Chapter 9

Sequence and Series

Exercise 9.3

Question 1: Find the 20^{th} and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Answer 1:

The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Here, a = first term = $\frac{5}{2}$

r = common ratio = $\frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{2(2)^{19}} = \frac{5}{(2)^{20}}$

$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{2(2)^{n-1}} = \frac{5}{(2)^n}$

Question 2: Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

Answer 2:

Common ratio, r = 0

Let a be the first term of the G.P.

$a_8 = ar^{8-1} = ar^7 = 192 \; a(2)^7 = (2)^6 (3)$

$= a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$

$a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$

Question 3: The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively. Show

that $q^2 = ps$.

Answer 3:

Let a be the first term and r be the common ratio of the G.P. according to the given condition,

$a_5 = a\,r^{5-1} = a\,r^4 = p \dots (1)$

$a_8 = a\,r^{8-1} = ar^7 = q \dots (2)$

$a_{11} = ar^{11-1} = ar^{10} = s \dots (3)$

dividing eq. (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$r^3 = \frac{q}{p} \dots (4)$

dividing eq. (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q} \dots (5)$$

Equation the value of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$= q^2 = ps$

Thus, the given result is proved.

Question 4: The 4th term of a G.P. is square of its second term, and the first term is –3. Determine its 7th term.

Answer 4:

Let, a be the first term and r be the common ratio of the G.P.

a = - 3

It is known that, an = arn-1

$a_4 = ar^3 = (-3)\ r^3$

$a_2 = a\ r^1 = (-3)\ r$

According to the given condition, $(-3)\ r^3 = [(-3)\ r]^2$

$= -\ 3r^3 = 9\ r^2 = r = -3\ a^7 = a\ r^{7-1} = a$

$r^{\ 6} = (-3)\ (-3)^6 = (-3)^7 = -2187$

thus, the 7th term of the G.P. is -2187.

Question 5: Which term of the following sequences:

(a) 2, $2\sqrt{2}$, 4, ... is 128? (b) $\sqrt{3}$, 3, $3\sqrt{3}$, ... is 729?

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots is\ \frac{1}{19683}$.?

Answer 5:

(a) the given sequence is 2, $2\sqrt{2}$, 4, … is 128?

Here, a = 2 and $r = (2\sqrt{2})/2 = \sqrt{2}$

Let, the nth term of the given sequence be 128.

$a_n = a\ r^{n-1}$

$= (2)\ \left(\sqrt{2}\right)^{n-1} = 128$

$= (2)\ (2)^{\frac{n-1}{2}} = (2)^7$

$= (2)^{\frac{n-1}{2}+1} = (2)^7$

$= \frac{n-1}{2} + 1 = 7$

$= \frac{n-1}{2} = 6$

$= n - 1 = 12$

$= n = 13$

Thus, the 13th term of the given sequence is 128.

(b) the given sequence is $\sqrt{3}$, 3, $3\sqrt{3}$, …

$a = \sqrt{3}$ and $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Let, the nth term of the given sequence be 729.

$a_n = a\, r^{n-1}$

$= a\, r^{n-1} = 729$

$= (\sqrt{3})\left(\sqrt{3}\right)^{n-1} = 729$

$= (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)6$

$= (3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)6$

$= \frac{1}{2} + \frac{n-1}{2} = 6$

$= \frac{1+n-1}{2} = 6$

$= n = 12$

Thus, the 12th term of the given sequence is 729.

(c) the given sequence is $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

Here, $a = \frac{1}{3}$ and $r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$

Let, the nth term of the given sequence be $\frac{1}{19683}$.

$a_n = ar^{n-1}$

$= a\, r^{n-1} = \frac{1}{19683}$

$= \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$

$= \left(\frac{1}{3}\right)^{n} = \left(\frac{1}{3}\right)^{9}$

= n = 9

Thus, the 9th term of the given sequence is $\frac{1}{19683}$.

Question 6: For what values of x, the numbers $\frac{-2}{7}$, x, $\frac{-7}{2}$are in G.P?

