, 2, 3) is ____________.

(a) $x+9y+11z-52=0$ (b) $x+9y+11z-20=0$

(c) $4x+y-2z+10=0$ (d) $4x+y-2z+1=0$

(iv) Distance of P_3 from origin is _____ units.

(a) 0 (b) 1

(c) $\frac{1}{\sqrt{5}}$ (d) $\frac{11}{10}$

Section–C (Assertion & Reason Type Questions)

26. **Assertion:** Lines $\vec{r}=\hat{i}-\hat{j}+\lambda\left(\hat{i}+\hat{j}-\hat{k}\right)$ and $\vec{r}=2\hat{i}-\hat{j}+\mu\left(\hat{i}+\hat{j}-\hat{k}\right)$ do not intersect.

Reason: Skew lines never intersect.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

27. **Assertion:** Lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda\left(3\hat{i}-\hat{j}\right)$ and $\vec{r}=4\hat{i}-\hat{j}+\mu\left(2\hat{i}+3\hat{k}\right)$ intersect.

Reason: If $\vec{b}\times\vec{d}=\vec{0}$, then lines $\vec{r}=\vec{a}+\lambda\vec{b}$ and $\vec{r}=\vec{c}+\lambda\vec{d}$ do not intersect.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

28. **Assertion:** The equation of two straight lines are $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{-3}$ and $\frac{x-2}{1}=\frac{y-1}{-3}=\frac{z+3}{2}$ are co planar

Reason: Two lines are said to be coplanar when they both lie on the same plane in a three-dimensional space

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

29. **Assertion:** Line $\frac{x-1}{1}=\frac{y-0}{2}=\frac{z+2}{-1}$ lies in the plane $2x-3y-4z-10=0$

Reason: If line $\vec{r}=\vec{a}+\lambda\vec{b}$ lies in the plane $\vec{r}.\vec{c}=n$ (where n is scalar), then $\vec{b}.\vec{c}=0$.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

30. **Assertion:** Let θ be the angle between the line $\frac{x-2}{2}=\frac{y-1}{-3}=\frac{z+2}{-2}$ and the plane $x+y-z=5$. Then $\theta=\sin^{-1}\left(\frac{1}{\sqrt{51}}\right)$.

Reason: The angle between a straight line and a plane is the complement of the angle between the line and the normal to the plane.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

EXERCISE – 3: Previous Year Questions

1. Write the distance of a point $P(a,b,c)$ from x-axis. **(Delhi 2014C)**

2. Find the distance of a point $(2,3,4)$ from the x-axis. **(Delhi 2010C)**

3. If a line makes angles 90°, 60° and θ with x, y and z-axis respectively, where θ is acute, then find θ. **(Delhi 2015)**

4. If a line makes angles α, β, γ with the positive direction of coordinate axes, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ **(Delhi 2015C)**

5. If a line has direction ratios $2, -1, -2,$ then what are its direction cosines? **(Delhi 2012)**

6. Write the direction cosines of the line joining the points $(1,0,0)$ and $(0,1,1)$. **(AI 2011)**

7. Write the direction cosines of a line equally inclined to the three coordinate axes. **(AI 2009)**

8. The equation of a line are $5x-3 = 15y+7 = 3-10z$. Write the direction cosines of the line. **(AI 2015)**

9. If the Cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line. **(AI 2014)**

10. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. **(Delhi 2013 C)**

11. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$. **(Foreign 2016)**

12. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection. **(Delhi 2014)**

13. Find the angles between the lines $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{j} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. **(Foreign 2014, AI 2008C)**

14. Write the equation of the straight line through the point (α, β, γ) and parallel to z-axis. **(AI 2014)**

15. Find the Cartesian equation of the line which passes through the point (-2, -4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. **(Delhi 2013)**

16. Write the vector equation of a line passing through the point $(1, -1, 2)$ and parallel to the line whose equation is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$. **(AI 2013C)**

