MATHEMATICS

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Introduction, Integral Power of Iota

Basic Level

1. $\sqrt{-2}\sqrt{-3} =$ **[Roorkee 1978]**

(a) $\sqrt{6}$ (b) $-\sqrt{6}$ (c) $i\sqrt{6}$ (d) None of these

2. The value of $(1+i)^5 \times (1-i)^5$ is **[Karnataka CET 1992]**

(a) – 8 (b) 8i (c) 8 (d) 32

3. $(1+i)^4 + (1-i)^4 =$ **[Karnataka CET 2001]**

(a) 8 (b) – 8 (c) 4 (d) – 4

4. The value of $(1+i)^8 + (1-i)^8$ is **[Rajasthan PET 2001]**

(a) 16 (b) – 16 (c) 32 (d) – 32

5. The value of $(1+i)^6 + (1-i)^6$ is **[Rajasthan PET 2002]**

(a) 0 (b) 2^7 (c) 2^6 (d) None of these

6. $(1+i)^{10}$, where $i^2 = -1$, is equal to **[AMU 2001]**

(a) 32 i (b) 64 + i (c) 24 i – 32 (d) None of these

7. If $i = \sqrt{-1}$, then $1 + i^2 + i^3 - i^6 + i^8$ is equal to **[Rajsthan PET 1995]**

(a) 2 – i (b) 1 (c) 3 (d) – 1

8. The value of $\dfrac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1 =$

(a) – 1 (b) – 2 (c) – 3 (d) – 4

9. If $i^2 = -1$, then sum $i + i^2 + i^3 + \ldots\text{to } 1000$ terms is equal to **[Kerala (Engg.) 2002]**

(a) 1 (b) – 1 (c) i (d) 0

10. If $(1-i)^n = 2^n$, then n **[Rajasthan PET 1990]**

(a) 1 (b) 0 (c) – 1 (d) None of these

11. If $\left(\frac{1+i}{1-i}\right)^m = 1,$ then the least integral value of m is **[IIT 1982; MNR 1984; UPSEAT 2001; MP PET 2002]**

(a) 2 (b) 4 (c) 8 (d) None of these

12. The least positive integer n which will reduce $\left(\frac{i-1}{i+1}\right)^n$ to a real number, is **[Roorkee 1998]**

(a) 2 (b) 3 (c) 4 (d) 5

13. $i^2 + i^4 + i^6 + ...$ upto (2n+1) terms = **[EAMCET 1980; DCE 2000]**

(a) i (b) – i (c) 1 (d) – 1

14. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1}),$ where $i = \sqrt{-1},$ equals **[IIT 1998]**

(a) i (b) i – 1 (c) – i (d) 0

15. The value of $i^{1+3+5+...+(2n+1)}$ is **[AMU 1999]**

(a) i if n is even, – i if n is odd (b) 1 if n is even, – 1 if n is odd

(c) 1 if n is odd, i if n is even (d) i if n is even, – 1 if n is odd

16. $i^{57} + \frac{1}{i^{125}},$ when simplified has the value **[Roorkee 1993]**

(a) 0 (b) 2 i (c) – 2 i (d) 2

17. The number $\frac{(1-i)^3}{1-i^3}$ is equal to **[Pb. CET 1991, Karnataka CET 1998]**

(a) i (b) – 1 (c) 1 (d) – 2

18. $(1+i)^6 + (1-i)^3 =$ **[Karnataka CET 1997; Kurukshetra CEE 1995]**

(a) 2 + i (b) 2 – 10 i (c) – 2 + i (d) – 2 – 10 i

19. If $(a+ib)^5 = \alpha + i\beta$ then $(b+ia)^5$ is equal to

(a) $\beta + i\alpha$ (b) $\alpha - i\beta$ (c) $\beta - i\alpha$ (d) $-\alpha - i\beta$

20. For a positive integer n, the expression $(1-i)^n\left(1-\frac{1}{i}\right)^n$ equals **[AMU 1992]**

(a) 0 (b) $2i^n$ (c) 2^n (d) 4^n

21. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = -1$ is **[Roorkee 1992]**

(a) 1 (b) 2 (c) 3 (d) 4

22. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer is **[Kurukshetra CEE 1992]**

(a) 2 (b) 4 (c) 8 (d) 16

Real and imaginary parts of complex numbers, Algebraic operations, Equality of two Complex numbers

Basic Level

23. The statement $(a+ib) < (c+id)$ is true for **[Rajasthan PET 2002]**

(a) $a^2 + b^2 = 0$ (b) $b^2 + c^2 = 0$ (c) $a^2 + c^2 = 0$ (d) $b^2 + d^2 = 0$

24. The true statement is **[Roorkee 1989]**

(a) $1 - i < 1+i$ (b) $2i + 1 > -2i + 1$ (c) $2i > 1$ (d) None of these

25. The complex number $\frac{1+2i}{1-i}$ lies in which quadrant of the complex plane **[MP PET 2001]**

(a) First (b) Second (c) Third (d) Fourth

26. If $|z|=1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then $\text{Re}(\omega)$ is **[IIT Screening 2003; Rajasthan PET 1997]**

(a) 0 (b) $-\frac{1}{|z+1|^2}$ (c) $\left|\frac{z}{z+1}\right|.\frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

27. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary, if θ = **[IIT 1976]**

(a) $2n\pi \pm \frac{\pi}{3}$ (b) $n\pi + \frac{\pi}{3}$ (c) $n\pi \pm \frac{\pi}{3}$ (d) None of these

[Where n is an integer]

28. If $z \neq 0$ is a complex numbers, then

(a) $\text{Re}(z)=0 \Rightarrow \text{Im}(z^2)=0$ (b) $\text{Re}(z^2)=0 \Rightarrow \text{Im}(z^2)=0$ (c) $\text{Re}(z)=0 \Rightarrow \text{Re}(z^2)=0$ (d) None of these

29. If z_1 and z_2 be two complex numbers, then Re $(z_1 z_2)$ =

(a) Re (z_1). Re (z_2) (b) Re (z_1). Im (z_2) (c) Im (z_1). Re (z_2) (d) None of these

30. The real part of $\frac{1}{1-\cos\theta+i\sin\theta}$ is equal to **[Karnataka CET 2001]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\tan\frac{\theta}{2}$ (d) $\frac{1}{1-\cos\theta}$

31. The multiplication inverse of a number is the number itself, then its initial value is **[Rajasthan PET 2003]**

(a) i (b) -1 (c) 2 (d) $-i$

32. If $z = 1+i$, then the multiplicative inverse of z^2 is (where $i = \sqrt{-1}$) **[Karnataka CET 1999]**

(a) $2i$ (b) $1-i$ (c) $-\frac{i}{2}$ (d) $\frac{i}{2}$

33. If $a = \cos\theta + i\sin\theta$, then $\frac{1+a}{1-a}$ =

(a) $\cot\theta$ (b) $\cot\frac{\theta}{2}$ (c) $i\cot\frac{\theta}{2}$ (d) $i\tan\frac{\theta}{2}$

34. If $z = x - iy$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to **[AIEEE 2004]**

(a) -2 (b) -1 (c) 2 (d) 1

35. If $(x+iy)^{1/3} = a+ib$, then $\frac{x}{a}+\frac{y}{b}$ is equal to **[IIT 1982; Karnataka CET 2000]**

(a) $4(a^2+b^2)$ (b) $4(a^2-b^2)$ (c) $4(b^2-a^2)$ (d) None of these

36. If $\sqrt{3}+i=(a+ib)(c+id)$, then $\tan^{-1}\left(\frac{b}{a}\right)+\tan^{-1}\left(\frac{d}{c}\right)$ has the value

(a) $\frac{\pi}{3}+2n\pi,\ n\in I$ (b) $n\pi+\frac{\pi}{6},\ n\in I$ (c) $n\pi-\frac{\pi}{3},\ n\in I$ (d) $2n\pi-\frac{\pi}{3},\ n\in I$

37. Additive inverse of 1 – i is

(a) 0 + 0 i (b) – 1 – i (c) – 1 + i (d) None of these

38. If $a^2 + b^2 = 1,$ then $\frac{1+b+ia}{1+b-ia} =$

(a) 1 (b) 2 (c) $b + ia$ (d) $a + ib$

39. $\left|(1+i)\frac{(2+i)}{(3+i)}\right| =$ **[MP PET 1995, 99]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) – 1

40. $\left\{\frac{2i}{1+i}\right\}^2 =$ **[BIT Ranchi 1992]**

(a) 1 (b) $2i$ (c) $1 - i$ (d) $1 - 2i$

41. If $Z_1 = (4, 5)$ and $Z_2 = (-3, 2)$, then $\frac{Z_1}{Z_2}$ equals **[Rajasthan PET 1996]**

(a) $\left(\frac{-23}{12}, \frac{-2}{13}\right)$ (b) $\left(\frac{2}{13}, \frac{-23}{13}\right)$ (c) $\left(\frac{-2}{13}, \frac{-23}{13}\right)$ (d) $\left(\frac{-2}{13}, \frac{23}{13}\right)$

42. If $x + \frac{1}{x} = 2\cos\theta,$ then x is equal to **[Rajasthan PET 2001]**

(a) $\cos\theta + i\sin\theta$ (b) $\cos\theta - i\sin\theta$ (c) $\cos\theta \pm i\sin\theta$ (d) $\sin\theta \pm i\cos\theta$

43. The number of real values of a satisfying the equation $a^2 - 2a \sin x + 1 = 0$ is

(a) Zero (b) One (c) Two (d) Infinite

44. Solving $3 - 2yi = 9^x - 7i$, where $i^2 = -1,$ for real x and y, we get **[AMU 2000]**

(a) $x = 0.5, y = 3.5$ (b) $x = 5, y = 3$ (c) $x = \frac{1}{2}, y = 7$ (d) $x = 0, y = \frac{3+7i}{2i}$

45. $\frac{1-i}{1+i}$ is equal to **[Rajasthan PET 1984]**

(a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$ (c) $\sin\frac{\pi}{2} + i\cos\frac{\pi}{2}$ (d) None of these

46. The values of x and y satisfying the equation $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i,$ are **[IIT 1980; MNR 1987, 88]**

(a) $x = -1, y = 3$ (b) $x = 3, y = -1$ (c) $x = 0, y = 1$ (d) $x = 1, y = 0$

47. If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta},$ then $x^2 + y^2$ is equal to

(a) $3x - 4$ (b) $4x - 3$ (c) $4x + 3$ (d) None of these

48. If $\frac{5(-8+6i)}{(1+i)^2} = a + ib,$ then (a, b) equals **[Rajasthan PET 1986]**

(a) (15, 20) (b) (20, 15) (c) (–15, 20) (d) None of these

49. If $x = -5 + 2\sqrt{-4},$ then the value of the expression $x^4 + 9x^3 + 35x^2 - x + 4$ is **[IIT 1972]**

(a) 160 (b) – 160 (c) 60 (d) – 60

50. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then (x, y) is **[MP PET 2000]**

(a) (3, 1) (b) (1, 3) (c) (0, 3) (d) (0, 0)

51. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib,$ then **[MP PET 1998]**

(a) $a=2, b=-1$ (b) $a=1, b=0$ (c) $a=0, b=1$ (d) $a=-1, b=2$

52. The real values of x and y for which the equation $(x+iy)(2-3i)=4+i$ is satisfied, are **[Roorkee 1978]**

(a) $x=\frac{5}{13}, y=\frac{8}{13}$ (b) $x=\frac{8}{13}, y=\frac{5}{13}$ (c) $x=\frac{5}{13}, y=\frac{14}{13}$ (d) None of these

53. The solution of the equation $|z|-z=1+2i$ is **[MP PET 1993, Kurukshetra CEE 1999]**

(a) $2-\frac{3}{2}i$ (b) $\frac{3}{2}+2i$ (c) $\frac{3}{2}-2i$ (d) $-2+\frac{3}{2}i$

54. Which of the following is not applicable for a complex number **[Kerala (Engg.) 1993; Assam JEE 1998; DCE 1999]**

(a) Addition (b) Subtraction (c) Division (d) Inequality

55. Multiplicative inverse of the non-zero complex number $x+iy$ $(x, y \in R)$ is

(a) $\frac{x}{x+y}-\frac{y}{x+y}i$ (b) $\frac{x}{x^2+y^2}-\frac{y}{x^2+y^2}i$ (c) $-\frac{x}{x^2+y^2}+\frac{y}{x^2+y^2}i$ (d) $\frac{x}{x+y}+\frac{y}{x+y}i$

56. The real value of α for which the expression $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real, is **[Kurukshetra CEE 1995]**

(a) $(n+1)\frac{\pi}{2}$, where n is an integer (b) $(2n+1)\frac{\pi}{2}$, where n is an integer

(c) $n\pi$, where n is an integer (d) None of these

57. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is **[Pb. CET 2000; IIIT Kolkata 2001]**

(a) $n\pi+\frac{\pi}{4}$ (b) $n\pi+(-1)^n\frac{\pi}{4}$ (c) $2n\pi\pm\frac{\pi}{2}$ (d) None of these

58. If $z(2-i)=3+i,$ then $z^{20}=$ **[Karnataka CET 2002]**

(a) $1-i$ (b) -1024 (c) 1024 (d) $1+i$

59. If $z_1=\sqrt{3}+i\sqrt{3}$ and $z_2=\sqrt{3}+i,$ then the complex number $\left(\frac{z_1}{z_2}\right)$ lies in the quadrant number **[AMU 1991]**

