Chapter 7
Alternating Current

1.What are phasors?

A phasor is a vector which rotates about the origin in anticlockwise direction with angular speed ω.

2. a)Obtain the expression for current when an AC voltage is applied to a resistor.
b)Draw the phasor representation for voltage and current in the circuit.

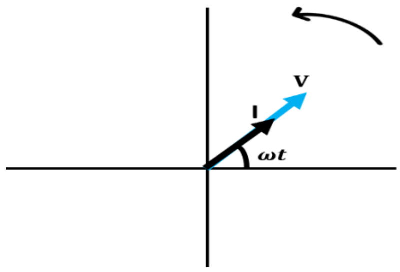
a)

$$v_m \sin \omega t = iR$$

$$i = \frac{v_m}{R} \sin \omega t$$

$$\mathbf{i = i_m \sin \omega t} \qquad \text{where } \mathbf{i_m = \frac{v_m}{R}}$$

b)



3.Obtain the expression for power dissipated in the resistor when an AC voltage is applied to it.

$$p = vi$$

$$p = v_m \sin \omega t \; i_m \sin \omega t$$

$$p = v_m i_m \sin^2 \omega t$$

Avearage power consumed over one complete cycle

$$\bar{p} = \; < v_m i_m \sin^2 \omega t >$$

$$\bar{p} = v_m i_m < \sin^2 \omega t >$$

$$< sn^2 \omega t > = \frac{1}{2}$$

$$\bar{p} = \frac{1}{2} v_m i_m$$

$$P = \left(\frac{v_m}{\sqrt{2}}\right)\left(\frac{i_m}{\sqrt{2}}\right)$$

$$\mathbf{P = VI}$$

4.Write the expression for rms current

$$\mathbf{I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m}$$

5.Write the expression for rms voltage

$$V_{rms} = \frac{v_m}{\sqrt{2}} = 0.707\, v_m$$

6.Why a shock from 220V ac is more fatal than that from 220Vdc?

The household line voltage of 220 V is an rms value.

$$V = 220V$$

Its peak voltage $v_m = \sqrt{2}\,V$

$$= 1.414 \times 220\text{ V} = 311\text{ V}$$

At some instant peak value of ac may reach upto 311V .So a shock from 220V ac is more fatal than that from 220Vdc.

7.A light bulb is rated at 100W for a 220 V supply. Find

(a) the resistance of the bulb
(b) the peak voltage of the source
(c) the rms current through the bulb.

(a) We are given P = 100 W and V = 220 V. The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220\text{ V})^2}{100\text{ W}} = 484\Omega$$

(b) The peak voltage of the source is

$$v_m = \sqrt{2}V = 311\text{ V}$$

(c) Since, $P = I\,V$

$$I = \frac{P}{V} = \frac{100\text{ W}}{220\text{ V}} = 0.450\text{A}$$

8. a) Show that the current lags the voltage by $\pi/2$ when an ac voltage applied to an inductor
b) Draw phasor representation for voltage and current in the circuit

a)

$$v_m \sin\omega t = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{v_m \sin\omega t}{L}$$

$$di = \frac{v_m}{L}\sin\omega t\, dt$$

$$i = \frac{v_m}{L}\int \sin\omega t\, dt$$

$$i = \frac{v_m}{L} \times \frac{-\cos\omega t}{\omega}$$

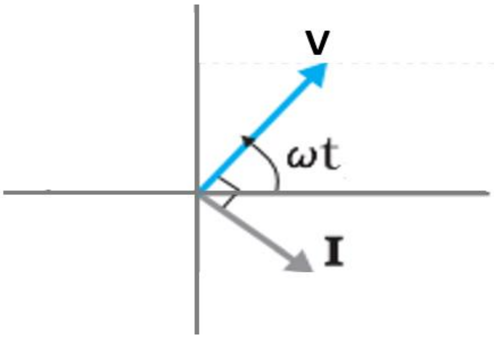
$$i = -\frac{v_m}{\omega L}\cos\omega t$$

$$i = -i_m \cos\omega t$$

$$\mathbf{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \textbf{where} \quad \mathbf{i_m = \frac{v_m}{\omega L}}$$

In a pure inductor, current lags the voltage by $\pi/2$ or one-quarter (1/4) cycle.

b)



