# THREE DIMENSIONAL GEOMETRY

# 27

## MCQs with One Correct Answer

1. The angle between the lines whose direction cosines are given by the equations

$3l + m + 5n = 0$ , $6nm - 2nl + 5lm = 0$ is

(a) $\cos^{-1}\left(\frac{1}{6}\right)$ (b) $\cos^{-1}\left(-\frac{1}{6}\right)$

(c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(-\frac{5}{6}\right)$

2. A line in the 3-dimensional space makes an angle $\theta\left(0<\theta\le\frac{\pi}{2}\right)$ with both the x and y axes. Then the set of all values of $\theta$ is the interval:

(a) $\left(0,\frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

(c) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$ (d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$

3. If $l_1$, $m_1$, $n_1$ and $l_2$, $m_2$, $n_2$ are DCs of the two lines inclined to each other at an angle $\theta$, then the DCs of the internal bisector of the angle between these lines are

(a) $\frac{l_1+l_2}{2\sin\theta/2}, \frac{m_1+m_2}{2\sin\theta/2}, \frac{n_1+n_2}{2\sin\theta/2}$

(b) $\frac{l_1+l_2}{2\cos\theta/2}, \frac{m_1+m_2}{2\cos\theta/2}, \frac{n_1+n_2}{2\cos\theta/2}$

(c) $\frac{l_1-l_2}{2\sin\theta/2}, \frac{m_1-m_2}{2\sin\theta/2}, \frac{n_1-n_2}{2\sin\theta/2}$

(d) $\frac{l_1-l_2}{2\cos\theta/2}, \frac{m_1-m_2}{2\cos\theta/2}, \frac{n_1-n_2}{2\cos\theta/2}$

4. If the straight lines $x = 1+s, y = -3-\lambda s, z = 1+\lambda s$ and $x = \frac{t}{2}, y = 1+t, z = 2-t,$ with parameters s and t respectively, are co-planar, then $\lambda$ equals.

(a) 0 (b) –1 (c) $-\frac{1}{2}$ (d) –2

5. $L_1$ and $L_2$ are two lines whose vector equations are $L_1 : \vec{\mathbf{r}} = \lambda[(\cos\theta+\sqrt{3})\hat{\mathbf{i}} + (\sqrt{2}\sin\theta)\hat{\mathbf{j}} + (\cos\theta-\sqrt{3})\hat{\mathbf{k}}]$ and $L_2 : \vec{\mathbf{r}} = \mu(a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}})$ where, $\lambda$ and $\mu$ are scalars and $\alpha$ is the acute angle between $L_1$ and $L_2$. If the angle $\alpha$ is independent of $\theta$, then the value of $\alpha$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

6. The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Then which of the following is not the side of the triangle?

(a) $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

(b) $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$

(c) $\frac{x-7}{3}=\frac{y-2}{5}=\frac{z-4}{-1}$

(d) None of these

**7.** The vertex $A$ of the triangle $ABC$ is on the line $\vec{r}=\hat{i}+\hat{j}+\lambda\hat{k}$ and the vertices $B$ and $C$ have respective position vectors $\hat{i}$ and $\hat{j}$. Let $\Delta$ be the area of the triangle and $\Delta \in \left[\frac{3}{2}, \frac{\sqrt{33}}{2}\right]$, then the range of values $\lambda$ corresponding to '$A$' is

(a) $[-8,-4]\cup[4,8]$ (b) $[-4,4]$

(c) $[-2,2]$ (d) $[-4,-2]\cup[2,4]$

**8.** A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point $Q$. The length of the line segment $PQ$ equals

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

**9.** Through a point $P\,(h,k,l)$ a plane is drawn at right angles to $OP$ to meet the co-ordinate axes in $A$, $B$ and $C$. If $OP=p$, then the area of $\Delta ABC$ is :

(a) $\frac{p^2hk}{l^2}$ (b) $\frac{p^3l}{3hk}$

(c) $\frac{p^2t^2}{2hk}$ (d) $\frac{p^5}{2hkl}$

**10.** Projection of the line $x+y+z-3=0=2x+3y+4z-6$ on the plane $z=0$ is

(a) $\frac{x}{-2}=\frac{y-6}{1}=\frac{z}{0}$ (b) $\frac{x}{1}=\frac{y-6}{-2}=\frac{z}{0}$

