# Linear Programming

## QUICK RECAP

### LINEAR PROGRAMMING

- Linear programming (LP) is an optimisation technique in which a linear function is optimised (*i.e.*, minimised or maximised) subject to certain restrictions which are in the form of linear inequalities.

### LINEAR PROGRAMMING PROBLEM

- A linear programming problem (LPP) is a problem that is concerned with finding the optimal value of a linear function subject to given constraints.

## WORKING RULE TO FORMULATE LPP

- The formulation of LPP as a mathematical model involves the following steps :

  **Step 1 :** Identify the aim or objective which is to be maximised or minimised and denote it by $Z$.

  **Step 2 :** Identify the decision variables and assign symbols $x, y \dots$ or $x_1, x_2 \dots$ to them.

  **Step 3 :** Identify all the restrictions or constraints in the problem and express them as linear inequalities or equations in terms of variables.

  **Step 4 :** Express the hidden conditions, generally involves non-negativity of variables.

- **Objective function :** The linear function $Z = ax + by$, which has to be optimised (maximised or minimised) is called the objective function.

- **Constraints :** The restrictions or inequalities in the linear programming problem.

- **Non-negativity constraints :** The assumption that negative values of variables are not possible in the solution. They are described as $x, y \geq 0$ or $x_1, x_2 \geq 0$.

## OPTIMISATION PROBLEM

- A problem which maximise or minimise a linear function subject to the given constraints.

- **Feasible Region :** The common region determined by all the constraints of an LPP is called the feasible region.

  The feasible region may be either bounded or unbounded.

  (i) **Bounded feasible region :** If the feasible region is enclosed within a circle, then it is called bounded feasible region.

  (ii) **Unbounded feasible region :** If the feasible region is not bounded, then it is called unbounded feasible region.

- **Feasible Solution :** The set of points, within or on the boundary of the feasible region is said to be the feasible solution.

  **Note :**

  (i) The region other than feasible region is called infeasible region.

  (ii) Any point outside the feasible region is called an infeasible solution.

- **Optimal Value :** The maximum or minimum value of the objective function is called optimal value.

- **Optimal Solution :** Any point in the feasible region which gives the optimal value is called optimal solution.

- **Corner Point :** The intersection point of two boundary lines of the feasible region.

## SOME IMPORTANT THEOREMS OF LPP

- **Theorem 1 :** Let $R$ be the feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. When $Z$ has an optimal value (maximum or minimum), where the variables $x$ and $y$ are subject to constraints described by linear inequalities, the optimal value must occur at a corner point of the feasible region.

- **Theorem 2 :** Let $R$ be the feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. If $R$ is bounded, then the objective function $Z$ has both maximum and minimum value on $R$ and each of these occurs at a corner point of $R$.

  **Remark :** If $R$ is unbounded, then a maximum or a minimum value of the objective function may not exist. However, if it exists, it must occur at a corner point of $R$.

## STEPS TO SOLVE LPP

- Here are the following steps to solve an LPP.

  **Step 1 :** Convert inequations into equations.

  **Step 2 :** Find the point of intersection.

  **Step 3 :** Draw the graph of inequations.

  **Step 4 :** Find the value of the objective function corresponding to each corner point.

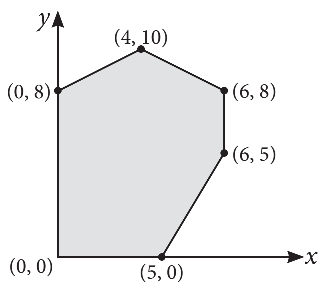
# Previous Years' CBSE Board Questions

## 12.2 Linear Programming Problem and its Mathematical Formulation

**VSA (1 mark)**

**1.** The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then
(a) $a = 2b$ (b) $2a = b$
(c) $a = b$ (d) $3a = b$ *(2020)*

**2.** The feasible region for an LPP is shown below :
Let $z = 3x - 4y$ be the objective function. Minimum of $z$ occurs at




(a) (0, 0) (b) (0, 8)
(c) (5, 0) (d) (4, 10) *(2020)*

**3.** The graph of the inequality $2x + 3y > 6$ is
(a) half plane that contains the origin
(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.
(c) whole $XOY$-plane excluding the points on the line $2x + 3y = 6$.
(d) entire $XOY$-plane. *(2020)*

**LA 1 (4 marks)**

**4.** Solve the following LPP graphically :
Minimize $z = 5x + 7y$
subject to the constraints
$2x + y \geq 8$
$x + 2y \geq 10$
$x, y \geq 0$ *(2020)*

**5.** Solve the following LPP graphically :
Minimise $Z = 5x + 10y$
Subject to constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$ and $x, y \geq 0$ *(Delhi 2017)*

**6.** Maximise $Z = x + 2y$
subject to the constraints:
$x + 2y \geq 100$
$2x - y < 0$
$2x + y \leq 200$
$x, y \geq 0$
Solve the above LPP graphically. *(AI 2017)*

**LA 2 (6 marks)**

**7.** Find graphically, the maximum value of $z = 2x + 5y$, subject to constraints given below:
$2x + 4y \leq 8$, $3x + y \leq 6$, $x + y \leq 4$; $x \geq 0$, $y \geq 0$ *(Delhi 2015)*

**8.** Maximise $z = 8x + 9y$ subject to the constraints given below :
$2x + 3y \leq 6$, $3x - 2y \leq 6$, $y \leq 1$; $x, y \geq 0$ *(Foreign 2015)*

## 12.3 Different Types of Linear Programming Problems

**SA (2 marks)**

**9.** A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate an LPP for finding how many of each should be produced daily to maximize the profit ? It is being given that at least one of each must be produced. *(Delhi 2017)*

**10.** Two tailors, $A$ and $B$, earn ₹ 300 and ₹ 400 per day respectively. $A$ can stitch 6 shirts and 4 pairs of trousers while $B$ can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP. *(AI 2017)*

## LA 1 (4 marks)

**11.** A furniture trader deals in only two items—chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1,000 and a table costs him ₹ 2,000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically. *(2020)*

**12.** A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type *A* requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type *A* souvenir is ₹ 100 each and for type *B* souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically. *(2020)*

## LA 2 (6 marks)

**13.** A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models *A* and *B* of an article. The making of one item of model *A* requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model *B* requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model *A* is ₹ 15 and on an item of model *B* is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit. *(Delhi 2019)*

**14.** A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. It costs ₹ 50 per kg to produce food I. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C and it costs ₹ 70 per kg to produce food II. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost. *(AI 2019)*

**15.** A factory manufactures two types of screws *A* and *B*, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws '*A*' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws '*B*'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws '*A*' at a profit of 70 paise and screws '*B*' at a profit of ₹ 1. Assuming that he can sell all the screws be manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit. *(2018)*

**16.** A manufacturer produces two products *A* and *B*. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product *A* requires 3 hours on both machines and each unit of product *B* requires 2 hours on first machine and 1 hour on second machine. Each unit of product *A* is sold at ₹7 profit and that of *B* at a profit of ₹4. Find the production level per day for maximum profit graphically. *(Delhi 2016)*

**17.** There are two types of fertilisers '*A*' and '*B*'. '*A*' consists of 12% nitrogen and 5% phosphoric acid whereas '*B*' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If '*A*' costs ₹ 10 per kg and '*B*' costs ₹ 8 per kg, then graphically determine how much of each type of fertiliser should be used so that the nutrient requirements are met at a minimum cost. *(AI 2016)*

**18.** In order to supplement daily diet, a person wishes to take $X$ and $Y$ tablets. The contents (in milligrams per tablet) of iron, calcium and vitamins in $X$ and $Y$ are given as below:

| Tablets | Iron | Calcium | Vitamin |
|---|---|---|---|
| $X$ | 6 | 3 | 2 |
| $Y$ | 2 | 3 | 4 |

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of $X$ and $Y$ is ₹ 2 and ₹ 1 respectively. How many tablets of each type should the person take in order to satisfy the above requirement at the minimum cost? Make an LPP and solve graphically. *(Foreign 2016)*

**19.** A company manufactures three kinds of calculators : $A$, $B$ and $C$ in its two factories I and II. The company has got an order for manufacturing atleast 6400 calculators of kinds $A$, 4000 of kind $B$ and 4800 of kind $C$. The daily output of factory I is of 50 calculators of kind $A$, 50 calculators of kind $B$ and 30 calculators of kind $C$. The Daily output of factory II is of 40 calculators of kind $A$, 20 of kind $B$ and 40 of kind $C$. The cost per day to run factory I is ₹ 12,000 and of factory II is ₹ 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve graphically. *(AI 2015)*

