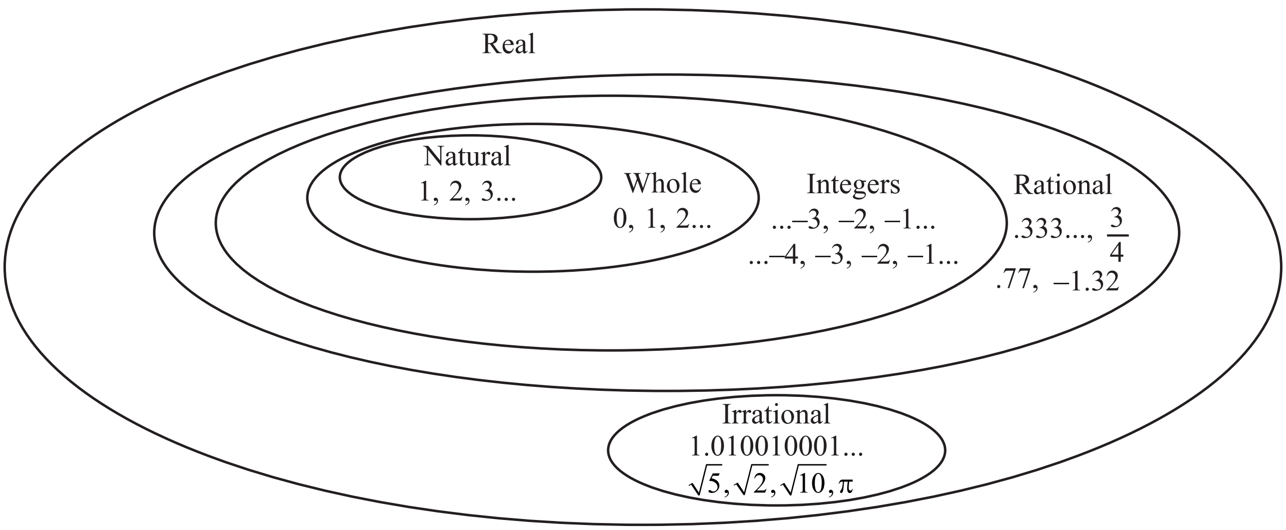
CHAPTER

1

REAL NUMBERS

Number System

You must have studied different types of numbers in your earlier classes such as natural numbers, whole numbers, integers, rational and irrational numbers. Let's revise them.



Real numbers are denoted by 'R' and consists of all rational and irrational numbers.

Note: Real numbers (R) include rational numbers (Q), which include integers (Z), which include whole numbers (W), which in turn include natural numbers (N).

- **Natural numbers:** Natural numbers are those which are used for counting and ordering i.e. N = {1, 2, 3, 4, ...} (excluding 0).
- **Whole numbers:** Whole numbers are collection of natural numbers including zero i.e. W = {0, 1, 2, 3, 4, ...}
- **Integers:** An integer is a number that can be written without fractional components. Integers consist of natural numbers, their negatives and zero i.e., Z = {....–4, –3, –2, –1, 0, 1, 2, 3, 4,....}.
- **Prime numbers:** A number which can only be divided either by 1 or by itself is called a prime number. For e.g. 2, 3, 5, 7, etc. Prime no's have only two factors.
- **Composite numbers:** A number which has more than two factors is known as a composite number. For e.g. 4, 6, 8, 10 etc.

Note: 1 is neither a prime nor a composite number.

- **Irrational numbers:** All real numbers which are not rational numbers are termed as irrational numbers. A non-terminating and non-repeating decimal is an example of irrational number. For e.g. $\sqrt{2}, \pi$ etc.
- **Rational numbers:** Rational numbers are those which can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Mind it

- Terminating or non-terminating and recurring (repeating) decimals are examples of rational numbers.
- Sum of a rational number and an irrational number always gives irrational number.
- Sum of two irrational numbers does not always give an irrational number.
- Product of a non-zero rational number with an irrational number always gives an irrational number.
- Product of two irrational numbers does not always give an irrational number.

- **Complex numbers:** Complex number is an imaginary number which can be written in the form of a + ib, where a and b are real numbers and $i = \sqrt{-1}$. For complex number (a + ib), a is known as real part and b is known as imaginary part. You will study more about complex numbers in class 11th.

Euclid's Division Lemma

Theorem 1: For two positive integers a and b, there always exist some unique integers q and r such that a = bq + r, where $0 \leq r < b$.

Here, when a is divided by b, q and r are obtained as quotient and remainder.

Let's have a look at some examples.

(i) Consider number 27 and 5, then:

$27 = 5 \times 5 + 2$

Comparing this with a = bq + r, we get

a = 27, b = 5, q = 5, r = 2

and $0 \leq r < b$ (as $0 \leq 2 < 5$).

(ii) Consider positive integers 14 and 4.

$14 = 4 \times 3 + 2$

Comparing this with a = bq + r, we get

a = 14, b = 4, q = 3, r = 2 and $0 \leq r < b$ (as $0 \leq 2 \leq 4$).

In the relation a = bq + r,

$0 \leq r < b$ is just a statement of long division of number 'a' by a number 'b' in which 'q' is obtained as quotient and 'r' is obtained as remainder.

Thus, dividend = divisor × quotient + remainder $\Rightarrow$ a = bq + r.

Example

1. How will you prove that every odd integer is of the form 2q + 1, where q is an integer.

Ans. Let a be any positive integer.

Applying division algorithm for a and b, where b = 2, we have

$a = 2q + r,$

Here q is an integer ≥ 0 and $0 \leq r < 2$.

As, $0 \leq r < 2$

($\therefore$ possible values of r can be 0 or 1)

For r = 0,

$a = 2q + r$

$\Rightarrow a = 2q$ (even integer)

and for r = 1,

$a = 2q + r$

$a = 2q + 1$

Here, since a is an odd integer, therefore a = 2q is not possible. Clearly every odd positive integer is of the form 2q + 1.

2. Prove that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.

Ans. As we know that for some integer m, any positive integer can be of the form 5m, 5m + 1, 5m + 2, 5m + 3 or 5m + 4.

Here $0 \leq r < 5$, $\because 0 \leq r < b$

Therefore, we have

$$(5m)^2 = 25m^2 = 5(5m^2) = 5q, \text{ q is an integer.}$$

[Here $q = 5m^2$]

$$(5m + 1)^2 = 25m^2 + 10m + 1 = 5(5m^2 + 2m) + 1 = 5q + 1, \text{ q is an integer.}$$

[Here $q = 5m^2 + 2m$]

$$(5m + 2)^2 = 25m^2 + 20m + 4 = 5(5m^2 + 4m) + 4 = 5q + 4, \text{ q is an integer.}$$

[Here $q = 5m^2 + 4m$]

$$\Rightarrow (5m + 3)^2 = 25m^2 + 30m + 9 = 25m^2 + 30m + 5 + 4 = 5(5m^2 + 6m + 1) + 4 = 5q + 4, \text{ q is an integer.}$$

[Here $q = 5m^2 + 6m + 1$]

$$\Rightarrow (5m + 4)^2 = 25m^2 + 40m + 16 = 25m^2 + 40m + 15 + 1 = 5(5m^2 + 8m + 3) + 1 = 5q + 1, \text{ q is an integer.}$$

[Here $q = 5m^2 + 8m + 3$]

This proves that for any integer q, the square of any positive integer cannot be of the form 5q + 2 or 5q + 3.

3. If x and y are both odd positive integers, then prove that $x^2 + y^2$ is even but not divisible by 4.

Ans. For some integers m and n,

Let $x = 2m + 1$ and $y = 2n + 1$

We have $x^2 + y^2$

$$\therefore \quad x^2 + y^2 = (2m + 1)^2 + (2n + 1)^2 = 4m^2 + 1 + 4m + 4n^2 + 1 + 4n$$

On rearranging, we get

$$= 4m^2 + 4n^2 + 4m + 4n + 1 + 1 = 4(m^2 + n^2) + 4(m + n) + 2 = 2[2(m^2 + n^2) + 2(m + n) + 1]$$

[Here $q = 2(m^2 + n^2) + 2(m + n) + 1$]

$$= 2q$$

As $x^2 + y^2$ is of the form 2q, this means that it is divisible by 2 and hence it is even.

Again $x^2 + y^2 = 4[(m^2 + n^2) + (m + n)\} + 2$

$$= 4q + 2$$

[Here $q = (m^2 + n^2) + (m + n)$]

$\Rightarrow x^2 + y^2$ is even and leaves remainder 2 on dividing by 4.

Therefore, $x^2 + y^2$ is not divisible by 4.

Euclid's Division Algorithm

Euclid's Algorithm is a technique to compute highest common factor (HCF) or the greatest common divisor (GCD). So, let's understand Euclid's division algorithm more clearly.

For obtaining HCF of two positive integers, say m and n, with m > n, follow the steps given below:

Step-1: Apply Euclid's division lemma to m and n, so that we can find whole numbers q and r, such that $m = nq + r$ $(0 \le r < n)$.

Step-2: If $r = 0$, n is the HCF of m and n. If $r \neq 0$, apply the division lemma to n and r.

Step-3: The process is continued till we get the remainder as zero. The divisor we get at this stage will be the required HCF.

Example

1. Explain the principle behind Euclid's division algorithm.

Ans. The principle behind Euclid's division algorithm is, if the smaller number is subtracted from the larger number, then the greatest common divisor of two numbers does not change.

Consider two numbers 252 and 105 whose HCF is 21.

$252 = 21 \times 12$ and $105 = 21 \times 5$

$\because 252 - 105 = 147$, the HCF (147, 105) is also 21.

Since, the larger of the two numbers is reduced, this process when repeated gives successively smaller numbers until zero is obtained. When that happens, the HCF is the remaining non-zero number.

Step-1: $252 = 105 \times 2 + 42$ $\quad \begin{Bmatrix} 252 - 105 = 147 \\ 147 - 105 = 42 \end{Bmatrix}$

Step-2: $105 = 42 \times 2 + 21$ $\quad \begin{Bmatrix} 105 - 42 = 63 \\ 63 - 42 = 21 \end{Bmatrix}$

Step-3: $42 = 21 \times 2 + 0$ $\quad \begin{Bmatrix} 42 - 21 = 21 \\ 21 - 21 = 0 \end{Bmatrix}$

$\therefore$ HCF (252, 105) = 21

2. Find the HCF of 420, 130 and 600, by using Euclid's division algorithm.

Ans. To find the HCF of 420, 130 & 600, we will first find the HCF of 420 & 130 by Euclid's division algorithm.

By using division algorithm, we get

$420 = 130 \times 3 + 30$

$130 = 30 \times 4 + 10$

$30 = 10 \times 3 + 0$

So, HCF (420, 130) = 10

Now, we will find the HCF of 10 and 600

$600 = 10 \times 60 + 0$

$\therefore$ HCF (10, 600) = 10

Hence, HCF (420, 130, 600) = 10.

Fundamental Theorem of Arithmetic

The fundamental theorem of arithmetic (FTA) which is also known as unique prime factorization theorem gives relationship between prime numbers and composite numbers. It states that:

Theorem 2: Every composite number can be expressed in the form of product of its prime factors and this factorization is unique, apart from the order in which the prime factorization occurs.

e.g. $12600 = 2^3.3^2.5^2.7$

Thus, the composite number 12600 is expressed as product of powers of primes in ascending order and this decomposition is unique.

H.C.F. and L.C.M. of numbers

H.C.F. (Highest common factor)

The H.C.F. of two or more numbers is the largest positive integer that divides each of the integers.

L.C.M. (Least common Multiple)

The L.C.M. of two or more numbers is the smallest positive integer which is exactly divisible by each one of the given numbers.

To find HCF and LCM of given numbers using Prime Factorisation Method or Fundamental Theorem of Arithmetic, first express each number in the form of product of prime factors, then

HCF = Product of the smallest powers of common factors.

LCM = Product of the greatest power of each prime factor involved in the numbers.

Important relations

- L.C.M × H.C.F = Product of two numbers.
- L.C.M of two or more prime numbers is equal to their product.
- H.C.F of two or more prime numbers is always 1.

Mind it

For three positive integers (a, b and c).

(i) HCF (a, b, c) × LCM (a, b, c) $\neq$ a × b × c, where a, b, c are positive integers.

(ii) $$\text{LCM}(a, b, c) = \frac{a \cdot b \cdot c \cdot \text{HCF}(a, b, c)}{\text{HCF}(a, b) \cdot \text{HCF}(b, c) \cdot \text{HCF}(a, c)}$$

(iii) $$\text{HCF}(a, b, c) = \frac{a \cdot b \cdot c \cdot \text{LCM}(a, b, c)}{\text{LCM}(a, b) \cdot \text{LCM}(b, c) \cdot \text{LCM}(a, c)}$$

For fractions $\frac{p}{q}$ and $\frac{a}{b}$,

(i) $$\text{L.C.M of fraction} = \frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$$

(ii) $$\text{H.C.F of fraction} = \frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$$

Example

1. On an evening walk, three people step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. How much minimum distance each should cover so that each can cover the same distance in complete steps?

Ans. $40 = 2^3 \times 5^1$

$42 = 2^1 \times 3^1 \times 7^1$

$45 = 3^2 \times 5^1$

As three people step off together, minimum distance covered by each one of them = LCM of 40, 42, 45

$= 2^3 \times 3^2 \times 5^1 \times 7^1$

$= 8 \times 9 \times 35 = 2520$ cm.

2. 7 × 11 × 13 + 13 is a composite number. Explain.

