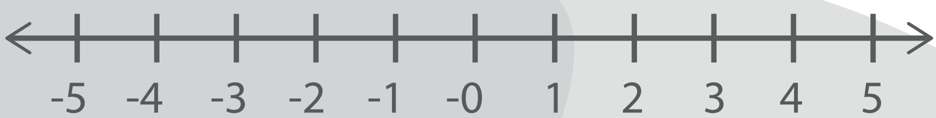
1

Real Numbers

Chapter Synopsis:

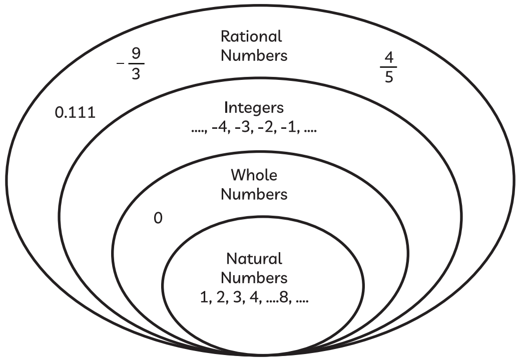


Objective Section MAP

REAL NUMBERS

RATIONAL NUMBERS

A number that can be expressed in the form of p/q, where p and q are integers and q ≠ 0, is called a rational numbers.



In Decimal Form

Terminating Rational Numbers

Eg: $\frac{4}{5} = 0.8$

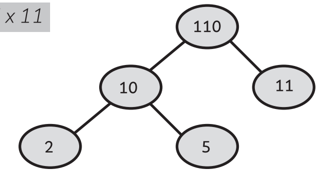
Non-Terminating Rational Numbers

Eg: $\frac{1}{3} = 0.333...$

THE FUNDAMENTAL THEOREM OF ARITHMETIC (PRIMEFACTORIZATION METHOD)

A composite number can be expressed (factorized) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.

Eg: *110 = 2 x 5 x 11*



Note: For two positive numbers a and b, HCF (a, b) x LCM (a, b) = a x b

Eg: $6 = 2^1 \times 3^1$ and $20 = 2^2 \times 5^1$
$LCM(6, 20) = 2^2 \times 3 \times 5 = 20$; $HCF(6, 20) = 2$
=> $HCF(6, 20) \times LCM(6, 20) = 6 \times 20$

HCF, LCM OF THREE INTEGERS

HCF(p,q,r) × LCM(p,q,r) ≠ p × q × r, where p, q, r are positive integers. However, the following results hold good for three numbers p, q and r:

$$LCM(p,q,r) = \frac{p.q.r.\ HCF(p,q,r)}{HCF(p,q).\ HCF(q,r).\ HCF(p,r)}$$

$$HCF(p,q,r) = \frac{p.q.r.\ LCM(p,q,r)}{LCM(p,q).\ LCM(q,r).\ LCM(p,r)}$$

IRRATIONAL NUMBERS

A number which can not be expressed in the form of p/q where p and q are integers and q ≠ 0 is called irrational numbers. The decimal form of irrational numbers is non-terminating and non-repeating.

Eg: $\sqrt{2}$, $5 + \sqrt{3}$, $\sqrt{7} - 4$, 0.8572984..., 0.10100100010..., π, etc.

RATIONAL NUMBER AND THEIR DECIMAL EXPANSIONS

- Let x = a/b, where a and b are coprimes, be a rational number whose decimal expansion terminates. Then the prime factorization of the denominator b has the powers of 2 or 5 or both like 2^n5^m, where n, m are non-negative integers.

  Eg: $\frac{35}{50} = \frac{70}{2^2 \times 5^2}$

- Let y = a/b be a rational number, if the prime factorization of the denominator b is not in the form of 2n5m, where n, m are non-negative integers then y has a non-terminating repeating decimal expansion.

  Eg: $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Note:
- 2^4 can be expressed as 2^45^0
- 5^4 can be expressed as 2^05^4

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The sum of exponents of prime factors in the prime-factorisation of 196 is:

(a) 3 (b) 4

(c) 5 (d) 6 **[CBSE 2020]**

Ans. *(b) 4*

Explanation: The prime factorisition of 196 is:

$$196 = 2^2 \times 7^2$$

So, sum of the exponents of prime factors 2 and 7 is 2 + 2 *i.e.*, 4

2. The total number of factors of a prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Ans. *(c) 2*

Explanation: Factors of a prime number are 1 and the number itself.

3. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3 **[CBSE 2020]**

Ans. *(c) 3, 420*

Explanation:

Here, $12 = 2^2 \times 3$

$21 = 3 \times 7$

$15 = 3 \times 5$

So, HCF = 3; LCM = $2^2 \times 3 \times 7$ *i.e.*, 420

4. The decimal representation of $\frac{11}{2^3 \times 5}$ will:

(a) terminate after 1 decimal place

(b) terminate after 2 decimal places

(c) terminate after 3 decimal places

(d) not terminate

Ans. *(c) terminate after 3 decimal places*

[CBSE Marking Scheme 2019]

5. The LCM of smallest two digit composite number and smallest composite number is:

(a) 12 (b) 4

(c) 20 (d) 44

Ans. *(c) 20* **[CBSE Marking Scheme 2019]**

6. If two positive integers *a* and *b* are written as $a = x^3y^2$ and $b = xy^3$, where *x* and *y* are prime numbers, then the HCF (*a*, *b*) is:

(a) xy (b) xy^2

(c) x^3y^3 (d) x^2y^2 **[Diksha]**

Ans. (b) xy^2

Explanation:

Given that $a = x^3y^2 = x \times x \times x \times y \times y$

and $b = xy^3 = x \times y \times y \times y$

$\Rightarrow$ HCF of *a* and *b* = HCF (x^3y^2, xy^3)

$= x \times y \times y = xy^2$

We know that HCF is the product of the smallest power of each common prime factor involved in the numbers.

7. If two positive integers *p* and *q* can be expressed as $p = ab^2$ and $q = a^3b$ where *a* and *b* are prime numbers, then the LCM (*p*, *q*) is:

(a) ab (b) a^2b^2

(c) a^3b^2 (d) a^3b^3 **[NCERT]**

Ans. (c) a^3b^2

Explanation: Given that

$p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

We know that LCM is the product of the greatest power of each Prime factor of the numbers.

$\Rightarrow$ LCM of *p* and *q* = LCM (ab^2, a^3b)

$= a \times b \times b \times a \times a = a^3b^2$

8. 7 × 11 × 13 × 15 + 15 is a:

(a) Composite number

(b) Whole number

(c) Prime number

(d) (a) and (b) both

Ans. *(d) (a) and (b) both*

Explanation: 7 × 11 × 13 × 15 + 15

= 15 (7 × 11 × 13 + 1) = 15 × 1002

Also 15 × 1002 is a whole number.

The number having factors more than two therefore, this is composite number and whole number.

9. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is

(a) 40 (b) 560

(c) 1120 (d) 1680

Ans. *(d) 1680*

Explanation: $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$

LCM = $2^4 \times 3 \times 5 \times 7 = 1680$

10. 1.23451326... is

(a) an integer

(b) an irrational number

(c) a rational number

(d) none of these

Ans. *(b) an irrational number*

Explanation: Number neither terminating nor repeated, therefore this is an irrational number.

11. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =

(a) 1 (b) 2

(c) 3 (d) 4

Ans. *(d) 4*

Explanation: We know that,

LCM $(a, b) \times$ HCF $(a, b) = a \times b$

$\Rightarrow \quad 36 \times 2 = a \times 18$

$\Rightarrow \quad a = \frac{36 \times 2}{18}$

$\Rightarrow \quad a = 4$

12. The product of a non–zero rational and an irrational number is:

(a) always irrational

(b) always rational

(c) rational or irrational

(d) one **[NCERT]**

Ans. *(a) always irrational*

Explanation: Product of a non–zero rational and an irrational number is always irrational.

For example:

$\frac{7}{9}$ is rational and $\sqrt{2}$ is irrational numbers.

Their product is an irrational number.

$\frac{7}{9} \times \sqrt{2} = \frac{7\sqrt{2}}{9}$, which is an irrational number.

13. The number of decimal places after which the decimal expansion of the rational number $\frac{9}{2^4 \times 5}$ will terminate, is:

(a) 1 (b) 2

(c) 3 (d) 4

Ans. *(d) 4*

Explanation: Number is

$\frac{9}{2^4 \times 5} = \frac{9 \times 5^3}{2^4 \times 5^4} = \frac{1125}{10^4} = 0.1125$

Therefore, number terminate after 4 decimal places.

14. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is:

(a) 10 (b) 100

(c) 504 (d) 2520 **[NCERT]**

Ans. *(d) 2520*

Explanation: As we require the least number, the problem is based on finding the LCM.

Factors of 1 to 10 numbers are as follows:

$1 = 1$

$2 = 1 \times 2$

$3 = 1 \times 3$

$4 = 1 \times 2 \times 2$

$5 = 1 \times 5$

$6 = 1 \times 2 \times 3$

$7 = 1 \times 7$

$8 = 1 \times 2 \times 2 \times 2$

$9 = 1 \times 3 \times 3$

$10 = 1 \times 2 \times 5$

LCM of number 1 to 10

= LCM (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

= 2520

15. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(a) one decimal place

(b) two decimal places

(c) three decimal places

(d) four decimal places

[CBSE 2017, 13]

Ans. *(d) four decimal places*

Explanation:

2	1250
5	625
5	125
5	25
5	5
	1

Simplifying the given fraction:

$\Rightarrow \quad \frac{14587}{1250} = \frac{14587}{5^4 \times 2}$

$= \frac{14587}{5^4 \times 2} \times \frac{2^3}{2^3}$

$= \frac{116696}{5^4 \times 2^4} = \frac{116696}{10^4}$

$= 11.6696$

Number is 11.6696

Hence, the given rational number will terminate after four decimal places.

