Area and Perimeter of Plane Figure

Perimeter and Semi-Perimeter

The concept of perimeter is applicable to many real-life situations. We need to calculate the perimeter: to find the length of barbed wire required for fencing the boundary of a garden; to find the length of the walkway around a swimming pool; and even to find the length of fabric required for stitching around the edge of a blanket or quilt.

Here, we will introduce a new term called **semi-perimeter** which, as the name itself implies, means half the perimeter.

In this lesson, we will learn how to find the perimeter and semi-perimeter of a triangle.

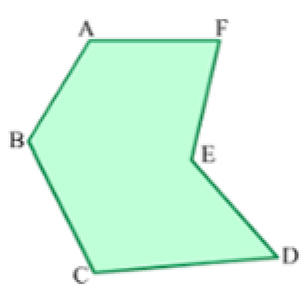
Perimeter of a Triangle

Perimeter is the length of the boundary of a closed figure.

The perimeter of a polygon is the sum of the lengths of all its sides.

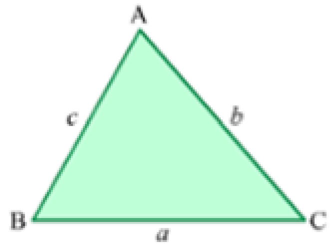
For example, in case of a polygon ABCDEF:

Perimeter of ABCDEF = AB + BC + CD + DE + EF + AF



In case of a triangle ABC, with sides of lengths *a*, *b* and *c* units:

Perimeter of ABC = AB + BC + AC = *a* + *b* + *c*



Did You Know?

Perimeter is the combination of the Greek words *peri*, which means ‘around’, and *meter*, which means ‘measure’.

Solved Examples

Easy

Example 1:

Two sides of a triangle measure 5 cm and 7 cm respectively. If the perimeter of the triangle is 20 cm, find the length of the third side.

Solution:

Let the measure of the three sides of the triangle be *a*, *b* and *c*.

We know that:

a = 5 cm

b = 7 cm

Perimeter = 20 cm

We also know that:

Perimeter = *a* + *b* + *c*

⇒ 20 cm = 5 cm + 7 cm + *c*

⇒ *c* = 20 cm − (5 + 7) cm

= 20 cm − 12 cm

= 8 cm

Thus, the length of the third side is 8 cm.

Example 2:

Ashish has a clock with a triangular frame. Its dimensions are 20 cm, 24 cm and 24 cm. One of the edges of the frame is cracked. Ashish puts tape along the boundary twice. What is the length of the tape put around the frame?

Solution:

Perimeter of the frame = 20 cm + 24 cm + 24 cm

= 68 cm

∴ Length of tape put around the frame = 2 × 68 cm

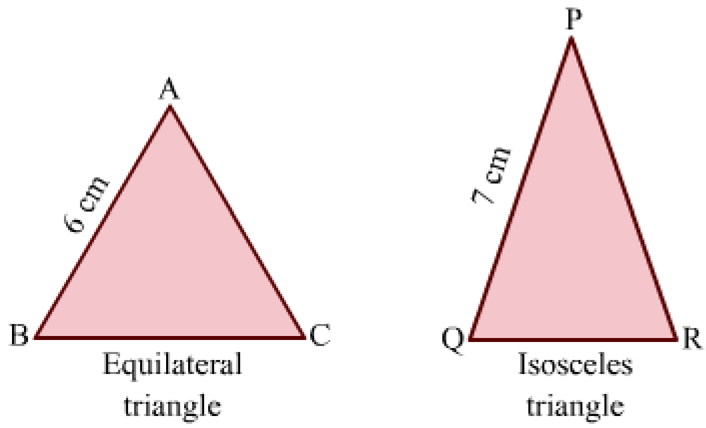
= 136 cm

= 1 m 36 cm

Medium

Example 1:

The two triangles shown in the figure have the same perimeter. What is the length of QR?



Solution:

We know that the sides of an equilateral triangle are equal in length.

∴ AB = BC = AC = 6 cm

Perimeter of ΔABC = AB + BC + AC

= 6 cm + 6 cm + 6 cm

= 18 cm

We know that two sides of an isosceles triangle are equal.

In ΔPQR, PQ = PR = 7 cm.

Perimeter of ΔPQR = PQ + QR + PR

= 7 cm + QR + 7 cm

= 14 cm + QR

It is given that:

Perimeter of ΔABC = Perimeter of ΔPQR

18 cm = 14 cm + QR

18 cm − 14 cm = QR

∴ QR = 4 cm

Example 2:

Ravi paid Rs 1200 for fencing his triangular park of dimensions 7 m, 8 m and 9 m. Find the cost of fencing per metre.

Solution:

Perimeter of the triangular park = 7 m + 8 m + 9 m

= 24 m

Cost of fencing 24 m = Rs 1200

∴ Cost of fencing 1 m = $\text{Rs}\,\frac{1200}{24}$

= Rs 50

Thus, the cost of fencing is Rs 50 per metre.

Activity

Follow these steps to verify the perimeter of a triangle.

(1) Take a wire of length, say, *l* cm.

(2) Bend the wire to make a triangle.

(3) Now, measure the length of each side of this triangle.

(4) Add the lengths to get the perimeter of the triangle.

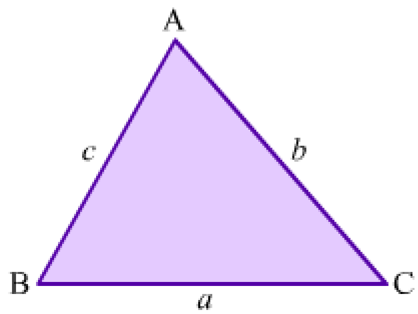
You will observe that the perimeter of the triangle is equal to the length of the wire.

Show a wire of length *l* cm which is bending to form a triangle.

Semi-Perimeter of a Triangle

The word 'semi' means 'half'. So, the semi-perimeter of a triangle is half the perimeter of the triangle.

The perimeter of a triangle with sides a, b and c is $(a + b + c)$.



Therefore, the semi-perimeter (*s*) of this triangle is $\frac{a+b+c}{2}$.

The semi-perimeter of a triangle is used for calculating the area of the triangle when the length of the altitude is not known.

Finding Semi-Perimeter of a Triangle

Solved Examples

Easy

Example 1:

The sides of a triangular field are 4 m, 7 m and 9 m. Calculate its semi-perimeter.

Solution:

Let the sides of the field be *a*, *b* and *c*.

We know that *a* = 4 m, *b* = 7 m and *c* = 9 m.

∴ Semi-perimeter, *s* $=\frac{a+b+c}{2}$

$$=\left(\frac{4+7+9}{2}\right)\text{m}$$
$$=10\,\text{m}$$

Thus, the semi-perimeter of the field is 10 m.

Example 2:

The semi-perimeter of a triangle is 25.5 cm. Two sides of this triangle are 8 cm and 11 cm. What is the length of the third side?

Solution:

Let the sides of the triangle be *a*, *b* and *c*.

It is given that *a* = 8 cm and *b* = 11 cm.

Also, semi-perimeter, *s* = 25.5 cm

We know that:

$$s = \frac{a+b+c}{2}$$

$$\Rightarrow 25.5\,\text{cm} = \frac{(8+11)\,\text{cm}+c}{2}$$
$$\Rightarrow 51\,\text{cm} = 19\,\text{cm}+c$$
$$\therefore c = (51-19)\,\text{cm}$$
$$= 32\,\text{cm}$$

Thus, the length of the third side of the triangle is 32 cm.

Medium

Example 1:

The sides of a triangle are in the ratio 5 : 3 : 4 and its semi-perimeter is 48 cm. Find the length of each side.

Solution:

Let the sides of the triangle be *a*, *b* and *c*.

It is given that the sides are in the ratio 5 : 3 : 4.

So, *a* = 5*x*, *b* = 3*x* and *c* = 4*x*.

Also, the semi-perimeter (*s*) of the triangle is 48 cm.

We know that:

$$s = \frac{a+b+c}{2}$$

$$\Rightarrow 48\,\text{cm} = \frac{5x+3x+4x}{2}$$
$$\Rightarrow 48\,\text{cm} \times 2 = 12x$$
$$\Rightarrow x = \frac{96}{12}\,\text{cm} = 8\,\text{cm}$$

Now,
$$5x = 5 \times 8\,\text{cm} = 40\,\text{cm}$$
$$3x = 3 \times 8\,\text{cm} = 24\,\text{cm}$$
$$4x = 4 \times 8\,\text{cm} = 32\,\text{cm}$$

Thus, the sides of the triangle are 40 cm, 24 cm and 32 cm.

Example 2:

The pairs of the sides of a triangle add up to 6 cm, 5 cm and 7 cm. What is its semi-perimeter?

Solution:

Let the sides of the triangle be *a*, *b* and *c*.

According to the given condition:

$$a + b = 6\,\text{cm} \quad ...(1)$$
$$b + c = 5\,\text{cm} \quad ...(2)$$
$$c + a = 7\,\text{cm} \quad ...(3)$$

On adding equations (1), (2) and (3), we get:

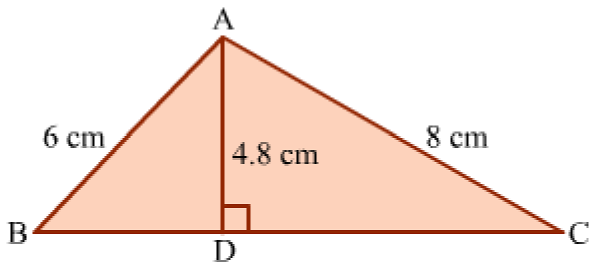
$$(a+b)+(b+c)+(c+a)=(6+5+7)\text{ cm}$$
$$\Rightarrow 2(a+b+c)=18\text{ cm}$$
$$\Rightarrow a+b+c=9\text{ cm}$$
$$\text{Semi-perimeter, } s=\frac{a+b+c}{2}$$
$$=\frac{9}{2}\text{ cm}$$
$$=4.5\text{ cm}$$

Thus, the semi-perimeter of the triangle is 4.5 cm.

