Wave Motion

Wave

It is a process by which transfer of energy and momentum takes from one portion of medium to another protion of medium, without any actual motion of particles of medium.

Wave Function

Any function of space and time which obeys

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$ represents a wave.

Travelling Wave or Progressive Wave

Any wave equation which is in the form of $y = f(\omega t \pm kx)$ or $y = f(x \pm vt)$ represents a progressive wave.

(a) If t and x are of opposite sign, wave is propagating along positive x-axis.

(b) If t and x are of same sign, the wave is propagating along negative x-axis.

(c) Wave speed $c = \frac{\omega}{k}$

(d) If $\omega t - kx$ = cosntant, then the slope of wave remains constant.

(e) Particle velocity $\quad v_P = \frac{dy}{dt}$

(f) $\text{slope} = \frac{dy}{dx}$

(g) For a wave, $\quad v_P = -v(\text{slope})$

Plane Harmonic Progressive or Travelling Wave

(a) The equation of plane harmonic progressive wave travelling along (+ve) x-axis is

$$y = A\sin(\omega t - kx)$$

$$y = A\sin(\omega t - kx + \phi)$$

(b) Wave moving along (–ve) x-axis

$$y = A\sin(\omega t + kx)$$

$$y = A\sin(\omega t + kx + \phi)$$

Speed of Transverse Wave on a stretched string

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$$

where T = tension in string

$\mu =$ mass per unit length

$\rho =$ density

$A =$ area of cross section

Stationary or Standing Wave

The superposition of two identical waves travelling in opposite directions along the same line gives rise to stationary waves. If two waves

$$y_1 = a\sin(\omega t - kx), \qquad y_2 = a\sin(\omega t + kx)$$

These two waves superpose to form stationary wave.

$$y = y_1 + y_2 = 2a\cos kx.\sin\omega t$$

Some important points regarding Standing Waves

1. Every particle of the medium vibrates in the same manner but amplitude depends on its position

   Amplitude in a starting wave
2. The point of medium with zero amplitude is a point of node and the point of medium with maximum amplitude is a point of antinode.
3. The particle of medium at node remains permanently at rest. Also nodes divide the medium into loops. All particle of a medium lying in a loop vibrate in the same phase with different amplitude.
4. Total energy of a loop remains constant.
5. At node, displacement is zero but pressure is maximum.
6. At anitnode, displacement is maximum but pressure is minimum.
7. The distance between nearest node and antinode is $\frac{\lambda}{4}$
8. The distance between successive nodes or antinodes is $\frac{\lambda}{2}$
9. Frequency of vibration f is

   $f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$, where l = length of string, μ = linear mass density, T= tension
10. If the string vibrates in P loops, p^{th} harmonic is produced, then frequency of p^{th} harmonic is

    $$f_p = \frac{P}{2l}\sqrt{\frac{T}{m}}$$
11. The average power transmitted by a wave is, $P = \frac{1}{2}\mu A^2\omega^2 c$
12. The instensity of wave is, $I = \frac{1}{2}\rho\omega^2 A^2 c$

SOUND WAVES

Longitudinal Wave

If a longitudinal wave is passing through a medium, the particles of medium oscillate about their mean position along the direction of propagation of wave. The propagation of transverse wave takes place in the form of crest and trough. But the propagation of longitudinal wave takes place in the form of rarefaction and compression.

Points

1. Mechanical transverese wave is not possible in gaseous and liquid medium. But longitudinal wave is possible in solid, liquid and gas.

2. In liquid and gas, sound is longitudinal wave
3. The velocity of longitudinal wave

$$\nu = \sqrt{\frac{E}{\rho}} \text{ , where } \quad E = \text{modulus of elasticity, } \rho = \text{density of medium}$$

VELOCITY OF SOUND

(a) In solid, $V = \sqrt{\frac{Y}{\rho}}$, Y = Young's modulus of elasticity

(b) In a fluid (gas or liquid) $V = \sqrt{\frac{B}{\rho}}$, B = Bulk modulus

SPEED OF SOUND IN AIR

Speed of sound in air is adiabatic. Rarefactors and compressions are so rapid that there is no exchange of heat. Modulus of elasticity involved would then be adiabatic back modulus.

$$E = B_{adiabatic} = \gamma p$$

The speed of sound in therefore,

$$= \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = 331.3 ms^{-1}$$

POINTS

1. The speed of sound does not change due to variation of pressure.
2. $$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$$
3. Due to change of temperature by 1°C, the speed of sound is changed by 0.61 ms^{-1}.
4. For small variation of temperature.

$$V_t = (V_0 + 0.61t) ms^{-1}$$

5. The speed of sound increase due to increase of humidity.

DISPLACEMENT WAVE AND PRESSURE WAVE

If the displacement wave equation is

$y = A\sin(\omega t - kx)$, then the pressure wave is

$$P = P_0 \cos(wt - kx)$$

$$P_0 = BAk$$

where, k = angular wave number

A = displacement amplitude

B = Bulk modulus of elasticity

ENERGY OF SOUND

The kinetic energy per unit volume of medium is $\frac{1}{2}\rho\omega^2 a^2 \cos^2(wt - kx)$,