17. If the equation of a line AB is $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$, find the direction cosines of a line parallel to AB. **(Delhi 2012C)**

18. The equation of a line is given by $\frac{4-x}{3} = \frac{y+3}{3} = \frac{z+2}{6}$. Write the direction cosines of a line parallel to the above line. **(AI 2012C)**

19. Find the equation of the line passing through the point $(-1, 3, -2)$ and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. **(Delhi 2012)**

20. Find the angle between the following pair of lines $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$. Also, check whether the lines are parallel or perpendicular. **(Delhi 2011)**

21. Find the equation of the perpendicular drawn from the point $(2, 4, -1)$ to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. **(AI 2019)**

22. Find the equation of a line, which passes through the point $(1, 2, 3)$ and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$. **(Delhi 2017)**

23. Find the distance between the lines l_1 and l_2 given by
$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$,
$l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$. **(Foreign 2014)**

24. Find the shortest distance between the two lines whose vector equations are

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and

$\vec{r}=\left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$. **(Delhi 2014C)**

25. Find the shortest distance between the following pair of lines: $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4};\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$ **(AI 2012C)**

26. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$ **(Delhi 2016)**

27. Find the sum of the intercepts cut off by the plane $2x+y-z=5$, on the coordinate axes. **(Foreign 2015)**

28. Find the unit vector perpendicular to the plane ABC where the position vectors of A,B and C are $2\hat{i}-\hat{j}+\hat{k},\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}+3\hat{k}$ respectively. **(AI 2014C)**

29. Find the vector and Cartesian equations of the plane passing through the line of a intersection of the planes $\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}\cdot(2\hat{i}+5\hat{j}-3\hat{k})=9$ such that the intercepts made by the plane on x-axis and z-axis are equal. **(AI 2015C)**

30. Find the vector equation of the plane passing through three points with position vectors $\hat{i}+\hat{j}-2\hat{k},2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ .Also find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$. **(Delhi 2013)**

31. Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ are coplanar. **(Delhi 2014)**

32. If lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines. **(Delhi 2015)**

33. Show that the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ are coplanar. **(AI 2013C)**

34. If the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar, find the equation of plane containing these lines. **(AI 2013C)**

35. If the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines. **(AI 2012)**

36. Write the vector equation of the line passing though $(1, 2, 3)$ and perpendicular to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$ **(AI 2015C)**

37. Write the vector equation of the plane, passing through the point (a, b, c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$ **(Delhi 2014)**

38. Find the vector equation of the plane through the points $(2,1,-1)$ and $(-1,3,4)$ and perpendicular to the plane $x-2y+4z=10$. **(AI 2013)**

39. Find the equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to each of the planes $2x+3y-2z=5$ and $x+2y-3z=8$. **(Delhi 2009C)**

40. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$. **(Delhi 2013, 2011)**

41. Find the length of the perpendicular drawn from the origin to the plane $2x-3y+6z+21=0$ **(AI 2013)**

42. Find the distance of the plane $3x-4y+12z=3$ from the origin. **(AI 2012)**

43. Find the distance of the point $P(3, 4, 4)$ from the point, where the line joining the points $A(3,-4,-5)$ and $B(2,-3,1)$ intersects the plane $2x+y+z=7$. **(AI 2015)**

44. Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$. **(AI 2014C)**

45. Find the position vector of the foot of perpendicular from the point P with position vector $2\hat{i}+3\hat{j}+4\hat{k}$ to the plane. **(AI 2016)**

46. Find the distance of the point $(1, -2, 3)$ from the plane $x-y+z=5$ measured parallel to the line whose direction cosines are proportional to $2, 3, -6$. **(Foreign 2015)**

47. Find the equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to both the planes $2x+3y-2z=5$ and $x+2y-3z=8$. Hence find the distance of point $P(-2, 5, 5)$ from the plane obtained above.