Hard

Example 1:

In a ΔABC, AC is 8 cm, AB is 6 cm and the length of the perpendicular drawn from A to D is 4.8 cm. Find the semi-perimeter of ΔABC if its area is 24 cm^2. Solution:



Area of ΔABC = 24 cm^2

$$\Rightarrow \frac{1}{2}\times \text{BC}\times \text{AD}=24\text{ cm}^2$$
$$\Rightarrow \frac{1}{2}\times \text{BC}\times 4.8\text{ cm}=24\text{ cm}^2$$
$$\therefore \text{BC}=10\text{ cm}$$

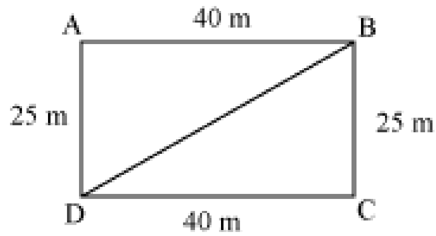
Semi-perimeter of **ΔABC,** $s=\dfrac{\text{AB}+\text{BC}+\text{AC}}{2}$

$$\Rightarrow s=\left(\frac{6+8+10}{2}\right)\text{cm}$$
$$=12\text{ cm}$$

Areas Of Triangles

A gardener wanted to grow roses and tulips in a garden. The garden was rectangular in shape and had a length of 40 m and breadth of 25 m. The gardener wanted to dedicate equal areas to grow these two types of flowers. He did not know anything about geometry, but even then, he thought that if he divided the garden diagonally, then it would be divided into two equal parts. Now, he could grow roses in one part of the garden and tulips in the other part. **Was he correct?**

Let ABCD be the garden.



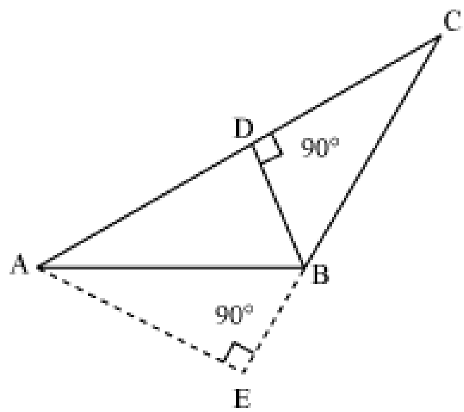
If it is divided along the diagonal BD, then two triangles ABD and BCD are formed.

Thus, the formula for the area of a triangle is:

Area of a triangle = $\frac{1}{2}$ × (Base × Height)

To use the formula, any side of a triangle can be taken as the base and the perpendicular drawn to the base from the opposite vertex is the corresponding height of the triangle.

Let us look at ΔABC drawn below. Side AC measures 10 cm and side BC measures 4 cm. AE is the perpendicular from vertex A to side BC and measures 8 cm.



Now, how do we find the area of ΔABC? Let us choose side BC as the base of ΔABC. Then, segment AE will be its corresponding height.

Therefore, area of ΔABC = $\frac{1}{2}$ × base × height $= \left(\frac{1}{2} \times 4 \times 8\right)$ cm^2 = 16 cm^2

That was easy! Now, can we also find the length of the perpendicular BD?

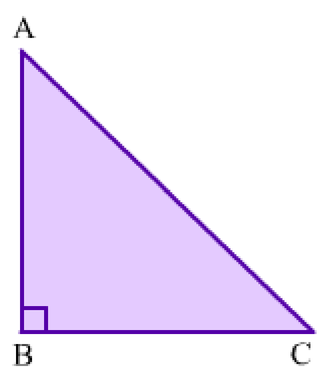
Note that if we choose side AC as the base of ΔABC, then segment BD will be its corresponding height.

Thus, the area of ΔABC can also be expressed as $\frac{1}{2} \times \text{AC} \times \text{BD}$.

$$\Rightarrow \frac{1}{2} \times 10 \text{ cm } \times \text{BD} = 16 \text{ cm}^2$$
$$\Rightarrow \text{BD} = \frac{16 \times 2}{10} \text{cm}$$
$$\Rightarrow \text{BD} = 3.2 \text{ cm}$$

Area of the right angled triangle:

Observe the right angled triangle given below:



It can be seen that ΔABC is right angled at B. So, side AB is perpendicular to side BC.

Thus, in ΔABC, side AB is the altitude or height and side BC is the base.

Now,

Area of ΔABC = $\frac{1}{2}$ × (Base × Height)

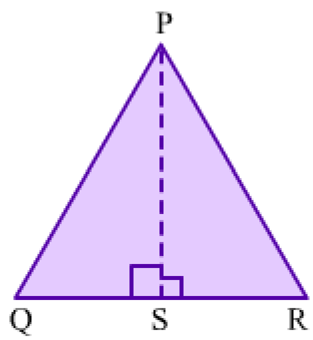
⇒ Area of ΔABC = $\frac{1}{2}$ × (BC × AB)

So, it can be concluded that:

Area of a right angled triangle = $\frac{1}{2}$ × Product of the lengths of the sides making right angle

Area of the equilateral triangle:

Let us take an equilateral triangle, say ΔPQR, such that PQ = QR = RP = *a*.



If we fold this triangle along side QR such that point Q coincides with point R, we will get two congruent triangles such as ΔPQS and ΔPRS.

In ΔPQS and ΔPRS, we obtain

$QS = RS = \frac{a}{2}$ and

∠PSQ = ∠PSR = 90°

By applying Pythagoras theorem on ΔPQS, we obtain

$$(PQ)^2 = (QS)^2 + (SP)^2$$

$$\Rightarrow a^2 = \left(\frac{a}{2}\right)^2 + (SP)^2$$

$$\Rightarrow (SP)^2 = a^2 - \frac{a^2}{4}$$

$$\Rightarrow (SP)^2 = \frac{3}{4}a^2$$

$$\Rightarrow SP = \frac{\sqrt{3}}{2}a$$

Now,

Area of ΔPQR = $\frac{1}{2}$ × (Base × Height)

$$\text{Area of } \Delta PQR = \frac{1}{2} \times QR \times SP$$
$$\Rightarrow \text{Area of } \Delta PQR = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a$$
$$\Rightarrow \text{Area of } \Delta PQR = \frac{\sqrt{3}}{4}a^2$$

Thus, it can be concluded that

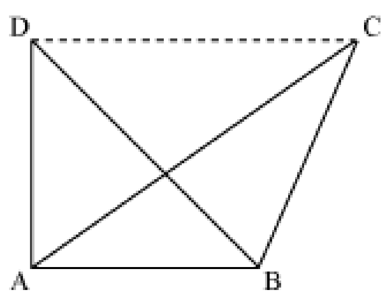
Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ × (Side)^2

One important point to remember about congruent triangles and their area is as follows.

We know that congruent figures are exactly alike. Thus, the areas of two congruent triangles will always be equal. However, its converse is not true. This means that two triangles, which have the same area, need not be congruent.

For example, look at triangles ABC and ABD drawn below. Both share the same base AB. Their height is also the same. Thus, the two triangles have the same area. However, it is clear from the figure that the two triangles are not congruent.

Thus, this proves the fact that two triangles having the same area need not be congruent.



Let us solve some examples using the above concepts.

Solved examples

Example 1:

Find the area of the triangle, which has a base of length 9 cm and a height of 11 cm.

Solution:

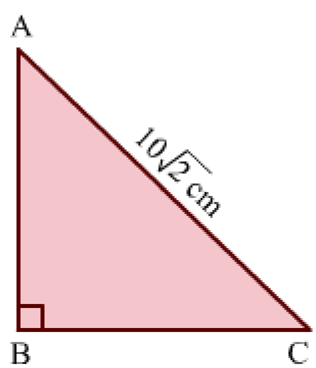
Area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 9 \text{ cm} \times 11 \text{ cm} = 49.5 \text{ cm}^2$

Example 2:

Find the area of the isosceles right angled triangle whose hypotenuse measures $10\sqrt{2}$ cm.

Solution:

Let ΔABC be the isosceles right angled triangle right angled at B whose hypotenuse measures $10\sqrt{2}$ cm.



In ΔABC, CA is the hypotenuse and AB = BC.

By applying Pythagoras theorem on ΔABC, we obtain

$(CA)^2 = (AB)^2 + (BC)^2$

$\left(10\sqrt{2}\right)^2 = (AB)^2 + (AB)^2 \qquad (\text{As } AB = BC)$

$\Rightarrow \left(10\sqrt{2}\right)^2 = 2(AB)^2$

$\Rightarrow 2(AB)^2 = 200$

$\Rightarrow (AB)^2 = 100$

$\Rightarrow AB = 10$

$\therefore AB = BC = 10\,\text{cm}$

Now,

Area of a right angled triangle = $\frac{1}{2}$ × Product of the lengths of the sides making right angle

⇒ Area of ΔABC = $\frac{1}{2}$ × AB × BC

⇒ Area of ΔABC = ($\frac{1}{2}$ × 10 × 10) cm^2 (AB = BC = 10 cm)

⇒ Area of ΔABC = 50 cm^2

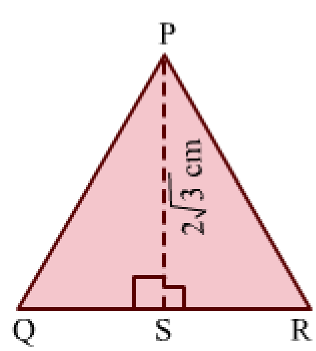
Thus, the area of the required triangle is 50 cm^2.

Example 3:

Find the area of the equilateral triangle whose height is $2\sqrt{3}$ cm.

Solution:

Let ΔPQR be the equilateral triangle whose height SP is $2\sqrt{3}$ cm.



In ΔPQR, we have

PQ = QR = RP,

QS = SP = $\frac{QR}{2}$ and

∠PSQ = PSR = 90°

By applying Pythagoras theorem on ΔPQS, we obtain

$$(PQ)^2 = (QS)^2 + (SP)^2$$

$$\Rightarrow (QR)^2 = \left(\frac{QR}{2}\right)^2 + \left(2\sqrt{3}\right)^2 \qquad \left(PQ = QR,\ QS = \frac{QR}{2} \text{ and } SP = 2\sqrt{3}\text{ cm}\right)$$

$$\Rightarrow (QR)^2 = \frac{(QR)^2}{4} + 12$$

$$\Rightarrow \frac{3}{4}(QR)^2 = 12$$

$$\Rightarrow (QR)^2 = 16$$

$$\Rightarrow QR = 4$$

Thus, PQ = QR = RP = 4 cm

Now,

Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ × (Side)^2

Area of ΔPQR= $\left(\frac{\sqrt{3}}{4} \times 4^2\right)\text{cm}^2$

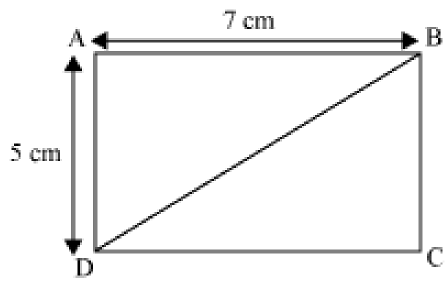
⇒ Area of ΔPQR = $4\sqrt{3}\text{ cm}^2$

Thus, the area of the required triangle is $4\sqrt{3}\text{ cm}^2$.

Example 4:

ABCD is a rectangle of length 7 cm and breadth 5 cm. What is the area of ΔABD?

Solution:



If a rectangle is divided diagonally, then the area of each triangle so obtained equals one-half the area of the rectangle.