16. **If HCF *(a, b)* = 45 and *a* × *b* = 30375, then LCM *(a, b)* is:**

(a) 1875 (b) 1350

(c) 625 (d) 675

Ans. *(d) 675*

We know that,

$$\text{LCM }(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

So, $$\text{LCM }(a, b) = \frac{30375}{45} = 675$$

17. **The cube of any positive integer is not of the form:**

(a) $9q$ (b) $9q + 1$

(c) $9q + 3$ (d) $9q + 8$

Ans. *(c) 9q + 3*

The cube of any positive integer is of the form $9q$ or $9q + 1$ or $9q + 8$. So, $9q + 3$ is incorrect.

18. **525 and 3000 are both divisible only by 3, 5, 15, 25 and 75, what is the HCF of (525, 3000)?**

(a) 25 (b) 125

(c) 75 (d) 15

Ans. HCF = 75

Explanation: Since 3, 5, 15, 25 and 75 are the only common factors of 525 and 3000, 75 is the HCF.

19. **If HCF of two numbers is 1, the numbers are called relatively .... or ....**

(a) Prime, co-prime (b) Composite, prime

(c) Both (a) and (b) (d) None of the above

[Diksha]

Ans. *(a) Prime, co-prime*

Explanation: Prime numbers are those numbers which have only two factors *i.e.,* 1 and itself. Example, 3, 5, 11 etc.

Co-prime numbers: Two numbers that have only 1 as a common factor.

Example, 35 and 39

35 = 1 × 5 × 7, 39 = 1 × 3 × 13

Here, common factor is 1.

Fill in the Blanks

Fill in the blanks/tables with suitable information:

20. $\left(\frac{2+\sqrt{5}}{3}\right)$ **is .... number. [CBSE 2020]**

Ans. irrational

Explanation: As $\sqrt{5}$ is irrational, $2+\sqrt{5}$ is irrational

21. **The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, the other number is ....**

Ans. *81*

Explanation: HCF of two numbers is 27 and their LCM is 162.

Let the other number be x.

Product of two numbers

$$= \text{HCF} \times \text{LCM} = 27 \times 162$$

$$\Rightarrow \quad 54x = 27 \times 162$$

$$\Rightarrow \quad x = 81$$

22. **If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$ then HCF (a, b) = ...**

Ans. *180*

Explanation:

$$a = (2^2 \times 3^3 \times 5^4)$$

$$b = (2^3 \times 3^2 \times 5)$$

$$\text{HCF}(a, b) = 2^2 \times 3^2 \times 5$$

$$= 4 \times 9 \times 5 = 180$$

23. **A decimal number $0.\overline{8}$ can be expressed in its simplest form as ....**

Ans. $x = \frac{8}{9}$

Explanation: Let x = 8.88888...

$$10x = 8.88888...$$

$$\Rightarrow \quad 10x - x = 8$$

$$\Rightarrow \quad 9x = 8$$

$$\Rightarrow \quad x = \frac{8}{9}$$

24. **Product of two numbers is 18144 and their HCF is 6, then their LCM is ....**

Ans. *324*

Explanation: Product of two numbers = 18144

HCF of two numbers is 6

Product of two numbers

$$= \text{HCF} \times \text{LCM} = 18144$$

$$\Rightarrow \quad 6 \times \text{LCM} = 18144$$

$$\Rightarrow \quad \text{LCM} = \frac{18144}{6} = 324$$

25. **The decimal expression of the rational number $\frac{23}{2^2 \times 5}$ will terminate after .... decimal place(s).**

Ans. *2*

Explanation: Here the power of 2 is 2 and the power of 5 is 1.

2 > 1

Hence, $\frac{23}{2^2 \times 5}$ has terminating decimal expansion which terminates after 2 places of decimals.

26. **The HCF of smallest composite number and the smallest prime number is ....**

Ans. *2*

Explanation: Smallest prime number = 2

Smallest composite number = 4

HCF (2, 4) = 2

27. **If a and b are positive integers, then $\frac{\text{HCF }(a,b) \times \text{LCM }(a,b)}{ab}$ = ...**

Ans. *1*

Explanation: HCF (a, b) × LCM (a, b) = ab

$\Rightarrow \frac{\text{HCF }(a,b) \times \text{LCM }(a,b)}{ab} = 1$

28. **.................... is the H.C.F. of two consecutive even numbers.**

Ans. *2*

Explanation: All even numbers are divisible by 2. Therefore, HCF of two consecutive numbers is 2.