(a) I (b) II (c) III (d) IV

60. If $\text{Re}\left(\frac{z-8i}{z+6}\right)=0,$ then z lies on the curve

(a) $x^2+y^2+6x-8y=0$ (b) $4x-3y+24=0$ (c) $x^2+y^2-8=0$ (d) None of these

Advance Level

61. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1,$ then $\frac{z_1}{z_2}$ is a number which is

(a) Positive real (b) Negative real (c) Zero or purely imaginary (d) None of these

62. If $z(1+a)=b+ic$ and $a^2+b^2+c^2=1,$ then $\frac{1+iz}{1-iz}=$

(a) $\frac{a+ib}{1+c}$ (b) $\frac{b-ic}{1+a}$ (c) $\frac{a+ic}{1+b}$ (d) None of these

63. Given that the equation $z^2+(p+iq)z+r+is=0$, where, p, q, r, s are real and non-zero has a real root, then **[DCE 1992]**

(a) $pqr=r^2+p^2s$ (b) $prs=q^2+r^2p$ (c) $qrs=p^2+s^2q$ (d) $pqs=s^2+q^2r$

64. If $\sum_{k=0}^{100} i^k = x+iy$, then the value of x and y are

(a) $x=-1, y=0$ (b) $x=1, y=1$ (c) $x=1, y=0$ (d) $x=0, y=1$

65. Let $\frac{1-ix}{1+ix}=a-ib$ and $a^2+b^2=1$, where a and b are real, then $x=$

(a) $\frac{2a}{(1+a)^2+b^2}$ (b) $\frac{2b}{(1+a)^2+b^2}$ (c) $\frac{2a}{(1+b)^2+a^2}$ (d) $\frac{2b}{(1+b)^2+a^2}$

66. If $\frac{(p+i)^2}{2p-i}=\mu+i\lambda$, then $\mu^2+\lambda^2$ is equal to

(a) $\frac{(p^2+1)^2}{4p^2-1}$ (b) $\frac{(p^2-1)^2}{4p^2-1}$ (c) $\frac{(p^2-1)^2}{4p^2+1}$ (d) $\frac{(p^2+1)^2}{4p^2+1}$

67. If $(1+i)(1+2i)(1+3i)......(1+ni)=a+ib$, then $2 \,.\, 5 \,.\, 10.........(1+n^2)$ is equal to **[Karnataka CET 2002; Kerala (Engg.) 2002]**

(a) a^2-b^2 (b) a^2+b^2 (c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

68. Given $z=\frac{q+ir}{1+p}$, then $\frac{p+iq}{1+r}=\frac{1+iz}{1-iz}$ if

(a) $p^2+q^2+r^2=1$ (b) $p^2+q^2+r^2=2$ (c) $p^2+q^2-r^2=1$ (d) None of these

Conjugate of a Complex Number

Basic Level

69. Conjugate of $1+i$ is **[Rajasthan PET 2003]**

(a) i (b) 1 (c) $1-i$ (d) $1+i$

70. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is **[MP PET 1994]**

(a) $\frac{7-26i}{25}$ (b) $\frac{-7-26i}{25}$ (c) $\frac{-7+26i}{25}$ (d) $\frac{7+26i}{25}$

71. The conjugate of $\frac{(2+i)^2}{3+i}$, in the form of $a+ib$, is **[Karnataka CET 2001]**

(a) $\frac{13}{2}+i\left(\frac{15}{2}\right)$ (b) $\frac{13}{10}+i\left(\frac{-15}{2}\right)$ (c) $\frac{13}{10}+i\left(\frac{-9}{10}\right)$ (d) $\frac{13}{10}+i\left(\frac{9}{10}\right)$

72. If $x+iy=\sqrt{\frac{a+ib}{c+id}}$, then $(x^2+y^2)^2=$ **[IIT 1979; Rajasthan PET 1997; Karnataka CET 1999; BIT Ranchi 1993]**

(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{a+b}{c+d}$ (c) $\frac{c^2+d^2}{a^2+b^2}$ (d) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^2$

73. If $(a+ib)(c+id)(e+if)(g+ih)=A+iB$, then $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2)$ is equal to **[MNR 1989]**

(a) A^2+B^2 (b) A^2-B^2 (c) A^2 (d) B^2

74. If z is a complex number, then $z \,.\, \bar{z}=0$ if and only if

(a) $z = 0$ (b) Re $(z) = 0$ (c) Im $(z) = 0$ (d) None of these

75. Let z_1, z_2 be two complex numbers such that z_1+z_2 and z_1z_2 both are real, then **[Rajasthan PET 1996]**

(a) $z_1 = -z_2$ (b) $z_1 = \bar{z}_2$ (c) $z_1 = -\bar{z}_2$ (d) $z_1 = z_2$

76. For any complex number z, $\bar{z} = \left(\frac{1}{z}\right)$ if and only if **[Rajasthan PET 1985]**

(a) z is a pure real number (b) $|z| = 1$

(c) z is a pure imaginary number (d) $z = 1$

77. If $\frac{c+i}{c-i} = a + ib$, where a, b, c are real, then $a^2 + b^2 =$ **[MP PET 1996]**

(a) 1 (b) – 1 (c) c^2 (d) $-c^2$

78. If $z = 3 + 5i$, then $z^3 + \bar{z} + 198 =$ **[EAMCET 2002]**

(a) $-3-5i$ (b) $-3+5i$ (c) $3+5i$ (d) $3-5i$

79. If a complex number lies in the IIIrd quadrant then its conjugate lies in quadrant number **[AMU 1986, 89]**

(a) I (b) II (c) III (d) IV

80. If $z = x + iy$ lies in IIIrd quadrant then $\frac{\bar{z}}{z}$ also lies in the IIIrd quadrant if **[AMU 1990; Kurukshetra CEE 1993]**

(a) $x > y > 0$ (b) $x < y < 0$ (c) $y < x < 0$ (d) $y > x > 0$

81. If $(1+i)z = (1-i)\bar{z}$ then z is

(a) $t(1-i), t \in R$ (b) $t(1+i), t \in R$ (c) $\frac{t}{1+i}, t \in R$ (d) None of these

82. The value of $(z+3)(\bar{z}+3)$ is equivalent to **[JMIEE 2000]**

(a) $|z+3|^2$ (b) $|z-3|$ (c) $z^2 + 3$ (d) None of these

83. The set of values of $a \in R$ for which $x^2 + i(a-1)x + 5 = 0$ will have a pair of conjugate complex roots is

(a) R (b) $\{1\}$ (c) $\{a | a^2 - 2a + 21 > 0\}$ (d) None of these

Advance Level

84. The equation $z^2 = \bar{z}$ has **[DCE 1995]**

(a) No solution (b) Two solutions

(c) Four solutions (d) An infinite number of solutions

85. If $z_1 = 9y^2 - 4 - 10ix, z_2 = 8y^2 - 20i$, where $z_1 = \bar{z}_2$, then $z = x + iy$ is equal to

(a) $-2+2i$ (b) $-2 \pm 2i$ (c) $-2 \pm i$ (d) None of these

86. If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root then

(a) $\alpha + \bar{\alpha} = 1$ (b) $\alpha + \bar{\alpha} = 0$

(c) $\alpha + \bar{\alpha} = -1$ (d) The absolute value of the real root is 1

Modulus of Complex Numbers

Basic Level

87. The value of $|z-5|$, if $z = x + iy$ is **[Rajasthan PET 1995]**

(a) $\sqrt{(x-5)^2+y^2}$ (b) $x^2+\sqrt{(y-5)^2}$ (c) $\sqrt{(x-y)^2+5^2}$ (d) $\sqrt{x^2+(y-5)^2}$

88. Modulus of $\left(\frac{3+2i}{3-2i}\right)$ is **[Rajasthan PET 1996]**

(a) 1 (b) 1/2 (c) 2 (d) $\sqrt{2}$

89. The product of two complex numbers each of unit modulus is also a complex number, of

(a) Unit modulus (b) Less than unit modulus (c) Greater than unit modulus (d) None of these

90. The moduli of two complex numbers are less than unity, then the modulus of the sum of these complex numbers

(a) Less than unity (b) Greater than unity (c) Equal to unity (d) Any

91. If z is a complex number, then which of the following is not true **[MP PET 1987]**

(a) $|z^2|=|z|^2$ (b) $|z^2|=|\bar{z}|^2$ (c) $z=\bar{z}$ (d) $\bar{z}^2=\overline{z^2}$

92. The values of z for which $|z+i| = |z-i|$ are **[Bihar CEE 1994]**

(a) Any real number (b) Any complex number (c) Any natural number (d) None of these

93. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then **[MP PET 1998, 2002]**

(a) $|z|=0$ (b) $|z|=1$ (c) $|z|>1$ (d) $|z|<1$

94. The minumum value of $|2z-1|+|3z-2|$ is **[Rajasthan PET 1997]**

(a) 0 (b) 1/2 (c) 1/3 (d) 2/3

95. If z_1 and z_2 are any two complex numbers then $|z_1+z_2|^2+|z_1-z_2|^2$ is equal to **[MP PET 1993]**

(a) $2|z_1|^2|z_2|^2$ (b) $2|z_1|^2+2|z_2|^2$ (c) $|z_1|^2+|z_2|^2$ (d) $2|z_1||z_2|$

96. If $\frac{2z_1}{3z_2}$ is a purely imaginary number, then $\left|\frac{z_1-z_2}{z_1+z_2}\right|$ is equal to **[MP PET 1993]**

(a) 3/2 (b) 1 (c) 2/3 (d) 4/9

97. If z_1, z_2, z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then $|z_1+z_2+z_3|$ is **[IIT Screening 2000]**

(a) Equal to 1 (b) Less than 1 (c) Greater than 3 (d) Equal to 3

98. If z_1 and z_2 are any two complex numbers, then $|z_1+\sqrt{z_1^2-z_2^2}|+|z_1-\sqrt{z_1^2-z_2^2}|$ is equal to

(a) $|z_1|$ (b) $|z_2|$ (c) $|z_1+z_2|$ (d) $|z_1+z_2|+|z_1-z_2|$

99. Find the complex number z satisfying the equations $\left|\frac{z-12}{z-8i}\right|=\frac{5}{3}, \left|\frac{z-4}{z-8}\right|=1$ **[Roorkee 1993]**

(a) 6 (b) $6\pm 8i$ (c) $6+8i,\ 6+17i$ (d) None of these

100. A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix}\right)=\alpha-i\beta\ (\alpha,\beta \text{ real})$, if **[Orissa JEE 2003]**

(a) $\alpha^2-\beta^2=-1$ (b) $\alpha^2-\beta^2=1$ (c) $\alpha^2+\beta^2=1$ (d) $\alpha^2-\beta^2=2$

101. The inequality $|z-4|<|z-2|$ represents the region given by **[IIT 1982; Rajasthan PET 1995, 98; AIEEE 2002; DCE 2002]**

(a) Re $(z)>0$ (b) Re $(z)<0$ (c) Re $(z)>2$ (d) None of these

74 Complex Numbers

102. If $z = 1 + i\tan\alpha$, where $\pi < \alpha < \frac{3\pi}{2}$, then | z | is equal to

(a) $\sec\alpha$ (b) $-\sec\alpha$ (c) $\text{cosec}\,\alpha$ (d) None of these

103. If z is a non-zero complex number then $\left|\frac{|\bar{z}|^2}{z\bar{z}}\right|$ is equal to

(a) $\left|\frac{\bar{z}}{z}\right|$ (b) 1 (c) $|\bar{z}|$ (d) None of these

104. If z is a complex number, then **[DCE 1997, Kurukshetra CEE 1995]**

(a) $|z^2| > |z|^2$ (b) $|z^2| = |z|^2$ (c) $|z^2| < |z|^2$ (d) $|z^2| \ge |z|^2$

105. If $z_1 \ne -z_2$ and $|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$ then

(a) At least one of z_1, z_2 is unimodular (b) Both z_1, z_2 are unimodular

(c) $z_1 . z_2$ is unimodular (d) None of these

106. Let z be a complex number of constant modulus such that z^2 is purely imaginary then the number of possible values of z is

(a) 2 (b) 1 (c) 4 (d) Infinite

107. Number of solutions of the equation $z^2 + |z|^2 = 0$ where $z \in C$ is **[Karnataka CET 1997; Pb. CET 2001]**

(a) 1 (b) 2 (c) 3 (d) Infinitely many

108. If | z | = Max. { | z – 2 |, | z + 2 | }, then

(a) $|z + \bar{z}| = 1$ (b) $z + \bar{z} = 2^2$ (c) $|z + \bar{z}| = 2$ (d) None of these

109. The modulus of $\sqrt{2i} - \sqrt{-2i}$ is **[EAMCET 1995]**

(a) 2 (b) $\sqrt{2}$ (c) 0 (d) $2\sqrt{2}$

Advance Level

110. If z is a complex number, then the minimum value of | z | + | z – 1 | is **[Roorkee 1992]**

(a) 1 (b) 0 (c) 1/2 (d) None of these

111. The maximum value of | z | where z satisfies the condition $\left|z + \frac{2}{z}\right| = 2$ is

(a) $\sqrt{3} - 1$ (b) $\sqrt{3} + 1$ (c) $\sqrt{3}$ (d) $\sqrt{2} + \sqrt{3}$

112. If $|z + 4| \le 3$, then the greatest and the least value of | z + 1 | are **[Rajasthan PET 2002; Karnataka CET 1995; DCE 1999]**

(a) 6, – 6 (b) 6, 0 (c) 7, 2 (d) 0, – 1

113. Let z be a complex number, then the equation $z^4 + z + 2 = 0$ cannot have a root, such that

(a) | z | < 1 (b) | z | = 1 (c) | z | > 1 (d) None of these

114. Let z and w be two complex numbers such that $|z| \le 1$, $|w| \le 1$ and $|z + iw| = |z - i\bar{w}| = 2$. Then z is equal to **[IIT 1995]**