Ans. $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13 = 78 \times 13$

$= 2 \times 3 \times 13 \times 13$

$= 2 \times 3 \times 13^2$

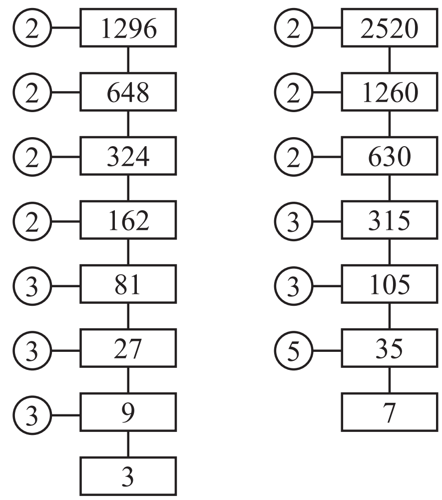
As the given number have more than two factors,

$\therefore$ $(7 \times 11 \times 13 + 13)$ is a composite number.

3. By using prime factorization method, find the L.C.M and H.C.F. of 1296 and 2520.

Ans. $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^4 \times 3^4$

$2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$



L.C.M. $(1296, 2520) = 2^4 \times 3^4 \times 5 \times 7 = 45360$

H.C.F. $(1296, 2520) = 2^3 \times 3^2 = 72$.

4. Find the greatest number which can divide 1251, 9377 and 15628 leaving 1, 2 and 3, as remainders respectively.

Ans. Numbers are: (1251 – 1), (9377 – 2) and (15628 – 3) or 1250, 9375 and 15625.

Required number is the HCF of 1250, 9375 and 15625.

Now, $1250 = 2 \times 5^4$

$9375 = 3 \times 5^5$

$15625 = 5^6$

$\text{HCF} = 5^4 = 625$

$\therefore$ 625 is the largest number that divides 1251, 9377 and 1568 leaving remainders 1, 2 and 3 respectively.

Irrational Numbers

An irrational number cannot be expressed as the ratio of two integers. Examples of irrational numbers are: $\sqrt{2}, \sqrt{3}, \sqrt{5}, -\sqrt{15}, \pi$, etc.

Also, non-terminating and non-repeating decimal is the decimal representation for irrational numbers. E.g. 0.101001000, 93.47803945....., 2.395634532.... .

Theorem 3: Let a be a prime number. If a divides p^2, then 'a' divides p, where p is a positive integer.

Proof: Let the prime factorisation of p be as follows:

$p = a_1a_2.....a_n$, where $a_1, a_2,.....a_n$ are primes numbers, not necessarily distinct.

Therefore,

$p^2 = (a_1a_2.....a_n)(a_1a_2 ..... a_n) = a_1^2a_2^2.....a_n^2$.

Now, we are given that a divides p^2. Therefore, a is one of the prime factors of p^2, according to the Fundamental Theorem of Arithmetic. However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we find that the only prime factors of p^2 are $a_1, a_2,..., a_n$. So a is one of $a_1, a_2,......, a_n$.

Now, since $p = a_1a_2 ...... a_n$

∴ we can conclude that a divides p as well.

Mind it

- If $\sqrt{pq}$ is irrational, then $\sqrt{p}+\sqrt{q}$ will definitely be irrational.
- If a is a positive prime number, then $\sqrt{a}$ is an irrational number.

Revisiting Rational Numbers

Rational number is any number which can be expressed in the form of $\frac{a}{b}$ where a & b are integers and $b \neq 0$.

Terminating or non-terminating but repeating decimal is the decimal representation for rational numbers.

e.g. $\frac{4}{7}$, 1, 0, –2, 4.1, 3.777..., 2.689689... etc.

Theorem 4: Number x can be expressed in the form $\frac{a}{b}$, if x is a rational number whose decimal expansion terminates. Here a and b are coprime and the prime factorization of b is of the form 2^m5^n, where m, n are non-negative integers.

Theorem 5: Let $x = \frac{a}{b}$ be a rational number in which the prime factorization of b is of the form $2^m 5^n$, where m, n are non-negative integers, then x has a decimal expansion which terminates.

Eg: $\frac{64}{125}=\frac{2^6}{5^3}=\frac{2^9}{(2\times5)^3}=\frac{512}{10^3}=0.512$

Eg: $\frac{9}{25}=\frac{9\times2^2}{5^2\times2^2}=\frac{36}{(2\times5)^2}=\frac{36}{(10)^2}=0.36$

So, a rational number of the form $\frac{a}{b}$, where b is of the form 2^m5^n can be converted to an equivalent rational number of the form $\frac{p}{q}$ where q is a power of 10. Therefore, the decimal expansion of such rational numbers terminate.

Theorem 6: Let $x = \frac{a}{b}$ be a rational number in which the prime factorization of b is not of the form 2^m5^n, where m, n are non-negative integers, then x has a decimal expansion which is recurring i.e. non-terminating repeating.

E.g: $\frac{1}{7} = 0.142857142857...$

As, denominator 7 is not of the form 2^m5^n, therefore this rational number will not terminate.

It can be concluded from the above discussion that the decimal expansion of every rational number is either terminating or non-terminating repeating.

Example

1. How will you prove that $\sqrt{2}$ is irrational?

Ans. Let's assume, to the contrary, that $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{a}{b}$ where a, b are integers having no common factor other than 1. i.e., a and b are co prime.

So, $\sqrt{2}b = a$.

On squaring both sides, we get

$a^2 = 2b^2$

$\therefore$ 2 divides a^2.

Now, using theorem 3, it follows that 2 divides a.

So, we can write a = 2c for some integer c.

Again on squaring both sides, we get

$$a^2 = 4c^2$$

$$2b^2 = 4c^2 \qquad [\because a^2 = 2b^2]$$

$$b^2 = 2c^2$$

This means that 2 divides b^2 and therefore 2 divides b (again using Theorem 3 with a = 2).

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b do not have a common factor other than 1.

This contradiction has arisen because of our wrong assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

2. Prove that $2+\sqrt{3}$ is irrational.

Ans. Let us assume, to the contrary, that $2+\sqrt{3}$ is a rational number equals to r

$\therefore 2+\sqrt{3} = r$

$\sqrt{3} = r - 2$

Here L.H.S. i.e. $\sqrt{3}$ is an irrational number while R.H.S. i.e. (r – 2) is rational.

$\therefore$ L.H.S $\neq$ R.H.S

Hence, it contradicts our assumption that $2+\sqrt{3}$ is rational.

$\therefore 2+\sqrt{3}$ is irrational.

3. Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

Ans. Let us assume, to the contrary, that $\sqrt{2}+\sqrt{3}$ is a rational number equals to 'x'

$\Rightarrow x = \sqrt{2}+\sqrt{3}$

On squaring both sides, we get

$x^2 = 2 + 3 + 2\sqrt{3}\cdot\sqrt{2} = 5 + 2\sqrt{6}$

$\Rightarrow x^2 = 5 + 2\sqrt{6} \Rightarrow \sqrt{6} = \frac{x^2-5}{2}$

As x, 5 and 2 are rational numbers, $\therefore \frac{x^2-5}{2}$ is a rational number.

Hence, $\sqrt{6} = \frac{x^2-5}{2}$ is a rational number, which is in contradiction to the fact that $\sqrt{6}$ is an irrational number.

Hence our assumption is wrong. So, we can conclude that $\sqrt{2}+\sqrt{3}$ is an irrational number.

4. Prove that $\sqrt{a}+\sqrt{b}$ is irrational, where a, b are primes.

Ans. Let us suppose, to the contrary, that $\sqrt{a}+\sqrt{b}$ is a rational number.

Let $\sqrt{a}+\sqrt{b} = r$, where r is rational.

Therefore $\sqrt{a} = r - \sqrt{b}$

On squaring both sides, we get

$a = r^2 + b - 2r\sqrt{b}$.

Therefore $\sqrt{b} = \dfrac{r^2 + b - a}{2r}$

Now r^2, b, a and r are all rational numbers. As rational numbers are closed under addition, subtraction, multiplication and division.

Note: Neglecting division by zero, rational numbers are also closed under division.

So, $\dfrac{r^2 + b - a}{2r}$ is a rational number.

But since b is a prime number, then b is not a perfect square and therefore $\sqrt{b}$ is an irrational number.

This contradicts the fact as an irrational number can never be equal to a rational number.

$\therefore$ Our assumption is wrong.

Hence $\sqrt{a} + \sqrt{b}$ is irrational.

5. Find terminating or non-terminating recurring decimals from given rational numbers.

(i) $\mathbf{\dfrac{17}{8}}$ **(ii)** $\mathbf{\dfrac{13}{3125}}$ **(iii)** $\mathbf{\dfrac{64}{455}}$ **(iv)** $\mathbf{\dfrac{29}{343}}$

(v) $\mathbf{\dfrac{15}{1600}}$

Ans.

(i) $\dfrac{17}{8} = \dfrac{17}{2^3} = \dfrac{17 \times 5^3}{(2 \times 5)^3} = \dfrac{17 \times 125}{(10)^3}$ = terminating

(ii) $\dfrac{13}{3125} = \dfrac{13}{(5)^5} = \dfrac{13 \times 2^5}{2^5 \times 5^5} = \dfrac{(13 \times 32)}{(10)^5}$ = terminating

(iii) $\dfrac{64}{455} = \dfrac{2^6}{5 \times 7 \times 13}$

($\because$ 7 & 13 cannot be removed from denominator)

$\therefore$ The number is non-terminating recurring.

(iv) $\dfrac{29}{343} = \dfrac{29}{(7)^3}$ = non-terminating recurring

(v) $\dfrac{15}{1600} = \dfrac{3 \times 5}{2^4 \times 10^2} = \dfrac{3 \times 5^5}{(2 \times 5)^4 \times 10^2} = \dfrac{3 \times 5^5}{10^6}$

= terminating

Important Concepts Regarding H.C.F & L.C.M of Numbers

- A number when divided by a_1 and a_2, leaves remainder r_1 and r_2, respectively. If the number is divided by $(a_1 \times a_2)$, then the remainder is $(a_1 \times r_2 + r_1.)$

  Ex: A number when divided by 5 and 7 leaves the remainder 3 and 4, respectively. Find the remainder when the same number is divided by 35.

  Sol. The required remainder $= a_1 \times r_2 + r_1$

  $= 5 \times 4 + 3$

  $= 23$

- To find how many numbers are divisible by a certain integer.

  Ex: (i) Find how many numbers up to 594 are divisible by 15?

  Sol. We divide 594 by 15.

  $594 = 39 \times 15 + 9$

  The quotient obtained is the required number. Therefore, there are 39 such numbers upto 594 which are divisible by 15.

  Ex: (ii) Find how many numbers up to 600 are divisible by 7 and 5 together?

  Sol. As, L.C.M. of 7 and 5 = 35

  $\therefore$ We divide 600 by 35

  $600 = 17 \times 35 + 5$

  Therefore, there are 17 such numbers upto 600 which are divisible by 7 and 5 together.

❐ Two numbers when get divided by a certain divisor leave remainders r_1 and r_2. When their sum is divided by the same divisor, gives remainder r_3. The divisor is given by $(r_1 + r_2 - r_3)$.

Ex: Two numbers when divided by a certain divisor leave remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Sol. The required divisor

= 437 + 298 – 236

= 499

Hence, the required divisor is 499.

❐ To find the largest number which will divide a, b and c leaving remainders x, y and z respectively.

Required number = H.C.F. of (a – x), (b – y) and (c – z).

Ex: Find the largest number which will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively.

Sol. The required greatest number

= H.C.F. of (148 – 4), (246 – 8) and (623 – 11),

= H.C.F. of 144, 240 and 612

= 12

Hence, the largest number which will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively is 12.

❐ To find the smallest number which when divided by a, b and c leave the remainders x, y and z respectively. It is always observed that (a – x) = (b – y) = (c – z) = k(say).

∴ Required number = (L.C.M. of a, b and c) – k.

Ex: Find the smallest number which when divided by 12, 15 and 21 leave the remainders 7, 10 and 16 respectively.

Sol. Since (12 – 7) = (15 – 10) = (21 – 16) = 5, therefore the required smallest number,

= (L.C.M of 12, 15 and 21) – 5

= 420 – 5

= 415.

❐ To find the smallest number which when divided by a, b and c leaves the same remainder r in each case.

Required number = (L.C.M of a, b and c) + r.

Ex: Find the smallest number which when divided by 16, 20 and 24, will leave a remainder 5 in each case.

Sol. The required smallest number

= (L.C.M of 16, 20 and 24) + 5

= 240 + 5

= 245.

❐ To find the largest number which will divide a, b and c leaving the same remainder in each case.

(a) When the value of remainder is not given:

Required number = H.C.F. of |(a – b)|, |(b – c)| and |(c – a)|

(b) When the value of remainder r is given:

Required number = H.C.F. of (a – r), (b – r) and (c – r).

Ex: (i) Find the largest number which on dividing 34, 90 and 104 leaves the same remainder in each case.

Sol. The required largest number.

= H.C.F. of $|(a-b)|$, $|(b-c)$ and $(c-a)|$

= H.C.F. of $|(34-90)|$, $|(90-104)|$ and $|(104-34)|$

= H.C.F. of 56, 14 and 70

= 14

Ex: (ii) Find the largest number which will divide 155 and 192 and leaves the remainder 7 in each case.