29. **If two positive integers p and q can be expressed as $p = a^2b^3$ and $q = a^4b$; a, b being prime numbers, then LCM (p, q) is....................**

Ans. a^4b^3

Very Short Answer Type Questions

30. **The LCM of two numbers is 182 and their HCF is 13. If one of the number is 26, find the other. [CBSE 2020]**

Ans. We know that HCF (a, b) × LCM (a, b) = $a \times b$

So, 13 × 182 = 26 × b

$\Rightarrow b = \frac{13 \times 182}{26} = 91$

Thus, the other number is 91.

31. **Given that HCF (135, 225) = 45, find the LCM (135, 225). [CBSE 2020]**

Ans. We know that

LCM × HCF = Product of two numbers

$\therefore \text{LCM }(135, 225) = \frac{\text{Product of 135 and 225}}{\text{HCF}(135, 225)}$

$= \frac{135 \times 225}{45}$

= 675

32. **After how many decimal places will the decimal representation of the rational number $\frac{229}{2^2 \times 5^7}$ terminate ? [CBSE 2020]**

Ans. Here, $\frac{229}{2^2 \times 5^7} = \frac{229 \times 2^5}{2^7 \times 5^7} = \frac{229 \times 2^5}{(10)^7}$

Hence, the given rational number will terminate after 7 decimal places.

33. **Are the smallest prime and the smallest composite numbers co-prime? Justify. [Diksha]**

Ans. *No.*

We know that,

Smallest prime number is 2 and smallest composite number is 4.

HCF of (2, 4) = 2

Since, there is a common factor 2.

So, they are not co-prime.

34. **The HCF of two numbers a and b is 5 and their LCM is 200. Find the product ab. [CBSE 2019]**

Ans. Given, HCF (a, b) = 5

LCM (a, b) = 200

HCF × LCM = Product of the numbers

$\Rightarrow a \times b = 5 \times 200$

$\Rightarrow ab = 1000$

Hence, the product of ab is 100.

35. **Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons. [NCERT]**

Ans. *No.*

We know that:

"The HCF of any two numbers must be a factor of the LCM of those numbers."

So, two numbers cannot have their HCF 18 and LCM 380, as 18 does not divide 380.

36. **Find a rational number between $\sqrt{2}$ and $\sqrt{7}$. [CBSE 2019]**

Ans. $\sqrt{2} = 1.414$

and $\sqrt{7} = 2.6$

Let the rational number be x.

$\therefore\ \sqrt{2} < x < \sqrt{7}$

or $1.4 < x < 2.6$

Hence, any rational number like 1.5, 2.0, 2.5, can be the answer.

37. Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$. [CBSE 2019]

Ans. Given, $2^2 \times 5^2 \times 5 \times 3^2 \times 17$

$= (2 \times 5)^2 \times 5 \times 3^2 \times 17$

[$\because$ on multiplying 2×5 we get 10]

$= (10)^2 \times 5 \times 3^2 \times 17$

The power of 10 in the given expression is 2.

Hence, the number of zeroes in the end will be = 2.

38. If the HCF of (336, 54) = 6, find the LCM (336, 54). [CBSE 2019]

Ans. The HCF of (336, 54) = 6.

We know that:

LCM × HCF = Product of two numbers

$\Rightarrow \quad \text{LCM} = \dfrac{336 \times 54}{6}$

$= 336 \times 9 = 3024$

Hence, the LCM of the two numbers is 3024.

39. Find a rational number betwen $\sqrt{2}$ and $\sqrt{3}$. [CBSE 2019]

Ans. Rational number between $\sqrt{2}$ (1.41 approx) and $\sqrt{3}$ (1.73 approx) can be 1.5, 1.6, 1.63 etc.

So, a required rational number may be 1.5.

40. Write one rational and one irrational number lying between 0.25 and 0.32. [CBSE 2020]

Ans. Rational number= 0.30

Irrational number = 0.3010203040...

Or any other correct rational and irrational number. **[CBSE Marking Scheme 2019]**

41. Write the exponent of 3 in the prime factorization of 144. [Diksha]

Ans. Prime factorization of 144 = $2^4 \times 3^2$

So, exponent of 3 = 2.

SHORT ANSWER (SA-I) Type Questions

[**2** marks]

42. Check whether 12^n can end with the digit 0 for any natural number n. [CBSE 2020]

Ans. Let, if possible, 12^n have a value which ends with the digit 0.

$\Rightarrow$ 10 is a factor of 12^n

$\Rightarrow$ 5 is a prime factor of 12^n

i.e., $12^n = 5 \times q$, where q is some natural number

$\Rightarrow \quad (2^2 \times 3)^n = 5 \times q$

or $\quad 2^{2n} \times 3^n = 5 \times q$

The assumption, 5 is a prime factor of $2^{2n} \times 3^n$, is not possible because $2^{2n} \times 3^n$ can have only 2 and 3 as prime factors.

Hence, our assumption is wrong and 12^n cannot end with the digit 0.

43. The product of the LCM and HCF of two natural numbers is 24. The difference of two numbers is 2. Find the numbers. [Diksha]

Ans. Let the natural numbers be p and q.

