CHAPTER

4

QUADRATIC EQUATIONS

Introduction to Quadratic Equations

You must have come across different types of equations in mathematics such as linear equations, linear equation in two variables and many more. This chapter will introduce you to a new set of equations called quadratic equations. The polynomial of the form $ax^2 + bx + c, a \neq 0$ is a quadratic polynomial. When we equate this quadratic polynomial to zero, we get a quadratic equation i.e. $ax^2 + bx + c = 0, a \neq 0$.

An equation of the form $ax^2 + bx + c = 0, a \neq 0$ where $a, b, c \in R$ is called a quadratic equation. The constants a, b, c is called the coefficients of equation. There will be two roots for a quadratic equation and can be found by solving the equation $ax^2 + bx + c = 0$. Roots are also called solutions or zeros of quadratic equations.

The study of quadratic equation is important because of its application to parabolas and other conic sections.

In this chapter, we will study about the quadratic equations and various ways of finding their roots.

Standard form of a Quadratic Equation:

The standard form of quadratic equation is $ax^2 + bx + c = 0, a \neq 0$ where $a, b, c \in R$. Quadratic equations, in general are of the following types:-

(i) When $b = 0, c \neq 0$ i.e., of the type $ax^2 + c = 0$, it becomes pure quadratic equation.

(ii) When $b \neq 0, c = 0$, i.e. of the type $ax^2 + bx = 0$ or when $b \neq 0, c \neq 0$ i.e., of the type $ax^2 + bx + c = 0$, it is an affected quadratic equation.

Mind it

- ❒ A quadratic equation has two and only two roots.
- ❒ A quadratic equation cannot have more than two different roots.
- ❒ If α be a root of the quadratic equation $ax^2 + bx + c = 0$ then $(x - \alpha)$ is a factor of quadratic equation $ax^2 + bx + c = 0$

Sum and Product of the Roots of a Quadratic Equations

Let α, β be the roots of a quadratic equation $ax^2 + bx + c = 0, a \neq 0$, then

$$\alpha + \beta = \frac{-b}{a} = -\left(\frac{\text{coefficient of } x}{\text{coefficient of } x^2}\right)$$

and, $$\alpha\beta = \frac{c}{a} = \left(\frac{\text{constant term}}{\text{coefficient of } x^2}\right)$$

Sign of the Roots

Sign of ($\alpha + \beta$)	Sign of ($\alpha \times \beta$)	Sign of the α, β
+ve	+ve	Both roots are positive.
–ve	+ve	Both roots are negative.
+ve	–ve	Roots are of opposite signs.
–ve	–ve	Roots are of opposite signs.

Mind it

- If the two roots α and β be reciprocal to each other, then a = c.
- If the two roots α and β be equal in magnitude and opposite in sign then b = 0.

Example

1. Which of the following are quadratic equations, give reason:

(i) $4x^2 - 5x + 13 = 0$

(ii) $x(x + 1) = x^2 + 5$

(iii) $2x + \frac{5}{x} = x^2$

(iv) $x^2 + \frac{2}{x^2} = 2$

Ans. (i) Since, $4x^2 - 5x + 13$ is of form $ax^2 + bx + c = 0$

$\Rightarrow 4x^2 - 5x + 13 = 0$, is a quadratic equation.

(ii) $x(x + 1) = x^2 + 5$

$x^2 + x = x^2 + 5$

i.e., $x - 5 = 0$

It is not of the form $ax^2 + bx + c = 0$

Therefore, the given equation is not a quadratic equation.

(iii) $2x + \frac{5}{x} = x^2$

$\Rightarrow 2x^2 + 5 = x^3$

$\Rightarrow x^3 - 2x^2 - 5 = 0$; which is cubic and not a quadratic equation.

(iv) $x^2 + \frac{2}{x^2} = 2$

$\Rightarrow x^4 + 2 = 2x^2$

$\Rightarrow x^4 - 2x^2 + 2 = 0$; which is biquadratic and not a quadratic equation.

2. Determine whether the given value of x is the solution of the given equation or not:

$(3x+8)(2x+5) = 0$; $x = 2\frac{2}{3}$.

Ans. Put $x = 2\frac{2}{3} = \frac{8}{3}$

$$(3x+8)(2x+5) = \left(3\times\frac{8}{3}+8\right)\left(2\times\frac{8}{3}+5\right)$$

$$= (8+8)\left(\frac{16}{3}+5\right) = 16\times\frac{31}{3} = \frac{496}{3} \neq 0$$

Therefore the given value of x is not the solution of the given equation.

3. In each of the following, determine whether the given values are solutions (roots) of the equation or not:

(i) $4x^2 - 2x - 2 = 0$; $x = 1$

(ii) $x^2 + \sqrt{2}x - 4 = 0$; $x = \sqrt{2}$, $x = -2\sqrt{2}$

Ans. (i) Given quadratic equation is $4x^2 - 2x - 2 = 0$

For $x = 1$, L.H.S $= 4x^2 - 2x - 2$

$= 4(1)^2 - 2(1) - 2$

$= 4 - 2 - 2 = 0 = $ R.H.S

$\therefore$ x = 1 is a solution of the given equation.

(ii) Given quadratic equation is $x^2 + \sqrt{2}x - 4 = 0$

For $x = \sqrt{2}$, L.H.S $= x^2 + \sqrt{2}x - 4$

$= \left(\sqrt{2}\right)^2 + \sqrt{2}\left(\sqrt{2}\right) - 4$

= 2 + 2 – 4 = 0 = R.H.S.

$\therefore$ $x = \sqrt{2}$ is a solution of the given equation.

For $x = -2\sqrt{2}$,

L.H.S $= \left(-2\sqrt{2}\right)^2 + \sqrt{2} \times \left(-2\sqrt{2}\right) - 4$

4 × 2 – 2 × 2 – 4 = 0 R.H.S.

$\therefore$ $x = -2\sqrt{2}$ is also a solution of the given equation.

4. **The sum of a fraction and 7 times of its reciprocal is 45/96. Represent the above statement in the form of a quadratic equation.**

Ans. Let the fraction be x. Then, according to the question,

$$x + \frac{7}{x} = \frac{45}{96} \Rightarrow 96x^2 - 45x + 672 = 0$$

This is the required quadratic equation.

5. **Ekom is selling tickets for a ride on a giant wheel. If he charges ₹ 30, he will sell 40 tickets. For each ₹ 1 decrease in price, 9 more tickets would get sold. Represent the above problem in the form of a quadratic equation.**

Ans. Let n denotes the number of ₹ 1 decrease.

Total revenue can be described as the product of price per ticket and the number of tickets sold.

Revenue = price per ticket × no. of tickets sold

$\therefore$ The current revenue is R = 30 × 40 because there are 40 tickets sold at price of ₹ 30 each.

According to the question,

The price decreases for each ₹ 1 decrement, the new price will be (30 – n)

For each ₹ 1 decrement, 9 more tickets would get sold. So, total tickets sold changes from 40 to (40 + 9n)

$\therefore$ New Revenue = (30 – n) (40 + 9n)

$= 1200 - 270n - 40n - 9n^2$

$= -9n^2 + 230n + 1200$

6. **A fruit grower has 400 crates of apples ready for market and will have 15 more crates for each day the shipment is delayed. The price per crate decreases by ₹ 3. The present price is ₹ 70. Form a quadratic equation to represent the above situation.**

Ans. Let n denotes the number of days by which shipment get delayed.

Revenue = price per crates × No. of crates sold

$\therefore$ The current revenue is R = 70 × 400

According to the question,

For 1 day delay in the shipment, the price per crate will be decreased by ₹ 3. Therefore, The new price will be (70 – 3n).

For each 1 day delayed in the shipment, the grower need to ship 15 more crates.

Therefore, amount of crates will be (400 + 15n).

$\therefore$ New revenue = (70 – 3n) (400 + 15n)

$= 28000 - 1050n - 1200n - 45n^2$

$= -45n^2 - 150n + 28000$

Solution of Quadratic Equations

The values of x which satisfies the given quadratic equation is known as its roots or zeros or solutions. The solutions of quadratic equations can be found using following methods:

(i) Factorisation Method

(ii) Completing the square

(iii) Quadratic Formula

Solution of Pure Quadratic Equations

Solutions of pure quadratic equations can be found by reducing it to the form $x^2 = k$ and then the values of x can be determined by taking the square root on both sides. The roots evidently will be $+\sqrt{k}$ or $-\sqrt{k}$ written as $\pm\sqrt{k}$.

Example

1. **Solve the given quadratic equations:**

   (i) $9x^2 - 28 = 100 + 7x^2$

   (ii) $(x + 6)(x - 6) = 13$

Ans. (i) Given, $9x^2 - 28 = 100 + 7x^2$

$\Rightarrow 9x^2 - 7x^2 = 128$

$\Rightarrow x^2 = \frac{128}{2} = 64$

This is of the form $x^2 = k$ i.e., this is a pure quadratic equation.

$\therefore x = \pm\sqrt{64}$

$\Rightarrow x = \pm 8$

(ii) $x^2 - 36 = 13$

$x^2 = 49$

This is of the form $x^2 = k$ i.e., this is a pure quadratic equation.

$\therefore x = \pm\sqrt{49}$

$\Rightarrow x = \pm 7$

2. **Solve the given quadratic equation for x:**

$$\mathbf{\frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{5}{2}}$$

Ans. $\frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{5}{2}$

by taking **L.C.M**

$2\left[(x+2)^2 + (x-2)^2\right] = 5(x-2)(x+2)$

$2(x + 2)^2 + 2(x - 2)^2 = 5(x^2 - 4)$

$\left[\because (a-b)(a+b) = a^2 - b^2\right]$

$2(x^2 + 4x + 4) + 2(x^2 - 4x + 4) = 5(x^2 - 4)$

$2x^2 + 8x + 8 + 2x^2 - 8x + 8 = 5x^2 - 20$

$4x^2 + 16 = 5x^2 - 20$

$x^2 = 36 \Rightarrow x = \pm 6$

3. **Solve for x:**

   $(x + 3)(x + 4) + (x - 2)(x - 5) = 30$

Ans. $x^2 + 7x + 12 + x^2 - 7x + 10 = 30$

$2x^2 + 22 = 30 \Rightarrow 2x^2 = 8 \Rightarrow x^2 = 4$

$x = \pm 2$

Solution of Adfected Quadratic Equations

1. **Factorisation method**

   Factorisation refers to the breaking up of an expression (polynomial expression) into smaller expressions. For e.g. $x^2 - 10x + 21$ is equal to $(x - 3)(x - 7)$. Here, a trinomial expression has been broken into binomial expressions.

   Let's see some more examples:

   1. $7x + 42$ is equal to $7(x + 6)$. Here, a binomial expression has been broken into a monomial and a binomial expression.

   2. $x^3 + x^2 - x - 1$

      $\Rightarrow x^2(x + 1) - 1(x + 1)$

      $\Rightarrow (x^2 - 1)(x + 1)$. Here, the given polynomial expression has been broken into binomial expressions.

   But the most used method in factoring the quadratic equations is known as splitting the middle term.

Algorithm:

Step (i) Let the quadratic equation be $ax^2 + bx + c$, $a \neq 0$. i.e., $2x^2 - 5x + 3 = 0$

Here $a = 2$, $b = -5$, and $c = 3$

Step (ii) Find the factors of the product of a and c.

Here the product of a and c is 6.

So, the factors of 6 are (1, 6), (2, 3), (3, 2), (6, 1), (–1, –6), (–2, –3), (–3, –2) and (–6, –1).

Step (iii) Express the coefficient of middle term as the sum or difference of the factors obtained in step (ii).

By splitting the middle term, we get

$$-5 = (-3) + (-2) \text{ or } -5 = (-2) + (-3)$$

Step (iv) Split the middle term in two parts obtained in step (iii).

Now, we get

$2x^2 + [(-2) + (-3)]x + 3 = 0$

$2x^2 - 2x - 3x + 3 = 0$

Step (v) Then group the terms to form pairs - the first two terms and the last two terms. Factor each pair by taking out the common factors.

i.e., $2x(x - 1) - 3(x - 1) = 0$

$(2x - 3)(x - 1) = 0$

$\therefore$ $x = 1$ and $x = \frac{3}{2}$ are the roots of given quadratic equation.

Example

1. Solve the quadratic equation: $x^2 - 3x - 10 = 0$

Ans. The given quadratic equation is $x^2 - 3x - 10 = 0$. By comparing it with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -3$ and $c = -10$.

Here, the product of a and c is (–10).

List down the factors of –10:

$1 \times (-10)$, $10 \times (-1)$, $2 \times (-5)$, $5 \times (-2)$

Middle term, $-3 = (-5) + 2$

So, we can split the middle term as follows:

$x^2 - 3x - 10 = 0$

$= x^2 - 5x + 2x - 10 = 0$

$= x(x - 5) + 2(x - 5)$

$= (x + 2)(x - 5)$

Equating each factor to zero gives;

$x + 2 = 0 \Rightarrow x = -2$

$x - 5 = 0 \Rightarrow x = 5$

Therefore, the solution are $x = -2$ and $x = 5$.

2. Solve the quadratic equation: $x^2 + 10x + 25 = 0$.

Ans. The given quadratic equation is $x^2 + 10x + 25 = 0$. By comparing it with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 10$ and $c = 25$.

Here, the product of a and c is 25.

List down the factors of 25: (1, 25), (5, 5), (25, 1), (–1, –25), (–5, –5) and (–25, –1).

Middle term, $10 = 5 + 5$

So, we can split the middle term as follows:

$x^2 + 10x + 25 = x^2 + 5x + 5x + 25$.

$= x(x + 5) + 5x + 25$

$= x(x + 5) + 5(x + 5)$

$= (x + 5)(x + 5)$

Therefore, $x = -5$ is the answer.

3. Solve $x^2 + 4x - 5 = 0$

Ans. $x^2 + 4x - 5 = 0$

$x^2 + 5x - x - 5 = 0$

$x(x + 5) - 1(x + 5) = 0$

$(x - 1)(x + 5) = 0$

Therefore, $x = 1$ and $x = -5$ are the solutions.

4. Factorize: $2x^2 + 24x + 64$.

Ans. $2x^2 + 24x + 64 = 0$

$\Rightarrow 2x^2 + 16x + 8x + 64 = 0$

$\Rightarrow 2x(x + 8) + 8(x + 8) = 0$

$\Rightarrow (2x + 8)(x + 8) = 0$

Therefore, $x = -4$ and $x = -8$ are the solutions.

5. Factorize: $2x^2 - 7x + 6 = 3$

Ans. $2x^2 - 7x + 6 - 3 = 0$

$2x^2 - 7x + 3 = 0$

$2x^2 - 6x - x + 3 = 0$

$2x(x - 3) - 1(x - 3) = 0$

$(2x - 1)(x - 3) = 0$

Therefore, $x = \frac{1}{2}$ and $x = 3$ are the solutions of the given quadratic equation.

6. Factorize $x^2 + (4 - 3y)x - 12y = 0$

Ans. Given equation is quadratic in x as highest power of x is 2. Take y as constant.

Expand the equation by opening brackets

$x^2 + 4x - 3xy - 12y = 0$

Factorize;

$\Rightarrow x(x + 4) - 3y(x + 4) = 0$

$(x + 4)(x - 3y) = 0$

$\Rightarrow x + 4 = 0$ or $x - 3y = 0$

$\Rightarrow x = -4$ or $x = 3y$

Thus, $x = -4$ or $x = 3y$ are the solutions of the given quadratic equation.

2. Completing the square

Completing the square is a method used to solve quadratic equations by adding constants to both sides of the equation so that the left side becomes a perfect square trinomial.

For e.g. the number that should be added to the equation $x^2 - 4x = 0$ in order to make it a perfect square trinomial is 4 i.e. $x^2 - 4x + 4 = 4 \Rightarrow (x - 2)^2 = 4$ $(\because a^2 - 2ab + b^2 = 0)$

Algorithm:

Step (i) Let the quadratic equation be $ax^2 + bx + c = 0, a \neq 0$.

Step (ii) If coefficient of x^2 is not unity, make it unity i.e., obtain $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Step (iii) Shift the constant term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step (iv) Take the coefficient of x, divide it by 2 and square whatever you get and add this to both sides of the equation i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obtain $x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$

Step (v) Write L.H.S. as the perfect square of a binomial expression and simplify R.H.S. to get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step (vi) Take square root of both sides to get $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Step (vii) Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS.

Example

1. Find the roots of the quadratic equation $2x^2 - 7x + 3 = 0$ by the method of completing the square.

Ans. The given quadratic equation is $2x^2 - 7x + 3 = 0$ is of the form $ax^2 + bx + c = 0$.

As coefficient of x^2 is not unity, first we make it unity.

$$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Next we shift the constant term on R.H.S

$$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$$

Now add half of the square of the coefficient of x to both the sides

$$\Rightarrow x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = -\frac{3}{2} + \left(\frac{7}{4}\right)^2$$

$$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$$

Next write L.H.S as a perfect square of the binomial expression.