Answer 6:

The given numbers are $\frac{-2}{7}$, x, $\frac{-7}{2}$

Common ratio = $\frac{x}{\frac{-2}{7}} = \frac{-7x}{2}$

Also, common ratio = $\frac{\frac{-7}{2}}{x} = \frac{-7}{2x}$

$= \frac{-7x}{2} = \frac{-7}{2x}$

$= x^2 = \frac{-2\times 7}{-2\times 7} = 1$

$= x = \sqrt{1}$

$= x = \pm 1$

Thus, for x = ± 1, the given numbers will be in G.P.

Question 7: Find the sum to 20 terms in the geometric progression 0.15, 0.015,

0.0015 ...

Answer 7:

The given G.P. is 0.15, 0.015, 0.00015, …

Here a = 0.15 and r = $\frac{0.015}{0.15}$ = 0.1

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{20} = \frac{0.15[1-(0.1)^{20}]}{1-0.1}$

= $\frac{0.15}{0.9}$ [1-(0.1)20]

= $\frac{15}{90}$ [1 – (0.1)20]

= $\frac{1}{6}$ [1 – (0.1)20]

Question 8: Find the sum to n terms in the geometric progression $\sqrt{7}$, $\sqrt{21}$, $3\sqrt{7}$, ...

Answer 8:

The given G.P. is $\sqrt{7}, \sqrt{21}$, $3\sqrt{7}$, …

Here, a = $\sqrt{7}$ and r = $\frac{\sqrt{21}}{7} = \sqrt{3}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_n = \frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}}$

$S_n = \frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$S_n = \frac{\sqrt{7}(\sqrt{3}+1)\left[1-(\sqrt{3})^n\right]}{1-3}$

$S_n = \frac{-\sqrt{7}(\sqrt{3}+1)\left[1-(\sqrt{3})^n\right]}{2}$

Question 9: Find the sum to n terms in the geometric progression 1, -a, a^2, $-a^3$ … (if a ≠ - 1)

Answer 9:

The given G.P. is 1, -a, a^2, $-a^3$, ….

Here, first term = a_1 = 1

Common ratio = r = - a

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_n = \frac{1[1-(-a)^n]}{1-(-a)} = \frac{[1-(-a)^n]}{1+a}$

Question 10: Find the sum to n terms in the geometric progression x^3, x^5, x^7, … (if $x \neq \pm 1$)

Answer 10:

The given G.P. is x^3, x^5, x^7, …

Here, a = x^3 and r = x^2

$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3\left[1-\left(x^2\right)^n\right]}{1-x^2} = \frac{x^3\left(1-x^{2n}\right)}{1-x^2}$

Question 11: Evaluate $\sum_{k=1}^{11}(2+3^k)$

Answer 11:

$\sum_{k=1}^{11}(2+3^k) = \sum_{k=1}^{11}(2) + \sum_{k=1}^{11} 3^k = 2\,(11) + \sum_{k=1}^{11} 3^k = 22 + \sum_{k=1}^{11} 3^k$ … (1)

$\sum_{k=1}^{11} 3^k$ = 31 +32 + 33 + … + 311

The terms of the sequence 3, 3^2, 3^3 …. Forms a G.P.

$S_n = \frac{a(r^n-1)}{r-1}$

$S_n = \frac{3\left[(3)^{11}-1\right]}{3-1}$

$S_n = \frac{3}{2}(3^{11}-1)$

$= \sum_{k=1}^{11} 3^k = \frac{3}{2}(3^{11} - 1)$

Substituting this value in eq. (1), we obtain

$\sum_{k=1}^{11}(2 + 3^k) = 22 + \frac{3}{2}(3^{11} - 1)$

Question 12: The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find

the common ratio and the terms.

Answer 12:

Let, $\frac{a}{r}$, a, ar be the first three terms of the G.P.

