Chapter 3

Pulse Modulation and Baseband Communications

CHAPTER HIGHLIGHTS

- *Pulse Modulations*
- *Sampling*
- *Natural Sampling*
- *Flat Top Sampling*
- *Analog Pulse Modulations*
- *Pulse Position Modulation*
- *Digital Pulse Modulation*
- *Pulse Code Modulation*
- *Quantization*
- *Non Uniform Quantization*
- *Encoding*
- *Non Return to Zero Signaling*
- *Return to Zero Signaling*
- *Bipolar Return to Zero Signaling*
- *Manchester Signaling*
- *Differential Encoding*
- *Differential Pulse Code Modulation*
- *Delta Modulation*
- *Base Band Pulse Transmission*
- *Matched Filter*
- *Inter Symbol Interference*
- *Ideal Nyquist Channel*
- *Raised Consine Spectrum*

PULSE MODULATIONS

Pulse modulations can be divided into following two families:

1. Analog pulse modulation
2. Digital pulse modulation

In the Analog pulse modulation, the message signal can be transmitted by varying one of the quantities of pulse train such as amplitude, width, or position in accordance with the message signal. These modulations are called pulse amplitude modulation, pulse width modulation, and pulse position modulation, respectively.

In the digital pulse modulation, the Analog signal is converted into digital signal. Pulse code modulation, differential pulse code modulation, and delta modulation are various techniques to convert the Analog signal into digital signal.

SAMPLING

Sampling is the technique to convert a continuous time continuous amplitude signal (Analog signal) into a discrete time continuous amplitude signal.

If the message signal $m(t)$ has a maximum frequency 'W', the sampling theorem guarantees that if we take the samples of $m(t)$ at a rate greater than 2 W, by using these samples we can completely reconstruct $m(t)$.

Sampling theorem can be proved by applying duality to the Fourier transform of periodic signals.

If $g(t)$ is periodic with period T_o, we can write

$$g_{T0}(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0)$$

The Fourier transform of periodic signal $g_{T0}(t)$ can be given as follows:

$$g_{T0}(t) \rightleftharpoons \sum_{n=-\infty}^{\infty} G(nf_0)\delta(f - nf_0)$$

Where $G(f)$ is the Fourier transform of generating signal $g(t)$ and $f_0 = \frac{1}{T_0}$

If $m(t)$ is the Analog signal, the sampled signal with sampling rate $f_s = \frac{1}{T_\delta}$] can be given by

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

To find out the frequency spectrum of $m_\delta(t)$, we can apply duality to above equation.

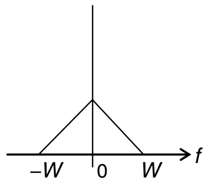
$$\sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) \Leftrightarrow f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)$$

The FT of the sampled signal is periodic in the frequency domain with a period f_s.

That is, if the signal is periodic in the time domain, the frequency spectrum has impulses separated by $f_0 = \frac{1}{T_0}$

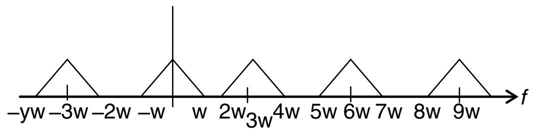
On the other hand, if the signal has impulses in the time domain, the spectrum is periodic in the frequency domain with a period f_s.

Let $M(f)$ =



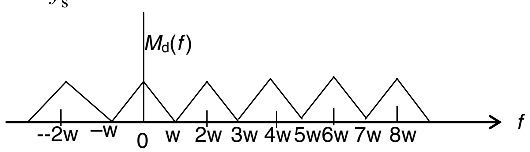
Case I: $f_s = 3$ W

$M_\infty(f)$

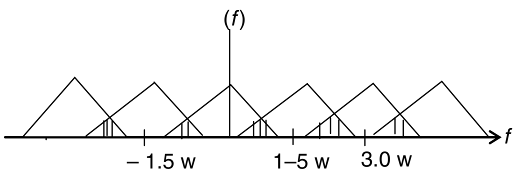


The spectrum of the samples is periodic with period 3 W. At the receiver, if we pass the samples through a low-pass filter of bandwidth W, exactly we can get back $M(f)$ and thus $m(t)$.

Case II: $f_s = 2$ W



The spectrum of the samples is periodic with a period 2 W. At the receiver, we can get back the original spectrum by using ideal low-pass filter of bandwidth W.



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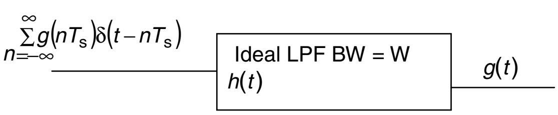
Case III: If $f_s = 1.5$ W, M_δ

In this case, the original spectrum of $M(f)$ cannot be regenerated from the sampled signal spectrum. This phenomenon is called aliasing.

Thus, to regenerate the original message signal $m(t)$ from the samples,

$$f_s \geq 2\text{ W}$$

$f_s = 2$ W is called Nyquist rate of sampling.



$$h(t) = 2\text{w sinc}(2wt)$$

$$g(t) = \left(\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t-nT_s)\right) \times 2\text{w sinc}(2wt)$$

$$= 2W\sum_{n=-\infty}^{\infty} g(nT_s)\sin c\left(2W\left(t-nT_s\right)\right).$$

sinc function is called interpolating function.

Let the maximum frequency in $g_1(t)$ and $g_2(t)$ is w_1 and w_2, respectively.

The Nyquist sampling rate for $g_1(t) = 2w_1$

Nyquist sampling rate for $g_2(t) = 2w_2$

Nyquist sampling rate for $g_1(t) + g_2(t) = 2$ Max (w_1, w_2)

Nyquist sampling rate for $g_1^{\text{n}}(t) = 2nw_1$

Nyquist sampling rate for $g_1^{\text{n1}}(t) + g_2^{\text{n2}}(t) = 2$ .max(n_1w_1, n_2w_2)

Nyquist sampling rate for $g_1(t).\ g_2(t) = 2(w_1 + w_2)$

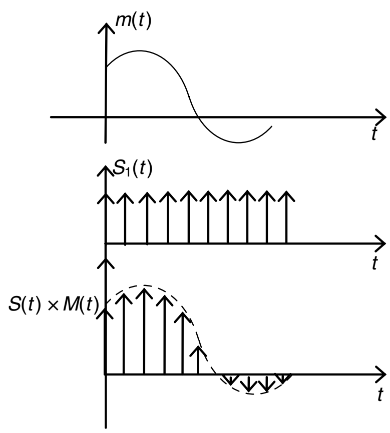
Nyquist sampling rate for $g_1(t) \times g_2(t) = 2\min(w_1, w_2)$

Types of Sampling

1. Ideal sampling or instantaneous sampling.
2. Rectangular pulse sampling or natural sampling.
3. Flat-top sampling.

Instantaneous Sampling

When a continuous signal $m(t)$ is sampled by a train of impulses, then it is called instantaneous sampling.

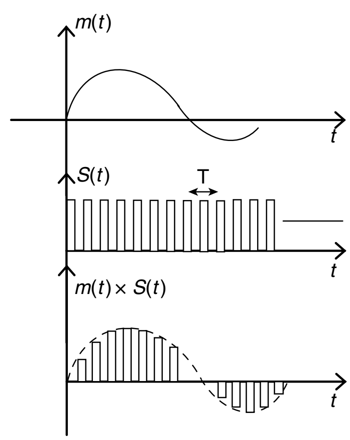


But this kind of instantaneous sampling is very complex to generate. It is possible to construct switches that could operate in an arbitrarily short time. We would never prefer to use them.

Disadvantage: Crosstalk is more in between the samples.

Natural Sampling

A reasonable manner of sampling is referred to as natural sampling. In this type of sampling, a train of pulses with duration τ are used for sampling.



In natural sampling, as in the instantaneous sampling, signal can also be fully recovered by its samples at the receiver side if the sampling has done at Nyquist rate.

At the receiver, the output signal

$$S_o(t) = \frac{\tau}{T_s} m(t)$$

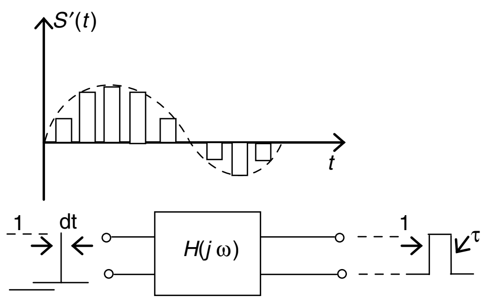
Where T_s is time period. Since pulses have finite duration, so it is not possible to eliminate completely the crosstalk generated in the channel. If N signals are to be multiplexed, then sample duration is T_s/N . To increase the signal level, τ should be as large as possible, but if τ is too large, then the guard time will decrease or eliminate between two samples. So there is compromise to choose the τ.