(a) 1 or i (b) i or – i (c) 1 or – 1 (d) i or – 1

115. If | z_1 | = | z_2 | = ... = | z_n | = 1, then the value of | $z_1 + z_2 + z_3 +$ .... $+ z_n$ | =

(a) 1 (b) $|z_1| + |z_2| + ... + |z_n|$ (c) $\left|\frac{1}{z_1}+\frac{1}{z_2}+.....+\frac{1}{z_n}\right|$ (d) None of these

116. If z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{(z_1+z_2)}{(z_1-z_2)}$ may be **[IIT 1986]**

(a) Purely imaginary (b) Real and positive (c) Real and negative (d) None of these

117. For any two complex numbers z_1 and z_2 and any real numbers a and b; $|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2 =$ **[IIT 1988]**

(a) $(a^2+b^2)(|z_1|+|z_2|)$ (b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$ (c) $(a^2+b^2)(|z_1|^2-|z_2|^2)$ (d) None of these

118. If $|a_k| < 1, \lambda_k \geq 0$ for k = 1, 2, ... n and $\lambda_1 + \lambda_2 + ... + \lambda_n = 1$, then the value of $|\lambda_1 a_1 + \lambda_2 a_2 + ... + \lambda_n a_n|$ is

(a) Equal to one (b) Greater than one (c) Zero (d) Less than one

119. If z_1, z_2, z_3, z_4 are roots of the equation $a_0 z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 0$,

where a_0, a_1, a_2, a_3 and a_4 are real, then

(a) $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$ are also roots of the equation (b) z_1 is equal to at least one of $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$

(c) $-\bar{z}_1, -\bar{z}_2, -\bar{z}_3, -\bar{z}_4$ are also roots of the equation (d) None of these

120. If z satisfies $|z + 1| < |z - 2|$, then $w = 3z + 2 + i$ **[MP PET 1998]**

(a) $|w + 1| < |w - 8|$ (b) $|w + 1| < |w - 7|$ (c) $w + \bar{w} > 7$ (d) $|w + 5| < |w - 4|$

121. $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$

(a) Is less than 6 (b) Is more than 3 (c) Is less than 12 (d) Lies between 6 and 12

122. If $|z - 4 + 3i| \leq 1$ and m and n be the least and greatest values of $|z|$ and K be the least value of $\frac{x^4+x^2+4}{x}$ on the interval $(0, \infty)$, then K =

(a) n (b) m (c) $m + n$ (d) None of these

123. The system of equations $|z + 1 - i| = \sqrt{2}$ and $|z| = 3$ has

(a) No solution (b) One solution (c) Two solutions (d) None of these

Amplitude (Argument) of Complex Numbers

Basic Level

124. The amplitude of 0 is **[Rajasthan PET 2000]**

(a) 0 (b) $\pi/2$ (c) π (d) None of these

125. The argument of the complex number $-1 + i\sqrt{3}$ is **[MP PET 1994]**

(a) – 60° (b) 60° (c) 120° (d) – 120°

126. Argument of $-1 - i\sqrt{3}$ is **[Rajasthan PET 2003]**

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $-\frac{2\pi}{3}$

127. The amplitude of $\frac{1+\sqrt{3}\,i}{\sqrt{3}+i}$ is **[DCE 1999]**

(a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) None of these

128. The amplitude of $\frac{1+\sqrt{3}\,i}{\sqrt{3}+1}$ is **[Karnataka CET 1992]**

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

129. The argument of the complex number $\frac{13-5i}{4-9i}$ is **[MP PET 1997]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{6}$

130. If $z=\frac{-2}{1+\sqrt{3}\,i}$ then the value of $arg(z)$ is **[Orissa JEE 2002]**

(a) π (b) $\pi/3$ (c) $2\pi/3$ (d) $\pi/4$

131. If $z=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$, then *arg* (*z*) = **[Roorkee 1990]**

(a) 60° (b) 120° (c) 240° (d) 300°

132. The amplitude of $\frac{1+\sqrt{3}\,i}{\sqrt{3}-i}$ is **[Rajasthan PET 2001]**

(a) 0 (b) $\pi/6$ (c) $\pi/3$ (d) $\pi/2$

133. If $z=1-\cos\alpha+i\sin\alpha$, then *amp z* =

(a) $\frac{\alpha}{2}$ (b) $-\frac{\alpha}{2}$ (c) $\frac{\pi}{2}+\frac{\alpha}{2}$ (d) $\frac{\pi}{2}-\frac{\alpha}{2}$

134. If $z=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$, then **[AMU 2002]**

(a) $|z|=1,\ arg z=\frac{\pi}{4}$ (b) $|z|=1,\ arg z=\frac{\pi}{6}$ (c) $|z|=\frac{\sqrt{3}}{2}, arg z=\frac{5\pi}{24}$ (d) $|z|=\frac{\sqrt{3}}{2}, arg z=\tan^{-1}\frac{1}{\sqrt{2}}$

135. Argument and modulus of $\frac{1+i}{1-i}$ are respectively **[Rajasthan PET 1984; MP PET 1987; Karnataka CET 2001]**

(a) $-\frac{\pi}{2}$ and 1 (b) $\frac{\pi}{2}$ and $\sqrt{2}$ (c) 0 and $\sqrt{2}$ (d) $\frac{\pi}{2}$ and 1

136. If *arg* (*z*) = θ, then *arg* $(\bar{z})$ = **[MP PET 1995]**

(a) θ (b) $-\theta$ (c) $\pi-\theta$ (d) $\theta-\pi$

137. If $arg z<0$ then $arg(-z)-arg(z)$ is equal to **[IIT Screening 2000]**

(a) π (b) $-\pi$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

138. Let *z* and *w* be the two non-zero complex numbers such that | *z* | = |*w*| and *arg z* + *arg w* = π. Then *z* is equal to

[IIT 1995; AIEEE 2002]

(a) *w* (b) – *w* (c) $\overline{w}$ (d) – $\overline{w}$

139. If *z* is a complex number, then the principal value of *arg* (*z*) lies between

(a) $-\frac{\pi}{4}$ and $\frac{\pi}{4}$ (b) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (c) $-\pi$ and π (d) None of these

140. The principal value of the argument of the complex number – 3*i* is

(a) 0 (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{2}$ (d) None of these

141. If $|z_1+z_2|=|z_1-z_2|$, then the difference in the amplitudes of z_1 and z_2 is **[EAMCET 1985]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 0

142. If z_1 and z_2 are two non-zero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then *arg* (z_1) – *arg* (z_2) is equal to

[IIT 1979, 87; EAMCET 1986; Rajasthan PET 1997; MP PET 1999, 2001]

(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) 0

143. If $z_1, z_2 .... z_n = z$, then $arg z_1 + arg z_2 + ... + arg z_n$ and *arg z* differ by a

(a) Multiple of π (b) Multiple of $\frac{\pi}{2}$ (c) Greater than π (d) Less than π

144. If z is a purely real number such that Re (z) < 0, then *arg* (z) is equal to

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $-\frac{\pi}{2}$

145. Let z be a purely imaginary number such that Im (z) < 0. then *arg* (z) is equal to

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $-\frac{\pi}{2}$

146. If $\bar{z}$ be the conjugate of the complex number z, then which of the following relations is false **[MP PET 1987]**

(a) $|z|=|\bar{z}|$ (b) $z.\bar{z}=|\bar{z}|^2$ (c) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ (d) $arg\ z = arg\ \bar{z}$

147. Let z_1 and z_2 be two complex numbers with α and β as their principal arguments such that $\alpha+\beta>\pi,$ then principal *arg* ($z_1 z_2$) is given by **[Roorkee 1989]**

(a) $\alpha+\beta+\pi$ (b) $\alpha+\beta-\pi$ (c) $\alpha+\beta-2\pi$ (d) $\alpha+\beta$

148. If z = –1, then the principal value of the *arg* $\left(z^{2/3}\right)$ is equal to **[IIT 1991, Kurukshetera CEE 1998]**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{10\pi}{3}$ (d) π

149. If z is any complex number satisfying | z – 1 | = 1, then which of the following is correct **[EAMCET 1999]**

(a) *arg* (z – 1) = 2 *arg z* (b) $2arg(z)=\frac{2}{3}arg(z^2-z)$ (c) $arg(z-1)=arg(z+1)$ (d) $arg z=2 arg(z+1)$

150. If $z = x + iy$ satisfies *amp* (z –1) = *amp* (z + 3i) then the value of (x – 1) : y is equal to

(a) 2 : 1 (b) 1 : 3 (c) – 1 : 3 (d) None of these

151. If $z(2-i2\sqrt{3})^2=i(\sqrt{3}+i)^4$ then amplitude of z is

(a) $\frac{5\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{7\pi}{6}$

Advance Level

152. If complex number $z=x+iy$ is taken such that the amplitude of fraction $\frac{z-1}{z+1}$ is always $\frac{\pi}{4}$, then

[UPSEAT 1999]

(a) $x^2+y^2+2y=1$ (b) $x^2+y^2-2y=0$ (c) $x^2+y^2+2y=-1$ (d) $x^2+y^2-2y=1$

153. If $z_1=10+6i$, $z_2=4+6i$ and z is a complex number such that *amp* $\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$, then the value of | z – 7 – 9i | is equal to

[IIT 1990]

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

154. If $z_1 = 8 + 4i$, $z_2 = 6 + 4i$ and $arg\left(\frac{z-z_1}{z-z_2}\right) = \frac{\pi}{4}$, then z satisfies **[IIT 1993]**

(a) $|z - 7 - 4i| = 1$ (b) $|z - 7 - 5i| = \sqrt{2}$ (c) $|z - 4i| = 8$ (d) $|z - 7i| = \sqrt{18}$

155. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then $arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right)$ equals

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

156. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $R(z_1\bar{z}_2) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies **[IIT 1985; UPSEAT 1996]**

(a) $|w_1| = 1$ (b) $|w_2| = 1$ (c) $R(w_1\bar{w}_2) = 0$ (d) All the above

157. If z_1, z_2, z_3 be three non-zero complex numbers, such that $z_2 \neq z_1$, $a = |z_1|$, $b = |z_2|$ and $c = |z_3|$.

Suppose that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then $arg\left(\frac{z_3}{z_2}\right)$ is equal to

(a) $arg\left(\frac{z_2-z_1}{z_3-z_1}\right)^2$ (b) $arg\left(\frac{z_2-z_1}{z_3-z_1}\right)$ (c) $arg\left(\frac{z_3-z_1}{z_2-z_1}\right)^2$ (d) $arg\left(\frac{z_3-z_1}{z_2-z_1}\right)$

158. If $amp\ \frac{z-2}{2z+3i} = 0$ and $z_0 = 3 + 4i$ then

(a) $z_0\bar{z} + \bar{z}_0 z = 12$ (b) $z_0 z + \bar{z}_0\bar{z} = 12$ (c) $z_0\bar{z} + \bar{z}_0 z = 0$ (d) None of these

159. The principal value of the $arg\,(z)$ and $|z|$ of the complex number $z = 1 + \cos\left(\frac{11\pi}{9}\right) + i\sin\frac{11\pi}{9}$ are respectively

(a) $\frac{11\pi}{8}, 2\cos\left(\frac{\pi}{18}\right)$ (b) $-\frac{7\pi}{18}, -2\cos\left(\frac{11\pi}{18}\right)$ (c) $\frac{2\pi}{9}, 2\cos\left(\frac{7\pi}{18}\right)$ (d) $-\frac{\pi}{9}, -2\cos\left(\frac{\pi}{18}\right)$

160. If $amp\,(z_1 z_2) = 0$ and $|z_1| = |z_2| = 1$ then

(a) $z_1 + z_2 = 0$ (b) $z_1 z_2 = 1$ (c) $z_1 = \bar{z}_2$ (d) None of these

161. If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then

(a) $\frac{z_1}{z_2}$ is purely real (b) $\frac{z_1}{z_2}$ is purely imaginary (c) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$ (d) $amp\ \frac{z_1}{z_2} = \frac{\pi}{2}$

162. Let $z_1 = \frac{(\sqrt{3}+i)^2.(1-\sqrt{3}\,i)}{1+i}, z_2 = \frac{(1+\sqrt{3}i)^2.(\sqrt{3}-i)}{1-i}$. Then

(a) $|z_1| = |z_2|$ (b) $amp\ z_1 + amp\ z_2 = 0$ (c) $3|z_1| = |z_2|$ (d) $3\ amp\ z_1 + amp\ z_2 = 0$

163. If z_1 and z_2 both satisfy $z + \bar{z} = 2|z - 1|$ and $arg\,(z_1 - z_2) = \frac{\pi}{4}$, then the imaginary part of $(z_1 + z_2)$ is

(a) 0 (b) 1 (c) 2 (d) None of these

164. If $z = \frac{(z_1 + \bar{z}_2)z_1}{z_2\bar{z}_1}$, where $z_1 = 1 + 2i$ and $z_2 = 1 - i$, then

(a) $|z| = \frac{1}{2}\sqrt{26}$, $arg\,z = -\pi + \tan^{-1}\frac{19}{17}$
(b) $|z| = \frac{1}{2}\sqrt{26}$, $arg\,z = \tan^{-1}\frac{19}{17}$
(c) $|z| = \frac{1}{2}\sqrt{15}$, $arg\,z = \tan^{-1}\frac{19}{17}$
(d) $arg\,z = -\pi + \tan^{-1}\frac{19}{17}$; $|z| = \frac{1}{3}\sqrt{26}$