According to question,

$p \times q = 24 \quad$ ...(i)

and $\quad p - q = 2$

$p = 2 + q \quad$ ...(ii)

From (i) and (ii)

$(q + 2) \times q = 24$

$q^2 + 2q - 24 = 0$

$q^2 + 6q - 4q - 24 = 0$

$(q + 6)(q - 4) = 0$

$q = -6, 4$

$q = 4$

[Since −6 is not a natural number]

So, the numbers are 4, 6.

44. Two alarm clocks ring their alarms at regular intervals of 72 seconds and 50 seconds if they first beep together at 12 noon, at what time will they beep again for the first time? [Diksha]

Ans. Here, we need to find the LCM of 72 and 50.

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$50 = 2 \times 5 \times 5$

LCM of 72 and 50 = $2^3 \times 3^2 \times 5^2 = 1800$

So, 1800 sec = 30 min

Hence, alarm clocks will beep again for the first time at 12.30 pm.

45. Find the HCF of 612 and 1314 using prime factorisation. [CBSE 2019]

Ans. Prime factors of 612 and 1314.

$612 = 2 \times 2 \times 3 \times 3 \times 17$

$1314 = 2 \times 3 \times 3 \times 73$

HCF (612, 1314) = 2 × 3 × 3

= 18

Hence, the HCF of 612 and 1314 is 18.

46. Write the smallest number which is divisible by both 306 and 657. [CBSE 2019]

Ans. Given numbers are 306 and 657.

The smallest number divisible by 306 and 657

= LCM(306, 657)

prime factors of 306 = 2 × 3 × 3 × 17

prime factors of 657 = 3 × 3 × 73

The LCM of (306, 657) = 2 × 3 × 3 × 17 × 73

= 22338

Hence, the smallest number divisible by 306 and 657 is 22,338.

47. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form $\frac{p}{q}$? Give reasons. [CBSE 2013]

Ans. As 327.7081 is a terminating decimal number, the denominator of the rational number must be of the form $2^m \times 5^n$.

Thus, $327.7081 = \frac{3277081}{10000}$

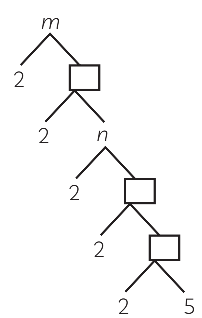
$= \frac{3277081}{10^4}$

$= \frac{3277081}{2^4 \times 5^4} = \frac{p}{q}$

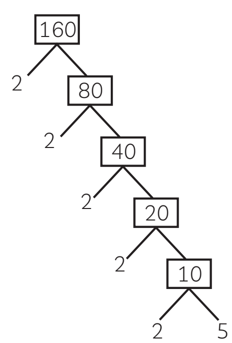
So, the prime factors of q are 2 and 5.

Here, q is of the form $2^m \times 5^n$, where m and n are natural numbers. The prime factors of p and q will be either 2 or 5 or both.

48. In the adjoining factor tree, find the numbers m and n.



Ans. Here, $m = 160$ and $n = 40$



49. Without actually performing the long division, write the decimal expansion of $\frac{11725}{2^3 \times 5^4}$.

Ans. $\frac{11725}{2^3 \times 5^4} = \frac{11725 \div 5}{2^3 \times 5^3} = \frac{2345}{(10)^3} = 2.345$

50. Write any two irrational numbers whose product is a rational number.

Ans. Consider two irrationals as, $5 - 2\sqrt{2}$ and $5 + 2\sqrt{2}$

Here,

$$(5 - 2\sqrt{2})(5 + 2\sqrt{2}) = 5^2 - (2\sqrt{2})^2$$

= 25 – 8 = 17 (a rational number)

51. Using prime factorisation method, find the HCF and LCM of 210 and 175. [CBSE 2011]

Ans. The prime factorisations of 210 and 175 are:

210 = 2 × 3 × 5 × 7

175 = 5 × 5 × 7

So, HCF (210, 175) = 5 × 7 = 35; and

LCM (210, 175) = 2 × 3 × 5 × 7 × 5 = 1050

52. Prove that the number 4^n, n being a natural number, can never end with the digit 0.

Ans. If 4^n ends with 0, then it must have 5 as a factor

But, $(4)^n = (2^2)^n = 2^{2n}$, *i.e.*, the only prime factor of 4^n is 2.

Also, we know from the Fundamental Theorem of Arithmetic that the prime factorisation of each number is unique.

∴ 4^n can never end with 0.

53. Find the two numbers which on multiplication with $\sqrt{360}$ gives a rational number. Are these numbers rational or irrational? [Diksha]

Ans.

$$\sqrt{360} = \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$$
$$= 6\sqrt{10}$$

If we multiply $6\sqrt{10}$ with $\sqrt{10}$ and 1.