Disadvantage: Difficult to implement the electronic circuit.

Flat-top Sampling

In natural sampling, tops of the pulses are following the contour of the signal $m(t)$ after sampling but in flat-top sampling, tops of the pulses do not follow contour of the signal instead tops are flat. In this type of sampling, the baseband signal $m(t)$ cannot be recovered completely by passing the samples through ideal low-pass filter. However, distortion is very less.

Advantages: It simplifies the circuit design used for generation or for sampling operation.



$S^1(t)$ = sampled signal

Disadvantage: Distortion occurs in the recovered signal and it is called aperture effect.

To remove the aperture effect, equalization is used in which we use a passive network transfer function is $\frac{X}{\sin x}$ or inverse of $H(j, \omega)$

Analog Pulse Modulations

Pulse Amplitude Modulation

In pulse amplitude modulation (PAM), the amplitude of pulse train is varied in proportion to the corresponding sample value of a continuous message signal.

The PAM signal can be represented as follows:

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s)$$

Where $h(t)$ is the rectangular pulse

$$h(t) = 1 \quad 0 < t < T$$
$$= 0 \quad t \notin (0, T)$$

PAM signal can be obtained by convolving $m_\delta(t)$ with $h(t)$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t-nT_s)$$

$$s(t) = m_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s)$$

$$s(f) = M_\delta(f) \,.\, H(f) = f_s\left(\sum_{n=-\infty}^{\infty} M(f-nf_s)\right).H(f)$$

At the receiver by using a reconstruction filter with transfer function $\frac{1}{H(f)}$ and passing the resultant through a ideal LPF, we can get back $M(f)$.

Pulse Position Modulation

In this modulation, the positions of pulses are varied in accordance with the message signal.

For PPM, the modulated signal is given by

$$s(t) = \sum_{n=-\infty}^{\infty} g\left(t - nT_s - K_p m(nT_s)\right)$$

The condition for distortionless PPM wave is

$$\text{Max}\left|K_P m(t)\right| < \frac{T_s}{2}.$$

Where T_s is distance between two pulses.

Pulses Width Modulation

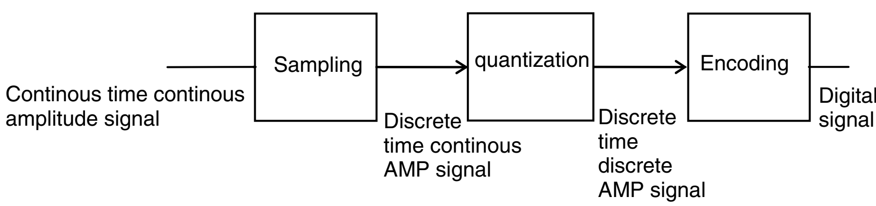
In this modulation, the width of pulses is varied in accordance with the message signal.

PPM and PWM exhibit same noise performance as PM and FM in Analog modulation. For both PPM and PWM, there exists a trade-off between transmission bandwidth and noise performance.

Digital Pulse Modulation

These modulations convert the Analog signal into discrete signal. PCM, DPCM, and DM are the digital pulse modulations.

Pulse Code Modulation



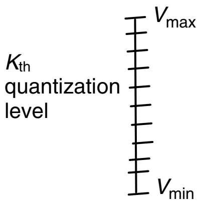
Quantization

The sampled $m(nT_s)$ is continous in amplitude. In a given finite amplitude interval, $m(nT_s)$ may take any amplitude with infinite possibilities.

$m(nT_s)$ = 2.314876……V. This sample amplitude has zero precision. We require infinite number of bits to represent this type of continuous amplitude or zero precision sample.

Quantization is the process of representing the continuous amplitudes into finite discrete amplitudes.

Let a signal $m(t)$ takes any value from $(-M, M)$. $m(t)$ may take any amplitude value form $-M$ to M and there exist infinite possibilities of amplitudes.



In quantization, we divide amplitude range into N equal parts. We assign a quantization level to each part.

If we represent each quantization level by R bits, total 2^R quantization levels can be represented.

Quantization interval size = step size

$$\Delta = \frac{2M}{2^R}$$

The quantization error = $m(nT_s) - m_\Delta(nT_s)$

In the midrise quantizer, the middle value of quantization interval is taken as quantization level.

Maximum quantization error in midrise quantizer is $\frac{\Delta}{2}$

The quantization error in midrise quantizer is in the range of $\frac{-\Delta}{2}$ to $\frac{\Delta}{2}$

If we assume uniform density for quantization error, i.e., $f_\Delta(\Delta) = \frac{1}{\Delta}$

The average quantization noise power = $E(q^2)$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\, dq$$

$$= \frac{\Delta^2}{12}$$

The noise performance of pulse digital modulations are measured by the quantity $(\text{SNR})_Q$ = signal to quantization noise power ratio

$$= \frac{\text{average signal power}}{\text{average quantization noise power}}$$

If signal power = P

$$(\text{SNR})_Q = \frac{p}{\frac{\Delta^2}{12}} = \frac{12p}{\Delta^2}$$

$$\text{If } m(t) = A_m \cos(2\pi f_m t)$$

$$\therefore P = \frac{A^2{}_m}{2}$$

$$\Delta = \frac{2_{Am}}{2^R}$$

$$(\text{SNR})_Q = \frac{\frac{A_m{}^2}{2}\times 12}{\frac{4A_m{}^2}{2^{2R}}} = \frac{3}{2}2^{2R}$$

$$(\text{SNR})_Q \text{ in dB} = 1.7 + 6R \text{ dB}$$

For the single tone, $(\text{SNR})_Q$ increases 6 dB for every 1bit increase in R(no of bits using to represent sample) or $(\text{SNR})_Q$ increase four times for every 1bit increase in R.

Non-Uniform Quantization

In uniform quantization, the quantization step size is equal throughout the dynamic range of $m(t)$.

In non-uniform quantizer, the step size is less where the signal present with a high probability and the step size is more where the signal $m(t)$ present with a low probability.

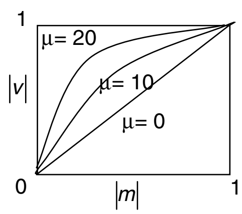
Non-uniform quantization can be performed by using signal compressor and then by using uniform quantizer.

There exist two famous signal compression techniques

(i) μ law
(ii) A law

μ Law

$$\text{The compressed voltage } |V| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

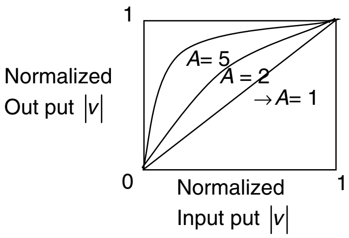


A Law

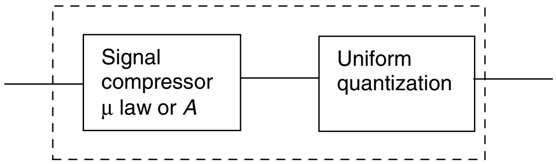
A law have a linear region and non-linear region

$$|V| = \frac{A|m|}{1+\log A}\quad 0 \le |m| \le \frac{1}{A}$$

$$= \frac{1+\log(A|m|)}{1+\log A}\quad \frac{1}{A} \le |m| \le 1$$



Non-uniform Quantizer



The process by non-uniform quantization by compressing the high-amplitude values is called companding.

Encoding

Encoding process translates the discrete set of sample values to a binary code. There exist several line codes to represent the binary symbols 1 and 0.

On–off Signalling

1 is represented by a pulse of constant amplitude and for the duration of a symbol and '0' is presented by switching off the pulse.

Non-return to Zero Signalling

'1' & 0 are represented by pulses of equal positive and negative amplitudes respectively and for the duration of symbol.

Return to Zero Signalling

'1' is represented by a pulse of half symbol width and '0' is represented by switching off the pulse

Bipolar Return to Zero Signalling

Positive and negative pulses of equal amplitude are used alternately for symbol 1, and no pulse is used to represent symbol '0'.

Manchester Signalling

In this symbol, '1' is represented by a positive pulse followed by a negative pulse with both pulses being of equal amplitude and half symbol width. To transmit symbol '0', the polarities of these two pulses reversed.

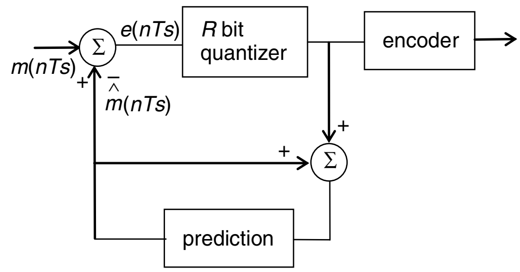
Differential Encoding

In this, a transition of present voltage level is used to designate symbol '0' and no transition is used to designate symbol 1.