165. If $(a_1 + ib_1)(a_2 + ib_2)....(a_n + ib_n) = A + iB$, then $\sum_{i=1}^{n} \tan^{-1}\left(\frac{b_i}{a_i}\right)$ is equal to

(a) $\frac{B}{A}$
(b) $\tan\left(\frac{B}{A}\right)$
(c) $\tan^{-1}\left(\frac{B}{A}\right)$
(d) $\tan^{-1}\left(\frac{A}{B}\right)$

Square Root of Complex Numbers

Basic Level

166. A square root of $2i$ is

(a) $\sqrt{2}\,i$
(b) $\sqrt{2}(1+i)$
(c) $1 + i$
(d) None of these

167. If $\sqrt{-8-6i} =$ **[Roorkee 1979; Rajasthan PET 1992]**

(a) $1 \pm 3i$
(b) $\pm(1-3i)$
(c) $\pm(1+3i)$
(d) $\pm(3-i)$

168. If $\sqrt{a+ib} = x + iy$, then possible value of $\sqrt{a-ib}$ is **[Kerala (Engg.) 2002]**

(a) $x^2 + y^2$
(b) $\sqrt{x^2 + y^2}$
(c) $x + iy$
(d) $x - iy$

169. If $(-7-24i)^{1/2} = x - iy$, then $x^2 + y^2 =$ **[Rajasthan PET 1989]**

(a) 15
(b) 25
(c) – 25
(d) None of these

170. If $\sqrt{x+iy} = \pm(a+ib)$, then $\sqrt{-x-iy}$ is equal to

(a) $\pm(b+ia)$
(b) $\pm(a-ib)$
(c) $\pm(b-ia)$
(d) None of these

171. A value of $\sqrt{i} + \sqrt{-i}$ is **[AMU 1985]**

(a) 0
(b) $\sqrt{2}$
(c) $-i$
(d) i

172. Given that the real parts of $\sqrt{5+12i}$ and $\sqrt{5-12i}$ are negative. Then the number $z = \frac{\sqrt{5+12i} + \sqrt{5-12i}}{\sqrt{5+12i} - \sqrt{5-12i}}$ reduces to **[Roorkee 1989]**

(a) $\frac{3}{2}i$
(b) $-\frac{3}{2}i$
(c) $-3 + \frac{2}{5}i$
(d) None of these

Representation of Complex Numbers

Basic Level

173. If $x + \frac{1}{x} = \sqrt{3}$, then $x =$ **[Rajasthan PET 2002]**

(a) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$ (b) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (c) $\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}$ (d) $\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$

174. $\sqrt{3}+i=$ **[MP PET 1999]**

(a) $\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$ (b) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ (c) $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ (d) None of these

175. If $(1+i\sqrt{3})^9=a+ib,$ then b is equal to **[Rajasthan PET 1995]**

(a) 1 (b) 256 (c) 0 (d) 9^3

176. If $x=\cos\theta+i\sin\theta$ and $y=\cos\phi+i\sin\phi$, then $x^m y^n+x^{-m}y^{-n}$ is equal to

(a) $\cos(m\theta+n\phi)$ (b) $\cos(m\theta-n\phi)$ (c) $2\cos(m\theta+n\phi)$ (d) $2\cos(m\theta-n\phi)$

177. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then **[MP PET 1997]**

(a) Re (z) = 0 (b) Im (z) = 0 (c) Re (z) > 0, Im (z) > 0 (d) Re (z) > 0, Im (z) < 0

178. If $z=re^{i\theta}$, then $|e^{iz}|=$

(a) $e^{r\sin\theta}$ (b) $e^{-r\sin\theta}$ (c) $e^{-r\cos\theta}$ (d) $e^{r\cos\theta}$

179. $\left(\frac{1+\cos\phi+i\sin\phi}{1+\cos\phi-i\sin\phi}\right)^n=$

(a) $\cos n\phi-i\sin n\phi$ (b) $\cos n\phi+i\sin n\phi$ (c) $\sin n\phi+i\cos n\phi$ (d) $\sin n\phi-i\cos n\phi$

180. If n is a positive integer, then $(1+i)^n+(1-i)^n$ is equal to **[Orissa JEE 2003]**

(a) $(\sqrt{2})^{n-2}\cos\left(\frac{n\pi}{4}\right)$ (b) $(\sqrt{2})^{n-2}\sin\left(\frac{n\pi}{4}\right)$ (c) $(\sqrt{2})^{n+2}\cos\left(\frac{n\pi}{4}\right)$ (d) $(\sqrt{2})^{n+2}\sin\left(\frac{n\pi}{4}\right)$

181. If $y=\cos\theta+i\sin\theta$, then the value of $y+\frac{1}{y}$ is **[Rajasthan PET 1995]**

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) $2\,\mathrm{cosec}\,\theta$ (d) $2\tan\theta$

182. The polar form of the complex number $(i^{25})^3$ is **[Tamilnadu Engg. 2002]**

(a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (b) $\cos\pi+i\sin\pi$ (c) $\cos\pi-i\sin\pi$ (d) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$

Advance Level

183. The amplitude of $e^{e^{-i\theta}}$ is equal to **[Rajasthan PET 1997]**

(a) $\sin\theta$ (b) $-\sin\theta$ (c) $e^{\cos\theta}$ (d) $e^{\sin\theta}$

184. The real part of $\sin^{-1}(e^{i\theta})$ is

(a) $\cos^{-1}(\sqrt{\sin\theta})$ (b) $\sinh^{-1}(\sqrt{\sin\theta})$ (c) $\sin^{-1}(\sqrt{\sin\theta})$ (d) $\sin^{-1}(\sqrt{\cos\theta})$

Logarithm of Complex Number

Basic Level

185. The real part of $(1-i)^{-i}$ is **[Rajasthan PET 1999]**

(a) $e^{-\pi/4}\cos\left(\frac{1}{2}\log 2\right)$ (b) $-e^{-\pi/4}\sin\left(\frac{1}{2}\log 2\right)$ (c) $e^{\pi/4}\cos\left(\frac{1}{2}\log 2\right)$ (d) $e^{-\pi/4}\sin\left(\frac{1}{2}\log 2\right)$

186. If $z=i\log(2-\sqrt{3})$, then $\cos z=$ **[Rajasthan PET 2001; Karnataka CET 2002; EAMCET 1991]**

(a) i (b) $2i$ (c) 1 (d) 2

187. The imaginary part of $\tan^{-1}\left(\frac{5i}{3}\right)$ is **[Rajasthan PET 1997]**

(a) 0 (b) ∞ (c) $\log 2$ (d) $\log 4$

188. The expression $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right]$ reduces to

(a) $\frac{ab}{a^2+b^2}$ (b) $\frac{2ab}{a^2-b^2}$ (c) $\frac{ab}{a^2-b^2}$ (d) $\frac{2ab}{a^2+b^2}$

189. If $\log_{\tan 30^o}\left(\frac{2|z|^2+2|z|-3}{|z|+1}\right)<-2$, then

(a) $|z|<3/2$ (b) $|z|>3/2$ (c) $|z|<2$ (d) $|z|>2$

190. If $\sin(\log i^i)=a+ib$, then a and b are respectively

(a) – 1, 0 (b) 0, –1 (c) 1, 0 (d) 0, 1

191. The general value of $\log_2(5i)$ is

(a) $\left\{\log 5+2\pi mi+\frac{i\pi}{2}\right\}$ (b) $\frac{1}{\log 2}\left\{\log 5+2\pi ni+\frac{i\pi}{2}\right\}$ (c) $-\frac{1}{\log 2}\left\{\log 5+2\pi ni-\frac{i\pi}{2}\right\}$ (d) None of these

Geometry of Complex Numbers, Rotation Theorem

Basic Level

192. $R(z^2)=1$ is represented by

(a) The parabola $x^2+y^2=1$ (b) The hyperbola $x^2-y^2=1$

(c) Parabola or a circle (d) All the above

193. If $z=x+iy$ and $w=\frac{1-iz}{z-i}$, then $|w|=1$ implies that **[Rajasthan PET 1985, 97; IIT 1983; DCE 2000, 01; UPSEAT 2003]**

(a) z lies on the imaginary axis (b) z lies on the real axis

(c) z lies on the unit circle (d) None of these

194. If $|z|=2$, then the points representing the complex numbers $-1+5z$ will lie on a

(a) Circle (b) Straight line (c) Parabola (d) None of these

195. The equation $\bar{b}z+b\bar{z}=c$, where b is a non-zero complex constant and c is real, represents

(a) A circle (b) A straight line (c) A parabola (d) None of these

196. If $z=x+iy$ and $|z-zi|=1$, then **[Rajasthan PET 1988, 1991]**

(a) z lies on x – axis (b) z lies on y – axis (c) z lies on circle (d) None of these

197. If three complex numbers are in A.P., then they lie on **[IIT 1985; DCE 1994, 2001]**

(a) A circle in the complex plane (b) A straight line in the complex plane

(c) A parabola in the complex plane (d) None of these

198. Length of the line segment joining the points – 1 – i and 2 + 3i is

(a) – 5 (b) 15 (c) 5 (d) 25

199. The equation $z\bar{z}+a\bar{z}+\bar{a}z+b=0,\ b\in R$ represents a circle if

(a) $|a|^2=b$ (b) $|a|^2>b$ (c) $|a|^2<b$ (d) None of these

200. If the complex numbers z_1,z_2,z_3 represent the vertices of an equilateral triangle such that $|z_1|=|z_2|=|z_3|$, then $z_1+z_2+z_3=$

[IIT 1984]

(a) 0 (b) 1 (c) – 1 (d) None of these

201. If z_1,z_2,z_3 are affixes of the vertices *A, B* and *C* respectively of a triangle *ABC* having centroid at *G* such that *z* = 0 is the mid point of *AG,* then

(a) $z_1+z_2+z_3=0$ (b) $z_1+4z_2+z_3=0$ (c) $z_1+z_2+4z_3=0$ (d) $4z_1+z_2+z_3=0$

202. For all complex numbers z_1,z_2 satisfying $|z_1|=12$ *and* $|z_2-3-4i|=5,$ the minimum value of $|z_1-z_2|$ is**[IIT Screening 200**

(a) 0 (b) 2 (c) 7 (d) 17

203. If the points z_1,z_2,z_3 are the vertices of an equilateral triangle in the complex plane, then the value of $z_1^2+z_2^2+z_3^2$ is equal to

(a) $\frac{z_1}{z_2}+\frac{z_2}{z_3}+\frac{z_3}{z_1}$ (b) $z_1z_2+z_2z_3+z_3z_1$ (c) $z_1z_2-z_2z_3-z_3z_1$ (d) $\frac{-z_1}{z_2}-\frac{z_2}{z_3}-\frac{z_3}{z_1}$

204. Let *z* be a complex number. Then the angle between vectors *z* and – *iz* is

(a) π (b) 0 (c) $-\frac{\pi}{2}$ (d) None of these

205. If $e^{i\theta}=\cos\theta+i\sin\theta$ then for the $\Delta ABC, e^{iA}.e^{iB}.e^{iC}$ is

(a) – i (b) 1 (c) – 1 (d) None of these

206. If $\text{Re}\left(\frac{z+4}{2z-i}\right)=\frac{1}{2},$ then *z* is represented by a point lying on

(a) A circle (b) An ellipse (c) A straight line (d) None of these

207. Let z_1 and z_2 be two complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1.$ then **[Kurukshetra CEE 1997]**

(a) z_1,z_2 are collinear (b) z_1,z_2 and the origin form a right angled triangle

(c) z_1,z_2 and the origin form an equilateral triangle (d) None of these

208. The equation not representing a circle is given by **[IIT 1991; DCE 1993]**

(a) $\text{Re}\left(\frac{1+z}{1-z}\right)=0$ (b) $z\bar{z}+iz-i\bar{z}+1=0$ (c) $arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$ (d) $\left|\frac{z-1}{z+1}\right|=1$

209. Let z_1 and z_2 be two non-real complex cube roots of unity and $|z-z_1|^2+|z-z_2|^2=\lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is

(a) 4 (b) 3 (c) 2 (d) $\sqrt{2}$

210. Let α and β be two fixed non-zero complex numbers and 'z' a variable complex number. If the lines $\alpha\bar{z}+\bar{\alpha}z+1=0$ and $\beta\bar{z}+\bar{\beta}z-1=0$ are mutually perpendicular, then

(a) $\alpha\beta+\bar{\alpha}\bar{\beta}=0$ (b) $\alpha\beta-\bar{\alpha}\bar{\beta}=0$ (c) $\bar{\alpha}\beta-\alpha\bar{\beta}=0$ (d) $\alpha\bar{\beta}+\bar{\alpha}\beta=0$

211. If P, P' represent the complex number z_1 and its additive inverse respectively, then the complex equation of the circle with PP' as a diameter is

(a) $\frac{z}{z_1}=\left(\frac{\bar{z}_1}{z}\right)$ (b) $z\bar{z}+z_1\bar{z}_1=0$ (c) $z\bar{z}_1+\bar{z}z_1=0$ (d) None of these

212. The triangle formed by the points 1, $\frac{1+i}{\sqrt{2}}$ and i as vertices in the Argand diagram is **[EAMCET 1995]**

(a) Scalene (b) Equilateral (c) Isosceles (d) Right-angled

213. If P, Q, R, S are represented by the complex numbers $4+i$, $1+6i$, $-4+3i$, $-1-2i$ respectively, then $PQRS$ is a **[Orissa JEE 2003]**