**20.** One kind of cake requires 200 g of flour and 25 g of fat, another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it an LPP and solve it graphically. (*Delhi 2015C, AI 2014C, 2011C*)

**21.** A manufacturer produces nuts and bolts. It take 2 hours work on machine $A$ and 3 hours on machine $B$ to produce a package of nuts. It takes 3 hours on machine $A$ and 2 hours on machine $B$ to produce a package of bolts. He earns a profit of ₹24 per package on nuts and ₹18 per package on bolts. How many package of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 10 hours a day. Make an LPP from above and solve it graphically? *(AI 2015C)*

**22.** A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him ₹ 360 and a manually operated sewing machine ₹ 240. He can sell an electronic sewing machine at a profit of ₹ 22 and a manually operated sewing machine at a profit of ₹ 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximise his profit? Make it as an LPP and solve graphically. *(Delhi 2014)*

**23.** A manufacturing company makes two types of teaching aids $A$ and $B$ of Mathematics for class XII. Each type of $A$ requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of $B$ requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type $A$ and ₹ 120 on each piece of type $B$. How many pieces of type $A$ and type $B$ should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week? (*AI 2014*)

**24.** A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes

1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹25 and that from a shade is ₹15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit? Formulate an LPP and solve it graphically. *(Foreign 2014)*

**25.** A housewife wishes to mix together two kinds of food, $X$ and $Y$, in such a way that the mixture contains at least 10 units of vitamin $A$, 12 units of vitamin $B$ and 8 units of vitamin $C$. The vitamin contents of one kg of food is given below:

| | **Vitamin *A*** | **Vitamin *B*** | **Vitamin *C*** |
|---|---|---|---|
| **Food *X*** | 1 | 2 | 3 |
| **Food *Y*** | 2 | 2 | 1 |

One kg of food $X$ costs ₹ 6 and one kg of food $Y$ costs ₹ 10. Formulate the above problem as a linear programming problem and find the least cost of the mixture which will produce the diet graphically. What value will you like to attach with this problem? *(Delhi 2014C)*

**26.** If a young man rides his motorcycle at 25 km per hour, he had to spend ₹2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to ₹5 per km and rate of pollution also increases. He has ₹100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question? *(Delhi 2014C, 2013C)*

**27.** A cooperative society of farmers has 50 hectares of land to grow two crops $A$ and $B$. The profits from crops $A$ and $B$ per hectare are estimated as ₹10,500 and ₹9,000 respectively. To control weeds a liquid herbicide has to be used for crops $A$ and $B$ at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximise the total profit? Form an LPP from the above information and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment? *(Delhi 2013)*

**28.** A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively; which he uses to produce two types of goods $A$ and $B$. To produce one unit of $A$, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of $B$. If $A$ and $B$ are priced at ₹100 and ₹120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as an LPP and solve graphically.
Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate? *(AI 2013)*

**29.** A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a cutting/grinding machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on sprayer to manufacture a pedestal lamp. It takes 1 hour on grinding/cutting machine and 2 hours on sprayer to manufacture a shade. On any day, to keep the environment pollution under minimum level, sprayer can be used for at the most 20 hours while grinding/cutting machine can be used for at the most 12 hours. The profit from selling a pedestal lamp is ₹ 5 and for selling a shade is ₹ 3. Assuming that it can sell all that it produces, how should it schedule its daily production to maximize its profit? Make it as an LPP and solve it graphically. Which value is described in this question? (*AI 2013C*)

**30.** A manufacturer produces nuts and bolts. It takes 1 hour of work on machine $A$ and 3 hours on machine $B$ to produce a package of nuts. It takes 3 hours on machine $A$ and 1 hour on machine $B$ to produce a package of bolts. He earns a profit of ₹ 17.50 per package on nuts and ₹ 7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? Form the above as a linear programming problem and solve it graphically. *(Delhi 2012)*

**31.** A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin $A$ and 10 units of vitamin $C$. Food I contains 2 units/kg of vitamin $A$ and 1 unit/kg of vitamin $C$ while food II contains 1 unit/kg of vitamin $A$ and 2 units/kg of vitamin $C$. It costs ₹ 5 per kg to purchase food I and ₹ 7 per kg to purchase food II. Determine the minimum cost of such a mixture. Formulate the above as an LPP and solve it graphically. *(AI 2012)*

**32.** A decorative item dealer deals in two items $A$ and $B$. He has ₹ 15,000 to invest and a space to store at the most 80 pieces. Item $A$ costs him ₹ 300 and item $B$ costs him ₹ 150. He can sell items $A$ and $B$ at respective profits of ₹ 50 and ₹ 28. Assuming he can sell all he buys, formulate the linear programming problem in order to maximise his profit and solve it graphically. *(Delhi 2012C)*

**33.** A manufacturer produces nuts and bolts. It takes 1 hour of work on machine $A$ and 3 hours on machine $B$ to produce a package of nuts while it takes 3 hours on machine $A$ and 1 hour on machine $B$ to produce a package of bolts. He earns a profit of ₹ 2.50 per package of nuts and ₹ 1.00 per package of bolts. How many packages of each type should he produce each day so as to maximise his profit, if he operates his machines for at most 12 hours a day? Formulate this problem as a linear programming problem and solve it graphically. *(AI 2012C)*

**34.** A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time. If the profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively, then find the number of tennis rackets and cricket bats that the factory must manufacture to earn maximum profit. Form it as an LPP and solve it graphically. *(Delhi 2011)*

**35.** A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost ₹ 25,000 and ₹ 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit, if he does not want to invest more than ₹ 70 lakhs and his profit on the desktop model is ₹ 4500 and on the portable model is ₹ 5,000. Form it as an LPP and solve it graphically. *(AI 2011)*

**36.** A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin $A$ and 10 units of vitamin $C$. Food I contains 2 units per kg of vitamin $A$ and 1 unit per kg of vitamin $C$. Food II contains 1 unit per kg of vitamin $A$ and 2 units per kg of vitamin $C$. It costs ₹ 50 per kg to purchase Food I and ₹ 70 per kg to purchase Food II. Formulate the problem as a linear programming problem to minimise the cost of such mixture and find the minimum cost graphically. *(Delhi 2011C)*

# Detailed Solutions

**1.** **(a) :** Since, maximum value of $z = ax + by$ occurs at both (2, 4) and (4, 0).

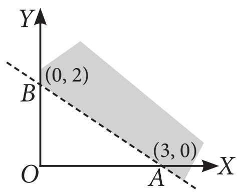
$\therefore \quad 2a + 4b = 4a + 0$

$\Rightarrow \quad 4b = 2a \Rightarrow 2b = a$

**2.** **(b) :** We know that minimum of objective function occurs at corner points.

| Corner points | Value of $z = 3x - 4y$ |
|---|---|
| (0, 0) | 0 |
| (5, 0) | 15 |
| (6, 5) | –2 |
| (6, 8) | –14 |
| (4, 10) | –28 |
| (0, 8) | –32 ← Minimum |

**3.** **(b) :** From the graph of inequality $2x + 3y > 6$. It is clear that it does not contain the origin nor the points of the line $2x + 3y = 6$.




**4.** We have, minimize $z = 5x + 7y$
subject to constraints,

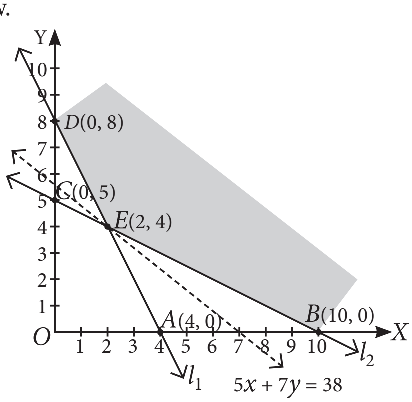
$2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$

To solve LPP graphically, we convert inequations into equations.

Now, $l_1 : 2x + y = 8$, $l_2 : x + 2y = 10$ and $x = 0, y = 0$

$l_1$ and $l_2$ intersect at $E(2, 4)$

Let us draw the graph of these equations as shown below.




The corner points of the feasible region are $D(0, 8)$, $B(10, 0)$ and $E(2, 4)$.

| Corner points | Value of $z = 5x + 7y$ |
|---|---|
| $D$ (0, 8) | 56 |
| $B$ (10, 0) | 50 |
| $E$ (2, 4) | 38 (Minimum) |

From the table, we find that 38 is the minimum value of $z$ at $E(2, 4)$. Since the region is unbounded, so we draw the graph of inequality $5x + 7y < 38$ to check whether the resulting open half plane has any point common with the feasible region. Since it has no point in common. So, the minimum value of $z$ is obtained at $E(2, 4)$ and the minimum value of $z = 38$.