Sol. The required largest number

= H.C.F. of $(155-7)$ and $(192-7)$

= H.C.F. of 148 and 185

= 37

Number of Factors of a Given Number

Let N be a composite number, and its prime factors be a, b, c, d,....... etc. and k, l, m, n...etc. be the powers of a, b, c, d..... etc. respectively i.e., N can be expressed as

$N = a^k.\ b^l.c^m.d^n...$

then, the number of total factors or divisors of N is $(k+1)(l+1)(m+1)(n+1)$......

Ex. Find the total number of factors of 1080:

Sol. $1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$540 = 2^3 \times 3^3 \times 5^1$

Therefore total number of factors of 1080 is $(3+1)(3+1)(1+1) = 32$

Sum of Factors of a Given Number

Let N be a composite number and its prime factors be a, b, c, d,....... etc. and k, *l*, m, n...etc. be the powers of a, b, c, d..... etc. respectively i.e., if N can be expressed as

$N = a^k.b^l.c^m.d^n...$

then the sum of all the factors or divisors of $N = \dfrac{(a^{k+1}-1)(b^{l+1}-1)(c^{m+1}-1)(d^{n+1}-1)}{(a-1)(b-1)(c-1)(d-1)}$

Ex: Find the sum of the factors of 180.

Sol. $180 = 2^2 \times 3^2 \times 5$

$\therefore$ Sum of the factors of 180 $= \dfrac{(2^{2+1}-1)(3^{2+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)}$

$= \dfrac{7 \times 26 \times 24}{1 \times 2 \times 4} = 546$

Product of Factors

Let's see the factors of 24:

$24 = 1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6$

$\therefore$ Product of factors of 24 are

$= (1 \times 24) \times (2 \times 12) \times (3 \times 8) \times (4 \times 6)$

$= 24 \times 24 \times 24 \times 24 = (24)^4$

Hence, the product of factors of a composite number N is given $N^{n/2}$, where n is the total number of factors of the composite number N.

Number of Odd Factors of a Given Number

To find the number of odd factors of a number N, firstly express the number N as

$N = (a_1^p \times a_2^q \times a_3^r \times ...) \times (e^x)$

(where, a_1, a_2, a_3, ... are the odd prime factors and e is the even prime factor)

Then the total number of odd factors = $(p + 1)(q + 1)(r + 1)$

Ex: The number of odd factors of 72 is ...

Sol. $72 = 2^3 \times 3^2$

$\therefore$ Number of odd factors = $(2 + 1) = 3$

Number of Even Factors of a Composite Number

Number of even factors of any number is given by:

Number of even factors of a number = (Total number of factors of the given number) – (Total number of odd factors of the given number).

Ex: (a) Find the number of even factors (or divisors) of 72

Sol. Total number of factors = $(3 + 1)(2 + 1)$

$= 12$

Number of odd factors = $(2 + 1)$

$= 3$

Number of even factors = Total number of factors – no. of odd factors

$= 12 - 3$

$= 9$

Summary

- **Euclid's Division Lemma:** For any two positive integers a and b, $a > b$ there exist unique integers q and r such that $a = bq + r$ $(0 \le r < b)$.
- **Euclid's Division Algorithm:** Euclid's algorithm provides step - wise procedure for computing the H.C.F of two natural numbers.
- **Fundamental Theorem of Arithmetic:** Every composite number can be expressed as a product of primes and their factorisation is unique.
- **Highest Common Factor (H.C.F):** H.C.F of two or more given positive integers is the largest positive integer that divides each one of them exactly.
  - HCF of two or more prime numbers is always 1.
  - LCM of two or more prime numbers is equal to their products.
  - HCF (a, b) × LCM (a, b) = a × b
- **An Irrational number** is a non - terminating and non - recurring decimal and cannot be expressed in the form p/q.
- **The rational numbers** are the numbers which can be expressed in the form p/q where p and q are integers and $q \ne 0$.
  - Let $x = p/q$, where p and q are co -prime and $q \ne 0$ and q has a prime factorisation of the form $q = 2^m 5^n$, where n and m are non - negative integers, then x has a **terminating decimal representation**.
  - Let $x = p/q$ be a rational number, such that the prime factorisation of q is not of the form $2^m 5^n$ where n and m are non - negative integers, then x has a decimal expansion which is **non - terminating repeating**.
- If p is a positive prime, then $\sqrt{p}$ is an irrational number.
- The sum of a rational number and an irrational number is always irrational.
- The product of a non - zero rational number and an irrational number is always irrational.
- If $\sqrt{ab}$ is an irrational numbers, then $\sqrt{a} + \sqrt{b}$ is also irrational.

Quick Recall

Fill in the blanks

1. 144 as a product of its prime factors is ....
2. Least value of r in the equation a = bq + r, is ....
3. If every positive even integer is of the form 2q, then every positive odd integer is of the form ...., where q is some integer.
4. In the prime factorisation of 72, the power of 2 is ....
5. For finding H.C.F of two positive numbers, an algorithm which is used is ....
6. A lemma is a proven statement used for ....
7. The product of two numbers is equal to the .... of their HCF and LCM.
8. $\frac{37}{50}$ is a .... decimal expansion.
9. An .... is a series of well defined steps which gives a procedure for solving a type of problem.
10. $\sqrt{2}$ is a/an .... number.

True and False Statements

1. For given positive integers a and b, there exist whole numbers q and r which satisfy equation, a = bq + r, where $0 \leq r < b$.
2. Every composite number can be expressed in the form of product of its prime factors, and this decomposition is unique, apart from the order in which the prime factors occur.
3. The sum of two prime numbers is always an even number.
4. $\sqrt{2}$ and $\sqrt{3}$ are rational numbers.
5. Every point on the number line is of the form $\sqrt{m}$.
6. The sum of two rational numbers is a rational number.
7. The quotient of two integers is always a rational number.
8. 1/0 is not a rational number.
9. If x = p/q be a rational number, such that the prime factorisation of q is of the form 2^m5^n, where m, n are non-negative integers, then x has a decimal expansion which terminates.
10. The number of irrational numbers between 15 and 18 is finite.

Match The Followings

1.

	Column I		Column II
(1)	Rational number is always	(A)	irrational number
(2)	Irrational number is always	(B)	rational number
(3)	$\frac{4}{7}$ is a	(C)	non-terminating and non-repeating
(4)	$\sqrt{5}$ is an	(D)	terminating or non-terminating recurring

a. 1-D 2-C 3-B 4-C
b. 1-A 2-B 3-C 4-D
c. 1-C 2-D 3-D 4-C
d. 1-B 2-C 3-A 4-D

2.

	Column I		Column II
(1)	H.C.F. of 306 and 657	(A)	2
(2)	H.C.F. of the smallest composite number and the smallest prime number	(B)	5
(3)	H.C.F. of 475 and 495	(C)	9 and 495

a. 1-A 2-B 3-C
b. 1-C 2-B 3-A
c. 1-C 2-A 3-B
d. 1-B 2-A 3-C

Answers

Fill in the Blanks:

1. $2^4 \times 3^2$
2. Zero
3. 2q + 1
4. 3
5. Euclid's division algorithm
6. Proving another statement
7. Product
8. Terminating
9. Algorithm
10. Irrational

True and False:

1. True
2. True
3. False
4. False
5. False
6. True
7. False, Since $\frac{1}{0}$ is not a rational number
8. True
9. True
10. False

Match the Followings

1. (a) 2. (c)

NCERT Exercise

Exercise-I

1. **Use Euclid's division algorithm to find the HCF of:**

   a. 135 and 225

   b. 196 and 38220

   c. 867 and 255

Exp: **(a)** 135 and 225

As we can see, 225 is greater than 135. Hence, on applying Euclid's division algorithm, we have

$225 = 135 \times 1 + 90$

Solving until we get remainder as 0.

$135 = 90 \times 1 + 45$

$90 = 45 \times 2 + 0$

We have obtained zero as remainder. The divisor of this step becomes our H.C.F.

Hence, the HCF of 135 and 225 is 45.

(b) Do it yourself.

(c) Do it yourself.

2. **Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.**

Exp: According to Euclid's Division Lemma if we have two positive integers a and b, then there exist unique integers q and r which satisfies the condition $a = bq + r$ where $0 \le r < b$.

Let a be any positive odd integer which when divided by 6 gives q as quotient and r as remainder.

According to the Euclid's division lemma

$a = 6q + r$ where $0 \le r < 6$.

So r can be either 0, 1, 2, 3, 4 and 5.

Therefore, a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5.

6q + 0: 6 is divisible by 2, so it is an even number.
6q + 1: 6 is divisible by 2, but 1 is not divisible by 2 so it is an odd number.

6q + 2: 6 is divisible by 2, and 2 is also divisible by 2 so it is an even number.

6q + 3: 6 is divisible by 2, but 3 is not divisible by 2 so it is an odd number.

6q + 4: 6 is divisible by 2, and 4 is also divisible by 2 so it is an even number.

6q + 5: 6 is divisible by 2, but 5 is not divisible by 2 so it is an odd number.

And therefore, any odd integer can be expressed in the form 6q + 1 or 6q + 3 or 6q + 5.

3. **An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?**

Exp: Given:

Number of members in army contingent = 616

Number of members in army band = 32

If the two groups have to march in the same number of columns, we need to find out the highest common factor (HCF) between the two groups.

We shall find the H.C.F using Euclid's algorithm

As, 616 is greater than 32, therefore

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

Now, we have obtained zero as remainder.

Therefore, HCF (616, 32) = 8.

Hence, the maximum number of columns in which the two groups can march is 8.

4. **Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m. [Hint: Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that it can be rewritten in the form 3m or 3m + 1.**

Exp: A similar question has been solved above.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Exp: Let us suppose, x be any positive integer and divisor = 3.

On using Euclid's division algorithm,

X = 3q + r, where $q \geq 0$ and $0 \leq r < 3$. Therefore r = 0, 1, 2.

Now, on putting the value of r, we get

$$X = 3q \text{ (if } r = 0)$$

or

$$X = 3q + 1 \text{ (if } r = 1)$$

or

$$X = 3q + 2 \text{ (if } r = 2)$$

On cubing both sides of all the three above expressions, we get

Case (i):

$$X^3 = (3q)^3$$
$$= 27q^3$$
$$= 9(3q^3)$$
$$= 9m; \quad [\text{where } m = 3q^3]$$

Case (ii):

$$X^3 = (3q + 1)^3$$
$$= (3q)^3 + 13 + 3 \times 3q \times 1(3q + 1)$$
$$= 27q^3 + 1 + 27q^2 + 9q$$
$$X^3 = 9(3q^3 + 3q^2 + q) +1$$
$$X^3 = 9m + 1;$$
$$[\text{where } m = 3q^3 + 3q^2 + q]$$

Case (iii):

$$X^3 = (3q + 2)^3$$
$$= (3q)^3 + 2^3 + 3 \times 3q \times 2(3q + 2)$$
$$= 27q^3 + 54q^2 + 36q + 8$$
$$X^3 = 9(3q^3 + 6q^2 + 4q) + 8$$
$$X^3 = 9m + 8$$
$$[\text{where } m = 3q^3 + 6q^2 + 4q]$$

Hence, the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Exercise-II

1. Express each number as a product of its prime factors:

a. 140 **b. 156**

c. 3825 **d. 5005**

e. 7429

Exp: Do it yourself.

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

a. 26 and 91

b. 510 and 92

c. 336 and 54

Exp: (a) 26 and 91

On expressing 26 and 91 as product of its prime factors, we have

$$26 = 2 \times 13 \times 1$$
$$91 = 7 \times 13 \times 1$$

Hence,

$$\text{LCM } (26, 91) = 2 \times 7 \times 13 \times 1$$
$$= 182$$

and

$$\text{HCF } (26, 91) = 13$$

Verification

As,

LCM × HCF = Product of two numbers

Product of 26 and 91 = 26 × 91

= 2366

Product of LCM and HCF = 182 × 13

= 2366

Clearly, LCM (26, 91) × HCF (26, 91) = Product of 26 and 91.

(b) Do it yourself.

(c) Do it yourself.

3. **Find the LCM and HCF of the following integers by applying the prime factorisation method.**

   a. 12, 15 and 21

   b. 17, 23 and 29

   c. 8, 9 and 25

Exp: Do it yourself.

4. **Given that HCF (306, 657) = 9, find LCM (306, 657).**

Exp: Do it yourself.

5. **Check whether 6^n can end with the digit 0 for any natural number n.**

Exp: For a number 6n to end with the digit zero (0), it must be divisible by 5, as we already know that any number having unit place as 0 or 5 is divisible by 5.

Prime factorization of 6n = (2 × 3)n

As we can see that, the prime factorization of 6n doesn't contain prime number 5.

Therefore, it is clear from above that, 6n is not divisible by 5 for any natural number n and hence, it proves that 6n can never end with the digit 0 for any natural number n.

6. **Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.**

Exp: Do it yourself.

7. **There is a circular path around a sports field. Monika takes 18 minutes to drive one round of the field, while Rohit takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?**

Exp: It is given that the time taken by Sonia is more than Ravi to complete one round.

We need to find the number of minutes after which they will meet again at the starting point. For this, there will be a smallest number that is divisible by both 18 and 12 and that will be the time when both meet again at the starting point. To find this we have to take LCM of both numbers.

Therefore, LCM (18, 12) = 2 × 3 × 3 × 2 × 1 = 36

Hence, Monika and Rohit will meet again at the starting point after 36 minutes.