(c) $\frac{x}{1}=\frac{y-6}{2}=\frac{z}{0}$ (d) none of these

**11.** A variable plane at a distance of the one unit from the origin cuts the coordinates axes at $A$, $B$ and $C$. If the centroid $D\,(x,y,z)$ of triangle $ABC$ satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the value $k$ is

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

**12.** The plane containing the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point:

(a) $(1,-2,5)$ (b) $(1,0,5)$

(c) $(0,3,-5)$ (d) $(-1,-3,0)$

**13.** The position vectors of points $a$ and $b$ are $\hat{i}-\hat{j}+3\hat{k}$ and $3\hat{i}+3\hat{j}+3\hat{k}$ respectively. The equation of a plane is $r\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9=0$. The points $a$ and $b$

(a) lie on the plane

(b) are on the same side of the plane

(c) are on the opposite side of the plane

(d) None of the above

**14.** The angle between the pair of planes represented by equation

$2x^2-2y^2+4z^2+6xz+2yz+3xy=0$ is

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(\frac{4}{21}\right)$

(c) $\cos^{-1}\left(\frac{4}{9}\right)$ (d) $\cos^{-1}\left(\frac{7}{\sqrt{84}}\right)$

**15.** Let $P=(-3,1,1)$ and $Q=(3,4,2)$. $R$ divides $\overline{PQ}$ in the ratio $PR:PQ=1:3$. Then, the equation of the plane perpendicular to $\overrightarrow{PQ}$ at $R$ is

(a) $18x+9y+3z=8$ (b) $18x+9y+3z=4$

(c) $9x+18y+3z=4$ (d) $3x+9y+18z=8$

**16.** Let $\sigma_1$, $\sigma_2$, $\sigma_3$ be planes passing through the origin. Assume that $\sigma_1$ is perpendicular to the vector $(1,1,1)$, $\sigma_2$ is perpendicular to a vector $(a,b,c)$, and $\sigma_3$ is perpendicular to the vector $(a^2,b^2,c^2)$.

What are all the positive values of $a$, $b$ and $c$ so that $\sigma_1 \cap \sigma_2 \cap \sigma_3$ is a single point?

(a) Any positive value of $a$, $b$, and $c$ other than 1

(b) Any positive values of $a$, $b$ and $c$ where either $a \neq b$, $b \neq c$ or $a \neq c$

(c) Any three distinct positive values of $a$, $b$, and $c$

(d) There exist no such positive real numbers $a$, $b$ and $c$

**17.** Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 5\hat{i} + \hat{j} - \hat{k}$ be three vectors. The area of the region formed by the set of points whose position vectors $\vec{r}$ satisfy the equations $\vec{r}.\vec{a} = 5$ and $|\vec{r} - \vec{b}| + |\vec{r} - \vec{c}| = 4$ is closest to the integer

(a) 4 (b) 9

(c) 14 (d) 19

## Numeric Value Answer

**18.** A line with direction ratios (2, 1, 2) intersects the lines $\vec{r} = -\hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = -\hat{i} + \mu(2\hat{i} + \hat{j} + \hat{k})$ at $A$ and $B$, respectively, then length of $AB$ is equal to

**19.** The shortest distance between the z-axis and the line, $x + y + 2z - 3 = 0$, $2x + 3y + 4z - 4 = 0$ is

**20.** The distance of the point (1, 0, –3) from the plane $x - y - z = 9$ measured parallel to the line $\frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$ is

**21.** Let the equation of the plane containing line $x - y - z - 4 = 0 = x + y + 2z - 4$ and parallel to the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$ be $x + Ay + Bz + C = 0$. Then the values of $|A + B + C - 4|$ is ........ .

**22.** Let $f$ be a one-one function with domain {–2, 1, 0} and range {1, 2, 3} such that exactly one of the following statements is true :

$f(-2) = 1, f(1) \neq 1, f(0) \neq 2$ and the remaining two are false. If the area of the triangle formed by (–2, 1, 0) and ($f$(–2), $f$(1), $f$(0)) and the origin is given by $\frac{\sqrt{k}}{2}$; then sum of digits of $k$ is.

**23.** If the equation of the plane through the intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$ is $ax + by + cz + 173 = 0$, then $b - 9(a + c)$ is equal to ........

**24.** If a line is passing through ($a$, $b$, $c$) and intersecting $y = 0$, $z^2 = 4\alpha x$ lies on the surface $(bz - cy)^2 = k\alpha(b - y)(bx - ay)$; then find the value of $k$.