∴ Area of ΔABD $= \frac{1}{2} \times \text{Area of rectangle ABCD}$

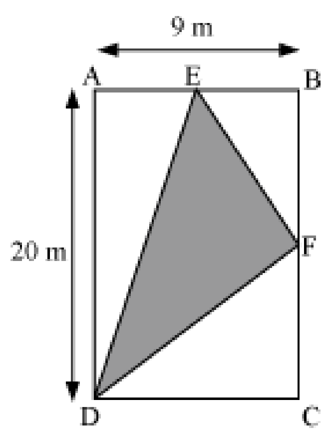
$= \frac{1}{2} \times \text{Length} \times \text{Width}$

$= \frac{1}{2} \times 7\text{ cm} \times 5\text{ cm}$

$= 17.5\text{ cm}^2$

Example 5:

The given figure shows a rectangle ABCD. E and F are the mid-points of sides AB and BC respectively. Find the area of the shaded portion in the given figure.



Solution:

Here, area of the shaded part = area of rectangle ABCD − (area of ΔADE + area of ΔBEF + area of ΔCDF)

Length AD of rectangle ABCD = 20 m

Width AB of rectangle ABCD = 9 m

∴ Area of rectangle ABCD = 9 m × 20 m = 180 m^2

E is the mid-point of side AB.

∴ AE = EB $= \frac{9}{2}\ \text{m} = 4.5\ \text{m}$

Also, F is the mid-point of side BC.

∴ BF = FC $= \frac{20}{2}\ \text{m} = 10\ \text{m}$

In ΔADE, length of base AD = 20 m

Length of corresponding height AE = 4.5 m

∴ Area of ΔADE $= \frac{1}{2} \times 20\ \text{m} \times 4.5\ \text{m} = 45\ \text{m}^2$

In ΔBEF, length of base BF = 10 m

Length of corresponding height BE = 4.5 m

∴ Area of ΔBEF $= \frac{1}{2} \times 10\ \text{m} \times 4.5\ \text{m} = 22.5\ \text{m}^2$

In ΔCDF, length of base CD = 9 m

Length of corresponding height CF = 10 m

∴ Area of ΔCDF $= \frac{1}{2} \times 9\ \text{m} \times 10\ \text{m} = 45\ \text{m}^2$

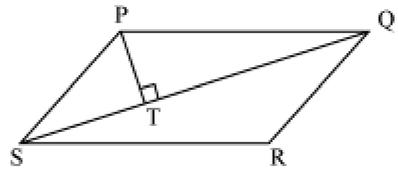
Thus, area of the shaded portion = [180 − (45 + 22.5 + 45)] m^2

= (180 − 112.5) m^2

= 67.5 m^2

Example 6:

The given figure shows a parallelogram PQRS that has an area of 16 cm^2. If the length of diagonal QS is 8 cm, then what is the length of perpendicular PT?



Solution:

Diagonal QS divides parallelogram PQRS into two triangles, ΔPQS and ΔQRS.

Thus, the area of ΔPQS is one-half the area of parallelogram PQRS.

∴ Area of ΔPQS $= \frac{1}{2} \times 16$ cm^2 = 8 cm^2

If we choose QS as the base of ΔPQS, then PT is its corresponding height.

Thus, the area of ΔPQS can also be expressed as $\frac{1}{2} \times \text{QS} \times \text{PT}$.

$$\therefore \frac{1}{2} \times \text{QS} \times \text{PT} = 8 \text{ cm}^2$$

$$\Rightarrow \frac{1}{2} \times 8 \text{ cm} \times \text{PT} = 8 \text{ cm}^2$$

$$\Rightarrow \text{PT} = \left(\frac{8 \times 2}{8}\right) \text{cm} = 2 \text{ cm}$$

Thus, the length of perpendicular PT is 2 cm.

Areas of Triangles Using Heron's Formula

Area of a Triangle

Kishan has a triangular field with sides 30 m, 30 m and 20 m. Can we find the area of his field using this information?

Yes, we can. The area of any triangle, when all its sides are known, can be calculated using Heron's formula.

We can use Heron's formula:

• To find the area of a triangle when the lengths of all its sides are given.

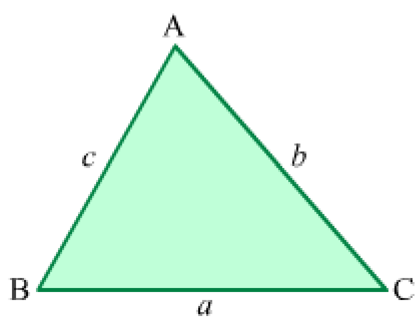
• To find the area of a quadrilateral by dividing it into two triangles.

• To calculate the area of a cyclic quadrilateral when the lengths of all its sides are given.

In this lesson, we will learn how to find the area of a triangle using Heron's formula.

Heron's Formula

Heron's formula can be used to find the area of any triangle in terms of the lengths of its sides. Let *a, b* and *c* denote the lengths of the sides of a ΔABC.



Perimeter of ΔABC = $a + b + c$

⇒ Semi-perimeter (s) of ΔABC = $\frac{a+b+c}{2}$

∴ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

Concept Builder

Area of a triangle:If any side of a triangle is taken as the base and a perpendicular is drawn to it from the opposite vertex, then the area of the triangle is given as follows:

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

Finding the Area of a Triangle Using Heron's Formula

Know Your Scientist

Heron

Born:10 AD **Died**:75 AD

Heron (or Hero) of Alexandria, Greece was a mathematician and engineer.

The proof of the formula named after him can be found in his book *Metrica*, written in 60 AD.

Heron has written so much on mathematics and physics that he can be described as an 'encyclopaedic writer' in these fields.

Solved Examples

Easy

Example 1:

Find the area of a triangle with sides 12 cm, 16 cm and 20 cm.

Solution:

Let the sides of the triangle be *a*, *b* and *c*.

In this case, *a* = 12 cm, *b* = 16 cm and *c* = 20 cm.

Semi-perimeter (*s*) of the triangle $= \frac{a+b+c}{2}$

$$= \left(\frac{12+16+20}{2}\right)\text{cm}$$
$$= 24 \text{ cm}$$

Using Heron's formula,

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

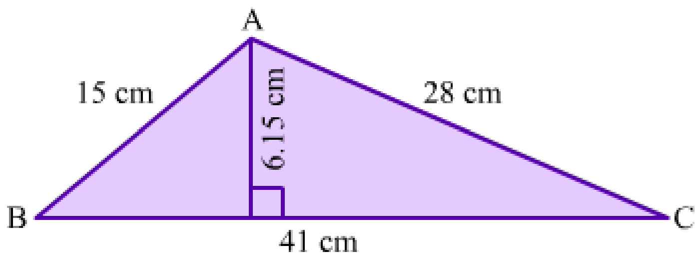
= $\sqrt{24(24-12)(24-16)(24-20)} \text{ cm}^2$

= $\sqrt{9216} \text{ cm}^2$

= 96 cm^2

Example 2:

Find the area of the triangle shown in the figure using the formula $\frac{1}{2} \times \text{Base} \times \text{Height}$ and Heron's formula. Compare the results obtained.



Solution:

Let the sides of ΔABC be *a*, *b* and *c*.

In this case, a = 15 cm, b = 41 cm and c = 28 cm.

Semi-perimeter (s) of ΔABC $= \frac{a+b+c}{2}$

$$= \left(\frac{15+41+28}{2}\right) \text{ cm}$$
$$= 42 \text{ cm}$$

Using Heron's formula,

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-15)(42-41)(42-28)} \text{ cm}^2$$
$$= \sqrt{42\times 27\times 1\times 14} \text{ cm}^2$$
$$= \sqrt{15876} \text{ cm}^2$$

= 126 cm^2

Now, we will calculate the area of ΔABC by using the formula: $\frac{1}{2}\times \text{Base}\times \text{Height}$

In this case,

Base = 41 cm

Height = 6.15 cm

$$\therefore \text{Area of } \Delta\text{ABC} = \frac{1}{2}\times 41\times 6.15 \text{ cm}^2$$

= 126.075 cm^2 ≈ 126 cm^2

Thus, the area of the triangle is found to be the same on using both the methods.

Medium

Example 1:

Two sides of a triangular field are 8 m and 11 m, and the semi-perimeter is 16 m. Find the area of the field.

Solution:

Let the sides of the field be *a*, *b* and *c*.

In this case, $a = 8$ m and $b = 11$ m.

It is given that the semi-perimeter (*s*) of the field is 16 m.

We know that $s = \frac{a+b+c}{2}$

$$\Rightarrow 16\,\text{m} = \frac{(8+11)\,\text{m} + c}{2}$$
$$\Rightarrow 32\,\text{m} = 19\,\text{m} + c$$
$$\therefore c = (32-19)\,\text{m} = 13\,\text{m}$$

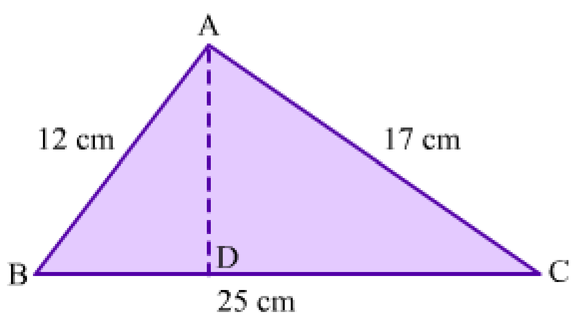
Using Heron's formula,

Area of the field = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-8)(16-11)(16-13)}\,\text{m}^2$$
$$= \sqrt{16\times8\times5\times3}\,\text{m}^2$$
$$= \sqrt{1920}\,\text{m}^2$$
$$= 8\sqrt{30}\,\text{m}^2$$

Example 2:

What is the height of the triangle shown in the figure?



Solution:

Let the sides of ΔABC be *a*, *b* and *c*.

In this case, $a = 12$ cm, $b = 25$ cm and $c = 17$ cm

$$\text{Semi-perimeter }(s)\text{ of }\Delta ABC = \frac{a+b+c}{2}$$
$$= \left(\frac{12+25+17}{2}\right)\text{cm}$$
$$= 27\text{cm}$$

Using Heron's formula,

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{27(27-12)(27-25)(27-17)}\ \text{cm}^2$$
$$= \sqrt{27\times15\times2\times10}\ \text{cm}^2$$
$$= \sqrt{8100}\ \text{cm}^2$$
$$= 90\ \text{cm}^2$$

We know that:

$$\text{Area of }\Delta ABC = \frac{1}{2}\times\text{Base}\times\text{Height}$$

$$\Rightarrow 90\ \text{cm}^2 = \frac{1}{2}\times 25\ \text{cm}\times AD$$
$$\Rightarrow AD = \frac{90\times2}{25}\ \text{cm} = 7.2\ \text{cm}$$

Thus, the height of ΔABC is 7.2 cm.

Hard

Example 1:

A floor is made up of 20 triangular tiles, each with sides 40 cm, 24 cm and 32 cm. Find the cost of polishing the floor at the rate of 25 paise per cm^2.