Differential Pulse Code Modulation

The $(\mathrm{SNR})_Q$ in PCM is depending in the dynamic range of the signal. For the same number of bits, if the dynamic range is less, $(\mathrm{SNR})_Q$ is more .

In DPCM, the dynamic range of quantization reduced by taking the quantization of difference between present sample and predicted version of the present sample by using previous samples.



That is, in DPCM, instead of quantizing $m(nT_s)$ directly, we are quantizing $e(nT_s) = m(nT_s) - \hat{m}(\mathrm{nTs})$.

The dynamic range of $e(nT_s)$ is much less than the dynamic range of $m(nT_s)$.

$$(\mathrm{SNR})_Q \text{ in DPCM} =$$

$$\frac{\text{input signal power to DPCM}}{\text{quit noise power}} = \frac{\sigma_M{}^2}{\sigma_Q{}^2}$$

$$(\mathrm{SNR})_Q \text{ in PCM} =$$

$$\frac{\text{signal power (power of input signal to quantizer)}}{\text{quant noise power}} = \frac{\sigma_E{}^2}{\sigma_Q{}^2}$$

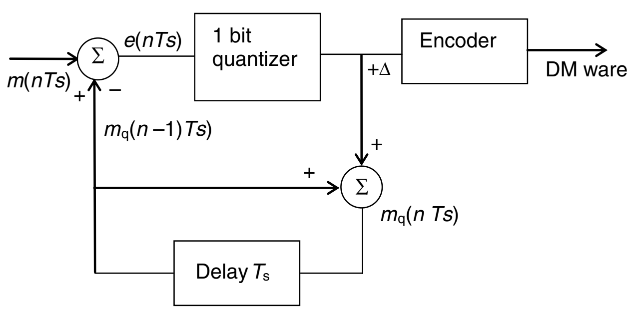
$$(\mathrm{SNR})_{Q\ \mathrm{DPCM}} = \frac{\sigma_M{}^2}{\sigma_E{}^2}.(SNR)_{Q\ \mathrm{PCM}}$$

$$= G_P \cdot (\mathrm{SNR})_{Q\ \mathrm{PCM}}$$

$G_P = \dfrac{\sigma_M{}^2}{\sigma_E{}^2}$ is called processing gain of DPCM indicates the amount of gain provided by DPCM over PCM.

Delta Modulation

In delta modulation, every sample is represented by one bit, either 1 or 0. In this modulation, every sample is compared with the quantized version of previous sample. If the present sample is greater than the quantized version of previous sample, the present sample is represented by 1; otherwise, it is represented by 0.



If e(nT_s) > 0, the o/p quantizer is Δ and the o/p of delta modulator is 1.

e(nT_s) < 0, the o/p of quantizer is $-\Delta$ and the o/p delta modulator is 0.

Delta modulation tracks the message samples $m(nT_s)$ with a step size of Δ for every T_s. The delta modulator can track the message signal if tracking speed (slope of delta modulator) is greater than the maximum slope of $m(t)$.

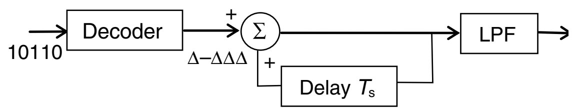
$$\text{i.e., } \therefore \frac{\Delta}{T_s} \geq \max \frac{dm(t)}{dt}$$

If the above condition is not met, delta modulator cannot track that particular $m(t)$.This distortion is called slope over load distortion.

If the signal is changing very slowly, the delta modulator oscillates between 1 and 0. This sequence of alternate 1's and 0's cannot represent the slowly varying signals accurately. This distortion is called Granular noise.

Receiver for DM

The DM receiver will have an accumulator to accumulate $\pm\Delta$ and tracks the message signal $m(t)$.



Solved Examples

Example 1

Consider the sampled signal $y(t) = 2 \times 10^{-5}\, x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ where $x(t) = 100 \sin(2{,}000\pi t)$ and T_s = 400 µs. When $y(t)$ is passed through an ideal low-pass filter with cut-off frequency 1.25 kHz, the output of the filter is

(A) $5\sin(2{,}000\pi t)$ (B) $100\sin(2{,}000\pi t)$
(C) $2 \times 10^{-3} \sin(2{,}000\pi t)$ (D) $2 \times 10^{-5} \sin(2{,}000\pi t)$

Solution

$y(t)$ is the sampled version of $x(t)$

$$y(t) = 2 \times 10^{-5}\, x(t - nT_s)$$

where T_s = 400 µs

$$f_s = \frac{1}{T_s} = 2{,}500$$

$$Y(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) = 2500 \times 2 \times 10^{-5} \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$= 5 \times 10^{-2} \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

If $Y(f)$ through a LPF of BW 1.25 kHz

The output $= 5 \times 10^{-2}\, X(f) \Leftrightarrow$

$= 5 \times 10^{-2} \times 100 \times \sin(2{,}000\pi t)$

$= 5 \sin(2{,}000\pi t)$

Example 2

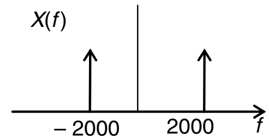
A signal $x(t) = 10 \cos(400\pi t)$ is ideally sampled with a sampling period of 40 µs and then passed through an ideal low-pass filter with a cut-off frequency 1 kHz. Which of the following frequencies is present at the output of the filter?

(A) 2,000 Hz (B) 1,000 Hz
(C) 1,500 Hz (D) 500 Hz

Solution

$$x(t) = 10\cos(4{,}000\pi t)$$

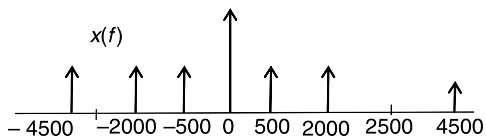
$$x(f) = 5[\delta(f - 2{,}000) + \delta(f + 2{,}000)]$$



$x(t)$ is sampled at sampling rate of 2,500 samples per second.

That is, the spectrum of $X(f)$ repeats for every 2,500.

$x_\delta(f)$



If we pass the samples through a LPF of 1 kHz BW, we get 500 Hz component.

Example 3

The Nyquist sampling frequency of a signal

$$\sin C^2(200t) \times \sin C^3(100t) \text{ is}$$

(A) 700 samples/s (B) 1,400 samples/s
(C) 800 samples/s (D) 300 samples/s

Solution

To find out Nyquist sampling rate, we have to find out the maximum frequency in the given signal.

$$\text{sinc}\,(200t) \rightleftharpoons \frac{1}{200}\text{rect}\left(\frac{f}{200}\right)$$

$\therefore$ The maximum frequency in sin c(200t) = 100

Max frequency in sin $C^2(200t)$ is 200.

Max frequency in sin C (100t) is 50.

Max frequency in sin $C^3(100t)$ is 150.

Max frequency in given signal = min(200, 50) = 150 Hz

Sampling rate = 300 samples second.

Example 4

The Nyquist sampling rate for the signal

$$\text{sinC}\,(1{,}100t) + \text{sinC}\,(1{,}400t) \text{ is}$$

(A) $\frac{1}{1100}$ seconds (B) $\frac{1}{1400}$ seconds

(C) $\frac{1}{2200}$ seconds (D) $\frac{1}{2800}$ seconds

Solution

Max frequency of sinc (100t) is 550

Max frequency of sinc (1,400t) is 700

Max frequency of sinc(1,100t) + sinc (1,400t) is 700

Sampling rate = 700 × 2 = 1,400 samples/s

Sampling interval = $\frac{1}{1400}$ seconds

Examples 5

In a PCM system, if the code word length is increased form 5bits to 7bits, the signal-to-quantization noise ratio improves by a factor of

(A) $\frac{7}{5}$ (B) $\left(\frac{7}{5}\right)^2$ (C) 4 (D) 16

Solution

$$(\text{SNR})_Q \ \alpha\ 2^{2R}$$

If R is increased to 7 bits form 5 bits $(\text{SNR})_Q$ will increase by $2^{2.2} = 2^4 = 16$ times

Example 6

A sinusoidal signal with peak to peak amplitude of 1.28 volts is quantized into 128 levels using a midrise uniform quantizer. The quantization noise power is

(A) 1.28 µW (B) 8 µW (C) 4 µW (D) 2 µW

Solution

$$\text{Quantization noise power} = \frac{\Delta^2}{12}$$

$$\Delta = \text{step size} = \frac{1.28}{128} = 0.01$$

$$\therefore \text{ Quantization noise power} = \frac{(0.01)^2}{12} = 8\ \mu\text{W}$$

Example 7

A sinusoidal signal is sampled at 10 kHz and it is quantized by using 10 bits uniform quantizer. The values of bit rate and $(\text{SNR})_Q$ in this scenario are

(A) 100 kbps, 61.6 dB (B) 100 kbps, 10 dB
(C) 50 kbps, 61.6 dB (D) 50 kbps, 10 dB

Solution

Data rate = No. of samples × No. of bits to represent each sample = 10,000 × 10 = 100 Kbps

$(\text{SNR})_Q$ for sinusoidal signal is given by (SNR)Q = 1.6 + 6R dB = 1.6 + 60 = 61.6 dB

Example 8

The minimum step size required for a delta modulator operating at 16 K samples/s to track the signal

$x(t) = 100(t(u(t) - u(t-1)) + (200 - 100t)(u(t-1) - u(t-2))$

By avoiding slope overload distortion would be,

(A) 10 mV (B) 8 mV (C) 20 mV (D) 6 mV

Solution

Given signal $x(t)$ is a triangular pulse with slope 100.