ρ = density of medium

a = displacement amplitude

Energy density = $(K.E.)_{max} = \frac{1}{2}\rho a^2\omega^2$

POWER

It is defined as rate of transmission of energy

$$\overline{P} = \frac{1}{2}\rho v w^2 a^2 A$$

INTENSITY

$$I = \frac{1}{2}\rho\omega^2 a^2 v = \frac{\overline{P}}{A} = \frac{P_0^2}{2\rho c}$$

Loudness = Intensity level = L

$$= 10\log_{10}\left(\frac{I}{I_0}\right) dB$$

Here, $I_0 = 10^{-12}$ w m^{-2} Threshold of hearing

For zero level sound, $I = I_0$

ORGAN PIPE

It is a cylindrical tube of uniform cross section.

(a) Closed End Organ Pipe : It's one end is closed

$$l = (2n-1)\frac{\lambda}{4}, \; f = \frac{(2n-1)}{4l}V$$

closed end pipe

where, V = speed of wave, l = length of tube, n = 1, 2, 3, ...

POINTS:

1. The closed end is always a point of displacement node and pressure antinode.
2. Open end of the closed end organ pipe is always a point of displacement antinode and pressure node.
3. The maximum possible wavelength is 4 l.
4. The fundamental frequency is $f_1 = \frac{V}{4l}$
5. Present harmonics: 1^{st}, 3^{rd}, 5^{th} and so on.
   Present overtones: Fundamental, 1^{st}, 2^{nd}, 3^{rd} and so on.
6. The modes of vibration of closed end organ pipe are similar to the modes of vibration of rod fixed or clamped at one end.

(b) Open End Organ Pipe: Its both ends are open. The frequency of vibration is

$$f = \frac{nv}{2l}, \qquad n = 1, 2, 3, ...$$

open end pipe

POINTS:

1. All harmonics are present
2. Open ends are points of displacement antinode and pressure node.
3. Possible harmonics: 1, 2, 3, 4, 5,

   Possible overtones: fundamental, 1, 2, 3, 4,
4. The maximum possible wavelength is 2l.
5. Fundamental frequency $f_1 = \frac{v}{2l}$
6. When an open end organ pipe is submerged in water upto half of its length it behaves as a closed end organ pipe. But frequency remains unchanged.
7. If the diameter of organ pipe decreases, frequency increases.

RESONATING AIR COLUMN EXPERIMENT

(a) At first resonance

$$L_1 + e = \frac{\lambda}{4}$$

At second resonance

$$L_2 + e = \frac{3\lambda}{4}$$

so, $L_2 - L_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$

or, $\lambda = 2(L_2 - L_1)$

(b) End correction $e = \frac{(L_2 - 3L_1)}{2}$

Beats

The superposition of two waves of small difference in frequency in same direction produces beats.

If $y_1 = a\sin\omega_1 t$ and $y_2 = a\sin\omega_2 t$

then $y = 2a\cos\omega t.\sin\omega_{av} t$

Here $\omega = \frac{\omega_1 - \omega_2}{2} = 2\pi\left(\frac{f_1 - f_2}{2}\right)$

$$\omega_{av} = \frac{\omega_1 + \omega_2}{2} = 2\pi\left(\frac{f_1 + f_2}{2}\right)$$

POINTS

1. The beat frequency = number of beats per second = $|f_1 - f_2|$
2. In the case of beats, the intensity at a point varies periodically
3. Due to waxing of tunning fork, frequency decreases.
4. Due to filing a tunning fork, frequency increases.

DOPPLER'S EFFECT OF SOUND

$$n' = \left(\frac{V - V_0}{V - V_s}\right)n$$

$V =$ velocity of sound in medium, $V_0 =$ velocity of observer in the medium

$V_s =$ velocity of source in the medium, $n' =$ apparent frequency, n = frequency of source

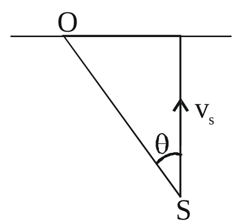
(a) When source moves towards stationary observer $n' = \left(\frac{V}{V - V_s}\right) n$

(b) When source moves away from observer. $n' = \left(\frac{V}{V + V_s}\right) n$

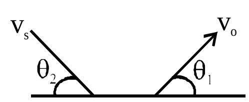
(c) When observer moves towards the stationary source $n' = \left(\frac{V + V_0}{V}\right) n$

(d) When observer moves away from the stationary source $n' = \left(\frac{V - V_0}{V}\right) n$

(e) $n' = \left(\frac{V}{V - V_s \cos\theta}\right) n$



(f) $n' = \left(\frac{V - V_0 \cos\theta_1}{V - V_s \cos\theta_2}\right) n$



(g) When source is at centre and observer is moving on the circular path.

$n' = n$