$\frac{a}{r} + a + ar = \frac{39}{10}$ … (1)

$\left(\frac{a}{r}\right)(a)(ar) = 1$ … (2)

From (2), we obtain a3 = 1

= a = 1 (considering real roots only)

Substituting a = 1 in eq. (1), we obtain

$\frac{1}{r} + 1 + r = \frac{39}{10}$

$= 1 + r + r^2 = \frac{39}{10} r$

$= 10 + 10r + 10r^2 – 39r = 0$

$= 10r^2 – 29r + 10 = 0$

$= 10r^2 – 25r – 4r + 10 = 0$

= 5r (2r – 5) – 2 (2r – 5) = 0

= (2r – 5) (5r – 2) = 0

$= r = \frac{2}{5}$ or $\frac{5}{2}$

Thus, the three terms if G.P. are $\frac{5}{2}$, 1 and $\frac{2}{5}$

Question 13: How many terms of G.P. 3, 3^2, 3^3 ... are needed to give the sum 120?

Answer 13:

The given G.P. is 3, 3^2, 3^3 ….

Let n terms of this G.P. be required to obtain the sum as 120.

$S_n = \frac{a(1-r^n)}{1-r}$

Here, a = 3 and r = 3

$S_n = 120 = \frac{3(3^n-1)}{3-1}$

$= 120 = \frac{3(3^n-1)}{2}$

$= \frac{120\times 2}{3} = 3n - 1$

= 3n – 1 = 80

= 3n = 81

= 3n = 34

= n = 4

Thus, four terms of the given G.P. are required to obtain the sum as 120.

Question 14: The sum of first three terms of a G.P. is 16 and the sum of the next three

terms are 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Answer 14:

Let, the G.P. be a, a r, ar^2, ar^3, … According to the given condition,

$a + a r + ar^2 = 16$ and $ar^3 + ar^4 + ar^5 = 128$

$= a (1 + r + r^2) = 16 \ldots (1)$

$= ar^3 (1 + r + r^2) = 128 \ldots (2)$

Dividing eq. (2) by (1), we obtain

$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$

$= r^3 = 8$

$= r = 2$

Substituting $r = 2$ in (1), we obtain $a (1 + 2 + 4) = 16$

$= a (7) = 16$

$= a = \frac{16}{7}$

$S_n = \frac{a(r^n-1)}{r-1}$

$S_n = \frac{16}{7}\frac{(2^n-1)}{2-1} = \frac{16}{7}(2^n - 1)$

Question 15: Given a G.P. with a = 729 and 7_{th} term 64, determine S7.

Answer 15:

$A = 729$ $a^7 = 64$

Let, r be the common ratio of the G.P. it is known that,

$= a_n = a\ rn^{-1}$

$= a_7 = ar^{7-1} = (729)\ r^6$

$= 64 = 729\ r^6$

$= r^6 = \frac{64}{729}$

$= r^6 = \left(\frac{2}{3}\right)^6$

$= r = \frac{2}{3}$

Also, it is known that,

$S_n = \frac{a(1-r^n)}{1-r}$

$S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$

$= 3 \times 729 \left[1 - \left(\frac{2}{3}\right)^7\right]$

$= (3)^7 \left[\frac{(3)^7-(2)^7}{(3)^7}\right]$

$= (3)^7 – (2)^7$

= 2187 – 128

= 2059

Question 16: Find a G.P. for which sum of the first two terms is –4 and the fifth term is 4 times the third term.

Answer 16:

Let, a be the first term and r be the common ratio of the G.P.

According to the given condition,

$S_2 = - 4 = \frac{a\left(1-r^2\right)}{1-r}$ … (1)

$= a_5 = 4 \times a^3$

$= a r^4 = 4 ar^2 = r^2 = 4$

$= r = \pm 2$

From (1), we obtain

$-4 = \frac{a[1-(2)^2]}{1-2}$ for r = 2

$= -4 = \frac{a(1-4)}{-1}$

$= -4 = a\,(3)$

$= a = \frac{-4}{3}$

Also, $-4 = \frac{a\left[1-(-2)^2\right]}{1-(-2)}$ for $r = -2$

$= -4 = \frac{a(1-4)}{1+2}$

$= -4 = \frac{a(-3)}{3}$

$= a = 4$

Thus, the required G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$ or 4, -8, 16, -32, …

Question 17: If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z is in G.P.