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{-24 + 49}{16}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

In next step, we need to take square root of both the sides.

$$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$$

$$\Rightarrow x - \frac{7}{4} = \frac{5}{4} \quad \text{or} \quad x - \frac{7}{4} = -\frac{5}{4}$$

$$\Rightarrow x = \frac{7}{4} + \frac{5}{4} \quad \text{or} \quad x = \frac{7}{4} - \frac{5}{4}$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = \frac{1}{2}$$

2. Find the roots of the quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ by the method of completing the square.

Ans. $4x^2 + 4\sqrt{3}x + 3 = 0$

$$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0$$

$$\Rightarrow x^2 + \sqrt{3}x = -\frac{3}{4}$$

$$\Rightarrow x^2 + 2 \times x \times \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}$$

$$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} - \frac{3}{4}$$

$$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$\therefore$ Roots are: $\dfrac{-\sqrt{3}}{2}$ and $\dfrac{-\sqrt{3}}{2}$

3. Find the roots of the quadratic equation $2x^2 + x + 4 = 0$ by the method of completing the square.

Ans. $2x^2 + x + 4 = 0$

$$\Rightarrow x^2 + \frac{x}{2} + 2 = 0$$

$$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = -2 + \left(\frac{1}{4}\right)^2$$

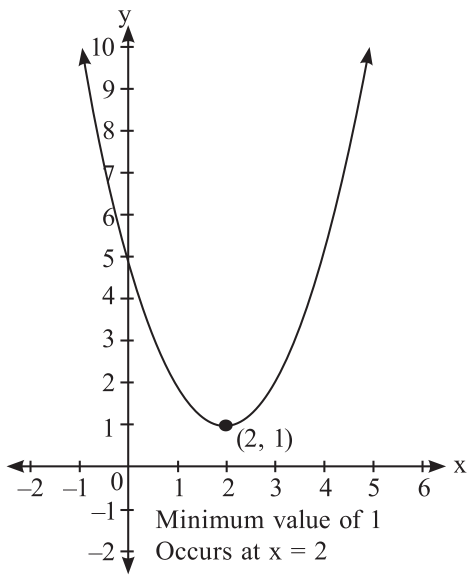
$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -2 + \frac{1}{16}$$

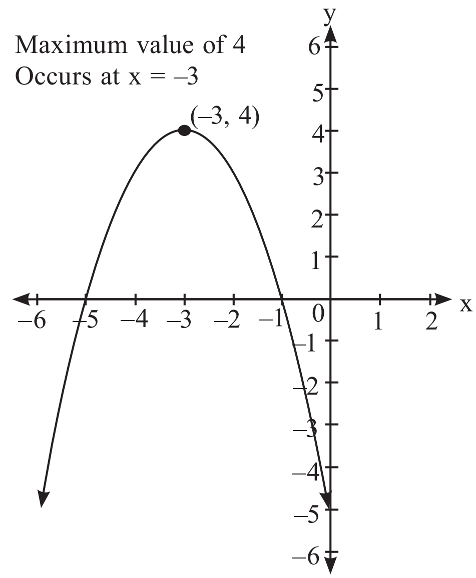
$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$$

Real roots of given quadratic equation doesn't exists as the square of a real number cannot be negative.

Maximum or Minimum Value of a Quadratic Equation

The maximum value of a function is the place where the graph has a vertex at its highest point whereas the minimum value of a function is the place where the graph has a vertex at its lowest point.





One way to find the maximum or minimum value of a quadratic function is to write the function in vertex form by completing the square. The vertex form of a quadratic function is $y = a(x - h)^2 + k$, where $a \neq 0$. The vertex of the graph is (h, k).

- The graph of f(x) or y opens downward when $a < 0$, which means f(x) or y has a maximum value equals to the y-coordinate of the vertex i.e. k.
- The graph of f(x) or y opens upward when $a > 0$, which means f(x) or y has a minimum value equals to the y-coordinate of the vertex i.e. k.

Example

1. Find the minimum value of $y = x^2 + 4x - 1$.

Ans. To obtain the minimum value of the function we should first convert it into the vertex form as follows:

$y = x^2 + 4x - 1$

$y + 1 = x^2 + 4x$

$y + 1 + 4 = x^2 + 4x + 4$ (completing the square for $x^2 + 4x$)

$y + 5 = (x + 2)^2$

$y = (x + 2)^2 - 5$

If you compare this to the vertex form of the equation i.e., $y = a(x - h)^2 + k$, we get $a = 1$, $h = -2$ and $k = -5$.

As a is positive ($a = 1$), the parabola opens up and the y-coordinate of the vertex is the minimum value.

So, the function has a minimum value of –5.

Note: No matter what you put here as the value of x, the minimum value of f(x) or y that you can get is –5.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$x^2 + 2 \times \frac{b}{2a} \times x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b^2}{4a^2}\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

On rearranging, we get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mind it

We get the maximum or minimum value of the quadratic equation $ax^2 + bx + c = 0$ at $x = \frac{-b}{2a}$.

- When $a > 0$, the minimum value of the given equation will be, $y = \frac{4ac - b^2}{4a}$.
- When $a < 0$, the maximum value of the given equation will be, $y = \frac{4ac - b^2}{4a}$.

3. Quadratic Formula (Sridharacharya Formula)

Sridharacharya was an Indian mathematician, Sanskrit pandit and a philosopher. He gave the quadratic formula to solve the quadratic equation, be it adfected or pure.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Algorithm:

Step (i) Find the value of a, b and c by comparing the given quadratic equation with general quadratic equation i.e., $ax^2 + bx + c = 0$

Step (ii) Find the discriminant of the quadratic equation, using $D = b^2 - 4ac$.

Step (iii) Now find the roots of the equation, using $x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$

Example

1. Solve the following quadratic equations by using quadratic formula:

(i) $x^2 - 7x + 12 = 0$

(ii) $2y^2 + 3y - 35 = 0$

Ans. (i) Comparing the given equation $x^2 - 7x + 12 = 0$ with standard quadratic equation $ax^2 + bx + c = 0$; we get : $a = 1$, $b = -7$ and $c = 12$

$\therefore D = b^2 - 4ac$

$= (-7)^2 - 4 \times 1 \times 12$

$= 1$

and $\sqrt{b^2 - 4ac} = \sqrt{1} = 1$

$\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{7 \pm 1}{2 \times 1}$

$= \frac{7+1}{2}$ or $\frac{7-1}{2}$

$= 4$ or 3

(ii) Comparing the given equation $2y^2 + 3y - 35 = 0$ with equation $ay^2 + by + c = 0$ we get: $a = 2$, $b = 3$ and $c = -35$

$\therefore D = b^2 - 4ac$

$= (3)^2 - 4(2)(-35)$

$= 289$

$\Rightarrow \sqrt{b^2 - 4ac} = \sqrt{289} = 17$

Hence, $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad y = \frac{-3 \pm 17}{2(2)}$

$= -5$ or $\frac{7}{2}$

2. Using the quadratic formula, solve the equation:

$a^2b^2x^2 - (4b^4 - 3a^4)\ x - 12a^2b^2 = 0$

Ans. Comparing the given equation with

$Ax^2 + Bx + C = 0$, we get:

$A = a^2b^2$, $B = -(4b^4 - 3a^4)$ and $C = -12a^2b^2$

$\therefore B^2 - 4AC = [-(4b^4 - 3a^4)]^2 - 4 \times a^2b^2 \times (-12a^2b^2)$

$= 16b^8 + 9a^8 - 24a^4b^4 + 48a^4b^4$

$= 16b^8 + 9a^8 + 24a^4b^4$

$= (4b^4 + 3a^4)^2$

$$\sqrt{B^2 - 4AC} = 4b^4 + 3a^4$$

$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{\left(4b^4 - 3a^4\right) \pm \left(4b^2 + 3a^2\right)}{2 \times a^2b^2}$$

$$= \frac{4b^4 - 3a^4 + 4b^4 + 3a^4}{2a^2b^2} \text{ or } \frac{4b^4 - 3a^4 - 4b^4 - 3a^4}{2a^2b^2}$$

$$= \frac{8b^4}{2a^2b^2} \text{ or } \frac{-6a^4}{2a^2b^2}$$

$$= \frac{4b^2}{a^2} \text{ or } \frac{-3a^2}{b^2}$$

3. For a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, prove that: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(Proof of Quadratic Formula)

Ans. Given, $ax^2 + bx + c = 0$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$x^2 + 2 \times \frac{b}{2a} \times x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b^2}{4a^2}\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

On rearranging, we get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of Roots

Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Nature of Roots

- Two distinct and real roots: $D = b^2 - 4ac > 0$
- Two equal and real roots: $D = b^2 - 4ac = 0$
- No real root: $D = b^2 - 4ac < 0$

Mind it

If a, b, c $\in$ Q and $p+\sqrt{q}$ is one root of quadratic equation then the other root must be the conjugate i.e., $p-\sqrt{q}$ and vice-versa. Here p and q are real numbers.

Example

1. Without solving for 'x', examine the nature of roots of the equations :

(i) $2x^2 + 2x + 3 = 0$

(ii) $x^2 + 5x + 6 = 0$

(iii) $x^2 - 5x - 2 = 0$

Ans. (i) Comparing $2x^2 + 2x + 3 = 0$ with $ax^2 + bx + c = 0$;

we get: a = 2, b = 2 and c = 3

$D = b^2 - 4ac$

$= (2)^2 - 4 \times 2 \times 3$

$= -20$; which is negative.

$\therefore$ The given equation has no real root.

(ii) Comparing $x^2 + 5x + 6 = 0$ with $ax^2 + bx + c = 0$;

we get : a = 1, b = 5 and c = 6

$D = b^2 - 4ac$

$= (5)^2 - 4(1)(6)$

$= 25 - 24 = 1$,

As D > 0, given equation has distinct and real roots.

(iii) Comparing $x^2 - 5x - 2 = 0$ with $ax^2 + bx + c = 0$;

we get : a = 1, b = – 5 and c = – 2

$D = b^2 - 4ac$

$= (-5)^2 - 4 \times 1 \times -2$

$= 25 + 8 = 33$;

which is positive but not a perfect square.

$\therefore$ The roots of the given equation are distinct and unequal.

2. Find the value of p, for which the roots of the equation $(3p + 1)x^2 + (11 + p)x + 9 = 0$ are equal.

Ans. Compare the given equation with $ax^2 + bx + c = 0$;

we get : a = 3p + 1, b = 11 + p and c = 9

$D = b^2 - 4ac$

$= (11 + p)^2 - 4(3p + 1) \times 9$

$= 121 + 22p + p^2 - 108p - 36$

$= p^2 - 86p + 85$

By factorising using middle terms splitting, we get

$= p^2 - 85p - p + 85$

$= (p - 85)(p - 1)$

Since the roots are equal i.e., D = 0

$\Rightarrow (p - 85)(p - 1) = 0$

$\Rightarrow p = 85$ or $p = 1$

Formation of Quadratic Equations

Suppose α and β are the two roots of the quadratic equation $ax^2 + bx + c = 0$, then the quadratic equation can be written as,

$$x^2 - (\text{sum of roots})\,x + (\text{product of roots}) = 0$$

i.e., $$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

Let 2 and 3 are the roots of some quadratic equation then, $x^2 - 5x + 6 = 0$ is the corresponding quadratic equation.

Example

1. Construct the quadratic equation whose roots are given below -

(i) 3, –3 (ii) $3+\sqrt{3}, 3-\sqrt{3}$

Ans. (i) The given roots are 3 and –3.

Sum of the roots = 3 + (–3) = 0

and, the product of the roots = 3(–3) = –9

∴ The required quadratic equation is :

x^2 – (sum of roots) x + (product of roots) = 0

$\Rightarrow x^2 - (0)x + (-9) = 0$, i.e., $x^2 - 9 = 0$

(ii) The given roots are $3+\sqrt{3}$ and $3-\sqrt{3}$.

Sum of the roots = $3+\sqrt{3}+3-\sqrt{3} = 6$

and, the product of the roots

$= (3+\sqrt{3})(3-\sqrt{3}) = 6$

∴ The required quadratic equation is :

x^2 – (sum of roots) x + (product of roots) = 0

$\Rightarrow x^2 - 6x + 6 = 0$

2. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ then find the values of:

(i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$

Ans. We know, sum of roots $(\alpha+\beta) = -\frac{b}{a}$

And, product of roots $(\alpha\beta) = \frac{c}{a}$; therefore:

(i) $(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$

$$\Rightarrow \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= \left(-\frac{b}{a}\right)^2 - \frac{2c}{a}$$

$$= \frac{b^2-2ac}{a^2}$$

(ii) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$$= \left(-\frac{b}{a}\right)\left(\frac{b^2-2ac}{a^2} - \frac{c}{a}\right)$$

$$= \left(-\frac{b}{a}\right)\left(\frac{b^2-2ac-ac}{a^2}\right) = -\frac{b(b^2-3ac)}{a^3}$$

Equations Reducible to Quadratic Equations

Some non-quadratic equations which can be converted into a quadratic equation by making a substitution are known as equations reducible to quadratic equations. These equations are usually difficult to solve but can be solved easily if we convert them into a quadratic form.

Type 1: Equations of the form $ax^4 + bx^2 + c = 0$

Method: Substitute $x^2 = y$ in these type of equations and then solve.

Example

1. Solve the following equations:

(i) $x^4 - 7x^2 + 12 = 0$

(ii) $z^4 - 10z^2 + 9 = 0$

Ans. (i) $x^4 - 7x^2 + 12 = 0$ is of the form $ax^4 + bx^2 + c = 0$

Therefore, substituting $x^2 = y$

$x^4 - 7x^2 + 12 = 0$

$\Rightarrow y^2 - 7y + 12 = 0$

Solving by middle term splitting as follows:

$\Rightarrow y^2 - 4y - 3y + 12 = 0$

$\Rightarrow y(y-4) - 3(y-4) = 0$

$\Rightarrow (y-3)(y-4) = 0$

i.e., y = 3 or 4

Putting these values in equation (i) we get,

$y = 3 \Rightarrow x^2 = 3$; $y = 4 \Rightarrow x^2 = 4$

$\Rightarrow x = \pm\sqrt{3}$; $x = \pm 2$

∴ Solution of the given equation is: $\pm\sqrt{3}$, ± 2.

(ii) Substituting $z^2 = x$

$z^4 - 10z^2 + 9 = 0$

$\Rightarrow x^2 - 10x + 9 = 0$

$\Rightarrow x^2 - 9x - x + 9 = 0$

$\Rightarrow x(x-9) - 1(x-9) = 0$

$\Rightarrow (x-9)(x-1) = 0$

i.e., x = 9 or 1

$x = 9 \Rightarrow z^2 = 9$; $x = 1 \Rightarrow z^2 = 1$

$\Rightarrow z = \pm 3$; $\Rightarrow z = \pm 1$

∴ Solution of the given equation is: ± 3, ± 1.

Type 2: Equation of the form: $px + \frac{q}{x} = r$

Method : Multiply each term by x and simplify.

Example

1. **Solve:**

   (i) $x+\frac{16}{x}=-8$

   (ii) $3y+\frac{5}{16y}=2$

Ans. (i) $x+\frac{16}{x}=-8$

$\Rightarrow x^2 + 16 = -8x$ [Multiplying each term by x]

$\Rightarrow x^2 + 8x + 16 = 0$

$\Rightarrow x^2 + 4x + 4x + 16 = 0$

$\Rightarrow x(x + 4) + 4(x + 4) = 0$

$\Rightarrow x = -4, -4$

$\therefore$ Required solution are $-4, -4$

(ii) $3y+\frac{5}{16y}=2$

$\Rightarrow 3y \times 16y + 5 = 2 \times 16y$

[Multiplying each term by 16y]

$\Rightarrow 48y^2 - 32y + 5 = 0$

$\Rightarrow 48y^2 - 12y - 20y + 5 = 0$

$\Rightarrow 12y(4y - 1) - 5(4y - 1) = 0$

$\Rightarrow (4y - 1)(12y - 5) = 0$

$\Rightarrow 4y = 1$ or $12y = 5$

$\Rightarrow y=\frac{1}{4}$ or $y=\frac{5}{12}$

$\Rightarrow$ Required solutions are $\frac{1}{4}, \frac{5}{12}$

Type 3: Equations involving one radical:

A radical equation is an equation in which a variable is under a radical i.e., $\sqrt{a-x^2}=bx+c$

Method:

1. Squaring both the sides to get $a - x^2 = (bx + c)^2$
2. Now simplify it to get a quadratic equation in the form of $ax^2 + bx + c = 0$.
3. Solve the quadratic equation obtained.

Example

1. **Solve $\sqrt{x}+2x=1$.**

Ans. $\sqrt{x}+2x=1 \Rightarrow \sqrt{x}=1-2x$

$\Rightarrow x = (1 - 2x)^2$

$\Rightarrow x = 1 + 4x^2 - 4x$

$\Rightarrow 4x^2 - 5x + 1 = 0$

$\Rightarrow 4x^2 - 4x - x + 1 = 0$

$\Rightarrow 4x(x - 1) - 1(x - 1) = 0$

$\Rightarrow (x - 1)(4x - 1) = 0$

$\Rightarrow x - 1 = 0$ or $4x - 1 = 0$

$\therefore x=1$ or $x=\frac{1}{4}$

❒ **Formation of a new quadratic equation by changing the roots of a given quadratic equation**

Let α, β be the roots of a quadratic equation $ax^2 + bx + c = 0$, then we can form a new quadratic equation as per the following rules-

☞ A quadratic equation whose roots are p more than the roots of the equation $ax^2 + bx + c = 0$ (i.e., the roots are $\alpha + p$ and $\beta + p$).

The required quadratic equation will be $a(x - p)^2 + b(x - p) + c = 0$

- A quadratic equation whose roots are less by p than the roots of the equation $ax^2 + bx + c = 0$, (i.e., the roots are $\alpha - p$ and $\beta - p$). The required equation will be $a(x + p)^2 + b(x + p) + c = 0$
- Let α and β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are p times the roots of the given equation can be written as $a\left(\frac{x}{p}\right)^2 + b\left(\frac{x}{p}\right) + c = 0$.
- Let α and β are the roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $1/\alpha$ and $1/\beta$ can be written as $a\left(\frac{1}{x}\right)^2 + b\left(\frac{1}{x}\right) + c = 0$ i.e., $cx^2 + bx + a = 0$
- A quadratic equation whose roots are 1/p times the roots of the equation $ax^2 + bx + c = 0$ (i.e., the roots are α/p and β/p)

  The required equation is $a(px)^2 + b(px) + c = 0$
- A quadratic equation whose roots are the negative of the roots of the equation $ax^2 + bx + c = 0$ (i.e., the root and $-\alpha$ and $-\beta$)

  The required equation is $ax^2 - bx + c = 0$
- A quadratic equation whose roots are the square of the roots of the equation $ax^2 + bx + c = 0$ (i.e., the roots are α^2 and β^2)

  The required equation will be $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

Summary

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation in x.

Roots of a Quadratic Equation

The value of x which satisfies the quadratic equation i.e., $ax^2 + bx + c = 0$ are called roots of a quadratic equation.