**25.** If the volume enclosed by the equation $|x| \leq 8, |y| \leq 8, |z| \leq 8$ and $|x + y + z| \leq 8$ is $t$, then $\frac{t}{512} =$

| ANSWER KEY | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | (b) | **4** | (d) | **7** | (d) | **10** | (b) | **13** | (c) | **16** | (c) | **19** | (2) | **22** | (7) | **25** | (4) |
| **2** | (c) | **5** | (a) | **8** | (c) | **11** | (d) | **14** | (c) | **17** | (a) | **20** | (b) | **23** | (6) | | |
| **3** | (b) | **6** | (c) | **9** | (d) | **12** | (b) | **15** | (b) | **18** | (3) | **21** | (7) | **24** | (4) | | |

# Hints & Solutions

## CHAPTER 27 Three Dimensional Geometry

**1.** **(b)** The given equations are $3l + m + 5n = 0$ ...(i)

and $6mn - 2nl + 5lm = 0$ ...(ii)

From (i), we have $m = -3l - 5n$. Putting $m = -3l - 5n$ in (ii), we get

$6(-3l - 5n)n - 2nl + 5l(-3l - 5n) = 0$

$\Rightarrow (n + l)(2n + l) = 0$

$\Rightarrow$ either $l = -n$ or $l = -2n$.

If $l = -n$, then putting $l = -n$ in (i), we obtain $m = -2n$.

If $l = -2n$, then putting $l = -2n$ in (i), we obtain $m = n$.

Thus, the direction ratios of two lines are $-n, -2n, n$ and $-2n, n, n$ i.e., $1, 2, -1$ and $-2, 1, 1$. Hence, the direction cosines are

$\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$ or $\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$. The angle $\theta$ between the lines is given by

$$\cos\theta = \frac{1}{\sqrt{6}} \times \frac{-2}{\sqrt{6}} + \frac{2}{\sqrt{6}} \times \frac{1}{\sqrt{6}} + \frac{-1}{\sqrt{6}} \times \frac{1}{\sqrt{6}} = \frac{-1}{6}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{6}\right).$$

**2.** **(c)** It makes $\theta$ with x and y-axes.

$l = \cos\theta,\ m = \cos\theta,\ n = \cos(\pi - 2\theta)$

we have $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2(\pi - 2\theta) = 1$

$\Rightarrow 2\cos^2\theta + (-\cos 2\theta)^2 = 1$

$\Rightarrow 2\cos^2\theta - 1 + \cos^2 2\theta = 0$

$\Rightarrow \cos 2\theta - [1 + \cos 2\theta] = 0$

$\Rightarrow \cos 2\theta = 0$ or $\cos 2\theta = -1$

$\Rightarrow 2\theta = \pi/2$ or $2\theta = \pi$

$\Rightarrow \theta = \pi/4$ or $\theta = \frac{\pi}{2} \Rightarrow \theta = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

**3.** **(b)** Let $OA$ and $OB$ be two lines with $DC$'s $l_1, m_1, n_1$ and $l_2, m_2, n_2$. Let $OA = OB = 1$. Then co-ordinates of $A$ and $B$ are $(l_1, m_1, n_1)$ and $(l_2, m_2, n_2)$ respectively. Let $OC\ Z$ be the bisector of $\angle AOB$ such that $C$ is the mid-point of $AB$ and so its co-ordinates are

$$\left(\frac{l_1 + l_2}{2}, \frac{m_1 + m_2}{2}, \frac{n_1 + n_2}{2}\right)$$

$\therefore$ $DR$'s of $OC$ are $\frac{l_1 + l_2}{2}, \frac{m_1 + m_2}{2}, \frac{n_1 + n_2}{2}$

$\therefore$ We have

$$OC = \sqrt{\left(\frac{l_1 + l_2}{2}\right)^2 + \left(\frac{m_1 + m_2}{2}\right)^2 + \left(\frac{n_1 + n_2}{2}\right)^2}$$