(Foreign 2014)

48. Find the length and the foot of the perpendicular from the point $P(7, 14, 5)$ to the plane $2x+4y-z=2$. Also find the image of point P in the plane.

(AI 2012)

49. Find the equation of the plane passing through the point $(3, -3, 1)$ and perpendicular to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$. Also find the coordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane.

(Delhi 2012C)

50. Find the distance of the point $(3, 4, 5)$ from the plane $x+y+z=2$ measured parallel to the line $2x=y=z$. **(Delhi 2012C)**

51. Find the coordinates of the foot of the perpendicular, the equation of the perpendicular and the perpendicular distance of the point $P(3, 2, 1)$ from the plane $2z-y+z+1=0$. Also find the image of the point Pin the plane. **(AI 2012C)**

52. Find the vector equation of the plane that contains the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$.Also find the length of perpendicular drawn from the point $(2, 1, 4)$ to the plane thus obtained.

(AI 2012C)

53. Find the distance of the point $(2, 4, 4)$ from the line measured parallel to the $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$ plane $3x+2y+2z-5=0$. **(AI 2019C)**

54. Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the *XZ* plane. Also find the angle which this line makes with the *XZ* plane.

(AI 2016)

55. Find the equation of the plane passing through the points $(-1, 2, 0)$, $(2, 2, -1)$ and parallel to the $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$ **(AI 2015)**

56. Find the Cartesian equation of the plane passing through the points $A(0, 0, 0)$ and $B(3, -1, 2)$ and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$. **(Delhi 2014)**

57. Find the equation of the plane through the $P(1, 1, 1)$ and containing the line $\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}-5\hat{k})$.
Also, show that the plane contains the line $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$. **(AI 2011)**

58. If a line makes angles $90°, 135°, 45°$ with the x, y and z axes respectively, find its direction cosines.

(Delhi 2019)

59. Find the vector equation of the line which passes through the point $(3,4,5)$ and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$. **(Delhi 2019)**

60. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not. **(Delhi 2019)**

61. Find the shortest distance between the lines $\vec{r}=\left(4\hat{i}-\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and $\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(2\hat{i}+4\hat{j}-5\hat{k}\right)$. **(2018)**

62. Find the vector equation of a line passing through the point $(2,3,2)$ and parallel to the line $\vec{r}=\left(-2\hat{i}+3\hat{j}\right)+\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$. Also, find the distance between these two lines. **(AI 2019)**

63. Find the acute angle between the planes $\vec{r}\cdot\left(\hat{i}-2\hat{j}-2\hat{k}\right)=1$ and $\vec{r}\cdot\left(3\hat{i}-6\hat{j}+2\hat{k}\right)=0$.

(AI 2019)

64. Find the vector and cartesian equations of the plane passing through the points $(2,2,-1),(3,4,2)$ and $(7,0,6)$. Also find the vector equation of a plane passing through $(4,3,1)$ and parallel to the plane obtained above. **(Delhi 2019)**

65. Find the vector equation of the plane determined by the points $A(3,-1,2), B(5,2,4)$ and $C(-1,-1,6)$. Hence, find the distance of the plane, thus obtained, from the origin. **(AI 2019)**

66. Find the vector equation of the plane that contains the line $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and the point $(-1,3,-4)$. Also find the length of the perpendicular drawn from the point $(2,1,4)$ to the plane, thus obtained. **(Delhi 2019)**

67. Find the coordinates of the foot of the perpendicular Q drawn from $P(3,2,1)$ to the plane $2x-y+z+1=0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.

(AI 2019)

68. Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

(Delhi 2018)

69. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is.

(AI 2020, Delhi 2020)

70. The vector equation of a line which through the points $(3,4,-7)$ and $(1,-1,6)$ is ________.

(AI, 2020, Delhi 2020)

71. Find the vector and cartesian equations of the line which perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{1} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point $(1,1,1)$. Also find the angle between the given lines.