Types of Quadratic Equations

Pure Quadratic Equation
$ax^2 + c = 0$

Affected Quadratic Equation
$ax^2 + bx + c = 0$ or $ax^2 + bx = 0$

Pure quadratic equations $ax^2 + c = 0$ can be solved by taking square root.

Solution of Adfected Quadratic Equations

Factorisation

Completing the Square

Quadratic Formula

Nature of Roots

Two distinct and real roots
$D = b^2 - 4ac > 0$

Two equal and real roots
$D = b^2 - 4ac = 0$

No real root
$D = b^2 - 4ac < 0$

Quick Recall

Fill in the blanks

1. If the product ac in the quadratic equation $ax^2 + bx + c = 0$ is negative, then the equation cannot have .... roots.
2. The values of k for which the equation $2x^2 + kx + x + 8 = 0$ will have real and equal roots are ....
3. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots of quadratic equation ....
4. The equation having maximum index of variable as 2 is called .... equation.
5. The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has no real roots if and only if discriminant is ....
6. The quadratic equation whose roots are the square of the sum of the roots and the square of the difference of the roots of the equation $x^2 - 3x + 2 = 0$ is ....
7. If the quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is ....
8. If a and b are the roots of $x^2 + x + 1 = 0$, then $a^2 + b^2$ = ...
9. If the discriminant of a quadratic equation is zero, then its roots are .... and ....
10. If D > 0, the roots of quadratic equations are real and ....

True and False Statements

1. Every quadratic equation has exactly one root.
2. The quadratic equation $x^2 + m = 0$ has no real roots.
3. If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
4. Sum of the reciprocals of the roots of the equation $x^2 + px + q = 0$ is 1/p.
5. 0.3 is a root of the equation $x^2 - 0.9 = 0$.
6. For the expression $ax^2 + 7x + 2$ to be quadratic, the possible values of a are non-zero real numbers.
7. If D = 0, then the quadratic equation $ax^2 + bx + c = 0$ has equal roots.
8. For $k = 2\sqrt{6}$, the quadratic equation $2x^2 + kx + 3 = 0$ has equal roots.
9. If sum of the roots is 5 and product is 2, then the quadratic equation is $x^2 - 2x + 5 = 0$
10. Every quadratic equation has at least two real roots.

Match The Followings

DIRECTION: Each question contains statements given in two Columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II

1. Column-II contains roots of quadratic equations given in Column-I.

Column I	Column II
(A) $3x^2 + 8x - 3 = 0$	(p) (–6, 4)
(B) $8x^2 + 16x + 10 = 202$	(q) (9, 36)
(C) $x^2 - 45x + 324 = 0$	(r) (3/5, –1)
(D) $5x^2 + 2x - 3 = 0$	(s) (1/3, –3)

a. A-s, B-p, C-q, D-r b. A-p, B-q, C-r, D-s
c. A-q, B-p, C-s, D-r d. A-r, B-q, C-p, D-s

2. Match the following columns.

Column I	Column II
(A) $6x^2 - 11x + 108 = 0$	(p) Fourth degree polynomial
(B) $(x + 2)^3 = 2x(x^2 - 1)$	(q) Quadratic equation
(C) $(3x - 7)^2 = 9x^2$	(r) Cubic equation
(D) $(2x^2 - 2)^2 = 3$	(s) Linear equation

a. A-s, B-q, C-r, D-p b. A-q, B-r, C-s, D-p
c. A-p, B-s, C-r, D-q d. A-p, B-r, C-q, D-s

Answers

Fill in the Blanks:

1. Non-real
2. 7 and –9
3. $a(x - h)^2 + b(x - h) + c = 0$
4. Quadratic
5. Less than
6. $x^2 - 10x + 9 = 0$
7. 4/3
8. –1
9. Real, equal
10. Unequal

True and False:

1. False
2. True
3. True
4. False
5. False
6. False
7. True
8. True
9. False
10. False

Match the Followings

1. (a) 2. (b)

NCERT Exercises

Exercise-I

1. Check whether the following are quadratic equations or not:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Exp. (i) Given,

$(x + 1)^2 = 2(x - 3)$

By using the formula for $(a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 7 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

(ii) Do it yourself.

(iii) Do it yourself.

(iv) Do it yourself.

(v) Do it yourself.

(vi) Do it yourself.

(vii) Do it yourself.

(viii) Do it yourself.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find speed of train.

Exp. (i) Let,

Breadth of the rectangular plot = x m

Hence, the length of the plot = (2x + 1) m.

Area of rectangle = length × breadth = 528 m²

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

(ii) Let, the first integer number = x

Hence, the next consecutive positive integer will be = x + 1

Product of two consecutive integers = 306

$\Rightarrow x(x + 1) = 306$

$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

(iii) Let,

Present age of Rohan = x years

Thus, Rohan's mother's age = x + 26

After 3 years, Age of Rohan = x + 3

Age of Rohan's mother will be = x + 26 + 3 = x + 29

The product of their ages after 3 years will be equal to 360, therefore

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

(iv) Let the original speed of the train be x km/h.

In 1st case, Distance = 480 km and speed = x km/h

$\Rightarrow$ Time taken = $\frac{\text{Distance}}{\text{Speed}} = \frac{480}{x}$ hrs

In 2nd case, the speed of the train is 8 km/h less than its original speed i.e. (x - 8) km/h and distance covered = 480 km.

And time taken to cover the same distance will be increased by 3 hrs.

Therefore,

Time taken to travel 480 km = $\left(\frac{480}{x} + 3\right)$ hrs

As we know,

Speed × Time = Distance

$$(x - 8)\left(\frac{480}{x} + 3\right) = 480$$

$\Rightarrow 480 + 3x - \frac{3840}{x} - 24 = 480$

$\Rightarrow 3x - \frac{3840}{x} = 24$

$\Rightarrow 3x^2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

Exercise-II

1. Find the roots of the following quadratic equations by factorisation:

(i) $\mathbf{x^2 - 3x - 10 = 0}$

(ii) $\mathbf{2x^2 + x - 6 = 0}$

(iii) $\mathbf{\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0}$

(iv) $\mathbf{2x^2 - x + \frac{1}{8} = 0}$

(v) $\mathbf{100x^2 - 20x + 1 = 0}$

Exp. (i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x - 5 = 0$ or $x + 2 = 0$

$\therefore x = 5$ or $x = -2$

(ii) Do it yourself.

(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}\, x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow \left(\sqrt{2}x + 5\right)\left(x + \sqrt{2}\right) = 0$

$\Rightarrow \sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$\therefore x = \frac{-5}{\sqrt{2}}$ or $-\sqrt{2}$

(iv) Do it yourself.

(v) Do it yourself.

2. Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had initially.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

Exp. (i) Let, the number of John's marble = x.

Therefore, number of marble Jivanti have = 45 – x

After losing 5 marbles each,

Number of marbles John have = x – 5

Number of jivanti's marble = 45 – x – 5 = 40 – x

Given, the product of their marbles is 124.

$\therefore (x - 5)(40 - x) = 124$

$\Rightarrow 40x - x^2 - 200 + 5x = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

$\Rightarrow x - 36 = 0$ or $x - 9 = 0$

$\Rightarrow x = 36$ or $x = 9$

Therefore,

If, John's marbles = 36,

Then, Jivanti's marbles = 45 – 36 = 9

And if John's marbles = 9, then jivanti has 36 marbles.

(ii) Let, number of toys produced in a day be x.

$\therefore$ Cost of production of each toy = ₹ (55 – x)

Given, total cost of production of the toys = ₹ 750

No. of toys × Cost of each toy

= Total cost of production.

$\therefore x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

$\Rightarrow x - 25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

$\therefore$ The number of toys produced in a day will be either 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

Exp. Do it yourself.

4. Find two consecutive positive integers, sum of whose squares is 365.

Exp. Let two consecutive positive integers be x and x + 1.

According to the question,

$x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$\Rightarrow (x + 14)(x - 13) = 0$

Thus, either, $x + 14 = 0$ or $x - 13 = 0$,

$\Rightarrow x = -14$ or $x = 13$

(–14 is rejected because it is a negative integer)

As, $x = 13$, therefore $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Exp. Do it yourself.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

Exp. Let the number of articles produced in a day be x.

Therefore, cost of production of each article = ₹ (2x + 3) ... (i)

Given, total cost of production is ₹ 90

No. of articles × Cost of production of each article

= Total Cost

$\therefore x(2x + 3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow (2x + 15)(x - 6) = 0$

$\Rightarrow 2x + 15 = 0$ or $x - 6 = 0$

$\Rightarrow x = -15/2$ or $x = 6$

As the number of articles produced can not be negative. Therefore, x can only be 6.

Hence, number of articles produced = 6

From equation (i)

Cost of each article = (2 × 6) + 3 = ₹ 15.

Exercise-III

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Exp. (i) $2x^2 - 7x + 3 = 0$

Dividing by 2 on both sides, we get

$$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation, we get

$$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49 - 24}{16} = \frac{25}{16}$$

$$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$$

$$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$$

$$\Rightarrow x = \frac{7}{4} + \frac{5}{4} \text{ or } x = \frac{7}{4} - \frac{5}{4}$$

$$\Rightarrow x = 3 \text{ or } \frac{1}{2}$$

(ii) Do it yourself.

(iii) Do it yourself.

(iv) Do it yourself.

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get, $a = 2$, $b = -7$ and $c = 3$

By using quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$\Rightarrow x = 3 \text{ or } \frac{1}{2}$$

(ii) Do it yourself.

(iii) Do it yourself.

(iv) Do it yourself.

3. Find the roots of the following equations:

(i) $\mathbf{x - \frac{1}{x} = 3,\ x \neq 0}$

(ii) $\mathbf{\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7}$

Exp. (i) Do it yourself.

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\Rightarrow \frac{x - 7 - x - 4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$\Rightarrow (x + 4)(x - 7) = -30$

$\Rightarrow x^2 - 7x + 4x - 28 = -30$

$\Rightarrow x^2 - 3x + 2 = 0$

We can solve this equation by factorization method now,

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 1$ or 2

4. The sum of the reciprocals of Rehman's ages (in years), 3 years ago and 5 years from now is 1/3. Find his present age.

Exp. Let present age of Rehman is x years. Three years ago, Rehman's age was $(x - 3)$ years. Five years after, his age will be $(x + 5)$ years.

According to the question:

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow \frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$

$\Rightarrow 3(2x + 2) = (x-3)(x+5)$

$\Rightarrow 6x + 6 = x^2 + 5x - 3x - 15$

$\Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow x(x - 7) + 3(x - 7) = 0$

$\Rightarrow (x - 7)(x + 3) = 0$

i.e., $x = 7$ or -3

As we know, age cannot be negative.

Therefore, Rehman's present age is 7 years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.

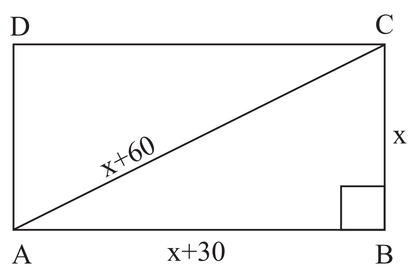
Exp. Do it yourself.

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Exp. Let the shorter side of the field be x m.

Then, larger side of the rectangle = (x + 30) m

As given, the length of the diagonal is = x + 60 m



On applying phythagoras therorm in Δ ABC, we get

$$BC^2 + AB^2 = AC^2$$

$\Rightarrow x^2 + (x + 30)^2 = (x + 60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$

$\Rightarrow x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow x(x - 90) + 30(x - 90) = 0$

$\Rightarrow (x - 90)(x + 30) = 0$

$\Rightarrow x = 90, -30$

As the side of the field cannot be negative, the length of the shorter side will be 90 m and the length of the larger side will be (90 + 30) m = 120 m.

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Exp. Do it yourself.

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Exp. Do it yourself.

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Exp. Let x hrs be the time taken by the smaller pipe to fill the tank.

$\therefore$ Time taken by the larger pipe to fill the tank will be (x–10) hrs.

if smaller pipe takes x hours to fill the pool, it will fill $\frac{1}{x}$ hours part in 1 hours.

Similarly, if larger pipe takes (x–10) hours to fill the pool, it will fill $\frac{1}{x-10}$ part in 1 hour.

Therefore, in 1 hours both the pipe can fill $\frac{1}{x}+\frac{1}{x-10}$ part of tank.

Now, it is given that

Time taken by both the taps to fill the tank

$$= 9\frac{3}{8}\text{ hrs} = \frac{75}{8}\text{hrs}$$

Thus, in 1 hour both the taps together can fill $\frac{8}{75}$ part of the tank.

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$\Rightarrow 75(2x - 10) = 8x^2 - 80x$

$\Rightarrow 150x - 750 = 8x^2 - 80x$

$\Rightarrow 8x^2 - 230x + 750 = 0$

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow 8x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(8x - 30) = 0$

$\Rightarrow x = 25, 30/8$

Time taken by the smaller pipe cannot be 30/8 hours, as the time taken by the larger pipe will become negative.

Hence, the smaller tap takes 25 hrs and larger tap takes $(x - 10)$. i.e., $(25 - 10) = 15$ hrs to fill the tank.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Exp. Let the average speed of passenger train = x km/h.

Then, the average speed of express train = (x + 11) km/h

We know, $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore$ Time taken by passenger train to cover 132 km $= \frac{132}{x}$

and time taken by express train to cover 132 km $= \frac{132}{x+11}$

According to the question, time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$$

$$\Rightarrow \frac{132(x+11-x)}{x(x+11)} = 1$$

$\Rightarrow 132 \times 11 = x(x + 11)$

$\Rightarrow x^2 + 11x - 1452 = 0$

$\Rightarrow x^2 + 44x - 33x - 1452 = 0$

$\Rightarrow x(x + 44) - 33(x + 44) = 0$

$\Rightarrow (x + 44)(x - 33) = 0$

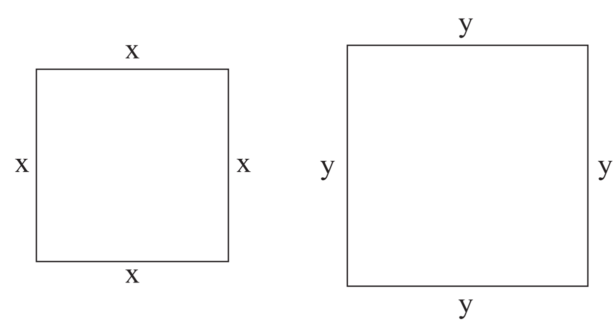
$\Rightarrow x = -44, 33$

Neglecting –44 as speed cannot be in negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

11. Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Exp. Let the side of smaller square be x and that of larger square be y.



Therefore, their perimeter will be 4x and 4y respectively and area of the squares will be x^2 and y^2 respectively.

According to the question, difference of perimeter is 24.

$4x - 4y = 24$

$\Rightarrow x - y = 6$

$\Rightarrow x = y + 6$

Also sum of the areas of square is 468.

$\Rightarrow x^2 + y^2 = 468$

$\Rightarrow (6 + y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$\Rightarrow y = -18, 12$

We know that side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m.

Exercise-IV

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Exp. (i) Given, $2x^2 - 3x + 5 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = -3$ and $c = 5$

We know, Discriminant $= b^2 - 4ac$

$= (-3)^2 - 4\,(2)\,(5)$

$= 9 - 40$

$= -31$

As you can see, $D < 0$

Therefore, no real root is possible for the given equation.

(ii) Do it yourself.

(iii) Do it yourself.

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

i. $2x^2 + kx + 3 = 0$

ii. $kx\,(x - 2) + 6 = 0$

Exp. (i) The given equation is $2x^2 + kx + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2$, $b = k$, $c = 3$

We know, Discriminant $= b^2 - 4ac$

$= k^2 - 4\,(2)\,(3)$

$= k^2 - 24$

For equal roots, $D = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24}$

$k = \pm 2\sqrt{6}$

Therefore, if this equation has two equal roots, k should be $+2\sqrt{6}$ and $-2\sqrt{6}$.

(ii) $kx(x - 2) + 6 = 0 \Rightarrow kx^2 - 2kx + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$

We know, Discriminant $= b^2 - 4ac$

$= (-2k)^2 - 4\,(k)\,(6)$

$= 4k^2 - 24k$

For equal roots, $D = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$

$\therefore$ $k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Exp. Do it yourself (Take help of Q5 given below).

4. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Exp. Let the present age of 1st friend = x

Given that

Present age of 1st friend + Present age of 2nd friend = 20

x + Present age of 2nd friend = 20

Present age of 2nd friend = 20 − x

4 years ago,

Age of 1st friend = x − 4

Age of 2nd friend = (20 − x) − 4 = 16 − x

Product of their ages 4 years ago was 48.

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get a = 1, b = –20, c = 112

We know that

$D = b^2 - 4ac$

$= (-20)^2 - (4 \times 1 \times 112)$

$= 400 - 448$

$= -48$

Since D < 0, the equation has no real roots.

Hence, the given situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth.

Exp. Let the length and breadth of the park be l and b.

Perimeter of the rectangular park = $2(l + b) = 80$

$\Rightarrow l + b = 40$

$\Rightarrow b = 40 - l$

Area of the rectangular park = length × breadth

$\Rightarrow l \times (40 - l) = 400 \Rightarrow 40l - l^2 = 400$

$\Rightarrow l^2 - 40l + 400 = 0$

By comparing the equation with $al^2 + bl + c = 0$, we get a = 1, b = –40, c = 400

Since, Discriminant = $b^2 - 4ac$

$= (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

As D = 0, The equation has equal real roots. Hence, the situation is possible.

Roots of the equation,

$l^2 - 40l + 400 = 0$

$l^2 - 20l - 20l + 400 = 0$

$\Rightarrow l(l - 20) - 20(l - 20) = 0$

$\Rightarrow l = 20$

Therefore, length of rectangular park, $l = 20$ m

And breadth of the park, $b = 40 - l = 40 - 20 = 20$ m.