Solution:

Let the sides of each tile be *a*, *b* and *c*.

In this case, a = 40 cm, b = 24 cm and c = 32 cm.

Semi-perimeter (s) of each tile $= \frac{a+b+c}{2}$

$$= \left(\frac{40+24+32}{2}\right)\text{cm}$$
$$= 48\,\text{cm}$$

Using Heron's formula,

Area of each tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{48(48-40)(48-24)(48-32)}\ \text{cm}^2$$
$$= \sqrt{48\times 8\times 24\times 16}\ \text{cm}^2$$
$$= \sqrt{147456}\ \text{cm}^2$$
$$= 384\text{cm}^2$$

∴ Area of 20 tiles = (384 × 20) cm^2 = 7680 cm^2

Thus, the area of the floor is 7680 cm^2.

Now, cost of polishing 1 cm^2 = 25 paise = Rs 0.25

∴ Total cost of polishing the floor = Rs (7680 × 0.25) = Rs 1920

Example 2:

The difference between the semi-perimeter of ΔABC and each of its sides are 8 cm, 7 cm and 5 cm. What is the area of ΔABC?

Solution:

Let the sides of ΔABC be a, b and c.

Semi perimeter (s) of ΔABC $= \frac{a+b+c}{2}$

It is given that:

$s - a = 8\,\text{cm} \quad ...(1)$

$s - b = 7\,\text{cm} \quad ...(2)$

$s - c = 5\,\text{cm} \quad ...(3)$

On adding equations (1), (2) and (3), we get:

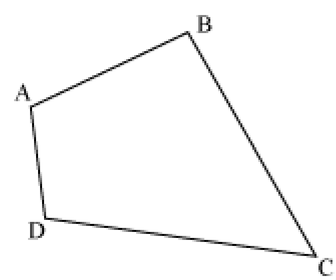
$$3s - (a + b + c) = 20\,\text{cm}$$
$$\Rightarrow 3s - 2s = 20\,\text{cm} \qquad (\because a + b + c = 2s)$$
$$\Rightarrow s = 20\,\text{cm}$$

Using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{20 \times 8 \times 7 \times 5}\,\text{cm}^2$$
$$= \sqrt{5600}\,\text{cm}^2$$
$$= 20\sqrt{14}\,\text{cm}^2$$

Area Of A General Quadrilateral

Look at the following quadrilateral ABCD.



Now, we have to find its area. Note that we cannot classify it as a rectangle, parallelogram or trapezium. Had we been able to do so, we could have easily applied the corresponding formula for the area. Now what do we do?

Example 1:

The length of the diagonal and the lengths of the perpendiculars from the opposite vertices to that diagonal of a quadrilateral are in the ratio 8: 3: 4. If the area of this quadrilateral is 448 m^2, then find the dimension of the diagonal and the perpendiculars.

Solution:

Let the length of the diagonal (d) of the quadrilateral be $8x$.

Thus, the lengths of the perpendiculars from opposite vertices are $h_1 = 3x$ and $h_2 = 4x$.

It is given that area of the quadrilateral = 448 m^2

$$\therefore \frac{1}{2}d\left(h_1 + h_2\right) = 448$$
$$\frac{1}{2}\times 8x\left(3x + 4x\right) = 448$$
$$4x \times 7x = 448$$
$$28x^2 = 448$$
$$x^2 = 16$$
$$x = \sqrt{16} = 4$$

$\therefore d = 8x = (8 \times 4)$ m = 32 m

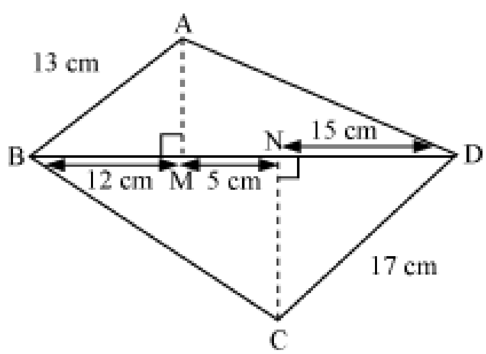
$h_1 = 3x = (3 \times 4)$ m = 12 m

$h_2 = 4x = (4 \times 4)$ m = 16 m

Thus, the length of the diagonal of the quadrilateral is 32 m, while the lengths of the perpendiculars on the diagonal from the opposite vertices of the quadrilateral are 12 m and 16 m.

Example 2:

Find the area of quadrilateral ABCD in the following figure.



Solution:

ΔABM is a right-angled triangle.

Applying Pythagoras theorem in ΔABM,

$AM^2 + BM^2 = AB^2$

$AM^2 + 12^2 = 13^2$

$AM^2 = 169 - 144 = 25$

$AM = \sqrt{25} = 5 \text{ cm}$

$\therefore h_1 = 5$ cm

Similarly, ΔCND is also a right-angled triangle.

Applying Pythagoras theorem in ΔCND,

$CN^2 + DN^2 = CD^2$

$CN^2 + 15^2 = 17^2$

$CN^2 = 289 - 225 = 64$

$CN = \sqrt{64} = 8 \text{ cm}$

$\therefore h_2 = 8$ cm

Length of diagonal BD = d = BM + MN + ND = (12 + 5 + 15) cm = 32 cm

Hence, area of the quadrilateral ABCD = $\frac{1}{2}d\left(h_1 + h_2\right)$

$$= \left[\frac{1}{2} \times 32 \times \left(5 + 8\right)\right] \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 32 \times 13\right) \text{ cm}^2$$

$= 208 \text{ cm}^2$

Area Of A Rhombus

How do we calculate the area of a rhombus? The given video will explain the formula required to calculate the area of a given rhombus.

Let us discuss some more examples based on the area of a rhombus.

Example 1:

A floor of a building consists of 5000 rhombus shaped tiles. The length of diagonals of each tile is 6 dm and 80 cm. If the cost of polishing the floor is Rs 70 per 3m^2, then find the total cost of polishing the floor.

Solution:

The diagonals of rhombus shaped marble tiles are

$$d_1 = 6\,\text{dm} = \frac{6}{10} = 0.6\,\text{m}\left(\because 1\,\text{dm} = \frac{1}{10}\,\text{m}\right)$$

$$\text{and } d_2 = 80\,\text{cm} = \frac{80}{100} = 0.8\,\text{m}\left(\because 1\,\text{cm} = \frac{1}{100}\,\text{m}\right)$$

Hence, area of each tile $= \frac{1}{2} d_1 \times d_2$

$$= \left(\frac{1}{2} \times 0.6 \times 0.8\right)\ \text{m}^2$$

Area of 5000 tiles = $\left(5000 \times \frac{1}{2} \times 0.6 \times 0.8\right)\ \text{m}^2$

$$= (2500 \times 0.6 \times 0.8)\ \text{m}^2$$
$$= 1200\ \text{m}^2$$

The rate of polishing the floor is Rs 70 per 3 m^2.

Hence, cost of polishing the floor $= \text{Rs } 1200 \times \frac{70}{3} = \text{Rs } 28000$

Example 2:

Area of a rhombus is $1600\sqrt{3}\ \text{dm}^2$. If one of its diagonals is $40\sqrt{3}\ \text{dm}$, then find the length of the other diagonal in terms of metre.

Solution:

Length of one diagonal $d_1 = 40\sqrt{3}\ \text{dm}$

Length of other diagonal = d_2

Given area of the rhombus $= 1600\sqrt{3}\ \text{dm}^2$

Therefore,

$$\frac{1}{2} d_1 \times d_2 = 1600\sqrt{3}$$
$$\frac{1}{2} \times 40\sqrt{3} \times d_2 = 1600\sqrt{3}$$
$$d_2 = \frac{1600\sqrt{3}}{20\sqrt{3}}$$
$$d_2 = 80\ \text{dm}$$
$$d_2 = \frac{80}{10}\,\text{m} = 8\,\text{m}\left(\because 1\,\text{dm} = \frac{1}{10}\ \text{m}\right)$$

Hence, the length of the other diagonal is 8 m.

Example 3:

If the ratio of the diagonals of a rhombus is 4:7 and the area of rhombus is 1400 m^2, then find the dimensions of the diagonals.

Solution:

Let the diagonals be $d_1 = 4x$ and $d_2 = 7x$.

Area of the rhombus = 1400 m^2.

Therefore,

$$\frac{1}{2} d_1 \times d_2 = 1400$$
$$\frac{1}{2} \times 4x \times 7x = 1400$$

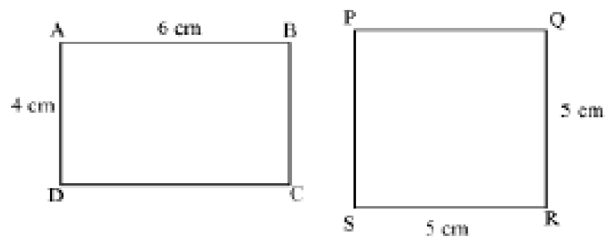
$$14x^2 = 1400$$
$$x^2 = 100$$
$$x = \sqrt{100} = 10$$
$$\text{Thus, } d_1 = 4x = 4 \times 10 = 40\,\text{m}$$
$$\text{and } d_2 = 7x = 7 \times 10 = 70\,\text{m}$$

Hence, the dimensions of the diagonals of the rhombus are 40 m and 70 m.

Area of Rectangle and Square

We usually come across the situations in our life when we need to find the area of various types of things such as area of a piece of land, area of wall to be painted, area of cloth required etc. The most common shapes that we see in our life are square and rectangle and thus, it becomes necessary for us to learn how to find their area.

Look at the figures given below.



Here, the first figure, i.e. ABCD, is a rectangle of length 6 cm and breadth 4 cm whereas the second figure, i.e. PQRS, is a square of side 5 cm.

Can we find which of the two shapes has the greater area?

It is difficult to answer this question by merely looking at the figures. To find the area of a rectangle or a square, we have to know the formula for each of them. Therefore, let us learn these formulae with the help of this video.

If the measure of the diagonal of the square is known, then its area can be calculated using the following formula.

$$\text{Area of square} = \frac{(\text{Diagonal})^2}{2}$$

Now, let us consider a real life situation to understand the concept better.

The owner of a paddy field decides to construct a 3 m wide path outside the field along its boundary. What will be the cost of constructing the path at the rate of Rs 500 per m^2?

Go through the given video to find the solution of this problem.

Now, let us discuss how to convert units of area.
As, we have already studied about the conversion of units for length and they are as follows:

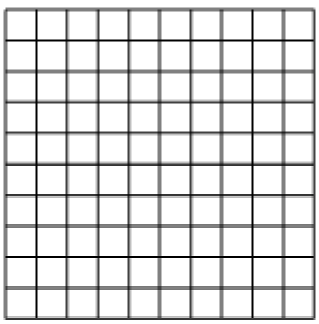
1 centimetre = 10 millimetres

1 metre = 100 centimetres

1 kilometre = 1000 metres

In the same manner, we can convert the unites of areas as well.