**5.** We have, Minimise $Z = 5x + 10y$
subject to constraints :

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

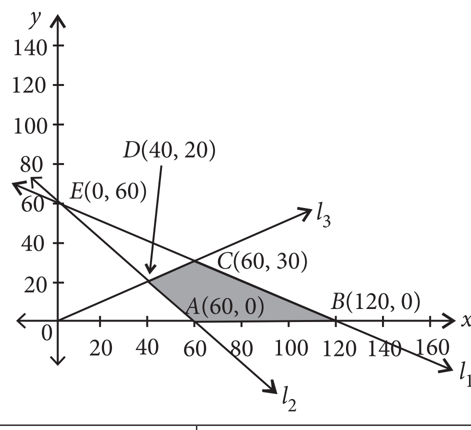
and $x, y \geq 0$

To solve L.P.P graphically, we convert inequations into equations.

$l_1 : x + 2y = 120$, $l_2 : x + y = 60$, $l_3 : x - 2y = 0$ and $x = 0, y = 0$

$l_1$ and $l_2$ intersect at $E(0, 60)$, $l_1$ and $l_3$ intersect at $C(60, 30)$, $l_2$ and $l_3$ intersect at $D(40, 20)$.

The shaded region $ABCD$ is the feasible region and is bounded. The corner points of the feasible region are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$.




| Corner points | Value of $Z = 5x + 10y$ |
|---|---|
| $A$(60, 0) | 300 ← (Minimum) |

| | |
|---|---|
| $B(120, 0)$ | 600 |
| $C(60, 30)$ | 600 |
| $D(40, 20)$ | 400 |

Hence, $Z$ is minimum at $A(60, 0)$ *i.e.*, 300.

**6.** Maximise $Z = x + 2y$

Subject to constraints : $x + 2y \geq 100$, $2x - y < 0$, $2x + y \leq 200$ and $x, y \geq 0$.

Converting the inequations into equations, we obtain the lines

$l_1 : x + 2y = 100$ ...(i)

$l_2 : 2x - y = 0$ ...(ii)

$l_3 : 2x + y = 200$ ...(iii)

$l_4 : x = 0$ ...(iv)

and $l_5 : y = 0$ ...(v)

By intercept form, we get

$$l_1 : \frac{x}{100} + \frac{y}{50} = 1$$

$\Rightarrow$ The line $l_1$ meets the coordinate axes at (100, 0) and (0, 50).

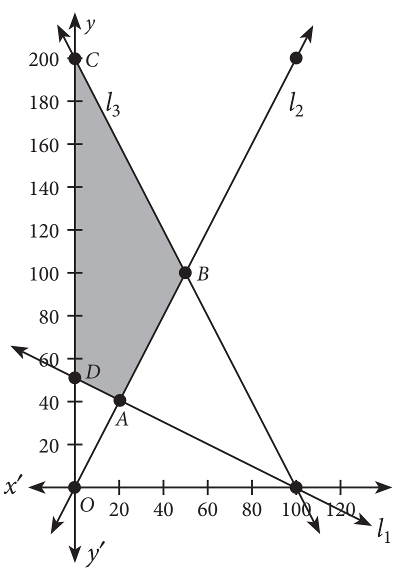
$l_2 : 2x = y$

$\Rightarrow$ The line $l_2$ passes through origin and (50, 100), (100, 200).

$$l_3 : \frac{x}{100} + \frac{y}{200} = 1$$

$\Rightarrow$ The line $l_3$ meets the coordinates axes at (100, 0) and (0, 200).

$l_4 : x = 0$ is the $y$-axis, $l_5 : y = 0$ is the $x$-axis




Now, plotting the above points on the graph, we get the feasible region of the LPP as shaded region $ABCD$. The coordinates of the corner points of the feasible region $ABCD$ are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$, $D(0, 50)$.

Now, $Z_A = 20 + 2 \times 40 = 100$

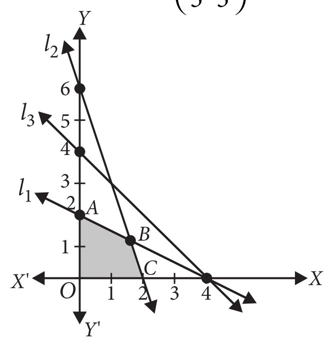
$Z_B = 50 + 2 \times 100 = 250$, $Z_C = 0 + 2 \times 200 = 400$

$Z_D = 0 + 2 \times 50 = 100$

$\therefore$ $Z$ is maximum at $C(0, 200)$ and having value 400.

**7.** Let $l_1 : 2x + 4y = 8$, $l_2 : 3x + y = 6$, $l_3 : x + y = 4$; $x = 0$, $y = 0$

Solving $l_1$ and $l_2$ we get $B\left(\frac{8}{5}, \frac{6}{5}\right)$



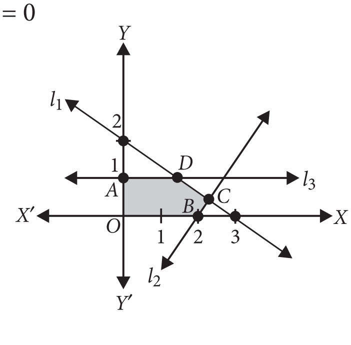

Shaded portion $OABC$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 2)$, $B\left(\frac{8}{5}, \frac{6}{5}\right)$, $C(2, 0)$

The value of objective function at these points are :

| Corner Points | Value of the objective function $z = 2x + 5y$ |
|---|---|
| $O(0, 0)$ | $2 \times 0 + 5 \times 0 = 0$ |
| $A(0, 2)$ | $2 \times 0 + 5 \times 2 = 10$ (Maximum) |
| $B\left(\frac{8}{5}, \frac{6}{5}\right)$ | $2 \times \frac{8}{5} + 5 \times \frac{6}{5} = 9.2$ |
| $C(2, 0)$ | $2 \times 2 + 5 \times 0 = 4$ |

$\therefore$ The maximum value of $z$ is 10, which is at $A(0, 2)$.

**8.** Let $l_1 : 2x + 3y = 6$, $l_2 : 3x - 2y = 6$, $l_3 : y = 1$; $x = 0$, $y = 0$



Solving $l_1$ and $l_3$, we get $D$ (1.5, 1)

Solving $l_1$ and $l_2$, we get $C\left(\frac{30}{13}, \frac{6}{13}\right)$

Shaded portion $OADCB$ is the feasible region, where coordinates of the corner points are $O(0, 0)$, $A(0, 1)$, $D(1.5, 1)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$, $B(2, 0)$.

The value of the objective function at these points are :

| Corner Points | Value of the objective function $z = 8x + 9y$ |
|---|---|
| $O$ (0, 0) | $8 \times 0 + 9 \times 0 = 0$ |
| $A$ (0, 1) | $8 \times 0 + 9 \times 1 = 9$ |
| $D$ (1.5, 1) | $8 \times 1.5 + 9 \times 1 = 21$ |
| $C\left(\frac{30}{13}, \frac{6}{13}\right)$ | $8\times\frac{30}{13}+9\times\frac{6}{13}=22.6$ (Maximum) |
| $B$ (2, 0) | $8 \times 2 + 9 \times 0 = 16$ |

The maximum value of $z$ is 22.6, which is at $C\left(\frac{30}{13}, \frac{6}{13}\right)$

**9.** Let $x$ necklaces and $y$ bracelets be manufactured per day to maximize the profit.

$\therefore$ Maximize $Z = 100x + 300y$

Subject to the constraints : $x + y \leq 24$,

$$(1)x + \left(\frac{1}{2}\right)y \leq 16 \Rightarrow 2x + y \leq 32$$

and $x \geq 1, y \geq 1 \Rightarrow x - 1 \geq 0$ and $y - 1 \geq 0$

**10.** Suppose tailors $A$ and $B$ work for $x$ and $y$ days respectively.

| Product \ Tailors | $A(x)$ | $B(y)$ | Requirements |
|---|---|---|---|
| Shirts | 6 | 10 | 60 |
| Trousers | 4 | 4 | 32 |

Then the L.P.P is

Minimise $Z = 300x + 400y$

Subject to constraints : $6x + 10y \geq 60$ or $3x + 5y \geq 30$, $4x + 4y \geq 32$ or $x + y \geq 8$, $x \geq 0$, $y \geq 0$.

**11.** Let the number of chairs be $x$ and the number of tables be $y$.

$\therefore \quad x + y \leq 35$

Cost of a chair is ₹ 1,000 and cost of a table is ₹ 2,000.

$\therefore \quad 1000x + 2000y \leq 50000$

$\Rightarrow \quad x + 2y \leq 50$

Now, total profit $Z = 150x + 250y$.