To avoid slope overload distortion

$$\frac{\Delta}{T_s} \geq \max \frac{dm(t)}{d(t)}$$

$$\Delta \times 16 \times 10^3 \geq 100$$

$$\Delta \geq \frac{1}{160}$$

$$\Delta \geq 6 \text{ mV}$$

Example 9

A delta modulation system require to maintain a minimum $(\text{SNR})_Q$ of 60 dB. The minimum sampling rate for $m(t) = 10 \cos(2{,}000\pi t)$ to avoid slope overload distortion is

(A) 10^4 samples/s (B) 10^6 samples/s
(C) 10^8 samples/s (D) 10^{10} samples/s

Solution

$$(\text{SNR})_Q = \frac{\text{signal power}}{\text{quantization noise power}} = 60 \text{ dB} = 10^6$$

$$= \frac{50}{\frac{\Delta^2}{12}} = 10^6 = \Delta = 0.6 \text{ mV}$$

For delta modulation

$$\frac{\Delta}{T_s} \geq \max \frac{d_M(t)}{d(t)}$$

$$\frac{0.6\text{mV}}{T_s} \geq 10 \times 2000\pi$$

$$\frac{1}{T_s} \geq \frac{10 \times 2000\pi}{0.6 \times 10^{-3}}$$

$$\frac{1}{T_s} \geq 10^8 \text{ samples/s}$$

Example 10

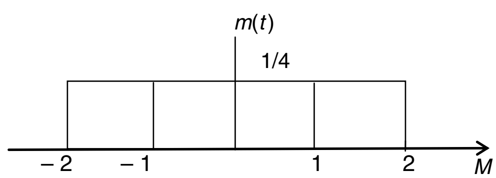
In a PCM system, the signal $m(t)$ is taking the values from (−2 V, 2 V) with uniform probability. Let the quatization scheme be as follows.

$m(t) \in (-2, -1): m_q = -1$
$m(t) \in (-1, 1): m_q = 0$
$m(t) \in (1, 2): m_q = 1$

The quantization error power in the above scheme is

(A) 1.0 (B) 0.5 (C) 1.5 (D) 2.0

Solution



Quantization error =

$$\frac{1}{4}\int_{-1}^{-2}(x+1)^2\,dx+\frac{1}{2}\int_{-1}^{1}(x-0)^2\,dx$$

$$\frac{1}{4}\int_{1}^{2}(x-1)^2\,dx$$

$$=\frac{1}{4}\left[\frac{(x+1)^3}{3}\right]_{-1}^{-2}+\frac{1}{2}\left[\frac{x^3}{2}\right]_{-1}^{1}+\frac{1}{4}\left[\frac{(x-1)^3}{3}\right]_{1}^{3}$$

$$=\frac{1}{12}+\frac{1}{3}+\frac{1}{12}\quad\text{[Quantization error is always positive]}$$

$$=\frac{1}{2}$$

Baseband Pulse Transmission

Baseband pulse transmission is the transmission of digital data over a baseband channel without any modulation.

Matched Filter

Matched filter is the filter used in the receiver of baseband communications to optimize the peak pulse signal to noise ratio (η).

$$\eta=\frac{\left|g_0(T)^2\right|}{E\left[n^2(t)\right]}$$

Where $\left|g_0{}^2(T)\right|$ is the instantaneous power of receiver filter output and $E(n^2(t))$ is the average noise power at the output.

Properties of Matched Filter

1. $h(t) = \text{kg}(T - t)$
   i.e., The impulse response of matched filter is the time reversed and delayed version of input pulse signal $g(t)$.
2. $H(f) = \text{KG} \times (f) \exp(-j2\pi fT)$
3. The peak pulse signal-to-noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of white noise.

$$\eta_{\max}=\frac{2E}{N_0}$$

Where E = Pulse signal energy

$$=\int_{-\infty}^{\infty}\left|g(t)\right|^2 dt=\int_{-\infty}^{\infty}\left|G(f)\right|^2 df$$

Error Rate in Baseband Communication

Probability of bit error in baseband communication is given by

$$P_e=\frac{1}{2}erf_c\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Where

$$erf_c(v)=\frac{2}{\sqrt{\pi}}\int_{v}^{\infty}\exp\left(-z^2\right)dz$$

$erf_c(v)$ is called complementary error function

E_b is the energy per bit and $\frac{N_0}{2}$ is the AWGN spectral density.

Inter Symbol Interference

In the baseband communications, because of finite bandwidth of channel, the impulse response of the channel does not decay in one T_b. Instead the impulse response of the channel spreads up to many T_b durations. Because of this effect, there would be interference among the adjacent pulses. This is called ISI.

Condition for ISI-free Channel

If $p(t)$ is the overall impulse response of the channel, transmit filter, and receiver filter, the condition for ISI-free channel is

$$\sum_{K=-\infty}^{\infty} p\left[(i-K)T_b\right] = 1 \text{ for } i = k$$

$$= 0 \text{ for } i \neq k$$

The frequency domain equivalent of ISI-free channel condition is

$$\sum_{n=-\infty}^{\infty} P(f-nR_b) = T_b$$

Where $R_b = 1/T_b$ is the data rate.

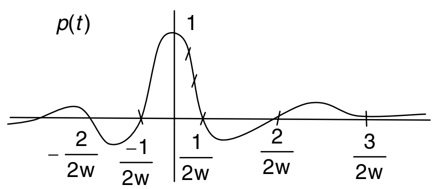
Ideal Nyquist Channel

$P(f) = \frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$ is called ideal Nyquist channel.

Ideal Nyquist channel satisfies the condition for interference-free channel.

Data rate Rb = 2 W in the ideal Nyquist channel, i.e., If we have ideal Nyquist channel with bandwidth 'W', then we can transmit baseband data at the rate of 2 W bits per second.

$$p(t) = \sin c(2\,wt)$$



At $\frac{n}{2W}$ or nT_b, $p(t)$ is zero.

i.e., $p(t)$ does not cause any ISI at $t = nT_b$.

Raised Cosine Spectrum

Ideal Nyquist channel has sharp cut off, thus practically not possible to design. Raised cosine channel solves this drawback of ideal Nyquist channel.

Raised cosine channel satisfies the equation

$$P(f) + P(f - 2w) + P(f + 2w) = \frac{1}{2W} \quad \text{for } f \in (-w, w)$$

Raised cosine spectrum is given by $P(f) =$

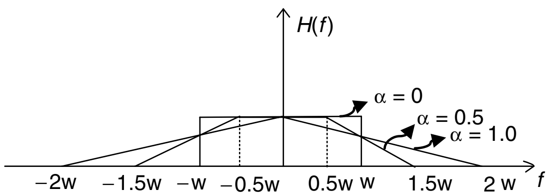
$$\begin{array}{ll} \dfrac{1}{2W} & 0 \le |f| < f_1 \\ \dfrac{1}{4W}\left\{1 - \sin\left[\dfrac{\pi(|f| - w)}{2w - 2f_1}\right]\right\} & f_1 \le |f| \le 2W - f_1 \\ 0 & |f| \ge 2W - f_1 \end{array}$$

The frequency parameter f_1 and bandwidth W are related by

$\alpha =$. The parameter α is called roll-off factor.