Let us consider a square of side 1 cm and divide that square into 100 small squares, each of side 1 mm.



It is evident from the figure that area of a square of side 1 cm will be equal to the areas of 100 small squares of side 1 mm.

$\Rightarrow 1\ cm^2 = 100 \times 1\ mm^2$

$\Rightarrow 1\ cm^2 = 100\ mm^2$

Similarly, we can say that $1\ m^2 = 1\ m \times 1\ m = 100\ cm \times 100\ cm = 10000\ cm^2$.

Now, to convert $1\ km^2$ into m^2, we will proceed as follows:

$1\ km^2 = 1\ km \times 1\ km = 1000\ m \times 1000\ m = 1000000\ m^2$.

It is quite significant to note that when we convert a unit of area to a smaller unit, the resulting number of units will be bigger.

The areas of land are usually measured in hectares where 1 hectare = Area of a square of side 100 m.

i.e. 1 hectare = 100 m × 100 m = 10000 m^2.

Let us have a look at some more examples to be clear with the concept.

Example 1:

A paddy field is in the form of a square with length 100 m. Find the area of the field.

Solution:

It is given that the length of a side of the square paddy field is 100 m.

∴ Area of the paddy field = $(\text{Side})^2$

= $(100 \text{ m})^2$

= 10000 m^2

Example 2:

A square park is 300 m long diagonally. What is the area of the park?

Solution:

Length of diagonal of square park = 300 m

$$\text{Area of square park} = \frac{(\text{Diagonal})^2}{2}$$

$$\Rightarrow \text{Area of square park} = \frac{(300)^2}{2} \text{ m}^2$$

$$\Rightarrow \text{Area of square park} = \frac{90000}{2} \text{ m}^2$$

⇒ Area of square park = 45000 m^2

Example 3:

All the four walls of a room have to be painted. If all the four walls have the equal length of 4 m and equal breadth of 3 m, then find the total cost of painting the walls at the rate of Rs 7 per m^2.

Solution:

Length of a wall = 4 m

Breadth of a wall = 3 m

Now, area of one wall = length × breadth

= 4m × 3m

= 12 m^2

∴ Total area of the four walls = 4 ×12 m^2

= 48 m^2

Cost of painting the walls = Rs 7 per m^2.

∴ Total cost of painting the four walls = Rs (7 × 48)

= Rs 336

Example 4:

A 90 cm long wire is bent into a rectangle of length 30 cm and an 80 cm long wire is bent in the form of a square. Which encloses more area − the rectangle or the square, and by how much?

Solution:

Perimeter of the rectangle = Length of the wire = 90 cm

Length of the rectangle = 30 cm

Let the breadth of the rectangle be *b*.

∴ Perimeter of the rectangle = 90 cm = 2 (30 cm + b)

⇒ 90 cm = 2 × 30 cm + 2 × b

⇒ 90 cm = 60 cm + $2b$

⇒ $2b$ = 90 cm − 60 cm = 30 cm

$$\Rightarrow b = \frac{30 \text{ cm}}{2} = 15 \text{ cm}$$

Thus, area enclosed by the rectangle = length × breadth = 30 cm × 15 cm = 450 cm^2

Perimeter of the square = Length of the wire = 80 cm

Let the length of the square be *l*.

∴ Perimeter of the square = 80 cm = 4*l*

$$\Rightarrow l = \frac{80 \text{ cm}}{4} = 20 \text{ cm}$$

Thus, area enclosed by the square = side × side = 20 cm × 20 cm = 400 cm^2

Therefore, the rectangle encloses 450 cm^2 − 400 cm^2 = 50 cm^2 more area than the square.

Example 5:

From a rectangular sheet of paper of 30 cm length and 600 cm^2 area, the biggest possible square is cut out. What is the area of the sheet of paper left?

Solution:

Length of the rectangular sheet of paper = 30 cm

Let the width of the rectangular sheet of paper be *w*.

Area of the sheet of paper = 600 cm^2 = 30 cm × *w*

$$\Rightarrow w = \frac{600 \text{ cm}^2}{30 \text{ cm}} = 20 \text{ cm}$$

∴ Width of the sheet = 20 cm

The biggest possible square that can be cut off from this sheet has each side of length equal to the width of the square i.e., 20 cm.

Thus, area of the square that is cut off = 20 cm × 20 cm = 400 cm^2

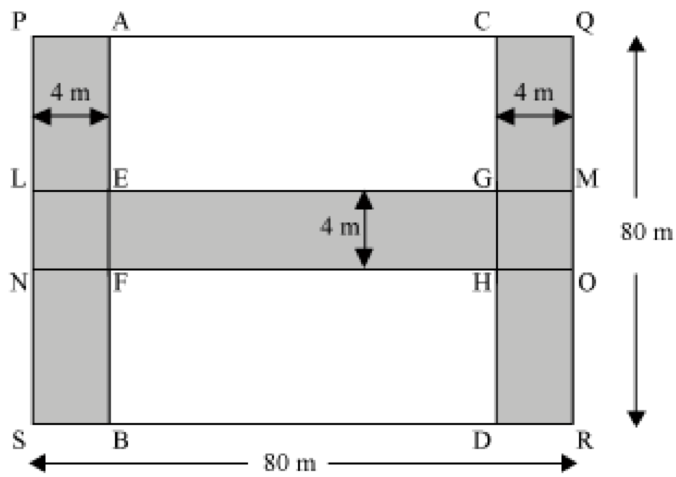
Thus, area of the sheet left = Original area of the sheet − Area of the square that is cut off

= 600 cm^2 − 400 cm^2

= 200 cm^2

Example 6:

Two jogging tracks, each of width 4 m, run along the two opposite sides inside a park. Another jogging track runs through the centre of the park and intersects the two tracks perpendicularly. What is the total area of the tracks?



Solution:

In the given figure, the shaded portion represents the jogging tracks.

Now, area of the roads

= area of PABS + area of CQRD + area of LMON − Area of LEFN − area of GMOH

= (80 m × 4 m) + (80 m × 4 m) + (80 m × 4 m) − (4 m × 4 m) − (4 m × 4 m)

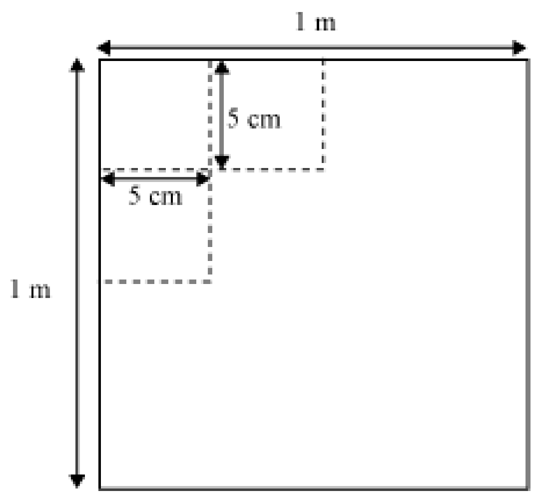
= 320 m^2 + 320 m^2 + 320 m^2 − 16 m^2 − 16 m^2

= 960 m^2 − 32 m^2

= 928 m^2

Example 7:

Raju was given a sheet of paper having all the sides of length 1 m. He was asked to draw squares with side 5 cm on the given sheet without leaving any space. How many squares can be drawn?



Solution:

Given, length of the sheet = 1 m = 100 cm

The length of all sides of the sheet is the same i.e., the sheet is in the form of a square.

Therefore, area of the sheet = $(\text{side})^2$

= $(100 \text{ cm})^2$

= 10000 cm^2

Now, side of each square to be drawn = 5 cm

∴ Area of one square = $(\text{Side})^2$

= $(5 \text{ cm})^2$

= 25 cm^2

Number of squares that can be drawn on the sheet

$$= \frac{\text{Area of the sheet}}{\text{Area of one square}}$$

$$= \frac{10000}{25} = 400$$

Thus, 400 squares can be drawn on the sheet.

Example 8:

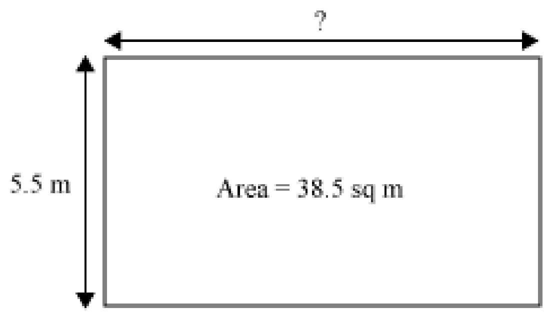
The area of a rectangular pond is 38.5 m^2. If the breadth of the pond is 5.5 m, then find the length of the pond.

Solution:

Given, area of the pond = 38.5 m^2

Breadth of the pond = 5.5 m

We have to find the length of the pond.



Since the pond is rectangular in shape,

Length × Breadth = Area of the pond

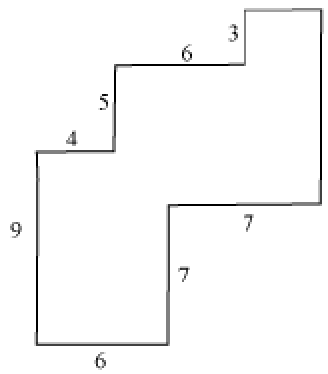
⇒ Length × 5.5 m = 38.5 m^2

⇒ Length = $\left(\frac{38.5}{5.5}\right) \text{m} = 7 \text{ m}$

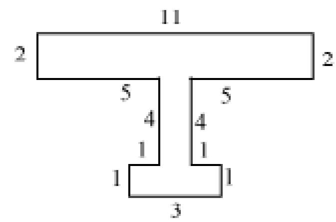
Thus, the length of the pond is 7 m.

Example 9:

By splitting the following figures into rectangles, find their areas. (The measures are given in centimetres)



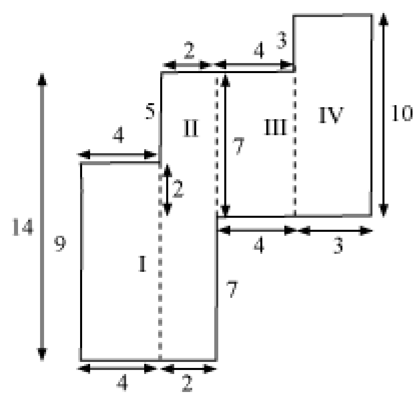
(a)



(b)

Solution:

(a) By splitting the given figure into rectangles, we will obtain the following figure.