We get,

$$6\sqrt{10} \times \sqrt{10} \times 1 = 60$$

Hence, numbers are $\sqrt{10}$ and 1.

Where, 1 is a rational number and $\sqrt{10}$ is an irrational number.

SHORT ANSWER (SA-II) Type Questions

[**3** marks]

54. Prove that $\sqrt{5}$ is an irrational number. **[CBSE 2014]**

Ans. Let us assume, to the contrary, that $\sqrt{5}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and b ≠ 0.

Now, $\sqrt{5} = \frac{a}{b}$

$\Rightarrow \quad 5 = \frac{a^2}{b^2}$

$\Rightarrow \quad 5b^2 = a^2$ ...(i)

$\Rightarrow$ a^2 is divisible by 5 [$\because$ $5b^2$ is divisible by 5]

$\Rightarrow$ a is divisible by 5 [$\because$ 5 is a prime number and divides $a^2 \Rightarrow$ 5 divides a]

Let $a = 5c$, for some integer 'c'

On substituting $a = 5c$ in (i), we get

$5b^2 = (5c)2$

$\Rightarrow \quad 5b^2 = 25c^2$

$\Rightarrow \quad b^2 = 5c^2$

$\Rightarrow$ b^2 is divisible by 5 [$\because 5c^2$ is divisible by 5]

$\Rightarrow$ b is divisible by 5

Since a and b are both divisible by 5, 5 is common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ is irrational.

55. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. **[CBSE 2019]**

Ans. Let us assume that $2 + 5\sqrt{3}$ is a rational number.

Therefore: $2 + 5\sqrt{3} = \frac{a}{b}$

(where, 'a' and 'b' are co-primes)

$\Rightarrow \quad 5\sqrt{3} = \frac{a}{b} - 2$

$\Rightarrow \quad 5\sqrt{3} = \frac{a - 2b}{b}$

$\Rightarrow \quad \sqrt{3} = \frac{a - 2b}{5b}$

Therefore, $\frac{a-2b}{5b}$ is in the form of $\frac{a}{b}$ which is a rational number.

But, this contradicts the fact that $\sqrt{3}$ is an irrational number.

Therefore, our assumption is wrong and $2 + 5\sqrt{3}$ is an irrational number.

56. Prove that $\sqrt{2}$ is an irrational number. **[CBSE 2016]**

Ans. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b as coprimes.

So, $\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad a^2 = 2b^2$

Thus, a^2 is a multiple of 2.

$\Rightarrow$ a is a multiple of 2. ...(i)

Let a = 2 m for some integer m.

Then, $b^2 = 2\text{ m}^2$

Thus, b^2 is a multiple of 2.

$\Rightarrow$ b is a multiple of 2. ...(ii)

From (i) and (ii,) 2 is a common factor of a and b.

This contradicts the fact that a and b are co-primes.

Hence, $\sqrt{2}$ is an irrational number.

57. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers. **[CBSE 2018]**

Ans. Given, numbers are 404 and 96.

Prime factorisation of both the number.

2	404
2	202
101	101
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

$\therefore \quad 404 = 2 \times 2 \times 101$

$96 = 2^5 \times 3$

HCF of 404 and 96 = $2^2 = 4$

LCM of 404 and 96 = $2^5 \times 3 \times 101$

= 9696

Now, HCF × LCM = 4 × 9696

= 38784 ...(i)

Product of two numbers = 404 × 96

= 38784 ...(ii)

From (i) and (ii) we get

HCF × LCM = Product of two numbers.

Hence, verified.

58. Write the denominator of rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Then, write its decimal expansion without actual division. [NCERT]

Ans. Denominator of the rational number $\frac{257}{5000}$ is 5000.

Now, $5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

$= (2)^3 \times (5)^4$

which is of the type $2^m \times 5^n$,

where $m = 3$ and $n = 4$ are non–negative integers.

Simplifying the given fraction:

$$\Rightarrow \frac{257}{5000} = \frac{257}{5^4 \times 2^3}$$

$$= \frac{257}{5^4 \times 2^3} \times \frac{2}{2}$$

$$= \frac{514}{5^4 \times 2^4} = \frac{514}{10^4}$$

$$= 0.0514$$

So, 0.0514 is the required decimal expansion of the rational number $\frac{257}{5000}$.

59. Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively, if they start tolling together, after what time will they next toll together? [Diksha]

Ans. The required time is the LCM of 12, 15 and 18.

$12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

$18 = 2 \times 3 \times 3$

LCM = $2^2 \times 3^2 \times 5 = 180$

So, next time the bells will ring together after 180 minutes or 3 hours.

60. Find if $\frac{987}{10500}$ will have terminating or non–terminating (repeating) decimal expansion. Give reasons for your answer. [CBSE 2010, 09]

Ans. *Yes, it will have a terminating decimal expansion.*

Simplified denominator has factor in the form of $2^m \times 5^n$.