Exercise-III

1. **Prove that $\sqrt{5}$ is irrational.**

Exp: Let us assume to the contrary that $\sqrt{5}$ is a rational number.

i.e. $\sqrt{5} = \frac{a}{b}$ (where, a and b are co-primes)

$b\sqrt{5} = a$

On squaring both sides, we have

$\left(b\sqrt{5}\right)^2 = a^2$

$\Rightarrow 5b^2 = a^2$ ...(1)

As, a^2 is divisible by 5, so a is also divisible by 5.

So, we can write a = 5c, for some integer c and on substituting the value of a in equation (1), we get

$5b^2 = (5c)^2$

$\Rightarrow b^2 = 5c^2$

As, b^2 is divisible by 5, it means b is also divisible by 5.

Therefore, a and b have at least 5 as a common factor. But this contradicts the fact that a and b are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ is an irrational number.

2. **Prove that $3+2\sqrt{5}$ is irrational.**

Exp: Let us assume to the contrary, that $3+2\sqrt{5}$ is rational.

i.e., we can find co-prime a and b (b ≠ 0) such that

$3+2\sqrt{5} = \frac{a}{b}$

On rearranging the equation, we get

$2\sqrt{5} = \frac{a}{b} - 3$

$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$

Since, a and b are integers, thus,

$\frac{1}{2}\left(\frac{a}{b} - 3\right)$ is a rational number, and therefore, $\sqrt{5}$ should also be a rational number.

But this contradicts the fact that $\sqrt{5}$ is irrational.

Hence, we conclude that $3+2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals:

a. $\frac{1}{\sqrt{2}}$

b. $7\sqrt{5}$

c. $6+\sqrt{2}$

Exp: Do it yourself.

Exercise-IV

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

a. $\frac{13}{3125}$ **b.** $\frac{17}{8}$

c. $\frac{64}{455}$ **d.** $\frac{15}{1600}$

e. $\frac{29}{343}$ **f.** $\frac{23}{\left(2^3 5^2\right)}$

g. $\frac{129}{\left(2^2 5^7 7^5\right)}$ **h.** $\frac{6}{15}$

i. $\frac{35}{50}$ **j.** $\frac{77}{210}$

Exp: Hint: A rational number has a terminating decimal expansion, if and only if the denominator contains factors of 2 and 5 or is in the form of $2^m \times 5^n$ where m and n are non-negative integers.

If the denominator contains factors other than 2 and 5 then it has a non-terminating decimal expansion.

2. Write down the decimal expansions of those rational numbers which have terminating decimal expansions.

a. $\frac{13}{3125}$ **b.** $\frac{17}{8}$

c. $\frac{64}{455}$ **d.** $\frac{15}{1600}$

e. $\frac{29}{343}$ **f.** $\frac{23}{\left(2^3 5^2\right)}$

g. $\frac{129}{\left(2^2 5^7 7^5\right)}$ **h.** $\frac{6}{15}$

i. $\frac{35}{50}$ **j.** $\frac{77}{210}$

Exp: (a) $\frac{13}{3125}$

```
3125 ) 13.00000 ( 0.00416
        0
       --------
        130
          0
       --------
        13000
       -12500
       --------
          5000
         -3125
       --------
          18750
          18750
       --------
          00000
       --------
```

$\frac{13}{3125} = 0.00416$

Rest do it yourself.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational and of the form p/q, what can you say about the prime factors of q?

a. **43.123456789**

b. **0.120120012000120000...**

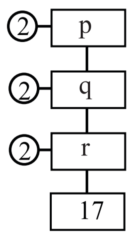
c. $43.\overline{123456789}$

Exp: Do it yourself.

Subjective Questions

Very Short Answer Type Questions

1. Find the prime factors of 1080.
2. Numbers p, q and r are missing in the following factorisation. Find them.



3. Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{4}$.
4. Find two irrational numbers in the range of 2 to 3.5.
5. There is a number 6^n, where n is a natural number. Check if there is any value of $n \in N$ exists, for which 6^n is divisible by 7.
6. Without dividing the number $\frac{7}{80}$, express it in decimal form.
7. Find the least possible positive value of P, if the square of a number 'P' is the sum of the square of the other two numbers 'Q' and 'R' where 5Q = 12R and Q, R are positive integers.
8. Find a number which is greater than 3 and when it gets divided by 4, 5 and 6 always leaves 3 as remainder.
9. For two numbers 150 and 100, it is given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100).
10. Find the HCF and LCM of the given fractions $\frac{8}{9}, \frac{16}{81}, \frac{2}{3}$ and $\frac{10}{27}$.

Short Answer Type Questions

1. Find the number which is nearest to 100000 and greater than 100000 which is exactly divisible by each of 8, 15 and 21.
2. Prove that $\left(3-\sqrt{5}\right)$ is an irrational number.
3. In a school 437 girls and 342 boys have been divided into groups, so that each group has the same number of students and no group has boys and girls mixed. What is the least number of groups needed?
4. Pooja multiplied a number 484 with a certain number to obtain the result 3823a. Find the value of a.
5. If first 100 multiples of 10 are multiplied with each other, then find the number of zeroes at the end of the product.
6. Three people running around a rectangular track, can complete one turn in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?
7. Find the value of:

$$\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2007}+\frac{1}{2008}\right)\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2007}\right)$$

$$-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2007}+\frac{1}{2008}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2007}\right)$$

8. 4 bells ring together at 9.00 a.m. They ring after 7, 8, 11 and 12 seconds respectively. How many times will they ring together again in the next 3 hours?
9. Find the greatest number which when divides 245 and 1029 leaves remainder 5 in each case.
10. Find the largest number which when divide 445, 572 and 699 leave remainders 4, 5 and 6 respectively.

Long Answer Type Questions

1. If a, b, c, d be positive rationals such that $a+\sqrt{b}=c+\sqrt{d}$, then prove that either a = c and b = d or b and d are squares of rationals.

2. Prove that $\sqrt{7}+\sqrt{11}$ is an irrational number.

3. Show that $(n^2 - 1)$ is divisible by 8, if n is an odd integer.

4. Show that only one of the number out of n, n + 4, n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.

5. For every positive integer n, prove that $n^2 - n$ is divisible by 2 [Hint: Take two cases of n = 2q and 2q + 1].

Integer Type Questions

1. How many number of positive integers n are there for which $\sqrt{n-1}+\sqrt{n+1}$ is a rational?

2. What will be the last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^5 + ... + 4^{99} + 9^{100}$.

3. Find HCF of 96 and 294, using Euclid's division algorithm.

4. A number on being divided by 342 leaves a remainder 47. What would be the remainder, if the same number is being divided by 19?

5. Two runners P and Q stop after 3 hrs and 5 hrs respectively while running. After ab hours (where ab is a two digit number) both of them will pause together for the first time, if they started running at the same time. Find the value of a + b.

6. If the L.C.M and H.C.F of two numbers are 168 and 28 respectively, then find the number of possible such pairs.

7. Find the total number of factors of 3025.

8. Find the HCF of the first 1000 even natural numbers.

9. Find the number of possible values of z, if a is a natural number, and $8^a - 7^a$ ends with a digit z.

10. What is the minimum possible sum of the numbers, if the L.C.M and the H.C.F of two numbers are 924 and 7 respectively?

Multiple Choice Questions

Level-I

1. The value of $a^2 + b^2 + c^2 - ab - bc - ca$ will be equal to which of the following options, if $a = 24$, $b = 26$, $c = 28$.

a. 0 b. 12
c. 10 d. 4

2. If natural number $n = (32)^{3/5}$, the value of n is equal to

a. 18 b. 6
c. 8 d. 4

3. LCM of 6, 72 and 120 is

a. 360 b. 260
c. 180 d. None of these

4. In the prime factorization of 144, the power of 2 is

a. 2 b. 4
c. 6 d. 3

5. The number of zeroes in number n, if $n = 2^3 \times 3^2 \times 5^2 \times 7$, is equal to

a. 2 b. 1
c. 5 d. 6

6. Between two rational numbers,

a. there are infinitely many rational numbers
b. there are exactly two irrational numbers
c. there are exactly four irrational numbers
d. None of these.

7. The value of $0.\overline{2}$ in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is

a. $\frac{2}{9}$ b. $\frac{1}{5}$
c. $\frac{6}{5}$ d. $\frac{3}{8}$

8. The value of 0.999 ... in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is

a. $\frac{4}{9}$ b. $\frac{8}{9}$
c. $\frac{9}{10}$ d. none of these

9. If we take approximate value of $\sqrt{2} = 1.4142$ and $\pi = 3.1415$, then the value of $\pi + 2\sqrt{2}$, upto four places of decimal is

a. 5.9408 b. 5.9999
c. 5.9699 d. none of these

10. If $\frac{1}{a} = 0.\overline{142857}$, then find the recurring decimal expansion of $2\frac{3}{a}$.

a. $2.\overline{428571}$ b. $2.\overline{46586}$
c. $2.\overline{1417628}$ d. none of these

11. In the rational number $\frac{14587}{1250}$, decimal expansion terminates after,

a. six decimal places b. four decimal places
c. eight decimal places d. two decimal places

12. In the rational number $\frac{33}{2^2 \cdot 5}$, decimal expansion terminates after,

a. two decimal places
b. one decimal place
c. more than 4 decimal places
d. more than 5 decimal places

13. The least number which can be divided by all the natural numbers from 1 to 10 (including both) is

a. 2520 b. 200
c. 604 d. 10

14. If $x = 2 + \sqrt{3}$, then $x - \frac{1}{x} =$

a. $2 + \sqrt{3}$ b. 4
c. $2\sqrt{3}$ d. none of these

15. For some integer n, every even integer is of the form

a. n b. 2n + 1
c. n + 1 d. 2n

16. For some integer n, every odd integer is of the form

a. n^2 b. 2n + 1
c. 2n d. n + 1

17. The product of an irrational number with a non-zero rational number is

a. rational or irrational b. always rational

c. always irrational d. None of these

18. Find the value of n, if the HCF of 65 and 117 is expressed as 65 n – 117.

a. 4 b. 3

c. 6 d. 2

19. Find the greatest number which when divides 70 and 125, leaves remainder 5 and 8 respectively.

a. 875 b. 10

c. 13 d. 1680

20. If two positive integers a and b can be expressed as $a = pq^2$ and $b = p^3q$; p, q being prime numbers, then LCM (a, b) is

a. p^3q^2 b. p^2q^2

c. pq d. p^3q^3

21. $(n^2 - 1)$ is divisible by 8, if n is

a. an integer

b. an odd integer greater than 1

c. a natural number

d. none of these

22. If a is a rational number, then $5^{2a} - 2^{2a}$ is divisible by

a. Both 3 and 7 b. 9

c. 3 d. None of these

23. Decimal expansion of a rational number is 327.7081. When the number is expressed in a/b form, what would be the prime factors of b?

a. 2 and 7 b. 3 and 8

c. 2 and 5 d. 2, 3 and 5

24. When a positive integer p is divided by 3, the values of the remainder r are

a. 0, 1, 2 b. 0, 1, 4

c. 0, 1, 2, 3 d. 1, 3, 4

25. $\frac{987}{10500}$ will have

a. Non-Terminating, non repeating decimal expansion

b. Terminating decimal expansion

c. Non-Terminating repeating decimal expansion

d. None of these

26. The H.C.F of 441, 567 and 693 is

a. 11 b. 221

c. 63 d. 126

27. On an evening walk, three people step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the least distance each should walk so that each can cover the same distance in complete steps?

a. 2555 cm b. 2825 cm

c. 2520 cm d. 2626 cm

28. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^p \times 5$ and LCM $(a, b, c) = 2^3 \times 3^2 \times 5$, then p =

a. 6 b. 4

c. 9 d. 2

29. The rational number of the form $\frac{a}{b}, b \neq 0$, a and b are positive integers, which represents $0.1\overline{34}$ i.e., (0.1343434...) is

a. $\frac{133}{990}$ b. $\frac{123}{990}$

c. $\frac{123}{999}$ d. $\frac{134}{999}$

30. Find the smallest number which is a perfect square and is divisible by each of 16, 20 and 24.

a. 600 b. 2800

c. 3600 d. 2400

31. Select the one having a terminating decimal expansion:

a. $\frac{77}{210}$ b. $\frac{22}{7}$

c. $\frac{23}{8}$ d. $\frac{125}{441}$

32. 1. The L.C.M. of a and 18 is 36.