Above LPP can be stated mathematically as :

Maximize , $Z = 150x + 250y$

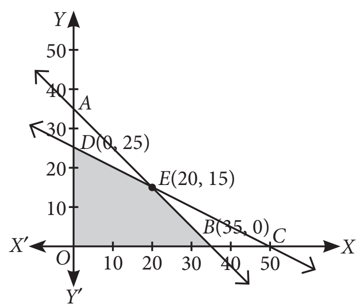
Subject to the constraints

$x + y \leq 35$, $x + 2y \leq 50$ and $x, y \geq 0$

To solve graphically, we convert the inequations into equations to obtain the following lines :

$x + y = 35$, $x + 2y = 50$, $x = 0$, $y = 0$

Let us draw the graph of these lines as shown below.




The feasible region is shown in the graph. We observe that the region is bounded.

The corner points of the feasible region $OBED$ are $O(0, 0)$, $B(35, 0)$, $E(20, 15)$ and $D(0, 25)$.

The value of the objective function at corner points of the feasible region are :

| **Corner points** | **Value of $Z = 150x + 250y$** |
|---|---|
| $O(0, 0)$ | 0 |
| $B(35, 0)$ | 5250 |
| $E(20, 15)$ | 6750 (Maximum) |
| $D(0, 25)$ | 6250 |

$\therefore \quad Z$ is maximum at $x = 20$, $y = 15$.

**12.** Let the number of souvenirs of type $A$ and type $B$ be $x$ and $y$ respectively. Then by the given data, the required LPP is

Maximize $Z = 100x + 120y$

Subject to constraints :

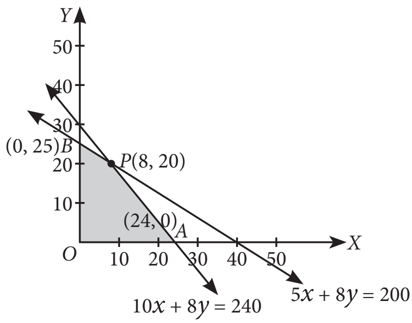
$5x + 8y \leq 200$, $10x + 8y \leq 240$, $x \geq 0$, $y \geq 0$

We have the lines

$l_1 : 5x + 8y = 200$ and $l_2 : 10x + 8y = 240$

These lines intersect at $P(8, 20)$.

Let us draw the graph of the lines.




The feasible region is $OAPB$ and the corner points are $O(0, 0)$, $A(24, 0)$, $P(8, 20)$ and $B(0, 25)$.

| **Corner points** | **Value of $Z = 100x + 120y$** |
|---|---|
| $O(0, 0)$ | 0 |
| $A(24, 0)$ | 2400 |
| $P(8, 20)$ | 3200 (Maximum) |
| $B(0, 25)$ | 3000 |

$\therefore$ Profit, $Z$ = ₹ 3200 will be maximum, when 8 souvenirs of type $A$ and 20 souvenirs of type $B$ are manufactured.

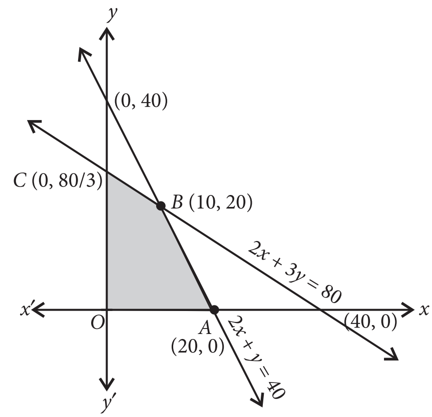
**13.** Let $x$ items of model $A$ and $y$ items of model $B$ be produced.

| | Skilled men (5) | Semi-skilled men (10) | Cost (in ₹) |
|---|---|---|---|
| Model $A$ ($x$) | 2 | 2 | 15 |
| Model $B$ ($y$) | 1 | 3 | 10 |
| Capacity | 8 | 8 | |

The inequations thus formed based on the given problem will be as follows :

$2x + y \le 40$ and $2x + 3y \le 80$

Let $z$ be the total profit of the models.
Then $z = 15x + 10y$
Mathematical formulation of LPP is :
Maximize $z = 15x + 10y$ subject to the constraints :
$2x + y \le 40$ $2x + 3y \le 80$, $x \ge 0$, $y \ge 0$
To solve the LPP graphically, we convert the inequations into equations to obtain the following lines :
$2x + y = 40$, $2x + 3y = 80$, $x = 0$ and $y = 0$
The point of intersection of lines $2x + y = 40$ and $2x + 3y = 80$ is $B$ (10, 20).




Shaded region $OABC$ is the feasible region, where $O(0, 0)$, $A(20, 0)$, $B$ (10, 20) and $C(0, 80/3)$.
The value of objective function at these points are as follows :

| Corner points | Value of objective function $z = 15x + 10y$ |
|---|---|
| $O(0, 0)$ | $15(0) + 10(0) = 0$ |
| $A(20, 0)$ | $15(20) + 10(0) = 300$ |
| $B(10, 20)$ | $15(10) + 10(20) = 350$ (maximum) |
| $C\left(0, \frac{80}{3}\right)$ | $15(0) + 10\left(\frac{80}{3}\right) = \frac{800}{3}$ |

$\therefore$ The maximum value of $z = 350$ which is at $B(10, 20)$.
Hence, manufacturer should produce 10 items of model $A$ and 20 items of model $B$.

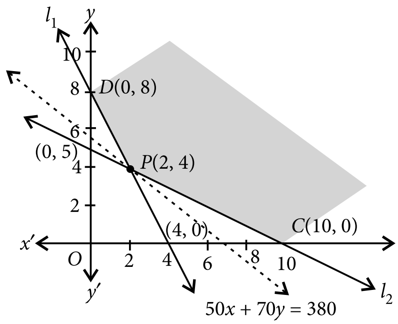
**14.** Let $x$ kg of food I and $y$ kg of food II be produced, then the given data can be represented in the tabular form as follows:

| Nutrient | Food I | Food II | Requirements |
|---|---|---|---|
| Vitamin A | 2 | 1 | 8 |
| Vitamin C | 1 | 2 | 10 |
| Cost (in ₹) | 50 | 70 | |

The given LPP is as follows:
Minimise $z = 50x + 70y$
Subject to the constraints:
$2x + y \ge 8$, $x + 2y \ge 10$, $x \ge 0$, $y \ge 0$
Now, we convert inequations into equations,

$l_1 : 2x + y = 8$, $l_2 : x + 2y = 10$

These lines intersect at $P(2, 4)$.




From the graph, the corner points of the feasible region are $C(10, 0)$, $P(2, 4)$ and $D(0, 8)$.

| Corner points | Value of $z = 50x + 70y$ |
|---|---|
| $C(10, 0)$ | $50(10) + 70(0) = 500$ |
| $P(2, 4)$ | $50(2) + 70(4) = 380$ (minimum) |
| $D(0, 8)$ | $50(0) + 70(8) = 560$ |

From the table, we find that 380 is the minimum value of $z$ at $P(2, 4)$. Since, the region is unbounded, we have to check the inequality $50x + 70y < 380$ in open half planes has any point in common or not. Since, it has no point in common, so, the minimum cost is ₹ 380 and this is attained at $P(2, 4)$ *i.e.*, when 2 units of food I and 4 units of food II are produced.

**15.** Let the manufacturer produce $x$ packages of screws $A$ and $y$ packages of screws $B$. Clearly, $x \geq 0, y \geq 0$. We make the following table from the given data.

| Machines | Time required to produce product | | Maximum machine mins. available |
|---|---|---|---|
| | $A$ | $B$ | |
| Automatic | 4 | 6 | $= 4 \times 60 = 240$ |
| Hand operated | 6 | 3 | $= 4 \times 60 = 240$ |
| Profit (in ₹) | 7 | 10 | |

Since the machine is available for at the most 4 hours a day, we have the constraints

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit $Z = 7x + 10y$

Hence the mathematical formulation of the problem is Maximise $Z = 7x + 10y$

subject to the constraints

$2x + 3y \leq 120$ ...(1)

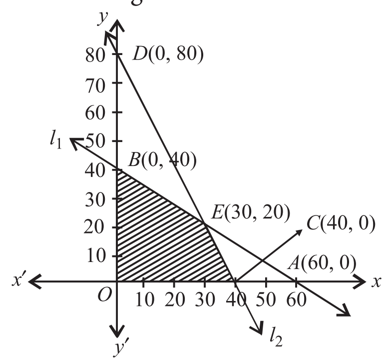
$2x + y \leq 80$ ...(2)

$x, y \geq 0$ ...(3)

Convert the inequations into equations, we get

$l_1 : 2x + 3y = 120, \quad l_2 : 2x + y = 80$

The feasible region is shown in the graph. We observe that the region is bounded.