So, this is a terminating decimal.

$$\begin{array}{r|r} 3 & 987 \\ \hline 7 & 329 \\ \hline & 47 \end{array} \qquad \begin{array}{r|r} 5 & 10500 \\ \hline 3 & 2100 \\ \hline 7 & 700 \\ \hline 5 & 100 \\ \hline 5 & 20 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

Now, $\frac{987}{10500} = \frac{3 \times 7 \times 47}{2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7}$

$$= \frac{47}{5^3 2^2} \times \frac{2}{2} = \frac{94}{5^3 2^3}$$

$$= \frac{94}{1000} = 0.094$$

And we know, if p/q is a rational number, such that the prime factorization of q is of the form $2^m \times 5^n$ where n and m are non–negative integers, than x has a decimal expansion which terminates.

Hence, it terminates.

61. On a morning walk, three people step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk, so that each can covers the same distance in complete steps? [CBSE 2015]

Ans. We know that the LCM is the product of the greatest power of each prime factor of the numbers.

We have to find the LCM of 40, 42 and 45 to get the required minimum distance.

For this, we find prime factorisation,

$40 = 2 \times 2 \times 2 \times 5$

$42 = 2 \times 3 \times 7$

$45 = 3 \times 3 \times 5$

LCM (40, 42, 45) = $2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 7$

= 2520

Hence, each person should walk a minimum distance of 2520 cm, so that each of them can cover the same distance in complete steps.

62. A merchant has 120 litres and 180 litres of two kinds of oil. He wants to sell oil by filling the two kinds of oil in tins of equal volumes. What is the greatest volume of such a tin?

Ans. In order to find volume of such a tin, we need to find the largest number which exactly divides 120 and 180 which is nothing but the HCF (120, 180).

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$
$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

H.C.F. (120, 180) = $2 \times 2 \times 3 \times 5 = 60$

Hence, the greatest volume of each tin is 60 litres.

63. Using prime factorisation, find HCF and LCM of 18, 45 and 60. Check if HCF × LCM = product of the numbers.

Ans. Here,

$$18 = 2 \times 3^2$$
$$45 = 3^2 \times 5$$

and $$60 = 2^2 \times 3 \times 5$$

So, HCF (18, 45, 60) = 3; and

LCM (18, 45, 60) = $2^2 \times 3^2 \times 5 = 180$

Clearly, HCF × LCM = $3 \times 180 = 540$

whereas, product of numbers

$$= 18 \times 45 \times 60 = 48600$$

Hence, HCF × LCM ≠ Product of numbers.

64. Show that the square of any positive odd integer is of the form $8m + 1$, for some integer m.

Ans. Let a be any positive integer.

So, it is of the form $2q + 1$, for some integer q

i.e. $a = 2q + 1$

$\Rightarrow$ $a^2 = (2q + 1)^2 = 4q^2 + 4q + 1$

$= 4q(q + 1) + 1$

Now, $q(q + 1)$ is either 0 or even. So, it is $2m$, where m is a whole number

$\therefore$ $a^2 = (2q + 1)^2$

$= 4 \times 2m + 1$ *i.e.* $8m + 1$

65. Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p and q are primes. [NCERT]

Ans. Let us suppose that $\sqrt{p}+\sqrt{q}$ is rational.

Let $\sqrt{p}+\sqrt{q} = a$, where a is a rational number,

$\Rightarrow$ $\sqrt{p} = a - \sqrt{q}$

On squaring both sides, we get

$\Rightarrow$ $p = a^2 + q - 2a\sqrt{q}$

[Using $(a - b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow$ $$\sqrt{q} = \frac{a^2+q-p}{2a}$$

Therefore, the above statement is a contradiction as the right hand side is a rational number, while the left hand side $\sqrt{q}$ is irrational, since p and q are prime numbers.

So, our assumption is wrong. Hence, $\sqrt{p}+\sqrt{q}$ is irrational.

66. Prove that $3 + 2\sqrt{5}$ is irrational number.

Ans. Let $3 + 2\sqrt{5}$ be a rational number.

So we can write this number as

$$3 + 2\sqrt{5} = \frac{a}{b}$$

Here a and b are two co-prime integers and $b \neq 0$.

Subtracting 3 from both sides, we get

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{(a - 3b)}{b}$$

On dividing both sides by 2, we get

$$\sqrt{5} = \frac{(a - 3b)}{2b}$$

Here a and b are integers, so $\frac{(a - 3b)}{2b}$ is a rational number which implies $\sqrt{5}$ should be a rational number, but $\sqrt{5}$ is an irrational number so it is a contradiction.

Hence, $3 + 2\sqrt{5}$ is an irrational number.

67. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. [CBSE 2020]

Ans. Let us assume, on the contrary, that $5+2\sqrt{7}$ is a rational number.

i.e., $$5+2\sqrt{7} = \frac{a}{b},$$

where 'a' and 'b' are co-prime numbers.