2. The H.C.F. of a and 18 is 2.

Find the value of number a.

a. 4 b. 5

c. 6 d. 1

33. The number $3^{13} - 3^{10}$ is divisible by

a. 3 and 5 b. 2, 3 and 13

c. 2, 3 and 10 d. 3 and 10

34. Among the following rational numbers, which one has a non-terminating repeating decimal expansion?

a. $\frac{45}{3125}$ b. $\frac{71}{512}$

c. $\frac{24}{200}$ d. None of these

35. Find the smallest number which on being divided by 15, leaves 5 as remainder, on being divided by 25, leaves a remainder of 15 and on being divided by 35 leaves a remainder of 25.

a. 1050 b. 525

c. 540 d. 515

Level-II

1. The number $10^A - 1$ is divisible by 11 for,

a. all value of A

b. even values of A

c. odd values of A

d. A must be multiple of 11

2. If a satisfies $|a - 1| + |a - 2| + |a - 3| \geq 6$, then

a. $0 \leq a \leq 2$ b. $a \geq -2$ or $a \leq 4$

c. $a \leq -2$ or $a \geq 4$ d. $a \leq 0$ or $a \geq 4$

3. A six digit number which consists of only one type of digits, either 1, 2, 3, 4, 5, 6, 7, 8 or 9, eg 111111, 222222... etc. This six digit number is always divisible by

a. 11 b. 13

c. 7 d. all of these

4. The difference of the squares of two odd natural numbers is divisible by

a. 8 b. 6

c. 14 d. 16

5. For any positive integer a, $a^3 - a$ is divisible by

a. 2 b. 6

c. 4 d. none of these

6. $p = 5 + 2\sqrt{6}$ and $q = \frac{1}{p}$, then $p^2 + q^2$ is equal to

a. 81 b. 100

c. 98 d. none of these

7. Find the greatest number which divides the three numbers 120, 124, 256.

a. 6 b. 8

c. 4 d. 18

8. Find the number of natural numbers lying between 200 and 400 which are divisible by 4 and 5.

a. 10 b. 10

c. 9 d. none of these

9. Evaluate $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + ...\infty}}}}$.

a. 3 b. 2.49

c. 2.89 d. 2.45

10. Find the number of perfect cubes in the sequence $1^1, 2^2, 3^3, 4^4, ..., 100^{100}$.

a. 32 b. 36

c. 37 d. 40

11. If $2^p = 3^q = 6^{-r}$, then $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$ is equal to

a. 0 b. –1

c. –2 d. 2

12. If two positive integers x and y are written as $x = p^3q^2$ and $y = pq^3$; where p and q are distinct primes, then find the HCF of x and y.

a. pq^2 b. p^2q^2

c. pq^3 d. None of these

13. If $|a - 2| < 3$, then

a. $1 < a < 3$ b. $-1 < a < 5$

c. $-1 < a < 2$ d. $0 < a < 6$

14. Find the largest number which will divide 398, 436 and 542 and leave 7, 11 and 15 as remainders, respectively.

a. 17 b. 16

c. 20 d. 19

15. If $a < 0 < b$, then

a. $\frac{1}{b} < \frac{1}{a}$ b. $\frac{1}{a^2} > \frac{1}{ab} > \frac{1}{b^2}$

c. $\frac{1}{a} > 1$ d. $\frac{1}{a} < \frac{1}{b}$

Assertion & Reason Type Questions

Direction (Q1 to Q5): In the following Questions, the Assertion and Reason have been put forward. Read the statements carefully and choose the correct alternative from the following:

(a) Both the Assertion and the Reason is correct and the Reason is the correct explanation of the Assertion.

(b) The Assertion and the Reason is correct but the Reason is not the correct explanation of the Assertion.

(c) Assertion is true but the Reason is false.

(d) The statement of the Assertion is false but the Reason is true.

1. **Assertion:** Rational number, $\frac{13}{3125}$ is a terminating decimal fraction.

   Reason: If b = $2^m.5^n$ where m, n are non-negative integers, then $\frac{a}{b}$ is a terminating decimal fraction.

2. **Assertion:** There is a decimal number 34.12345, when it is expressed in the form $\frac{a}{b}$, $b \neq 0$, it is of the form $2^m \times 5^n$, where m, n are non-negative integers.

   Reason: Decimal number 34.12345 is a terminating decimal fraction.

3. **Assertion:** If H.C.F. of two numbers is 16 and their product is 3072, then their L.C.M. is 162.

   Reason: If p, q are two positive integers, then H.C.F. × L.C.M. = p × q.

4. **Assertion:** 2 is an example of a rational number.

   Reason: The square roots of all positive integers are irrational numbers.

5. **Assertion:** For any two positive integers p and q, HCF (p, q) × LCM (p, q) = p × q

   Reason: If the HCF of two numbers is 5 and their product is 150, then their LCM is 40.

Case-Based Type Questions

Case-Based-I: The department of Mathematics is conducting an International Seminar. In the seminar, the number of participants in Mathematics, Physics and Computer Science are 60, 84 and 108 respectively. The arrangement was made such that, the same number of participants, all of them being in the same subject, are to be seated in each room. A separate room was alloted for all the official other than participants.

1. Total number of participants will be:

   a. 160 b. 84
   c. 220 d. none of these

2. Find the LCM of 60, 84 and 108.

   a. 3780 b. 840
   c. 544320 d. 12

3. If in each room, the same number of participants are to be seated and all of them being in the same subject, then find the least number of rooms required for them.

   a. 16 b. 20
   c. 14 d. 21

Case-Based-II: Priya works as a librarian in Bright scholar International School in Pune. She ordered books on English, Science and Mathematics. She received 96 English books, 240 Science Books and 336 Maths books. She wants to arrange these books in stacks such that each stack contains the same number of books and that too of only one subject. She also wants to keep the number of stacks minimum.

1. The number of books in each stack will be:

   a. 46 b. 54
   c. 48 d. 68

2. Number of stacks formed will be:

   a. 14 b. 15
   c. 7 d. 18

3. Find the height of each stack of English books if the thickness of each English book is 3 cm.

   a. 160 cm b. 144 cm
   c. 180 cm d. 124 cm

Multi Correct MCQ's

1. Select the rational number having terminating decimal expansion:

   a. $\frac{343}{2^3 \times 5^3 \times 7^3}$ b. $\frac{22}{7}$
   c. $\frac{11}{25000}$ d. none of these

2. If $p = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $q = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, then $\frac{q^2 - p^2}{\sqrt{5}}$ is divisible by:

   a. 12 b. 2
   c. 6 d. 17

3. If A = $55^3 + 17^3 - 72^3$, then A is divisible by:

   a. 15 b. 11
   c. 3 d. 17

4. The sum of three consecutive even numbers is always divisible by

a. 4 b. 3

c. 2 d. 12

Olympiad & NTSE Type Questions

1. Number 1146600 can be written as the product of two factors, in how many ways?

a. 100 b. 273

c. 216 d. 108

2. When 2^{39} is divided by 39, the remainder obtained is

a. 8 b. 6

c. 0 d. 12

3. If $a = \frac{1}{2-\sqrt{3}}$, the value of $a^3 - 2a^2 - 7a + 5$ is

a. –6 b. 3

c. 0 d. 8

4. If a, b, c are positive real numbers, then

$\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} =$

a. 1 b. –1

c. $\frac{1}{\sqrt{abc}}$ d. $\frac{1}{abc}$

5. The rationalizing factor of $\frac{1}{\sqrt{x} \pm \sqrt{y}}$ is

a. $\frac{1}{\sqrt{x} \mp \sqrt{y}}$ b. $\frac{1}{\sqrt{x} \pm \sqrt{y}}$

c. $\sqrt{x} \pm \sqrt{y}$ d. None of these

6. Find the greatest number which when divides 969 and 2059, leaves remainder as 9 and 11 respectively.

a. 118 b. 64

c. 32 d. 124

7. What is the remainder when $14^{15^{18}}$ is divided by 5?

a. 1 b. 2

c. 4 d. 3

8. Find unit's digit in $a = 7^{17} + 7^{34}$.

a. 7 b. 8

c. 10 d. 6

9. Last two digits of 33^{288} will be

a. 41 b. 81

c. 56 d. 93

10. Find the unit's digit of $(9^0 + 9^1 + 9^2 + 9^3 + ... + 9^{2009})$.

a. 9 b. 5

c. 0 d. 7

11. The 288th term of the series a, b, b, c, c, c, d, d, d, d, ... is?

a. u b. x

c. w d. b

12. Find the greatest power of 7 contained in 926!

a. 148 b. 1078

c. 152 d. none of these

13. Find the remainder when $3^{57} + 27$ is divided by 28.

a. 26 b. 20

c. 6 d. 32

14. How any zeroes will be there at the end of $36!^{36!}$.

a. 6! b. 8 × 36!

c. 36! d. $8^3 \times 36!$

15. Find the value of n for which $2^{200} - 2^{192}.31 + 2^n$ is a perfect square.

a. 198 b. 208

c. 232 d. 146

Explanations

Subjective Questions

Very Short Answer Type Questions

1.
(2) 1080
(2) 540 — 1080 divided by 2 gives 540
(2) 270 — 540 divided by 2 gives 270
(3) 135 — 270 divided by 2 gives 135
(3) 45 — 135 divided by 3 gives 45
(3) 15 — 45 divided by 3 gives 15
(5) 5 — 15 divided by 3 gives 5

$\therefore$ 1080 = 2 × 2 × 2 × 3 × 3 × 3 × 5
= $2^3 \times 3^3 \times 5$

2. r = 17 × 2
= 34
q = r × 2
= 34 × 2
= 68
and p = q × 2
= 68 × 2
= 136
i.e., p = 136, q = 68 and r = 34.

3. As we know that, if a and b are two different positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.

$\therefore$ Irrational number between $\sqrt{2}$ and $\sqrt{4}$ is $\sqrt{\sqrt{2} \times \sqrt{4}} = \sqrt{\sqrt{8}} = 8^{1/4}$

Irrational number between $\sqrt{2}$ and $8^{1/4}$ is $\sqrt{\sqrt{2} \times 8^{1/4}} = 2^{1/4} \times 8^{1/8}$

Hence, required irrational numbers are $8^{1/4}$ and $2^{1/4} \times 8^{1/8}$.

4. $\sqrt{ab}$ is an irrational number lying between a and b, if a and b are two different positive rational numbers such that ab is not a perfect square of a rational number.

$\therefore$ Irrational number between 2 and 3.5 is

$\sqrt{2 \times 3.5} = \sqrt{7}$

Similarly, irrational number between 2 and $\sqrt{7}$ is $\sqrt{2 \times \sqrt{7}}$

Hence, required numbers are $\sqrt{7}$ and $\sqrt{2 \times \sqrt{7}}$.

5. The prime factorisation of number 6:

6 = 2 × 3

The prime factorisation of given number 6^n:

$6^n = 2^n \times 3^n$

For the number 6^n to be divisible by 7, the prime factorisation of 6^n should contain the prime number 7. As prime factorisation of 6^n contains only numbers 2 and 3.

$\therefore$ 6^n is not divisible by 7.

6. $\frac{7}{80} = \frac{7}{2^4 \times 5}$

$= \frac{7 \times 5^3}{2^4 \times 5^4}$

$= \frac{875}{10^4}$

$= 0.0875$

7. It is given that $P^2 = Q^2 + R^2$

Also, $\frac{Q}{R} = \frac{12}{5}$ (Given : 5Q = 12R)

Therefore, the least possible values of Q and R are 12 and 5 respectively. Hence

$P^2 = 12^2 + 5^2$

$\Rightarrow$ $P^2 = 169$

$\Rightarrow$ P = 13.

8. Required number = LCM (4, 5 and 6) + 3

= 60 + 3

= 63

9. Given L.C.M. (150, 100) = 300

Since, the product of numbers 150 and 100

= 150 × 100

= 1500

And, we know:

$$\text{H.C.F} \times \text{L.C.M} = \text{Product of two numbers}$$

$$\text{H.C.F. (150, 100)} = \frac{\text{Product of 150 and 100}}{\text{L.C.M.(150,100)}}$$

$$= \frac{150 \times 100}{300}$$

$$= 50$$

∴ H.C.F (150, 100) = 50

10. $$\text{H.C.F of fraction} = \frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$$

$$\text{HCF of given fractions} = \frac{\text{HCF of } (2, 8, 16, 10)}{\text{LCM of } (3, 9, 81, 27)} = \frac{2}{81}$$

$$\text{L.C.M of fraction} = \frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$$

$$\text{L.C.M of given fractions} = \frac{\text{LCM of } (2, 8, 16, 10)}{\text{HCF of } (3, 9, 81, 27)} = \frac{80}{3}$$

Short Answer Type Questions

1. $8 = (2^3)$,

15 = (3)(5),

21 = (3)(7)

∴ LCM of (8, 15, 21) = $(2^3)(3)(5)(7)$ = 840.

Required number = Multiple of 840 which is nearest to 100000

= 100800. [∵ 100800 = (840)(120)].

Hence, the required number is 100800.

2. Let us suppose, to the contrary that $3-\sqrt{5}$ is a rational number i.e.,

$3-\sqrt{5} = \frac{a}{b}$, a and b are co-prime and b ≠ 0.

$\Rightarrow \sqrt{5} = 3 - \frac{a}{b}$

Since, a and b are integers, Thus, $3-\frac{a}{b}$ is a rational number.

$\Rightarrow \sqrt{5}$ is also a rational number.

But, this contradicts the fact that $\sqrt{5}$ is irrational.

Hence, $3-\sqrt{5}$ is an irrational number.

3. To find the maximum number of students in a group, we need to find H.C.F of 437 and 342.

H.C.F (437, 342) = 19

∴ number of groups = $\frac{437}{19} + \frac{342}{19} = 41$ groups

4. Since 3823a is obtained after multiplying 484 with a certain number.

It is clear that 3823a is divisible by 484. Therefore 484 is a factor of 3823a.

The prime factorisation of number 484:

484 = 2 × 2 × 11 × 11

Hence the number 484 has only 2 prime factors i.e., 2 and 11.

Therefore, 3823a is divisible by both 2 and 11.

Also, we know that if the last digit of a whole number is even, then the entire number is divisible by 2.

The last digit of given number is 'a', so we can take any value from 0, 2, 4, 6 or 8 for 'a'.

So, the possible numbers are 38230, 38232, 38234, 38236 and 38238.

Out of all these numbers, only 38236 is divisible by 11.

Therefore, value of a is 6.

5. First 100 multiples of 10 are 10, 20, 30, 40, ..., 990, 1000.

We have to find the number of zeroes in 10 × 20 × 30 × ..... 990 × 1000.

It can be written as:

$10^{100}(1 \times 2 \times 3 \times 4 \times ... \times 99 \times 100)$

Now the number of zeroes in (1 × 2 × 3 × 4 × ... × 99 × 100).