For rectangle I,

Length = 4 cm and breadth = 9 cm

∴ Area of rectangle I = Length × Breadth = (4 × 9) sq cm = 36 cm^2

For rectangle II,

Length = 14 cm and breadth = 2 cm

∴ Area of rectangle II = Length × Breadth = (14 × 2) sq cm = 28 cm^2

For rectangle III,

Length = 7 cm and breadth = 4 cm

∴ Area of rectangle III = Length × Breadth = (7 × 4) sq cm = 28 cm^2

For rectangle IV,

Length = 10 cm and breadth = 3 cm

∴ Area of rectangle IV = Length × Breadth = (10 × 3) sq cm = 30 cm^2

∴ Area of the given figure = Area of rectangle I + Area of rectangle II +

Area of rectangle III + Area of rectangle IV

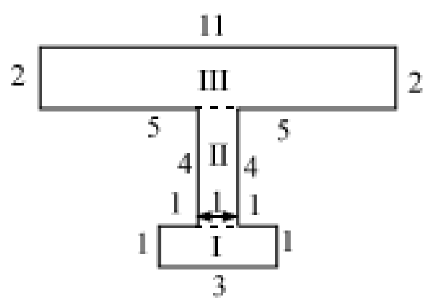
= 36 cm^2 + 28 cm^2 + 28 cm^2 + 30 cm^2

= (36 + 28 + 28 + 30) cm^2

= 122 cm^2

Thus, the area of the given figure is 122 cm^2.

(b) By splitting the given figure into rectangles, we will obtain the following figure.



For rectangle I,

Length = 3 cm and breadth = 1 cm

∴ Area of rectangle I = Length × Breadth = (3 × 1) cm^2 = 3 cm^2

For rectangle II,

Length = 4 cm and breadth = 1 cm

∴ Area of rectangle II = Length × Breadth = $(4 \times 1)\ cm^2 = 4\ cm^2$

For rectangle III,

Length = 11 cm and breadth = 2 cm

∴ Area of rectangle III = Length × Breadth = $(11 \times 2)\ cm^2 = 22\ cm^2$

∴ Area of the given figure = Area of rectangle I + Area of rectangle II +

Area of rectangle III

$= (3 + 4 + 22)\ cm^2$

$= 29\ cm^2$

Thus, the area of the given figure is $29\ cm^2$.

Perimeter of Rectangle

Rectangle is a quadrilateral with opposite sides equal. Let us try to find the general formula for perimeter of any rectangle with given length and breadth with the help of an example.

Let us discuss some more examples based on the perimeter of a rectangle.

Example 1:

Find the cost of fencing a rectangular park of length 217 m and breadth 183 m at the rate of Rs 12.50 per metre.

Solution:

Length of the rectangle = 217 m

Breadth of the rectangle = 183 m

To find the cost of fencing the rectangular park, we have to find out the perimeter of the rectangular park.

Now, perimeter of the rectangular park = 2 × (length + breadth)

= 2 × (217 m + 183 m)

= 2 × (400 m)

= 800 m

Cost of fencing = Rs 12.50 per metre

∴Cost of fencing 800 m = Rs (12.50 × 800) = Rs 10000

Thus, the cost of fencing the whole rectangular park is Rs 10000.

Example 2:

The lid of a rectangular box of size 60 cm by 20 cm is sealed all around with tape. Find the length of the tape required.

Solution:

Length of the rectangular box = 60 cm

Breadth of the rectangular box = 20 cm

The rectangular box is sealed all around with tape i.e., the tape covers the boundary of the rectangular box.

∴ Length of tape required = Perimeter of the rectangular box

= 2 × (Length + Breadth)

= 2 × (60 cm + 20 cm)

= 2 × (80 cm)

= 160 cm

Example 3:

A rectangular piece of land measures 0.75 km by 0.5 km. Each side is to be fenced with 6 rows of wires. Find the length of the wire required.

Solution:

Length of the rectangular land = 0.75 km

Breadth of the rectangular land = 0.5 km

It is given that each side of the land is to be fenced with 6 rows of wires.

Therefore, the total length of wire required for fencing is 6 times the perimeter of the land.

Perimeter of the rectangular land = 2 × (Length + Breadth)

= 2 × (0.75 km + 0.5 km)

= 2 × (1.25 km)

= 2.5 km

∴ Length of wire required = (6 × 2.5 km) = 15 km

Example 4:

Chulbul takes 10 rounds of a rectangular park, which is 65 m long and 35 m wide. Find the total distance covered by her.

Solution:

Length of the rectangular park = 65 m

Breadth of the rectangular park = 35 m

Total distance covered by Chulbul in one round is the perimeter of the rectangular park.

Perimeter of the rectangular park = 2 × (Length + Breadth)

= 2 × (65 m + 35 m)

= 2 × (100 m)

= 200 m

Therefore, total distance covered by Chulbul in 1 round = 200 m

∴ Distance covered by Chulbul in 10 rounds = (10 × 200 m) = 2000 m

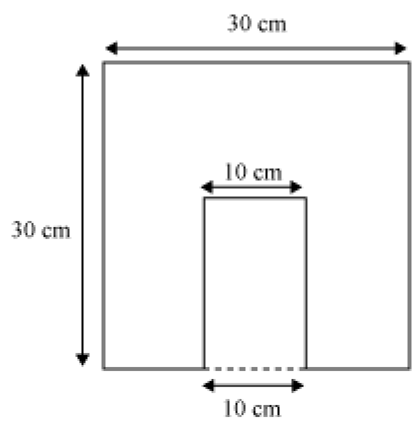
However, $1 \text{ m} = \left(\frac{1}{1000}\right) \text{ km}$

$$\therefore 2000 \text{ m} = \left(2000 \times \frac{1}{1000}\right) \text{ km} = 2 \text{ km}$$

Thus, the distance covered by Chulbul is 2 km.

Example 5:

A rectangular portion is cut off from one side of a square sheet of paper as shown in the figure. If the length of the rectangular portion is 15 cm, then what will be the difference between the perimeters of the sheet of paper before and after cutting off the rectangular portion?



Solution:

Original perimeter of the sheet of paper = 4 × 30 cm = 120 cm

Length of the rectangular portion that is cut off = 15 cm

Breadth of the rectangular portion that is cut off = 10 cm

After cutting off the rectangular portion, three sides of the sheet of paper remain the same, but one side gets changed.

Thus, perimeter of the sheet of paper after the rectangular portion is cut off

= 30 cm + 30 cm + 30 cm + (30 − 10) cm + 15 cm + 10 cm + 15 cm

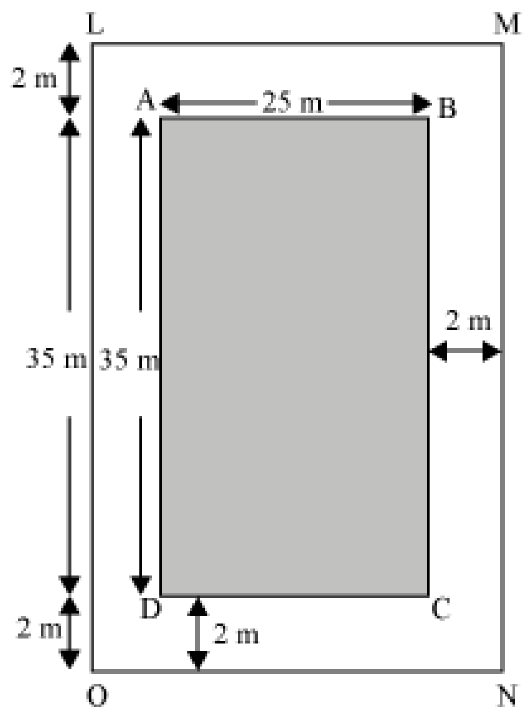
= 30 cm + 30 cm + 30 cm + 20 cm + 15 cm + 10 cm + 15 cm

= 150 cm

Thus, difference between the two perimeters = 150 cm − 120 cm = 30 cm

Example 6:

What is the difference between the perimeters of rectangles ABCD and LMNO?



Solution:

Length of rectangle ABCD = 35 m

Breadth of rectangle ABCD = 25 m

∴ Perimeter of rectangle ABCD = 2 (35 m + 25 m) = 2 × 60 m = 120 m

Length of rectangle LMNO = 2m + 35 m + 2 m = 39 m

Breadth of rectangle LMNO = 2 m + 25 m + 2 m = 29 m

∴ Perimeter of rectangle LMNO = 2 (39 m + 29 m) = 2 × 68 m = 136 m

∴ Difference between the perimeters of the two rectangles = 136 m − 120 m = 16 m

Example 7:

The perimeters of a square and a rectangle are equal. If the length of the rectangle is 9 cm and its breadth is 3 cm less than its length, then what is the length of each side of the square?

Solution:

Length (l) of the rectangle = 9 cm

Thus, breadth (b) of the rectangle = (9 − 3) cm = 6 cm

∴ Perimeter of the rectangle = 2 × (l + b) = 2 × (9 + 6) cm = 2 × 15 cm = 30 cm

It is given that the perimeters of the square and the rectangle are equal.

Therefore, perimeter of the square = 4 × length of the square = 30 cm

Thus, length of each side of the square = $\frac{30\text{ cm}}{4} = 7.5\text{ cm}$

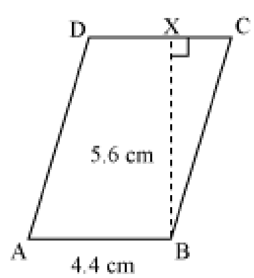
Areas Of Parallelograms

Ramesh and his brother Suresh once got into an argument. Suresh said that the area of a parallelogram is the product of its base and height while Ramesh said that this formula is for the area of a rectangle. Who do you think is right?

Ramesh, of course, is right as the area of a rectangle is the product of its base and height. However, this formula also represents the formula for the area of a parallelogram. Let us consider the following parallelogram. Can we convert it into a rectangle? Let us see how.

Thus, area of a parallelogram = base × height

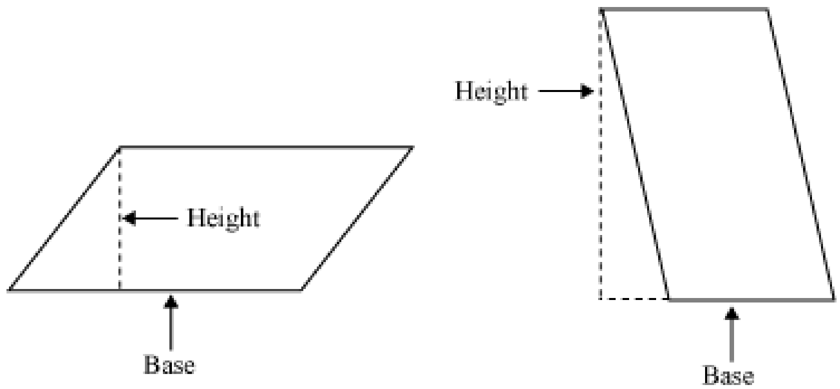
Using this formula, let us calculate the area of parallelogram ABCD drawn below.