9. a)Show that the current leads the voltage by $\pi/2$ when an ac voltage applied to an capacitor

b)Draw the phasor representation for voltage and current in the circuit

a)

$$v_m \sin \omega t = \frac{q}{C}$$

$$q = C\, v_m \sin \omega t$$

$$i = \frac{d}{dt}(C\, v_m \sin \omega t)$$

$$i = C\, v_m \frac{d}{dt}(\sin \omega t)$$

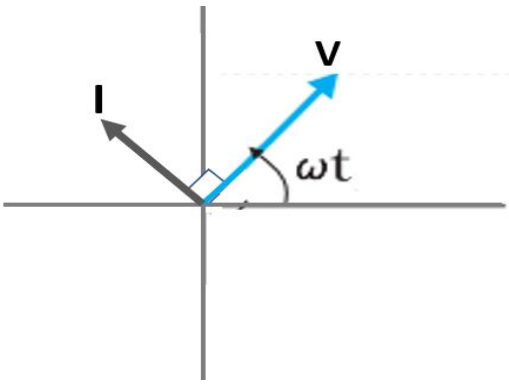
$$i = C\, v_m\, \omega \cos \omega t$$

$$i = \omega C\, v_m \cos \omega t$$

$$i = i_m \cos \omega t$$

$$\mathbf{i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)} \quad \textbf{where } \mathbf{i_m = \omega C\, v_m}$$

b)



10.Write the equation for inductive reactance

$$\mathbf{X_L = \omega L = 2\pi f L}$$

11.A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

$$\text{Inductive reactance, } X_L = \omega L = 2\pi f L$$
$$= 2\text{x } 3.14\text{x}50\text{x}25\text{x}10^{-3}$$
$$= 7.85\Omega$$

The rms current in the circuit is, $I = \frac{V}{X_L}$

$$I = \frac{220}{7.85} = 28A$$

12.Write the equation for capacitive reactance

$$\mathbf{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

13.A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

$$\text{The capacitive reactance } X_C = \frac{1}{\omega C}$$
$$= \frac{1}{2\pi f C}$$
$$= \frac{1}{2\text{x } 3.14\text{x}50\text{x}15\text{x}10^{-6}} = 212\Omega$$

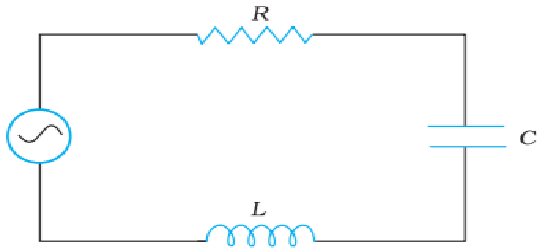
The rms current is , $I = \frac{V}{X_C}$

$$I = \frac{220}{212} = 1.04A$$

$$\text{The peak current is } i_m = \sqrt{2}\ I$$
$$= 1.414 \text{ x}1.04$$
$$= 1.47A$$

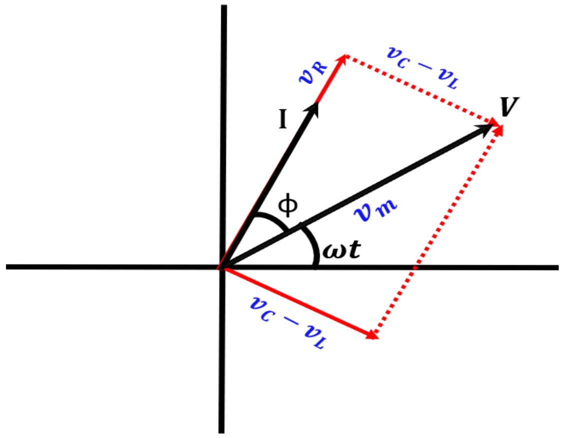
If the frequency is doubled, the capacitive reactance is halved , and consequently, the current is doubled.

