DPP – 06
CLASS –12th
TOPIC – PRINCIPAL VALUE OF ITFs

Q.1 Evaluate:

$$\cot\left(\sin^{-1}\frac{3}{4}+\sec^{-1}\frac{4}{3}\right)$$

Q.2 Evaluate:

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right) \text{ for } x<0$$

Q.3 Evaluate:

$$\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right) \text{ for } x>0$$

Q.4 Evaluate:

$\cot(\tan^{-1}a + \cot^{-1}a)$

Q.5 Evaluate:

$$\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right), |x|\geq 1$$

Q.6 If $\cos^{-1}x+\cos^{-1}y=\frac{\pi}{4}$, find the value of $\sin^{-1}x+\sin^{-1}y$.

Q.7 If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{3}$ and $\cos^{-1}x - \cos^{-1}y = \frac{\pi}{6}$,

Find the value of x and y.

Q.8 If $\cot\left(\cos^{-1}\frac{3}{5}+\sin^{-1}x\right)=0$, find the values of x.

Q.9 If $\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2=\frac{17x^2}{36}$, find x.

Q.10 Solve:

$$\sin\left\{\sin^{-1}\frac{1}{5}+\cos^{-1}x\right\}=1$$

Q.11 Solve:

$$\sin^{-1}x=\frac{\pi}{6}+\cos^{-1}x$$

Q.12 Solve:

$4\sin^{-x} = \pi - \cos^{-1}x$

Q.13 $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$

Q.14 Solve:

$5\tan^{-1}x + 3\cot^{-1}x = 2\pi$

SOLUTION

(MATHS) INVERSE TRIGONOMETRIC FUNCTIONS

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Sol.1 $\cot\left(\sin^{-1}\frac{3}{4}+\sec^{-1}\frac{4}{3}\right)$

$=\cot\left(\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right)$

$=\cot\left(\frac{\pi}{2}\right)$

$=0$

Sol.2 $\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$

$=\sin\left(-\pi+\tan^{-1}\left(\frac{x+\frac{1}{x}}{1-\frac{x}{x}}\right)\right)$

$=\sin\left(-x+\tan^{-1}(\infty)\right)$

$=\sin\left(-\pi+\frac{\pi}{2}\right)$

$=-1$

Sol.3 $\sin\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$

$=\sin\left(\pi+\tan^{-1}\left(\frac{x+\frac{1}{x}}{1-\frac{x}{x}}\right)\right)$

$=\sin\left(\pi+\tan^{-1}(\infty)\right)$

$=\sin\left(\pi+\frac{\pi}{2}\right)$

$=-\sin\left(\frac{\pi}{2}\right)$

$=-1$

Sol.4 $\cot(\tan^{-1}a+\cot^{-1}a)$

$c=\cot\left(\frac{x}{2}\right)$

$=0$

Sol.5 $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right)$

$=\cos\left(\frac{\pi}{2}\right)$

$=0$

Sol.6 $[\pi/2 - \sin^{-1}x] + [\pi/2 - \sin^{-1}y] = \pi/4$

$\sin^{-1}x + \sin^{-1}y = \pi - \pi/4$

$\sin^{-1}x + \sin^{-1}y = 3\pi/4$

Sol.7 $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{3} \ldots\ldots\ldots\ldots (i)$

$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{6} \ldots\ldots\ldots\ldots (ii)$

On adding both the equation

$\pi/2 + \sin^{-1}y - \cos^{-1}y = \pi/2$

$[\pi/2 - \cos^{-1}y] - \cos^{-1}y = 0$

$\cos^{-1}y = \pi/4$

$y = 1/\sqrt{2}$

on putting $y = 1/\sqrt{2}$ in 2nd equation

$\cos^{-1}x - \pi/4 = \pi/6$

$\cos^{-1}x = \pi/4 + \pi/6$

$x = \cos(\pi/4 + \pi/6)$

$x = \cos(\pi/4 + \pi/6)$

$x = \cos(\pi/4)\cos(\pi/6) - \sin(\pi/4)\sin(\pi/6)$

$x = (\sqrt{3}-1)/2\sqrt{2}$

Sol.8 $\cot(z) = 0$ means $z = \pi/2, 3\pi/2, 5\pi/2 \ldots\ldots\ldots\ldots$

$\cos^{-1}(3/5) + \sin^{-1}x = n\pi + \pi/2$

$\sin^{-1}x\ n\pi + \pi/2 - \cos^{-1}(3/5)$

$\sin^{-1}x = n\pi + \sin^{-1}(3/5)$

$x = \sin(n\pi + \sin^{-1}(3/5)) = (-1)^n \sin(\sin^{-1}(3/5))$

$x = (-1)^n\, 3/5$

Sol.9 $[\pi/2 - \cos^{-1}x]^2 + (\cos^{-1}x)^2 = 17\pi^2/36$

$\pi^2/4 - \pi\cos^{-1}x + 2(\cos^{-1}x)^2 = 17\pi^2/36$

Let, $\cos^{-1}x - u$

$2u^2 - \pi u + \pi^2/4 - 17\pi^2/36 = 0$

$2u^2 - \pi u - 2\pi^2/9 = 0$

$18u^2 - 9\pi u - 2\pi^2 = 0$

On factorizing

$18u^2 - 12\pi u + 3\pi u - 2\pi^2 = 0$

$6u(3u - 2\pi) + \pi(3u - 2\pi) = 0$

$(3u - 2\pi)(6u + \pi) = 0$

$u = -\pi/6,\ 2\pi/3$

i.e. $\cos^{-1}x = -\pi/6, 2\pi/3$

but range of $\cos^{-1}x$ is $[0, \pi]$

$x = \cos(\pi/2 + \pi/6)$

$x = -1/2$

Sol.10 $\sin^{-1}(1/5) + [\pi/2 - \sin^{-1}x] = \sin^{-1}1$

$\sin^{-1}(1/5) + \pi/2 - \sin^{-1}x = \pi/2$

$\sin^{-1}(1/5) - \sin^{-1}x = 0$

$x = 1/5$

Sol.11 $\pi/2 - \cos^{-1}x = \pi/6 + \cos^{-1}x$

$\pi/3 = 2\cos^{-1}x$

$\cos^{-1}x = \pi/6$

$x = \sqrt{3}/2$

Sol.12 $4\sin^{-1}x + \cos^{-1}x = \pi$

$3\sin^{-1}x + \sin^{-1}x + \cos^{1}x = \pi$

$3\sin^{-1}x = \pi/2$ $[\sin^{-1}x + \cos^{-1}x = \pi/2]$

$\sin^{-1}x = \pi/6$

$x = \sin\pi/6 = 0.5$

Sol.13 $\text{Tan}^{-1}x + \cos^{-1}x = \pi/2$ so the above equation reduces to

$\cot^{-1}x = 2\pi/3 - \pi/2 = \pi/6$

$x = \cot\pi/6 = \sqrt{3}$

Sol.14 $2\tan^{-1}x + 3(\pi/2) = 2\pi$

$2\tan^{-1}x = 2\pi - 3\pi/2 = \pi/3$

$\tan^{-1}x = \pi/6$

$x = \tan\pi/6 = 1/\sqrt{3}$