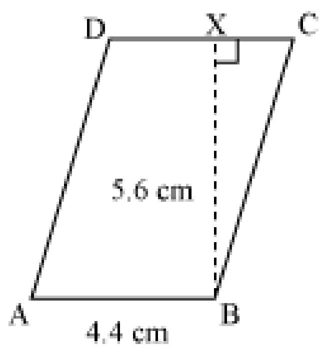
Here, AB is the base of the parallelogram and its corresponding height is BX.

Thus, area of the parallelogram = AB × BX = 4.4 cm × 5.6 cm = 24.64 cm^2

A point to note is that any side of a parallelogram can be taken as a base and the perpendicular drawn to the base from the opposite vertex is the height of the parallelogram.



In the example cited above, let us draw a perpendicular from vertex B to side CD.



Now, we can take the base of the parallelogram as CD and its corresponding height as BX.

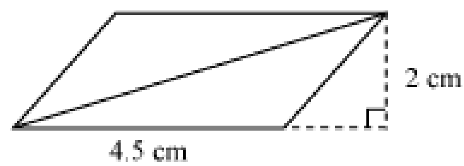
Thus, the area of the parallelogram is also equal to the product of base CD and its corresponding height BX.

Let us solve some examples to understand the concept better.

Solved examples

Example 1:

What is the area of the following parallelogram?



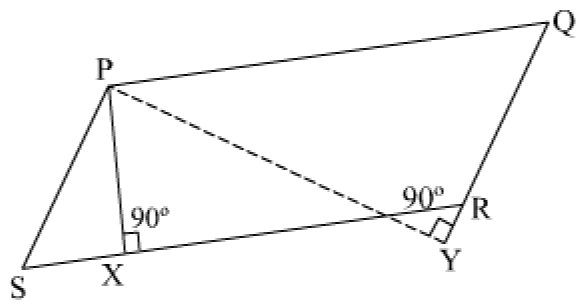
Solution:

Base of the parallelogram = 4.5 cm

Height of the parallelogram = 2 cm

Therefore, area of PQRS = height × base = 4.5 cm × 2 cm = 9 cm^2

Example 2:



PQRS is a parallelogram.

The lengths of $\overline{SR}$ and $\overline{PX}$ are 7 cm and 2.5 cm respectively. If the length of $\overline{PY}$ is 5 cm, then what will be the length of $\overline{RQ}$?

Solution:

It is given that $\overline{SR}$ = 7 cm and $\overline{PX}$ = 2.5 cm.

Therefore, area of parallelogram PQRS = height × base = $\overline{SR}$ × $\overline{PX}$ = 7 × 2.5 cm^2 = 17.5 cm^2

If we choose $\overline{RQ}$ as the base, then its corresponding height will be $\overline{PY}$.

Now, height × base = area of parallelogram PQRS

⇒ $\overline{PY} \times \overline{RQ}$ = area of PQRS

⇒ 5 cm × $\overline{RQ}$ = 17.5 cm^2

⇒ $\overline{RQ} = \frac{17.5}{5}$ cm = 3.5 cm

Area Of A Trapezium

Namita has a garden in front of her house. The garden is in the shape of a trapezium. The lengths of the parallel sides of the garden are 15 m and 20 m and the distance between these parallel sides is 12 m. She wants to spread fertilizer in the garden, which costs Rs 5 per square metre. Namita is wondering what it will cost her to spread the fertilizer in the entire garden. Can we help her out?

Let us now discuss some examples based on the areas of trapeziums.

Example 1:

What is the area of a trapezium (in m^2) whose parallel sides are 125 cm and 7.5 dm and the perpendicular distance between them is 90 cm.

Solution:

The parallel sides are $a = 125 \text{ cm} = \frac{125}{100} = 1.25 \text{ m}$ and

$b = 7.5 \text{ dm} = \frac{7.5}{10} = 0.75 \text{ m}$

$\left(\text{As } 1 \text{ cm} = \frac{1}{100} \text{ m and } 1 \text{ dm} = \frac{1}{10} \text{ m}\right)$

Perpendicular distance between the parallel sides = h = 90 cm $= \frac{90}{100} = 0.9 \text{ m}$

But we know that area of a trapezium $= \frac{1}{2} h(a+b)$

$$=\left[\frac{1}{2}\times 0.9\left(1.25+0.75\right)\right]\text{m}^2$$
$$=\left(\frac{1}{2}\times 0.9\times 2\right)\text{m}^2$$

= 0.9 m^2

Hence, the area of the given trapezium is 0.9 m^2.

Example 2:

Area of a trapezium is 285 dm^2. If length of one of the parallel sides is 24 dm and its height is 15 dm, then what is the length of the other parallel side?

Solution:

Given that area of trapezium = 285 dm^2

Length of one of the parallel sides = a = 24 dm

Height = h = 15 dm

We have to find another parallel side i.e. 'b'.

But we know that area of a trapezium $=\frac{1}{2}h\left(a+b\right)$

$$285=\frac{1}{2}\times 15\left(24+b\right)$$
$$24+b=\frac{285\times 2}{15}$$

24 + b = 38

b = 38 − 24

b = 14 dm

Hence, the length of the other parallel side of the trapezium is 14 dm.

Example 3:

The two parallel sides of a trapezium are in the ratio 3:5 and its height is 9 cm. What are the dimensions of the parallel sides of the trapezium if its area is 90 cm^2?

Solution:

Let the two parallel sides of the trapezium be $a = 3x$ and $b = 5x$.

Given that, height = h = 9 cm

And area of trapezium = 90 cm^2

But we know that area of trapezium $= \frac{1}{2}h(a+b)$

$$90 \text{ cm}^2 = \left[\frac{1}{2}\times 9(3x+5x)\right] \text{ cm}$$
$$90 \text{ cm}^2 = \left[\frac{1}{2}\times 9\times 8x\right] \text{ cm}$$

$$x = \frac{90\times 2}{9\times 8} \text{ cm}$$
$$x = \frac{5}{2} = 2.5 \text{ cm}$$

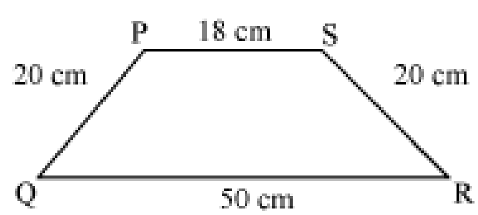
But, $a = 3x = 3 \times 2.5$ cm = 7.5 cm

$b = 5x = 5 \times 2.5$ cm = 12.5 cm

Hence, the dimensions of the parallel sides of the given trapezium are 7.5 cm and 12.5 cm.

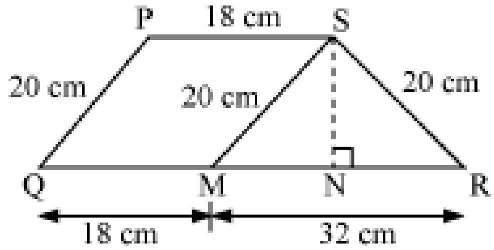
Example 4:

The following figure PQRS is a trapezium, where PS || QR. If PS = 18 cm, QR = 50 cm, PQ = RS = 20 cm, then what is the area of trapezium PQRS?



Solution:

In the trapezium PQRS, let us draw SM || PQ which intersects QR at M.



Let us also draw SN ⊥ MR.

As PS || QR

Therefore, PS || QM……………… (1)

From our construction, SM || PQ………… (2)

Thus, from equation (1) and equation (2), PQMS is a parallelogram.

(Opposite sides of a parallelogram are parallel)

Hence, SM = PQ (opposite sides of a parallelogram are equal)

and QM = PS

SM = 20 cm and QM = 18 cm

But MR = QR − QM = 50 − 18 = 32 cm

Triangle SMR is an isosceles triangle (as SM and SR = 20 cm each)

Now, $NR = \frac{1}{2} MR = \frac{1}{2} \times 32 = 16 \text{ cm}$

Now, the triangle SNR is a right-angled triangle.

Hence, $SR^2 = SN^2 + NR^2$ (Using Pythagoras theorem)

$20^2 = SN^2 + 16^2$

$SN^2 = 400 - 256 = 144$

SN = $\sqrt{144}$ = 12 cm

This is the height of trapezium.

Hence, we obtain *a* = 18 cm, *b* = 50 cm and *h* = 12 cm.

Area of the trapezium

$$= \frac{1}{2} h(a+b)$$
$$= \left[\frac{1}{2} \times 12(18+50)\right] \text{ cm}^2$$

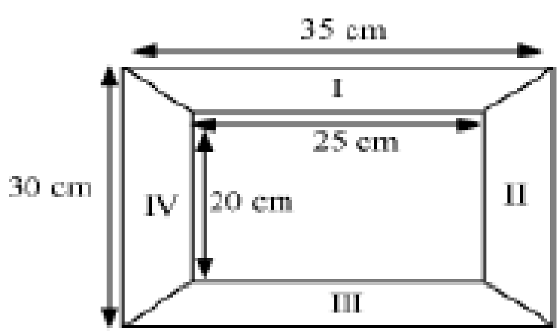
= (6 × 68) cm^2

= 408 cm^2

Hence, the area of the given trapezium is 408 cm^2.

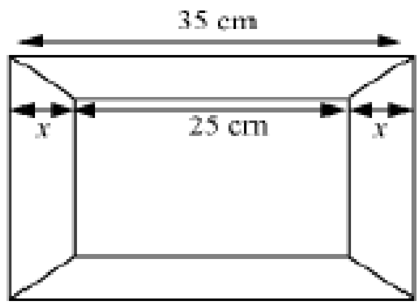
Example 5:

Diagram of the following picture frame has outer dimensions of 35 cm × 30 cm and inner dimensions of 25 cm × 20 cm. If the width of each section is the same, then find the area of each outer section of the frame.



Solution:

Let the width of the given picture frame be *x* cm.



From the figure, it is clear that

$x + 25 + x = 35$

$2x = 35 - 25 = 10$

$x = 5$ cm

Hence, the width of the frame is 5 cm.

This is the height for the outer sections I, II, III and IV which are in the shape of a trapezium.

Thus, area of the trapezium I = $\frac{1}{2}h(a+b)$

$$= \left[\frac{1}{2} \times 5(35+25)\right] \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 5 \times 60\right) \text{ cm}^2$$
$$= 150 \text{ cm}^2$$

Thus, area of the trapezium II = $\frac{1}{2}h(a+b)$

$$= \left[\frac{1}{2} \times 5(30+20)\right] \text{ cm}^2$$
$$= \left(\frac{1}{2} \times 5 \times 50\right) \text{ cm}^2$$
$$= 125 \text{ cm}^2$$

Thus, area of the trapezium III = $\frac{1}{2}h\left(a+b\right)$

$$=\left[\frac{1}{2}\times 5\left(35+25\right)\right]\ \text{cm}^2$$
$$=\left(\frac{1}{2}\times 5\times 60\right)\ \text{cm}^2$$
$$=150\ \text{cm}^2$$

Thus, area of the trapezium IV = $\frac{1}{2}h\left(a+b\right)$

$$=\left[\frac{1}{2}\times 5\left(30+20\right)\right]\ \text{cm}^2$$
$$=\left(\frac{1}{2}\times 5\times 50\right)\ \text{cm}^2$$
$$=125\ \text{cm}^2$$

Total area of the outer section = sum of the areas of all the trapeziums

= 150 + 125 + 150 + 125

= 550 cm^2.