(a) Rectangle (b) Square (c) Rhombus (d) Parallelogram

214. Let A, B and C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number

(a) $z_1+z_2-z_3$ (b) $z_2+z_3-z_1$ (c) $z_3+z_1-z_2$ (d) $z_1+z_2+z_3$

215. Multiplying a complex numbers by i rotates the vector representing the complex number through an angle of

(a) 180° (b) 90° (c) 60° (d) 360°

Advance Level

216. Let z be a complex number satisfying $|z-5i|\le 1$ such that *amp z* is minimum. Then z is equal to

(a) $\frac{2\sqrt{6}}{5}+\frac{24i}{5}$ (b) $\frac{24}{5}+\frac{2\sqrt{6}i}{5}$ (c) $\frac{2\sqrt{6}}{5}-\frac{24i}{5}$ (d) None of these

217. If ω is a complex number satisfying $\left|\omega+\frac{1}{\omega}\right|=2,$ then maximum distance of ω from origin is

(a) $2+\sqrt{3}$ (b) $1+\sqrt{2}$ (c) $1+\sqrt{3}$ (d) None of these

218. If $|z-25i|\le 15$, then | max. *amp* (z) – min. *amp* (z) | =

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\pi-2\cos^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{\pi}{2}+\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)-\cos^{-1}\left(\frac{3}{5}\right)$

219. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1-z_2}{z_1+z_2}\right|=1$ and $iz_1=kz_2$, where $k\in R,$ then the angle between z_1-z_2 and z_1+z_2 is

(a) $\tan^{-1}\left(\frac{2k}{k^2+1}\right)$ (b) $\tan^{-1}\left(\frac{2k}{1-k^2}\right)$ (c) $-2\tan^{-1}k$ (d) $2\tan^{-1}k$

220. If at least one value of the complex number $z=x+iy$ satisfy the condition $|z+\sqrt{2}|=a^2-3a+2$ and the inequality $|z+i\sqrt{2}|<a^2$, then

(a) a > 2 (b) a = 2 (c) a < 2 (d) None of these

221. The maximum distance from the origin of coordinates to the point z satisfying the equation $\left|z+\frac{1}{z}\right|=a$ is

(a) $\frac{1}{2}(\sqrt{a^2+1}+a)$ (b) $\frac{1}{2}(\sqrt{a^2+2}+a)$ (c) $\frac{1}{2}(\sqrt{a^2+4}+a)$ (d) None of these

222. Let a be a complex number such that $|a| < 1$ and z_1, z_2 .... be vertices of a polygon such that $z_k = 1 + a + a^2 + ... + a^{k-1}$. Then the vertices of the polygon lie within a circle

(a) $|z - a| = a$ (b) $\left|z - \frac{1}{1-a}\right| = |1 - a|$ (c) $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$ (d) $|z - (1-a)| = |1-a|$

223. If a, b, c and u, v, w are complex numbers representing the vertices of two traingles such that $c = (1-r)a + rb$ and $w = (1-r)u + rv$, where r is a complex number, then the two triangles

(a) Have the same area (b) Are similar (c) Are congruent (d) None of these

224. If z_1, z_2, z_3, z_4 are the affixes of four points in the Argand plane and z is the affix of a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then z_1, z_2, z_3, z_4 are

(a) Concyclic (b) Vertices of a parallelogram (c) Vertices of a rhombus (d) In a straight line

225. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represents the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number

(a) $3 - \frac{1}{2}i$ or $1 - \frac{3}{2}i$ (b) $\frac{3}{2} - i$ or $\frac{1}{2} - 3i$ (c) $\frac{1}{2} - i$ or $1 - \frac{1}{2}i$ (d) None of these

226. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$. If $Z_1 = 1 + i\sqrt{3}$, then values of Z_3 and Z_2 are respectively **[IIT 1994]**

(a) $-2, 1 - i\sqrt{3}$ (b) $2, 1 + i\sqrt{3}$ (c) $1 + i\sqrt{3}, -2$ (d) None of these

227. If a and b are real numbers between 0 and 1 such that the points $z_1 = a + i, z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then

[IIT 1989]

(a) $a = b = 2 + \sqrt{3}$ (b) $a = b = 2 - \sqrt{3}$ (c) $a = 2 - \sqrt{3}, b = 2 + \sqrt{3}$ (d) None of these

228. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that origin, z_1 and z_2 form an equilateral triangle. Then **[AIEEE 2003]**

(a) $a^2 = b$ (b) $a^2 = 2b$ (c) $a^2 = 3b$ (d) $a^2 = 4b$

229. If z_1, z_2, z_3, z_4 are represented by the vertices of a rhombus taken in the anticlockwise order then

(a) $z_1 - z_2 + z_3 - z_4 = 0$ (b) $z_1 + z_2 = z_3 + z_4$ (c) $amp \frac{z_2 - z_4}{z_1 - z_3} = \frac{\pi}{2}$ (d) $amp \frac{z_1 - z_2}{z_3 - z_4} = \frac{\pi}{2}$

230. The join of $z_1 = a + ib$ and $z_2 = \frac{1}{-a + ib}$ passes through

(a) Origin (b) $z = 1 + i\,0$ (c) $z = 0 + i$ (d) $z = 1 + i$

231. If A, B, C are three points in the Argand plane representing the complex numbers z_1, z_2, z_3 such that $z_1 = \frac{\lambda z_2 + z_3}{\lambda + 1}$, where $\lambda \in R$, then the distance of A from the line BC is

(a) λ (b) $\frac{\lambda}{\lambda + 1}$ (c) 1 (d) 0

232. The roots of the equation $1 + z + z^3 + z^4 = 0$ are represented by the vertices of

(a) A square (b) An equilateral triangle (c) A rhombus (d) None of these

233. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C, then

[DCE 1998]

(a) $(z_1-z_3)^2=2(z_1-z_2)(z_3-z_2)$ (b) $(z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$

(c) $(z_1+z_2)^2=2(z_1-z_2)(z_3+z_2)$ (d) $(z_1+z_3)^2=2(z_1+z_2)(z_3+z_2)$

234. *ABCD* is a square, vertices being taken in the anticlockwise sense. If *A* represents the complex number *z* and the intersection of the diagonals is the origin then

(a) *B* represents the complex number *iz* (b) *D* represents the complex number $i\bar{z}$

(c) *B* represents the complex number $i\bar{z}$ (d) *D* represents the complex number –*iz*

235. The angle that the vector representing the complex number $\frac{1}{(\sqrt{3}-i)^{25}}$ makes with the positive direction of the real axis is

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

236. If z_0, z_1 represent points *P, Q* on the locus $|z-1|=1$ and the line segment *PQ* subtends an angle $\pi/2$ at the point $z=1$ then z_1 is equal to

(a) $1+i(z_0-1)$ (b) $\frac{i}{z_0-1}$ (c) $1-i(z_0-1)$ (d) $i(z_0-1)$

237. If $z^n\sin\theta_0+z^{n-1}\sin\theta_1+z^{n-2}\sin\theta_2+.....+z\sin\theta_{n-1}+\sin\theta_n=2$, then all the roots of the equation lies

(a) Outside the circle $|z|=\frac{1}{2}$ (b) Inside the circle $|z|=\frac{1}{2}$ (c) On the circle $|z|=\frac{1}{2}$ (d)

238. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle circumscribing the circle $|z|=1$. If $z_1=1+\sqrt{3}\,i$ and z_1, z_2, z_3 are in the anticlockwise sense, then z_2 is

(a) $1-\sqrt{3}i$ (b) 2 (c) $\frac{1}{2}(1-\sqrt{3}i)$ (d) None of these

239. In the Argand plane, the vector $z=4-3i$ is turned in the clockwise sense through 180° and streatched three times. The complex number represented by the new vector is

(a) $12+9i$ (b) $12-9i$ (c) $-12-9i$ (d) $-12+9i$

240. The vector $z=3-4i$ is turned anticlockwise through an angle of 180° and stretched 2.5 times. The complex number corresponding to the newly obtained vector is

(a) $\frac{15}{2}-10i$ (b) $\frac{-15}{2}+10i$ (c) $\frac{-15}{2}-10i$ (d) None of these

Triangle Inequilities, Area of Triangle and Collinearity

Basic Level

241. If z_1 and z_2 are any two complex numbers, then which of the following is true

[Rajasthan PET 1985; MP PET 1987; Kerala (Engg.) 2002]

(a) $|z_1+z_2|=|z_1|+|z_2|$ (b) $|z_1-z_2|=|z_1|-|z_2|$ (c) $|z_1+z_2|\le|z_1|+|z_2|$ (d) $|z_1-z_2|\le|z_1|-|z_2|$

242. Which of the following are correct for any two complex numbers z_1 and z_2 [MP PET 1994; Roorkee 1998]

(a) $|z_1z_2|=|z_1|\ |z_2|$ (b) $arg(z_1z_2)=(arg z_1)(arg z_2)$ (c) $|z_1+z_2|=|z_1|+|z_2|$ (d) $|z_1-z_2|\ge|z_1|-|z_2|$

243. If $z_1, z_2\in C$, then [MP PET 1995]

(a) $|z_1+z_2|\ge|z_1|+|z_2|$ (b) $|z_1-z_2|\ge|z_1|+|z_2|$ (c) $|z_1-z_2|\le|z_1|-|z_2|$ (d) $|z_1+z_2|\ge|z_1|-|z_2|$

244. Which one of the following statement is true [Rajasthan PET 2002]

(a) $|x-y|=|x|-|y|$ (b) $|x+y|\le|x|-|y|$ (c) $|x-y|\ge|x|-|y|$ (d) $|x+y|\ge|x|-|y|$

245. The value of $|z_1+z_2|^2+|z_1-z_2|^2$ is **[Rajasthan PET 1997]**

(a) $\frac{1}{2}[|z_1|^2+|z_2|^2]$ (b) $2[|z_1|^2+|z_2|^2]$ (c) $2[|z_1|^2-|z_2|^2]$ (d) $\frac{1}{2}[|z_1|^2-|z_2|^2]$

246. If z, iz and $z+iz$ are the vertices of a triangle whose area is 2 units, then the value of $|z|$ is **[Rajasthan PET 2000]**

(a) – 2 (b) 2 (c) 4 (d) 8

247. If the area of the triangle formed by the points $z, z+iz$ and iz on the complex plane is 18, then the value of $|z|$ is **[MP PET 2001]**

(a) 6 (b) 9 (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

248. If *A, B, C* are represented by $3+4i, 5-2i, -1+16i,$ then *A, B, C* are **[Rajasthan PET 1986]**

(a) Collinear (b) Vertices of equilateral triangle

(c) Vertices of isosceles triangle (d) Vertices of right angled triangle

249. If $z_1=1+i, z_2=-2+3i$ and $z_3=ai/3$, where $i^2=-1,$ are collinear then the value of a is **[AMU 2001]**

(a) – 1 (b) 3 (c) 4 (d) 5

250. The area of the triangle whose vertices are the points, represented by the complex numbers z_1, z_2, z_3 on the Argand diagram is

[DCE 1997]

(a) $\frac{\sum |z_2-z_3|\ |z_1|^2}{4iz_1}$ (b) $\frac{1}{2}|z_1|\ |z_2|$ (c) $\frac{1}{3}|z_1|^2$ (d) $\sum \frac{z_1-z_3}{4iz_1}$

251. Area of the triangle formed by 3 complex numbers 1 + *i*, *i* – 1, 2*i* in the Argand plane is **[EAMCET 1993]**

(a) 1/2 (b) 1 (c) $\sqrt{2}$ (d) 2

252. The area of the triangle whose vertices are represented by the complex numbers 0, z, $ze^{i\alpha}$, $(0<\alpha<\pi)$ equals **[AMU 20**

(a) $\frac{1}{2}|z|^2\cos\alpha$ (b) $\frac{1}{2}|z|^2\sin\alpha$ (c) $\frac{1}{2}|z|^2\sin\alpha\cos\alpha$ (d) $\frac{1}{2}|z|^2$

253. If the roots of $z^3+iz^2+2i=0$ represent the vertices of a ΔABC in the argand plane, then the area of the triangle is

(a) $\frac{3\sqrt{7}}{2}$ (b) $\frac{3\sqrt{7}}{4}$ (c) 2 (d) None of these

254. If $2z_1-3z_2+z_3=0$ then z_1, z_2, z_3 are represented by

(a) Three vertices of a triangle (b) Three collinear points (c) Three vertices of a rhombus (d) None of these

Standard Loci in the Argand Plane

Basic Level

255. The complex numbers $z=x+iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right|=1$ lie on **[IIT 1982; Pb. CET 1998]**

(a) Real axis (x-axis) (b) The line y = 5

(c) A circle passing through the origin (d) None of these

256. If $z = x + iy$ is a complex number satisfying $\left|z + \frac{i}{2}\right|^2 = \left|z - \frac{i}{2}\right|^2$, then the locus of z is **[EAMCET 2002]**

(a) $2y = x$ (b) $y = x$ (c) y-axis (d) x-axis

257. If $arg\ (z - a) = \frac{\pi}{4}$, where $a \in R$, then the locus of $z \in C$ is a **[MP PET 1997]**

(a) Hyperbola (b) Parabola (c) Ellipse (d) Straight line

258. The locus of z given by $\left|\frac{z-1}{z-i}\right| = 1$, is **[Roorkee 1990]**

(a) A circle (b) An ellipse (c) A straight line (d) A parabola

259. Locus of the point z satisfying the equation $|iz - 1| + |z - i| = 2$ is **[Roorkee 1999]**

(a) A straight line (b) A circle (c) An ellipse (d) A pair of straight lines

260. If the imaginary part of $\frac{2z+1}{iz+1}$ is – 2, then the locus of the point representing z in the complex plane is **[DCE 2001]**