(AI 2020, Delhi 2020)

72. Find the coordinates of the point where the line through $(-1,1,-8)$ and $(5,-2,10)$ crosses the *ZX*-plane.

(AI 2020)

73. Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect. Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

(AI 2020)

74. The image of the point $(2,-1,4)$ in the *YZ*-plane is ________.

(AI 2020)

75. The distance of the origin $(0,0,0)$ from the plane $-2x+6y-3z=-7$ is ________.

(AI 2020)

Answer Key

EXERCISE-1:

Basic Subjective Questions

1. 1 **2.** $\frac{2}{7}$

3. $6x+4y+3z=12$

4. $\vec{r}=3\hat{i}+4\hat{j}-7\hat{k}+\lambda\left(-2\hat{i}-5\hat{j}+13\hat{k}\right)$

5. $x+y-z=2$ **6.** $\pm\frac{1}{\sqrt{3}}$

7. $\frac{1}{5\sqrt{2}}$ **8.** $\frac{\hat{i}}{\sqrt{14}}+\frac{2\hat{j}}{\sqrt{14}}+\frac{3\hat{k}}{\sqrt{14}}$

9. $-2,\frac{4}{3}$ and $-\frac{4}{5}$ **10.** $\sin^{-1}\frac{9}{2\sqrt{39}}$

11. $\cos^{-1}\left(\frac{5}{2\sqrt{7}}\right)$ **12.** $\frac{x-5}{2}=\frac{y+2}{1}=\frac{z-4}{3}$

13. $\vec{r}\cdot\left(5\hat{i}-3\hat{j}-2\hat{k}\right)=38$

15. $5\hat{i}+5\sqrt{2}\hat{j}+5\hat{k}$ **16.** $\frac{x-1}{3}=\frac{y+2}{-2}=\frac{z-3}{6}$

17. $\cos^{-1}\left(\frac{19}{21}\right)$ **19.** $x+y+2z=19$

20. $3x-2y+6z-27=0$

21. $7x+3y-z=17$

22. 7 units **23.** $\frac{x-3}{-2}=\frac{y}{1}=\frac{z-1}{3}=\lambda$

24. $\frac{3\sqrt{34}}{17}$ units **25.** $\frac{\pi}{3}$

26. $(2,6,-2)$; $3\sqrt{5}$ units

27. $\left(0,\frac{5}{2},0\right)$; $\sqrt{6}$ units

28. $18x+17y+4z=49$

29. 14 units

EXERCISE-2:

Basic Objective Questions

1. (c)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (b)
6. (c)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (a)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (a)
16. (b)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (a)
21. (a)	**22.** (b)			

23. (i) (a) **(ii)** (b) **(iii)** (c) **(iv)** (a)

24. (i) (c) **(ii)** (b) **(iii)** (d) **(iv)** (a)

25. (i) (b) **(ii)** (c) **(iii)** (a) **(iv)** (a)

26. (b)	27. (b)	28. (a)	29. (b)	30. (a)

EXERCISE-3:

Previous Year Questions

1. $\sqrt{b^2+c^2}$ units

2. 5 units

3. $\frac{\pi}{6}$

4. 2

5. $\frac{2}{3},\frac{-1}{3},\frac{-2}{3}$

6. $\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

7. $\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}$

8. $\frac{6}{7},\frac{2}{7},-\frac{3}{7}$

9. $\vec{r}=(3\hat{i}-4\hat{j}+3\hat{k})+\lambda(-5\hat{i}+7\hat{j}+2\hat{k})$

10. $\frac{2}{7},\frac{-6}{7},\frac{3}{7}$

13. $\cos^{-1}\left(\frac{19}{21}\right)$

14. $\frac{x-\alpha}{0}=\frac{y-\beta}{0}=\frac{z-\gamma}{1}$

15. $\frac{x+2}{3}=\frac{y+4}{-5}=\frac{z+5}{6}$

16. $\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

17. $\frac{3}{7},\frac{-2}{7},\frac{6}{7}$

18. $\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}$

19. $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$

20. $\frac{\pi}{2}$

21. $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$

22. $\frac{x-1}{-2}=\frac{y-2}{14}=\frac{z-3}{3}$

23. $\frac{\sqrt{293}}{7}$ units.

