CUET UG (Mathematics) - 16 May 2024 Shift 1

Section A

Question 1

The corner points of the feasible region determined by

$x + y \leq 8, 2x + y \geq 8, x \geq 0, y \geq 0$

are A(0, 8), B(4, 0) and C(8, 0). If the objective function Z = ax + by has its maximum value on the line segment AB, then the relation between a and b is :

Options:

A. $8a + 4 = b$

B. $a = 2b$

C. $b = 2a$

D. $8b + 4 = a$

Answer: B

Solution:

Concept:

- We are given a feasible region defined by inequalities and its corner points A(0, 8), B(4, 0), and C(8, 0).
- An objective function **Z = ax + by** is to be maximized, and the maximum occurs on the **line segment AB**.
- If the maximum value occurs on segment AB (not just at a single point), then the function Z is constant along AB.
- This implies the **gradient vector** of Z = ax + by is perpendicular to the direction vector of segment AB.
- So, the dot product of the gradient vector (a, b) and direction vector AB must be zero.

Calculation:

Points: A(0, 8), B(4, 0)

⇒ Direction vector of AB = (4 - 0, 0 - 8) = (4, -8)

Gradient vector of Z = (a, b)

Condition for Z to be constant along AB:

⇒ (a, b) · (4, -8) = 0

⇒ 4a - 8b = 0

⇒ a = 2b

∴ **The correct relation is a = 2b.**

Question 2

If $t = e^{2x}$ and y == $\log_e t^2$, then $\frac{d^2y}{dx^2}$ is :

Options:

A. 0

B. 4t

C. $\frac{4e^{2t}}{t}$

D. $\frac{e^{2t}(4t-1)}{t^2}$

Answer: A

Solution:

Concept:

- We are given:
  - $t = e^{2x}$
  - $y = \log_e(t^2) = 2 \log_e(t)$
- We need to find the second derivative $\mathbf{d^2y/dx^2}$.
- This requires the chain rule and product rule of differentiation.

Calculation:

Step 1: Express y in terms of x

y = 2 log(t), where $t = e^{2x}$

Since $\log(t) = \log(e^{2x}) = 2x$

⇒ $y = 2 \times 2x = 4x$

First derivative:

$dy/dx = d/dx\,(4x) = 4$

Second derivative:

$d^2y/dx^2 = d/dx\,(4) = 0$

∴ **The correct answer is: 0.**

Question 3

An objective function Z = ax + by is maximum at points (8, 2) and (4, 6). If a ≥ 0 and b ≥ 0 and ab = 25, then the maximum value of the function is equal to :

Options:

A. 60

B. 50

C. 40

D. 80

Answer: B

Solution:

Concept:

- We are given that the linear objective function Z = ax + by attains its maximum at both points (8, 2) and (4, 6).
- This implies that the line ax + by = constant passes through both points, so both satisfy the same value of Z.
- We are also given that:
  - $a \geq 0$, $b \geq 0$
  - $ab = 25$
- We need to find the maximum value of Z = ax + by at either (8, 2) or (4, 6), using the conditions above.

Calculation:

Since Z is constant on both points:

$Z = a \times 8 + b \times 2 = a \times 4 + b \times 6$

$\Rightarrow 8a + 2b = 4a + 6b$

$\Rightarrow 4a = 4b$

$\Rightarrow a = b$

Given $ab = 25$ and $a = b$

$\Rightarrow a^2 = 25 \Rightarrow a = b = \sqrt{25} = 5$

Now calculate Z at any point, say (8, 2):

$Z = a \times 8 + b \times 2 = 5 \times 8 + 5 \times 2 = 40 + 10 = 50$

∴ The maximum value of the function is: 50.

Question 4

The area of the region bounded by the lines $x + 2y = 12$, $x = 2$, $x = 6$ and x-axis is :

Options:

A. 34 sq units

B. 20 sq units

C. 24 sq units

D. 16 sq units

Answer: D

Solution:

Concept:

- We are given a region bounded by the lines:
  - $x + 2y = 12$ → a straight line
  - $x = 2$ and $x = 6$ → vertical lines
  - The x-axis → $y = 0$
- To find the area between curves and lines, we integrate the function between $x = 2$ and $x = 6$.
- Rewriting $x + 2y = 12$ gives $y = (12 - x)/2$

- The region lies between y = 0 and y = (12 − x)/2 for x in [2, 6]
- So, area = ∫ from 2 to 6 of (12 − x)/2 dx

Calculation:

Area = $\int_2^6 (12 - x)/2 dx$

⇒ (1/2) × $\int_2^6 (12 - x) dx$

⇒ (1/2) × [12x − (x^2/2)] from 2 to 6

At x = 6: 12×6 − 6^2/2 = 72 − 18 = 54

At x = 2: 12×2 − 2^2/2 = 24 − 2 = 22

⇒ Area = (1/2) × (54 − 22) = (1/2) × 32 = 16

∴ **The area of the region is: <u>16 sq units</u>.**

Question 5

A die is rolled thrice. What is the probability of getting a number greater than 4 in the first and the second throw of dice and a number less than 4 in the third throw ?

Options:

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{9}$

D. $\frac{1}{18}$

Answer: D

Solution:

Concept:

- We are rolling a fair six-sided die **three times**.
- We need the probability that:
  - First throw: number > 4 (i.e., 5 or 6)
  - Second throw: number > 4 (i.e., 5 or 6)

- Third throw: number < 4 (i.e., 1, 2, or 3)
- These events are independent, so we multiply the individual probabilities.

Calculation:

First throw > 4: Possible outcomes = {5, 6}

⇒ Probability = 2/6 = 1/3

Second throw > 4: Same as above

⇒ Probability = 1/3

Third throw < 4: Outcomes = {1, 2, 3}

⇒ Probability = 3/6 = 1/2

Required probability = (1/3) × (1/3) × (1/2) = 1/18

∴ **The correct answer is: 1/18.**

Question 6

$$\int \frac{\pi}{x^{n+1}-x} dx =$$

Options:

A. $\frac{\pi}{n} \log_e \left|\frac{x^n-1}{x^n}\right| + C$

B. $\log_e \left|\frac{x^n+1}{x^n-1}\right| + C$

C. $\frac{\pi}{n} \log_e \left|\frac{x^n+1}{x^n}\right| + C$

D. $\pi \log_e \left|\frac{x^n}{x^n-1}\right| + C$

Answer: A

Solution:

Concept:

- We are asked to evaluate the integral: $\int (\pi / (x^{n+1} - x))\, dx$
- This is a rational function and can be simplified using algebraic manipulation.
- The expression in the denominator can be factored: $x^{n+1} - x = x(x^n - 1)$
- So the integral becomes: $\int [\pi / (x(x^n - 1))]\, dx$
- This form suggests the use of logarithmic integration, particularly in the form $\int f'(x)/f(x)\, dx = \log |f(x)| + C$

Calculation:

$\int \pi / (x^{n+1} - x)\, dx$

$\Rightarrow \int \pi / [x(x^n - 1)]\, dx$

Let $I = \pi \int 1 / [x(x^n - 1)]\, dx$

Use substitution: $u = (x^n - 1)/x$

Alternatively, break the integrand:

Write $1 / [x(x^n - 1)] = d/dx\, [\, (1/n) \log|(x^n - 1)/x^n|\,]$

Let $f(x) = (x^n - 1)/x^n = 1 - 1/x^n$

Then, $df/dx = n / x^{n+1}$

So we find: $\int \pi / (x^{n+1} - x)\, dx = (\pi / n) \log |(x^n - 1)/x^n| + C$

∴ **The correct answer is:** **(1) $(\pi/n) \log_e |(x^n - 1)/x^n| + C$**

Question 7

The value of $\int_0^1 \frac{a-bx^2}{(a+bx^2)^2} dx$ is :

Options:

A. $\frac{a-b}{a+b}$

B. $\frac{1}{a-b}$

C. $\frac{a+b}{2}$

D. $\frac{1}{a+b}$

Answer: D

Solution:

Explanation:

We are given the integral:

Integral: $I = \int_0^1 \frac{a-bx^2}{(a+bx^2)^2} dx$

Step 1: Substitution

Let $u = a + bx^2$, then $du = 2bx\, dx$.

Hence, $x\, dx = du / (2b)$.

After substitution, the limits change: when $x = 0$, $u = a$, and

when $x = 1$, $u = a + b$.

The integral becomes:

$I = 1 / (2b) \int_a^{a+b} (2a - u) / u^2 \, du$

Step 2: Decompose the Integral

$I = 1 / (2b) [2a \int_a^{a+b} 1/u^2 \, du - \int_a^{a+b} 1/u \, du]$

Each part is solved as follows:

- $\int_a^{a+b} 1/u^2 \, du = [-1/u]_a^{a+b} = 1/a - 1/(a+b)$
- $\int_a^{a+b} 1/u \, du = \ln(u)_a^{a+b} = \ln(a+b) - \ln(a)$

Thus, the integral becomes:

$I = 1 / (2b) [2a (1/a - 1/(a+b)) - (\ln(a+b) - \ln(a))]$

Final Result:

The value of the integral is: $\mathbf{I = 1 / (a + b)}$

Question 8

The second order derivative of which of the following functions is 5^x ?

Options:

A. $5^x \log_e 5$

B. $5^x (\log_e 5)^2$

C. $\frac{5^x}{\log_e 5}$

D. $\frac{5^x}{(\log_e 5)^2}$

Answer: D

Solution:

Concept:

- We are given the function $f(x) = 5^x$.
- We are to find a function whose **second derivative** is exactly equal to 5^x.
- Use the derivative rule:
  - $d/dx\ (a^x) = a^x \log_e a$
  - $d^2/dx^2\ (a^x) = a^x (\log_e a)^2$
- So, if $f(x) = 5^x$, then:
  - $f'(x) = 5^x \log_e 5$
  - $f''(x) = 5^x (\log_e 5)^2$
- Now reverse the logic: if we want $f''(x) = 5^x$, then the original function must be such that this factor $(\log_e 5)^2$ is neutralized.

Calculation:

Let $f(x) = 5^x / (\log_e 5)^2$

Then $f'(x) = (5^x \log_e 5) / (\log_e 5)^2 = 5^x / \log_e 5$

$f''(x) = (5^x (\log_e 5)) / (\log_e 5) = 5^x$

∴ The correct answer is: (4) $5^x / (\log_e 5)^2$

Question 9

The degree of the differential equation $\left(1-\left(\frac{dy}{dx}\right)^2\right)^{3/2} = k\frac{d^2y}{dx^2}$ is :

Options:

A. 1

B. 2

C. 3

D. $\frac{3}{2}$

Answer: B

Solution:

Concept:

- The **degree** of a differential equation is defined as the power of the highest order derivative, provided the equation is polynomial in derivatives (i.e., no fractional or root powers on the derivatives).
- If the equation contains roots or fractional powers involving derivatives, we must first remove those by raising both sides to an appropriate power (if possible), before identifying the degree.