(a) A circle (b) A straight line (c) A parabola (d) None of these

261. The locus represented by | z – 1 | = | z + i | is **[EAMCET 1991]**

(a) A circle of radius 1 (b) An ellipse with foci at (1, 0) and (0, –1)

(c) A straight line through the origin (d) A circle on the line joining (1, 0), (0, 1) as diameter

262. If $z^2 + z|z| + |z|^2 = 0$, then the locus of z is

(a) A circle (b) A straight line (c) A pair of straight lines (d) None of these

263. If $z = x + iy$ and $|z - 2 + i| = |z - 3 - i|$, then locus of z is **[Rajasthan PET 1999]**

(a) $2x + 4y - 5 = 0$ (b) $2x - 4y - 5 = 0$ (c) $x + 2y = 0$ (d) $x - 2y + 5 = 0$

264. If the amplitude of $z - 2 - 3i$ is $\pi/4$, then the locus of $z = x + iy$ is **[EAMCET 2003]**

(a) $x + y - 1 = 0$ (b) $x - y - 1 = 0$ (c) $x + y + 1 = 0$ (d) $x - y + 1 = 0$

265. If $z = x + iy$ and $arg\left|\frac{z-2}{z+2}\right| = \frac{\pi}{6}$, then locus of z is **[Rajasthan PET 2002]**

(a) A straight line (b) A circle (c) A parabola (d) An ellipse

266. If $z = (\lambda + 3) + i\sqrt{5 - \lambda^2}$, then the locus of z is a

(a) Circle (b) Straight line (c) Parabola (d) None of these

267. A complex number z is such that $arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$. The points representing this complex number will lie on **[MP PET 200[1]**

(a) An ellipse (b) A parabola (c) A circle (d) A straight line

268. The equation $|z - 5i| \div |z + 5i| = 12$, where $z = x + iy$, represents a/an **[AMU 1999]**

(a) Circle (b) Ellipse (c) Parabola (d) No real curve

269. If $\frac{|z-2|}{|z-3|} = 2$ represents a circle, then its radius is equal to **[Karnataka CET 1990; Kurukshetra CEE 1998]**

(a) 1 (b) 1/3 (c) 3/4 (d) 2/3

270. A point z moves on Argand diagram in such a way that $|z - 3i| = 2$, then its locus will be **[Rajasthan PET 1992; MP PET 20**

(a) y – axis (b) A straight line (c) A circle (d) None of these

271. A circle whose radius is r and centre z_0, then the equation of the circle is **[Rajasthan PET 2000]**

(a) $z\bar{z} - z\bar{z}_0 - \bar{z}z_0 + z_0\bar{z}_0 = r^2$ (b) $z\bar{z} + z\bar{z}_0 - \bar{z}z_0 + z_0\bar{z}_0 = r^2$

(c) $z\bar{z} - z\bar{z}_0 + \bar{z}z_0 - z_0\bar{z}_0 = r^2$ (d) None of these

272. If $|z+\bar{z}|+|z-\bar{z}|=2,$ then z lies on

(a) A straight line (b) A square (c) A circle (d) None of these

273. If $z=x+iy,$ then $z\bar{z}+2(z+\bar{z})+c=0$ implies **[Rajasthan PET 1998; Pb. CET 2002]**

(a) A circle (b) Straight line (c) Parallel (d) Point

274. The equation $|z+1-i|=|z+i-1|$ represents **[EAMCET 1996]**

(a) A straight line (b) A circle (c) A parabola (d) A hyperbola

275. The equation $z\bar{z}+(2-3i)z+(2+3i)\bar{z}+4=0$ represents a circle of radius **[Kurukshetra CEE 1996]**

(a) 2 (b) 3 (c) 4 (d) 6

276. In the Argand diagram all the complex number z satisfying $|z-4i|+|z+4i|=10$ lie on a **[EAMCET 1996]**

(a) Straight line (b) Circle (c) Ellipse (d) Parabola

Advance Level

277. When $\frac{z+i}{z+2}$ is purely imaginary, the locus described by the point z in the Argand diagram is a

(a) Circle of radius $\frac{\sqrt{5}}{2}$ (b) Circle of radius $\frac{5}{4}$ (c) Straight line (d) Parabola

278. If $\log_{\sqrt{3}}\left(\frac{|z|^2-|z|+1}{2+|z|}\right)<2,$ then the locus of z is **[Karnataka CET 1996]**

(a) $|z|=5$ (b) $|z|<5$ (c) $|z|>5$ (d) None of these

279. The region of Argand plane defined by $|z-1|+|z+1|\le 4$ is

(a) Interior of an ellipse (b) Exterior of a circle

(c) Interior and boundary of an ellipse (d) None of these

280. The equation $|z+i|-|z-i|=k$ represent a hyperbola if

(a) $-2<k<2$ (b) $k>2$ (c) $0<k<2$ (d) None of these

281. The equation $|z-i|-|z+i|=k$, $k>0$, can represent an ellipse if k is

(a) 1 (b) 2 (c) 4 (d) None of these

282. If $|z|=2$ and locus of $5z-1$ is the circle having radius a and $z_1^2+z_2^2-2z_1z_2\cos\theta=0,$ then $|z_1|:|z_2|=$

(a) $a:1$ (b) $2a:1$ (c) $a:10$ (d) None of these

283. The locus of the complex number z in an argand plane satisfying the inequality $\log_{\left(\frac{1}{2}\right)}\left(\frac{|z-1|+4}{3|z-1|-2}\right)>1$ is, (where $|z-1|\ne\frac{2}{3}$)

(a) A circle (b) An interior of a circle (c) The exterior of the circle (d) None of these

284. Let $z=1-t+i\sqrt{t^2+t+2},$ where t is a real parameter. The locus of z in the Argand plane is

(a) A hyperbola (b) An ellipse (c) A straight line (d) None of these

285. The locus of the centre of a circle which touches the circle $|z-z_1|=a$ and $|z-z_2|=b$ externally (z, z_1 and z_2 are complex numbers) will be **[AIEEE 2002]**

(a) An ellipse (b) A hyperbola (c) A circle (d) None of these

De' Moivre's Theorem

Basic Level

286. The value of $i^{1/3}$ is **[UPSEAT 2002]**

(a) $\frac{\sqrt{3}+i}{2}$ (b) $\frac{\sqrt{3}-i}{2}$ (c) $\frac{1+i\sqrt{3}}{2}$ (d) $\frac{1-i\sqrt{3}}{2}$

287. Given $z=(1+i\sqrt{3})^{100}$, then $\frac{\text{Re}(z)}{\text{Im}(z)}$ equals **[AMU 2002]**

(a) 2^{100} (b) 2^{50} (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

288. $(-1+i\sqrt{3})^{20}$ is equal to **[Rajasthan PET 2003]**

(a) $2^{20}(-1+i\sqrt{3})^{20}$ (b) $2^{20}(1-i\sqrt{3})^{20}$ (c) $2^{20}(-1-i\sqrt{3})^{20}$ (d) None of these

289. $(-\sqrt{3}+i)^{53}$ where $i^2=-1$ is equal to **[AMU 2000]**

(a) $2^{53}(\sqrt{3}+2i)$ (b) $2^{52}(\sqrt{3}+i)$ (c) $2^{53}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$ (d) $2^{53}(\sqrt{3}-i)$

290. If $z=\frac{\sqrt{3}+i}{2}$, then the value of z^{69} is **[Rajasthan PET 2002]**

(a) $-i$ (b) i (c) 1 (d) -1

291. If $a=\sqrt{2i}$, then which of the following is correct **[Roorkee 1989]**

(a) $a=1+i$ (b) $a=1-i$ (c) $a=-(\sqrt{2})i$ (d) None of these

292. If $z=\cos\theta+i\sin\theta$ then the value of $z^n+\frac{1}{z^n}$ is

(a) $\cos 2n\theta$ (b) $2\cos n\theta$ (c) $2\sin n\theta$ (d) None of these

293. The value of $(-i)^{1/3}$ is **[Roorkee 1995]**

(a) $\frac{1+\sqrt{3}\,i}{2}$ (b) $\frac{1-\sqrt{3}\,i}{2}$ (c) $\frac{-\sqrt{3}-i}{2}$ (d) $\frac{\sqrt{3}-i}{2}$

294. $(\sin\theta+i\cos\theta)^n$ is equal to **[Rajasthan PET 2001]**

(a) $\cos n\theta+i\sin n\theta$ (b) $\sin n\theta+i\cos n\theta$

(c) $\cos n\left(\frac{\pi}{2}-\theta\right)+i\sin n\left(\frac{\pi}{2}-\theta\right)$ (d) None of these

295. The product of all the roots of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ is **[MNR 1984; EAMCET 1985]**

(a) -1 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

296. $\left[\frac{1+\cos(\pi/8)+i\sin(\pi/8)}{1+\cos(\pi/8)-i\sin(\pi/8)}\right]^8$ is equal to **[Rajasthan PET 2001]**

(a) -1 (b) 0 (c) 1 (d) 2

297. $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4$ equals **[Rajasthan PET 1996]**

(a) $\sin 8\theta - i\cos 8\theta$ (b) $\cos 8\theta - i\sin 8\theta$ (c) $\sin 8\theta + i\cos 8\theta$ (d) $\cos 8\theta + i\sin 8\theta$

298. $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$ is equal to **[MNR 1985; UPSEAT 2000]**

(a) $\cos\theta - i\sin\theta$ (b) $\cos 9\theta - i\sin 9\theta$ (c) $\sin\theta - i\cos\theta$ (d) $\sin 9\theta - i\cos 9\theta$

299. $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^5} =$ **[Rajasthan PET 1992, 96, 2002; UPSEAT 2000]**

(a) $\cos(4\alpha+5\beta)+i\sin(4\alpha+5\beta)$ (b) $\cos(4\alpha+5\beta)-i\sin(4\alpha+5\beta)$

(c) $\sin(4\alpha+5\beta)-i\cos(4\alpha+5\beta)$ (d) None of these

300. We express $\frac{(\cos 2\theta - i\sin 2\theta)^4(\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 3\theta - i\sin 3\theta)^{-9}}$ in the form of $x + iy$, we get **[Karnataka CET 2001]**

(a) $\cos 49\theta - i\sin 49\theta$ (b) $\cos 23\theta - i\sin 23\theta$ (c) $\cos 49\theta + i\sin 49\theta$ (d) $\cos 21\theta + i\sin 21\theta$

301. If $\left(\frac{1+\cos\theta + i\sin\theta}{i+\sin\theta + i\cos\theta}\right)^4 = \cos n\theta + i\sin n\theta$, then n is equal to **[EAMCET 1986]**

(a) 1 (b) 2 (c) 3 (d) 4

302. The value of $\frac{(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)}{(\cos\gamma + i\sin\gamma)(\cos\delta + i\sin\delta)}$ is **[Rajasthan PET 2001]**

(a) $\cos(\alpha+\beta-\gamma-\delta)-i\sin(\alpha+\beta-\gamma-\delta)$ (b) $\cos(\alpha+\beta-\gamma-\delta)+i\sin(\alpha+\beta-\gamma-\delta)$

(c) $\sin(\alpha+\beta-\gamma-\delta)-i\cos(\alpha+\beta-\gamma-\delta)$ (d) $\sin(\alpha+\beta-\gamma-\delta)+i\cos(\alpha+\beta-\gamma-\delta)$

303. The value of $\left[\frac{1-\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}}{1-\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}}\right]^{10} =$ **[Karnataka CET 2001]**

(a) 0 (b) – 1 (c) 1 (d) 2

304. If $z = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$, then $(\bar{z})^{100}$ lies in **[AMU 1999]**

(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

305. The following in the form of $A + iB$

$(\cos 2\theta + i\sin 2\theta)^{-5}(\cos 3\theta - i\sin 3\theta)^6(\sin\theta - i\cos\theta)^3$ is **[MNR 1991]**

(a) $(\cos 25\theta + i\sin 25\theta)$ (b) $i(\cos 25\theta + i\sin 25\theta)$ (c) $i(\cos 25\theta - i\sin 25\theta)$ (d) $(\cos 25\theta - i\sin 25\theta)$

306. $A+iB$ form of $\frac{(\cos x + i\sin x)(\cos y + i\sin y)}{(\cot u + i)(1+i\tan v)}$ is **[Roorkee 1980]**

(a) $\sin u\cos v[\cos(x+y-u-v)+i\sin(x+y-u-v)]$ (b) $\sin u\cos v[\cos(x+y+u+v)+i\sin(x+y+u+v)]$

(c) $\sin u\cos v[\cos(x+y+u+v)-i\sin(x+y+u+v)]$ (d) None of these

307. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$ is **[IIT 1987; DCE 2000; Karnataka CET 2002]**

(a) – 1 (b) 0 (c) $-i$ (d) i

308. If $x_n = \cos\left(\frac{\pi}{3^n}\right) + i\sin\left(\frac{\pi}{3^n}\right)$, then $x_1.x_2.x_3....x_\infty$ is equal to **[Rajasthan PET 2002; Kurukshetra CEE 2002]**

(a) 1 (b) – 1 (c) i (d) $-i$

309. If $x_n = \cos\left(\frac{\pi}{4^n}\right) + i\sin\left(\frac{\pi}{4^n}\right)$, then $x_1.x_2.x_3....\infty =$ **[EAMCET 2002]**

(a) $\frac{1+i\sqrt{3}}{2}$ (b) $\frac{-1+i\sqrt{3}}{2}$ (c) $\frac{1-i\sqrt{3}}{2}$ (d) $\frac{-1-i\sqrt{3}}{2}$

310. The value of infinite product $(\cos\theta + i\sin\theta)\ (\cos\frac{\theta}{2} + i\sin\frac{\theta}{2})\ (\cos\frac{\theta}{2^2} + i\sin\frac{\theta}{2^2})....$ is **[Rajasthan PET 1999]**