The corner points of the feasible region $OCEB$ are $O(0, 0)$, $C(40, 0)$, $E(30, 20)$ and $B(0, 40)$.

The value of the objective function at these points is given as :

| Corner Points | Value of $Z = 7x + 10y$ |
|---|---|
| $O$ (0, 0) | 0 |
| $C$ (40, 0) | 280 |
| $E$ (30, 20) | 410 (Maximum) |
| $B$ (0, 40) | 400 |

We find that the maximum value of $Z$ is 410 at $E(30, 20)$. Hence the factory must sell 30 packages of screw $A$ and 20 packages of screw $B$ to realise maximum profit and maximum profit is ₹ 410.

**16.** The given information can be represented in the tabular form as below:

| Machines | Time required to produce product | | Maximum machine hours available |
|---|---|---|---|
| | $A$ | $B$ | |
| First machine | 3 | 2 | 12 |
| Second machine | 3 | 1 | 9 |
| Profit (in ₹) | 7 | 4 | |

Let the manufacturer produces $x$ units of product $A$ and $y$ units of product $B$ per day.

$\therefore \quad 3x + 2y \leq 12$ and $3x + y \leq 9$

Let $Z$ denote the total profit.

$\therefore \quad Z = 7x + 4y$

Clearly $x \geq 0$ and $y \geq 0$.

Above LPP can be stated mathematically as:

Maximise $Z = 7x + 4y$

subject to the constraints

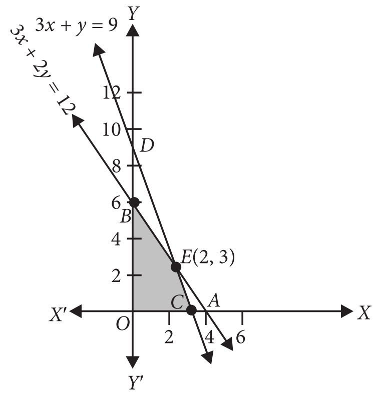
$3x + 2y \le 12$, $3x + y \le 9$ and $x, y \ge 0$

To solve graphically, we convert the inequations into equations to obtain the following lines:

$3x + 2y = 12$, $3x + y = 9$, $x = 0$, $y = 0$

The line $3x + 2y = 12$ meets the coordinate axes at $A(4, 0)$ and $B(0, 6)$. Similarly $3x + y = 9$ meets the coordinate axes at $C(3, 0)$ and $D(0, 9)$

The point of intersection of lines $3x + 2y = 12$ and $3x + y = 9$ is $E(2, 3)$.




Coordinates of the corner points of the feasible region $OCEB$ are $O(0, 0)$, $C(3, 0)$, $E(2, 3)$, $B(0, 6)$

Values of the objective function at corner points of the feasible region are

| Corner Points | Value of $Z = 7x + 4y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $C(3, 0)$ | 21 + 0 = 21 |
| $E(2, 3)$ | 14 + 12 = 26 (Maximum) |
| $B(0, 6)$ | 0 + 24 = 24 |

$\therefore$ $Z$ is maximum at $x = 2$, $y = 3$

So, for maximum profit the manufacturer should manufacture 2 units of product $A$ and 3 units of product $B$.

**17.** Let the requirement of fertiliser $A$ by the farmer be $x$ kg and that of $B$ be $y$ kg.

| | Fertiliser $A$ | Fertiliser $B$ | Minimum requirement (in kg) |
|---|---|---|---|
| Nitrogen (in %) | 12 | 4 | 12 |
| Phosphoric acid/(in %) | 5 | 5 | 12 |
| Cost (in ₹ kg) | 10 | 8 | |

The inequations thus formed based on the given problem will be as follows:

$$\frac{12x}{100} + \frac{4y}{100} \ge 12 \Rightarrow 12x + 4y \ge 1200$$

$\Rightarrow$ $3x + y \ge 300$

Also, $\dfrac{5x}{100} + \dfrac{5y}{100} \ge 12$

$\Rightarrow$ $5x + 5y \ge 1200$ $\Rightarrow$ $x + y \ge 240$ and $x \ge 0, y \ge 0$.

Let $Z$ be the total cost of the fertilisers. Then

$Z = 10x + 8y$

The LPP can be stated mathematically as

Minimise $Z = 10x + 8y$

subject to constraints $3x + y \ge 300$, $x + y \ge 240$, $x \ge 0$, $y \ge 0$.

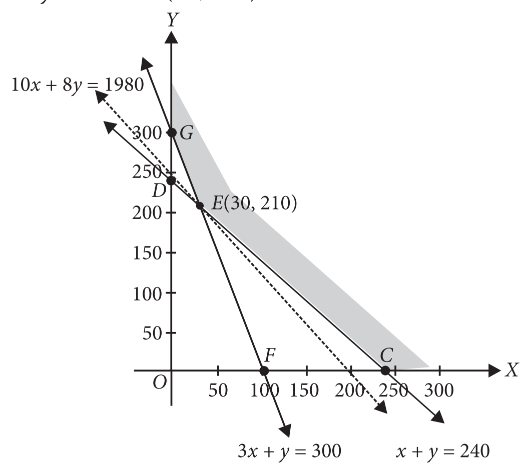
To solve the LPP graphically, we convert the inequations into equations to obtain the following lines:

$3x + y = 300$, $x + y = 240$, $x = 0$ and $y = 0$

Equation $3x + y = 300$ meets the coordinate axes at points $F(100, 0)$ and $G(0, 300)$

Equation $x + y = 240$ meets the coordinate axes at points $C(240, 0)$ and $D(0, 240)$.

The point of intersection of lines $3x + y = 300$ and $x + y = 240$ is $E(30, 210)$



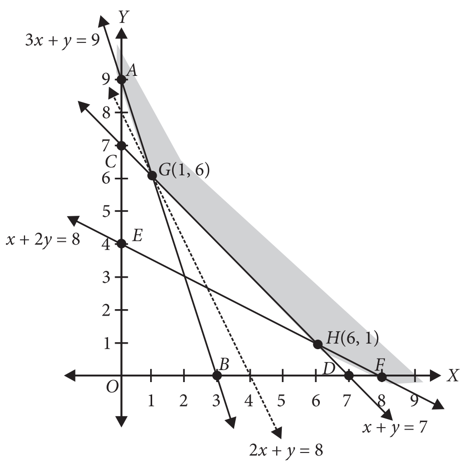

The shaded region $GEC$ represents the feasible region of given LPP and it is unbounded.

| Corner points | Value of $Z = 10x + 8y$ |
|---|---|
| $G(0, 300)$ | 2400 |
| $C(240, 0)$ | 2400 |
| $E(30, 210)$ | 1980 (Minimum) |

From the table, we find that 1980 is the minimum value of $Z$ at $E(30, 210)$. Since the region is unbounded, we have check that the inequality $10x + 8y < 1980$ in open half plane has any point in common or not. Since, it has no point in common.

So, the minimum value of $Z$ is obtained at $E(30, 210)$ and the minimum value of $Z$ is 1980.
So, the minimum requirement of fertiliser of type $A$ will be 30 kg and that of type $B$ will be 210 kg.

**18.** Let the person takes $x$ tablets of type $X$ and $y$ tablets of type $Y$.
According to the given conditions, we have
$6x + 2y \geq 18 \Rightarrow 3x + y \geq 9,$
$3x + 3y \geq 21 \Rightarrow x + y \geq 7$
$2x + 4y \geq 16 \Rightarrow x + 2y \geq 8$
Let $z$ be the total cost of tablets.
$\therefore \quad z = 2x + y$
Hence, the given LPP is
Minimise $Z = 2x + y$ subject to the constraints
$3x + y \geq 9, x + y \geq 7, x + 2y \geq 8$ and $x, y \geq 0$
To solve graphically, we convert the inequations into equations.
$3x + y = 9, x + y = 7, x + 2y = 8, x = 0, y = 0$
The line $3x + y = 9$ meets the coordinate axes at $A(0, 9)$ and $B(3, 0)$. Similarly, $x + y = 7$ meets the coordinate axes at $C(0, 7)$ and $D(7, 0)$. Also, line $x + 2y = 8$ meets the coordinate axes at $E(0, 4)$, $F(8, 0)$
The point of intersection of the lines $3x + y = 9$ and $x + y = 7$ is $G(1, 6)$. Also, the point of intersection of the lines $x + y = 7$ and $x + 2y = 8$ is $H(6, 1)$.