Subjective Questions

Very Short Answer Type Questions

1. Solve $36x^2 - 676 = 0$.
2. Find the solutions of the quadratic equation $2x^2 - x - 6 = 0$.
3. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square.
4. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$.
5. Solve for x: $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$.
6. Find the nature of the roots of the quadratic equation $7x^2 - 9x + 2 = 0$.
7. Find the value (s) of p so that the equation $x^2 + 4px + p^2 - p + 2 = 0$ has equal roots.
8. Find a quadratic equation, whose one root is $\frac{1}{3+\sqrt{8}}$.
9. If p and q are positive, then find the nature of roots of the equation $x^2 - px - q = 0$.
10. Find the nature of the roots of the equation $(b + c)x^2 - (a + b + c)x + a = 0$, $(a,b,c \in Q)$.

Short Answer Type Questions

1. Solve the following quadratic equation by factorisation method: $x^2 - 2ax + a^2 - b^2 = 0$.
2. The sum of two positive integers is 16. Also, twice the square of the larger part exceeds the square of the smaller part by 164.
3. Solve $x^2 + 3x + 1 = 0$ using the method of completing the square.
4. A motor boat, whose speed is 15 km/hr in still water, goes 30 km downstream and comes back in 4 hours 30 minutes. Determine the speed of the stream.
5. The difference of roots of equation $(k - 2)x^2 - (k - 4)x - 2 = 0$ is 3. Find the value of k.
6. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, show that $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$
7. Solve: $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$.
8. A truck travels a distance of 300 km at a uniform speed. If the speed of the truck is increased by 5 km per hour, the journey would have taken two hours less. Find the original speed of the truck.
9. In a cinema hall, the number of columns was equal to the number of seats in each column. If the number of columns is doubled and the number of seats in each column is reduced by 5, then the total number of seats is increased by 375. How many columns were there ?
10. Find the value of $\frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2}$ where α and β are the roots of the equation $ax^2 + bx + c = 0$.

Long Answer Type Questions

1. Rashmi can row her boat at a speed of 5 km/h in still water. If she takes 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the boat in upstream.
2. To fill a swimming pool two pipes are used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find how much long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter?

3. Out of a group of total students of class X, $\frac{7}{2}$ times the square root of the total number are playing cricket. The two remaining ones are playing badminton. Find the total number of students.

4. A boat trip costs ₹ 60. If the trip was 2 km shorter and each km costs ₹ 1 more, the cost would remain unchanged. Find the length of the trip.

5. A plane left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away on time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Integer Type Questions

1. Find the value of k if one root of the equation $5x^2 + 13x + k = 0$ is reciprocal of the other.

2. Find the number of real roots of the equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$.

3. Find the minimum value of $x^2 + 2x + 4$.

4. If roots of equation $x^2 - px + q = 0$ are consecutive integers. Find the discriminant of the equation.

5. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then find the value of ab.

Multiple Choice Questions

Level-I

1. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as –6 and 1 respectively. The correct roots are

a. 6, 1 b. –6, 1

c. –6, –1 d. 6, –1

2. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

a. ab = c b. a = b

c. ac = 1 d. a = c

3. Find the value of k, if $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$.

a. 2 b. –2

c. 0 d. $\frac{1}{2}$

4. The number of real roots of the equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$, when $a \neq b$ is

a. 2 b. 1

c. 0 d. None of these

5. If the equation $6 + x - x^2 > 0$, then value of x lies between

a. $-1 < x < 4$ b. $-2 < x < 3$

c. $-2 < x < 5$ d. $-2 < x < 4$

6. The value of m, if one root of the equation $(x - 1)(7 - x) = m$ is three times the other is

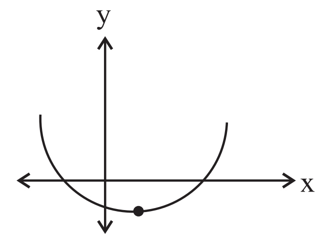
a. 2 b. 4

c. 1 d. 5

7. Graph of $y = ax^2 + bx + c$ is given adjacently. What conclusions can be drawn from the graph?

(i) $a > 0$ (ii) $b < 0$

(iii) $c < 0$ (iv) $b^2 - 4ac > 0$



a. (i) and (iv) b. (ii) and (iii)

c. (i), (ii) and (iii) d. (i), (ii), (iii) and (iv)

8. α, β are the roots of a quadratic equation such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$. Then the quadratic equation having α and β as its roots is

a. $x^2 - 12x + 28 = 0$ b. $x^2 - 24x + 126 = 0$

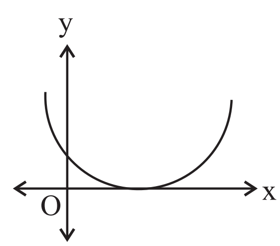
c. $x^2 - 24x + 128 = 0$ d. $x^2 - 24x - 128 = 0$

9. If the roots x_1 and x_2 of the quadratic equation $x^2 - 2x + c = 0$ also satisfy the equation $7x_2 - 4x_1 = 47$, then which of the following is true?

a. $c = -15$ b. $x_1 = 5, x_2 = 3$

c. $x_1 = 4.5, x_2 = -2.5$ d. None of these

10. What does the following graph represent?



a. Quadratic polynomial has just one root.

b. Quadratic polynomial has equal roots.

c. Quadratic polynomial has no real root.

d. Quadratic polynomial has equal roots and constant term is non-zero.

11. If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then the common factor is

a. x – 3 b. x – 6

c. x – 8 d. x – 2

12. If p and q are roots of $x^2 + px + q = 0$, then

a. $p = 1, q = -2$ b. $p = 1, q = 2$

c. $p = -1, q = 2$ d. $p = -2, q = 1$

13. The value of $(1 + \alpha)(1 + \beta)$ where α and β are the roots of $ax^2 + bx + c$ is

a. $\frac{a+b+c}{a}$ b. $\frac{a-b+c}{a}$

c. $\frac{a+b-c}{a}$ d. $\frac{b+c-a}{a}$

14. The discriminant of the quadratic equation $5x^2 - 12\sqrt{3}x + 7 = 0$ is

a. 572 b. 292

c. –572 d. None of these

15. If the roots of quadratic equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, then

a. $\frac{a}{d} = \frac{b}{c}$ b. $\frac{a^2+b^2}{c^2} = \frac{b^2+c^2}{d^2}$

c. $\frac{a}{b} = \frac{c}{d}$ d. $\frac{a}{b} = \frac{d}{c}$

16. If sum of roots of the quadratic equation $x^2 - (k + 6)x + 2(2k - 1) = 0$ is half of their product, then the value of k is

a. 6 b. 7

c. 4 d. 5

17. If sum of the squares of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40, then the value of k is

a. 10 b. –10

c. 12 d. –12

18. The value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is, where α and β are the roots of equation $x^2 - 5x + 4$.

a. $\frac{27}{4}$ b. $-\frac{27}{4}$

c. $\frac{20}{27}$ d. $\frac{4}{27}$

19. The number of zeros of equation $2x^2 + 5x + 3 = 0$ is/are

a. 1 b. 0

c. 2 d. 3

20. The minimum value of $x^2 + 4x + 7$ is

a. 1 b. 2

c. 3 d. 0

21. The roots of quadratic equation $2x^2 - 12x + 15 = 0$ are

a. real b. imaginary

c. equal d. real & equal

22. The values of k such that the equation $x^2 + 5kx + 16 = 0$ has no real roots is

a. $-\frac{6}{5} < k < \frac{16}{5}$ b. $-\frac{6}{5} < k < \frac{9}{5}$

c. $-\frac{8}{5} < k < \frac{8}{5}$ d. $\frac{-7}{5} < k < \frac{7}{5}$

23. The roots of the equation $(\ell^2 - m^2)x^2 + (m^2 - n^2)x + (n^2 - \ell^2) = 0$ are

a. $1, \frac{n^2-\ell^2}{\ell^2-m^2}$ b. $1, \frac{\ell^2-n^2}{\ell^2-m^2}$

c. $1, \frac{m^2-n^2}{\ell^2-m^2}$ d. $\frac{n^2-\ell^2}{\ell^2-m^2}, \frac{m^2-n^2}{\ell^2-m^2}$

24. The value of $\frac{b-d}{c-a}$ such that $(x - 2)$ is a factor of $x^2 + ax + b$ and $x^2 + cx + d$ is

a. 3 b. –1

c. 2 d. –2

25. The value of 'a' such that the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, is

a. $a < 2$ b. $a \leq 3$

c. $3 < a \leq 4$ d. $a > 4$

26. The roots of quadratic equation $2x^2 - 8x + 8 = 0$ are

a. Real & equal b. Real & unequal

c. Not real d. Non-real and equal

27. The number of real roots of equation $(x+1)^2 - x^2 = 0$ is/are

a. 1 b. 2

c. 3 d. 0

28. The value of k such that 5/2 is a root of the equation $14x^2 - 27x + k = 0$ is

a. 2 b. –2

c. 3 d. –1

29. A natural number which when increased by 12 is equal to 160 times of its reciprocal. Find the number.

a. 3
b. 8
c. 4
d. 2

30. The product of two successive natural integral multiples of 5 is 300. Then the multiples are

a. 15, 20
b. 10, 15
c. –15, –20
d. both a & c

31. The value of $\sqrt{6+\sqrt{6+\sqrt{6.........}}}$ is

a. 3.5
b. 4
c. 3
d. –3

32. Find the value of x, if $p^2x^2 - q^2 = 0$.

a. $\pm q/p$
b. $\pm p/q$
c. p
d. q

33. Find the positive root of equation $\sqrt{3x^2+6} = 9$.

a. 3
b. 5
c. 4
d. 7

34. If $x^2(a^2 + b^2) + 2x(ac + bd) + c^2 + d^2 = 0$ has no real roots, then

a. $ad \neq bc$
b. $ad < bc$
c. $ad > bc$
d. All of these

35. If Preeti scored 10 more than the actual marks in a science test out of 30, then 9 times of these marks is equal to the square of her actual marks. Find the marks she got in the test.

a. 14
b. 16
c. 15
d. 18

Level-II

1. If the roots of the equation $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}x + q = 0$ be real, then

a. $q^2 = p^2r$
b. $q^2 = pr$
c. $p^2 = qr$
d. $r^2 = pq$

2. If $x = \sqrt{7+4\sqrt{3}}$, then find the value of $x + \frac{1}{x}$.

a. 4
b. 8
c. 3
d. 2

3. Find the condition for which one root of the quadratic equation $ax^2 + bx + c = 0$ is twice the other.

a. $b^2 = 4ac$
b. $2b^2 = 9ac$
c. $c^2 = 4a + b^2$
d. $c^2 = 9a - b^2$

4. Find the real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$.

a. 1, 8
b. –1, –2
c. –1, 8
d. 1, –8

5. The equation $2x^2 + 2(p + 1)x + p = 0$, where p is real, always has roots that are

a. Equal

b. Equal in magnitude but opposite in sign

c. Irrational

d. Real

6. Which constant must be added or subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

a. $\frac{1}{24}$
b. $\frac{1}{576}$
c. $\frac{9}{64}$
d. $\frac{3}{64}$

7. Find the condition for which the equation $(m^2 + n^2)x^2 - 2(mp + nq)x + p^2 + q^2 = 0$ has equal roots.

a. $mp = nq$
b. $mq = np$
c. $mp = n^2$
d. $mq = \sqrt{np}$

8. If each root of the equation $x^2 - bx + c = 0$ is decreased by 2, then the resulting equation is $x^2 - 2x + 1 = 0$. Find the value of b and c.

a. b = 6, c = 9
b. b = –6, c = 9
c. b = 2, c = –1
d. b = –4, c = 3

9. One root of the equation $ax^2 - bx + c = 0$ is square of the other, then the value of $ac(a + c + 3b)$ is

a. a^3
b. b^3
c. c^3
d. ab^2

10. The value of 4q + 1 such that the roots of the equation $x^2 - px + q = 0$ differ by unity.

a. p
b. p^2
c. p + 1
d. None of these

11. If both roots of the equation $x^2 + mx + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ are common, then

a. $m = -2$
b. $m = -1$
c. $m = 2$
d. $m = 1$

12. The sum of two natural numbers is 8. If the sum of their reciprocals is $\frac{8}{15}$, then the numbers are

a. 2, 6
b. –3, –5
c. 3, 5
d. –3, 5

13. Find the value of k such that $\alpha^2 + \beta^2 = 24$, where α and β are the roots of the equation $kx^2 + 4x + 4 = 0$

a. 1
b. $-\frac{2}{3}$
c. –1
d. $\frac{3}{2}$

14. Solution of the equation $\left(\frac{2x-1}{x+1}\right)-15\left(\frac{x+1}{2x-1}\right)=-2$ is

a. 4
b. –4
c. $-\frac{4}{7}$
d. Both (b) and (c)

15. Students of class X planned to go on a trip. The budget for the trip was ₹ 480. But eight of these failed to go and thus the cost of trip for each member increased by ₹ 10. Find the total number of students who went for the trip.

a. 13
b. 32
c. 24
d. 16

Assertion & Reason Type Questions

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) is true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) is true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true

1. **Assertion:** $3x^2 - 6x + 3 = 0$ has repeated roots.

   Reason: The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

2. **Assertion:** $(2x - 1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.

   Reason: An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, where $a, b, c \in R$ is called a quadratic equation.

3. **Assertion:** If one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3, then the value of k is 2.

   Reason: The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has almost two roots.

4. **Assertion:** The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary

   Reason: If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

5. **Assertion:** Sum and product of roots of $5x^2 + 9x - 6 = 0$ are –9/5 and –6/5 respectively.

   Reason: If α and β are the roots of $ax^2 + bx + c = 0$, $a \neq 0$, then sum of roots $= \alpha+\beta=\frac{b}{a}$ and product of roots $= \alpha\beta=-\frac{c}{a}$

Cased-Based Type Questions

Cased-Based-I: A survey was conducted over the students of class X. It was found that α number of students like cricket and β number of students like football, such that $\alpha > b$ and α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 7x + 10$ then answer the following questions.

1. Name the type of polynomial in the above statement?

   a. Quadratic
   b. Cubic
   c. Linear
   d. Bi-quadratic

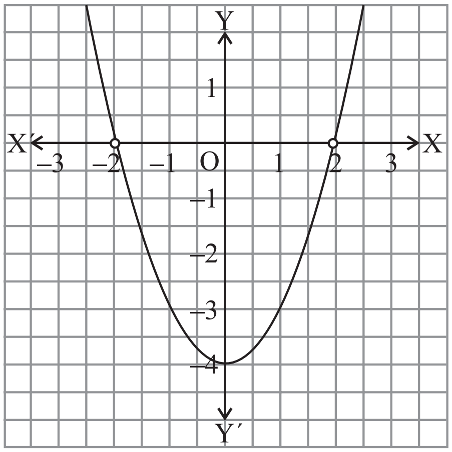
2. Find the number of students who like cricket.

   a. 5
   b. 2
   c. 7
   d. None of these

3. Find the quadratic polynomial whose zeroes are –3 and –4.

   a. $x^2 + 4x + 2$
   b. $x^2 - 4x - 2$
   c. $x^2 - 7x + 2$
   d. None of these

Cased-Based-II: Look at the graph given below and answer the following questions.



1. The zeroes of the polynomial are:

a. –2, 3 b. –2, 2

c. –2, 0 d. –1, –2

2. Name the shape of curve given in graph.

a. Spiral b. Ellipse

c. Linear d. Parabola

3. What will be the expression of the given polynomial?

a. $x^2 + 4$ b. $x^2 – 4$

c. $x + 4$ d. $(x – 2)$

Multi Correct MCQ's

1. If $a + b + c = 0$, then roots of the quadratic equation $ax^2 + bx + c = 0$ is/are

a. $\frac{b}{a}$ b. 1

c. $\frac{c}{a}$ d. $\frac{a}{c}$

2. The sum of a number and its reciprocal is $2\frac{1}{30}$, the number(s) is/are

a. $\frac{5}{6}$ b. $1\frac{1}{5}$

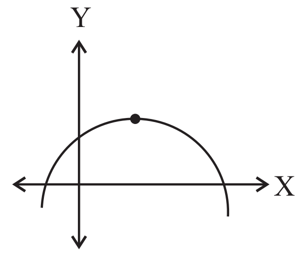
c. $\frac{9}{4}$ d. $\frac{4}{9}$

3. The sum of the squares of two consecutive natural number is 313. The number(s) is/are

a. 12 b. –13

c. 11 d. None of these

4. The adjoining figure shows the graph of $y = ax^2 + bx + c$, then which of the following is/are correct.



a. $a > 0$ b. $b > 0$

c. $c > 0$ d. $b^2 < 4ac$

Olympiad & NTSE Type Questions

1. If $x^2 + y^2 = 25$, $xy = 12$, then $x =$

a. {3, 4} b. {3, 5, –5, –3}

c. {3, 4, –3, 4} d. {3, –4}

2. If the ratio of the roots of the equations $x^2 + bx + c = 0$ is the same as that of $x^2 + qx + r = 0$, then b^2r is equal to

a. q^2c b. qc

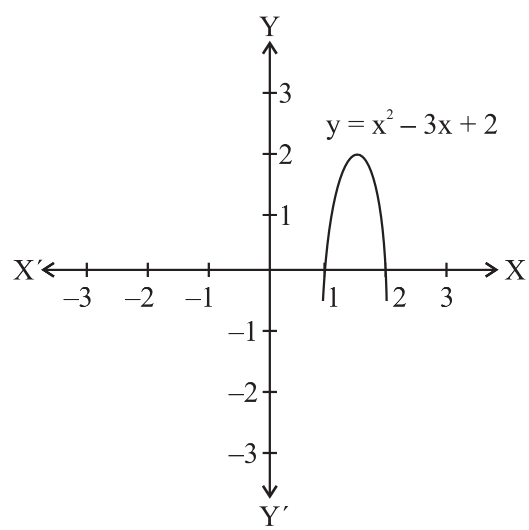
c. qc^2 d. r^2b

3. A factory kept increasing its output by the same percentage every year. Find the percentage, such that the output is doubled in the last 2 years.