(a) $\cos 2\theta - i\sin 2\theta$ (b) $\cos 2\theta + i\sin 2\theta$ (c) $\sin 2\theta - i\cos 2\theta$ (d) $\sin 2\theta + i\cos 2\theta$

311. The value of expression $\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)$..... to ∞ is **[Kurukshetra CEE 1998]**

(a) – 1 (b) 1 (c) 0 (d) 2

312. If $z_i = \cos\frac{i\pi}{10} + i\sin\frac{i\pi}{10}$, then $z_1 z_2 z_3 z_4$ is equal to **[DCE 1998]**

(a) – 1 (b) 1 (c) – 2 (d) 2

313. If $2\cos\alpha = a + \frac{1}{a}$ and $2\cos\beta = b + \frac{1}{b}$, then the value of $ab + \frac{1}{ab}$ is **[Rajasthan PET 1992, Pb. CET 2002]**

(a) $2\cos(\alpha+\beta)$ (b) $2\sin(\alpha+\beta)$ (c) $2\cos(\alpha-\beta)$ (d) $4\cos\alpha\cos\beta$

314. $\frac{(\sin\pi/8 + i\cos\pi/8)^8}{(\sin\pi/8 - i\cos\pi/8)^8} =$ **[EAMCET 1994]**

(a) – 1 (b) 0 (c) 1 (d) $2i$

Advance Level

315. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)........(\cos n\theta + i\sin n\theta) = 1$, then the value of θ is **[Karnataka CET 1992; Kurukshetra CEE 2002]**

(a) $4m\pi$ (b) $\frac{2m\pi}{n(n+1)}$ (c) $\frac{4m\pi}{n(n+1)}$ (d) $\frac{m\pi}{n(n+1)}$

316. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$ equals to **[Karnataka CET 2000]**

(a) 0 (b) $\cos(\alpha+\beta+\gamma)$ (c) $3\cos(\alpha+\beta+\gamma)$ (d) $3\sin(\alpha+\beta+\gamma)$

317. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ equals **[Rajasthan PET 2000]**

(a) $2\cos(\alpha+\beta+\gamma)$ (b) $\cos 2(\alpha+\beta+\gamma)$ (c) 0 (d) 1

318. If $\sin\alpha + \sin\beta + \sin\gamma = 0 = \cos\alpha + \cos\beta + \cos\gamma$, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is **[Rajasthan PET 1999]**

(a) 2/3 (b) 3/2 (c) 1/2 (d) 1

319. If $a = \cos(2\pi/7) + i\sin(2\pi/7)$, then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$ is **[Rajasthan PET 2000]**

(a) $x^2 - x + 2 = 0$ (b) $x^2 + x - 2 = 0$ (c) $x^2 - x - 2 = 0$ (d) $x^2 + x + 2 = 0$

320. If $x^2 - x + 1 = 0$ then the value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$ is

(a) 8 (b) 10 (c) 12 (d) None of these

321. If n_1, n_2 are positive integers, then $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$ is a real number iff **[IIT 1996]**

(a) $n_1 = n_2 + 1$ (b) $n_1 + 1 = n_2$

(c) $n_1 = n_2$ (d) n_1, n_2 are any two +ve integers

322. If $a = \cos\alpha + i\sin\alpha,\ b = \cos\beta + i\sin\beta,\ c = \cos\gamma + i\sin\gamma$ and $\frac{b}{c}+\frac{c}{a}+\frac{a}{b}=1,$ then $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)$ is equal to

[Rajasthan PET 1993, 2001]

(a) 3/2 (b) – 3/2 (c) 0 (d) 1

323. If $\cos A + \cos B + \cos C = 0,\ \sin A + \sin B + \sin C = 0$ and $A + B + C = 180^o$, then the value of $\cos 3A + \cos 3B + \cos 3C$ is

[EAMCET 1995]

(a) 3 (b) – 3 (c) $\sqrt{3}$ (d) 0

324. The value of z satisfying the equation $\log z + \log z^2 + ... + \log z^n = 0$ is

(a) $\cos\frac{4m\pi}{n(n+1)} + i\sin\frac{4m\pi}{n(n+1)},\ m = 1, 2, ..$

(b) $\cos\frac{4m\pi}{n(n+1)} - i\sin\frac{4m\pi}{n(n+1)},\ m = 1, 2, ..$

(c) $\sin\frac{4m\pi}{n} + i\cos\frac{4m\pi}{n},\ m = 1, 2, ..$

(d) 0

Cube Roots of Unity, n^{th} Roots of Unity

Basic Level

325. The product of cube roots of –1 is equal to

(a) 0 (b) 1 (c) – 1 (d) None of these

326. One of the cube roots of unity is **[MP PET 1994, 2003]**

(a) $\frac{-1+i\sqrt{3}}{2}$ (b) $\frac{1+i\sqrt{3}}{2}$ (c) $\frac{1-i\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}-i}{2}$

327. The two numbers such that each one is square of the other, are **[MP PET 1987]**

(a) ω, ω^3 (b) $-i, i$ (c) –1, 1 (d) ω, ω^2

328. If $1, \omega, \omega^2$ are the cube roots of unity, then their product is **[Karnataka CET 1999, 2001]**

(a) 0 (b) ω (c) – 1 (d) 1

329. The value of $(8)^{1/3}$ is **[Rajasthan PET 2003]**

(a) $-1+i\sqrt{3}$ (b) $-1-i\sqrt{3}$ (c) 2 (d) All of these

330. If $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n$ is an integer, then n is **[UPSEAT 2002]**

(a) 1 (b) 2 (c) 3 (d) 4

331. $\left(\frac{\sqrt{3}+i}{2}\right)^6+\left(\frac{i-\sqrt{3}}{2}\right)^6$ is equal to **[Rajasthan PET 1997]**

(a) – 2 (b) 0 (c) 2 (d) 1

332. If $\frac{1+\sqrt{3}\,i}{2}$ is a root of equation $x^4-x^3+x-1=0$, then its real roots are **[EAMCET 2002]**

(a) 1, 1 (b) – 1, – 1 (c) 1, – 1 (d) 1, 2

333. If $z=\frac{\sqrt{3}+i}{-2}$, then z^{69} is equal to **[Rajasthan PET 2001]**

(a) 1 (b) – 1 (c) i (d) – i

334. If ω is a complex cube root of unity, then for positive integral value of *n*, the product of $\omega.\omega^2.\omega^3.........\omega^n$ will be **[Roorkee 1991]**

(a) $\frac{1-i\sqrt{3}}{2}$ (b) $-\frac{1-i\sqrt{3}}{2}$ (c) 1 (d) (b) and (c) both

335. If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7=A+B\omega,$ then *A* and *B* are respectively, the numbers **[IIT 1995]**

(a) 0, 1 (b) 1, 0 (c) 1, 1 (d) – 1, 1

336. If ω is a cube root of unity, then $(1+\omega-\omega^2)(1-\omega+\omega^2)=$ **[MNR 1990; UPSEAT 1999; MP PET 1993, 02]**

(a) 1 (b) 0 (c) 2 (d) 4

337. If cube root of 1 is ω, then the value of $(3+\omega+3\omega^2)^4$ is **[MP PET 2001]**

(a) 0 (b) 16 (c) $16\,\omega$ (d) $16\,\omega^2$

338. If $1,\omega\ \omega^2$ are the three cube roots of unity, then $(3+\omega^2+\omega^4)^6=$ **[MP PET 1995]**

(a) 64 (b) 729 (c) 2 (d) 0

339. The value of $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ will be **[Ranchi BIT 1989; Orissa JEE 2003]**

(a) 1 (b) – 1 (c) 2 (d) – 2

340. If ω is a non real cube root of unity, then $(a+b)(a+b\omega)(a+b\omega^2)$ is **[Kerala (Engg.) 2002]**

(a) a^3+b^3 (b) a^3-b^3 (c) a^2+b^2 (d) a^2-b^2

341. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals **[IIT 1998; MP PET 2000]**

(a) $128\,\omega$ (b) $-128\,\omega$ (c) $128\,\omega^2$ (d) $-128\,\omega^2$

342. If ω is the cube root of unity, then $(3+5\omega+3\omega^2)^2+(3+3\omega+5\omega^2)^2=$ **[MP PET 1999]**

(a) 4 (b) 0 (c) – 4 (d) None of these

343. If ω is an imaginary cube root of unity, then the value of $\sin\left[(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right]$ is **[IIT Screening 1994]**

(a) $-\sqrt{3}/2$ (b) $-1/\sqrt{2}$ (c) $1/\sqrt{2}$ (d) $\sqrt{3}/2$

344. If $1,\omega,\ \omega^2$ are three cube roots of unity, then $(a+b\omega+c\omega^2)^3+(a+b\omega^2+c\omega)^3$ is equal to, if $a+b+c=0$ **[WB JEE 1992]**

(a) 27*abc* (b) 0 (c) 3*abc* (d) None of these

345. The value of $(1-\omega+\omega^2)(1-\omega^2+\omega)^6$, where ω,ω^2 are cube roots of unity **[DCE 2001]**

(a) 128ω (b) $-128\omega^2$ (c) -128ω (d) $128\omega^2$

346. If ω is a cube root of unity, then the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5=$ **[IIT 1965; MP PET 1997; Rajasthan PET 1997]**

(a) 16 (b) 32 (c) 48 (d) – 32

347. If ω is a complex cube root of unity, then $225+(3\omega+8\omega^2)^2+(3\omega^2+8\omega)^2=$ **[EAMCET 2003]**

(a) 72 (b) 192 (c) 200 (d) 248

348. If $x=a, y=b\omega, z=c\omega^2$, where ω is a complex cube root of unity, then $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=$ **[AMU 1983]**

(a) 3 (b) 1 (c) 0 (d) None of these

349. If $x=a+b, y=a\omega+b\omega^2, z=a\omega^2+b\omega,$ then the value of $x^3+y^3+z^3$ is equal to **[Roorkee 1977; IIT 1970]**

(a) a^3+b^3 (b) $3(a^3+b^3)$ (c) $3(a^2+b^2)$ (d) None of these

350. If ω is an nth root of unity, other than unity, then the value of $1+\omega+\omega^2+...+\omega^{n-1}$ is **[Karnataka CET 1999]**

(a) 0 (b) 1 (c) – 1 (d) None of these

351. If ω is a complex cube root of unity, then $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)......$ to $2n$ factors = **[AMU 2000]**

(a) 0 (b) 1 (c) – 1 (d) None of these

352. Find the value of $(1+2\omega+\omega^2)^{3n}-(1+\omega+2\omega^2)^{3n}$ is **[UPSEAT 2002]**

(a) 0 (b) 1 (c) ω (d) ω^2

353. If α and β are imaginary cube roots of unity, then the value of $\alpha^4+\beta^{28}+\frac{1}{\alpha\beta}$, is **[MP PET 1998]**

(a) 1 (b) – 1 (c) 0 (d) None of these

354. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-2)^3+27=0$ are **[Kurukshetra CEE 1998]**

(a) – 1, – 1, – 1 (b) $-1, -\omega, -\omega^2$ (c) $-1, 2+3\omega, 2+3\omega^2$ (d) $-1, 2-3\omega, 2-3\omega^2$

355. If α, β are non-real cube roots of unity, then $\alpha\beta+\alpha^5+\beta^5$ is equal to **[Kurukshetra CEE 1999]**

(a) 1 (b) 0 (c) – 1 (d) 3

356. If α and β are imaginary cube roots of unity, then $\alpha^4+\beta^4+\frac{1}{\alpha\beta}=$ **[IIT 1977]**

(a) 3 (b) 0 (c) 1 (d) 2

357. If ω is a cube root of unity, then a root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}=0$ is **[MNR 1990; MP PET 1999, 2002]**

(a) $x=1$ (b) $x=\omega$ (c) $x=\omega^2$ (d) $x=0$

358. If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta=\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to **[AIEEE 2003]**

(a) 0 (b) 1 (c) ω (d) ω^2

359. If $\omega(\neq 1)$ is a cube root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$ is equal to **[IIT 1995]**

(a) 0 (b) 1 (c) ω (d) i

360. If ω is a complex root of the equation $z^3 = 1$, then $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....\right)}$ is equal to **[Roorkee 2000]**

(a) – 1 (b) 0 (c) 9 (d) i

361. The product of *n*, *n*th roots of unity is

(a) 1 (b) – 1 (c) $(-1)^{n-1}$ (d) $(-1)^n$

362. Let $\omega_n = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right), i^2 = -1$, then $(x + y\omega_3 + z{\omega_3}^2)(x + y{\omega_3}^2 + z\omega_3)$ is equal to **[AMU 2001]**

(a) 0 (b) $x^2 + y^2 + z^2$

(c) $x^2 + y^2 + z^2 - yz - zx - xy$ (d) $x^2 + y^2 + z^2 + yz + zx + xy$

363. If *p* is not a multiple of *n*, then the sum of *p*th powers of *n*th roots of unity is

(a) 0 (b) 1 (c) n (d) p

364. If *n* is a positive integer greater than unity and *z* is a complex number satisfying the equation $z^n = (z+1)^n$, then

(a) Re (*z*) < 0 (b) Re (*z*) > 0 (c) Re (*z*) = 0 (d) None of these

365. If z_1, z_2, z_3, z_4 are the roots of the equation $z^4 = 1$, then the value of $\sum_{i=1}^{4} z_i^3$ is **[Kurukshetra CEE 1996]**

