ELLIPSE

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is -

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{4}{5}$

2. If the eccentricity of an ellipse be 5/8 and the distance between its foci be 10, then its latus rectum is -

(A) $\frac{39}{4}$ (B) 12 (C) 15 (D) $\frac{37}{2}$

3. The curve represented by x = 3(cost + sint), y = 4(cost – sint), is -

(A) ellipse (B) parabola (C) hyperbola (D) circle

4. If the distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2, then the eccentric angle is-

(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

5. An ellipse having foci at (3, 3) and (–4, 4) and passing through the origin has eccentricity equal to-

(A) $\frac{3}{7}$ (B) $\frac{2}{7}$ (C) $\frac{5}{7}$ (D) $\frac{3}{5}$

6. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major & minor axes in points A & B respectively. If C is the centre of the ellipse then the area of the triangle ABC is :

(A) 12 sq. units (B) 24 sq. units (C) 36 sq. units (D) 48 sq. units

7. The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ included between the co-ordinate axes is the curve-

(A) $9x^2 + 16y^2 = 4x^2y^2$ (B) $16x^2 + 9y^2 = 4x^2y^2$

(C) $3x^2 + 4y^2 = 4x^2y^2$ (D) $9x^2 + 16y^2 = x^2y^2$

8. An ellipse is drawn with major and minor axes of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of the circle being outside the ellipse. The radius of the circle is-

(A) $\sqrt{3}$ (B) 2 (C) $2\sqrt{2}$ (D) $\sqrt{5}$

9. Which of the following is the common tangent to the ellipses $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$ & $\frac{x^2}{a^2}+\frac{y^2}{a^2+b^2}=1$?

(A) $ay = bx + \sqrt{a^4-a^2b^2+b^4}$ (B) $by = ax - \sqrt{a^4+a^2b^2+b^4}$

(C) $ay = bx - \sqrt{a^4+a^2b^2+b^4}$ (D) $by = ax - \sqrt{a^4-a^2b^2+b^4}$

10. Angle between the tangents drawn from point (4, 5) to the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is -

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

11. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and its corresponding point Q on the auxiliary circle meet on the line -

(A) x = a/e (B) x = 0 (C) y = 0 (D) none

12. An ellipse is such that the length of the latus rectum is equal to the sum of the lengths of its semi principal axes. Then -

(A) Ellipse becomes a circle (B) Ellipse becomes a line segment between the two foci

(C) Ellipse becomes a parabola (D) none of these

13. The equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the positive end of latus rectum is -

(A) $x + ey + e^2a = 0$ (B) $x - ey - e^3a = 0$ (C) $x - ey - e^2a = 0$ (D) none of these

14. The eccentric angle of the point where the line, $5x - 3y = 8\sqrt{2}$ is a normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is -

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\tan^{-1}2$

15. PQ is a double ordinate of the ellipse $x^2 + 9y^2 = 9$, the normal at P meets the diameter through Q at R, then the locus of the mid point of PR is -

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

16. The equation of the chord of the ellipse $2x^2 + 5y^2 = 20$ which is bisected at the point (2, 1) is -

(A) $4x + 5y + 13 = 0$ (B) $4x + 5y = 13$ (C) $5x + 4y + 13 = 0$ (D) $4x + 5y = 13$

17. If F_1 & F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5}+\frac{y^2}{3}=1$ on the tangent at any point P on the ellipse , then $(S_1F_1)\ .\ (S_2F_2)$ is equal to

(A) 2 (B) 3 (C) 4 (D) 5

18. If $\tan\theta_1.\ \tan\theta_2 = -\frac{a^2}{b^2}$ then the chord joining two points θ_1 & θ_2 on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtend a right angle at -

(A) focus (B) centre (C) end of the major axis (D) end of the minor axis

19. The number of values of c such that the straight line $y = 4x + c$ touches the curve $(x^2 / 4) + y^2 = 1$ is -

(A) 0 (B) 1 (C) 2 (D) infinite **[JEE 98]**

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

20. If $x - 2y + k = 0$ is a common tangent to $y^2 = 4x$ & $\frac{x^2}{a^2}+\frac{y^2}{3}=1$ $(a > \sqrt{3})$, then the value of a, k and other common tangent are given by -

(A) $a = 2$ (B) $a = -2$ (C) $x + 2y + 4 = 0$ (D) $k = 4$

21. All ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(0 < b < a)$ has fixed major axis. Tangent at any end point of latus rectum meet at a fixed point which can be -

(A) (a, a) (B) (0, a) (C) (0, –a) (D) (0, 0)

22. Eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance $\sqrt{3}$ units from the centre of the ellipse is -

(A) $\frac{5\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) $\frac{2\pi}{3}$

23. For the ellipse $9x^2 + 16y^2 - 18x + 32y - 119 = 0$, which of the following is/are true -

(A) centre is (1, –1)