Hence, the total area of the outer section is 550 cm^2.

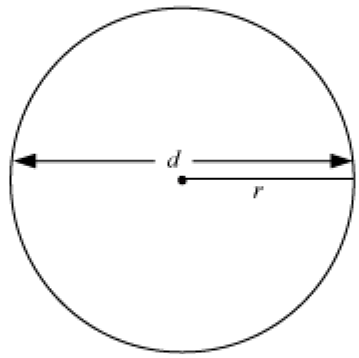
Circumference of Circle

A car moved certain distance. To cover this distance, its wheel of radius 21 cm made 50 revolutions. Can we calculate this distance covered by the car?

To understand how we calculate the distance, look at the following video.

We can find the distance covered by the car, if we can find the circumference of the circular wheel. For this, we have to know the formula for the circumference of a circle from its radius. Let us learn how to find the circumference of the circle from its radius.

For this, let us consider a circle of radius *r* and diameter *d*.



The circumference *C* of the circle is given by the formula

$$C = \pi d \text{ or } C = 2\pi r$$

Here, *d* = 2*r* and the approximate value of π (read as ‘pi’) is 3.14 or 22/7.

Using this formula for the circumference of a circle, let us find the distance covered by the car.

It is given that the radius of the wheel, *r* = 21 cm

We have, distance covered = 50 × circumference of the circle

$$= 50 \times 2\pi r$$

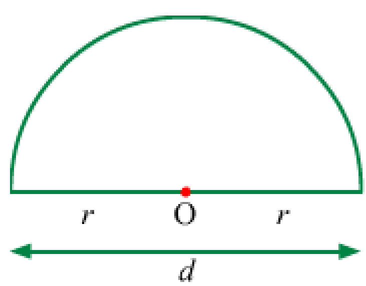
$$= \left(50 \times 2 \times \frac{22}{7} \times 21\right) \text{ cm} \qquad (\text{Putting values of } r \text{ and } \pi)$$

$$= 6600 \text{ cm}$$

$$= \left(\frac{6600}{100}\right) \text{ m} \qquad \left[1 \text{ cm} = \left(\frac{1}{100}\right) \text{ m}\right]$$

$$= 66 \text{ m}$$

In case of semicircle, we find the perimeter *P* as follows:



$P = \pi r + 2r$

or

$P = \pi r + d$

Now, let us discuss some examples based on circumference of a circle. In each of the examples, assume the value of π as $\frac{22}{7}$, unless mentioned otherwise.

Example 1:

What is the difference between the circumferences of two circles if the diameter of one is 21 cm and radius of the other is 7 cm?

Solution:

Circumference of the circle of diameter 21 cm = $\pi d = \frac{22}{7} \times 21 \text{ cm} = 66 \text{ cm}$

Circumference of the circle of radius 7 cm = $2\pi r = 2 \times \frac{22}{7} \times 7 \text{ cm} = 44 \text{ cm}$

Thus, difference between the circumference of the two circles = (66 − 44) cm = 22 cm

Example 2:

The wheel of a vehicle made 150 revolutions to cover a distance of 132 m. Find the diameter of the wheel.

Solution:

Distance covered by the wheel = 132 m = 13200 cm

Number of revolutions made by the wheel = 150

$$\text{Now, distance covered in one revolution} = \frac{\text{Total distance covered by the vehicle}}{\text{Number of revolutions}}$$
$$= \frac{13200 \text{ cm}}{150}$$
$$= 88 \text{ cm}$$

We know that the distance covered by the wheel in one revolution equals its circumference.

∴ Circumference of the wheel = 88 cm

We know, the circumference of the wheel = π*d*, where *d* is the diameter.

$$\Rightarrow \frac{22}{7} \times d = 88 \text{ cm}$$
$$\Rightarrow d = \left(88 \times \frac{7}{22}\right) \text{ cm} = 28 \text{ cm}$$

Thus, the diameter of the wheel is 28 cm.

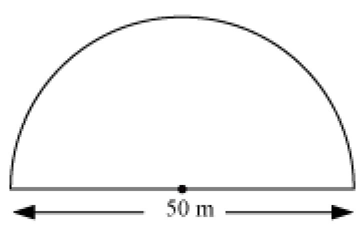
Example 3:

A gardener wants to fence a semi-circular garden of diameter 50 m with three rounds of wire. Find the length of the wire that he needs to purchase. Also find the costs of the wire, if it costs Rs 6 per metre. (Take π = 3.14)

Solution:

It is given that the diameter of the circle, *d* = 50 m

To find the length of wire required, we have to find the perimeter of the semi-circular field. The perimeter of the semi-circular field includes the circumference of the semi-circle and the diameter.



$$\begin{aligned}\text{Now, perimeter of the semi-circular field} &= \text{Circumference of the semi-circle} \\ &\quad + \text{ diameter of the circle} \\ &= \frac{1}{2} \times \pi d + d \\ &= \left(\frac{1}{2} \times 3.14 \times 50 + 50\right) \text{ m} \\ &= (78.5 + 50) \text{ m} \\ &= 128.5 \text{ m}\end{aligned}$$

1 round of wire requires 128.5 m of wire.

∴3 rounds of wire requires (128.5 × 3) m = 385.5 m wire

Therefore, the gardener has to purchase 385.5 m of wire.

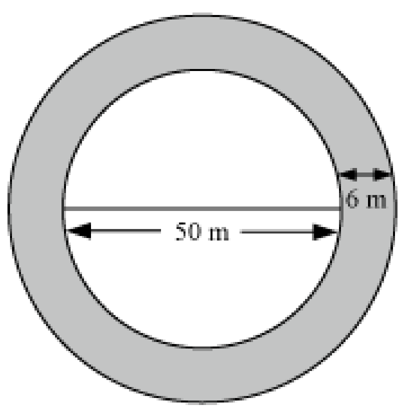
It is given that cost of 1 m of wire = Rs 6

∴ Cost of 385.5 m wire = Rs (385.5 × 6) = Rs 2313

Therefore, the cost of the wire is Rs 2313.

Area of Circle

The figure drawn below shows a circular park of diameter 50 m. The shaded region represents a 6 m wide jogging track, which runs along the boundary of the park.



This path requires to be gravelled at the rate of Rs 12 per square metre. Now, how do we calculate the required amount? The given video will help you understand the application of formula used to calculate the area of a circle.

In many cases, we have to find the area of semicircular shapes. Area of a semicircle is exactly half the area of the corresponding circle.

Mathematically, we can calculate the same using the following formula.

Area of semicircle = $\frac{1}{2} \times \pi r^2$

Let us discuss some examples based on the area of circles and semicircles in order to understand the concept better.

Example 1:

The circumference of a circle is 77 cm. Find its area.

Solution:

Let *r* be the radius of the circle.

It is given that $2\pi r = 77$ cm.

$$\Rightarrow r = \frac{77}{2 \times \frac{22}{7}}$$

$$\Rightarrow r = \frac{7 \times 7}{2 \times 2}$$

$$\Rightarrow r = \frac{49}{4}$$

$$\text{Area of circle } = \pi r^2$$

$$= \frac{22}{7} \times \left(\frac{49}{4}\right)^2$$

$$= 471.625 \text{ cm}^2$$

Thus, area of the circle is 471.625 cm^2.

Example 2:

Rajeev wants to paint the semi-circular face of a wooden article. If the diameter of the semi-circle is 3.5 m, then find the cost of painting it at the rate of Rs 100/m^2?

$$\left(\text{Use } \pi = \frac{22}{7}\right)$$

Solution:

Diameter of the semi-circular face, $d = 3.5$ m

We know that the diameter of a circle, $d = 2r$, where r is the radius of the circle.

$\therefore 2r = 3.5$ m

$$r = \left(\frac{3.5}{2}\right) \text{ m} = 1.75 \text{ m}$$

$$\text{Area of the semi-circular face} = \frac{1}{2} \times \pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times (1.75)^2\right) m^2$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times 1.75 \times 1.75\right) m^2 = 4.8125 \ m^2$$

Now, cost of painting 1 m^2 area = Rs 100

Therefore, cost of painting 4.8125 m^2 area = Rs (100 × 4.8125) = Rs 481.25

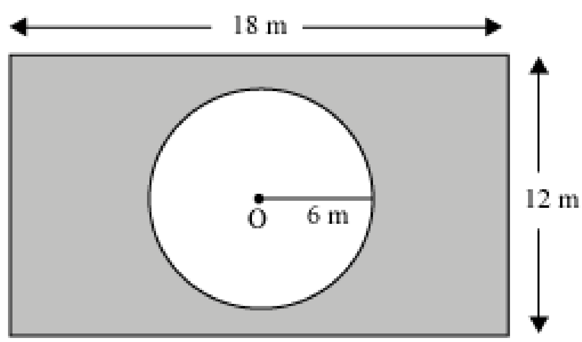
Thus, the cost of painting the top face of the wooden block is Rs 481.25.

Example 3:

A rectangular garden is 18 m long and 12 m wide. A sprinkler is used to water the garden and covers a radius of 6 m. What area of the garden is not watered by the sprinkler? [Use π = 3.14]

Solution:

The given information can be represented as



Here, the rectangle represents the garden, while point O shows the sprinkler.

The portion of the garden that is not watered by the sprinkler is shown by the shaded part.

It is clear that, area of the garden that is not watered by the sprinkler

= Area of the garden − area of circle with radius 6 m

Area of the garden = length × breadth = 18 m × 12 m = 216 m^2

Also, we know area of a circle = πr^2, where r is the radius of the circle.

∴Area of the circle with radius 6 m = $\pi \times (6\text{ m})^2 = (3.14 \times 36)\text{ m}^2 = 113.04\text{ m}^2$

Thus, area of the garden that is not watered by the sprinkler = $(216 - 113.04)\text{ m}^2$

= 102.96 m^2

Example 4:

The second's hand of a circular clock is 10 cm long. What area does the tip of the second's hand cover in half an hour? (Take π = 3.14)

Solution:

Radius of the second's hand, r = 10 cm

The second's hand of a clock takes 1 minute to complete one revolution.

We know half an hour = 30 minutes

Therefore, number of revolution made by the second's hand in half an hour = 30

Area covered by the second's hand in one revolution = $\pi r^2 = (3.14 \times 10 \times 10)\text{ cm}^2$

= 314 cm^2

Thus, area covered by the second's hand in 30 revolutions = $(30 \times 314)\text{ cm}^2 = 9420\text{ cm}^2$