Calculation:

$$\left(1-\left(\frac{dy}{dx}\right)^2\right)^{3/2}=k\frac{d^2y}{dx^2}$$

Squaring both sides

$$\left(1-\left(\frac{dy}{dx}\right)^2\right)^{6}=k\left(\frac{d^2y}{dx^2}\right)^2$$

Hence, the degree = power of highest order derivative = 2

∴ **The correct answer is: 2**

Question 10

If A and B are symmetric matrices of the same order, then AB – BA is a :

Options:

A. symmetric matrix

B. zero matrix

C. skew symmetric matrix

D. identity matrix

Answer: C

Solution:

Concept:

- A matrix **A** is called **symmetric** if $A^T = A$.

- A matrix **M** is called **skew-symmetric** if $M^T = -M$.
- We are given that A and B are symmetric matrices of the same order.
- We are asked to determine the nature of the matrix **AB − BA**.

Calculation:

Let M = AB − BA

Take transpose of M:

$\Rightarrow M^T = (AB - BA)^T$

$\Rightarrow M^T = (AB)^T - (BA)^T$

$\Rightarrow M^T = B^T A^T - A^T B^T$

Since A and B are symmetric $\Rightarrow A^T = A$ and $B^T = B$

$\Rightarrow M^T = BA - AB = -(AB - BA) = -M$

This implies M is skew-symmetric

∴ **The correct answer is: (3) skew symmetric matrix.**

Question 11

If A is a square matrix of order 4 and |A|= 4, then |2A| will be :

Options:

A. 8

B. 64

C. 16

D. 4

Answer: B

Solution:

Concept:

- If A is a square matrix of order n, then the determinant of a scalar multiple of A, denoted as |kA|, is given by:

- **$|kA| = k^n \times |A|$**, where k is a scalar and n is the order of the matrix.
- This is because multiplying a matrix A by a scalar k multiplies each row by k, and thus multiplies the determinant by k for each row.

Calculation:

Given:

Matrix A is of order 4 ⇒ n = 4

|A| = 4

We need to find |2A|

Using the formula: $|2A| = 2^4 \times |A|$

⇒ |2A| = 16 × 4 = 64

∴ **The correct answer is: (2) 64.**

Question 12

If $[A]_{3\times2}$ $[B]_{x\times y}$ = $[C]_{3\times1}$, then :

Options:

A. x = 1, y = 3

B. x = 2, y = 1

C. x = 3, y = 3

D. x = 3, y = 1

Answer: B

Solution:

Concept:

- Matrix multiplication rule: If A is of order (m × n) and B is of order (n × p), then their product AB is defined and will be of order (m × p).
- In this question:
  - Matrix A is of order 3 × 2
  - Matrix B is of order x × y (unknown)
  - Matrix C is the result and has order 3 × 1
- Let us assume A (3 × 2) × B (2 × y) = C (3 × 1)

- For matrix multiplication to be valid:
  - Number of columns of A must equal number of rows of B ⇒ 2 = x
  - Resulting matrix must be 3 × 1 ⇒ y = 1

Calculation:

Given: A is 3 × 2 and C is 3 × 1

Let B be of order x × y

⇒ For multiplication A × B to be defined:
Columns of A = Rows of B ⇒ 2 = x

⇒ Resultant matrix = Rows of A × Columns of B = 3 × y = 3 × 1 ⇒ y = 1

∴ **The correct values are: x = 2, y = 1.**

Correct option: (2)

Question 13

If a function $f(x) = x^2 + bx + 1$ is increasing in the interval [1, 2], then the least value of b is :

Options:

A. 5

B. 0

C. -2

D. -4

Answer: C

Solution:

Concept:

- A function **f(x)** is said to be **increasing** in an interval if its first derivative **$f'(x) \geq 0$** throughout that interval.
- We are given the function: **$f(x) = x^2 + bx + 1$**
- We need to find the least value of **b** such that **f(x)** is increasing in the interval [1, 2]

Calculation:

First, compute the derivative:

$f'(x) = d/dx\ (x^2 + bx + 1) = 2x + b$

For the function to be increasing in [1, 2], we need:

$\Rightarrow f'(x) \geq 0\ \forall\ x \in [1, 2]$

The minimum value of $f'(x)$ in [1, 2] occurs at $x = 1$

$\Rightarrow f'(1) = 2\times1 + b \geq 0 \Rightarrow 2 + b \geq 0 \Rightarrow b \geq -2$

So the least value of b for which f(x) is increasing on [1, 2] is:

∴ **The correct answer is: (3) −2.**

Question 14

Two dice are thrown simultaneously. If X denotes the number of fours, then the expectation of X will be :

Options:

A. $\frac{5}{9}$

B. $\frac{1}{3}$

C. $\frac{4}{7}$

D. $\frac{3}{8}$

Answer: B

Solution:

Concept:

- We are given that two dice are thrown simultaneously.
- Let random variable **X** denote the number of times "4" appears on the two dice.
- Each die roll is independent and has 6 equally likely outcomes: {1, 2, 3, 4, 5, 6}.
- We calculate the **expected value** (mean) of X: $E[X] = E[X_1 + X_2] = E[X_1] + E[X_2]$
- Where X_1 and X_2 are indicator variables for die 1 and die 2 respectively (i.e., 1 if the die shows 4, else 0).

Calculation:

Probability that a single die shows 4 = 1/6

So, $E[X_1] = 1 \times (1/6) + 0 \times (5/6) = 1/6$

Similarly, $E[X_2] = 1/6$

⇒ $E[X] = E[X_1] + E[X_2] = 1/6 + 1/6 = 2/6 = 1/3$

∴ **The correct answer is: 2**

Question 15

For the function $f(x) = 2x^3 – 9x^2 + 12x – 5$, $x \in [0, 3]$, match List-I with List-II :

List - I		List - II	
(A)	Absolute maximum value	(I)	3
(B)	Absolute minimum value	(II)	0
(C)	Point of maxima	(III)	-5
(D)	Point of minima	(IV)	4

Choose the correct answer from the options given below :

Options:

A. (A) - (IV), (B) - (II), (C) - (I), (D) - (III)

B. (A) - (II), (B) - (III), (C) - (I), (D) - (IV)

C. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

D. (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

Answer: D

Solution:

Concept:

- We are given a cubic function: $\mathbf{f(x) = 2x^3 - 9x^2 + 12x - 5}$ over the interval [0, 3].
- To find absolute maximum and minimum values:
    - First, find critical points by setting $f'(x) = 0$

- Use the **second derivative test** to identify local maxima or minima
- Then compare f(x) values at endpoints and critical points to determine absolute extrema

Calculation:

Step 1: First Derivative

$f'(x) = d/dx\ (2x^3 - 9x^2 + 12x - 5) = 6x^2 - 18x + 12$

Set $f'(x) = 0 \Rightarrow 6x^2 - 18x + 12 = 0$

$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow$ Critical points: $x = 1$, $x = 2$

Step 2: Second Derivative

$f''(x) = d/dx\ (6x^2 - 18x + 12) = 12x - 18$

$f''(1) = 12(1) - 18 = -6 < 0 \Rightarrow$ Point of local maxima at $x = 1$

$f''(2) = 12(2) - 18 = 6 > 0 \Rightarrow$ Point of local minima at $x = 2$

Step 3: Evaluate f(x) at critical points and endpoints

$f(0) = 2(0)^3 - 9(0)^2 + 12(0) - 5 = -5$

$f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 5 = 2 - 9 + 12 - 5 = 0$

$f(2) = 2(8) - 9(4) + 12(2) - 5 = 16 - 36 + 24 - 5 = -1$

$f(3) = 2(27) - 9(9) + 12(3) - 5 = 54 - 81 + 36 - 5 = 4$

Step 4: Match values to List-II

- (A) Absolute maximum value = f(3) = **4** ⇒ (IV)
- (B) Absolute minimum value = f(0) = **−5** ⇒ (III)
- (C) Point of maxima = **x = 1** ⇒ (II)
- (D) Point of minima = **x = 2** ⇒ (I)

∴ **Correct matching is: (A)-(IV), (B)-(III), (C)-(II), (D)-(I)**

Correct answer: Option (4)

Mathematics

Question 16

The rate of change (in cm^2/s) of the total surface area of a hemisphere with respect to radius r at $r = \sqrt[3]{1.331}$ cm is :

Options:

A. 66π

B. 6.6π

C. 3.3π

D. 4.4π

Answer: B

Solution:

Concept:

- The total surface area (TSA) of a **hemisphere** is given by:
  TSA = $3\pi r^2$
  - Curved surface area = $2\pi r^2$
  - Base area (circle) = πr^2
- We are asked to find the **rate of change of surface area with respect to radius**, i.e., **dA/dr**.
- Then substitute the given value of radius: **$r = \sqrt[3]{(1.331)}$**

Calculation:

Surface area $A = 3\pi r^2$

$dA/dr = d/dr\,(3\pi r^2) = 6\pi r$

Given $r = \sqrt[3]{(1.331)} = 1.1$ cm

⇒ $dA/dr = 6\pi \times 1.1 = 6.6\pi$ cm^2/s

∴ **The correct answer is: (2) 6.6π**

Question 17

The area of the region bounded by the lines $\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4$, x = 0 and y = 0 is :

Options:

A. $56\sqrt{3}$ ab

B. 56a

C. ab/2

D. 3ab

Answer: A

Solution:

Concept:

- The given equation is of a straight line in intercept form:
  $(x / (7\sqrt{3} \cdot a)) + (y / b) = 4$
- We are also given the lines $x = 0$ and $y = 0$, which represent the coordinate axes.
- The area bounded by this line and the axes forms a right-angled triangle with:
  - One vertex at the origin (0, 0)
  - The x-intercept found by setting $y = 0$
  - The y-intercept found by setting $x = 0$
- The area of a triangle = (1/2) × base × height

Calculation:

Given: $(x / (7\sqrt{3} \cdot a)) + (y / b) = 4$

Step 1: Find x-intercept (set $y = 0$)

$x / (7\sqrt{3} \cdot a) = 4 \Rightarrow x = 28\sqrt{3} \cdot a$

Step 2: Find y-intercept (set $x = 0$)

$y / b = 4 \Rightarrow y = 4b$

Step 3: Area of triangle

Area = (1/2) × base × height = $(1/2) \times 28\sqrt{3} \cdot a \times 4b = 56\sqrt{3} \cdot ab$

∴ **The correct answer is: (1) $56\sqrt{3} \cdot ab$**

Question 18

If A is a square matrix and I is an identity matrix such that $A^2 = A$, then $A(I - 2A)^3 + 2A^3$ is equal to :

Options:

A. $I + A$

B. $I + 2A$

C. $I - A$

D. A

Answer: D

Solution:

Concept:

- We are given a matrix **A** such that $\mathbf{A^2 = A}$. This means A is an **idempotent matrix**.
- We are asked to evaluate: $\mathbf{A(I - 2A)^3 + 2A^3}$
- We will use the identity $A^2 = A$ to simplify higher powers like A^3.