(B) length of major and minor axis are 8 and 6 respectively

(C) $e=\frac{\sqrt{7}}{4}$

(D) foci are $(1\pm\sqrt{7}, -1)$

24. With respect to the ellipse $4x^2 + 7y^2 = 8$, the correct statement(s) is/are -

(A) length of latus rectum $\frac{8\sqrt{2}}{7}$

(B) the distance between the directrix $4\sqrt{\frac{7}{3}}$

(C) tangent at $\left(\frac{1}{2}, 1\right)$ is $2x + 7y = 8$

(D) Area of Δ formed by foci and one end of minor axis is $\frac{4\sqrt{3}}{7}$

25. On the ellipse, $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are - **[JEE 99]**

(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	A	B	C	B	A	B	B	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	B	B	C	B	B	B	C	A,B,C,D
Que.	21	22	23	24	25					
Ans.	B,C	A,B,D	A,B,C,D	A,C,D	B,D					

EXTRA PRACTICE QUESTIONS ON ELLIPSE

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ & $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$. Then the value of b and the other common tangent are given by -

(A) $b = \sqrt{3}$; $x + 2y + 4 = 0$ (B) $b = 3$; $x + 2y + 4 = 0$

(C) $b = \sqrt{3}$; $x + 2y - 4 = 0$ (D) $b = \sqrt{3}$; $x - 2y - 4 = 0$

2. The tangent at any point P on a standard ellipse with foci as S & S' meets the tangents at the vertices A & A' in the points V & V', then -

(A) $l(AV).l(A'V') = b^2$ (B) $l(AV).l(A'V') = a^2$

(C) $\angle V'SV = 90°$ (D) V'S' VS is a cyclic quadrilateral

3. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\pi/4$ is -

(A) $\frac{(a^2 - b^2)ab}{a^2 + b^2}$ (B) $\frac{(a^2 + b^2)ab}{a^2 - b^2}$ (C) $\frac{(a^2 - b^2)}{ab(a^2 + b^2)}$ (D) $\frac{(a^2 + b^2)}{(a^2 - b^2)ab}$

4. Q is a point on the auxiliary circle of an ellipse. P is the corresponding point on ellipse. N is the foot of perpendicular from focus S, to the tangent of auxiliary circle at Q. Then -

(A) SP = SN (B) SP = PQ (C) PN = SP (D) NQ = SP

5. The line, $lx + my + n = 0$ will cut the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in points whose eccentric angles differ by $\pi/2$ if -

(A) $x^2 l^2 + b^2 n^2 = 2m^2$ (B) $a^2 m^2 + b^2 l = 2n^2$

(C) $a^2 l^2 + b^2 m^2 = 2n^2$ (D) $a^2 n^2 + b^2 m^2 = 2l$

6. A circle has the same centre as an ellipse & passes through the foci F_1 & F_2 of the ellipse, such that the two curves intersect in 4 points. Let 'P' be any one of their point of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the foci is -

(A) 11 (B) 12 (C) 13 (D) none

7. The normal at a variable point P on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of eccentricity e meets the axes of the ellipse in Q and R then the locus of the mid-point of QR is a conic with an eccentricity e' such that -

(A) e' is independent of e (B) e' = 1

(C) e' = e (D) e' = 1/e

8. The length of the normal (terminated by the major axis) at a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is -

(A) $\frac{b}{a}(r + r_1)$ (B) $\frac{b}{a}|r - r_1|$ (C) $\frac{b}{a}\sqrt{rr_1}$ (D) independent of r, r_1

where r and r_1 are the focal distance of the point.

9. Point 'O' is the centre of the ellipse with major axis AB and minor axis CD. Point F is one focus of the ellipse. If OF = 6 and the diameter of the inscribed circle of triangle OCF is 2, then the product (AB)(CD) is equal to -

(A) 65 (B) 52 (C) 78 (D) none

10. If P is a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then -

(A) PS + PS' = 2a, if a > b

(B) PS + PS' = 2b, if a < b

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}$

(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}[a-\sqrt{a^2-b^2}]$ when a > b

11. If the chord through the points whose eccentric angles are θ & ϕ on the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the focus, then the value of tan (θ/2) tan (ϕ/2) is -

(A) $\frac{e+1}{e-1}$ (B) $\frac{e-1}{e+1}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{1-e}{1+e}$

12. If point P(α + 1, α) lies between the ellipse $16x^2 + 9y^2 - 16x = 0$ and its auxiliary circle, then -

(A) [α] = 0 (B) [α] = –1

(C) no such real α exist (D) [α] = 1

where [.] denotes greatest integer function.

13. If latus rectum of an ellipse $\frac{x^2}{16}+\frac{y}{b^2}=1$ {0< b < 4}, subtend angle 2θ at farthest vertex such that cosecθ = $\sqrt{5}$, then -

(A) $e=\frac{1}{2}$ (B) no such ellipse exist

(C) b = $2\sqrt{3}$ (D) area of Δ formed by LR and nearest vertex is 6 sq. units

14. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) & (x_3, y_3) - **[JEE 99]**

(A) lie on a straight line (B) lie on an ellipse

(C) lie on a circle (D) are vertices of a triangle.

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A,C,D	A	A	C	C	C	C	A	A,B,C
Que.	11	12	13	14						
Ans.	A,B	A,B	A,C,D	A						