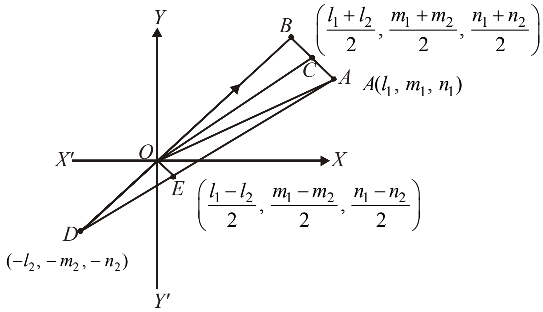
$$= \frac{1}{2}\sqrt{(l_1^2 + m_1^2 + n_1^2) + (l_2^2 + m_2^2 + n_2^2) + 2(l_1 l_2 + m_1 m_2 + n_1 n_2)}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta}\ [\because \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2]$$

$$= \frac{1}{2}\sqrt{2(1 + \cos\theta)} = \cos\left(\frac{\theta}{2}\right).$$

$\therefore$ $DC$s of $\overline{OC}$ are $\frac{l_1 + l_2}{2(OC)}, \frac{m_1 + m_2}{2(OC)}, \frac{n_1 + n_2}{2(OC)}$

i.e., $\frac{l_1 + l_2}{2\cos\theta/2}, \frac{m_1 + m_2}{2\cos\theta/2}, \frac{n_1 + n_2}{2\cos\theta/2}$




**4.** **(d)** The given lines are

$$x-1=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s \qquad \text{..........(i)}$$

and $2x=y-1=\frac{z-2}{-1}=t$ .........(ii)

The lines are coplanar, if

$$\begin{vmatrix} 0-(-1) & -1-3 & -2-(-1) \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix}=0$$

$$C_2 \to C_2+C_3;\begin{vmatrix} 1 & -5 & -1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix}=0$$

$$\Rightarrow 5(-1-\frac{\lambda}{2})=0 \Rightarrow \lambda=-2$$

**5.** **(a)** Both the lines pass through origin.

Line $L_1$ is parallel to the vector

$$\vec{V}_1=(\cos\theta+\sqrt{3})\hat{\mathbf{i}}+(\sqrt{2}\sin\theta)\hat{\mathbf{j}}+(\cos\theta-\sqrt{3})\hat{\mathbf{k}}$$

and $L_2$ is parallel to the vector

$$\vec{V}_2=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$$

$$\therefore \cos\alpha=\frac{\vec{V}_1\cdot\vec{V}_2}{|\vec{V}_1||\vec{V}_2|}$$

$$=\frac{a(\cos\theta+\sqrt{3})+(b\sqrt{2})\sin\theta+c(\cos\theta-\sqrt{3})}{\sqrt{a^2+b^2+c^2}\sqrt{(\cos\theta+\sqrt{3})^2+2\sin^2\theta+(\cos\theta-\sqrt{3})^2}}$$

$$=\frac{(a+c)\cos\theta+b\sqrt{2}\sin\theta+(a-c)\sqrt{3}}{\sqrt{a^2+b^2+c^2}\sqrt{2+6}}$$

In order that cos α in independent of θ

$a+c=0$ and $b=0$

$$\therefore \cos\alpha=\frac{2a\sqrt{3}}{a\sqrt{2}\cdot 2\sqrt{2}}=\frac{\sqrt{3}}{2}\Rightarrow \alpha=\frac{\pi}{6}$$

**6.** **(c)** Given one vertex $A(7, 2, 4)$ and line

$$\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$$

General point on above line,

$B\equiv(5\lambda-6,\ 3\lambda-10,\ 8\lambda-14)$

Direction ratios of line $AB$ are

$<5\lambda-13,\ 3\lambda-12,\ 8\lambda-18>$

Direction ratios of line $BC$ are $<5, 3, 8>$

Since, angle between $AB$ and $BC$ is $\frac{\pi}{4}$.

$$\cos\frac{\pi}{4}=\frac{(5\lambda-3)5+3(3\lambda-12)+8(8\lambda-18)}{\sqrt{5^2+3^2+8^2}\cdot\sqrt{(5\lambda-13)^2+(3\lambda-12)^2+(8\lambda-18)^2}}$$

Squaring and solving, we have λ = 3, 2.
Hence, equation of lines are

$$\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6} \text{ and}$$

$$\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}.$$

**7.** **(d)** $\Delta = \frac{1}{2}\left|(\hat{j}+\lambda\hat{k})\times(\hat{i}+\lambda\hat{k})\right|$

$= \frac{1}{2}\left|-\hat{k}+\lambda\hat{i}+\lambda\hat{j}\right| = \frac{1}{2}\sqrt{2\lambda^2+1}$

$\Rightarrow \frac{9}{4} \le \frac{1}{4}(2\lambda^2+1) \le \frac{33}{4}$

$\Rightarrow 4 \le \lambda^2 \le 16 \Rightarrow 2 \le |\lambda| \le 4.$

**8.** **(c)** The line has +ve and equal direction cosines, these are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ or direction ratios are 1, 1, 1. Also the lines passes through $P$ $(2, -1, 2)$.