14.Obtain the expression for current when an ac voltage applied to a series LCR circuit using phasor diagram



$$v_m \sin \omega t = iR + L\frac{di}{dt} + \frac{q}{C}$$

Phasor-diagram solution



From phasor diagram, current leads the voltage by an angle ϕ

$$\mathbf{i = i_m \sin(\omega t + \phi)}$$

To find the value of i_m

$$v_m^2 = v_R{}^2 + (v_C - v_L)^2$$

$$v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$v_m^2 = i_m^2 \, [(R)^2 + (X_C - X_L)^2]$$

$$i_m^2 = \frac{v_m^2}{(R)^2 + (X_C - X_L)^2}$$

$$i_m = \frac{v_m}{\sqrt{(R)^2 + (X_C - X_L)^2}}$$

$$\mathbf{i_m = \frac{v_m}{Z}}$$

where $\mathbf{Z=\sqrt{(R)^2 + (X_C - X_L)^2}}$ is called impedance

15.Write the expression for Impedance of a series LCR circuit

$$Z = \sqrt{(R)^2 + (X_C - X_L)^2}$$

or

$$Z = \sqrt{(R)^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

16.A sinusoidal voltage of peak value 283 V and frequency Hz is applied to a series LCR circuit in which R=3Ω, L=25.48 mH and C=796μF. Find the impedance of the circuit.

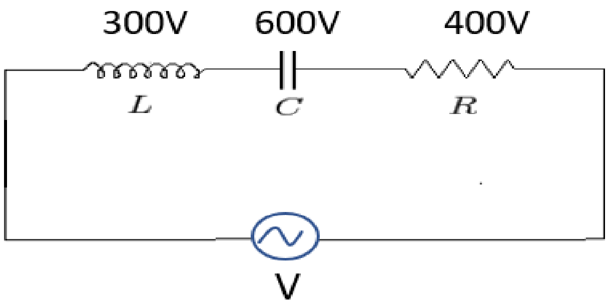
$$\mathbf{Z} = \sqrt{(\mathbf{R})^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

$$= \sqrt{(R)^2 + \left(\frac{1}{2\pi f C} - 2\pi f L\right)^2}$$

$$= \sqrt{(3)^2 + \left(\frac{1}{2x3.14x50x796x10^{-6}} - 2x3.14x50x25.48x10^{-3}\right)^2}$$

$$= \sqrt{9 + (4-8)^2} = \sqrt{9+16} = 5\ \Omega$$

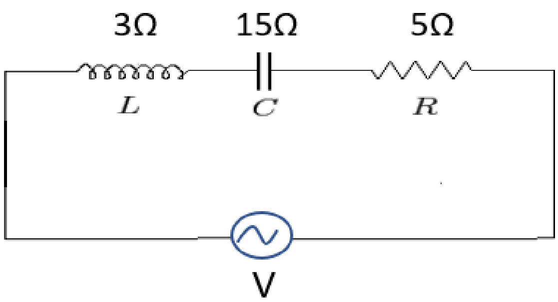
17.In the following circuit,find the impedance



$$V = \sqrt{{v_R}^2 + (v_C - v_L)^2}$$

$$= \sqrt{400^2 + (600-300)^2} = 500V$$

18. In the following circuit ,find the impedance



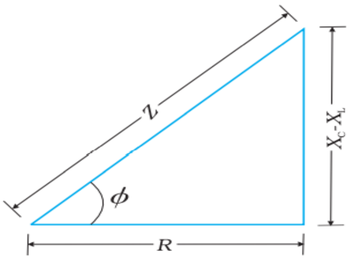
$$Z = \sqrt{(R)^2 + (X_C - X_L)^2}$$

$$= \sqrt{5^2 + (15-3)^2} = \sqrt{169} = 13\Omega$$

19.Write ant two factors on which the impedance of a series LCR crcuit depends.

Resistance, Capacitance, Inductance, Frequency of AC

20.Using Impedance diagram(impedance triangle) obtain the expression for phase difference between voltage and current in a series LCR circuit



$$\tan\phi = \frac{X_C - X_L}{R}$$

$$\phi = \tan^{-1}\frac{X_C - X_L}{R}$$

21.What is the principle behind tuning of radio or TV Resonance or At what condition a series LCR circuit is used for tuning.