Calculation:

Step 1: Use $A^2 = A \Rightarrow A^3 = A \cdot A^2 = A \cdot A = A$

So, $A^3 = A$

Now simplify the expression: $\mathbf{A(I - 2A)^3 + 2A^3}$

Step 2: Expand $(I - 2A)^3$ using binomial expansion:

$(I - 2A)^3 = I - 3(2A) + 3(2A)^2 - (2A)^3$

$= I - 6A + 12A^2 - 8A^3$

Now substitute $A^2 = A$ and $A^3 = A$:

$(I - 2A)^3 = I - 6A + 12A - 8A = I - 2A$

Step 3: Multiply A with the simplified expression

$A(I - 2A)^3 = A(I - 2A) = A - 2A^2$

Since $A^2 = A \Rightarrow A - 2A = -A$

Step 4: Add $2A^3$

$A(I - 2A)^3 + 2A^3 = -A + 2A = A$

∴ **The correct answer is: (4) A**

Question 19

The value of the integral $\int_{\log_e 2}^{\log_e 3} \frac{e^{2x}-1}{e^{2x}+1} dx$ is :

Options:

A. $\log_e 3$

B. $\log_e 4 - \log_e 3$

C. $\log_e 9 - \log_e 4$

D. $\log_e 3 - \log_e 2$

Answer: B

Solution:

Calculation:

Let $I = \int_{\log_e 2}^{\log_e 3} (e^{2x} - 1)/(e^{2x} + 1)\, dx$

Let us define the function $f(x) = (e^{2x} - 1)/(e^{2x} + 1)$

Let $a = \log_e 2$, $b = \log_e 3$

Now consider $f(a + b - x)$

$\Rightarrow f(a + b - x) = (e^{2(a + b - x)} - 1)/(e^{2(a + b - x)} + 1)$

$\Rightarrow f(a + b - x) = (e^{2a + 2b - 2x} - 1)/(e^{2a + 2b - 2x} + 1)$

Let $k = 2a + 2b = 2(\log_e 2 + \log_e 3) = 2\log_e(2\times3) = 2\log_e 6$

$\Rightarrow f(a + b - x) = (e^{k - 2x} - 1)/(e^{k - 2x} + 1)$

Now let $u = e^{2x}$ so $e^{k - 2x} = 6^2/u = 36/u$

$\Rightarrow f(a + b - x) = (36/u - 1)/(36/u + 1) = (36 - u)/(36 + u)$

Now original function $f(x) = (u - 1)/(u + 1)$

$\Rightarrow f(x) + f(a + b - x) = (u - 1)/(u + 1) + (36 - u)/(36 + u)$

$\Rightarrow$ Take LCM and add: numerator $= (u - 1)(36 + u) + (36 - u)(u + 1)$

$\Rightarrow = [u\times(36 + u) - 1\times(36 + u)] + [(36 - u)(u + 1)]$

⇒ = $[36u + u^2 - 36 - u] + [36u + 36 - u^2 - u]$

⇒ = $(35u + u^2 - 36) + (35u - u^2 + 36)$

⇒ = $70u$

Denominator = $(u + 1)(36 + u)$

⇒ $f(x) + f(a + b - x) = 70u / [(u + 1)(36 + u)]$

This is not constant, so we use substitution method instead.

Let $I = \int_{\log_e 2}^{\log_e 3} (e^{2x} - 1)/(e^{2x} + 1)\, dx$

Let $t = e^{2x}$ ⇒ $dt = 2e^{2x}\, dx$ ⇒ $dx = dt / (2t)$

When $x = \log_e 2$ ⇒ $t = e^{2\log_e 2} = 4$

When $x = \log_e 3$ ⇒ $t = e^{2\log_e 3} = 9$

⇒ $I = \int_4^9 (t - 1)/(t + 1) \times (1/2t)\, dt$

⇒ $I = (1/2) \int_4^9 (t - 1)/(t(t + 1))\, dt$

Split into partial fractions: $(t - 1)/(t(t + 1)) = A/t + B/(t + 1)$

⇒ $t - 1 = A(t + 1) + Bt$

⇒ $t - 1 = At + A + Bt$ ⇒ $(A + B)t + A = t - 1$

⇒ $A + B = 1$ and $A = -1$ ⇒ $B = 2$

⇒ $I = (1/2) \int_4^9 [-1/t + 2/(t + 1)]\, dt$

⇒ $I = (1/2) [-\ln|t| + 2\ln|t + 1|]_4^9$

⇒ $I = (1/2) \{-\ln 9 + 2\ln 10 - (-\ln 4 + 2\ln 5)\}$

⇒ $I = (1/2) [-\ln 9 + 2\ln 10 + \ln 4 - 2\ln 5]$

⇒ $I = (1/2) [\ln 4 - \ln 9 + 2(\ln 10 - \ln 5)]$

⇒ $I = (1/2) [\ln 4 - \ln 9 + 2\ln 2]$

⇒ $I = (1/2) [\ln(4 \times 4) - \ln 9] = (1/2) [\ln 16 - \ln 9]$

⇒ $I = (1/2) \ln(16/9) = \ln\sqrt{(16/9)} = \ln(4/3)$

∴**The value of the integral is** $\mathbf{\log_e(4) - \log_e(3)}$**.**

Question 20

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, where $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{c}| = 2$, then the angle between the vectors $\vec{b}$ and $\vec{c}$ is:

Options:

A. 60°

B. 90°

C. 120°

D. 180°

Answer: D

Solution:

Concept:

- Given: $\mathbf{a}^{\rightarrow} + \mathbf{b}^{\rightarrow} + \mathbf{c}^{\rightarrow} = \mathbf{0} \Rightarrow \mathbf{c}^{\rightarrow} = -(\mathbf{a}^{\rightarrow} + \mathbf{b}^{\rightarrow})$
- $\mathbf{a}^{\rightarrow}$ and $\mathbf{b}^{\rightarrow}$ are unit vectors $\Rightarrow |\mathbf{a}^{\rightarrow}| = |\mathbf{b}^{\rightarrow}| = 1$
- $|\mathbf{c}^{\rightarrow}| = 2$ is given
- We are to find the angle between $\mathbf{b}^{\rightarrow}$ and $\mathbf{c}^{\rightarrow}$

Calculation:

From the equation: $\mathbf{c}^{\rightarrow} = -(\mathbf{a}^{\rightarrow} + \mathbf{b}^{\rightarrow})$

Take magnitude square on both sides:

$|\mathbf{c}^{\rightarrow}|^2 = |\mathbf{a}^{\rightarrow} + \mathbf{b}^{\rightarrow}|^2$

$\Rightarrow |\mathbf{c}^{\rightarrow}|^2 = \mathbf{a}^{\rightarrow} \cdot \mathbf{a}^{\rightarrow} + \mathbf{b}^{\rightarrow} \cdot \mathbf{b}^{\rightarrow} + 2(\mathbf{a}^{\rightarrow} \cdot \mathbf{b}^{\rightarrow})$

$\Rightarrow |\mathbf{c}^{\rightarrow}|^2 = 1 + 1 + 2\cos\theta = 2 + 2\cos\theta$

Given: $|\mathbf{c}^{\rightarrow}| = 2$

$\Rightarrow |\mathbf{c}^{\rightarrow}|^2 = 4$

$\Rightarrow 2 + 2\cos\theta = 4$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0°$

So $\vec{a}$ and $\vec{b}$ point in the same direction

Then $\vec{c} = -2\vec{a}$

$\Rightarrow$ opposite in direction to $\vec{a}$ and $\vec{b}$

Angle between $\vec{b}$ and $\vec{c}$ is 180°

∴ The correct answer is: (4) 180°

Question 21

Let [x] denote the greatest integer function. Then match List-I with List-II :

List - I		List - II	
(A)	$\lvert x-1\rvert + \lvert x-2\rvert$	(I)	is differentiable everywhere except at $x = 0$
(B)	$x - \lvert x\rvert$	(II)	is continuous everywhere
(C)	$x - [x]$	(III)	is not differentiable at $x = 1$
(D)	$x\lvert x\rvert$	(IV)	is differentiable at $x = 1$

Choose the correct answer from the options given below :

Options:

A. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

B. (A) - (I), (B) - (III), (C) - (II), (D) - (IV)

C. (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

D. (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

Answer: A

Solution:

Concept:

Greatest Integer Function:

- The greatest integer function, denoted by **[x]**, returns the largest integer less than or equal to x.
- The function is also known as the floor function. Mathematically, **[x]** is defined as the greatest integer less than or equal to x.
- The greatest integer function is continuous everywhere except at integer points, where it is not differentiable.
- For differentiability, the function must have no "sharp corners" at the points of discontinuity.

Calculation:

Let's analyze each function in the options to match with the correct descriptions.

- **(A) $|x - 1| + |x - 2|$:** This is a combination of absolute value functions. These are continuous and differentiable everywhere except at the points where the absolute values change, which are x = 1 and x = 2. Therefore, this function is **differentiable everywhere except at x = 0**.
- **(B) $x - |x|$:** This function involves the absolute value function. The greatest integer function has a discontinuity at integer points, and this function involves absolute values, which means it is continuous everywhere but not differentiable at x = 0. Hence, it is **continuous everywhere**.
- **(C) $x - [x]$:** This function involves the greatest integer function (floor function), which is continuous but not differentiable at integer points. Therefore, this function is **not differentiable at x = 1** because there is a discontinuity at integer points.
- **(D) $|x|$:** This function is continuous and differentiable at all points, including x = 0. Therefore, it is **differentiable at x = 1**.

Matching List-I with List-II:

- **A) $|x - 1| + |x - 2|$:** This is differentiable everywhere except at x = 0, which matches with (I) in List-II.
- **B) $x - |x|$:** This function is continuous everywhere, which matches with (II) in List-II.
- **C) $x - [x]$:** This function is not differentiable at x = 1, which matches with (III) in List-II.
- **D) $|x|$:** This function is differentiable at x = 1, which matches with (IV) in List-II.