$\therefore$ Equation of line is

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-2}{1} = \lambda \text{ (say)}$$

Let $Q(\lambda+2, \lambda-1, \lambda+2)$ be a point on this line where it meets the plane

$$2x+y+z=9$$

Then $Q$ must satisfy the eq$^n$ of plane

i.e. $2(\lambda+2)+\lambda-1+\lambda+2=9 \Rightarrow \lambda=1$

$\therefore$ $Q$ has coordinates (3, 0, 3)

Hence the length of line segment

$= \sqrt{(2-3)^2+(-1-0)^2+(2-3)^2} = \sqrt{3}$

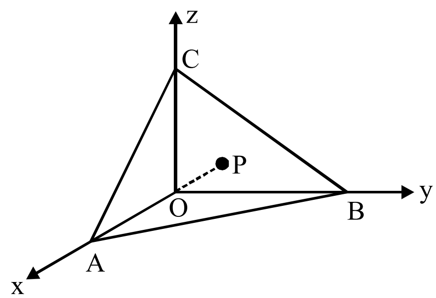
**9.** **(d)** Here $OP = \sqrt{h^2+k^2+l^2} = p$

$\therefore$ $DR$s of $OP$ are :

$$\frac{h}{\sqrt{h^2+k^2+l^2}}, \frac{k}{\sqrt{h^2+k^2+l^2}}, \frac{l}{\sqrt{h^2+k^2+l^2}}$$

or $\frac{h}{p}, \frac{k}{p}, \frac{l}{p}$

Since $OP$ is normal to the plane, therefore, equation of plane is




$\frac{h}{p}x + \frac{k}{p}y + \frac{l}{p}z = p$ or $hx+ky+lz = p^2$

$\therefore A\left(\frac{p^2}{h}, 0, 0\right), B\left(0, \frac{p^2}{k}, 0\right), C\left(0, 0, \frac{p^2}{l}\right)$

Now, Area of $\Delta ABC$, $\Delta = \sqrt{A_{xy}^2 + A_{yz}^2 + A_{zx}^2}$

where, $A_{xy}^2$ is area of projection of $\Delta ABC$ on xy plane = area of $\Delta AOB$

Now, $A_{xy} = \frac{1}{2}\begin{vmatrix} p^2/h & 0 & 1 \\ 0 & p^2/k & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{p^4}{2|hk|}$

Similarly, $A_{yz} = \frac{p^4}{2|kl|}$ and $A_{zx} = \frac{p^4}{2|lh|}$

$\therefore \Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2, \quad \Delta = \frac{p^5}{2hkl}$

**10.** **(b)** A plane containing the given lines is

$2x+3y+4z-6+\lambda(x+y+z-3)=0$ … (i)

This plane is perpendicular to plane $z=0$

if $4+\lambda=0 \Rightarrow \lambda=-4$

So, the equation (i) becomes

$-2x-y+6=0 \Rightarrow 2x+y-6=0$ … (ii)

Equation of the projection will be the line of intersection of plane (2) and the plane z = 0. If the line has d.c. proportional to $\ell$, $m$, $n$ then $2\ell + m = 0$ and $n = 0$

$\Rightarrow \ell : m : n = 1 : -2 : 0$. Obviously (0, 6, 0) is a point on both the planes, hence lies on the line as well.

$\therefore$ Equation of the line is $\frac{x}{1} = \frac{y-6}{-2} = \frac{z}{0}$

**11.** **(d)** Let the eq$^n$ of variable plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ which meets the axes at

$A\,(a, 0, 0)$, $B\,(0, b, 0)$ and $C\,(0. 0, c)$.

$\therefore$ Centroid of $\Delta\,ABC$ is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

and it satisfies the relation

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k \;\Rightarrow\; \frac{9}{a^2} + \frac{9}{b^2} + \frac{9}{c^2} = k$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{z^2} = \frac{k}{9} \qquad ...(i)$$

Also given that the distance of plane

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ from (0, 0, 0) is 1 unit.