The shaded region $AGHF$ represents the feasible region of the given LPP. The corner points of the feasible region are $A(0, 9)$, $G(1, 6)$, $H(6, 1)$ and $F(8, 0)$.
The values of the objective function at these points are given in the following table :

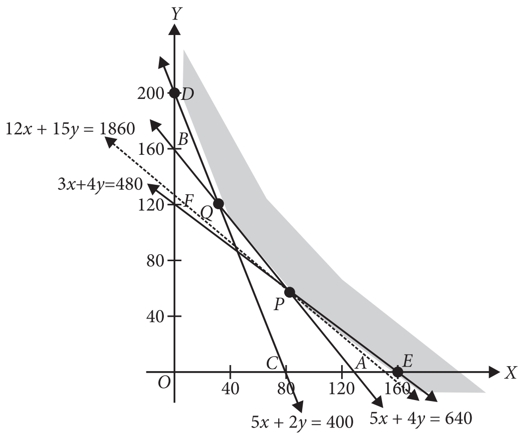
| Corner Points | Value of $Z = 2x + y$ |
|---|---|
| $A(0, 9)$ | $2 \times 0 + 9 = 9$ |
| $G(1, 6)$ | $2 \times 1 + 6 = 8$ (Minimum) |
| $H(6, 1)$ | $2 \times 6 + 1 = 13$ |
| $F(8, 0)$ | $2 \times 8 + 0 = 16$ |

From the table, we find that 8 is the minimum value of $Z$ at $G(1, 6)$. Since the region is unbounded we have to check that the inequality $2x + y < 8$ in open half plane has any point in common or not. Since, it has no point in common. So, $Z$ is minimum at $G$ (1, 6) and the minimum value of $Z$ is 8.
Hence, the person should take 1 tablet of type $X$ and 6 tablets of type $Y$ in order to meet the requirements at the minimum cost.

**19.** Let $x$ and $y$ be the number of days, factory I and factory II have to be in operation to produce the order, respectively.

| Calculator | Factory I | Factory II | Requirement |
|---|---|---|---|
| $A$ | 50 | 40 | 6400 |
| $B$ | 50 | 20 | 4000 |
| $C$ | 30 | 40 | 4800 |
| Cost (in ₹) | 12000 | 15000 | |

The inequations thus formed based on the given problem are as follows :
$50x + 40y \geq 6400 \Rightarrow 5x + 4y \geq 640,$
$50x + 20y \geq 4000 \Rightarrow 5x + 2y \geq 400,$
$30x + 40y \geq 4800 \Rightarrow 3x + 4y \geq 480; x, y \geq 0$
Let $Z$ be the total cost of production.
$\therefore \quad Z = 12000x + 15000y$
So, the given LPP can be mathematically stated as
Minimise $Z = 12000x + 15000y$
Subject to constraints
$5x + 4y \geq 640, 5x + 2y \geq 400, 3x + 4y \geq 480; x, y \geq 0$
To solve the LPP graphically, we convert inequations into equations to obtain the following lines :
$5x + 4y = 640, 5x + 2y = 400, 3x + 4y = 480, x = 0$ and $y = 0$
The line $5x + 4y = 640$ meets the coordinate axes at $A(128, 0)$ and $B(0, 160)$
The line $5x + 2y = 400$ meets the coordinate axes at $C(80, 0)$ and $D(0, 200)$
The line $3x + 4y = 480$ meets the coordinate axes at $E(160, 0)$ and $F(0, 120)$
The point of intersection of lines $5x + 4y = 640$ and $5x + 2y = 400$ is $Q(32, 120)$.
The point of intersection of lines $5x + 4y = 640$ and $3x + 4y = 480$ is $P(80, 60)$.




The shaded region $DQPE$ represents the feasible region of the given LPP.
The corner points of the feasible region are
$D(0, 200)$, $Q(32, 120)$, $P(80, 60)$ and $E(160, 0)$
The values of the objective function at these points are given as follows:

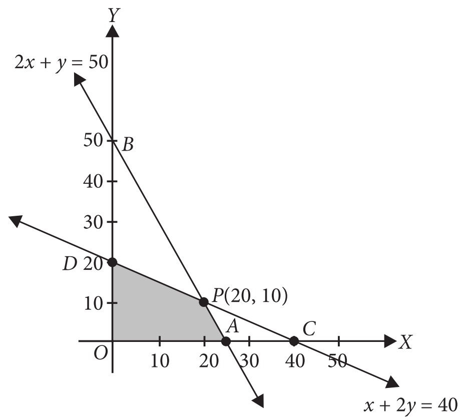
| Corner Points | Value of $Z = 12000x + 15000y$ |
|---|---|
| $D(0, 200)$ | 30,00,000 |
| $Q(32, 120)$ | 21,84,000 |
| $P(80, 60)$ | 18,60,000 (Minimum) |
| $E(160, 0)$ | 19,20,000 |

From the table, we find that 1860000 is the minimum value of $Z$ at $P(80, 60)$. Since the region is unbounded we have to check that the inequality $12000x + 15000y < 1860000$ in open half plane has any point in common or not. Since it has no point in common. So, minimum value of $Z$ is at $P(80, 60)$ and the minimum value of $Z$ is 18,60,000.

**20.** Let $x$ be the number of cakes of I kind and $y$ be the number of cakes of II kind.

| | Flour | Fat |
|---|---|---|
| Cake I | 200 g | 25 g |
| Cake II | 100 g | 50 g |
| Availability | 5 kg | 1 kg |

The required LPP is
Maximise $Z = x + y$ subject to constraints
$200x + 100y \leq 5,000 \Rightarrow 2x + y \leq 50$
$25x + 50y \leq 1,000 \Rightarrow x + 2y \leq 40; x \geq 0, y \geq 0$
To solve LPP graphically, we convert inequations into equations.
$2x + y = 50$ ...(i), $x + 2y = 40$ ...(ii)
Lines (i) and (ii) intersect at $P(20, 10)$.

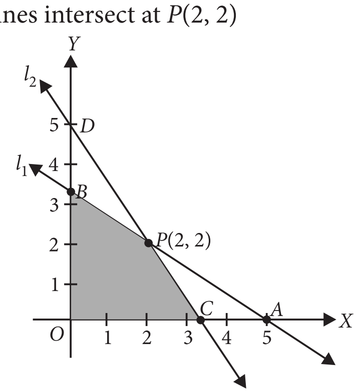



Shaded region is the feasible region *i.e.* $OAPD$. The corner points of the feasible region are $O(0, 0)$, $A(25, 0)$, $P(20, 10)$, $D(0, 20)$.

| Corner Points | Value of $Z = x + y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $A(25, 0)$ | 25 |
| $P(20, 10)$ | 30 (Maximum) |
| $D(0, 20)$ | 20 |

Clearly, the number of cakes is maximum at $P(20, 10)$ *i.e.,* when 20 cakes of I kind and 10 cakes of II kind are made.

**21.** Let $x$ and $y$ be the number of packages of nuts and bolts manufactured respectively by the manufacturer. Then by the given data, the required LPP is
Maximize $Z = 24x + 18y$
Subject to constraints
$2x + 3y \leq 10$, $3x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.
We have the lines
$l_1 : 2x + 3y = 10$ and $l_2 : 3x + 2y = 10$
These lines intersect at $P(2, 2)$




The feasible region is $OCPB$ and the corner points are $O(0, 0)$, $B\left(0, \frac{10}{3}\right)$, $C\left(\frac{10}{3}, 0\right)$, $P(2, 2)$.

| Corner points | Value of $Z = 24x + 18y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $C\left(\frac{10}{3}, 0\right)$ | 80 |
| $P(2, 2)$ | 84 (Maximum) |
| $B\left(0, \frac{10}{3}\right)$ | 60 |

$\therefore$ Profit, $Z$ = ₹84 will be maximum, when 2 packages nuts and 2 packages of bolts are manufactured.

**22.** Let $x$ be the number of electronic sewing machines and $y$ be the number of manually operated sewing machines, the dealer sells. The given problem can be formulated as

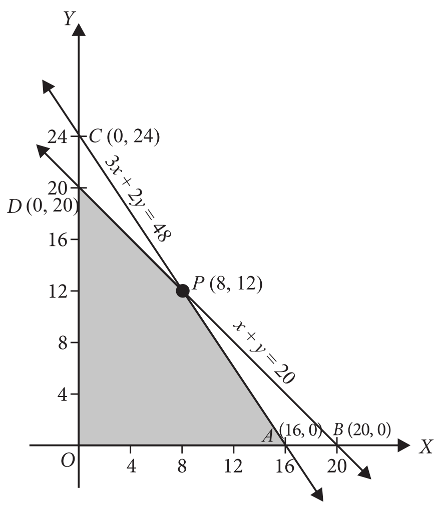
Maximise $Z = 22x + 18y$

Subject to constraints

$x + y \leq 20$, $360x + 240y \leq 5760$

$\Rightarrow 3x + 2y \leq 48$ and $x, y \geq 0$

To solve LPP graphically, we convert the inequations into equations *i.e.*, $x + y = 20$, $3x + 2y = 48$, $x = y = 0$




The shaded region $APDO$ is the feasible region. The corner points of the feasible region are $A(16, 0)$, $P(8, 12)$, $D(0, 20)$ and $O(0, 0)$.

| Corner Points | Value of $Z = 22x + 18y$ |
|---|---|
| $A(16, 0)$ | 352 |
| $P(8, 12)$ | 392 (Maximum) |
| $D(0, 20)$ | 360 |
| $O(0, 0)$ | 0 |

We see that the point $P(8, 12)$ is giving the maximum value of $Z$.