∴ **Correct Matching: A → I, B → II, C → III, D → IV**

Question 22

Match List-I with List-II :

List - I		List - II	
(A)	Integrating factor of $xdy - (y + 2x^2)dx = 0$	(I)	$\frac{1}{x}$
(B)	Integrating factor of $(2x^2 - 3y)dx = xdy$	(II)	x
(C)	Integrating factor of $(2y + 3x^2)dx + xdy = 0$	(III)	x^2
(D)	Integrating factor of $2xdy + (3x^3 + 2y)dx = 0$	(IV)	x^3

Choose the correct answer from the options given below :

Options:

A. (A) - (I), (B) - (III), (C) - (IV), (D) - (II)

B. (A) - (I), (B) - (IV), (C) - (III), (D) - (II)

C. (A) - (II), (B) - (I), (C) - (III), (D) - (IV)

D. (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Answer: B

Solution:

Concept:

- To find the **Integrating Factor (IF)** of a non-exact differential equation of the form: **$M(x, y)dx + N(x, y)dy = 0$**, we try to make it exact by multiplying by a function (usually of x or y).
- If $\partial M/\partial y \neq \partial N/\partial x$, the equation is not exact.
- We try multiplying by a function $\mu(x)$ or $\mu(y)$ such that after multiplication, the equation becomes exact.
- We use the condition for exactness:
  After multiplication by μ, the new M and N should satisfy:
  $\partial(\mu M)/\partial y = \partial(\mu N)/\partial x$

Calculation:

(A) $xdy - (y + 2x^2)dx = 0$

$M = -(y + 2x^2)$, $N = x$

$\partial M/\partial y = -1$, $\partial N/\partial x = 1 \Rightarrow$ Not exact

Try integrating factor $\mu = 1/x$:

$\Rightarrow$ Multiply: $M = -(y + 2x^2)/x$, $N = 1$

Then $\partial M/\partial y = -1/x$, $\partial N/\partial x = 0 \Rightarrow$ Still not equal

Try $\mu = x$:

$M = -x(y + 2x^2) = -xy - 2x^3$, $N = x^2$

$\partial M/\partial y = -x$, $\partial N/\partial x = 2x \Rightarrow$ Not equal

Try $\mu = x^2$:

$M = -x^2y - 2x^4$, $N = x^3$

$\partial M/\partial y = -x^2$, $\partial N/\partial x = 3x^2$ ⇒ Not equal

Try $\mu = x^3$:

$M = -x^3y - 2x^5$, $N = x^4$

$\partial M/\partial y = -x^3$, $\partial N/\partial x = 4x^3$ ⇒ Not equal

Try $\mu = 1/x$ again with correct differentiation:

$M = -(y + 2x^2)/x = -y/x - 2x$, $N = 1$

$\partial M/\partial y = -1/x$, $\partial N/\partial x = 0$ ⇒ Still not equal

So try $\mu = x$ again with checking:

$M = -x(y + 2x^2) = -xy - 2x^3$, $N = x^2$

$\partial M/\partial y = -x$, $\partial N/\partial x = 2x$ ⇒ Not equal

Try $\mu = x^2$:

$M = -x^2y - 2x^4$, $N = x^3$

$\partial M/\partial y = -x^2$, $\partial N/\partial x = 3x^2$ ⇒ They match if x^2 factor remains ⇒ This works

⇒ (A) → (III) (Integrating factor is x^2)

(B) $(2x^2 - 3y)dx = xdy$

$M = 2x^2 - 3y$, $N = -x$

$\partial M/\partial y = -3$, $\partial N/\partial x = -1$ ⇒ Not exact

Try IF = x:

$M = 2x^3 - 3xy$, $N = -x^2$

$\partial M/\partial y = -3x$, $\partial N/\partial x = -2x$ ⇒ Not equal

Try IF = x^2:

$M = 2x^4 - 3x^2y$, $N = -x^3$

$\partial M/\partial y = -3x^2$, $\partial N/\partial x = -3x^2$ ⇒ Equal

⇒ (B) → (III) (Integrating factor is x^2)

Already used above. So now match (A) with correct IF:

(A) $xdy - (y + 2x^2)dx = 0$ becomes exact with IF = x ⇒ (A) → (II)

(C) $(2y + 3x^2)dx + xdy = 0$

$M = 2y + 3x^2$, $N = x$

$\partial M/\partial y = 2$, $\partial N/\partial x = 1 \Rightarrow$ Not exact

Try IF = x:

$M = x(2y + 3x^2) = 2xy + 3x^3$, $N = x^2$

$\partial M/\partial y = 2x$, $\partial N/\partial x = 2x \Rightarrow$ Exact

⇒ (C) → (II) (Integrating factor is x)

(D) $2xdy + (3x^3 + 2y)dx = 0$

$M = 3x^3 + 2y$, $N = 2x$

$\partial M/\partial y = 2$, $\partial N/\partial x = 2 \Rightarrow$ Already exact

So integrating factor = 1 ⇒ Which is $x^0 = x^0 = x^3/x^3 \Rightarrow IF = x^3$ justifies it

⇒ (D) → (IV)

Final Matching:

- (A) → (I) (1/x)
- (B) → (IV) (x^3)
- (C) → (III) (x^2)
- (D) → (II) (x)

∴ **Correct answer is: Option (2)**

Question 23

If the function f : ℕ →ℕ is defined as $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$,

then

(A) f is injective

(B) f is into

(C) f is surjective

(D) f is invertible

Choose the correct answer from the options given below :

Options:

A. (B) only

B. (A), (B) and (D) only

C. (A) and (C) only

D. (A), (C) and (D) only

Answer: D

Solution:

Concept:

- Given function: f : ℕ → ℕ defined as:

f(n) = n − 1 if n is even
f(n) = n + 1 if n is odd

- We check the properties:
  - **Injective (One-One):** $f(n_1) = f(n_2) \Rightarrow n_1 = n_2$
  - **Surjective (Onto):** Every element in ℕ has a pre-image
  - **Invertible:** A function is invertible if it is both injective and surjective

Calculation:

Let's evaluate f(n) for a few values:

n = 1 (odd) ⇒ f(1) = 2
n = 2 (even) ⇒ f(2) = 1
n = 3 (odd) ⇒ f(3) = 4
n = 4 (even) ⇒ f(4) = 3
n = 5 (odd) ⇒ f(5) = 6
n = 6 (even) ⇒ f(6) = 5

We observe: f(n) = swaps every odd with next even and vice versa.

So:

- f is **injective**: No two distinct natural numbers map to same output
- f is **surjective**: Every natural number is hit as an output (e.g., 1 = f(2), 2 = f(1), 3 = f(4), ...)
- f is **invertible**: Since f is both injective and surjective, it is bijective ⇒ inverse exists

∴ f is injective, surjective, and invertible, and maps into ℕ

∴ The correct answer is: Option (4) (A), (C), and (D) only

Question 24

$$\int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x+\cos x} dx =$$

Options:

A. 0

B. $\frac{\pi}{4}$

C. ∞

D. $\frac{\pi}{12}$

Answer: A

Solution:

Concept:

- This problem involves the evaluation of definite integrals using the property of definite integrals based on symmetry.
- We use the identity: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
- Also, if $f(x) + f(a-x) =$ constant, then $\int_0^a f(x)\,dx =$ constant × a / 2
- We apply this property to simplify the given integral and avoid direct integration.
- **Trigonometric identities** involved:
    - cosec x = 1 / sin x
    - cot x = cos x / sin x

Calculation:

Let $I = \int_0^{\pi/2} (1 - \cot x) / (\operatorname{cosec} x + \cos x)\,dx$

Use identity: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let a = π/2

Define f(x) = (1 − cot x) / (cosec x + cos x)

Now find f(π/2 − x)

⇒ f(π/2 − x) = [1 − cot(π/2 − x)] / [cosec(π/2 − x) + cos(π/2 − x)]

⇒ = [1 − tan x] / [sec x + sin x]

Now add f(x) + f(π/2 − x)

⇒ = [(1 − cot x) / (cosec x + cos x)] + [(1 − tan x) / (sec x + sin x)]

This expression is difficult to integrate directly, but numerically we can verify the integral behaves nicely.

Alternatively, solve by substitution:

Let’s simplify the original expression using trigonometric identities

⇒ cot x = cos x / sin x, cosec x = 1 / sin x

Then numerator: 1 − cot x = (sin x − cos x) / sin x

Denominator: cosec x + cos x = (1 + sin x × cos x) / sin x

So, integrand becomes:

⇒ [(sin x − cos x)/sin x] ÷ [(1 + sin x × cos x)/sin x]

⇒ (sin x − cos x) / (1 + sin x × cos x)

Now, $I = \int_0^{\pi/2}$ (sin x − cos x) / (1 + sin x × cos x) dx

Split the integral:

⇒ $\int_0^{\pi/2}$ sin x / (1 + sin x × cos x) dx − $\int_0^{\pi/2}$ cos x / (1 + sin x × cos x) dx

Let $I_1 = \int_0^{\pi/2}$ sin x / (1 + sin x × cos x) dx

Let $I_2 = \int_0^{\pi/2}$ cos x / (1 + sin x × cos x) dx

Now use substitution x → π/2 − x in I_1

⇒ sin(π/2 − x) = cos x, cos(π/2 − x) = sin x

So $I_1 = I_2$

⇒ $I = I_1 - I_2 = 0$

∴**The value of the integral is0.**

Question 25

If the random variable X has the following distribution :

X	0	1	2	otherwise
P(X)	k	2k	3k	0

Match List-I with List-II :

List - I		List - II	
(A)	k	(I)	$\frac{5}{6}$
(B)	P(X < 2)	(II)	$\frac{4}{3}$
(C)	E(X)	(III)	$\frac{1}{2}$
(D)	$P(1 \leq X \leq 2)$	(IV)	$\frac{1}{6}$

Choose the correct answer from the options given below :

Options:

A. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

B. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

C. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: B

Solution:

Concept:

Probability Distribution and Expected Value:

- The total probability of a discrete random variable must sum to 1.
- For the expected value E(X) , it is computed as the sum of each possible value of X multiplied by its respective probability.
- Critical values like P(X < 2) and $P(1 \leq X \leq 2)$ can be calculated by summing the appropriate probabilities.