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = 1$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1 \qquad ...(ii)$$

From (i) and (ii), we get $\frac{k}{9} = 1$ i.e. $k = 9$

**12.** **(b)** Equation of the plane containing the line

$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ is

$a\,(x - 1) + b\,(y - 2) + c\,(z - 3) = 0$ ....... (i)

where $a.1 + b.2 + c.3 = 0$

i.e., $a + 2b + 3c = 0$ ....... (ii)

Since the plane (i) parallel to the line

$$\frac{x}{1} = \frac{y}{1} = \frac{z}{4}$$

$\therefore$ $a.1 + b.1 + c.4 = 0$

i.e., a + b + 4c = 0 ....... (iii)

From (ii) and (iii),

$$\frac{a}{8-3} = \frac{b}{3-4} = \frac{c}{1-2} = k \text{ (let)}$$

$\therefore$ $a = 5k$, $b = -k$, $c = -k$

On putting the value of $a$, $b$ and $c$ in equation (i),

$5(x - 1) - (y - 2) - (z - 3) = 0$

$\Rightarrow$ $5x - y - z = 0$ ....... (iv)

when $x = 1$, $y = 0$ and $z = 5$; then

L.H.S. of equation (iv) $= 5x - y - 2$

$= 5 \times 1 - 0 - 5$

$= 0$

= R.H.S. of equation (iv)

Hence coordinates of the point (1, 0, 5) satisfy the equation plane represented by equations (iv), Therefore the plane passes through the point (1,0,5).

**13.** **(c)** The position vectors of two given points are $a = \hat{i} - \hat{j} + 3\hat{k}$ and $b = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and the equation of the given plane is

$r = (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$ or $r \cdot n + d = 0$

We have

$a \cdot n + d = (\hat{i} - \hat{j} + 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9$

$= 5 - 2 - 21 + 9 < 0$

and

$b \cdot n + d = (3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9$

$= 15 + 6 - 21 + 9 > 0$

So, the points $a$ and $b$ are on the opposite sides of the plane.

**14.** **(c)** $2x^2 - 2y^2 + 4z^2 + 6xz + 2yz + 3xy = 0$

or $2x^2 + x(6z + 3y) - 2y^2 + 4z^2 + 2yz = 0$

$$\therefore x = \frac{-(6z+3y) \pm \sqrt{36z^2 + 9y^2 + 36yz - 8(-2y^2 + 4z^2 + 2yz)}}{4}$$

$$\therefore x = \frac{-(6z+3y) \pm \sqrt{(2x+5y)^2}}{4}$$

$$\therefore x = \frac{-(6z+3y) \pm (2z+5y)}{4}$$

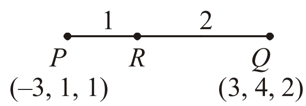
or $2x - y + 2z = 0,\ x + 2y + 2z = 0$

$\therefore$ Angle between planes

$$\theta = \cos^{-1} \frac{(2)(1) + (-1)(2) + (2)(2)}{\sqrt{(2)^2 + (-1)^2 + (2)^2}\sqrt{(1)^2 + (2)^2 + (2)^2}}$$

$$= \cos^{-1}\left(\frac{4}{9}\right)$$

**15.** **(b)** $PR : PQ = 1 : 3 \Rightarrow 3PR = PQ$




$\Rightarrow 3PR = PR + RQ \Rightarrow 2PR = RQ$

Therefore, $PR : RQ = 1 : 2$. Hence

$$R = \left(\frac{-6+3}{1+2}, \frac{2+4}{3}, \frac{2+2}{3}\right) = \left(-1, 2, \frac{4}{3}\right)$$

The normal to the required plane is

$\overline{PQ} = (6, 3, 1)$. Hence, the equation of the required plane is

$$6(x+1) + 3(y-2) + 1\left(z - \frac{4}{3}\right) = 0$$

$$\Rightarrow 18x + 9y + 3z - 4 = 0$$

**16.** **(c)** $\sigma_1$ is perpendicular to $(\hat{i} + \hat{j} + \hat{k})$

$\sigma_2$ is perpendicular to $(a\hat{i} + b\hat{j} + c\hat{k})$

and $\sigma_3$ is perpendicular to

$(a^2\hat{i} + b^2\hat{j} + c^2\hat{k})$

Then, the planes are

$\sigma_1 : x + y + z = 0$

$\sigma_2 : ax + by + cz = 0$

$\sigma_3 : a^2x + b^2y + c^2z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

So, for unique solution, $\Delta \neq 0$

$\Rightarrow \Delta = (a-b)(b-c)(c-a) \neq 0$

$\Rightarrow a \neq b, b \neq c, c \neq a$

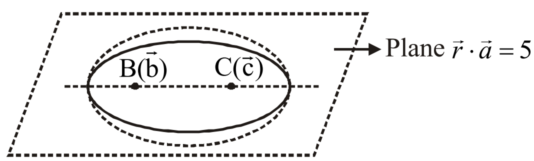
**17.** **(a)** The equation of plane is $\vec{r}.\vec{a} = 5$

$\because\ |\vec{r} - \vec{b}| + |\vec{r} - \vec{c}| = 4$

$\Rightarrow$ sum of distances of a point $(\vec{r})$ from two fixed points with position vector $\vec{b}$ and $\vec{c}$ is constant.