a. $100\left(-1-\sqrt{2}\right)$ b. $100\left(-1+\sqrt{2}\right)$

c. $100\left(1-\sqrt{2}\right)$ d. None of these

4. A graph of a quadratic polynomial is given below



If we rotate the axes at an angle of 90° in anti-clockwise direction, the figure remains at the same position. Find the equation of the graph.

a. $y^2 + 3y + 2$ b. $y^2 - 3y + 2$

c. $x^2 + 3x + 2$ d. $y^3 - 3y + 2$

5. Find the value of b^2 such that sin α and cos α are the roots of the equation $ax^2 + bx + c = 0$.

a. $a^2 + 3ac$ b. $a^2 + ac$

c. $a^2 + 2ac$ d. $c^2 + 3ac$

6. If $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$, then extraneous root of this equation is

a. 26 b. –9

c. –26 d. 13

7. A man has certain number of flowers. One-fourth of the total flowers is offered to the first temple. $\frac{1}{9}$ of total flowers along with $\frac{1}{4}$ as well as 7 times the square root of total number is offered to the second temple. He left with 56 flowers at last. Find the total number of flowers.

a. 576 b. 567

c. 556 d. 557

8. The roots of the equation $2^{2x} - 10 \cdot 2^x + 16 = 0$ are

a. x = 3, 2 b. x = 2, 1

c. x = 3, 1 d. None of these

9. If α and β are roots of $ax^2 + bx + c = 0$, then $\frac{\alpha}{\beta}(a\alpha + b) + \frac{\beta}{\alpha}(a\beta + b)$ is

a. a b. b

c. ab d. (–2, –5)

10. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then bc^2, ca^2, ab^2 are in

a. A.P b. G.P

c. H.P d. none of these

11. If a, b, c be real numbers with a ≠ 0 and α, β be the roots of the equation $ax^2 + bx + c = 0$, then the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β are

a. $\alpha\beta$ and $\frac{\beta}{\alpha}$ b. $\frac{\alpha}{\beta}$ and $\alpha\beta$

c. $\alpha^2\beta$ and $\alpha\beta$ d. $\alpha^2\beta$ and $\alpha\beta^2$

12. If I had walked 1 km/hr faster, I would have taken 10 minutes less to walk 2 km, then the rate of my walking is

a. 3 km/hr b. 4 km/hr

c. 5 km/hr d. 6 km/hr

13. The roots of the equation (x – b)(x – c) + (x – c)(x – a) + (x – a)(x – b) = 0 are always

a. Positive b. Negative

c. Real d. Complex

14. Find the value of $x^3 - 6x^2 + 6x$ such that $x = 2 + 2^{1/3} + 2^{2/3}$.

a. 3 b. 2

c. 1 d. 6

Explanations

Subjective Questions

Very Short Answer Type Questions

1. We have $36x^2 - 676 = 0$

$$36x^2 = 676$$
$$x^2 = \frac{676}{36}$$
$$x = \pm\sqrt{\frac{676}{36}}$$
$$x = \pm\frac{26}{6}$$
$$x = \pm\frac{13}{3}$$

Thus, $x = \frac{13}{3}$ and $\frac{-13}{3}$ are solutions of the given equation.

2. By comparing the given equation i.e., $2x^2 - x - 6 = 0$ with $ax^2 + bx + c = 0$, we get $a = 2, b = -1, c = -6$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+48}}{4} = \frac{1 \pm 7}{4}$$

$x = 2$ or $-\frac{3}{2}$

3. The given quadratic equation is $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$.

Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = (4 - k)$, $b = (2k + 4)$ and $c = (8k + 1)$.

As the given equation is a perfect square, then

$D = b^2 - 4ac = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (2k + 4)^2 - 4(4 - k)(8k + 1) = 0$

$\Rightarrow 4(k + 2)^2 - 4(4 - k)(8k + 1) = 0$

$\Rightarrow 4[(k + 2)^2 - (4 - k)(8k + 1)] = 0$

$\Rightarrow [(k^2 + 4k + 4) - (-8k^2 + 31k + 4)] = 0$

$\Rightarrow 9k^2 - 27k = 0 \Rightarrow 9k(k - 3) = 0$

i.e., $k = 0$ or $k = 3$

Hence, the given equation is a perfect square, if $k = 0$ or $k = 3$.

4. The given quadratic equation is $(b - c)x^2 + (c - a)x + (a - b) = 0$.

Comparing the given equation with $Ax^2 + Bx + C = 0$, we get $A = (b - c)$, $B = (c - a)$, and $C = (a - b)$.

For equal roots, $D = B^2 - 4AC = 0$

$$B^2 - 4AC = 0$$

$\Rightarrow (c - a)^2 - 4(b - c)(a - b) = 0$

$\Rightarrow c^2 + a^2 - 2ac + 4b^2 - 4ab + 4ac - 4bc = 0$

$\Rightarrow c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow c^2 + a^2 + (2b)^2 + 2ac - 2a(2b) - 2(2b)c = 0$

As we know,

$x^2 + y^2 + z^2 + 2xz - 2xy - 2yz = (x - y + z)^2$

$\Rightarrow (c + a - 2b)^2 = 0$

$\Rightarrow c + a = 2b$ **Hence Proved.**

5. The given quadratic equation is:

$12abx^2 - (9a^2 + 8b^2)x - 6ab = 0$

$\Rightarrow 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$\Rightarrow 3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$\Rightarrow (4bx - 3a)(3ax + 2b) = 0$

i.e., $4bx - 3a = 0$ or $3ax + 2b = 0$

$\therefore$ $x = \frac{3a}{4b}$ or $x = -\frac{2b}{3a}$

6. The given quadratic equation is $7x^2 - 9x + 2 = 0$. Comparing with $ax^2 + bx + c = 0$, we get $a = 7$, $b = -9$ and $c = 2$

$D = b^2 - 4ac$
$= (-9)^2 - 4(7)(2)$
$= 81 - 56$
$= 25$

As $D > 0$ and a perfect square. So roots are real and distinct.

7. Comparing $x^2 + 4px + p^2 - p + 2 = 0$

with $ax^2 + bx + c = 0$,

we get: $a = 1$, $b = 4p$ and $c = p^2 - p + 2$

Since, the roots are equal; the discriminant, $D = 0$

$b^2 - 4ac = 0$

$\Rightarrow (4p)^2 - 4(p^2 - p + 2) = 0$

$\Rightarrow 16p^2 - 4p^2 + 4p - 8 = 0$

$\Rightarrow 12p^2 + 4p - 8 = 0$

$\Rightarrow 3p^2 + p - 2 = 0$

$\Rightarrow 3p^2 + 3p - 2p - 2 = 0$

$\Rightarrow$ 3p(p + 1) – 2(p + 1) = 0

$\Rightarrow$ (3p – 2) (p + 1) = 0

$\Rightarrow p = -1 \text{ or } \frac{2}{3}$

8. The given root is $\frac{1}{3+\sqrt{8}}$.

By Rationalising,

$$\frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} = \frac{3-\sqrt{8}}{3^2-\left(\sqrt{8}\right)^2}$$

[(a + b) (a – b) = $a^2 - b^2$]

$$= 3-\sqrt{8}$$

So, the other root will be $3+\sqrt{8}$

Sum of roots = $3+\sqrt{8}+3-\sqrt{8}=6$

Products of roots = $\left(3+\sqrt{8}\right)\left(3-\sqrt{8}\right)=1$

$\therefore$ The requried quadratic equation is

x^2 – (sum of roots)x + product of roots = 0

$\Rightarrow x^2 - 6x + 1 = 0$

9. The given quadratic equation is $x^2 - px - q = 0$

Comparing with $ax^2 + bx + c = 0$, we get a = 1, b= –p, and c = –q.

Here, Discriminant = $b^2 - 4ac$

$= (-p)^2 - 4(1)(-q)$

$= p^2 + 4q$

Since p, q > 0

Therefore, Discriminant = $p^2 + 4q > 0$

Hence, the roots are real and distinct.

10. The discriminant of the equation is $b^2 - 4ac$

$= (a + b + c)^2 - 4(b + c)(a)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4(b+c)a$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4ab - 4ac$

$= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

$= (a - b - c)^2 > 0$

So, roots are real and distinct.

Short Answer Type Questions

1. Factors of $(a^2 - b^2)$ are (a – b) and (a + b). Coefficient of the middle term = –2a = –[(a – b) + (a + b)]

$\Rightarrow x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow x^2 - \{(a - b) + (a + b)\}x + (a - b)(a + b) = 0$

$\Rightarrow x^2 - (a - b)x - (a + b)x + (a - b)(a + b) = 0$

$\Rightarrow x\{x - (a - b)\} - (a + b)\{x - (a - b)\} = 0$

$\Rightarrow \{x - (a - b)\}\{x - (a + b)\} = 0$

$\Rightarrow x - (a - b) = 0$ or $x - (a + b) = 0$

$\Rightarrow x = a - b$ or $x = a + b$

2. Let larger part be x, therefore the smaller part will be 16 – x.

According to the question, twice the square of larger part exceed the square of smaller part by 164.

$\Rightarrow 2x^2 - (16 - x)^2 = 164$

$\Rightarrow 2x^2 - 256 - x^2 + 32x - 164 = 0$

$\Rightarrow x^2 + 32x - 420 = 0$

$\Rightarrow x^2 + 42x - 10x - 420 = 0$

$\Rightarrow (x + 42)(x - 10) = 0$

i.e., x = – 42 or x = 10

Neglecting x = –42, as integers are positive.

$\therefore$ x = 10

Hence the larger part = 10 and the smaller part = 16 – x = 16 – 10 = 6

3. Given, $x^2 + 3x + 1 = 0$

$\Rightarrow x^2 + 3x = -1$

$$\Rightarrow x^2 + 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2 - 1$$

$$\left(x+\frac{3}{2}\right)^2 = \frac{9}{4} - 1$$

$$\Rightarrow \left(x+\frac{3}{2}\right)^2 = \frac{5}{4}$$

$$\Rightarrow \left(x+\frac{3}{2}\right)^2 = \left(\frac{\sqrt{5}}{2}\right)^2$$

$$\Rightarrow x+\frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$

This gives $x = \frac{-\left(3+\sqrt{5}\right)}{2}$ or $x = \frac{-3+\sqrt{5}}{2}$

4. Let the speed of the stream be x km/hr

$\Rightarrow$ The speed of the boat downstream = (15 + x) km/hr

and the speed of the boat upstream = (15 – x) km/hr.

As we know, $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Now, time taken to go 30 km downstream

$= \frac{30}{15+x}$ hrs.

and time taken to come back 30 km upstream

$= \frac{30}{15-x}$

Given : Total time for journey = $4\frac{1}{2}$ hrs

$$\therefore \frac{30}{15+x} + \frac{30}{15-x} = 4\frac{1}{2}$$

$$\Rightarrow \quad \frac{30(15-x)+30(15+x)}{(15+x)(15-x)}=\frac{9}{2}$$

i.e., $\frac{450-30x+450+30x}{225-x^2}=\frac{9}{2}$

$\Rightarrow 2 \times 900 = 9\ (225 - x^2)$

$\Rightarrow 2 \times 100 = 225 - x^2$

i.e., $x^2 = 225 - 200$

$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

Neglecting $x = -5$, as speed cannot be negative.

$\therefore$ Speed of stream is 5 km/hr.

5. Sum of roots, $\alpha+\beta=\frac{k-4}{k-2}$

Product of roots, $\alpha\beta=\frac{-2}{k-2}$

We know, $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$
$= \alpha^2 + \beta^2 - 2\alpha\beta + 2\alpha\beta - 2\alpha\beta$
$= \alpha^2 + \beta^2 + 2\alpha\beta - 4\alpha\beta$
$= (\alpha + \beta)^2 - 4\alpha\beta$

$\therefore\ (\alpha-\beta)^2=\left(\frac{k-4}{k-2}\right)^2+\frac{8}{(k-2)}$

$\Rightarrow 3^2=\frac{k^2+16-8k+8(k-2)}{(k-2)^2}$

$\Rightarrow 9(k - 2)^2 = k^2 + 16 - 8k + 8k - 16$

$\Rightarrow 9k^2 + 36 - 36k = k^2$

$\Rightarrow 8k^2 - 36k + 36 = 0$

$\Rightarrow 2k^2 - 9k + 9 = 0$

$\Rightarrow 2k^2 - 6k - 3k + 9 = 0$

$\Rightarrow 2k(k - 3) - 3(k - 3) = 0$

$\therefore\ k = 3$ or $\frac{3}{2}$

6. Since the roots of the given equation are equal, so discriminant will be equal to zero.

$\therefore\ D = 0$

$\Rightarrow\ b^2(c - a)^2 - 4a(b - c)\ c(a - b) = 0$

$\Rightarrow\ b^2(c^2 + a^2 - 2ac) - 4ac(ba - ca - b^2 + bc) = 0$

$\Rightarrow\ b^2c^2 + b^2a^2 - 2acb^2 - 4a^2bc + 4a^2c^2 + 4acb^2 - 4abc^2 = 0$

On rearranging the terms we have,

$\Rightarrow\ a^2b^2 + b^2c^2 + 4a^2c^2 + 4b^2ac - 2b^2ac - 4a^2bc - 4abc^2 = 0$

$\Rightarrow\ a^2b^2 + b^2c^2 + 4a^2c^2 + 2b^2ac - 4a^2bc - 4abc^2 = 0$

As we know,

$a^2 + b^2 - c^2 + 2ab - 2bc - 2ca = (a + b - c)^2$

$\Rightarrow\ (ab + bc - 2ac)^2 = 0$

$\Rightarrow\ ab + bc - 2ac = 0 \Rightarrow ab + bc = 2ac$

Divide by abc both sides

$\Rightarrow\ \frac{1}{c}+\frac{1}{a}=\frac{2}{b}$

$\Rightarrow\ \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$

7. Given, $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0$

$\Rightarrow \frac{2x(2x+3)+(x-3)+3x+9}{(x-3)(2x+3)}=0$

$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$\Rightarrow 4x^2 + 10x + 6 = 0$

$\Rightarrow 2x^2 + 5x + 3 = 0$

$\Rightarrow (2x + 3)(x + 1) = 0$

i.e., $2x + 3 = 0$ or $x + 1 = 0$

But according to the question, $2x + 3$ cannot be 0 as it makes the equation undefined.

$\therefore\ x + 1 = 0 \Rightarrow x = -1$

8. Let the original speed of the truck be x km/hr.

In 1st case, Distance = 300 km and speed = x km/hr.

$\Rightarrow$ Time taken $=\frac{\text{distance}}{\text{speed}}=\frac{300}{x}$ hrs.

In 2nd case, Distance = 300 km and speed = (x +5) km/hr.

$\Rightarrow$ Time taken $=\frac{\text{distance}}{\text{speed}}=\frac{300}{x+5}$ hrs.

According to the question,

$\frac{300}{x}-\frac{300}{x+5}=2$

$\Rightarrow\ \frac{300(x+5)-300x}{x(x+5)}=2$

$\Rightarrow\ \frac{300x+1500-300x}{x^2+5x}=2$

$\Rightarrow 2(x^2 + 5x) = 1500$

$\Rightarrow x^2 + 5x - 750 = 0$

$\Rightarrow x^2 + 30x - 25x - 750 = 0$

i.e., $(x + 30)\ (x - 25) = 0$

$\therefore\ x = -30$ or $x = 25$

Neglecting $x = -30$, as speed can't be negative

$\therefore$ speed of truck is 25 km/hr.

9. Let the number of columns be x

No. of seats in each columns = x

Total number of seats = Number of seats in each column × number of columns = $x \times x = x^2$

Now, the new no. of columns = 2x and the new no. of seats in each column = x – 5

The total number of seats now will be = 2x(x – 5).

According to the question:

$2x(x - 5) - x^2 = 375$

$\Rightarrow 2x^2 - 10x - x^2 = 375$

$\Rightarrow x^2 - 10x - 375 = 0$

$\Rightarrow x^2 - 25x + 15x - 375 = 0$

$\Rightarrow x(x - 25) + 15(x - 25) = 0$

$\Rightarrow (x - 25)(x + 15) = 0$

$x = 25$ or $x = -15$

Neglecting x = – 15, as number of column cannot be negative.

$\therefore$ Number of columns = 25

10. Since α, β are the roots of the equation $ax^2 + bx + c = 0$

$f(\alpha) = 0$ and $f(\beta) = 0$

$a(\alpha)^2 + b(\alpha) + c = 0$	$a(\beta)^2 + b(\beta) + c = 0$
$\Rightarrow \alpha(a\alpha + b) + c = 0$	$\Rightarrow \beta(a\beta + b) + c = 0$
$\Rightarrow (a\alpha + b) = -c/\alpha$ ...(1)	$\Rightarrow (a\beta + b) = -c/\beta$ ...(2)

$$\therefore \frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2} = \frac{1}{(-c/\alpha)^2} + \frac{1}{(-c/\beta)^2}$$

$$= \frac{\alpha^2}{c^2} + \frac{\beta^2}{c^2} = \frac{\alpha^2+\beta^2}{c^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{c^2}$$

$$= \frac{b^2/a^2 - 2c/a}{c^2} = \frac{b^2 - 2ac}{a^2c^2}$$

Long Answer Type Questions

1. Let the speed of the stream be x km/h

$\therefore$ Speed of the boat in upstream = (5 – x)km/h

Speed of the boat in downstream = (5 + x)km/h

We know, $\text{Time} = \frac{\text{distance}}{\text{speed}}$

Time for going 5.25 km upstream = $\frac{5.25}{5-x}$

Time for returning 5.25 km downstream $= \frac{5.25}{5+x}$

Therefore, according to the question,

$$\frac{5.25}{5-x} - \frac{5.25}{5+x} = 1$$

$$\Rightarrow \frac{525}{100}\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$$

$$\Rightarrow \frac{21}{4}\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$$

$$\Rightarrow 21\left(\frac{5+x-5+x}{25-x^2}\right) = 4$$

$\Rightarrow 21(2x) = 4(25 - x^2)$

$\Rightarrow 42x = 100 - 4x^2$

$\Rightarrow 4x^2 + 42x - 100 = 0$

$\Rightarrow 2x^2 + 21x - 50 = 0$

$\Rightarrow 2x^2 + 25x - 4x - 50 = 0$

$\Rightarrow (2x + 25)(x - 2) = 0$

i.e., $x = 2$ or $-\frac{25}{2}$

Since, speed cannot be negative, so we reject $x = \frac{-25}{2}$.