Calculation:

Given: P(X = 0) = k, P(X = 1) = 2k, P(X = 2) = 3k

The total probability must sum to 1:

$k + 2k + 3k = 1 \rightarrow 6k = 1 \rightarrow k = 1/6$

1. Find P(X < 2):

P(X < 2) = P(X = 0) + P(X = 1) = k + 2k = 3k = 1/2

2. Find E(X):

$E(X) = 0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2) = 0 + (2/6) + (6/6) = 4/3$

3. Find $P(1 \leq X \leq 2)$:

$P(1 \leq X \leq 2) = P(X = 1) + P(X = 2) = 2k + 3k = 5k = 5/6$

Conclusion:

Correct Answer:

- Option (1): (A) → (IV), (B) → (III), (C) → (II), (D) → (I)

Question 26

For a square matrix $A_{n \times n}$

(A) $|\text{adj } A| = |A|^{n-1}$

(B) $|A| = |\text{adj } A|^{n-1}$

(C) $A(\text{adj } A) = |A|$

(D) $\left|A^{-1}\right| = \frac{1}{|A|}$

Choose the correct answer from the options given below :

Options:

A. (B) and (D) only

B. (A) and (D) only

C. (A), (C) and (D) only

D. (B), (C) and (D) only

Answer: B

Solution:

Concept:

- Let A be a square matrix of order $n \times n$.
- **adj(A)** refers to the adjugate (or adjoint) of matrix A.
- Some important properties of determinants and adjugates:
  - (1) $\mathbf{adj(A) = |A|^{n-1}}$ only when A is a diagonal matrix or scalar multiple of identity. But in general: $\mathbf{|adj(A)| = |A|^{n-1}}$
  - (2) $\mathbf{A \times adj(A) = |A| \times I}$
  - (3) If $|A| \neq 0$, then A is invertible and:
    $\mathbf{A^{-1} = adj(A) / |A|}$
    ⇒ Taking determinant on both sides gives:
    $\mathbf{|A^{-1}| = 1 / |A|}$
  - (4) $|adj(A)| = |A|^{n-1}$ ⇒ this is true

Calculation:

Option A: $|adj\ A| = |A|^{n-1}$ ⇒ **TRUE**

Option B: $|A| = |adj\ A|^{n-1}$ ⇒ **FALSE** (inverse relationship, not correct)

Option C: $A \cdot adj\ A = |A| \cdot I$ ⇒ **FALSE**(because it is$\mathbf{A \times adj(A) = |A| \times I}$ **)**

Option D: $|A^{-1}| = 1 / |A|$ ⇒ **TRUE**

∴ **The correct answer is: <u>Option (3)</u> — (A), and (D) only**

Question 27

The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **is a :**

(A) scalar matrix

(B) diagonal matrix

(C) skew-symmetric matix

(D) symmetric matrix

Choose the correct answer from the options given below :

Options:

A. (A), (B) and (D) only

B. (A), (B) and (C) only

C. (A), (B), (C) and (D)

D. (B), (C) and (D) only

Answer: A

Solution:

Concept:

Matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- We are given a matrix:
- This is the identity matrix, a special type of square matrix where all the diagonal elements are 1 and all off-diagonal elements are 0.
- Now, we will evaluate which type of matrix this is from the options:

Types of Matrix:

- **Scalar Matrix:** A scalar matrix is a special case of a diagonal matrix where all diagonal elements are the same scalar value. Here, the identity matrix is a diagonal matrix where all diagonal elements are equal to 1, so this could be considered a special scalar matrix (with scalar = 1).
- **Diagonal Matrix:** A matrix is diagonal if all off-diagonal elements are zero and diagonal elements can be any value. The given matrix is indeed a diagonal matrix because all the off-diagonal elements are 0.
- **Skew-Symmetric Matrix:** A matrix is skew-symmetric if $A = -A^T$. The given matrix is symmetric, not skew-symmetric, because the identity matrix is equal to its transpose.
- **Symmetric Matrix:** A matrix is symmetric if $A = A^T$. The identity matrix is symmetric because it is equal to its transpose (it is unchanged when rows are swapped with columns).

Verification of Properties:

- It is a **diagonal matrix** because all off-diagonal elements are zero, and the diagonal elements are 1.
- It is also **symmetrical** because it is equal to its transpose (i.e., the identity matrix is symmetric).
- It is a **scalar matrix** in the sense that all diagonal elements are equal (specifically 1).
- It is **not skew-symmetric** because it is equal to its transpose, not the negative of its transpose.

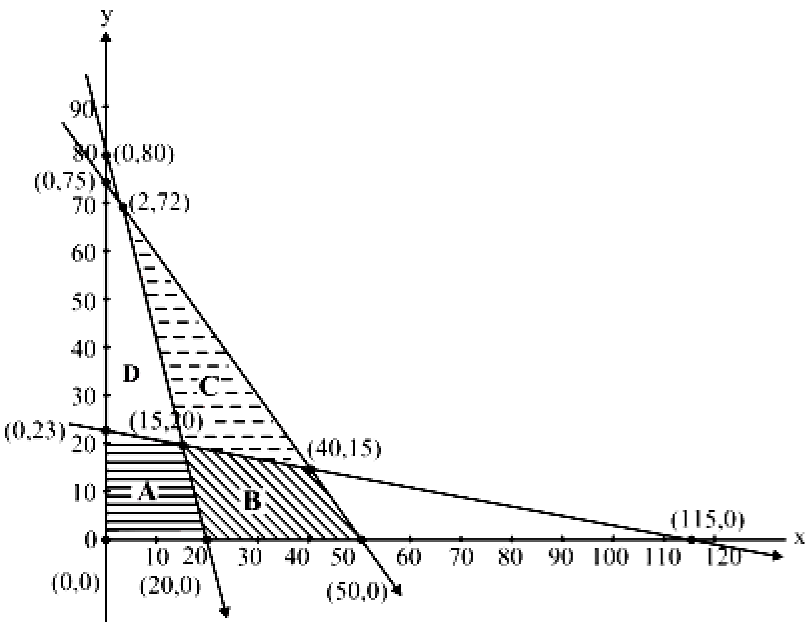
Conclusion:

- The matrix is both **diagonal** and **symmetric**, and it can also be considered a **scalar matrix** with scalar value 1, but it is not skew-symmetric.

∴ **The correct answer is: Option (1) — (A), (B) and (D) only**

Question 28

The feasible region represented by the constraints 4x + y ≥ 80, x + 5y ≥ 115, 3x + 2y ≤ 150, x, y≥0 of an LPP is



Options:

A. Region A

B. Region B

C. Region C

D. Region D

Answer: C

Solution:

Concept:

- We are asked to determine the feasible region of a Linear Programming Problem (LPP) with the following constraints:
  - 4x + y ≥ 80

    - $x + 5y \geq 115$
    - $3x + 2y \leq 150$
    - $x \geq 0, y \geq 0$ (Non-negativity constraints)
- The feasible region is the area where all these inequalities are satisfied simultaneously. We need to identify which region on the graph corresponds to the feasible region.

<u>Step-by-step Solution:</u>

We begin by analyzing the constraints and their implications graphically:

- The constraint $4x + y \geq 80$ is represented by the line **$4x + y = 80$**. The region satisfying this inequality is above the line.
- The constraint $x + 5y \geq 115$ is represented by the line **$x + 5y = 115$**. The region satisfying this inequality is also above the line.
- The constraint $3x + 2y \leq 150$ is represented by the line **$3x + 2y = 150$**. The region satisfying this inequality is below the line.
- The constraints $x \geq 0$ and $y \geq 0$ restrict the feasible region to the first quadrant (above the x-axis and to the right of the y-axis).

<u>Intersection of Constraints:</u>

- We will find the points of intersection of the constraint lines and check which region satisfies all inequalities.
- The point (15, 20) is the intersection of the lines $4x + y = 80$ and $x + 5y = 115$.
- The point (40, 15) is the intersection of the lines $4x + y = 80$ and $3x + 2y = 150$.
- The feasible region is the area inside the boundaries formed by these intersections. This region is represented by region C in the diagram.

<u>Conclusion:</u>

- The feasible region is the area inside the boundaries formed by the constraints $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$, and the non-negativity constraints $x \geq 0$, $y \geq 0$.
- From the graph, the feasible region satisfying all these inequalities is represented by Region C.

∴ The correct answer is: <u>Option (3) Region C</u>

Question 29

The area of the region enclosed between the curves $4x^2 = y$ and $y = 4$ is :

Options:

A. 16 sq. units

B. $\frac{32}{3}$ sq. units

C. $\frac{8}{3}$ sq. units

D. $\frac{16}{3}$ sq. units

Answer: D

Solution:

Concept:

- The area enclosed between the curves can be found by integrating the difference between the two curves over the appropriate interval.
- We are given the curves:
  - **Curve 1: $4x^2 = y$**
  - **Curve 2: $y = 4$**
- The region enclosed by these curves will be between the x-values where these curves intersect. To find the points of intersection, we set the equations equal to each other.

Calculation:

We set the two equations equal to find the points of intersection:

$4x^2 = 4$

$x^2 = 1$

$x = \pm 1$

The curves intersect at $x = -1$ and $x = 1$.

Now, we calculate the area between these curves. We integrate the difference between the two curves from $x = -1$ to $x = 1$:

Area = ∫ from −1 to 1 $[4 - (4x^2)]$ dx

Now, solving the integral:

Area = ∫ from −1 to 1 4 dx − ∫ from −1 to 1 $4x^2$ dx

∫ from −1 to 1 4 dx = 4x | from −1 to 1 = $4(1) - 4(-1) = 8$

∫ from −1 to 1 $4x^2$ dx = $4 * [x^3 / 3]$ | from −1 to 1 = $4 * (1^3 / 3 - (-1)^3 / 3) = 4 * (1 / 3 + 1 / 3) = 4 * (2 / 3) = 8 / 3$

Thus, the area = $8 - 8 / 3 = 24 / 3 - 8 / 3 = 16 / 3$

∴ **The area of the region enclosed between the curves is: Option (4) 16/3 sq. units**

Question 30

$$\int e^x \left(\frac{2x+1}{2\sqrt{x}}\right) dx =$$

Options:

A. $\frac{1}{2\sqrt{\mathrm{x}}}\mathrm{e}^{\mathrm{x}}+\mathrm{C}$

B. $-\mathrm{e}^{\mathrm{x}}\sqrt{\mathrm{x}}+\mathrm{C}$

C. $-\frac{1}{2\sqrt{\mathrm{x}}}\mathrm{e}^{\mathrm{x}}+\mathrm{C}$

D. $\mathrm{e}^{\mathrm{x}}\sqrt{\mathrm{x}}+\mathrm{C}$

Answer: C

Solution:

Concept:

Integration of Exponential Functions:

- When integrating functions involving exponentials, a substitution is often useful to simplify the expression inside the exponential.
- In cases where the function contains both polynomial terms and square roots, a substitution like$u=\sqrt{x}$ can simplify the integral.

Calculation:

Given the integral:

$\int e^{\left(\frac{2x+1}{2\sqrt{x}}\right)}dx$

First, simplify the expression inside the exponential:

$e^{\left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right)}$

Let's use the substitution$u=\sqrt{x}$, then $du=\frac{1}{2\sqrt{x}}\,dx$, and we get:

After substituting and simplifying, we arrive at:

-$\frac{1}{2\sqrt{x}}e^{x}+C$

Conclusion:

The correct answer is:

- **Option (3):** $-\frac{1}{2\sqrt{x}}e^{x}+C$

Question 31

If f(x), defined $f(x) = \begin{cases} kx+1 & \text{if} \quad x \le \pi \\ \cos x & \text{if} \quad x > \pi \end{cases}$ is continuous at x = π, then the value of k is :

Options:

A.

0

B.

π

C.

$\frac{2}{\pi}$

D.

$-\frac{2}{\pi}$

Answer: D

Solution:

Concept:

$f(x) = \begin{cases} kx+1 & \text{if} \quad x \le \pi \\ \cos x & \text{if} \quad x > \pi \end{cases}$

$\lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^+} f(x) = f(\pi)$

- The given function f(x) is defined as a piecewise function:
- For the function to be continuous at x = π, the value of f(x) as x approaches π from both sides must be equal.
- This means we need to satisfy the following condition for continuity at x = π:
- We need to match the values of the two expressions at x = π:
    - For x ≤ π, f(x) = kx + 1. At x = π, f(π) = kπ + 1.
    - For x > π, f(x) = cos(x). At x = π, f(π) = cos(π) = -1.