$\Rightarrow$ such points lies on ellipsoid.

Now points with position vector $\vec{b}$ and $\vec{c}$ satisfies the equation of plane $\vec{r}.\vec{a} = 5$, then $\vec{b}.\vec{a} = 5$ and $\vec{c}.\vec{a} = 5$




Area in the plane constitutes an ellipse

Distance between $\vec{b}$ and $\vec{c}$

$= 2 \times$ (semi major axis) $\times\, e = \sqrt{14}$

$2ae = \sqrt{14}$ ...(i)

Sum of distance = constant = major axis = 4

$2a = 4$ ...(ii)

From eq$^n$ (i) and (ii)

$e = \frac{\sqrt{14}}{4} \Rightarrow b = a\sqrt{(1-e^2)} = \frac{1}{\sqrt{2}}$ (semi minor axis)

Area of ellipse $= \pi.a.b.$

$$= \pi.2.\frac{1}{\sqrt{2}} = \sqrt{2}\pi \approx 4.443$$

**18.** **(3)** $L_1 : \frac{x-0}{1} = \frac{y+1}{1} = \frac{z-0}{1} = \lambda$

$L_2 : \frac{x+1}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \mu$

Hence any point on $L_1$ and $L_2$ can be

$(\lambda, \lambda-1, \lambda)$ and $(2\mu-1, \mu, \mu)$, respectively.

According to the question,

$$\frac{2\mu-1-\lambda}{2} = \frac{\mu-\lambda+1}{1} = \frac{\mu-\lambda}{2}$$

On solving, we get $\mu = 1$ and $\lambda = 3$

$\therefore A = (3, 2, 3)$ $B = (1, 1, 1)$ $\therefore AB = 3.$

**19.** **(2)** The equation of any plane passing through given line is :

$(\text{x}+\text{y}+2\text{z}-3)+\lambda(2\text{x}+3\text{y}+4\text{z}-4) =$

$\Rightarrow (1+2\lambda)\text{x}+(1+3\lambda)\text{y}+(2+4\lambda)\text{z}-(3+4\lambda) = 0$ ....(i)

If this plane is parallel to z-axis whose direction cosines are 0, 0, 1; then the normal to the plane will be perpendicular to z-axis

$\therefore (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1) = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

Put in eq (i), the required plane is

$(\text{x}+\text{y}+2\text{z}-3)-\frac{1}{2}(2\text{x}+3\text{y}+4\text{z}-4) = 0$

$\Rightarrow \text{y}+2 = 0$ ...(ii)

$\therefore$ S.D. = distance of any point say (0, 0, 0) on z-axis from plane (ii) $= \frac{2}{\sqrt{(1)^2}} = 2$

**20.** **(7)**

**21.** **(7)** A plane containing line of intersection of the given planes is

$x-y-z-4+\lambda(x+y+2z-4) = 0$

i.e., $(\lambda+1)x+(\lambda-1)y+(2\lambda-1)z-4(\lambda+1) = 0$

vector normal to it

$V = (\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(2\lambda-1)\hat{k}$

Now the vector along the line of intersection of the planes

$2x+3y+z-1=0$

and $x+3y+2z-2=0$ is given by

$$n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3(\hat{i}-\hat{j}+\hat{k})$$

As $n$ is parallel to the plane (i), therefore,

$n \cdot V = 0$

$(\lambda+1)-(\lambda-1)+(2\lambda-1) = 0$

$\Rightarrow 2+2\lambda-1 = 0 \Rightarrow \lambda = \frac{-1}{2}$

Hence, the required plane is

$\frac{x}{2}-\frac{3y}{2}-2z-2 = 0$

$\Rightarrow x-3y-4z-4 = 0$

Hence, $|A+B+C-4| = 7$

**22.** **(7)** Under the given conditions the possible situation is $f(-2) = 2; f(0) = 3; f(1) = 1$.

{where $f(-2) = 1$ is false, $f(0) \neq 2$ is true and $f(1) \neq 1$ is false}. The triangle formed is with vertices

$A(-2, 1, 0), B(2, 1, 3)$ and $O(0, 0, 0)$

$$\text{Area of } \Delta AOB = \frac{1}{2}\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ -2 & 1 & 0 \end{matrix} \right\|$$

$=\frac{1}{2}|-3\hat{i}-6\hat{j}+4\hat{k}|=\frac{1}{2}\sqrt{61}$ square units

$=\frac{\sqrt{k}}{2}$; so $k=61$.