Thus, the speed of the stream is 2 km/h.

And speed of boat in upstream = 5 – x = 5 – 2 = 3 km/h.

2. Let x and y hours be the time taken by larger pipe and smaller pipe to fill the swimming pool respectively.

Given, y – x = 10 or y = x + 10

If smaller pipe takes x hours to fill the pool, it will fill 1/x part in 1 hour

Similarly if larger pipe takes y hours to fill the pool, it will fill 1/y part in 1 hour

According to the question, if the larger pipe is used for 4 hours and the smaller pipe is used for 9 hours only half of the pool can be filled.

$$4 \times \frac{1}{x} + 9 \times \frac{1}{y} = \frac{1}{2}$$

$$\Rightarrow \frac{4}{x} + \frac{9}{y} = \frac{1}{2}$$

Now putting y = x + 10

$$\Rightarrow \frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$

$$\Rightarrow \frac{4(x+10)+9x}{x(x+10)} = \frac{1}{2}$$

$$\Rightarrow \frac{4x+40+9x}{x(x+10)} = \frac{1}{2}$$

$\Rightarrow 26x + 80 = x^2 + 10x \Rightarrow x^2 - 16x - 80 = 0$

$\Rightarrow x^2 - 20x + 4x - 80 = 0$

$\Rightarrow (x - 20)(x + 4) = 0$

i.e., x = 20 or –4

Neglecting –4, as time can't be negative.

$\therefore$ time taken by larger pipe to fill the pool is 20 hrs.

Then time taken by smaller pipe will be 30 hrs.

3. Let the number of students be x.

Number of students playing cricket $= \frac{7}{2}\sqrt{x}$

Number of students remaining = 2

According to the question,

$$\Rightarrow \ x = \frac{7}{2}\sqrt{x} + 2$$

$$\Rightarrow \ x - 2 = \frac{7}{2}\sqrt{x}$$

Squaring both sides,

$$\Rightarrow \ (x-2)^2 = \left(\frac{7}{2}\right)^2 x$$

$\Rightarrow 4(x^2 - 4x + 4) = 49x$

$\Rightarrow 4x^2 - 65x + 16 = 0$

$\Rightarrow 4x^2 - 64x - x + 16 = 0$

$\Rightarrow 4x(x - 16) - 1(x - 16) = 0$

$\Rightarrow (x - 16)(4x - 1) = 0$

This gives x = 16 or $x = \frac{1}{4}$

We reject $x = \frac{1}{4}$ as number of students cannot be in fraction.

Hence, the total number of students is 16.

4. Let "x" be the length of the given trip and costs ₹ 60.

Then, the cost of 1 km trip, c = $\frac{60}{x}$

According to the question, if trip was shorter by 2 km and each km costs ₹ 1 more, then the total cost would remain unchanged.

Total km covered × cost/km = total cost

$$(x-2)\left(\frac{60}{x} + 1\right) = 60$$

$\Rightarrow (x - 2)(60 + x) = 60x$

$\Rightarrow 60x + x^2 - 120 - 2x = 60x$

$\Rightarrow x^2 - 2x - 120 = 0$

$\Rightarrow x^2 - 12x + 10x - 120 = 0$

$\Rightarrow x(x - 12) + 10(x - 12) = 0$

$\Rightarrow (x + 10)(x - 12) = 0$

$\therefore$ x = –10 or 12

Because distance of trip cannot be a negative number, we can ignore "–10".

So, the length of the given trip is 12 km.

5. Let the usual speed be x km/hr

As we know, $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\therefore$ time taken to cover 1500 km with usual speed

$= \frac{1500}{x}$ hrs.

To reach the destination on time, speed has to be increased by 250 km/hr.

$\therefore$ New speed = x + 250

Then, time taken to cover 1500 km with increased speed = $\frac{1500}{x+250}$

According to the question, the plane takes half an hour less to reach at its destination if it flies by the speed of (x + 250) km/hr.

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$\Rightarrow \frac{1500(x+250) - 1500x}{x(x+250)} = \frac{1}{2}$$

$$\Rightarrow \frac{1500 \times 250}{x(x+250)} = \frac{1}{2}$$

$\Rightarrow x^2 + 250x - 750000 = 0$

$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow (x + 1000)(x - 750) = 0$

i.e., x = –1000 or 750

Neglecting –1000, as speed can not be negative.

$\therefore$ Usual speed of the plane is 750 km/hr

Integer Type Questions

1. Let the roots be α and $\frac{1}{\alpha}$.

Product of roots = $\frac{c}{a}$

$$\alpha \times \frac{1}{\alpha} = \frac{k}{5}$$

$\therefore$ k = 5

2. $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

$$\Rightarrow \ \frac{x - (x-1) - 2}{x-1} = \frac{x-1-2}{x-1}$$

$\Rightarrow x(x - 1) - 2 = x - 1 - 2$

$\Rightarrow x^2 - x - 2 = x - 3$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow x^2 - x - x + 1 = 0$

$\Rightarrow x(x - 1) - 1(x - 1) = 0$

$\Rightarrow (x - 1)(x - 1) = 0$

i.e., x = 1

But x = 1 is not possible as it makes our equation not defined. So, no real roots are possible.

3. Given polynomial is $x^2 + 2x + 4$.

By comparing with $ax^2 + bx + c$, we get $a = 1$, $b = 2$, $c = 4$

Using the formula of minimum value,

Minimum value $= \dfrac{4ac - b^2}{4a} = \dfrac{4(1)(4) - 2^2}{4(1)} = \dfrac{12}{4} = 3$

4. Let the roots be α and $\alpha + 1$.

Sum of roots $= 2\alpha + 1 = -\dfrac{(-p)}{1}$

$$\alpha = \frac{p-1}{2}$$

Product of roots $= \alpha(\alpha + 1) = q$

$$\frac{p-1}{2}\left(\frac{p-1}{2} + 1\right) = q$$

$$\frac{(p-1)}{2}\left(\frac{p+1}{2}\right) = q$$

$$p^2 - 1 = 4q$$

$$p^2 - 4q = 1 \quad \text{...(i)}$$

Now, $D = b^2 - 4ac = (-p)^2 - 4q = p^2 - 4q = 1$

5. Given, 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$

Putting $y = 1$ in both the equations, we get

$a(1)^2 + a(1) + 3 = 0 \Rightarrow a = \dfrac{-3}{2}$

Also, $(1)^2 + 1 + b = 0 \Rightarrow b = -2$

So, the value of $ab = \dfrac{-3}{2}(-2) = 3$

Multiple Choice Questions

Level-I

1. **(d)** Let the incorrect equation having roots 2 and 3 is $ax^2 + bx + c = 0$.

As constant term does not affect the sum of roots,

$\therefore$ Sum of roots $= -\dfrac{b}{a}$

$\Rightarrow 3 + 2 = -\dfrac{b}{a}$

$\Rightarrow \dfrac{b}{a} = -5$ ...(i)

The other one copied the constant term and coefficient of x^2 correctly as -6 and 1.

$\therefore$ $a = 1$ and $c = -6$

Also, $b = -5$ [using (i)]

So, the equation is

$x^2 - 5x - 6 = 0$

The roots of the equation are 6 and -1

2. **(d)** The quadratic equation is $ax^2 + bx + c = 0$

Given, If one root is α, then the other will be $\dfrac{1}{\alpha}$.

$\Rightarrow \alpha \cdot \dfrac{1}{\alpha} =$ product of roots $= \dfrac{c}{a}$

$\Rightarrow 1 = \dfrac{c}{a} \Rightarrow a = c$

3. **(a)** Since $\dfrac{1}{2}$ is a root of the quadratic equation

$x^2 + kx - \dfrac{5}{4} = 0$.

$\therefore \left(\dfrac{1}{2}\right)^2 + k\left(\dfrac{1}{2}\right) - \dfrac{5}{4} = 0$

$\Rightarrow \dfrac{1}{4} + \dfrac{k}{2} - \dfrac{5}{4} = 0 \Rightarrow \dfrac{1 + 2k - 5}{4} = 0$

$\Rightarrow 2k - 4 = 0 \Rightarrow k = 2$

4. **(c)** $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

$D = b^2 - 4ac$

$D = 4(a + b)^2 - 4(2)(a^2 + b^2)(1)$

$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$= -4a^2 - 4b^1 + 8ab$

$= -4(a^2 + b^2 - 2ab)$

$= -4(a - b)^2$

As value of $(a - b)^2$ cannot be less than zero. Therefore, the value of $-4(a - b)^2$ will always be less than or equal to zero.

That's why, D will always be less than or equal to zero

Given equation has real roots only if $D = 0$

$\Rightarrow -4(a - b)^2 = 0$

$\Rightarrow a = b$

$\therefore$ the given equation does not have real roots if $a \neq b$

5. **(b)** $6 + x - x^2 > 0$

$\Rightarrow -x^2 + 3x - 2x + 6 > 0$

$\Rightarrow -x(x - 3) - 2(x - 3) > 0$

$\Rightarrow (x - 3)(-x - 2) > 0$

$\therefore$ Zeroes for the inequality are $x = 3$ and $x = -2$.

So, between 3 and -2. The function will either be always greater than zero or always less than zero.

Let's pick a value in between and test it.

At $x = 1$: $6 + x - x^2$

$= 6 + 1 - (1)^2$

$= 6 > 0$

$\therefore$ x lies between -2 and 3.

$-2 < x < 3$ option (b).

6. **(d)** Given equation is $(x - 1)(7 - x) = m$

$\Rightarrow 7x - x^2 - 7 + x = m$

$\Rightarrow x^2 - 8x + 7 + m = 0$ ...(i)

Given, one root is 3 times the other.

Let the roots be α, 3α

$\therefore$ Sum of roots $= -\frac{b}{a}$

$\Rightarrow (\alpha + 3\alpha) = 8$

$\Rightarrow \alpha = 2$

Putting $\alpha = 2$ in the equation (i)

$4 - 16 + 7 + m = 0$

$\Rightarrow m = 16 - 11 = 5$

7. **(d)** The number of zeroes of p(x) is the number of times the curve intersects the x-axis. Here, the polynomial p(x) meets the x-axis at 2 points.

From the graph we can conclude that the equation has two real and distinct roots.

For real and distinct roots, $D > 0$

$D = b^2 - 4ac > 0$

As the graph is open upwards, so $a > 0$.

We can also conclude from the graph that both the roots are of opposite signs.

$\therefore$ Product of zeros $\left(= \frac{c}{a}\right)$ must be less than zero.

$\frac{c}{a} < 0$

$\Rightarrow c < 0$

Also magnitude of +ve roots is larger

$\therefore$ Sum of roots > 0

$\Rightarrow \frac{-b}{a} > 0$

Hence all the options are correct.

8. **(c)** Given, $\alpha + \beta = 24$ ...(i)

$\alpha - \beta = 8$ ...(ii)

Adding (i) and (ii), we get

$2\alpha = 32$

$\Rightarrow \alpha = \frac{32}{2} = 16$

Putting $\alpha = 16$ in (i), we get $\beta = 24 - 16 = 8$

Now, $\alpha + \beta = 24$ (given)

and $\alpha\beta = 16 \times 8 = 128$

Hence the required quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e. $x^2 - 24x + 128 = 0$

9. **(a)** Given, x_1 and x_2 are the roots of the equation, $x^2 - 2x + c$.

Sum of roots $= (\alpha + \beta) = - b/a$

$\therefore \quad x_1 + x_2 = 2$ ...(i)

Product of roots $= \alpha\beta = c/a$

$x_1 x_2 = c$ ...(ii)

Also, x_1 and x_2 satisfy the equation $7x_2 - 4x_1 = 47$ ...(iii)

By solving equation (i) and (iii), we get

$x_1 = -3$ and $x_2 = 5$

Using equation (ii)

$c = x_1 x_2 = (-3)(5) = -15$

10. **(d)** Graph touches x-axis means polynomial has equal roots and graph cuts the y-axis means polynomial has a constant term.

11. **(c)** Let the common factor be $(x - \alpha)$

$\therefore$ α is the root of both the equations.

Let β be the another root of the equation $x^2 - 11x + a$ and r be the another root of the equation $x^2 - 14x + 2a$.

In the expression $x^2 - 11x + a$

$\therefore \alpha + \beta = 11$ ...(i)

And $\alpha\beta = a$ ...(ii)

Similarly, in the equation $x^2 - 14x + 2a$.

$\therefore \alpha + r = 14$ ...(iii)

And $\alpha r = 2a$ ...(iv)

using equation (i) and (iii), we can write

$(11 - \beta) + r = 14$

$r - \beta = 3$ ...(v)

Using equation (ii) and (iv), we have

$\alpha r = 2(\alpha\beta)$

$r = 2\beta$ ...(vi)

By solving equation (v) and (vi) we get,

$\beta = 3$ and $r = 6$

From (i), we get $\alpha = 8$

So, common factor is $(x - 8)$

12. **(a)** Given, p and q are the roots of the equation $x^2 + px + q = 0$

$\therefore$ Sum of roots, $p + q = -p$

$\Rightarrow 2p = -q$ ...(i)

Product of roots, $pq = q$

$\left(\frac{-q}{2}\right)q = q$ using (i)

$q = -2$

$\therefore p = 1$

13. **(b)** $(1 + \alpha)(1 + \beta)$

$= 1 + (\alpha + \beta) + \alpha\beta$

$= 1 + \left(\frac{-b}{a}\right) + \left(\frac{c}{a}\right)$

$= \frac{a - b + c}{a}$

14. **(b)** $D = b^2 - 4ac$

$= \left(12\sqrt{3}\right)^2 - 4 \times 7 \times 5$

$= 144 \times 3 - 140 = 432 - 140 = 292$

15. **(c)** We know D = 0 for equal roots

i.e., $b^2 = 4ac$

$\Rightarrow 4(ac + bd)^2 = 4 \times (c^2 + d^2)(a^2 + b^2)$

$\Rightarrow 4(a^2c^2 + b^2d^2 + 2acbd)$

$= 4(a^2c^2 + a^2d^2 + c^2b^2 + d^2b^2)$

$\Rightarrow a^2d^2 + c^2b^2 - 2acbd = 0$

$\Rightarrow (ad - cb)^2 = 0$

$\Rightarrow ad = cb \Rightarrow \frac{a}{b} = \frac{c}{d}$

16. **(b)** The given quadratic equation is $(a^2 + b^2)$ $x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$

Let α and β be the roots of the equation.

Given, $\alpha + \beta = \frac{\alpha\beta}{2}$

$\Rightarrow k + 6 = \frac{2(2k - 1)}{2}$

$\Rightarrow k + 6 = 2k - 1$

$\Rightarrow 2k - k = 7 \Rightarrow k = 7$

17. **(c)** Let the roots of quadratic equation $x^2 - 8x + k = 0$ be α and β.

Given, $\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow (8)^2 - 2k = 40$ $\quad [\because \alpha + \beta = 8 \text{ and } \alpha\beta = k]$

$\Rightarrow 64 - 2k = 40$

$\Rightarrow 2k = 24 \Rightarrow k = 12$

18. **(b)** Sum of roots $= \alpha + \beta = 5$

Product of roots $= \alpha\beta = 4$

So, $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$= \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta$

$= \frac{5}{4} - 2(4) = \frac{-27}{4}$

19. **(c)** $2x^2 + 5x + 3 = 0$

$D = b^2 - 4ac = 5^2 - 4(2)(3) = 25 - 24 = 1$

As D > 0, given equation has two distinct zeros.

20. **(c)** By comparing the equation $x^2 + 4x + 7 = 0$ with $ax^2 + bx + c = 0$, we get

a = 1, b = 4 and c = 7

As we know, the minimum value of given equation can be denoted by $y = \frac{4ac - b^2}{4a}$

$y = \frac{4(1)(7) - (4)^2}{4(1)}$

$= \frac{28 - 16}{4} = \frac{12}{4} = 3$

So, the minimum value of given quadratic equation is 3.

21. **(a)** Given, $2x^2 - 12x + 15 = 0$

$D = b^2 - 4ac = (-12)^2 - 4(2)(15)$

$= 144 - 120 = 24$

As D > 0, the equation has distinct real roots

22. **(c)** We have been given the quadratic equation:

$x^2 + 5kx + 16 = 0$

No real roots implies D < 0.

$\therefore D = b^2 - 4ac < 0$

$\Rightarrow (5k)^2 - 4(1)(16) < 0$

$\Rightarrow 25k^2 - 64 < 0$

$\Rightarrow k^2 < \frac{64}{25}$

$\Rightarrow k > -\frac{8}{5}$ and $k < \frac{8}{5}$

23. **(a)** We have been given the quadratic equation:

$(l^2 - m^2)x^2 + (m^2 - n^2)x + (n^2 - l^2) = 0$

$\Rightarrow l^2x^2 - m^2x^2 + m^2x - n^2x + n^2 - l^2 + l^2x - l^2x = 0$

On rearranging the terms, we get

$l^2x^2 - m^2x^2 + m^2x - l^2x + l^2x - n^2x + n^2 - l^2 = 0$

$\Rightarrow (l^2 - m^2)x^2 + (m^2 - l^2)x + (l^2 - n^2)x + (n^2 - l^2) = 0$

$\Rightarrow (l^2 - m^2)(x^2 - x) + (l^2 - n^2)(x - 1) = 0$

$\Rightarrow (l^2 - m^2)x(x - 1) + (l^2 - n^2)(x - 1) = 0$

$\Rightarrow (x - 1)[(l^2 - m^2)x + (l^2 - n^2)] = 0$

i.e., $(x - 1) = 0$ or $(l^2 - m^2)x + (l^2 - n^2) = 0$

$x = 1$ or $x = \frac{n^2 - l^2}{l^2 - m^2}$

24. **(c)** $(x - 2)$ is a factor of $x^2 + ax + b$ and $x^2 + cx + d$ i.e., 2 is a root of both the equations, therefore on putting 2 in both the equations, we get

$4 + 2a + b = 0$ ...(i)

$4 + 2c + d = 0$ ...(ii)

Subtract (ii) from (i)

$\Rightarrow 2a + b - 2c - d = 0$

$\Rightarrow b - d = 2c - 2a$

$\Rightarrow \dfrac{b-d}{c-a} = 2$

25. **(a)** The given quadratic equation is

$x^2 - 2ax + a^2 + a - 3 = 0$

Let α and β be the roots of the given equation.