Hence, the dealer should purchase 8 electronic sewing machines and 12 manually operated sewing machines to obtain the maximum profit under the given conditions.

**23.** Let $x$ and $y$ be the number of teaching aids of type $A$ and $B$ respectively to be manufactured per week. Then the LPP can be stated mathematically as

Maximise $Z = 80x + 120y$

subject to constraints

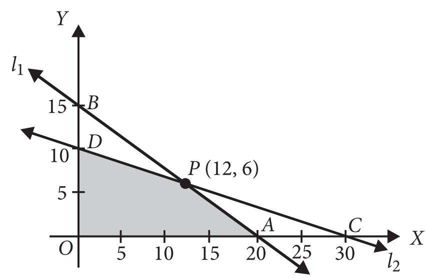
$9x + 12y \leq 180$, $x + 3y \leq 30$; $x \geq 0$, $y \geq 0$

To solve LPP graphically, we convert inequations into equations

$l_1 : 9x + 12y = 180 \Rightarrow 3x + 4y = 60$

and $l_2 : x + 3y = 30$

Both the lines intersect at $P(12, 6)$.




The feasible region is $OAPDO$.

| Corner Points | Value of $Z = 80x + 120y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $A(20, 0)$ | 1600 |
| $P(12, 6)$ | 1680 (Maximum) |
| $D(0, 10)$ | 1200 |

$\therefore$ The profit is maximum at $P(12, 6)$ *i.e.*, when the teaching aids of types $A$ and $B$ are 12 and 6 respectively.

Also, maximum profit = ₹ 1680 per week

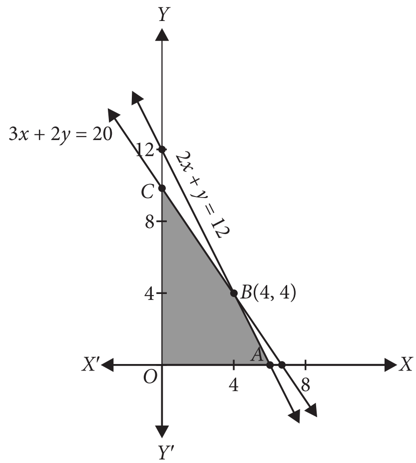
**24.** Let the cottage industry manufactures $x$ pedestal lamps and $y$ wooden shades. Then the LPP can be stated mathematically as

Maximize $Z = 25x + 15y$

Subject to constraints :

$2x + y \leq 12$, $3x + 2y \leq 20$, $x \geq 0$, $y \geq 0$.

Both the lines intersect at $B(4, 4)$.




The corner points of feasible region are :
$A(6, 0)$, $B(4, 4)$, $C(0, 10)$ & $O(0, 0)$

| Corner Points | Value of $Z = 25x + 15y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $A(6, 0)$ | 150 |
| $B(4, 4)$ | 160 (Maximum) |
| $C(0, 10)$ | 150 |

Clearly, $Z$ is maximum at $B(4, 4)$
So, maximum profit of ₹160 is obtained when 4 pedestal lamps and 4 wooden shades are manufactured.

**25.** *Refer to answer 18.*

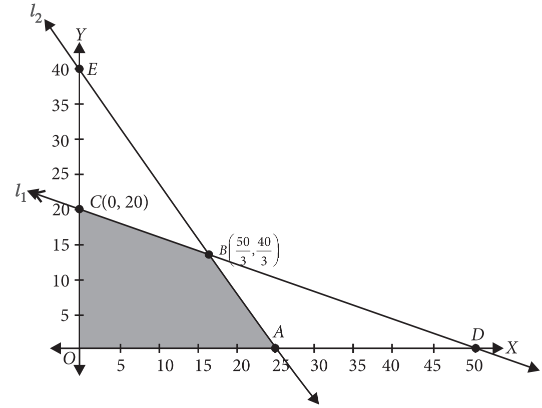
**26.** Suppose that the young man rides $x$ km at 25 km per hour and $y$ km at 40 km per hour. Then, the given problem can be formulated as
Maximize $Z = x + y$.
Subject to the constraints,
$x \geq 0$, $y \geq 0$, $2x + 5y \leq 100$,

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$\Rightarrow \quad 8x + 5y \leq 200$
Now, we convert the system of the inequations into equations.
$l_1 : 2x + 5y = 100$ and $l_2 : 8x + 5y = 200$

Both the lines intersect at $B\left(\frac{50}{3}, \frac{40}{3}\right)$

The solution set of the given system is the shaded region $OABC$.

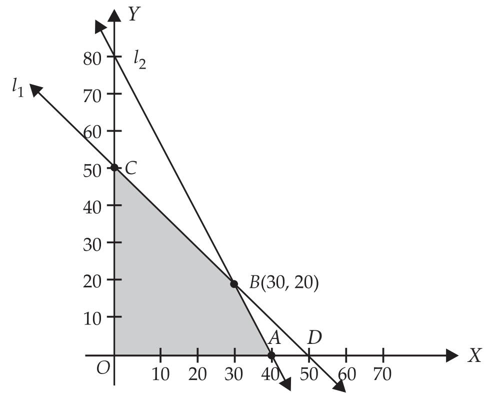



The coordinates of corner points $O$, $A$, $B$, $C$ are $(0, 0)$, $(25, 0)$, $\left(\frac{50}{3}, \frac{40}{3}\right)$ and $(0, 20)$ respectively.

| Corner Points | Value of $Z = x + y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $A(25, 0)$ | 25 |
| $B\left(\frac{50}{3}, \frac{40}{3}\right)$ | 30 (Maximum) |
| $C(0, 20)$ | 20 |

So, $Z = x + y$ is maximum when $x = \frac{50}{3}$ and $y = \frac{40}{3}$.
Thus, the student can cover the maximum distance of 30 km, if he rides $\frac{50}{3}$ km at 25 km/hr and $\frac{40}{3}$ km at 40 km/hr.
The value indicated in this question is that maximum distance is covered in one hour with less pollution.

**27.** Let $x$ hectare of land to be allocated to crop $A$ and $y$ hectare to $B$.
Thus, the LPP can be formulated as
Maximise $Z = 10{,}500x + 9{,}000y$
subject to the constraints
$x + y \leq 50$, $20x + 10y \leq 800 \Rightarrow 2x + y \leq 80$, $x \geq 0$, $y \geq 0$
To solve LPP graphically, we convert inequations into equations.
$l_1 : x + y = 50$, $l_2 : 2x + y = 80$
$x = 0$, $y = 0$
Both the lines intersect at $B(30, 20)$




$OABC$ is the feasible region which is bounded.
The corner points are $O(0, 0)$, $A(40, 0)$, $B(30, 20)$, $C(0, 50)$.

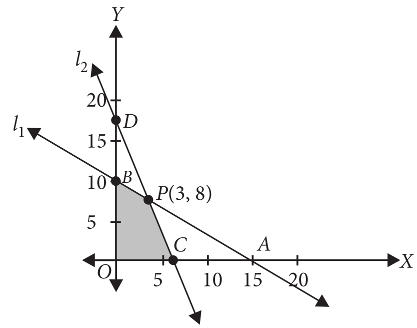
| Corner Points | Value of $Z = 10{,}500x + 9{,}000y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $A(40, 0)$ | 420000 |
| $B(30, 20)$ | 495000 (Maximum) |
| $C(0, 50)$ | 450000 |

Hence society will allocate 30 hectares of land to crop $A$ and 20 hectares of land to crop $B$ to maximise the total profit.
Yes, protection of wildlife is necessary to preserve balance in environment because it will be a loss of biodiversity. The wild animals would get extinct ultimately if we would not provide protection to them. Wild life animals are being killed for their valuable ivory, skin, fur etc. to make products such as leather, meat etc.