**23.** **(6)** The required plane is of the form

$(x+2y+3z-4)+\lambda(2x+y-z+5)=0$

whose normal is $(1+2\lambda, 2+\lambda, 3-\lambda)$. This plane is perpendicular to the plane $5x+3y+6z+8=0$. So we have

$5(1+2\lambda)+3(2+\lambda)+6(3-\lambda)=0$

$\Rightarrow 7\lambda=-29 \Rightarrow \lambda=\frac{-29}{7}$

Therefore, the required plane is

$(x+2y+3z-4)-\frac{29}{7}(2x+y-z+5)=0$

$\Rightarrow 51x+15y-50z+173=0$

Comparing this with $ax+by+cz+173=0$ we get $a=51, b=15, c=-50$

so that, $b-9(a+c)=15-9=6$.

**24.** **(4)** Any point on parabola $z^2=4\alpha x, y=0$ is given by $Q(\alpha t^2, 0, 2\alpha t)$

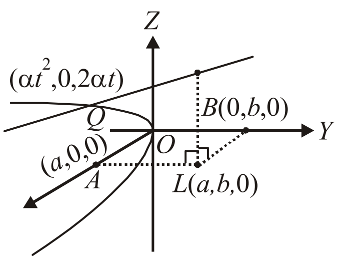
Now equation of line joining $P(a, b, c)$ and $Q$ $(\alpha t^2, 0, 2\alpha t)$ is given by

$\frac{x-a}{a-\alpha t^2}=\frac{y-b}{b-0}=\frac{z-c}{c-2\alpha t}=\lambda$ (say)

$\Rightarrow x=a+\lambda(a-\alpha t^2);\ y=b+b\lambda;\ z=c+\lambda;$

$z=c+\lambda(c-\alpha t)$ by given condition point $Q$

lies on $(bz-cy)^2=k\alpha(b-y)(bx-ay)$




$\Rightarrow [bc+b\lambda, (c-2\alpha t)-cb-cb\lambda]^2$

$=k\alpha(-b\lambda)(ba+b\lambda)(a-\alpha t^2)-(ab-ab\lambda)$

$\Rightarrow [bc+b\lambda c-2\alpha b\lambda t-cb-cb\lambda)^2$

$=-kb\alpha\lambda(ba+ab\lambda-ab\lambda t^2-ab-ab\lambda)$

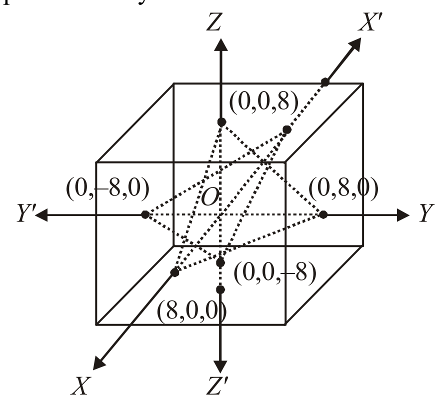
$\Rightarrow 4\alpha^2b^2\lambda^2t^2=-kb\alpha\lambda(-\alpha b\lambda t^2)$

$\Rightarrow 4\alpha^2b^2\lambda^2t^2=k^2b^2\alpha^2\lambda^2t^2 \Rightarrow k=4$.

**25.** **(4)** $|x|\le 8 \Rightarrow x\in[-8, 8]$ similarly for $y$ and $z$.

This represent a cube of side 16 units with centre at origin.

Now, $-8\le x+y+z\le 8$ gives space between two panes namely




$x+y+z+8=0$ and $x+y+z-8=0$ subject to the limits of $x,\ y,\ z\in[-8, 8]$

This will take out $\left(\frac{1}{4}+\frac{1}{4}\right)$ volume of the cube

$\Rightarrow$ The required volume $=\frac{1}{2}\times(16)^3=2018$.

$\Rightarrow \text{t}=2048 \Rightarrow \frac{\text{t}}{512}=4$