The roll-off factor indicates the excess bandwidth required over the ideal bandwidth W

Transmission bandwidth required = $W(1 + \alpha)$



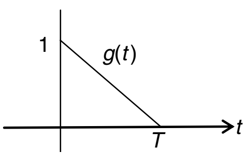
If $\alpha = 1$ the channel is called full - cosine roll off.

$$P(f) = \begin{array}{ll} \dfrac{1}{4W}\left[1 + \cos\left(\dfrac{\pi f}{2w}\right)\right] & 0 < |f| < 2w \\ 0 & \text{else} \end{array}$$

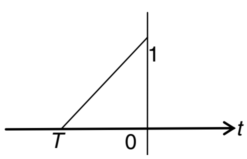
$$\text{And } p(t) = \frac{\sin c(4wt)}{1 - 16w^2t^2}$$

Example 11

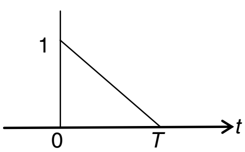
Consider the pulse shape $g(t)$ as shown. The impulse response $h(t)$ of the filter matched to this filter is



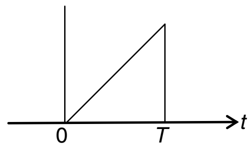
(A)



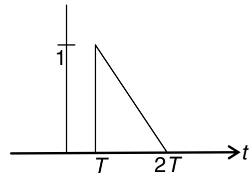
(B)



(C)



(D)

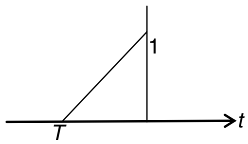


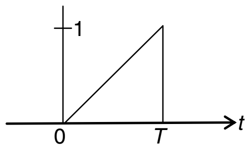
Solution

Matched filter impulse response = $h(t)$

$= g(T - t)\ g(-t)$ is given by

$G(T - t)$ is given by





Example 12

The input to a matched filter is given by

$s(t) = 20\sin(2\pi \times 10^5 t)$ for $0 < t < 10^{-3}$s

$= 0$ else

The peak amplitude of the filter output is

(A) 20 V (B) 2 V (C) 0.2 V (D) 10 V

Solution

$$s(t) = 20\sin\,(2\pi \times 10^{-5} t)$$

$$T = 10^{-3}$$

$$h(t) = g(T - t) \text{ or } s(T - t)$$

$$= 20\ \sin(2\pi \times 10^{-5}(10^{-3} - t))$$

$$= -20\ \sin(2\pi \times 10^{-5} t)$$

The matched filter will be integration of the response of filter form 0 to $T = 10^{-3}$

$$g_0(t) = s(t) \times h(t)$$

Peak value of output =

$$\int_0^{10^{-3}} 400 \sin^2(2\pi \times 10^{-5} t)$$

$$= 200 \times 10^{-3}$$

$$= 0.2 \text{ V}$$

Example 13

If E_b is the energy per bit of a binary digital signal is 10^{-4} watt sec and the power spectral density of white noise $N_0 = 10^{-5}$W/Hz, then the output SNR, of the matched filter is

(A) 10 dB (B) 13 dB
(C) 20 dB (D) 26 dB

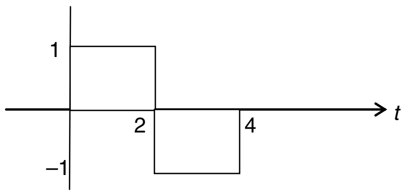
Solution

$$\text{Output SNR of the matched filter} = \frac{2E}{N_0}$$

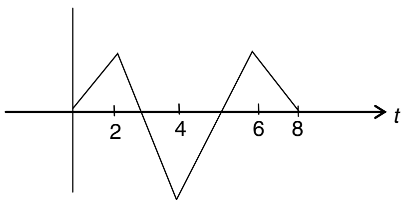
$$= \frac{2\times10^{-4}}{10^{-5}} = 20 = 13 \text{ dB}$$

Example 14

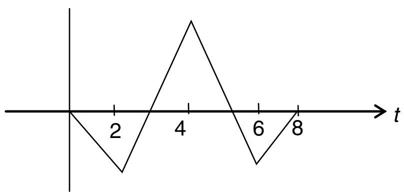
A signal $g(t)$ shown below is applied to a matched filter. The matched filter output is given by



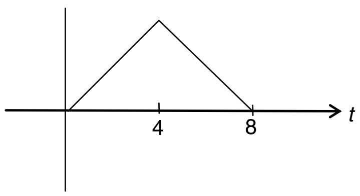
(A)



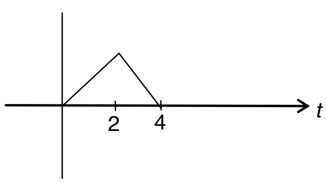
(B)



(C)



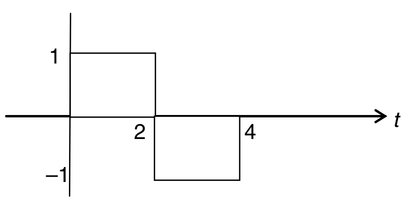
(D)

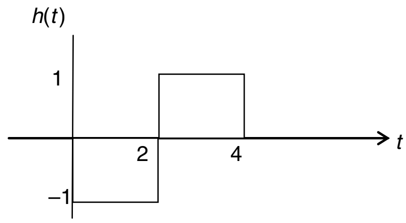


Solution

$$h(t) = g(T - t)$$

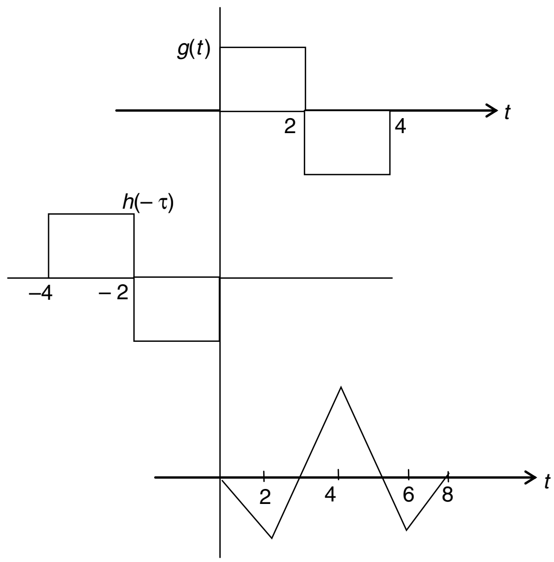
$$g(t) =$$





Output of matched filter

$$g_0(t) = g(t) \times h(t) = \int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau$$



Example 15

A baseband communication system requires to transmit a data rate of 100 kbps through a raised cosine channel with roll off factor $\alpha = 0.25$. The BW required in this scihorio is

(A) 50 kHz (B) 100 kHz
(C) 62.5 kHz (D) 75 kHz

Solution

$$\text{BW} = W(1 + \alpha)$$

$$W = \frac{R_b}{2} = 50 \text{ kHz}$$

$$\text{BW} = 50\ (1 + 0.25) \text{ kHz}$$

$$= 62.5 \text{ kHz}$$

Example 16

The raised cosine pulse $p(t)$ is used for zero ISI in digital communications. The expression for impulse response with $\alpha = 1$ is given by

$$p(t) = \frac{\sin(4\pi wt)}{4\pi\ wt(1-16w^2t^2)}$$

The value of $p(t)$ at $t = \frac{1}{4W}$ & $\frac{1}{2W}$ are given by

(A) 0, 0 (B) 0.5, 0 (C) 0, 0.5 (D) 0, 1.0

Solution

$$\underset{t \to \frac{1}{4W}}{Lt}\ p(t) = \frac{0}{0}$$

If we apply Hospital rule

$$\underset{t \to \frac{1}{4w}}{Lt} = \frac{4\pi w \cos(4\pi wt)}{4\pi w - 4\pi w \cdot 16w^2 \cdot 3t^2}$$

$$= 0.5$$

$$\text{At } t = \frac{1}{2w}$$

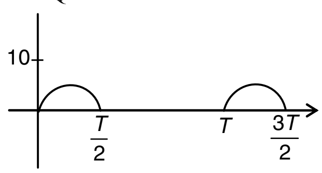
$$p(t) = 0$$

Exercises

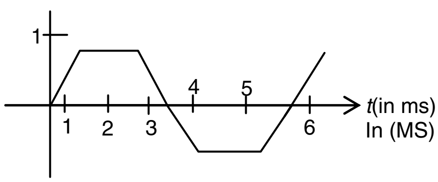
Practice Problems I

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. If $m(t) = |10 \cos (2{,}000\pi t)|$. is pulse code modulated by using $P = 8$ bits to represent each sample, signal-to-quantization noise ratio $(SNR)_Q$ is given by
   (A) 30 dB. (B) 44 dB. (C) 56 dB. (D) 60 dB.
2. If $m(t)$ is the half wave rectified wave as given below is pulse code modulated by using 8 bits to represent each sample, $(SNR)_Q$ is given by



   (A) 53 dB. (B) 56 dB. (C) 30 dB. (D) 60 dB.
3. The following signal $m(t)$ is delta modulated. If the quantization noise power in this modulation should be less than 40 μW, the minimum sampling rate to avoid slope overload distortion is