As we know that the trailing zero is formed when a multiple of 5 is multiplied with a multiple of 2. Now, all we have to do is count the number of 5's and 2's in the multiplication.

Let's count the 5's first. 5, 10, 15, 20, 25 and so on making a total of 20. However, there is more to this. Since 25, 50, 75 and 100 have two 5's in each of them (25 = 5 * 5, 50 = 2 * 5 * 5, …), you have to count them twice. This makes the grand total 24.

Moving on to count the number of 2's. 2, 4, 6, 8, 10 and so on. Total of 50 multiples of 2's, 25 multiples of 4's (count these once more), 12 multiples of 8's (count these once more) and so on… The grand total comes out to 97.

Each pair of 2 and 5 will cause a trailing zero. Since we have only 24 5's, we can only make 24 pairs of 2's and 5's thus the number of trailing zeros in (1 × 2 × 3 × 4 × ... × 99 × 100) is 24.

Therefore, the total umber of zeros in the product = 100 + 24

= 124

∴ 124 number of zeroes will be there at the end of the product.

6. As we know that,

2 hours = 120 minutes

4 hours = 240 minutes

5.5 hours = 330 minutes

i.e. 120 min, 240 min and 330 min.

We need to find the time after which all the three people will meet again at the starting point. For this, there will be a number that is divisible by 120, 240 and 330 and that will be the time when all the three meet again at the starting point. To find this we have to take LCM of all three numbers.

L.C.M. (120, 240 and 330) = 2640 minutes

Time = 2640/60 hours

= 44 hours.

7. Let $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2007}=p$

Then, $\left(p+\frac{1}{2008}\right)(1+p)-\left(1+p+\frac{1}{2008}\right)(p)$

$=p+\frac{1}{2008}+p^2+\frac{p}{2008}-p-p^2-\frac{p}{2008}$

$=\frac{1}{2008}$

8. We need to find the L.C.M of 7, 8, 11 and 12.

L.C.M (7, 8, 11, 12) = 1848

Therefore, after every 1848 seconds bells will ring together.

In 3 hours, there are 3600 × 3 = 10800 secs.

number of times the bell would ring = $\frac{10800}{1848}=5$

Therefore, bells will ring together 5 times in the next 3 hours.

9. To get the required number, we need to find the H.C.F of 245 and 1029 after subtracting 5 from each number:

HCF = (245 – 5, 1029 – 5)

= HCF(240, 1024)

240 = $2^4 \times 3 \times 5$

1024 = 2^{10}

∴ HCF (240, 1024) = 2^4

= 16

Hence, the required number is 16.

10. 445 – 4 = 441

572 – 5 = 567

699 – 6 = 693

H.C.F. of 441, 567 and 693 is,

441 = 3 × 3 × 7 × 7

567 = 3 × 3 × 3 × 3 × 7

693 = 3 × 3 × 7 × 11

The common factors are 3 × 3 × 7 = 63.

∴ H.C.F (441, 567, 693) = 63

Hence, 63 is the required number.

Long Answer Type Questions

1. If a = c then

$a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow \sqrt{b}=\sqrt{d}$

$\Rightarrow b=d$

If a ≠ c then there exist a positive rational number k such that a = c + k

Now, $a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow c+k+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow k+\sqrt{b}=\sqrt{d}$ ..(i)

On squaring both sides, we get:

$\Rightarrow \left(k+\sqrt{b}\right)^2=\left(\sqrt{d}\right)^2$

$\Rightarrow k^2+2\sqrt{b}k+b=d$

$\Rightarrow d-k^2-b=2k\sqrt{b}$

$\Rightarrow \sqrt{b}=\frac{d-k^2-b}{2k}$

$\Rightarrow \sqrt{b}$ is rational

$\because$ d, k, b are rational numbers

$\therefore \dfrac{d-k^2-b^2}{2k}$ is rational

$\therefore$ b is the square of a rational number.

From equation (i), we have

$\sqrt{d}=k+\sqrt{b}$

$\Rightarrow \sqrt{d}$ is rational

$\therefore$ d is the square of rational number

Therefore, either a = c and b = d; or b and d are the squares of rationals.

2. Let us assume, to the contrary, that $\sqrt{7}+\sqrt{11}$ be a rational number $\dfrac{a}{b}$ (a, b are integers, $b \neq 0$)

$\therefore \sqrt{7}+\sqrt{11}=\dfrac{a}{b}$

On squaring both sides:

$$7+11+2\sqrt{77}=\frac{a^2}{b^2}$$

$\Rightarrow \sqrt{77}=\dfrac{1}{2}\left(\dfrac{a^2}{b^2}-18\right)$

$\Rightarrow \sqrt{77}=\dfrac{a^2-18b^2}{2b^2}$

As a and b are integers,

$\therefore \dfrac{a^2-18b^2}{2b^2}$ is a rational number,

$\Rightarrow \sqrt{77}$ is a rational number.

But this contradicts the fact that $\sqrt{77}$ is irrational.

$\therefore$ Our assumption is wrong.

Hence $\sqrt{7}+\sqrt{11}$ is an irrational number.

3. As we already know that any odd positive integer is of the form 4q + 1 or 4q + 3 for some integer q.

Therefore, we have the following two cases.

Case-I: When n = 4q + 1

Here, we have n = 4q + 1

On subtracting 1 from both sides after squaring, we have

$$\begin{aligned} n^2 - 1 &= (4q + 1)^2 - 1 \\ &= 16q^2 + 8q + 1 - 1 \\ &= 8q(2q + 1) \\ &= 8r \end{aligned}$$

where r = q(2q + 1), which is an integer

$\therefore n^2 - 1$ is divisible by 8.

Case-II: When n = 4q + 3

Here, we have n = 4q + 3

On subtracting 1 from both sides after squaring, we have

$$\begin{aligned} n^2 - 1 &= (4q + 3)^2 - 1 \\ &= 16q^2 + 24q + 9 - 1 \\ &= 16q^2 + 24q + 8 \\ &= 8(2q^2 + 3q + 1) \\ &= 8(2q + 1)(q + 1) \\ &= 8r \end{aligned}$$

where r = (2q + 1)(q + 1), which is an integer.

$\therefore n^2 - 1$ is divisible by 8.

Hence $n^2 - 1$ is divisble by 8.

4. We know that for some integer q, any positive integer is of the form 5q, 5q + 1, 5q + 2, 5q + 3 or 5q + 4 and only one out of these possibilities can occur. Therefore, we have the following cases:

Case-I: When n = 5q

Here, we have

n = 5q, which is divisible by 5

Now, n = 5q

$\therefore$ n + 4 = 5q + 4

i.e. n + 4 leaves remainder 4 when divided by 5

Hence, n + 4 is not divisible by 5.

$$\begin{aligned} \text{Now} \quad n + 8 &= 5q + 8 \\ &= 5(q + 1) + 3 \\ &= 5m + 3, \end{aligned}$$

where m is an integer (q + 1 = m)

Clearly, n + 8 is also not divisible by 5.

$$\begin{aligned} \text{Again, } n + 12 &= 5q + 12 \\ &= 5(q + 2) + 2 \\ &= 5m + 2, \end{aligned}$$

where m is an integer (q + 2 = m)

Clearly n + 12 is also not divisible by 5.

$$\begin{aligned} \text{Now} \quad n + 16 &= 5q + 16 \\ &= 5(q + 3) + 1 \\ &= 5m + 1 \end{aligned}$$

where m is an integer (q + 3 = m)

$\therefore$ n + 16 is also not divisible by 5

Thus, n = 5q is the only one out of n, n + 4, n + 8, n + 12 and n + 16 which is divisible by 5.

In the same way, this result can be proved for the rest of the cases.

5. **Case I:** Let us suppose, n be an even positive integer.

$\Rightarrow \quad n = 2q$

Squaring and then subtracting n from both sides, we have

$$n^2 - n = (2q)^2 - 2q$$
$$= 4q^2 - 2q$$
$$= 2q(2q - 1)$$
$$n^2 - n = 2r,$$

where $\quad r = q(2q - 1)$

$\therefore$ $(n^2 - n)$ is divisible by 2.

Case II: Let us suppose, n be an odd positive integer.

$\Rightarrow \quad n = 2q + 1$

Squaring and then subtracting n from both sides, we have

$$n^2 - n = (2q + 1)^2 - (2q + 1)$$
$$= 4q^2 + 1 + 4q - 2q - 1$$
$$= 4q^2 + 2q$$
$$= 2q\,(2q + 1)$$
$$n^2 - n = 2r,$$

where $\quad r = q(2q + 1)$

$(n^2 - n)$ is divisible by 2.

Here, $(n^2 - n)$ is divisible by 2 for every positive value of integer n.

Integer Type Questions

1. Let $\sqrt{n-1} + \sqrt{n+1} = \dfrac{a}{b}$ $\quad \left(\dfrac{a}{b} \text{ is rational}\right)$

On squaring both sides:

$$\frac{a^2}{b^2} = 2n + 2\sqrt{n^2 - 1}$$

$$\frac{a^2}{b^2} - 2n = 2\sqrt{n^2 - 1}$$

For all values of n greater than 1,

n2 – 1 is not a perfect square

so, $\sqrt{n^2 - 1}$ is an irrational number

but $\dfrac{a^2}{b^2} - 2n$ is a rational

$\therefore$ Our assumption is wrong.

Hence, number of positive integers for which 'n' is a rational is zero

2. $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + ... + 4^{99} + 9^{100}$

$$\underbrace{4 + 9^2}_{\text{last digit}=5} + \underbrace{4^3 + 9^4}_{\text{last digit}=5} + \underbrace{4^5 + 9^6}_{\text{last digit}=5} + ... + \underbrace{4^{99} + 9^{100}}_{\text{last digit}=5}$$

$\Rightarrow$ 5 + 5 + ... + 5 (50 times i.e. even no. of times)

$\therefore$ unit's digit = 0

3. Let N be any positive integer which when divided by 342 gives q as quotient and 47 as remainder.

According to the Euclid's division lemma

$$N = 342\,q + 47$$
$$= 342\,q + 38 + 9$$
$$= 19\,(18q + 2) + 9$$

$\therefore$ The given number when divided by 19 gives $(18q + 2)$ as quotient and 9 as remainder.

4. We know, dividend = divisor × quotient + remainder

By using division algorithm, we get

$$294 = 96 \times 3 + 6$$
$$96 = 6 \times 16 + 0$$

$\therefore$ HCF of 96 and 294 is 6.

5. For finding the time, when both runners will meet, we need to find the L.C.M of 3 and 5.

$$\text{L.C.M}\,(3, 5) = 3 \times 5$$
$$= 15 \text{ hrs.}$$

As ab is two digit number $\therefore$ a = 1 & b = 5

$\Rightarrow \quad a + b = 1 + 5$

$\quad\quad = 6$

6. Let the two numbers be a and b.

Since their HCF is 28,

$\therefore$ a = 28x, b = 28y, such that x, y are co-primes.

As, we already know that,

$$\text{LCM} \times \text{HCF} = a \times b$$
$$168 \times 28 = 28x \times 28y$$
$$xy = 6$$

$\Rightarrow \quad xy = 1 \times 6$

or $\quad xy = 2 \times 3$

$\therefore$ Only two pairs of numbers are there.

Hence, the correct answer is 2.

7. The prime factorisation of 3025:

$$3025 = 5 \times 5 \times 11 \times 11$$
$$3025 = 5^2 \times 11^2$$

Therefore total number of factors of 3025 is (2+1)(2+1) = 9.

8. HCF of the first 1000 even natural numbers is 2.

Therefore, the correct answer is 2.

9. $8^1 = 8$ ends with 8.

$8^2 = 64$ ends with 4

$8^3 = 512$ ends with 2

$8^4 = 4096$ ends with 6

and after that it starts repeating.

Let 'a' be any number which when divided by 4 gives q as quotient and r as remainder where $0 \le r < 4$ i.e., r = 0, 1, 2, 3.

According to the Euclid's division lemma

$a = 4q + r$

$\therefore$ 8^a can be written as 8^{4q+r}.

For 8^{4q+1} ends with 8.

For 8^{4q+2} ends with 4.

For 8^{4q+3} ends with 2.

For 8^{4q+0} ends with 6.

7 follows the similar logic and ends with 7, 9, 3, 1

For 7^{4q+1} ends with 7.

For 7^{4q+2} ends with 9.

For 7^{4q+3} ends with 3.

For 7^{4q+0} ends with 1.

$\therefore$ $8^{4q+1} - 7^{4q+1}$ ends with 1.

$8^{4q+2} - 7^{4q+2}$ ends with 5.

$8^{4q+3} - 7^{4q+3}$ ends with 9.

$8^{4q} - 7^{4q}$ ends with 5.

$\therefore$ $8^a - 7^a$ must end with the digits either 1, 5, or 9.

Here, there are 3 possible units digit.

Therefore, the correct answer is 3.

10. Given: LCM = 924 and HCF = 7

Let the two numbers be 7a and 7b, where 7 is the HCF of 7a and 7b,

i.e., a and b are co-primes.

$$\begin{aligned} \text{LCM} &= 924 \\ 7ab &= 924 \\ ab &= 132 \\ \Rightarrow \quad ab &= 1 \times 132 \\ \text{or} \quad &= 3 \times 44 \\ \text{or} \quad &= 4 \times 33 \\ \text{or} \quad &= 11 \times 12 \end{aligned}$$

$\therefore$ The minimum possible sum

$$\begin{aligned} &= (Ha + Hb) \qquad [H = HCF] \\ &= 7 \times 11 + 7 \times 12 \\ &= 161 \end{aligned}$$

Therefore, the correct answer is 161.