Since, both roots are less than 3.

$\Rightarrow \alpha < 3, \beta < 3$

Therefore, $\alpha + \beta < 6$ and $\alpha\beta < 9$

Now, Sum of roots $= \alpha + \beta = 2a < 6$

$a < 3.$ ...(i)

Again, product, $P = \alpha\beta = 2a < 9$

$\Rightarrow \alpha < 3, \beta < 3$

$\Rightarrow \alpha\beta = a^2 + a - 3 < 9$

$a^2 + a - 12 < 0$

$a^2 + 4a - 3a - 12 < 0$

$a(a + 4) - 3(a + 4) < 0$

$(a - 3)(a + 4) < 0$

$-4 < a < 3$ ...(ii)

Also, roots of the equations are real i.e., $D \geq 0$

$D = b^2 - 4ac \geq 0$

$(-2a)^2 - 4(1)(a^2 + a - 3) \geq 0$

$4a^2 - 4a^2 - 4a + 12 \geq 0$

$-4a \geq -12$

$4a \leq 12$

$a \leq 3$ ...(iii)

From eq. (i), (ii) and (iii), we get

$a < 2$

26. **(a)** Here discriminant $= b^2 - 4ac$

$= (-8)^2 - 4(2)(8)$

$= 64 - 64 = 0$

For $D = 0$, roots are real and equal.

27. **(a)** Since $(x + 1)^2 - x^2 = 0$

$\Rightarrow x^2 + 1 + 2x - x^2 = 0$

$\Rightarrow 1 + 2x = 0$

$\Rightarrow x = -1/2$

This gives only 1 real value of x.

28. **(a)** As one root of the equation $14x^2 - 27x + k = 0$ is 5/2

Putting $x = 5/2$ in the equation $14x^2 - 27x + k = 0$

$\Rightarrow 14\left(\dfrac{5}{2}\right)^2 - 27\left(\dfrac{5}{2}\right) + k = 0$

$\Rightarrow 14\left(\dfrac{25}{4}\right) - \dfrac{135}{2} + k = 0$

$\Rightarrow \dfrac{175}{2} - \dfrac{135}{2} + k = 0$

$\Rightarrow k = \dfrac{135 - 175}{2} = -20$

29. **(b)** Let the number be x

Then according to the question,

$x + 12 = 160/x$

$x^2 + 12x - 160 = 0$

$x^2 + 20x - 8x - 160 = 0$

$(x + 20)(x - 8) = 0$

$x = -20, 8$

Here -20 is not a natural number therefore $x = 8$ is our answer.

30. **(a)** Let the consecutive integral multiples of 5 be 5n and $5(n + 1)$, where n is a positive integer.

According to the question, product of these two consecutive multiples is 300.

$\therefore 5n \times 5(n + 1) = 300$

$\Rightarrow n^2 + n - 12 = 0$

$\Rightarrow n^2 + 4n - 3n - 12 = 0$

$\Rightarrow (n - 3)(n + 4) = 0$

i.e., $n - 3 = 0$ or $n + 4 = 0$

$\Rightarrow n = 3$ and $n = -4$.

We neglect $n = -4$ as n is a natural number.

Therefore the multiples are 15 and 20.

31. **(c)** Let $\sqrt{6 + \sqrt{6 + \sqrt{6 \ldots\ldots}}} = y$

$\Rightarrow \sqrt{6 + y} = y$

Squaring both sides, we get

$6 + y = y^2$

$\Rightarrow y^2 - y - 6 = 0$

$\Rightarrow y^2 - 3y + 2y - 6 = 0$

$\Rightarrow y(y - 3) + 2(y - 3) = 0$

$\Rightarrow (y - 3)(y + 2) = 0$

$\Rightarrow y = 3, -2$

32. **(a)**

$p^2x^2 - q^2 = 0$

$\Rightarrow p^2x^2 = q^2$

$\Rightarrow x = \pm q/p$

33. **(b)** We have been given the equation: $\sqrt{3x^2 + 6} = 9$

Squaring both sides, we get

$\Rightarrow 3x^2 + 6 = 81$

$\Rightarrow 3x^2 = 75$

$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

So, the positive root of the given equation is 5.

34. **(d)** If equation has no real roots, then discriminant of the equation must be less than zero.

i.e., $D = b^2 - 4ac < 0$

$\Rightarrow 2^2(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) < 0$

$\Rightarrow 4a^2c^2 + 4b^2d^2 + 8acbd < 4a^2c^2 + 4b^2d^2 + 4a^2d^2 + 4b^2c^2$

$\Rightarrow 2acbd < a^2d^2 + b^2c^2$

$\Rightarrow 2acbd < (ad - bc)^2 + 2acbd$

$\Rightarrow (ad - bc)^2 > 0$

Above relation will not hold good only for $ad = bc$.

$\therefore$ $ad \neq bc$, then $ad < bc$ or $ad > bc$ are the correct options.

35. **(c)** Let actual marks of preeti be x

According to the question.,

$9(x + 10) = x^2$

$\Rightarrow x^2 - 9x - 90 = 0$

$\Rightarrow x^2 - 15x + 6x - 90 = 0$

$\Rightarrow x(x - 15) + 6(x - 15) = 0$

$\Rightarrow (x + 6)(x - 15) = 0$

Therefore $x = -6$ or $x = 15$

Since x is the marks obtained, $x \neq -6$.

Therefore, $x = 15$.

Hence, preeti got 15 marks in her test.

Level-II

1. **(b)** Equation $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}x + q = 0$ have real roots, then from first

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow 4q^2 - 4pr \geq 0$

$\Rightarrow q^2 \geq pr$ ...(i)

and from second

$4(pr) - 4q^2 \geq 0$ (for real root)

$pr \geq q^2$ ...(ii)

From (i) and (ii), we get

$q^2 = pr$

2. **(a)** Given, $x = \sqrt{7 + 4\sqrt{3}}$

$x = \sqrt{4 + 3 + 2\left(2\sqrt{3}\right)}$

$x = \sqrt{\left(2 + \sqrt{3}\right)^2}$

$x = 2 + \sqrt{3}$

Now, $\frac{1}{x} = \frac{1}{2 + \sqrt{3}}$

$= \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$

$= \frac{2 - \sqrt{3}}{4 - \left(\sqrt{3}\right)^2} = 2 - \sqrt{3}$

[using $(a + b)(a - b) = a^2 - b^2$]

$\therefore$ $x + \frac{1}{x} = 2 + \sqrt{3} + 2 - \sqrt{3}$

$= 4$

3. **(b)** The given quadratic equation is $ax^2 + bx + c = 0$

Let one of the roots be α and the other be 2α

$\therefore$ Sum of roots $= 3\alpha = \frac{-b}{a}$ ...(i)

Product of roots $= \alpha(2\alpha) = \frac{c}{a}$

$2\alpha^2 = \frac{c}{a}$

$\Rightarrow$ $2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a}$ (using (i))

$\Rightarrow$ $\frac{2b^2}{9a^2} = \frac{c}{a}$

$\Rightarrow$ $2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$.

4. **(d)** The given equation is

$x^{2/3} + x^{1/3} - 2 = 0$

Put $x^{1/3} = y$

then $y^2 + y - 2 = 0$

$y^2 + 2y - y - 2 = 0$

$y(y + 2) - 1(y + 2) = 0$

$(y - 1)(y + 2) = 0$

$y - 1 = 0$ or $y + 2 = 0$

$y = 1$, $y = -2$

$\therefore$ $x^{1/3} = 1 \Rightarrow x = 1$

Also, $x^{1/3} = -2 \Rightarrow x = -8$

Hence, the real roots of the given equation are 1, –8

5. **(d)** The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$.

Here, $a = 2$, $b = 2(p + 1)$ and $c = p$

$b^2 - 4ac = [2(p + 1)]^2 - 4(2p)$

$= 4(p + 1)^2 - 8p$

$= 4[(p + 1)^2 - 2p]$

$= 4[(p^2 + 2p + 1) - 2p]$

$= 4(p^2 + 1)$

For any real value of p, $4(p^2 + 1)$ will always be positive as p^2 cannot be negative for real p.

Hence, the discriminant $b^2 - 4ac$ will always be positive. When the discriminant is greater then '0' or is positive, then the roots of a quadratic equation will be real.

6. **(b)** The given quadratic equation is $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$

$$x^2 + \frac{3}{36}x - \frac{\sqrt{2}}{9} = 0$$

$$x^2 + \frac{1}{12}x = \frac{\sqrt{2}}{9}$$

$$x^2 + \frac{1}{12}x + \left(\frac{1}{24}\right)^2 = \frac{\sqrt{2}}{9} + \left(\frac{1}{24}\right)^2$$

Thus, $\frac{1}{576}$ must be added and subtracted to solve the given equation.

7. **(b)** The given equation has real roots only when D = 0

i.e., $b^2 = 4ac$

$\Rightarrow 4(mp + nq)^2 = 4(m^2 + n^2)(p^2 + q^2)$

$\Rightarrow m^2p^2 + n^2q^2 + 2mpnq = m^2p^2 + m^2q^2 + n^2p^2 + n^2q^2$

$\Rightarrow m^2q^2 + n^2p^2 - 2mnpq = 0$

$\Rightarrow (mq - np)^2 = 0$

$\Rightarrow mq - np = 0 \Rightarrow mq = np$

8. **(a)** Let the roots of the equation $x^2 - bx + c = 0$ be α and β.

$\therefore\ \alpha + \beta = b$ ...(i)

and $\alpha\beta = c$ ...(ii)

Resulting equation if each root of the above equation is decreased by 2 is $x^2 - 2x + 1 = 0$

and its roots are $(\alpha - 2)$ and $(\beta - 2)$

$\therefore$ Sum of roots = 2

Product of roots = 1

$(\alpha - 2)(\beta - 2) = 1$

$\alpha\beta - 2\alpha - 2\beta + 4 = 1$

$\alpha\beta - 2(\alpha + \beta) + 4 = 1$

$c - 2b = -3$ ...(iii) [using (i) and (ii)]

Also, $(\alpha - 2) + (\beta - 2) = 2$

$\alpha + \beta = 6$

$\therefore\ b = 6$ ...(iv) [using (i)]

By substituting (iv) in (iii), we get

$c - 2(6) = -3$

$\therefore\ c = 9$

9. **(b)** Let α, α^2 be the roots of the equation $ax^2 - bx + c = 0$

$\therefore$ Sum of roots = $\alpha + \alpha^2 = \frac{b}{a}$ ...(i)

Also, product of roots = $\alpha^3 = \frac{c}{a}$

Taking cube on both side of equation (i), we get

$$\Rightarrow \left(\alpha + \alpha^2\right)^3 = \frac{b^3}{a^3}$$

Using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$\Rightarrow \alpha^3 + \alpha^6 + 3\alpha^3\left(\alpha + \alpha^2\right) = \frac{b^3}{a^3}$$

$$\Rightarrow \alpha^3 + (\alpha^3)^2 + 3\alpha^3(\alpha + \alpha^2) = \frac{b^3}{a^3}$$

$$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + 3\cdot\frac{c}{a}\cdot\frac{b}{a} = \frac{b^3}{a^3} \qquad \left(\because \alpha + \alpha^2 = \frac{b}{a}; \alpha^3 = \frac{c}{a}\right)$$

$$\Rightarrow \frac{ac + c^2 + 3bc}{a^2} = \frac{b^3}{a^3}$$

$$\Rightarrow ac + c^2 + 3bc = \frac{b^3}{a}$$

$$\Rightarrow a\left(ac + c^2 + 3bc\right) = b^3$$

$\Rightarrow a^2c + ac^2 + 3abc = b^3$

$\Rightarrow ac(a + c + 3b) = b^3$

10. **(b)** Let α, β be the roots of $x^2 - px + q = 0$

$\therefore$ sum of roots = $\alpha + \beta = p$

Also, product of roots = $\alpha\beta = q$

Given, $\alpha - \beta = 1$

$\Rightarrow (\alpha - \beta)^2 = 1$

Using $(a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow p^2 - 4q = 1$

$\Rightarrow p^2 = 4q + 1$

11. **(a)** Let α and β are the roots of the equation $x^2 + mx + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$

$\therefore$ Sum of roots = $(\alpha + \beta) = -\frac{b}{a}$

Sum of roots of eq. 1 = Sum of roots of eq. 2

$$-m = -\frac{(c - a)}{b - c} \qquad ...(i)$$

Also, product of roots = $\alpha\beta = \frac{c}{a}$

Product of roots of eq. 1 = Product of roots of eq. 2

$$1 = \frac{a - b}{b - c}$$

$\Rightarrow b - c = a - b$ ...(ii)

$\Rightarrow 2b = a + c$ ...(iii)

Now from (i)

$m = \dfrac{c-a}{b-c}$

$m = \dfrac{c+a-a-a}{b-c}$

$m = \dfrac{2b-2a}{a-b}$ [using (ii) and (iii)]

$m = -2\dfrac{(a-b)}{a-b}$

$m = -2$

12. **(c)** Let the two numbers be a and b.

Given, $a + b = 8$ ...(i)

$\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{8}{15}$

$\therefore \dfrac{1}{ab}=\dfrac{1}{15}$ [using (i)]

$\Rightarrow ab = 15 \Rightarrow a=\dfrac{15}{b}$ ...(ii)

By substituting (ii) in (i), we get

$a+\dfrac{15}{a}=8$

$\Rightarrow a^2 + 15 = 8a$

$\Rightarrow a^2 - 8a + 15 = 0$

$\Rightarrow a^2 - 5a - 3a + 15 = 0$

$\Rightarrow a(a - 5) - 3(a - 5) = 0$

$\Rightarrow (a - 5)(a - 3) = 0$

$\therefore a = 5$ or 3

If $a = 5$, then $b = 3$ and if $a = 3$, then $b = 5$

$\therefore$ Two required numbers are 3 and 5.

13. **(c)** Given α and β are the roots of quadratic equation $kx^2 + 4x + 4 = 0$

$\therefore \alpha + \beta = -\dfrac{4}{k}$ ...(i)

Also, $\alpha\beta=\dfrac{4}{k}$ ...(ii)

By squaring equation (i), we get

$(\alpha + \beta)^2 = \left(\dfrac{-4}{k}\right)^2$

$\alpha^2+\beta^2+2\alpha\beta=\dfrac{16}{k^2}$

$\alpha^2+\beta^2=\dfrac{16}{k^2}-\dfrac{8}{k}$ (using (ii))

$24k^2 = 16 - 8k$ (given, $\alpha^2 + \beta^2 = 24$)

$3k^2 + k - 2 = 0$

$3k^2 + 3k - 2k - 2 = 0$

$3k(k + 1) - 2(k + 1) = 0$

$(3k - 2)(k + 1) = 0$

i.e., $k = \dfrac{2}{3}$ or $k = -1$

14. **(d)** Let $\dfrac{2x-1}{x+1}=y$ ...(i)

$\therefore \quad y-\dfrac{15}{y}=-2$

$\Rightarrow \quad y^2 - 15 = -2y$

$\Rightarrow \quad y^2 + 2y - 15 = 0$

$\Rightarrow y^2 + 5y - 3y - 15 = 0$

$y(y + 5) - 3(y + 5) = 0$

$y = 3, -5$

Putting $y = 3$ and $y = -5$ in equation (i)

$\dfrac{2x-1}{x+1}=3$, $\dfrac{2x-1}{x+1}=-5$

$2x - 1 = 3x + 3$ $\quad$ $2x - 1 = -5x - 5$

$3x - 2x = -3+1$ $\quad$ $7x = -4$

$x = -4$ $\quad$ $x = -4/7$

15. **(d)** Let the total number of students who planned the trip be x.

Then, the share of each student = ₹ $\dfrac{480}{x}$

$\therefore$ no. of students who went for the trip = $x - 8$

New share of each student = $\dfrac{480}{x-8}$

Total budget of the trip = ₹ 480

Then, according to the question if eight of the students failed to go on the trip then the share of each student gets increased by ₹ 10.

$\dfrac{480}{x-8}-\dfrac{480}{x}=10$

$\Rightarrow 480x - 480(x - 8) = 10x(x - 8)$

$\Rightarrow 480x - 480x + 3840 = 10x^2 - 80x$

$\Rightarrow x^2 - 8x - 384 = 0$

$\Rightarrow x^2 - 24x + 16x - 384 = 0$

$\Rightarrow x(x - 24) + 16(x - 24) = 0$

i.e., $x = 24$ or -16

As no. of students can't be negative.

Therefore, no. of students who planned the trip was 24 and the students who attended the trip was $x - 8 = 24 - 8 = 16$.

Assertion & Reason Type

1. **(c)** Assertion (A) is true but reason (R) is false.

   Assertion $3x^2 - 6x + 3 = 0$

   $$D = b^2 - 4ac$$
   $$= (-6)^2 - 4(3)(3)$$
   $$= 36 - 36 = 0$$

   Roots are repeated as $D = 0$.

   Reason is false, as roots are equal for $D = 0$.

2. **(b)** Both assertion (A) and reason (R) is true and reason (R) is the correct explanation of assertion (A).