   (A) 54 K samples/s (B) 45 K samples/s
   (C) 64 K samples/s (D) 36 K samples/s
4. If the sinusoidal signal $A_m \cos (2\pi f_m t)$ is pulse code modulated, the value of bits per sample (R) required to maintain an $(SNR)_Q$ of 60 dB is
   (A) 8. (B) 9. (C) 10. (D) 6.
5. The Nyquist sampling rate for the signal
   $M(t) = 10 \text{ sinc}^2 (2{,}000\ t) + 20 \text{ sinc}^3 (500\ t)$ is
   (A) 8,000 samples/s (B) 6,000 samples/s
   (C) 4,000 samples/s (D) 5,000 samples/s
6. A signal $m(t) = 10 \cos (2{,}000\pi t) + 15 \sin (1{,}500\pi t)$ is sampled by a sampling rate of 2,500 samples per second. The resultant sampled signal is passed through a low pass filter of BW 2 kHz. The frequency components at the output of the LPF are
   (A) 750, 1,000, 1,500
   (B) 750, 1,000
   (C) 750, 1,000, 1,500, 1,750
   (D) 750, 1,000, 1,500, 1,750
7. If $m(t) = 10 \cos(2{,}000\pi t)$ mV is ideally sampled with a sampling rate of 2,500 samples/s and then passed the sampled signal through a ideal low-pass filter of bandwidth 1,500 Hz, the output of LPF is
   (A) $25 \cos(2{,}000\pi t)$V. (B) $10 \cos(2{,}000\pi t)$mV.
   (C) $250 \cos(2{,}000\pi t)$V. (D) $10 \cos(2{,}000\pi t)$V.
8. A PCM quantizer is charapterized by $(SNR)_Q = 20$ dB for a given signal input is used in a DPCM modulator with processing gain of 30 dB. $(SNR)_Q$ of DPCM modulator is given by
   (A) 20 dB. (B) 30 dB. (C) 50 dB. (D) 60 dB.
9. A binary source uses RZ signalling to transmit it's data. If 5 V is used to represent '1' and '0'v is used to represent '0' and the source emitting '1' with probability 0.6 and '0' with probability 0.4, the average power transmitted by the source is
   (A) 2.5W. (B) 12.5W. (C) 25W. (D) 15W.
10. If a speech signal $g(t)$ takes the amplitude (0,5V) with uniform probability throughout the range, the power of the $g(t)$ is given by
    (A) 12.5 W. (B) 8.33 W. (C) 25 W. (D) 10 W.
11. Let a speech signal $m(t)$ takes the voltage in the range of 2 V to 14.8 V. $m(t)$ is transmitted by PCM by using 7 bits per sample and if the power of the signal $m(t)$ is given by 30 W, $(SNR)_Q$ in this modulation is given by
    (A) 45 dB (B) 55 dB (C) 35 dB (D) 65 dB
12. Let a random variable X is taking the values in the range of (−2, 2) with uniform density. If R_v X is represented by two quantization levels as given below:

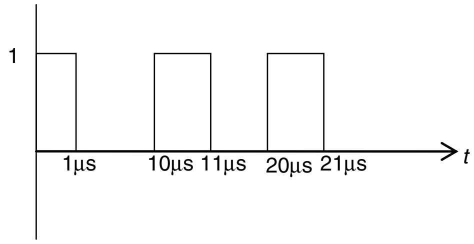
    $X_q = -0.5$ if $X \in (-2, 0)$

    $X_q = 0.5$ if $X \in (0, 2)$

    The quantization noise power with this quantizer is
    (A) 1.0 W. (B) 0.5 W. (C) 2.0 W. (D) 0.8 W

13. A sinusoidal signal $m(t) = 10 \cos(2{,}000\pi t)$ is required to transmit by using pulse position modulation with a sensitivity modulation with a sensitivity $K_p = 1$ μs/volt. The maximum frequency of pulse train is
(A) 2,000 pulses per second.
(B) 20,000 pulses per second.
(C) 50,000 pulses per second.
(D) 10^5 pulses per second.

14. A speech signal $m(t)$ is pulse amplitude modulated with pulse train mentioned below.



To regenerate the speech signal $m(t)$ at the receive, the transfer function of regeneration filter is

(A) $\dfrac{1}{10^{-6}\,\mathrm{sin}c(10^{-6} f)}$. (B) 10^{-6} sinc $(10^{-6} f)$.

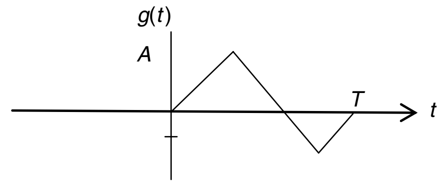
(C) 10^{-5} sinc$(10^{-5} f)$. (D) $\dfrac{1}{10^{-5}\,\mathrm{sin}c(10^{-5} f)}$.

15. A PCM system uses a uniform quantizer followed by 8-bit binary encoder. The bit rate of the system is equivalent to 50×10^6 b/s. The maximum message bandwidth for which the system operates satisfactorily is
(A) 2 MHz (B) 3.75 MHz
(C) 3.125 MHz (D) 4 MHz

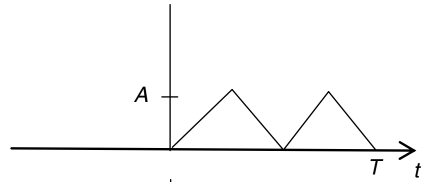
16. A baseband channel requires to transmit 10 kbps data by using raised cosine spectrum with roll of factor $\alpha = 0.5$. The base width requirement is
(A) 5 kHz (B) 10 kHz
(C) 7.5 kHz (D) 20 kHz

17. The impulse response of ideal Nyquist channel is
(A) sinc function (B) Impulse function
(C) Exponential function (D) Cosine function

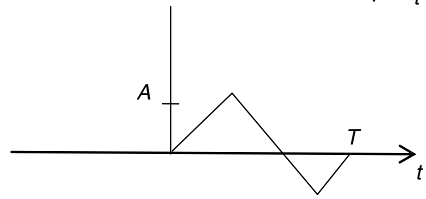
18. If $g(t)$ is given below, the impulse response $h(t)$ of the filter matched to $g(t)$ is



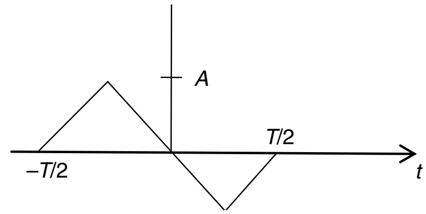
(A)



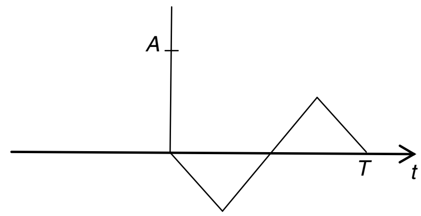
(B)



(C)



(D)



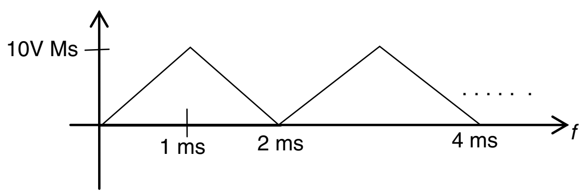
19. If $g(t) = \cos^3(2{,}000\ \pi t) + \sin^2(3{,}000\ \pi t)$, the Nyquist rate of sampling is
(A) 2,000 samples/s (B) 4,000 samples/s
(C) 6,000 samples/s (D) 8,000 samples/s

20. The Nyquist sampling rate for the signal $g(t) = \cos(4{,}500\ \pi t) \cos(9{,}000\ \pi t)$ is
(A) 4,500 samples/s (B) 5,500 samples/s
(C) 9,000 samples/s (D) 11,000 samples/s

Practice Problems 2

Direction for questions 1 to 18: Select the correct alternative from the given choices.

1. A message signal $m(t)$ is given by



$m(t)$ is required to be transmitted by using pulse code modulation with 256 quantization levels. $(\mathrm{SNR})_Q$ in this modulation is given by
(A) 60 dB (B) 52.5 dB
(C) 47.5 dB (D) 42.5 dB

2. Nyquist sampling rate for the signal $\sin^5(2{,}000\pi t) \sin^8(1{,}500\pi t)$ is
(A) 24 K samples/s (B) 22 K samples/s
(C) 20 K samples/s (D) 28 K samples/s

3. Let a signal $m(t) = 25 \cos(10t) + 15 \sin(5t)$ is required to transmit by using delta modulator with a sampling rate of 100 samples/s. The minimum step size to avoid slope overload distribution in the above delta modulator is
(A) 0.4 V. (B) 4.0 V.
(C) 3.25 V. (D) 0.325 V.

4. If $(SNR)_Q$ in a DPCM modulator is 50 dB. If the predictor in this DPCM is replaced by more accurate predictor, then the $(SNR)_Q$ of the new $(DPCM)_{modulator}$ is
(A) >50 dB. (B) = 50 dB.
(C) <50 dB. (D) cannot say.