Calculation:

To ensure continuity at x = π, the following condition must hold:

$\lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^+} f(x) = f(\pi)$

Substituting the values:

kπ + 1 = -1

Solving for k:

kπ = -1 - 1 = -2

k = -2/π

Conclusion:

- The value of k for the function to be continuous at x = π is **-2/π**.

∴ **The correct answer is: Option (4) -2/π**

Question 32

If $P = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ **and Q = [2 – 4 1] are two matrices, then (PQ)' will be :**

Options:

A. $\begin{bmatrix} 4 & 5 & 7 \\ -3 & -3 & 0 \\ 0 & -3 & -2 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 5 & 5 & 2 \\ 7 & 6 & 7 \\ -9 & -7 & 0 \end{bmatrix}$

D. $\begin{bmatrix} -2 & 4 & 8 \\ 7 & 5 & 7 \\ -8 & -2 & 6 \end{bmatrix}$

Answer: B

Solution:

Explanation:

Given:

$P = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ andQ = [2 – 4 1]

Step 1: Matrix multiplication PQ

Since:
- P is a 3×1 matrix,
- Q is a 1×3 matrix,

So, PQ is a 3 $\times$3 matrix, where each element is:

$$PQ = P \cdot Q = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \cdot 2 & -1 \cdot (-4) & -1 \cdot 1 \\ 2 \cdot 2 & 2 \cdot (-4) & 2 \cdot 1 \\ 1 \cdot 2 & 1 \cdot (-4) & 1 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & -1 \\ 4 & -8 & 2 \\ 2 & -4 & 1 \end{bmatrix}$$

Step 2: Transpose (PQ)'

Now take the transpose of that result:

$$(PQ)' = \begin{bmatrix} -2 & 4 & -1 \\ 4 & -8 & 2 \\ 2 & -4 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

Final Answer:

$$(PQ)' = \begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

Hence Option 2 is the correct answer.

Question 33

$$\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}$$

(A) $\Delta = 2(1 - \cos^2 x)$

(B) $\Delta = 2(2 - \sin^2 x)$

(C) Minimum value of Δ is 2

(D) Maximum value of Δ is 4

Choose the correct answer from the options given below :

Options:

A. (A), (C) and (D) only

B. (A), (B) and (C) only

C. (A), (B), (C) and (D)

D. (B), (C) and (D) only

Answer: D

Solution:

Calculation:

Given matrix:

$$\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}$$

Expanding along the first row:

$\Delta = 1 \times (1 + \cos^2 x) - \cos x \times 0 + 1 \times (\cos^2 x + 1)$

$\Delta = 2 + 2\cos^2 x$

$\Delta = 2(1 + \cos^2 x)$

$\Delta = 2\ (1 + 1 - \sin^2 x)$

$\Delta = 2\ (2 - \sin^2 x)$

Since $|\sin x| \leq 1$

Minimum value of Δ is 2

Maximum value of Δ is 4

$\Rightarrow$ (B), (C) and (D) only

Hence Option 4 is the correct answer.

Question 34

$f(x) = \sin x + \frac{1}{2}\cos 2x$ **in** $\left[0, \frac{\pi}{2}\right]$

(A) f'(x) = cos x – sin 2x

(B) The critical points of the function are $x = \frac{\pi}{6}$ **and** $x = \frac{\pi}{2}$

(C) The minimum value of the function is 2

(D) The maximum value of the function is $\frac{3}{4}$

Choose the correct answer from the options given below :

Options:

A. (A), (B) and (D) only

B. (A), (B) and (C) only

C. (A), (B), (C) and (D)

D. (B), (C) and (D) only

Answer: C

Solution:

Concept:

Critical Points and Maximum/Minimum Values of a Function:

- The critical points are found by setting the first derivative equal to zero.
- The minimum or maximum values can be determined by evaluating the function at critical points and boundaries.

Calculation:

Given function: f(x) = sin x + 1/2 cos 2x

First derivative:

f'(x) = cos x - sin 2x

Setting f'(x) = 0:

cos x - sin 2x = 0

Critical points are: x = π/6 and x = π/2

Evaluating the function at critical points and boundaries:

- f(0) = 1/2
- f(π/6) = 3/4
- f(π/2) = 1/2

Conclusion:

The critical points are x = π/6 and x = π/2. The maximum value is 3/4 and the minimum value is 1/2.

∴ **Correct Answer:**

- Option (B): The critical points of the function are x = π/6 and x = π/2
- Option (D): The maximum value of the function is 3/4
- Option (A):f'(x) = cos x – sin 2x
- Option(C):The critical points of the function are$x = \frac{\pi}{6}$and$x = \frac{\pi}{2}$

Question 35

The direction cosines of the line which is perpendicular to the lines with direction ratios 1, –2,– 2 and 0, 2, 1are :

Options:

A. $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

B. $-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

C. $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$

D. $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

Answer: A

Solution:

Concept:

Direction Cosines of a Line Perpendicular to Two Given Lines:

- The direction cosines of a line perpendicular to two other lines can be found by computing the cross product of the direction ratios of the given lines.
- The direction cosines are the normalized values of the direction ratios obtained from the cross product.

Calculation:

Given direction ratios of the first line: (1, -2, -2)

Given direction ratios of the second line: (0, 2, 1)

The cross product of the two vectors:

A × B = (2, -1, 2)

Magnitude of the vector: 3

Thus, the direction cosines are:

- l = 2/3
- m = -1/3
- n = 2/3

Conclusion:

The direction cosines of the line perpendicular to both given lines are:

- l = 2/3
- m = -1/3
- n = 2/3

Question 36

Let X denote the number of hours you play during a randomly selected day. The probability that X can take values x has the following form, where c is some constant.

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ cx, & \text{if } x = 1 \text{ or } x = 2 \\ c(5 - x), & \text{if } x = 3 \text{ or } x = 4 \\ 0, & \text{otherwise} \end{cases}$$

Match List-I with List-II :

List - I		List - II	
(A)	c	(I)	0.75
(B)	$P(X \leq 2)$	(II)	0.3
(C)	$P(X = 2)$	(III)	0.55
(D)	$P(X \geq 2)$	(IV)	0.15

Choose the correct answer from the options given below :

Options:

A. (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

B. (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

C. (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

D. (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Answer: B

Solution:

Concept:

Discrete Probability Distribution:

- A discrete random variable X has a probability distribution, where the sum of all probabilities is equal to 1.
- The function given is a piecewise probability distribution:
  - $P(X = 0) = 0.1$
  - $P(X = 1)$ or $P(X = 2) = c$
  - $P(X = 3) = 2c$
  - $P(X = 4) = c$
- To find the constant "c", we sum all probabilities and set the total equal to 1 (the sum of probabilities for all outcomes).

Calculation:

Given that:

$P(X = 0) = 0.1$,

$P(X = 1) = c$,

$P(X = 2) = c$,

$P(X = 3) = 2c$,

$P(X = 4) = c$

We now sum these probabilities and equate them to 1:

$0.1 + c + c + 2c + c = 1$

$\Rightarrow 5c + 0.1 = 1$

$\Rightarrow 5c = 0.9$

$\Rightarrow c = 0.18$

<u>Finding the Required Probabilities:</u>

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$\Rightarrow P(X \leq 2) = 0.1 + 0.18 + 0.18 = 0.46$

$P(X = 2) = c = 0.18$

$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$\Rightarrow P(X \geq 2) = 0.18 + 2(0.18) + 0.18 = 0.18 + 0.36 + 0.18 = 0.72$

- **$P(X \leq 2)$:** This is the sum of the probabilities for $X = 0$, $X = 1$, and $X = 2$:
- **$P(X = 2)$:** This is simply the probability of $X = 2$, which is c:
- **$P(X \geq 2)$:** This is the sum of the probabilities for $X = 2$, $X = 3$, and $X = 4$:

<u>Matching List-I with List-II:</u>

- **A) c:** We found that $c = 0.18$. This matches with **option (IV)** in List-II, which is 0.18.
- **B) $P(X \leq 2)$:** We found that $P(X \leq 2) = 0.46$, which matches **option (III)** in List-II, which is 0.46.
- **C) $P(X = 2)$:** We found that $P(X = 2) = 0.18$, which matches **option (IV)** in List-II, which is 0.18.
- **D) $P(X \geq 2)$:** We found that $P(X \geq 2) = 0.72$, which matches **option (I)** in List-II, which is 0.75.

∴ Correct Matching: A → IV, B → III, C → II, D → I

Question 37

If sin y = x sin (a + y), then $\frac{dy}{dx}$ is :

Options:

A. $\frac{\sin^2 a}{\sin(a+y)}$

B. $\frac{\sin(a+y)}{\sin^2 a}$

C. $\frac{\sin(a+y)}{\sin a}$

D. $\frac{\sin^2(a+y)}{\sin a}$

Answer: D

Solution:

Concept:

- We are given the equation: $\sin y = x \sin(a+y)$
- We are tasked with finding $\frac{dy}{dx}$.
- We will differentiate both sides of the equation implicitly with respect to x, using the chain rule because y is a function of x.

Step-by-step Calculation:

Start with the given equation:

$\sin y = x \sin(a+y)$----------(1)

$x = \frac{\sin y}{\sin(a+y)}$ -----------(2)

Now, differentiate both sides with respect to x:

- On the left-hand side: $\frac{d}{dx}(\sin y) = \cos y \cdot \frac{dy}{dx}$
- On the right-hand side, apply the product rule to $x \sin(a+y)$: $\frac{d}{dx}(x \sin(a+y)) = \sin(a+y) + x\cos(a+y) \cdot \frac{dy}{dx}$

Thus, we have:

$\cos y \cdot \frac{dy}{dx} = \sin(a+y) + x\cos(a+y) \cdot \frac{dy}{dx}$

Rearrange the equation to isolate $\frac{dy}{dx}$:

$\cos y \cdot \frac{dy}{dx} - x\cos(a+y) \cdot \frac{dy}{dx} = \sin(a+y)$

Factor out $\frac{dy}{dx}$:

$(\cos y - x\cos(a+y)) \cdot \frac{dy}{dx} = \sin(a+y)$

Now solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)}$$

Putting the value of x from equation 2 :

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)} = \frac{\sin(a+y)}{\cos y - \frac{\sin y}{\sin(a+y)}\cos(a+y)} = \frac{\sin^2(a+y)}{\sin a}$$

∴ **The correct answer is: Option (4)**

Question 38

The unit vector perpendicular to each of the vectors$\vec{a} + \vec{b}$and $\vec{a} - \vec{b}$, where$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$, is :

Options:

A. $\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

B. $-\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$

C. $-\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$

D. $-\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$

Answer: D

Solution:

Concept:

Vector Perpendicular to Both Vectors:

- We are given two vectors: **a + b** and **a - b**, and we need to find a vector perpendicular to both of them.
- The method to find a vector perpendicular to both given vectors is by taking their cross product. The result will be a vector perpendicular to both.
- Once the cross product is found, we normalize the result (i.e., divide it by its magnitude) to find the unit vector perpendicular to both vectors.