Multiple Choice Questions

Level-I

1. (b) $\because$ $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

$$= \frac{1}{2}\left[(-2)^2 + (-2)^2 + (4)^2\right]$$

$$= 12$$

2. (c)
$$\begin{aligned} n &= (32)^{3/5} \\ &= (2^5)^{3/5} \\ &= (2)^3 \\ &= 8 \end{aligned}$$

3. (a)
$$\begin{aligned} 6 &= 2 \times 3, \\ 72 &= 2^3 \times 3^2, \\ 120 &= 2^3 \times 3 \times 5 \end{aligned}$$

$\therefore$ LCM (6, 72, 120) = 360

4. (b) $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

$\therefore$ power of 2 = 4

5. (a) $n = (3^2 \times 7) \times (8 \times 25)$

$n = (3^2 \times 7) \times (2 \times 4 \times 25)$

$= (9 \times 7 \times 2) \times 100$

$= 126 \times 100$

$\therefore$ two zeroes.

6. (a) Between two rational numbers, we can insert as many rational number as we want.

7. (a)
$$A = 0.\overline{2}$$

$$A = 0.2222 \qquad \text{...(i)}$$

Multiplying by 10 on both side:

$$10A = 2.222222 \qquad \text{...(ii)}$$

On subtracting equation (i) from (ii)

$$\begin{aligned} 10A - A &= 2.2222 - 0.2222... \\ 9A &= 2 \\ A &= \frac{2}{9} \end{aligned}$$

8. (d) let $a = 0.9999$ ...(i)

On multiplying equation (i) by 10

$$10a = 9.9999 \qquad \text{...(ii)}$$

On subtracting equation (i) form (ii)

$$\begin{aligned} 9a &= 9 \\ a &= \frac{9}{9} \\ &= 1 \end{aligned}$$

9. (c) Let
$$\begin{aligned} A &= \pi + 2\sqrt{2} \\ &= 3.1415 + 2 \times 1.4142 \\ &= 3.1415 + 2.8284 \\ &= 5.9699 \end{aligned}$$

10. (a) $\frac{1}{a} = 0.\overline{142857}$

$\frac{3}{a} = 0.\overline{428571}$

$2\frac{3}{a} = 2.\overline{428571}$

11. (b) $\frac{14587}{1250} = \frac{14587}{(5)^4 \times 2}$

$$= \frac{14587}{(5)^4 \times (2)^4} \times 2^3$$

$$= \frac{14587 \times 2^3}{10000}$$

Therefore, the given number will terminate after 4 decimal places.

12. (a) The power of 2 in the prime factorization of denominator determines the termination of any rational number.

13. (a) The least number divisible by all the numbers from 1 to 10 will be the L.C.M of the following number,

1 = 1

2 = 2 × 1

3 = 3 × 1

4 = 2 × 2

5 = 5 × 1

6 = 2 × 3

7 = 7 × 1

8 = 2 × 2 × 2

9 = 3 × 3

10 = 2 × 5

So, the L.C.M. of these number is
1 × 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

Hence, the least number divisible by all the numbers from 1 to 10 is 2520

14. (c) $x = 2 + \sqrt{3}$

$$\Rightarrow \frac{1}{x} = \frac{1 \times (2 - \sqrt{3})}{(2 + \sqrt{3}) \times (2 - \sqrt{3})} = \frac{(2 - \sqrt{3})}{(4 - 3)} = (2 - \sqrt{3})$$

$$\Rightarrow x - \frac{1}{x} = 2 + \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3}$$

15. (d) If n is any integer irrespective of even or odd, it will become even when it gets multiplied by 2. Therefore, general expression for even integer is 2n.

16. (b) As we know that the number 2n will always be even, therefore, if 1 is added to it, then the number will always be odd.

17. (c) Product of an irrational number with a non-zero rational number is always irrational.

eg. $\sqrt{2} \times \frac{6}{8}$ = (irrational) × (rational) = irrational

18. (d) On using Euclid's division algorithm,

b = aq + r

117 = 65 × 1 + 52

65 = 52 × 1 + 13

52 = 13 × 4 + 0

∴ H.C.F (65, 117) = 13

H.C.F = 65n – 117 [Given]

∴ 65n – 117 = 13

⇒ 65n = 130

⇒ n = 2

∴ value of n is 2.

19. (c) As, the required number on dividing 70 and 125, leaves remainder 5 and 8 respectively, therefore after subtracting these remainders from the numbers, we have the numbers:

65 = (70 − 5), 117 = (125 − 8) which is divisible by the required number.

Required number = H.C.F of (65, 117)

117 = 65 × 1 + 52

65 = 52 × 1 + 13

52 = 13 × 4 + 0

H.C.F (65, 117) = 13

Therefore, 13 is the largest number which divides 70 and 125 leaving remainders 5 and 8 respectively.

20. (a) a = p × q × q

b = p × p × p × q

As, L.C.M is the product of the highest power of each prime factor involved in the numbers

Therefore, L.C.M of a and b = p^3q^2.

21. (b) n can be even or odd

Case 1: If n is even

n = 2a

Then,

p = $(2a)^2 - 1$

p = $4a^2 - 1$

For a = –1

$4(-1)^2 - 1 = 3$, not divisible by 8

Case 2: If n is odd

n = 2a + 1

Then,

p = $(2a + 1)^2 - 1$

p = $4a^2 + 4a + 1 - 1$

p = $4a^2 + 4a$

For a = 1

p = $4a^2 + 4a = 8$, which is divisible by 8.

Similarly we can check for any integer.

22. (a) $5^{2a} - 2^{2a}$ is of the form $a^{2a} - b^{2a}$ which is divisible by both (a + b) and (a – b).

So, $5^{2a} - 2^{2b}$ is divisible by both 7, 3.

23. (c) This can be explained as,

$$327.7081 = \frac{3277081}{10000} = \frac{a}{b}$$

$\therefore$ b = 10000
$= 10^4$
$= (2 \times 5)^4$
$= 2^4 \times 5^4$

24. (a) According to Euclid's division lemma,

p = 3q + r, where $0 < r < 3$

$\therefore$ Possible value of r will be 0, 1 or 2.

25. (b) After simplification,

$$\frac{987}{10500} = \frac{47}{500} = \frac{47}{5^3 \times 2^2}$$

As factors of denominator contain $5^3 \times 2^2$ and which is of the form $5^m \times 2^n$. Therefore, this is a terminating decimal expansion.

26. (c) $441 = 3 \times 3 \times 7 \times 7$

$567 = 3 \times 3 \times 3 \times 3 \times 7$

$693 = 3 \times 3 \times 7 \times 11$

Hence, H.C.F of 693, 567 and 441 is 63.

27. (c) To find the required minimum distance, we need to find the L.C.M of 40, 42 and 45 cm.

$40 = 2 \times 2 \times 2 \times 5$

$42 = 2 \times 3 \times 7$

$45 = 3 \times 3 \times 5$

L.C.M. $= 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 7 = 2520$

28. (d) Here $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^p \times 5$

LCM (a, b, c) = maximum power of 2 × maximum power of 3 × maximum power of 5

$= 2^3 \times 3^p \times 5$

According to the question,

LCM (a, b, c) $= 2^3 \times 3^2 \times 5$

$\Rightarrow 2^3 \times 3^p \times 5 = 2^3 \times 3^2 \times 5$

Comparing both sides, the value of p = 2

29. (a) $0.1\overline{34} = \frac{134-1}{990} = \frac{133}{990}$

30. (c) L.C.M. of 16, 20 and 24 is 240. The smallest multiple of 240 which is a perfect square is 3600. Therefore, 3600 is the least number which is a perfect square and is divisible by each of 16, 20 and 24.

31. (c) Condition for terminating decimal expansion is that the denominator must contain only 2 or only 5 or 2 and 5 as factors.

Here, $\frac{23}{8} = \frac{23}{(2)^3}$

(only 2 as a factor of denominator. Hence, terminating)

32. (a) We know L.C.M. × H.C.F. = Product of two numbers

$\therefore 36 \times 2 = 18 \times a$

Hence, required number $= \frac{36 \times 2}{18} = 4$

Therefore, value of a is 4.

33. (b) $3^{13} - 3^{10} = 3^{10}(3^3 - 1)$
$= 3^{10}(26)$
$= 2 \times 13 \times 3^{10}$

Therefore, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

34. (d) From the given options, denominators of all fractions i.e. 3125, 512, 200 contain factors of the form $2^m \times 5^n$, (where m and n are whole numbers). So given fractions have a terminating decimal expansion.

35. (d) Among the given options 515 is short by 10 for complete division by 15, 25 or 35.

Level-II

1. (b) Since $(10^A - 1) = (10^{2a} - 1) = (10^2)^a - 1$

$= 100^a - 1^a$ [let A = 2a]

$= (100 - 1)(z)$

$= 99(z)$

Therefore, for any even power it is divisible by 11.

i.e. A = 2a.

2. (d) Since |a| = a or –a

$\therefore |a - 1| + |a - 2| + |a - 3| \geq 6$

$\Rightarrow 3a - 6 \geq 6$ or $6 - 3a \geq 6$

$\Rightarrow a \geq 4$ or $a \leq 0$

3. (d) Option (d) is the correct answer, as 7, 11 and 13, all are the factors of such a number.

4. (a) Let two odd natural no's be (2a + 1) and (2b + 1), then

$(2a + 1)^2 - (2b + 1)^2$

$= (2a + 1 + 2b + 1)(2a + 1 - 2b - 1)$

$= 4(a - b)(a + b + 1)$

= (4 × even number × odd number) is a multiple of 8.

5. (b) $a^3 - a = (a - 1)a(a + 1)$ is the product of 3 consecutive integers and is divisible by 6.

6. (c) $p = \left(5 + 2\sqrt{6}\right)$

$$q = \frac{1}{p} = \frac{1 \times \left(5 - 2\sqrt{6}\right)}{\left(5 + 2\sqrt{6}\right) \times \left(5 - 2\sqrt{6}\right)}$$

$$=\frac{\left(5-2\sqrt{6}\right)}{(5)^2-\left(2\sqrt{6}\right)^2}$$

$$=\frac{5-2\sqrt{6}}{1}$$

$$p^2 + q^2 = \left(5+2\sqrt{6}\right)^2+\left(5-2\sqrt{6}\right)^2$$

$$=(25+24)+20\sqrt{6}+25+24-20\sqrt{6}$$

$$=98$$

7. **(c)** To find the greatest number which divides all the given three numbers, we need to find the H.C.F.

Required number is HCF of 120, 124 and 256.

To find the HCF of 120, 124 and 256, we will first find the HCF of 120 and 124 by Euclid's division algorithm

By using division algorithm, we get

$$124 = 120 \times 1 + 4$$

$$120 = 4 \times 40 + 0$$

So, HCF (120, 124) = 4

Now, we will find the HCF of 4 and 256.

$$256 = 4 \times 64 + 0$$

∴ HCF (4, 256) = 4

Hence, HCF (120, 124, 256) = 4.

8. **(c)** Number which is divisible by 4 and 5 is the number divisible by 20.

So total numbers between 200 and 400 which are divisible by 20 is 9 (excluding 200 and 400) i.e. 220 .... 380 = (9) terms.

∴ there are 9 natural numbers between 200 and 400, which are divisible by 4 and 5.

9. **(a)** Let $x = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$ ....

$$x=\sqrt{6+x}$$

On squaring both sides, we have

$$x^2 = 6 + x$$

$\Rightarrow x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$\Rightarrow \quad x = 3$ or $x = -2$

($x = -2$ is not possible as the sum of positive numbers ie., $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$ .... cannot be negative).

$\therefore x = 3$

10. **(b)** $1^1, 2^2, 3^3, 4^4 \ldots 100^{100}$

Here, all the numbers having power which are multiple of 3 will be perfect cube, moreover 1, 8 and 64 will also be a perfect cube.

Hence, total number of perfect cubes = 33 + 3 = 36.

11. **(b)** $2^p = 3^q = 6^{-r}$

since $2^p = 6^{-r}$

$\Rightarrow 2 = 6^{-(r/p)}$ ...(i)

Similarly, $3 = 6^{-(r/q)}$ ...(ii)

On multiplying equation (i) and (ii)

$2 \times 3 = 6^{-(r/p)} \times 6^{-(r/q)}$

$$\Rightarrow 6^1 = 6^{-\frac{r}{p}-\frac{r}{q}}$$

$$\Rightarrow -\frac{r}{p}+\frac{(-r)}{q}=1$$

$$\Rightarrow \frac{1}{p}+\frac{1}{q}=-\left(\frac{1}{r}\right)$$

$$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=0$$

12. **(a)**

$$x = p^3q^2$$
$$= p \times p \times p \times q \times q$$
$$y = pq^3$$
$$= p \times q \times q \times q$$

Therefore,

HCF of (x, y) = pq^2

13. **(b)** $|a - 2| < 3$

$\Rightarrow (a - 2) < 3$ or $2 - a < 3$

$\Rightarrow a < 5$ or $a > -1$

$\Rightarrow -1 < a < 5$

14. **(a)** Required number is HCF of (398 – 7), (436 – 11) and (542 – 15)

HCF of (391, 425 and 527) = 17

15. **(d)** $a < 0 \Rightarrow \frac{1}{a} < 0$

And $b > 0 \Rightarrow \frac{1}{b} > 0$

$$\Rightarrow \frac{1}{a} < \frac{1}{b}$$

Assertion & Reason Type

1. **(a)** As, the factors of the denominator 3125 is of the form $2^0 \times 5^5$. i.e. $2^m \times 5^n$

$\therefore \frac{13}{3125}$ is a terminating decimal fraction.