5. Let $m(t) = 10 \cos(2{,}000\pi t) \sin(1{,}000\pi t)$ is sampled with a impulse train of $T_s = 0.25$ ms. The sampled signal $M_\delta(t)$ is passed through BPF with pass band 24 kHz to 26 kHz. The frequency components at the output of BPF are
(A) 24.5 kHz
(B) 25.5 kHz
(C) 24.5 kHz and 25.5 kHz
(D) 24.5 kHz and 25.0 kHz

6. A PCM modulation system has 256 quantization levels, and it is operating at $(SNR)_Q$ of 50 dB. If the number of quantization levels are increased to 1,024 keeping all other parameters of the modulator the same, the $(SNR)_Q$ of new modulator is
(A) 50 dB. (B) 56 dB. (C) 62 dB. (D) 44 dB.

7. A speech signal $g(t)$ is taking the value in the range of (−2, 8 V). If $g(t)$ is required to be transmitted by using pulse position modulation with sensitivity K_p = 10 μs/volt, maximum frequency of pulse train that can be used without overlap of pulses is
(A) 10 kHz (B) 5 kHz
(C) 8 kHz (D) 6.25 kHz

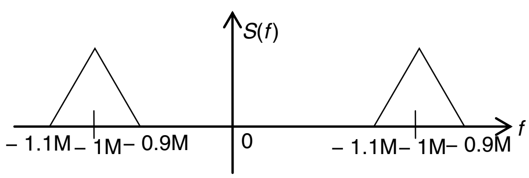
8. A PCM system uses a uniform quantizer followed by 10-bit binary encoder. The bit rate of the system is 200 kbps. The maximum message bandwidth for which the system operates satisfactory is
(A) 10 kHz (B) 15 kHz
(C) 20 kHz (D) 25 kHz

9. A bandpass signal $(S)(t)$ is given

$S(t) = 100 \cos(2\pi \times 10^5 t).\ \sin(2\pi \times 10^3\ t).$

Nyquist sampling rate for the signal $S(t)$ is given by
(A) 200×10^3 samples/s (B) 202×10^3 samples/s
(C) 2×10^3 samples/s (D) 4×10^3 samples/s

10. The spectrum of a pass band signal is given as follows:



The Nyquist sampling rate required to represent $s(t)$ is (in samples/s)
(A) 200×10^3 (B) 400×10^3
(C) $2{,}200 \times 10^3$ (D) $2{,}000 \times 10^3$

11. A delta modulator is operating at $(SNR)_q = 40$ dB and at a sampling rate of 100 K samples/s. If the requirement is to half the sampling rate, to avoid slope over load distortion if the step size is increased, the new $(SNR)_Q$ of the system is
(A) 40 dB. (B) 46 dB.
(C) 37 dB. (D) 34 dB.

12. If a message signal is taking the amplitude in the range (−4, 6 V) and if 1,024 quantization levels are used to represent this message signal, the maximum value of quantization error by midrise quainter is
(A) 10 mv. (B) 5 mv.
(C) 2.5 mv. (D) 20 mv.

13. In a PCM system, if the quantization levels are increased from 32 to 256, the relative bandwidth requirement to transmit PCM signal will
(A) Increase by 2 times.
(B) Increase by 1.5 times.
(C) Increase by 1.6 times.
(D) Decrease by 1.5 times.

14. The output signal to quantization ratio of a 10-bit PCM is 20 dB. The desired $(SNR)_Q$ is 38 dB. If the $(SNR)_Q$ is increased by increasing encoding bits to represent each sample, the percentage increase in bandwidth to represent the new PCM signal is
(A) 30%. (B) 50%. (C) 100%. (D) 90%.

15. A low-pass Analog signal of 4 kHz bandwidth is sampled at the Nyquist rate and quantized and encoded with 256 quantization levels. A synchronization bit is added at the end of each quantized code word. The bit rate in the resulting PCM signal will be
(A) 32 kbps. (B) 36 kbps.
(C) 64 kbps. (D) 72 kbps.

16. If $m(t) = 10 \cos t(200\ \pi t)$ is pulse code modulated with eight bits after sampling it with Nyquist rate. The data rate and quantization noise power, respectively, are
(A) 8 kbps, 507 μW (B) 16 kbps, 507 μW
(C) 8 kbps, 302 μW (D) 16 kbps, 302 μW

17. The signal to quantization noise power in the above question is
(A) 50 dB (B) 40 dB (C) 60 dB (D) 30 dB

18. Let $m(t) = |10\cos 2t|$ is represented by PCM modulation with 256 level quantization. The signal-to-quantization ratio is
(A) 44 dB (B) 24 dB
(C) 32 dB (D) 55.7 dB

Previous Years' Questions

1. In a PCM system, if the code word length is increased from 6 to 8 bits, the signal-to-quantization noise ratio improves by the factor. **[2004]**
(A) 8 (B) 12 (C) 16 (D) 4

2. In the output of a DM speech encoder, the consecutive, pulses are of opposite polarity during time interval $t_1 \le t \le t_2$. This indicates that during this interval. **[2004]**
(A) The input to the modulator is essentially constant.
(B) The modulator is going through slope overload.
(C) The accumulator is in saturation.
(D) The speech signal is being sampled at the Nyquist rate.

3. Select the correct one from among the alternatives **[2004]**

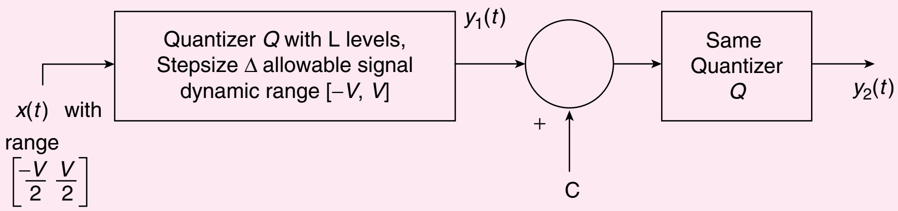
	Group 1		Group 2
1.	FM	P	Slope over load
2	DM	Q	μ law
3	PSK	R	Envelope detector
4	PCM	S	Capture effect
		T	Hilbert transform
		U	Matched filter

(A) 1 – T 2 – P 3 – U 4 – S
(B) 1 – S 2 – R 3 – P 4 – T
(C) 1 – S 2 – P 3 – U 4 – Q
(D) 1 – U 2 – R 3 – S 4 – Q

4. Three Analog signals, having bandwidths 1,200 Hz, 600 Hz, and 600 Hz are sampled at their respective Nyquist rates, encoded with 12-bit words, and time division multiplexed. The bit rate for the multiplexed signal is **[2004]**
(A) 115.2 kbps (B) 28.8 kbps
(C) 57.6 kbps (D) 38.4 kbps

5. The minimum step size required for a delta modulator operating at 32 k samples/s to track the signal (here $u(t)$ is the unit-step function)
$x(t) = 125t(u(t) - u(t-1)) + (250 - 125t)(u(t-1) - u(t-2))$
So that slope over load is avoided and would be **[2006]**
(A) 2^{-10} (B) 2^{-8} (C) 2^{-6} (D) 2^{-4}

6. In the following figure, the minimum value of the constant 'C', which is to be added to $y_1(t)$ such that $y_1(t)$ and $y_2(t)$ are different, is **[2006]**



(A) Δ (B) $\frac{\Delta}{2}$ (C) $\frac{\Delta^2}{12}$ (D) $\frac{\Delta}{L}$

7. In delta modulation, the slope overload distortion can be reduced by **[2007]**
(A) decreasing the step size
(B) decreasing the granular noise
(C) decreasing the sampling rate
(D) increasing the step size

8. Four messages band limited to W, W, 2W, and 3W, respectively are to be multiplexed using time division multiplexing (TDM). The minimum bandwidth required for the transmission of this TDM signal is **[2008]**
(A) W (B) 3W (C) 6W (D) 7W

Direction for questions 9, 10 and 11:
A speech signal, band limited to 4 kHz and peak voltage varying between +5 V and −5 V is sampled at the Nyquist rate. Each sample is quantized and represented by 8 bits.