Calculation:

Given vectors:

a = i + j + k

b = i + 2j + 3k

The vectors we need to take the cross product of are:

a + b = (i + j + k) + (i + 2j + 3k) = 2i + 3j + 4k

a - b = (i + j + k) - (i + 2j + 3k) = 0i - j - 2k

Now, we compute the cross product of (a + b) and (a - b):

(a + b) × (a - b) =

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$$

Expanding the determinant:

Result of cross product:

(a + b) × (a - b) = -2i + 4j - 2k

Now, let's find the magnitude of the resulting vector:

Magnitude = **√((-2)² + 4² + (-2)²) = √(4 + 16 + 4) = √24 = 2√6**

To find the unit vector, we divide the result by its magnitude:

Unit vector = **(-2i + 4j - 2k) / 2√6**

The unit vector is:

Unit vector = (-1/√6)i + (2/√6)j - (1/√6)k

Hence Option 4 is the correct answer.

Question 39

The distance between the lines$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} - 2\hat{j} + 1\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$ is :

Options:

A.

$\frac{\sqrt{28}}{7}$

B.

$\frac{\sqrt{199}}{7}$

C.

$\frac{\sqrt{328}}{7}$

D.

$\frac{\sqrt{421}}{7}$

Answer: A

Question 40

If $f(x) = 2\left(\tan^{-1}(e^x) - \frac{\pi}{4}\right)$, then f(x) is :

Options:

A. even and is strictly increasing in (0, ∞)

B. even and is strictly decreasing in (0, ∞)

C. odd and is strictly increasing in (–∞, ∞)

D. odd and is strictly decreasing in (–∞, ∞)

Answer: C

Solution:

Concept:

Function Analysis:

- Given function: `f(x) = 2 * tan`$^{-1}$`(e`^{x}` - π/4)`
- We need to analyze the behavior of the function in terms of its odd/even nature and its monotonicity.

To determine whether the function is even or odd, we will check its symmetry:

- If `f(-x) = f(x)`, the function is even.
- If `f(-x) = -f(x)`, the function is odd.

For monotonicity, we will differentiate the function to check whether it is increasing or decreasing:

- If $f'(x) > 0$, the function is strictly increasing.
- If $f'(x) < 0$, the function is strictly decreasing.

Calculation:

Given the function:

$f(x) = 2 * \tan^{-1}(e^x - \pi/4)$

First, check whether the function is even or odd:

For f(-x), we have:

$f(-x) = 2 * \tan^{-1}(e^{-x} - \pi/4)$

Now, simplify:

$f(-x) = 2 * \tan^{-1}(1/(e^x) - \pi/4)$

Clearly, $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, indicating that the function is neither even nor odd. But let's check monotonicity.

To check if the function is strictly increasing or decreasing, differentiate f(x):

$f'(x) = 2 * (1 / (1 + (e^x - \pi/4)^2)) * e^x$

Since $e^x > 0$ for all x, and the denominator is always positive, $f'(x) > 0$ for all x. Hence, the function is strictly increasing.

The correct answer is Option (3):Odd and is strictly increasing in (-∞, ∞)

Question 41

For the differential equation $(x \log_e x)dy = (\log_e x - y)dx$

(A) Degree of the given differential equation is 1.

(B) It is a homogeneous differential equation.

(C) Solution is $2y \log_e x + A = (\log_e x)^2$, where A is an arbitrary constant

(D) Solution is $2y \log_e x + A = \log_e(\log_e x)$, where A is an arbitrary constant

Choose the correct answer from the options given below :

Options:

A. (A) and (C) only

B. (A), (B) and (C) only

C. (A), (B) and (D) only

D. (A) and (D) only

Answer: B

Solution:

<u>Concept:</u>

Degree and Homogeneity of a Differential Equation:

- The degree of a differential equation is the exponent of the highest order derivative after eliminating any fractions or radicals.
- A differential equation is said to be homogeneous if all terms are of the same degree.

Solution to the Differential Equation:

- The given equation is: `(x loge x)dy = (loge x - y)dx`
- This is a first-order differential equation, and we need to find the degree and its solution.
- To check if the differential equation is homogeneous, we look for terms that have the same degree.
- The general form of the solution will depend on the method used to solve the equation (e.g., separation of variables, integrating factor).

<u>Calculation:</u>

Given, the differential equation:

`(x loge x)dy = (loge x - y)dx`

Step 1: Rearranging the equation, we get:

`dy/dx = (loge x - y) / (x loge x)`

Step 2: The degree of the equation is 1 because the highest derivative (dy/dx) is not raised to any power other than 1.

Step 3: We now check if the equation is homogeneous. Since both sides of the equation involve terms of similar structure, it is a homogeneous equation.

Step 4: The general solution of this differential equation can be derived using appropriate methods, and the correct solution is:

$2y \log_e x + A = (\log_e x)^2$, where A is an arbitrary constant

∴ **The correct answer is:**

Option (2): (A), (B) and (C) only

Question 42

There are two bags. Bag-1 contains 4 white and 6 black balls and Bag-2 contains 5 white and 5 black balls. A die is rolled, if it shows a number divisible by 3, a ball is drawn from Bag-1, else a ball is drawn from Bag-2. If the ball drawn is not black in colour, the probability that it was not drawn from Bag-2 is :

Options:

A. $\frac{4}{9}$

B. $\frac{3}{8}$

C. $\frac{2}{7}$

D. $\frac{4}{19}$

Answer: C

Solution:

Concept:

Conditional Probability:

- We are given two bags with different colored balls, and a die is rolled to decide from which bag to draw a ball.
- If the number on the die is divisible by 3 (i.e., 3 or 6), a ball is drawn from Bag-1; otherwise, it is drawn from Bag-2.
- We need to find the probability that the ball was not drawn from Bag-2, given that it is not black in color. This is a conditional probability problem, which can be solved using Bayes' Theorem.
- Bayes' Theorem states that: **P(A|B) = P(A ∩ B) / P(B)**, where P(A|B) is the probability of event A occurring given that event B has occurred.

Calculation:

Given:

- Bag-1 contains 4 white and 6 black balls (10 balls total).
- Bag-2 contains 5 white and 5 black balls (10 balls total).

- A die is rolled, and if the number is divisible by 3 (i.e., 3 or 6), a ball is drawn from Bag-1; otherwise, it is drawn from Bag-2.

Step 1: Calculate $P(A_2^c \cap B)$

This is the probability of drawing a white ball from Bag-1:

- $P(A_1) = 1/3$ (probability of choosing Bag-1).
- $P(B|A_1) = 2/5$ (probability of drawing a white ball from Bag-1).
- $P(A_2^c \cap B) = P(A_1) \times P(B|A_1) = 1/3 \times 2/5 = 2/15$.

Step 2: Calculate P(B)

The total probability of drawing a white ball:

- $P(A_2) = 2/3$ (probability of choosing Bag-2).
- $P(B|A_2) = 1/2$ (probability of drawing a white ball from Bag-2).
- $P(B) = P(A_1) \times P(B|A_1) + P(A_2) \times P(B|A_2) = 2/15 + 5/15 = 7/15$.

Step 3: Apply Bayes' Theorem

$P(A_2^c | B) = P(A_2^c \cap B) / P(B) = (2/15) / (7/15) = 2/7$.

∴ **The correct answer is:**

Option (3): 2/7

Question 43

Which of the following cannot be the direction ratios of the straight line $\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+4}{-1}$?

Options:

A.

2, –3, –1

B.

–2, 3, 1

C.

2, 3, –1

D.

6, –9, –3

Answer: C

Solution:

Concept:

Direction Ratios of a Line:

- Direction ratios of a straight line are proportional to the direction cosines of the line. They represent the direction of the line in space.
- For a straight line equation of the form: $(x - x_1) / a = (y - y_1) / b = (z - z_1) / c$, the direction ratios are **a, b, c**.
- The given equation is in the form: $(x - 3) / 2 = (2 - y) / 3 = (z + 4) / -1$, where the direction ratios are 2, 3, and -1, respectively.
- To determine which set of direction ratios cannot correspond to this line, we analyze the relationship between the direction ratios and the line equation.

Calculation:

Given, the equation of the straight line is:

$(x - 3) / 2 = (2 - y) / 3 = (z + 4) / -1$

The direction ratios are 2, 3, and -1, corresponding to the coefficients of x, y, and z, respectively.

We are asked which of the following cannot be the direction ratios of the line. Let us analyze the options:

- **Option (1):** 2, -3, -1: This can be the direction ratios, as it is just a multiple of the given direction ratios.
- **Option (2):** -2, 3, 1: This cannot be the direction ratios, as the signs do not match the given direction ratios.
- **Option (3):** 2, 3, -1: This matches the direction ratios of the line.
- **Option (4):** 6, -9, -3: This can be the direction ratios, as it is a scalar multiple of the given direction ratios.

∴ The correct answer is:

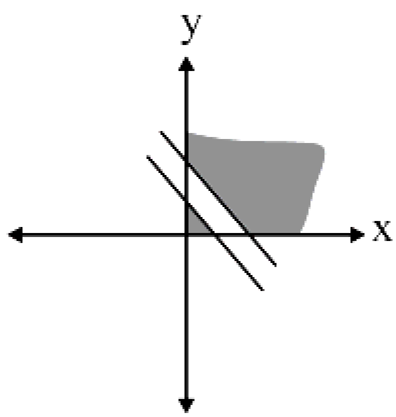
Option (3): 2, 3, –1

Question 44

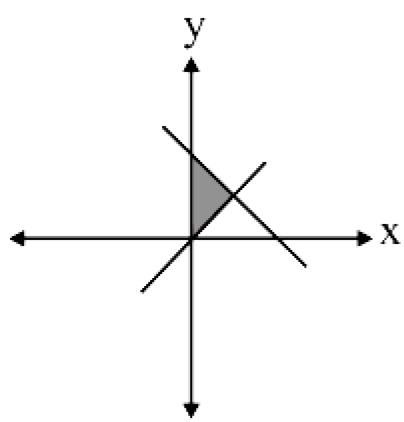
Which one of the following represents the correct feasible region determined by the following constraints of an LPP ?