**28.** Let $x$ units of the goods $A$ and $y$ units of goods $B$ be produced to maximise the total revenue.

| | Workers | Capital (in units) | Revenue per unit (in ₹) |
|---|---|---|---|
| Goods $A$ | 2 | 3 | 100 |
| Goods $B$ | 3 | 1 | 120 |
| Total units | 30 | 17 | |

The LPP is given by
Maximise $Z = 100x + 120y$
Subject to the constraints
$2x + 3y \le 30$, $3x + y \le 17$, $x \ge 0$, $y \ge 0$
To solve LPP graphically, we convert the inequations into equations.
$l_1 : 2x + 3y = 30$, $l_2 : 3x + y = 17$ and $x = 0$, $y = 0$
These lines meet at $P(3, 8)$.




The feasible region $OCPB$ has been shaded.
The corner points of the feasible region are $O(0, 0)$, $C(5.6, 0)$, $P(3, 8)$, $B(0, 10)$

| Corner Points | Value of $Z = 100x + 120y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $C(5.6, 0)$ | 560 |
| $P(3, 8)$ | 1260 (Maximum) |
| $B(0, 10)$ | 1200 |

Clearly, the maximum revenue is obtained at $P(3, 8)$, *i.e.*, when 3 units of good $A$ and 8 units of good $B$ are produced.
Yes, I agree with the view of the manufacturer. Men and women workers should be equally paid so that they can do their work efficiently and accurately.

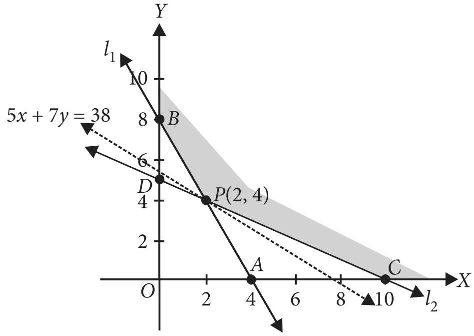
**29.** *Refer to answer 24.*

**30.** *Refer to answer 21.*

**31.** Let $x$ kg of food I and $y$ kg of food II be purchased, then the given data can be represented in the tabular form as follows :

| Nutrients | Food I | Food II | Requirements |
|---|---|---|---|
| Vitamin $A$ | 2 | 1 | 8 |
| Vitamin $C$ | 1 | 2 | 10 |
| Cost (in ₹) | 5 | 7 | |

$\therefore$ The given LPP is as follows.
Minimise $Z = 5x + 7y$
Subject to the constraints
$2x + y \ge 8$, $x + 2y \ge 10$, $x \ge 0$, $y \ge 0$
To solve LPP graphically, we convert inequations into equations.
$l_1 : 2x + y = 8$, $l_2 : x + 2y = 10$
These lines intersect at $P(2, 4)$




From the graph, the corner points of the feasible region are $C(10, 0)$, $P(2, 4)$ and $B(0, 8)$.

| Corner Points | Value of $Z = 5x + 7y$ |
|---|---|
| $C(10, 0)$ | 50 |
| $P(2, 4)$ | 38 (Minimum) |
| $B(0, 8)$ | 56 |

From the table, we find that 38 is the minimum value of $Z$ at $P(2, 4)$. Since the region is unbounded, we have to check that the inequality $5x + 7y < 38$ in open half plane has any point in common or not. Since, it has no point in common. So the minimum cost is ₹ 38 and this is attained at $P(2, 4)$, *i.e.*, when 2 units of food I and 4 units of food II are purchased.

**32.** Let the number of items of the type $A$ and $B$ be $x$ and $y$ respectively. Then the LPP is

Maximise $Z = 50x + 28y$

Subject to the constraints,

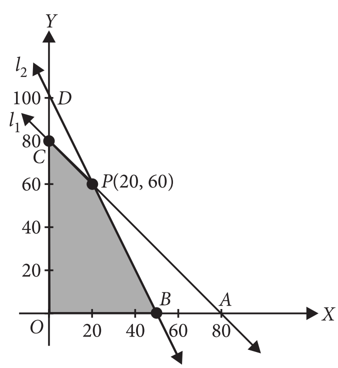
$x + y \leq 80$,

$300x + 150y \leq 15000 \Rightarrow 2x + y \leq 100$,

$x \geq 0, y \geq 0$

To solve LPP graphically, we convert inequations into equations.

$l_1 : x + y = 80, l_2 : 2x + y = 100$




Their point of intersection is $P(20, 60)$. The feasible region $OBPC$ is shown shaded in the figure with corner points $O(0, 0)$, $B(50, 0)$, $P(20, 60)$ and $C(0, 80)$

| Corner Points | Value of $Z = 50x + 28y$ |
|---|---|
| $O(0, 0)$ | 0 |
| $C(0, 80)$ | 2240 |
| $P(20, 60)$ | 2680 (Maximum) |
| $B(50, 0)$ | 2500 |

Thus, the maximum profit of ₹ 2680 is at $P(20, 60)$ *i.e.*, when 20 items of type $A$ and 60 items of type $B$ are purchased and sold.

**33.** *Refer to answer 21.*

**34.** Let the factory makes '$x$' tennis rackets and '$y$' cricket bats.

We make the following table from the given data

| | Tennis Rackets | Cricket Bats | Availability |
|---|---|---|---|
| Machine time (in hrs) | 1.5 | 3 | 42 |
| Craftsman's time (in hrs) | 3 | 1 | 24 |
| Profit (in ₹) | 20 | 10 | |

Hence, the mathematical formulation of the problem is

Maximize $Z = 20x + 10y$

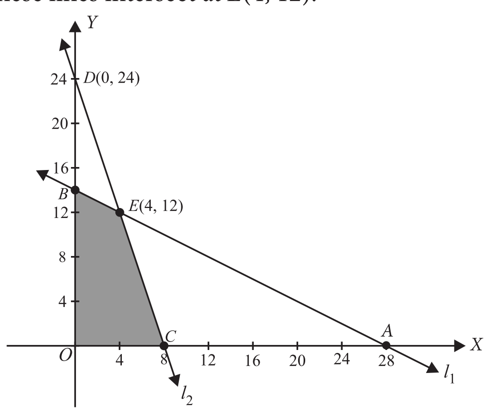
subject to the constraints

$1.5x + 3y \leq 42 \Rightarrow x + 2y \leq 28$, $3x + y \leq 24$; $x, y \geq 0$

Convert the given inequations into equations, we have

$l_1 : x + 2y = 28$, $l_2 : 3x + y = 24$ and $x = 0, y = 0$

These lines intersect at $E(4, 12)$.

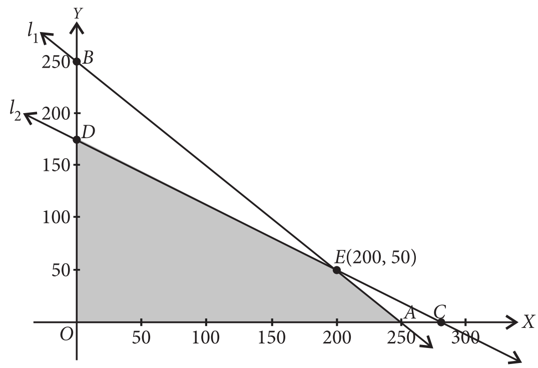


The feasible region $OCEB$ is shown in the graph and the corner points are $O(0, 0)$, $C(8, 0)$, $E(4, 12)$, $B(0, 14)$

| Corner Points | Value of $Z = 20x + 10y$ |
|---|---|
| $O$ (0, 0) | 0 |
| $C$ (8, 0) | 160 |
| $E$ (4, 12) | 200 (Maximum) |
| $B$ (0, 14) | 140 |

Hence, the profit is maximum *i.e.*, ₹ 200 when 4 tennis rackets and 12 cricket bats are manufactured.

**35.** Let the merchant stocks $x$ desktop model computers and $y$ portable model computers.
Hence, the mathematical formulation of the problem is
Maximize $Z = 4500x + 5000y$
subject to the constraints
$x + y \leq 250$, $25000x + 40000y \leq 7000000$,
$5x + 8y \leq 1400$; $x, y \geq 0$
To solve LPP graphically we convert inequations into equations
$l_1 : x + y = 250$, $l_2 : 5x + 8y = 1400$ and $x = 0$, $y = 0$
These lines intersect at $E(200, 50)$




The feasible region $OAED$ is shown in the graph. Here we observe that the feasible region is bounded. The corner points are $O(0, 0)$, $A(250, 0)$, $E(200, 50)$ and $D(0, 175)$.

| Corner Points | Value of $Z = 4500x + 5000y$ |
|---|---|
| $O$ (0, 0) | 0 |
| $A$ (250, 0) | 1125000 |
| $E$ (200, 50) | 1150000 (Maximum) |
| $D$ (0, 175) | 875000 |

We find that maximum value of $Z$ is 1150000 at $E(200, 50)$. Hence the merchant should stock 200 units of desktop model and 50 units of portable model to realise maximum profit and maximum profit is ₹ 1150000.

**36.** *Refer to answer 31.*