Resonance

22.What is the condition for resonance in a series LCR circuit

$$\mathbf{X_C = X_L}$$

$$\frac{1}{\omega_0 C} = \omega_0 L$$

23.Obtain the expression for resonant frequency

At resonance ,$X_C = X_L$

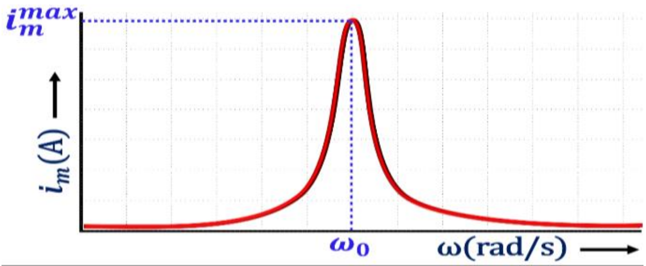
$$\frac{1}{\omega_0 C} = \omega_0 L$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

ω_0 is called Resonant frequency

24.Draw the graphical variation of **current amplitude i_m** with **frequency ω**



25.Obtain the expression for power in an ac circuit

$$p = v\,i$$

$$p = v_m \sin\omega t \; i_m \sin(\omega t + \phi)$$

$$P = \frac{v_m i_m}{2}\langle\cos\phi - \cos(2\omega t + \phi)\rangle$$

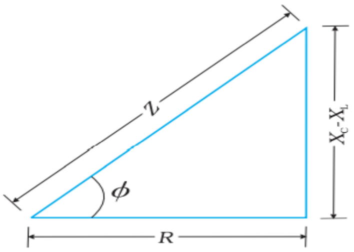
$$P = \frac{v_m i_m}{2}\cos\phi$$

$$P = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$P = V\ I\cos\phi$$

The quantity cosϕ is called the **power factor.**

26.Draw the Impedance diagram(impedance triangle) and write the expression for power factor from it.

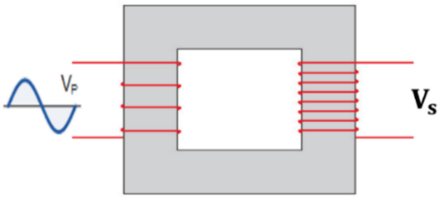


$$\cos \phi = \frac{R}{Z}$$

27.What is the principle of transformer

Mutual Induction

28.Explain with the help of a neat diagram the working of a transformer



Transformer works on the Principle of Mutual Induction

When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it.

$$V_P = -N_P \frac{d\phi}{dt}$$

$$V_s = -N_s \frac{d\phi}{dt}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \text{ ------------------- (1)}$$

If the transformer is 100% efficient

Power input= power output

$$I_P V_P = I_S V_S$$

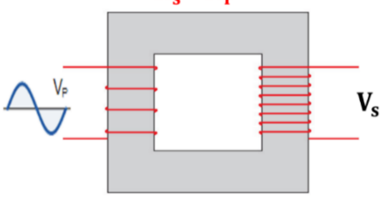
$$\frac{I_P}{I_S} = \frac{V_S}{V_P} \text{ -------------(2)}$$

From eq(1) and (2)

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

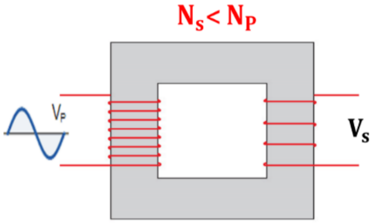
29.What is a step-up transformer

$N_s > N_P$



- For a step up transformer the number of turns in the secondary will be greater than that in the primary($N_s > N_P$)
- $V_s > V_P$)
- $I_s < I_P$)

30.What is a step-down transformer



- For a step down transformer the number of turns in the secondary will be less than that in the primary($N_S < N_P$)
- $V_S < V_P$)
- $I_S > I_P$)

31.A power transmission line feeds input power at 3300 V to a step-down transformer with its primary windings having 6000 turns. What should be the number of turns in the secondary in order to get output power at 220 V?

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$N_S = \frac{V_S}{V_P} x N_P$$

$$N_S = \frac{220}{3300} x6000 = 400$$

32.Explain briefly the energy losses in a transformer and the method to minimise these losses.

(i)Flux Leakage:

- Not all of the flux due to primary passes through the secondary .
- It can be reduced by winding the primary and secondary coils one over the other.

(ii)Resistance of the windings :

- The energy is lost due to heat produced in the wire as I^2R.
- these are minimised by using thick wire.

(iii)Eddy currents loss:

- The alternating magnetic flux induces eddy currents in the iron core and causes heating.
- The effect is reduced by having a laminated core.

(iv)Hysteresis loss:

- The repeated magnetisation and demagnetisation of the core produces hysteresis loss as heat.
- This can be minimised by using soft iron as core which has a low hysteresis loss.