Also, the assertion follows from reason.

2. **(a)** Hence, assertion is true.

As we can see 34.12345 has a terminating decimal expansion

Hence, reason is also true.

Also, the assertion follows from reason.

$$\text{As, } 34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5 \times 5^4}$$

Its denominator is of the form $2^m \times 5^n$

[m = 5, n = 4 are non-negative integers]

Hence, assertion is true

As we can see 34.12345 has a terminating decimal expansion.

Hence, reason is also true.

Also, the assertion follows from reason.

3. **(d)** Here reason is true [standard result]

Assertion is false.

As, H.C.F × L. C. M = Product of two numbers

$$\therefore \quad \text{L.C.M} = \frac{\text{Product of two numbers}}{\text{H.C.F}} = \frac{3072}{16}$$

$$= 192$$

$$\neq 162$$

4. **(c)** Here, reason is false. As $\sqrt{16} = \pm 4,$ which is not an irrational number.

5. **(c)** We have,

LCM (p, q) × HCF (p, q) = p × q

LCM × 5 = 150

$$\text{LCM} = \frac{150}{5} = 30 \neq 40$$

∴ Reason is false.

Cased-Based Type Questions

Case-Based-I

1. **(d)** Total number of participants

= 60 + 84 + 108

= 252

2. **(a)** L.C.M of 60, 84 and 108 is:

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

LCM(60, 84, 108) = $2^2 \times 3^3 \times 5 \times 7$

= 3780

3. **(d)** If in a room all of the participants are of same subject then maximum number of participants to be seated is HCF of 60, 84 and 108.

First we find the HCF of 60 and 84 by using Euclid's Division algorithm.

84 = 60 × 1 + 24

60 = 24 × 2 + 12

24 = 12 × 2 + 0

So, HCF (84, 60) = 12

Now, we will find the HCF of 12 and 108.

108 = 12 × 9 + 0

∴ HCF (12, 108) = 12

Hence, HCF (60, 84 and 108) = 12

Least number of rooms required for all the participants =252/12

= 21

Case-Based-II

1. **(c)** To find the number of books in each stack, we need to find the H.C.F of 96, 240 and 336.

$96 = 2^5 \times 3$

$240 = 2^4 \times 3 \times 5$

$336 = 2^4 \times 3 \times 7$

∴ H.C.F. (96, 240, 336) = $2^4 \times 3$

= 16 × 3

= 48.

2. **(a)** Total number of books = 96 + 240 + 336

= 672

Number of books in each stack = 48

$$\therefore \text{ Number of stacks formed } = \frac{672}{48}$$

= 14

Hence, 14 stacks will be formed.

3. **(b)** Number of English books in each stack = 48

Thickness of each english book = 3 cm

∴ Height of each stack of english books = (48 × 3) cm

= 144 cm

Multi Correct MCQ's

1. **(a, c)** $\frac{343}{2^3 \times 5^3 \times 7^3} = \frac{1}{2^3 \times 5^3}$

$$= \frac{1}{(10)^3}$$

$$= (0.001)$$

$$\frac{11}{25000} = \frac{11}{5^2 \times 10^2} = 0.00044$$

2. (a, b, c)

$$p=\frac{2-\sqrt{5}}{2+\sqrt{5}}, q=\frac{2+\sqrt{5}}{2-\sqrt{5}}$$

$p^2 - q^2 = (p + q)(p - q)$

$$=\left(\frac{2-\sqrt{5}}{2+\sqrt{5}}+\frac{2+\sqrt{5}}{2-\sqrt{5}}\right)\left(\frac{2-\sqrt{5}}{2+\sqrt{5}}-\frac{2+\sqrt{5}}{2-\sqrt{5}}\right)$$

$$=\left(\frac{\left(2-\sqrt{5}\right)^2+\left(2+\sqrt{5}\right)^2}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}\right)\left(\frac{\left(2-\sqrt{5}\right)^2-\left(2+\sqrt{5}\right)^2}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}\right)$$

$$=\frac{(9+9)}{-1}\left(\frac{-4\sqrt{5}-4\sqrt{5}}{-1}\right)=18\times\left(-8\sqrt{5}\right)=-144\sqrt{5}$$

Now, $\frac{q^2-p^2}{\sqrt{5}}=144$, which is divisible by 12, 2 and 6.

3. (a, b, c, d)

If $p + q + r = 0$

$\Rightarrow \quad p^3 + q^3 + r^3 = 3pqr$

$\Rightarrow 55^3 + 17^3 - 72^3 = 3 \times 55 \times 17 \times (-72)$

$= -(3 \times 5 \times 11 \times 17 \times 2^3 \times 3^2)$

$\Rightarrow$ A is divisible by 3, 11, 15 and 17.

4. (a, b, c, d) $a + (a + 2) + (a + 4)$

$= 3a + 6 = 3(a + 2)$

where, $\quad a = 2, 4, 6, ...$

Therefore, it is always divisible by 12 and its factors.

Olympiad & NTSE Type Questions

1. (d) $1146600 = 2^3 \times 3^2 \times 5^2 \times 7^2 \times 13$

$\therefore$ total number of factors

$= (3 + 1)(2 + 1)(2 + 1)(2 + 1)(1 + 1)$

$= 216$

Therefore, the total number of ways in which the number 1146600 can be written as a product of two factors

$= \frac{216}{2}$

$= 108$

2. (b) $$\frac{2^{39}}{39}\to\frac{\left(2^6\right)^6\times 2^3}{39}\to\frac{64^6\times 8}{39}\to\frac{(39+25)^6\times 8}{39}\to$$

$$\frac{25^6\times 8}{39}\to\frac{(625)^3\times 8}{39}\to\frac{(624)^3\times 8}{39}\to\frac{1^3\times 8}{39}\to\frac{8}{39}.$$

Hence, the remainder is 8.

3. (b) $$a=\frac{1}{2-\sqrt{3}}=\frac{1\times\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}=\left(\frac{2+\sqrt{3}}{1}\right)$$

Now $a^3 - 2a^2 - 7a + 5$

$= a^2(a - 2) - 7a + 5$

$= \left(2+\sqrt{3}\right)^2\left(2+\sqrt{3}-2\right)-7\left(2+\sqrt{3}\right)+5$

$= \left(7+4\sqrt{3}\right)\left(\sqrt{3}\right)-14-7\sqrt{3}+5$

$= 7\sqrt{3}+12-14-7\sqrt{3}+5$

$= 3$

4. (a) $\sqrt{a^{-1}b}\cdot\sqrt{b^{-1}c}\cdot\sqrt{c^{-1}a}$

$=\sqrt{\frac{b}{a}}\times\sqrt{\frac{c}{b}}\times\sqrt{\frac{a}{c}}$

$=1$

5. (a) $n=\frac{1}{\sqrt{x}\pm\sqrt{y}}$, if $n=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)}$

Now, in order to rationalize we have to multiply numerator and denominator by $\left(\frac{1}{\sqrt{x}-\sqrt{y}}\right)$

$$n=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)}\times\frac{1}{\sqrt{x}-\sqrt{y}}=\left(\frac{1}{x-y}\right)$$

In the similar way, if $n=\left(\frac{1}{\sqrt{x}-\sqrt{y}}\right)$ its rationalizing factor will be $\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)}$

So rationalizing factor of $\frac{1}{\left(\sqrt{x}\pm\sqrt{y}\right)}$ will be $\frac{1}{\left(\sqrt{x}\mp\sqrt{y}\right)}$.

6. (b) Largest number which divides 969 leaves 9 as remainder.

Therefore, 969 – 9 = 960 will be a multiple of given number.

In the similar way, 2059 – 11 = 2048 will be a multiple of given number.

Now, to find the required number we need to find the HCF of 960 and 2048 i.e.

By using Euclid's division algorithm

$2048 = 960 \times 2 + 128$

$960 = 128 \times 7 + 64$

$128 = 64 \times 2 + 0$

$\therefore$ HCF (960, 2048) = 64

Therefore, the required number is 64.

7. (c) The dividend is $14^{15^{18}}$ and divisor is 5. As we know that if (na + b) is divided by a then remainder is b.

Hence when 14 i.e., (2 × 5 + 4) is divided by 5 then remainder is 4.

And when $14^{15^{18}}$ divided by 5 then remainder obtained is the same as $4^{15^{18}}$ divided by 5.

We can find this using Euclid division lemma.

The remainder when 4^1 is divided by 5 is 0 × 5 + 4 i.e., 4.

The remainder when 4^2 is divided by 5 is 3 × 5 + 1 i.e., 1.

The remainder when 4^3 is divided by 5 is 12 × 5 + 4 i.e, 4.

The remainder when 4^4 is divided by 5 is 51 × 5 + 1 i.e., 1.

So the cycle is repeating after 2 steps.

It means when 4^n is divided by 5 gives 4 as remainder when n is odd and 1 as remainder when n is even.

As 15^1 = 15 i.e., odd

15^2 = 225 i.e., odd

15^3 = 3375 i.e., odd

Similarity 15^{18} is also an odd number.

Now when 14 is divided by 5 leaves the remainder 4.

∵ 15^{18} times the cycle is repeating.

It means the required remainder is equal to the remainder when $4^{15^{18}}$ is divided by 5.

As 15^{18} is an odd number

∵ Remainder when $4^{15^{18}}$ i.e., 4^{odd} is divided by 5 is 1.

8. (d) $a = 7^{17} + (7)^{34}$

$= (7^4)^4 \times 7^1 + (7^4)^8 \times 7^2.$

= 1 × 7 + 1 × 9 (digits at unit's place)

= 7 + 9

= 16

Therefore, unit's is digit in $a = 7^{17} + 7^{34}$ is 6.

9. (a) $33^{288} = (33^4)^{72}.$

$= (33^2 \times 33^2)^{72}$

$= (1089 \times 1089)^{72}$

$= (1185921)^{72}$

The unit's digit of this number will always be 1 as 1^n is 1.

Note: Now to get tens digit, multiply the tens digit of the number (2 here) with the last digit of the exponent (2 here) i.e., 2 × 2 = 4.

Hence, last two digits of $(1185921)^{72}$ is 41.

10. (c) The unit's digit of 9^1 is 9.

The unit's digit of 9^2 is 1.

The unit's digit of 9^3 is 9.

The unit's digit of 9^4 is 1.

So we can conclude that unit's digit of 9^{odd} is 9 and of 9^{even} is 1.

$9^0 + 9^1 + 9^2 + ... + 9^{2009}$

Unit's digit are 1 + 9 + 1 + ... + 9

As in the given expression there are total 2010 terms out of which 1005 terms will be of even powers and 1005 terms will be of odd powers i.e., unit's digit will be 1005 × 1 + 1005 × 9

$\underset{\text{unit's digit}}{5} + \underset{\text{unit's digit}}{5} = \underset{\text{unit's digit}}{10}$

Therefore, unit's digit will be 0.

11. (b) The given series is

a, b, b, c, c, c, d, d, d ....

Here 'a' comes once, 'b' come twice, 'c' come thrice and so on.

So, we can write

a = 1, b = 2, c = 3, ... , z = 28

∴ The series will become,

1, 2, 3, 4 ......

The given series forms an A.P having first term (a) = 1

Common difference (d) = 1

For n = 23

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{23} = \frac{23}{2}[2(1) + (23-1)1]$$

$$= \frac{23}{2}[2 + 22]$$

$$= \frac{23}{2} \times 24$$

$$= 276$$

It means 276^{th} term of series will be 23^{rd} alphabet i.e., W.

Then, the alphabet at the position 24^{th} will repeat itself from 277 to (276 + 27) i.e., 277 to 300.

Since alphabet at 24^{th} place is X.

Hence, answer is X.

12. **(c)** Let m = 926!

In order to find the maximum power of 7 in 926!, the greatest integer value method can be used i.e.

$$\frac{926}{7}+\frac{926}{(7)^2}+\frac{926}{(7)^3}$$

$$\downarrow \quad \downarrow$$

Greatest integer: 132 + 18 + 2 = 152

13. **(a)** Let m = $3^{57} + 27$, if m is divided by 28, then remainder will be given by:

$$\begin{aligned} \text{Remainder} &= (3^3)^{19} + 27 \\ &= (27)^{19} + 27 \\ &= (28 - 1)^{19} + 27 \\ &= (-1)^{19} + 27 \\ &= -1 + 27 \\ &= 26 \text{ (remainder)} \end{aligned}$$

Therefore, remainder obtained will be 26.

14. **(b)** In 36! power of 5 is given as:

$$\frac{36}{5}+\frac{36}{(5)^2}$$

Greatest integer: 7 + 1 = (8)

Therefore, number of zeroes in $36!^{36!}$ is 8 × 36!

15. **(a)**

$$\begin{aligned} q &= 2^{200} - 2^{192}.31 + 2^n \\ &= 2^{192}(2^8 - 31) + 2^n \\ &= 2^{192} \times 225 + 2^n \\ &= 2^{192}(225 + 2^{n-192}) \\ &= 2^{192}(225 + y) \end{aligned}$$

Now, in order to make (225 + y) a perfect square number the minimum values of y should be:

$$2^{n-192} = 2^6$$

$$\Rightarrow \quad n - 192 = 6$$

So n = 192 + 6 = 198