Answer 17:

Let, a be the first term and r be the common ratio of the G.P.

According to the given condition,

$= a_4 = a\,r^3 = x$ …. (1)

$= a_{10} = a\,r^9 = y$ …. (2)

$= a_{16} = a\,r^{15} = z$ … (3)

Dividing (2) by (1), we obtain

$\frac{y}{x} = \frac{ar^9}{ar^3} = \frac{y}{x} = r^6$

Dividing (3) by (2), we obtain

$\frac{z}{y} = \frac{ar^{15}}{ar^9} = \frac{z}{y} = r^6$

$= \frac{y}{x} = \frac{z}{y}$

Thus, x, y, z is in G.P.

Question 18: Find the sum to n terms of the sequence, 8, 88, 888, 8888...

Answer 18:

The given sequence is 8, 88, 888, 8888...

This sequence is not a G.P. however, it can be changed to G.P.by writing the terms as

$S_n = 8 + 88 + 888 + 8888 + \ldots. + n$ terms

$= \frac{8}{9}$ [9 + 99 + 999 + 9999 + … to n terms]

$= \frac{8}{9}$ [(10 – 1) + (10^2 – 1) + (10^3 – 1) + (10^4 – 1) + … to n terms]

$= \frac{8}{9}$ [(10 + 10^2 + …. n terms) – (1 + 1 + 1 +n terms)]

$= \frac{8}{9}\left(\frac{10(10^n-1}{10-1} - n\right)$

$= \frac{8}{9}\left[\frac{10(10^n-1)}{9} - n\right]$

$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$

Question 19: Find the sum of the products of the corresponding terms of the sequences2, 4, 8, 16, 32 and 128, 32, 8, 2, 1⁄2.

Answer 19:

Required sum = $2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$

$= 64\left[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2}\right]$

Here, 4, 2, 1, $\frac{1}{2}, \frac{1}{2^2}$ is a G.P.

First term, a = 4

Common ratio r = $\frac{1}{2}$

It is known that,

$S_n = \frac{a(1-r^n)}{1-r}$

$S_5 = \frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = \frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}} = 8\left(\frac{32-1}{32}\right) = \frac{31}{4}$

Required sum = 64 × $\frac{31}{4}$ = (16) (31) = 496

Question 20: Show that the products of the corresponding terms of the sequences form

a, a r, ar^2, … ar^{n-1} and A, AR, AR^2, … AR^{n-1} a G.P, and find the common ratio.

Answer 20:

It has to be proved that the sequence: aA, a r AR, ar^2AR^2, …$ar^{n-1}AR^{n-1}$, forms a G.P.

$\frac{second\ term}{first\ term} = \frac{arAR}{aA}$ = Rr

$\frac{third\ term}{secind\ term} = \frac{ar^2AR^2}{arAR}$ = Rr

Thus, the above sequence forms a G.P. and the common ratio is rR.

Question 21:Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Answer 21:

Let, a be the first term and r be the common ratio of the G.P.

= a_1 = a, a_2 = a r, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition

$= a_3 = a_1 + 9 = ar^2 = a + 9 \ldots (1)$

$= a_2 = a_4 + 18 = a\,r = ar^3 + 18 \ldots (2)$

From eq. (1) and (2), we obtain

$= a\,(r^2 - 1) = 9 \ldots (3)$

$= a\,r\,(1 - r^2) = 18 \ldots (4)$

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$

$= -r = 2$

$= r = -2$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$= 3a = 9$

$= a = 3$

Thus, the first four numbers of the G.P. are 3, 3 (-2), $3(-2)^2$ and $3(-2)^3$ i.e., 3, -6, 12, and -24.