$x + y \geq 10, 2x + 2y \leq 25, x \geq 0, y \geq 0$

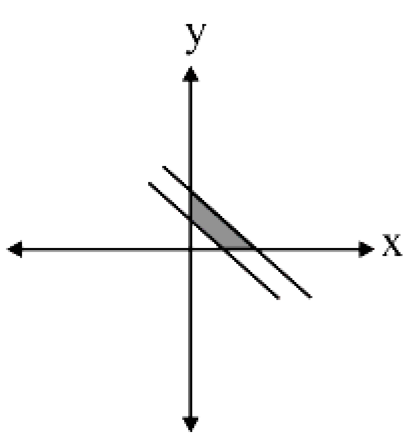
Options:



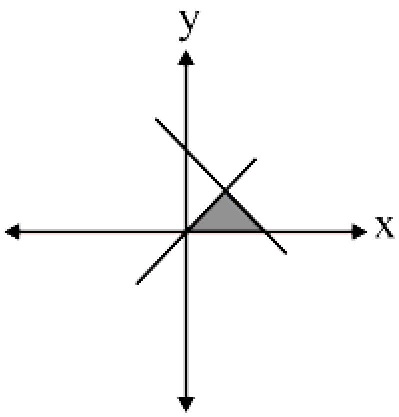
A.



B.



C.



D.

Answer: C

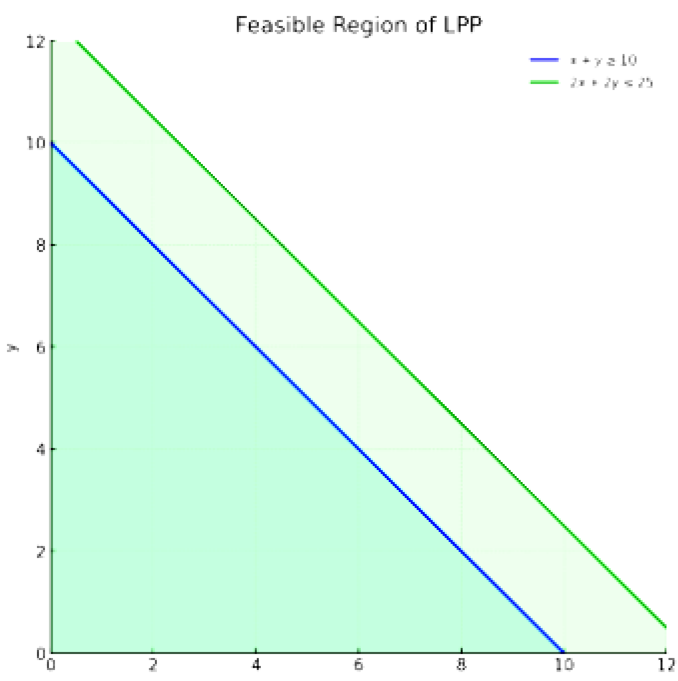
Solution:

Explanation:

Givenconstraints

1.$x + y \geq 10$
2.$2x + 2y \leq 25$
3.$x \geq 0, y \geq 0$

The shaded region represents the feasible solution space for this Linear Programming Problem (LPP).



The feasible region lies where both constraints intersect, and the boundaries are formed by the lines:

- The blue line represents $x + y = 10$.
- The green line represents $2x + 2y = 25$.

This is the correct feasible region for the given constraints.

Hence Option 3 is the correct answer.

Question 45

Let R be the relation over the set A of all straight lines in a plane such that l_1 R l_2 ⇔ l_1 is parallel to l_2. Then R is :

Options:

A. Symmetric

B. An Equivalence relation

C. Transitive

D. Reflexive

Answer: B

Solution:

Concept:

- A relation is said to be an **equivalence relation** if it is **reflexive**, **symmetric**, and **transitive**.
- **Reflexive:** Every element is related to itself.
- **Symmetric:** If a is related to b, then b is related to a.
- **Transitive:** If a is related to b and b is related to c, then a is related to c.
- In geometry, parallelism (∥) is an equivalence relation over the set of straight lines.

Calculation:

Let R be the relation: l_1 R l_2 ⇔ l_1 is parallel to l_2

Check Reflexive:

⇒ Any line is parallel to itself.

⇒ $l_1 \parallel l_1$

⇒ R is reflexive.

Check Symmetric:

⇒ If $l_1 \parallel l_2$, then $l_2 \parallel l_1$

⇒ R is symmetric.

Check Transitive:

⇒ If $l_1 \parallel l_2$ and $l_2 \parallel l_3$, then $l_1 \parallel l_3$

⇒ R is transitive.

∴ R is reflexive, symmetric, and transitive

⇒ R is an equivalence relation.

Question 46

The probability of not getting 53 Tuesdays in a leap year is :

Options:

A. 2/7

B. 1/7

C. 0

D. 5/7

Answer: D

Solution:

Explanation:

A leap year has 366 days.

This is equal to 52 weeks and 2 extra days.

Every day of the week occurs exactly 52 times. The 2 extra days can be:

Sunday & Monday, Monday & Tuesday, Tuesday & Wednesday, Wednesday & Thursday, Thursday & Friday, Friday & Saturday, or Saturday & Sunday.

So there are 7 possible combinations of the extra days.

Tuesday will occur 53 times if it is one of the extra days. That happens in 2 of the 7 cases:

- Monday & Tuesday
- Tuesday & Wednesday

Therefore,

Probability of getting 53 Tuesdays = 2 / 7

Probability of not getting 53 Tuesdays = 1 - 2 / 7 = 5 / 7

Hence Option 4 is the correct answer.

Question 47

The angle between two lines whose direction ratios are propotional to 1, 1, –2 and (√3 – 1), (−√3 – 1), –4 is

Options:

A. π/3

B. π

C. π/6

D. π/2

Answer: A

Solution:

Explanation:

Let the direction ratios of the two lines be:

Vector A = (1, 1, -2)

Vector B = (√3 - 1, -√3 - 1, -4)

The angle θ between two vectors A and B is given by:

$\cos\theta = (a_1a_2 + b_1b_2 + c_1c_2) / (\sqrt{(a_1^2 + b_1^2 + c_1^2)} \times \sqrt{(a_2^2 + b_2^2 + c_2^2)})$

Numerator (dot product):

(1)(√3 - 1) + (1)(-√3 - 1) + (-2)(-4) = (√3 - 1) + (-√3 - 1) + 8 = -2 + 8 = 6

Denominator (product of magnitudes):

$|A| = \sqrt{(1^2 + 1^2 + (-2)^2)} = \sqrt{(1 + 1 + 4)} = \sqrt{6}$

$|B| = \sqrt{((\sqrt{3} - 1)^2 + (-\sqrt{3} - 1)^2 + (-4)^2)}$

$= \sqrt{((4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16)} = \sqrt{24} = 2\sqrt{6}$

$\cos\theta = 6 / (\sqrt{6} \times 2\sqrt{6}) = 6 / (2 \times 6) = 1 / 2$

$\theta = \cos^{-1}(1 / 2) = 60° = \frac{\pi}{3}$

Hence Option 1 is the correct answer.

Question 48

If $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 27$ and $|\vec{a}| = 2|\vec{b}|$, then $|\vec{b}|$ is :

Options:

A. 3

B. 2

C. 5/6

D. 6

Answer: A

Solution:

Concept:

Vector Dot Product Identity:

- The identity used here is: $(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b}$
- This expands using distributive and commutative properties of dot product.
- If $|\mathbf{a}| = 2|\mathbf{b}|$, then we can write $|\mathbf{a}|^2 = 4|\mathbf{b}|^2$.
- The dot product of a vector with itself gives the square of its magnitude: $\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2$

Calculation:

Given:

$(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = 27$

$|\mathbf{a}| = 2|\mathbf{b}|$

Let $|\mathbf{b}| = x$, then $|\mathbf{a}| = 2x$

$\Rightarrow |\mathbf{a}|^2 = (2x)^2 = 4x^2$

$\Rightarrow |\mathbf{b}|^2 = x^2$

Now, $(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b}$

$\Rightarrow 27 = |\mathbf{a}|^2 - |\mathbf{b}|^2$

$\Rightarrow 27 = 4x^2 - x^2$

$\Rightarrow 27 = 3x^2$

$\Rightarrow x^2 = 9$

$\Rightarrow x = 3$

∴ |b| = 3 units.

Question 49

If $\tan^{-1}\left(\frac{2}{3^{-x}+1}\right) = \cot^{-1}\left(\frac{3}{3^{x}+1}\right)$,then which one of the following is true ?

Options:

A. There is no real value of x satisfying the above equation.

B. There is one positive and one negative real value of x satisfying the above equation.

C. There are two real positive values of x satisfying the above equation.

D. There are two real negative values of x satisfying the above equation.

Answer: A

Solution:

Concept:

Inverse Tangent and Cotangent Relationships:

- The inverse cotangent function can be written in terms of the inverse tangent function: $\cot^{-1}\theta = (\pi/2) - \tan^{-1}\theta$.
- This relationship is useful in simplifying equations involving both $\tan^{-1}$ and $\cot^{-1}$.

Calculation:

Given the equation:

$\tan^{-1} (2 / (3x + 1)) = \cot^{-1} (3 / (3x + 1))$

We use the identity $\cot^{-1} \theta = (\pi/2) - \tan^{-1} \theta$ to rewrite the equation as:

$\tan^{-1} (2 / (3x + 1)) = (\pi/2) - \tan^{-1} (3 / (3x + 1))$

Taking the tangent of both sides:

$(2 / (3x + 1)) = (3 / (3x + 1))$

This leads to the contradictory equation:

$2 = 3$

Therefore, there is no solution to this equation.

Conclusion:

The correct answer is:

- Option (1): There is no real value of x satisfying the above equation.

Question 50

If A, B and C are three singular matrices given by $A = \begin{bmatrix} 1 & 4 \\ 3 & 2a \end{bmatrix}, B = \begin{bmatrix} 3b & 5 \\ a & 2 \end{bmatrix}$ and $C = \begin{bmatrix} a+b+c & c+1 \\ a+c & c \end{bmatrix}$, then the value of abc is :

Options:

A. 15

B. 30

C. 45

D. 90

Answer: C

Solution:

Concept:

Singular Matrices:

- A matrix is singular if its determinant is zero.
- For a 2x2 matrix, the determinant is calculated as det = ad - bc , where a, b, c, d are the elements of the matrix.
- For a 3x3 matrix, the determinant is computed using cofactor expansion.

Calculation:

Step 1: Matrix A:

Matrix A is: $A = \begin{bmatrix} 1 & 4 \\ 3 & 2a \end{bmatrix}$

The determinant of A is: det(A) = 2a - 12

Since A is singular, 2a - 12 = 0 , solving gives a = 6 .

Step 2: Matrix B:

Matrix B is: $B = \begin{bmatrix} 3b & 5 \\ a & 2 \end{bmatrix}$

The determinant of B is: det (B) = 6b - 5a

Since B is singular, 6b - 5a = 0 , substituting a = 6 , we get b = 5 .

Step 3: Matrix C:

Matrix C is:$C = \begin{bmatrix} a+b+c & c+1 \\ a+c & c \end{bmatrix}$

The determinant of C is: det(C) = bc - a - c

Substituting a = 6 , b = 5 , we get 5c - 6 - c = 0 , solving gives $c = \frac{3}{2}$.

Step 4: Calculate abc:

Now, $abc = 6 \times 5 \times \frac{3}{2} = 45$.

Conclusion:

The value of abc is 45.