(a) 0 (b) 1 (c) i (d) 1 + i

366. If α is an imaginary cube root of unity, then for $n \in N$, the value of $\alpha^{3n+1} + \alpha^{3n+3} + \alpha^{3n+5}$ is **[MP PET 1996]**

(a) – 1 (b) 0 (c) 1 (d) 3

367. If $\alpha \neq 1$ is any n^{th} root of unity, then $S = 1 + 3\alpha + 5\alpha^2 + ...$ upto n terms, is equal to

(a) $\frac{2n}{1-\alpha}$ (b) $-\frac{2n}{1-\alpha}$ (c) $\frac{n}{1-\alpha}$ (d) $-\frac{n}{1-\alpha}$

368. The common roots of the equations $x^{12} - 1 = 0,\ x^4 + x^2 + 1 = 0$ are **[EAMCET 1989]**

(a) $\pm\omega$ (b) $\pm\omega^2$ (c) $\pm\omega, \pm\omega^2$ (d) None of these

369. Which of the following is a fourth root of $\frac{1}{2} + \frac{i\sqrt{3}}{2}$ **[Karnataka CET 2003]**

(a) $cis\left(\frac{\pi}{2}\right)$ (b) $cis\left(\frac{\pi}{12}\right)$ (c) $cis\left(\frac{\pi}{6}\right)$ (d) $cis\left(\frac{\pi}{3}\right)$

370. If ω is a complex root of unity, then **[T.S. Rajendra 1991, Kurukshetra CEE 2000]**

(a) $\omega^4 = 1$ (b) $\omega^{14} = \omega^2$ (c) $\omega^6 = \omega$ (d) $\omega^5 = 1$

371. If ω is an imaginary cube root of unity, then the value of $\frac{1}{1+2\omega} + \frac{1}{2+\omega} - \frac{1}{1+\omega}$ is **[Karnataka CET 1998]**

(a) – 2 (b) – 1 (c) 1 (d) 0

372. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is **[Kurukshetra CEE 1994, EAMCET 1995]**

(a) 2 (b) – 2 (c) 1 (d) 0

373. If the roots of the equation $x^3 - 1 = 0$ are 1, ω and ω^2, then the value of $(1-\omega)(1-\omega^2)$ is **[MNR 1992]**

(a) 0 (b) 1 (c) 2 (d) 3

374. If $i=\sqrt{-1}$, ω = non-real cube root of unity then $\frac{(1+i)^{2n}-(1-i)^{2n}}{(1+\omega^4-\omega^2)(1-\omega^4+\omega^2)}$ is equal to

(a) 0 if n is even (b) 0 for all $n\in Z$ (c) $2^{n-1}-i$ for all $n\in N$ (d) None of these

375. If $z+z^{-1}=1$, then $z^{100}+z^{-100}$ is equal to **[UPSEAT 2001]**

(a) i (b) $-i$ (c) 1 (d) – 1

376. If α is nonreal and $\alpha=\sqrt[5]{1}$ then the value of $2^{|1-\alpha+\alpha^2+\alpha^{-2}-\alpha^{-1}|}$ is equal to

(a) 4 (b) 2 (c) 1 (d) None of these

377. Which of the following statements are true **[Tamilnadu Engg. 2002]**

(1) The amplitude of the product of complex numbers is equal to the product of their amplitudes

(2) For any polynomial $f(x)=0$ with real co-efficients, imaginary roots occur in conjugate pairs.

(3) Order relation exists in complex numbers whereas it does not exist in real numbers.

(4) The values of ω used as a cube root of unity and as a fourth root of unity are different

(a) (1) and (2) only (b) (2) and (4) only (c) (3) and (4) only (d) (1), (2) and (4) only

378. If $x=a+b$, $y=a\alpha+b\beta$ and $z=a\beta+b\alpha$, where α and β are complex cube roots of unity, then xyz =

[IIT 1978; Roorkee 1989; Rajasthan PET 1997]

(a) a^2+b^2 (b) a^3+b^3 (c) a^3b^3 (d) a^3-b^3

Advance Level

379. $\frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}}+\frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}$ is equal to **[AMU 2000]**

(a) – 64 (b) – 32 (c) – 16 (d) $\frac{1}{16}$

380. $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$.......... .. to $2n$ factors is **[EAMCET 1988; AMU 1997]**

(a) 2^n (b) 2^{2n} (c) 0 (d) 1

381. If $1,\omega,\omega^2,\omega^3......,\omega^{n-1}$ are the n, n^{th} roots of unity, then $(1-\omega)(1-\omega^2)......(1-\omega^{n-1})$ equals **[MNR 1992; IIT 1984; DCE 2001]**

(a) 0 (b) 1 (c) n (d) n^2

382. The value of the expression $1.(2-\omega)(2-\omega^2)+2.(3-\omega)(3-\omega^2)+$.......... .. $+(n-1).(n-\omega).(n-\omega^2)$, where ω is an imaginary cube root of unity, is **[IIT 1996]**

(a) $\frac{1}{2}(n-1)n(n^2+3n+4)$ (b) $\frac{1}{4}(n-1)n(n^2+3n+4)$ (c) $\frac{1}{2}(n+1)n(n^2+3n+4)$ (d) $\frac{1}{4}(n+1)n(n^2+3n+4)$

383. If α,β,γ are the cube roots of $p\,(p<0)$, then for any x, y and z, $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}=$ **[IIT 1989]**

(a) $\frac{1}{2}(-1+i\sqrt{3})$ (b) $\frac{1}{2}(1+i\sqrt{3})$ (c) $\frac{1}{2}(1-i\sqrt{3})$ (d) None of these

384. Common roots of the equations $z^3+2z^2+2z+1=0$ and $z^{1985}+z^{100}+1=0$ are

(a) ω,ω^2 (b) ω,ω^3 (c) ω^2,ω^3 (d) None of these

385. If $z_1,z_2,z_3,$.......... ...z_n are n, nth roots of unity, then for $k=1,2,$.......... , n

(a) $|z_k|=k|z_{k+1}|$ (b) $|z_{k+1}|=k|z_k|$ (c) $|z_{k+1}|=|z_k|+|z_{k+1}|$ (d) $|z_k|=|z_{k+1}|$

386. Let z_1 and z_2 be n^{th} roots of unity which are ends of a line segment that subtend a right angle at the origin. Then n must be of the form **[IIT Screening 2001; Karnataka CEE 2002]**

(a) $4k+1$ (b) $4k+2$ (c) $4k+3$ (d) $4k$

387. The cube roots of unity when represented on the Argand plane form the vertices of an **[IIT 1988]**

(a) Equilateral triangle (b) Isosceles triangle (c) Right angled triangle (d) None of these

388. If $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}+\frac{1}{d+\omega}=\frac{2}{\omega}$, where a, b, c are real and ω is a non-real cube root of unity, then

(a) $\frac{1}{a+\omega^2}+\frac{1}{b+\omega^2}+\frac{1}{c+\omega^2}+\frac{1}{d+\omega^2}=-\frac{2}{\omega^2}$ (b) $abc+bcd+abd+acd=4$

(c) $a+b+c+d=-2abcd$ (d) $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}+\frac{1}{1+d}=2$

389. If z is a complex number satisfying $z+z^{-1}=1$ then z^n+z^{-n}, $n \in N$, has the value

(a) $2(-1)^n$, when n is a multiple of 3 (b) $(-1)^{n-1}$, when n is not a multiple of 3

(c) $(-1)^{n+1}$, when n is a multiple of 3 (d) 0, when n is not a multiple of 3

390. If z be a complex number satisfying $z^4+z^3+2z^2+z+1=0$ then $|z|$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) 1 (d) None of these

391. If the fourth roots of unity are z_1, z_2, z_3, z_4 then $z_1^2+z_2^2+z_3^2+z_4^2$ is equal to

(a) 1 (b) 0 (c) i (d) None of these

392. If $1, \alpha, \alpha^2, ..., \alpha^{n-1}$ are the nth roots of unity, then $\sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$ is equal to

(a) $(n-2).2^n$ (b) $\frac{(n-2)\,2^{n-1}+1}{2^n-1}$ (c) $\frac{(n-2)\,2^{n-1}}{2^n-1}$ (d) None of these

393. If $z_1+z_2+z_3=A$, $z_1+z_2\omega+z_3\omega^2=B$, $z_1+z_2\omega^2+z_3\omega=C$, where $1, \omega, \omega^2$ are the three cube roots of unity, then $|A|^2+|B|^2+|C|^2=$

(a) $3(|z_1|^2+|z_2|^2+|z_3|^2)$ (b) $2(|z_1|^2+|z_2|^2+|z_3|^2)$

(c) $|z_1|^2+|z_2|^2+|z_3|^2$ (d) None of these

394. For complex numbers $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$, if $\sin\theta=\frac{x_1y_2-x_2y_1}{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}}$ where θ is the angle between z_1 and z_2, then the angle between the roots of the equation $z^2+2z+3=0$ is

(a) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$ (c) $\sin^{-1}\left(\frac{1}{3}\right)$ (d) None of these

Miscellaneous Problems

Basic Level

395. $\sinh ix$ is **[EAMCET 2002]**

(a) $i\sin(ix)$ (b) $i\sin x$ (c) $-i\sin x$ (d) $\sin(ix)$

396. The value of $\sec h(i\pi)$ is **[Rajasthan PET 1999]**

(a) – 1 (b) i (c) 0 (d) 1

397. The imaginary part of $\cosh(\alpha + i\beta)$ is **[Rajasthan PET 2000]**

(a) $\cosh\alpha \cos\beta$ (b) $\sinh\alpha \sin\beta$ (c) $\cos\alpha \cosh\beta$ (d) $\cos\alpha \cos\beta$

398. $\cosh(\alpha + i\beta) - \cosh(\alpha - i\beta)$ is equal to **[Rajasthan PET 2000]**

(a) $2\sinh\alpha \sinh\beta$ (b) $2\cosh\alpha \cosh\beta$ (c) $2i\sinh\alpha \sin\beta$ (d) $2\cosh\alpha \cos\beta$

399. If $\cos(u + iv) = \alpha + i\beta,$ then $\alpha^2 + \beta^2 + 1$ equals **[Rajasthan PET 1999]**

(a) $\cos^2 u + \sinh^2 v$ (b) $\sin^2 u + \cosh^2 v$ (c) $\cos^2 u + \cosh^2 v$ (d) $\sin^2 u + \sinh^2 v$

400. If $\tan^{-1}(\alpha + i\beta) = x + iy,$ then $x =$ **[Rajasthan PET 2002]**

(a) $\frac{1}{2}\tan^{-1}\left(\frac{2\alpha}{1-\alpha^2-\beta^2}\right)$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{2\alpha}{1+\alpha^2+\beta^2}\right)$ (c) $\tan^{-1}\left(\frac{2\alpha}{1-\alpha^2-\beta^2}\right)$ (d) None of these

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Answer Sheet

Complex Numbers ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
b	d	b	c	a	a	a	b	d	b	b	a	d	b	c	a	d	d	a	c
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	c	d	d	b	a	c	a	d	b	b	c	c	a	b	b	c	c	c	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
c	c	c	a	b	b	b	a	b	d	b	c	c	d	b	c	c	b	a	a
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	a	d	c	b	d	b	a	c	b	c	a	a	a	b	b	a	c	b	c
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	a	b	c	b	a,c,d	a	a	a	d	c	a	b	c	b	b	a	d	c	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
d	b	a,b	b	c	c	d	c	a	a	b	b	a	c	c	a	b	d	b	a
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	a	a	d	c	d	a	a	b	c	c	d	d	b	d	b	a	d	c	c
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
c	d	a	a	d	d	c	b	a	b	b	d	c	b	a	d	c	b	b	b,c
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b,c,d	d	c	a	c	c	b	d	b	c	b	b	d	b	c	c	b	b	b	c
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
a	d	b	a	a	d	c	b	d	a	b	b	b	a	b	c	b	c	b	a
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
d	b	b	c	c	c	c	d	b	d	a	c	b	d	b	a	b	b	c	a
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
c	c	b	a	a	a	b	c	c	a	d	b	b	d	d	c	a	d	d	b
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
c	d	d	c,d	b	b	a	a	d	a	b	b	c	b	a	d	d	c	a	b
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
c	c	a	d	b	a	c	a	d	c	a	b	a	a	b	c	a	b	c	a
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**	**295**	**296**	**297**	**298**	**299**	**300**
c	c	c	a	b	a	c	d	c	a	a	b	c,d	c	b	a	d	d	c	a
301	**302**	**303**	**304**	**305**	**306**	**307**	**308**	**309**	**310**	**311**	**312**	**313**	**314**	**315**	**316**	**317**	**318**	**319**	**320**
d	b	b	c	c	a	d	c	a	b	a	a	a	c	c	c	c	b	d	a
321	**322**	**323**	**324**	**325**	**326**	**327**	**328**	**329**	**330**	**331**	**332**	**333**	**334**	**335**	**336**	**337**	**338**	**339**	**340**
d	d	b	a	c	a	d	d	d	c	a	c	c	d	c	d	c	a	b	a
341	**342**	**343**	**344**	**345**	**346**	**347**	**348**	**349**	**350**	**351**	**352**	**353**	**354**	**355**	**356**	**357**	**358**	**359**	**360**
d	c	c	a	c	b	d	c	b	a	b	a	c	d	b	b	d	a	a	a
361	**362**	**363**	**364**	**365**	**366**	**367**	**368**	**369**	**370**	**371**	**372**	**373**	**374**	**375**	**376**	**377**	**378**	**379**	**380**
c	c	a	a	a	b	b	c	b	b	d	a	d	a	d	a	b	b	a	b

381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400
c	b	a	a	d	d	a	d	a	c	b	b	a	a	b	a	b	c	c	a