Question 22: If p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that

$A^{q-r}.b^{r-p}.\ c^{p-q} = 1.$

Answer 22:

Let, A be the first term and R be the common ratio of the G.P.

According to the given information,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

$= a^{q-r}. b^{r-p}. c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Thus, the given result is proved.

Question 23: If the first and the n^{th} term of a G.P. are a ad b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Answer 23:

The first terms of the A.P. is a and the last term is b.

Therefore, the G.P. is a, a r, ar^2, ar^3 … ar^{n-1}, where r is common ratio

$= b = ar^{n-1}$ … (1)

= p = product of n terms

$= (a)(ar)(ar^2) \ldots (ar^{n-1})$

$= (a \times a \times a)(r \times r^2 \times \ldots r^{n-1})$

= an r 1 + 2 + … (n – 1) … (2)

Here, 1, 2, … (n – 1) is an A.P.

= 1 + 2 + … + (n – 1)

$= \frac{n-1}{2}[2 + (n-1-1) \times 1] = \frac{n-1}{2}[2 + n - 2] = \frac{n(n-1)}{2}$

$P = a^n r^{\frac{n(n-1)}{2}}$

$P^2 = a^{2n} r^{n(n-1)}$

$= \left[a^2 r^{(n-1)}\right]^n$

$= [a \times ar^{n-1}]^n$

$= (ab)^n$ [using eq. (1)]

Thus, the given result id proved.

Question 24: Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Answer 24:

Let, a be the first term and r be the common ratio of the G.P.

Sum of first n terms = $\frac{a(1-r^n)}{1-r}$

Since there are n terms from $(n + 1)^{th}$ to $(2n)^{th}$ term,

Sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term

$S_n = \frac{a_{n+1}(1-r^n)}{1-r}$

Thus, required ratio = $\frac{a(1-r^n)}{1-r} \times \frac{1-r}{ar^n(1-r^n)} = \frac{1}{r^n}$

Thus, the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ terms is $\frac{1}{r^n}$

Question 25: If a, b, c and d are in G.P. show that:

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc – cd)^2$

Answer 25:

a, b, c and d are in G.P. therefore,

= bc = ad … (1)

= b2 = ac … (2)

= c2 = bd … (3)

It has to be proved that,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc – cd)^2$

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [using (1)]

$= [ab + d(a + c)]^2$

$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2d^2 + c^2 \times c^2 + c^2d^2$

[using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$ = L.H.S.

L.H.S. = R.H.S.

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc – cd)^2$

Question 26: Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Answer 26:

Let, G_1 and G_2 be two numbers between 3 and 81 such that the series, 3, G_1, G_2, 81, forms a G.P.

Let, a be the first term and r be the common ratio of the G.P.

$81 = (3)(r)^3$

$= r^3 = 27$

= r = 3 (taken real roots only)

For r = 3

G_1 = a r = (3) (3) = 9

G_2 = a r^2 = (3) $(3)^2$ = 27

Thus, the required two numbers are 9 and 27.

Question 27: Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Answer 27:

A.M. is a and b is $\sqrt{ab}$

By the given condition: $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

Squaring both sides, we obtain

$= \frac{\left(a^{n+1}+b^{n+1}\right)^2}{(a^n+b^n)^2} = ab$

$= a^{2n+2} + 2a^{n+1} b^{n+1} + b^{2n+2} = (ab) (a^{2n} + 2a^n b^n + b^{2n})$

$= a^{2n+2} + 2a^{n+1} b^{n+1} + b^{2n+2} = a^{2n+1} b + 2a^{n+1} b^{n+1} + ab^{2n+1}$

$= a^{2n+2} + b^{2n+2} = a^{2n+1} b + ab^{2n+1}$

$= a^{2n+2} – a^{2n+1} b = ab^{2n+1} – b^{2n+2}$

$= a^{2n+1} (a – b) = b^{2n+1} (a – b)$

$= \left(\frac{a}{b}\right)^{2n+1} = 1 = \left(\frac{a}{b}\right)^0$

= 2n + 1 = 0

$= n = \frac{-1}{2}$

Question 28: The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$

Answer 28:

Let, the two numbers be a and b.