   Assertion $(2x - 1)^2 - 4x^2 + 5 = 0$

   $\Rightarrow$ $-4x + 6$

   Reason: The above equation is not of the form $ax^2 + bx + c = 0$.

   $\therefore$ above equation is not a quadratic equation.

3. **(b)** Both assertion (A) and reason (R) is true and reason (R) is not the correct explanation of assertion (A).

   As one root is $\frac{2}{3}$, $x = \frac{2}{3}$

   $$6\times\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$
   $$6\times\frac{4}{9} - \frac{2}{3} = k$$
   $$k = \frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$$
   $$k = 2$$

   So, both A and R are correct but R does not explain A.

4. **(a)** Both assertion (A) and reason (R) is true and reason (R) is the correct explanation of assertion (A).

   $$x^2 + 2x + 2 = 0$$

   Discriminant, $D = b^2 - 4ac$
   $$= (2)^2 - 4 \times 1 \times 2$$
   $$= 4 - 8 = -4 < 0$$

   As we know roots are imaginary when $D < 0$.

   So, both A and R are correct and R explains A.

5. **(c)** Assertion (A) is true but reason (R) is false.

   Assertion $5x^2 + 9x - 6 = 0$

   $$\alpha + \beta = \frac{-b}{a} = \frac{-9}{5}$$

   and $\alpha\beta = \frac{c}{a} = \frac{-6}{5}$

Case-Based Type Questions

Case-Based-I

1. **(a)** Quadratic
2. **(a)** Now, $f(x) = x^2 - 7x + 10 = 0$

   $\Rightarrow x^2 - 5x - 2x + 10 = 0$

   $\Rightarrow x(x - 5) - 2(x - 5) = 0$

   $\Rightarrow (x - 5)(x - 2) = 0$

   $\Rightarrow x = 5, 2$

   As no. of students who like cricket is greater than the students who like football.

   $\therefore$ there are 5 students who like cricket.
3. **(d)** Sum of Zeroes $= -3 - 4 = -7$,

   Product of Zeroes $= -3 \times -4 = 12$

   $x^2 -$ (Sum of Zeroes)x + (Product of Zeroes)

   $= x^2 - (-7)x + 12 = x^2 + 7x + 12$

Case-Based-II

1. **(b)** $-2, 2$
2. **(d)** Parabola
3. **(b)** Sum of Zeroes $= -2 + 2 = 0$,

   Product of Zeroes $= -2 \times 2 = -4$

   $x^2 -$ (Sum of Zeroes)x + (Product of Zeroes)

   $= x^2 - (0)x - 4 = x^2 - 4$

Multi Correct MCQs

1. **(b, c)**

   $a + b + c = 0$

   $b = -a - c$

   now $ax^2 + bx + c = 0$

   $ax^2 - ax - cx + c = 0$ $(\because b = -a - c)$

   $ax(x - 1) - c(x - 1) = 0$

   $$x = 1, \frac{c}{a}$$
2. **(a, b)**

   let the number be x, then according to the question

   $$x + \frac{1}{x} = 2\frac{1}{30}$$

   $\Rightarrow 30x^2 + 30 = 61x$

   $\Rightarrow 30x^2 - 61x + 30 = 0$

   $\Rightarrow 30x^2 - 36x - 25x + 30 = 0$

   $\Rightarrow 6x(5x - 6) - 5(5x - 6) = 0$

   $\Rightarrow (6x - 5)(5x - 6) = 0$

   $$x = \frac{6}{5}, \frac{5}{6}$$

3. **(a)** Let two consecutive natural number be 'a' and 'a + 1', then according to the question

$a^2 + (a + 1)^2 = 313$

$\Rightarrow a^2 + a^2 + 2a + 1 = 313$

$\Rightarrow 2a^2 + 2a = 312$

$\Rightarrow a^2 + a - 156 = 0$

$\Rightarrow a^2 + 13a - 12a - 156 = 0$

$\Rightarrow a(a + 13) - 12(a + 13) = 0$

$\Rightarrow a = 12$ or -13

Neglecting –13 as no.'s are natural.

4. **(b,c)** The number of zeroes of p(x) is the number of times the curve intersects the x-axis. Here, the polynomial p(x) meets the x-axis at 2 points.

From the graph we can conclude that the equation has two real and distinct roots.

For real and distinct roots, $D > 0$

$D = b^2 - 4ac > 0$

$\Rightarrow b^2 > 4ac$

As the graph is open downwards, so $a < 0$.

We can also conclude from the graph that both the roots are of opposite signs.

$\therefore$ Product of zeros $\left(=\frac{c}{a}\right)$ must be less than zero.

$\frac{c}{a} < 0$

As a is less than zero. Therefore $\left(\frac{c}{a}\right)$ can be less than zero only when $c > 0$.

Also magnitude of +ve roots is larger

$\therefore$ Sum of roots > 0

$\Rightarrow \frac{-b}{a} > 0$

As a is less than zero i.e negative.

$\therefore$ To make $\frac{-b}{a} > 0$, b should be positive i.e. $b > 0$

Hence options (b) and (c) are correct.

Olympiad & NTSE Type

1. **(c)** Given, $x^2 + y^2 = 25$ ...(i)

$xy = 12 \Rightarrow y = \frac{12}{x}$

Putting the value of y in equation (i), we get

$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$

$\Rightarrow \quad x^4 + 144 - 25x^2 = 0$

$\Rightarrow \quad x^4 - 16x^2 - 9x^2 + 144 = 0$

$\Rightarrow \quad x^2(x^2 - 16 - 9(x^2 - 16) = 0$

$\Rightarrow \quad (x^2 - 16)(x^2 - 9) = 0$

Hence, $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9 \Rightarrow x = \pm 3$

2. **(a)** Let α and β are the roots of the equation $x^2 + bx + c = 0$, while r and s are the roots of equation $x^2 + qx + r = 0$

According to the question,

$\frac{\alpha}{\beta} = \frac{r}{s} = k$

$\Rightarrow \alpha = \beta k$ ...(i)

and $r = sk$ ...(ii)

$\therefore$ Sum of roots $= \beta k + \beta = -b$ ...(using (i))

$\beta(1 + k) = -b$ ...(iii)

Product of roots $= (\beta k)\beta = c$

$\beta^2 k = c$ ...(iv)

Dividing equation (iii) by (iv), we get

$$\frac{(1+k)^2}{k} = \frac{b^2}{c} \quad ...(v)$$

Similarly, $\frac{(1+k)^2}{k} = \frac{q^2}{r}$ ...(vi)

Using (v) and (vi)

$\frac{b^2}{c} = \frac{q^2}{r} \Rightarrow b^2 r = q^2 c$

3. **(b)** Let P be the production 2 years ago and the increase in product every year be x%

Then product at the end of 1st year $= p + \frac{px}{100}$

Product at the end of 2nd year,

$= p + \frac{px}{100} + \frac{x}{100}\left(p + \frac{px}{100}\right)$

$= \left(p + \frac{px}{100}\right)\left(1 + \frac{x}{100}\right)$

$= p\left(1 + \frac{x}{100}\right)^2$

According to the question,

$= p\left(1 + \frac{x}{100}\right)^2 = 2p$

$\Rightarrow \left(1 + \frac{x}{100}\right)^2 = 2$

Using quadratic formula,

$\Rightarrow x^2 + 200x - 10000 = 0$

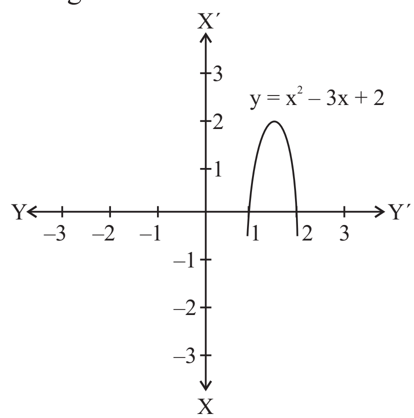
$$\Rightarrow x = \frac{-200 \pm \sqrt{(200)^2 + 40000}}{2}$$

$$= -100 \pm 100\sqrt{2} = 100\left(-1 \pm \sqrt{2}\right)$$

x cannot be negative as production can't be negative.

$$x = 100\left(-1 + \sqrt{2}\right)$$

4. (a) When we rotate the axes at an angle of 90° in anti-clockwise direction, then the new graph is same as shown alongside.



Here, we see that graph is shown on negative of Y – axis. So, we replace y by x and x by –y in the original equation

$$y = x^2 - 3x + 2$$

We get, $x = (-y)^2 - 3(-y) + 2$

$$x = y^2 + 3y + 2$$

5. (c)

Given equation is, $ax^2 + bx + c = 0$

Since, $\sin\alpha$ and $\cos\alpha$ are the roots of this equation.

Sum of the roots, $\sin\alpha + \cos\alpha = \frac{-b}{a}$ ...(1)

and product of the roots, $\sin\alpha\cos\alpha = \frac{c}{a}$ ...(2)

On squaring both sides of Eq. (1), we get

$$(\sin\alpha + \cos\alpha)^2 = \left(\frac{-b}{a}\right)^2$$

$$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$$

$$\Rightarrow 1 + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow 2\sin\alpha\cos\alpha = \frac{b^2}{a^2} - 1$$

$$\Rightarrow 2 \times \left(\frac{c}{a}\right) = \frac{b^2 - a^2}{a^2} \qquad \text{[From Eq. 2]}$$

$$\Rightarrow 2ac = b^2 - a^2$$

$$\Rightarrow b^2 = a^2 + 2ac$$

6. (b) Given, $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$ ...(i)

Put $\sqrt{x+10} = y$

$$y - \frac{6}{y} = 5$$

$$\Rightarrow y^2 - 5y - 6 = 0$$

$$\Rightarrow y^2 - 6y + y - 6 = 0$$

$$\Rightarrow y(y - 6) + 1(y - 6) = 0$$

$$\Rightarrow (y + 1)(y - 6) = 0$$

i.e., $y = 6$ or -1

As $y = \sqrt{x+10}$

$\sqrt{x+10} = 6$ or $\sqrt{x+10} = -1$

Squaring both sides we get

$x + 10 = 36$ or $x + 10 = 1$

$x = 26$ or $x = -9$

On putting $x = -9$ in eq. (1), we get

$$\sqrt{-9+10} - \frac{6}{\sqrt{-9+10}} = 5$$

$$1 - 6 = 5$$

$-5 = 5$ (which is not true)

Hence, extraneous root of given equation is –9.

Note: Extraneous root is a solution which looks correct but after analysing it (by substituting it into the original equation) turns out as incorrect.

7. (a) Let the total number of flowers be x

No. of flowers offered to first temple $= \frac{x}{4}$

No. of flowers offered to 2nd temple $= \frac{x}{4} + \frac{x}{9} + 7\sqrt{x}$

No. of flowers he is left with = 56

According to the question:

$$\frac{x}{4} + \left(\frac{x}{9} + \frac{x}{4} + 7\sqrt{x}\right) + 56 = x$$

$$\Rightarrow x - \frac{x}{4} - \frac{x}{9} - \frac{x}{4} - 7\sqrt{x} - 56 = 0$$

$$\Rightarrow \frac{36x - 9x - 4x - 9x}{36} - 7\sqrt{x} - 56 = 0$$

$$\Rightarrow \frac{14x}{36} - 7\sqrt{x} - 56 = 0$$

$$\Rightarrow \frac{x}{18} - \sqrt{x} - 8 = 0 \qquad \text{[dividing both sides by 7]}$$

Put $\sqrt{x} = y$, then above equation becomes

$$y^2 - 18y - 144 = 0$$

$$y^2 - 24y + 6y - 144 = 0$$

$y(y - 24) + 6(y - 24) = 0$

$(y - 24)(y + 6) = 0$

$\Rightarrow y = 24$ or -6

But y cannot be -6 [as no. of flowers can't be negative]

as $\sqrt{x} = y$

$\Rightarrow \sqrt{x} = 24 \Rightarrow x = 576$

Hence, total number of flowers is 576.

8. **(c)** Given, $2^{2x} - 10.2^x + 16 = 0$

$\Rightarrow (2^x)^2 - 10.2^x + 16 = 0$

Put $2^x = y$

$\Rightarrow y^2 - 10y + 15 = 0$

$\Rightarrow y^2 - 8y - 2y + 16 = 0$

$\Rightarrow y(y - 8) - 2(y - 8) = 0$

$\Rightarrow (y - 2)(y - 8) = 0$

i.e., $y = 2$ or $y = 8$

As $y = 2^x = 2$ or $2^x = 8$

$2^x = (2)^1$ or $2^x = (2)^3$

So, roots are 1 and 3.

9. **(b)** As α and β are the roots of $ax^2 + bx + c = 0$.

Also, $\alpha + \beta = \frac{-b}{a}$

and $\alpha\beta = \frac{c}{a}$

$\therefore \frac{\alpha}{\beta}(a\alpha + b) + \frac{\beta}{\alpha}(a\beta + b)$

$\Rightarrow \frac{\alpha^2}{\beta}a + \frac{\alpha}{\beta}b + \frac{\beta^2}{\alpha}a + \frac{\beta}{\alpha}b$

$\Rightarrow \left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)a + \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)b$

$\Rightarrow \frac{a(\alpha^3 + \beta^3) + b(\alpha^2 + \beta^2)}{\alpha\beta}$

As we know,

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$a^2 + b^2 = (a + b)^2 - 2ab$

$$\Rightarrow \frac{a\left[(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)\right] + b\left[(\alpha + \beta)^2 - 2\alpha\beta\right]}{\alpha\beta}$$

$$\Rightarrow \frac{a\left[\left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)\right] + b\left[\left(-\frac{b}{a}\right)^2 - \frac{2c}{a}\right]}{\frac{c}{a}}$$

$$\Rightarrow \frac{a\left[-\frac{b^3}{a^3} + \frac{3bc}{a^2}\right] + b\left[\frac{b^2}{a^2} - \frac{2c}{a}\right]}{\frac{c}{a}}$$

$$\Rightarrow \frac{\frac{-b^3}{a^2} + \frac{3bc}{a} + \frac{b^3}{a^2} - \frac{2bc}{a}}{\frac{c}{a}} = b$$

10. **(a)** Note: If a, b and c are in A.P., then $2b = a + c$.

Let α, β be roots of the equation $ax^2 + bx + c = 0$, then

$\alpha + \beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

Given that $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

$\Rightarrow \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$$\Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

$\Rightarrow -\frac{b}{a} \times \frac{c^2}{a^2} = \frac{b^2}{a^2} - \frac{2c}{a} \Rightarrow 2a^2c = b^2a + c^2b$

$\therefore ab^2, a^2c, bc^2$ are in A.P.

11. **(d)** α, β are roots of $ax^2 + bx + c = 0$

then $\alpha + \beta = \frac{-b}{a}$ ...(i)

$\alpha\beta = \frac{c}{a}$ ...(ii)

Let α_1, β_1 be roots of $a^3x^2 + abcx + c^3 = 0$

$\alpha_1 + \beta_1 = -\frac{abc}{a^3}$

$= -\frac{bc}{a^2}$

$= \left(-\frac{b}{a}\right)\left(\frac{c}{a}\right)$

$= (\alpha + \beta)(\alpha\beta)$ using (i) and (ii)

$\therefore \alpha_1 + \beta_1 = \alpha^2\beta + \alpha\beta^2$ ...(iii)

$\alpha_1\beta_1 = \frac{c^3}{a^3}$

$$= (\alpha\beta)^3$$
$$= \alpha^3\beta^3$$
$$= (\alpha^2\beta)(\alpha\beta^2)$$

$\therefore\ \alpha_1\beta_1 = (\alpha^2\beta)(\alpha\beta^2)$ ...(iv)

From (iii) and (iv), $\alpha_1 = (\alpha^2\beta)$ $\beta_1 = (\alpha\beta^2)$

Roots of $a^3x^2 + abcx + c^3 = 0$ are $\alpha^2\beta$ and $\alpha\beta^2$.

12. (a) Let the original speed be x km/h

Distance covered (d) = 2 km

Time $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{2}{x}$

If he/she had walked 1 km/h faster, then the speed would have been (x + 1) km/h.

$\therefore$ Time $= \dfrac{2}{x+1}$

According to the question,

$$\Rightarrow \frac{2}{x} - \frac{2}{x+1} = \frac{10}{60}$$

$$\Rightarrow \frac{x+1-x}{x(x+1)} = \frac{1}{12}$$

$\Rightarrow 12 = x^2 + x$

$\Rightarrow x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x + 4) - 3(x + 4) = 0$

$\Rightarrow (x - 3)(x + 4) = 0$

i.e., $x = 3$ or $x = -4$

As speed can't be negative.

$\therefore$ speed = 3km/hr.

13. (c) $(x - b)(x - c) + (x - c)(x - a) + (x - a)(x - b) = 0$

$x^2 - bx - cx + bc + x^2 - cx - ax + ac + x^2 - ax - bx + ab = 0$

$3x^2 - (2b + 2c + 2a)x + ab + bc + ca = 0$

To know the nature of roots, we need to find discriminant.

$D = b^2 - 4ac$

$= (2b + 2c + 2a)^2 - 12(ab + bc + ca)$

$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) - 12ab - 12bc - 12ca$

$= 4a^2 + 4b^2 + 4c^2 - 4ab - 4bc - 4ca$

$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

On rearranging, we get

$= 2(a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca)$

$= 2[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$D \geq 0$

So the roots are always real.

14. (b) Given

$$\Rightarrow x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$$

$$(x - 2) = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right) \quad \text{...(i)}$$

On cubing both sides, we get

$$\Rightarrow (x - 2)^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$$

As we know,

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$\Rightarrow x^3 - 8 - 6x(x - 2) = 2 + 4 + 3 \times 2^{1/3} \times 2^{2/3}(2^{1/3} + 2^{2/3})$

$\Rightarrow x^3 - 8 - 6x^2 + 12x = 6 + 3 \times 2(x - 2)$

[From (i) $2^{1/3} + 2^{2/3} = x - 2$]

$\Rightarrow x^3 - 6x^2 + 12x = 14 + 6x - 12$

$\Rightarrow x^3 - 6x^2 + 6x = 2$