$\Rightarrow$ $$2\sqrt{7} = \frac{a}{b}-5$$

$\Rightarrow$ $$\sqrt{7} = \frac{a-5b}{2b}$$

Since $\frac{a-5b}{2b}$ is a rational number, $\sqrt{7}$ is also a rational number,

which is contradiction to the given results.

Hence, $5+2\sqrt{7}$ is irrational.

LONG ANSWER Type Questions

[**4** marks]

68. Show that the square of any positive integer cannot be of the form $(5q + 2)$ or $(5q + 3)$ for any integer q. [CBSE 2020]

Ans. Let 'a' be any positive integer. Then, it is of the form $5p$, or $5p + 1$ or $5p + 2$ or $5p + 3$ or $5p + 4$

Case 1 When $a = 5p$

$\Rightarrow a^2 = 25p^2 = 5(5p^2) = 5q$, where $q = 5p^2$

Case 2 When $a = 5p + 1$

$\Rightarrow a^2 = 25p^2 + 10p + 1 = 5(5p^2 + 2p) + 1 = 5q + 1$, where $q = 5p^2 + 2p$.

Case 3 When $a = 5p + 2$

$\Rightarrow a^2 = 25p^2 + 20p + 4 = 5(5p^2 + 4p) + 4 = 5q + 4$, where $q = 5p^2 + 4p$.

Case 4 When $a = 5p + 3$

$\Rightarrow a^2 = 25p^2 + 30p + 9 = 5(5p^2 + 6p + 1) + 4 = 5q + 4$, where $q = 5p^2 + 6p$.

Case 5 When $a = 5p + 4$

$\Rightarrow a^2 = 25p^2 + 40p + 16 = 5(5p^2 + 8p + 3) + 1 = 5q + 1$, where $q = 5p^2 + 8p +$

Thus, square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$, for any integer n.

69. Prove that one of every three consecutive positive integers is divisible by 3. [CBSE 2020]

Ans. Let n, $n + 1$ and $n + 2$ be three consecutive positive integers

Also, we know that a positive integer n is of the form $3q$, $3q + 1$ or $3q + 2$

Case I: When $n = 3q$

Here n is clearly divisible by 3.

But $(n + 1)$ and $(n + 2)$ are not divisible by 3.

[When $(n + 1)$ is divided by 3, remainder is 1 and when $(n + 2)$ is divided by 3, the remainder is 2]

Case II: When $n = 3q + 1$

Here $n + 2 = 3q + 3 = 3(q + 1)$

Clearly, it is divisible by 3.

But n and $(n + 1)$ are not divisible by 3.

Case III: when $n = 3q + 2$

Here, $n + 1 = 3q + 3 = 3(q + 1)$

clearly, $(n + 1)$ is divisible by 3

But n and $(n + 2)$ are not divisible by 3

Hence, one of every three consecutive positive integers is divisible by 3.

70. Prove that $\sqrt{n}$ is not a rational number, if n is not perfect square.

Ans. Let $\sqrt{n}$ be a rational number.

$\therefore \sqrt{n} = \frac{p}{q}$, where p and q are co-prime integers, $q \neq 0$.

On squaring both sides, we get

$\Rightarrow \quad n = \frac{p^2}{q^2}$

$p^2 = nq^2$ ...(i)

$\Rightarrow n$ divides p^2

[Let p be a prime number. If p divided a^2 then p divides a, where a is a positive integer]

$\Rightarrow n$ divides p ...(ii)

Let $p = nm$, where m is any integer.

$\Rightarrow \quad p^2 = n^2 m^2$

(i) $\Rightarrow \quad n^2 m^2 = nq^2$

$\Rightarrow \quad q^2 = nm^2$

$\Rightarrow n$ divides q^2

$\Rightarrow n$ divides q ...(iii)

[Let p be a prime number. If p divided a^2 then p divides a, where a is a positive integer]

From (ii) and (iii), n is a common factor of both p and q which contradicts the assumption that p and q are co-prime integer.

So, our supposition is wrong, √n is an irrational number.

71. The decimal expansions of some real numbers are given below. In each case, decide whether they are rational or not. If they are rational, write it in the form $\frac{p}{q}$. What can you say about the prime factors of q?

(A) 0.140140014000140000 ...

(B) $0.\overline{16}$

Ans. (A) We have, 0.140140014000140000... a non-terminating and non-repeating decimal expansion. So it is irrational. It cannot be written in the form of $\frac{p}{q}$.

(B) We have, $0.\overline{16}$ a non-terminating but repeating decimal expansion. So it is rational.

Let $x = 0.\overline{16}$

Then, $\quad x = 0.1616...$ ...(i)

$100x = 16.1616$ ...(ii)

On subtracting (i) from (ii), we get

$100x - x = 16.1616 - 0.1616$

$\Rightarrow 99x = 16 \Rightarrow x = \frac{16}{99}$

The denominator (q) has factors other than 2 or 5.