G.M. = $\sqrt{ab}$

According to the given condition,

$= a + b = 6\sqrt{ab}$ … (1)

$= (a + b)^2 = 36\,(ab)$

Also, $(a - b)^2 = (a + b)^2 - 4ab = 36ab - 4ab = 32ab$

$= a - b = \sqrt{32}\sqrt{ab}$

$= 4\sqrt{2}\sqrt{ab}$ … (2)

Adding (1) and (2), we obtain

$2a = (6 + 4\sqrt{2})\,\sqrt{ab}$

$= a = (3 + 2\sqrt{2})\,\sqrt{ab}$

Substituting the value of a in (1), we obtain

$= b = 6\,\sqrt{ab} - (3 + 2\sqrt{2})\,\sqrt{ab}$

$= b = (3 - 2\sqrt{2})\,\sqrt{ab}$

$= \frac{a}{b} = \frac{(3+2\sqrt{2})\sqrt{ab}}{(3-2\sqrt{2})\sqrt{ab}} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

Thus, the required ratio is $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$

Question 29: If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm\sqrt{(A+G)(A-G)}$

Answer 29:

It is given that A and G are A.M. and G.M. between two positive numbers.

Let, these two positive numbers be a and b.

= A.M. = A = $\frac{a+b}{2}$ … (1)

G.M. = G = $\sqrt{ab}$ … (2)

From (1) and (2), we obtain

= a + b = 2A … (3)

= ab = G^2 …. (4)

Substituting the value of a and b from (3) and (4) in the identity

$(a - b)^2 = (a + b)^2 - 4ab$. We obtain

$(a - b)^2 = 4A^2 - 4G^2 = 4 (A^2 - G^2)$

$(a - b)^2 = 4 (A + G) (A - G)$

$(a - b) = 2\sqrt{(A+G)(A-G)}$ … (5)

From (3) and (5), we obtain

$2a = 2A + 2\sqrt{(A+G)(A-G)}$

= $a = A + \sqrt{(A+G)(A-G)}$

Substituting the value of a in (3), we obtain

= $b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$

Thus, the two numbers are $A \pm\sqrt{(A+G)(A-G)}$

Question 30: The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and an hour?

Answer 30:

It is given that the number of bacteria doubles every hour. Therefore, the numbers of bacteria after every hour will form a G.P.

Here, a = 30 and r = 2

$= a_3 = ar^2 = (30)\ (2)^2 = 120$

Therefore, the numbers of bacteria at the end of 2nd hour will be 120.

$= a_5 = ar^4 = (30)\ (2)^4 = 480$

The number of bacteria at the end of 4th hour will be 480.

$= a_{n+1} = a\ r^n = (30)\ 2^n$

Thus, numbers of bacteria at the end of nth hour will be $30\ (2)^n$

Question 31: What will ₹500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Answer 31:

The amount deposited in the bank is ₹500.

At the end of first year, amount = ₹500$\left(1+\frac{1}{10}\right)$ = rs500(1.1)

At the end of 2nd year, amount = ₹500 (1.1) (1.1)

At the end of 3rd year, amount = ₹500 (1.1) (1.1) (1.1) and so on.

Amount at the end of 10 years = ₹500 (1.1) (1.1) … (10 times) = Rs500 $(1.1)^{10}$

Question 32: If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Answer 32:

Let, the root of the quadratic equation be a and b.

According to the given condition,

A.M. = $\frac{a+b}{2}$ = 8 = a + b = 16 … (1)

G.M. = $\sqrt{ab}$ = 5 = ab = 25 … (2)

The quadratic equation is given by,

x^2 – x (sum of roots) + (product of roots) = 0

x^2 – x (a + b) + (ab) = 0

x^2 – 16x + 25 = 0 [using (1) and (2)]

thus, the required quadratic equation is x^2 – 16x + 25 = 0