24. $\frac{3}{\sqrt{19}}$ units

25. $\frac{1}{\sqrt{6}}$ units

26. $\vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})=35$

27. $\frac{5}{2}$

28. $\frac{1}{\sqrt{14}}\left(3\hat{i}+2\hat{j}-\hat{k}\right)$

29. $\vec{r}\cdot\left(3\hat{j}\right)=2$

30. $\vec{r}.\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$, $(1,\ 1,-2)$

32. $k=\frac{9}{2}$; $5x-2y-z-6=0$

34. $x-2y+z=0$

35. $k=2$; $-22x+19y+5z=31.$

36. $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}+2\hat{j}-5\hat{k})$

37. $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$

38. $\vec{r}\cdot(18\hat{i}+17\hat{j}+4\hat{k})=49$

39. $5x-4y-z-7=0$

40. $\vec{r}\cdot(33\hat{i}+45\hat{j}+50\hat{k})=41$

41. 3 units

42. $\frac{3}{13}$ units

43. 7 units

44. $(-3,\ 5,\ 2)$

45. $3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}$

46. 1 unit

47. $5x-4y-z-7=0$; $\sqrt{42}$ units

48. $(1,\ 2, 8)$; $3\sqrt{21}$ units ; (-5,-10,11)

49. $x+5y-6z+18=0$; $\left(-\frac{9}{31},\frac{-45}{31},\frac{54}{31}\right)$;

$\frac{x-0}{1}=\frac{y-0}{5}=\frac{z-0}{-6}$; $6\sqrt{\frac{2}{31}}$ units

50. 6 units

51. $(1,\ 3,\ 0)$; $\frac{6}{\sqrt{6}}$ units ; $(-1, 4, -1)$

52. $\vec{r}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)=0$; $\sqrt{3}$ units

53. $\sqrt{33}$ units

54. $\left(\frac{17}{3},0,\frac{23}{3}\right)$; $\sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$

55. $x+2y+3z=3$

56. $x-19y-11z=0$

57. $\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k})=0$

58. $0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$

59. $\vec{r}=\left(3\hat{i}+4\hat{j}+5\hat{k}\right)+\lambda\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$

60. $\lambda=7$

61. $\frac{6}{\sqrt{5}}$ units

62. $\vec{r}=\left(2\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$; $\frac{20}{7}$ units

63. $\cos^{-1}\left(\frac{11}{21}\right)$

64. $5x+2y-3z-17=0$; $\vec{r}\cdot\left(5\hat{i}+2\hat{j}-3\hat{k}\right)=17$

$\vec{r}\cdot\left(5\hat{i}+2\hat{j}-3\hat{k}\right)=23$

65. $12x-16y+12z=76$; $\frac{19}{\sqrt{34}}$ units

66. $\vec{r}\cdot\left(-5\hat{i}+5\hat{j}+5\hat{k}\right)=0$; $\sqrt{3}$ units

67. $(1,3,0)$; $\sqrt{6}$ units ;(-1, 4, -1)

68. 13 units

69. $k=\frac{-2}{3}$

70. $\vec{r}=(3-2\lambda)\hat{i}+(4-5\lambda)\hat{j}+(-7+13\lambda)\hat{k}$

71. $\frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1}$; $\cos^{-1}\frac{24}{\sqrt{21}\sqrt{29}}$

72. (1, 0, -2)

73. $(1,-1,2)$; $2x-y+z=5$

74. $(-2,-1,4)$

75. 1 unit