9. If the bits 0 and 1 are transmitted using bipolar pulses, the minimum bandwidth required for distortion free transmission is **[2008]**
(A) 64 kHz (B) 32 kHz
(C) 8 kHz (D) 4 kHz

10. Assuming the signal to be uniformly distributed between its peak values, the signal-to-noise ratio at the quantizer output is **[2008]**
(A) 16 dB (B) 32 dB
(C) 48 dB (D) 64 dB

11. The number of quantization levels required to reduce the quantization noise by a factor of 4 would be **[2008]**
(A) 1,024 (B) 512 (C) 256 (D) 64

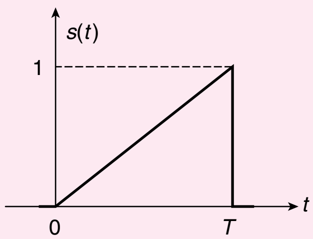
Direction for questions 12 and 13:
The amplitude of a random signal is uniformly distributed between −5 V and 5 V

12. If the signal-to-quantization noise ratio required in uniformly quantizing the signal is 43.5 dB, the step size of the quantization is approximately **[2009]**
(A) 0.0333 V (B) 0.05 V
(C) 0.0667 V (D) 0.10 V

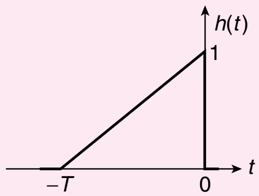
13. If the positive values of the signal are uniformly quantized with a step size of 0.05 V, and the negative values are uniformly quantized with a step size of 0.1 V, the resulting signal to quantization noise ratio is approximately **[2009]**

(A) 46 dB (B) 43.8 dB
(C) 42 dB (D) 40 dB

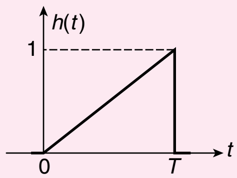
14. Consider the pulse shape $s(t)$ as shown. The impulse response $h(t)$ of the filter matched to this pulse is **[2010]**



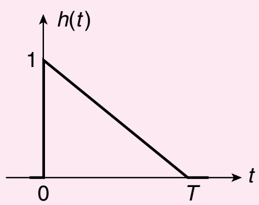
(A)



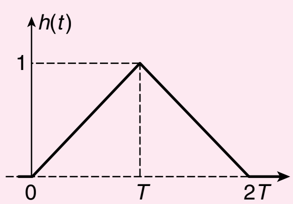
(B)



(C)



(D)



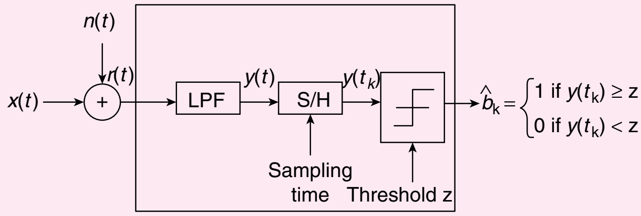
Direction for questions 15 and 16:

Consider a baseband binary PAM receiver shown below. The additive channel noise $n(t)$ is white with power spectral density $S_N(f) = N_0/2 = 10^{-20}$ W/Hz. The low-pass filter is ideal with unity gain and cut-off frequency 1 MHz. Let Y_k represent the random variable $y(t_k)$.

$Y_k = N_k$ if transmitted bit $b_k = 0$

$Y_k = a + N_k$ if transmitted bit $b_k = 1$

Where N_k represents the noise sample value. The noise sample has a probability density function, $P_{Nk}(n) = 0.5\alpha e^{-\alpha|n|}$ (This has mean zero and variance $2/\alpha^2$). Assume transmitted bits to be equiprobable and threshold z is set to $a/2 = 10^{-6}$ V.



15. The value of the parameter α (in V^{-1}) is **[2010]**

(A) 10^{10} (B) 10^7
(C) 1.414×10^{-10} (D) 2×10^{-20}

16. The probability of bit error is **[2010]**

(A) $0.5 \times e^{-3.5}$ (B) $0.5 \times e^{-5}$
(C) $0.5 \times e^{-7}$ (D) $0.5 \times e^{-10}$

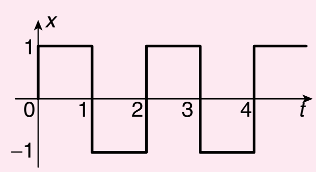
17. In a baseband communications link, frequencies up to 3,500 Hz are used for signalling. Using a raised cosine pulse with 75% excess bandwidth and for no inter symbol interference, the maximum possible signalling rate in symbols per second is **[2012]**

(A) 1,750 (B) 2,625
(C) 4,000 (D) 5,250

18. A band-limited signal with a maximum frequency of 5 kHz is to be sampled. According to the sampling theorem, the sampling frequency which is not valid is **[2013]**

(A) 5 kHz (B) 12 kHz
(C) 15 kHz (D) 20 kHz

19. Consider the periodic square wave in the figure shown. **[2014]**



The ratio of the power in the 7th harmonic to the power in the 5th harmonic for this waveform is closest in value to _______.

20. In a PCM system, the signal $m(t) = \{\sin(100\pi t) + \cos(100\pi t)\}$ V is sampled at the Nyquist rate. The samples are processed by a uniform quantizer with step size 0.75 V. The minimum data rate of the PCM system in bits per second is ____________. **[2014]**

21. A sinusoidal signal of 2 kHz frequency is applied to a delta modulator. The sampling rate and step-size Δ of the delta modulator are 20,000 samples per second and 0.1 V, respectively. To prevent slope overload, the maximum amplitude of the sinusoidal signal (in volts) is **[2015]**

(A) $\frac{1}{2\pi}$ (B) $\frac{1}{\pi}$

(C) $\frac{2}{\pi}$ (D) π

22. Consider binary data transmission at a rate of 56 kbps using base band binary pulse amplitude modulation (PAM) that is designed to have a raised cosine spectrum. The transmission bandwidth (in kHz) required for a roll off factor of 0.25 is __________ . **[2016]**

23. An analog pulse $s(t)$ is transmitted over an additive white Gaussian noise (AWGN) channel. The received signal is $r(t) = s(t) + n(t)$, where n(t) is additive white Gaussian noise with power spectral density $\frac{N_0}{2}$. The received signal is passed through a filter with impulse response $h(t)$. Let E_s and E_h denote the energies of the pulse $s(t)$ and the filter $h(t)$ respectively. When the signal to noise ratio (SNR) is maximized at the output of the filter (SNR_{max}) which of the following holds? **[2016]**

(A) $E_s = E_h$; $SNR_{max} = \frac{2E_s}{N_0}$

(B) $E_s = E_h$; $SNR_{max} = \frac{E_s}{2N_0}$

(C) $E_s > E_h$; $SNR_{max} > \frac{2E_s}{N_0}$

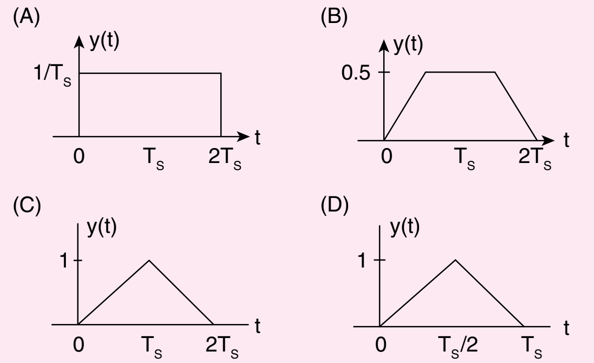
(D) $E_s < E_h$; $SNR_{max} = \frac{2E_h}{N_0}$

24. A speech signal is sampled at 8 kHz and encoded into PCM format using 8 bits/sample. The PCM data is transmitted through a baseband channel via 4-level PAM. The minimum bandwidth (in kHz) required for transmission is __________. **[2016]**

25. A binary baseband digital communication system employs the signal

$$p(t) = \begin{cases} \frac{1}{\sqrt{T_s}}, 0 \le t \le T_s, \text{otherwise} \\ 0 \end{cases}$$

for transmission of bits. The graphical representation of the matched filter output $y(t)$ for this signal will be: **[2016]**



26. A voice-grade AWGN (additive white Gaussian noise) telephone channel has a bandwidth of 4.0 kHz and two sided noise power spectral density $\frac{\eta}{2} = 2.5 \times 10^{-5}$ Watt per Hz. If information at the rate of 52 kbps is to be transmitted over this channel with arbitrarily small bit error rate, then the minimum bit-energy E_b(in mJ/bit) necessary is __________ . **[2016]**

Answer Keys

Exercises

Practice Problems 1

1. C	**2.** A	**3.** B	**4.** C	**5.** C	**6.** C	**7.** C	**8.** C	**9.** D	**10.** B
11. A	**12.** B	**13.** C	**14.** A	**15.** C	**16.** C	**17.** A	**18.** D	19 C	**20.** B

Practice Problems 2

1. B	**2.** B	**3.** C	**4.** A	**5.** C	**6.** C	**7.** D	**8.** A	**9.** C	**10.** A
11. D	**12.** B	**13.** C	**14.** A	**15.** D	**16.** B	**17.** A	**18.** D		

Previous Years' Questions

1. C	**2.** A	**3.** C	**4.** C	**5.** B	**6.** B	**7.** D	**8.** C	**9.** B	**10.** C
11. B	**12.** C	**13.** B	**14.** C	**15.** B	**16.** D	**17.** C	**18.** A	**19.** 0.5 to 0.52	
20. 200	**21.** A	**22.** 35	**23.** A	**24.** 16	**25.** C	**26.** 31.503			