Chapter 4
Determinants

Exercise 4.5

Q. 1

Find adjoint of each of the matrices.

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

Answer:

Adjoint of the matrix A = $[a_{ij}]$n × n is defined as the transpose of the matrix [Aij]n × n where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, A_{11} = 4, A_{12} = -3, A_{21} = -2, A_{22} = 1.

$$\therefore \text{Adj A} = \begin{vmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{vmatrix}$$

$$= \begin{vmatrix} 4 & -2 \\ -3 & 1 \end{vmatrix}$$

Q. 2

Find adjoint of each of the matrices.

$$\begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$$

Answer:

Adjoint of the matrix A = $[a_{ij}]$n × n is defined as the transpose of the matrix $[A_{ij}]$n × n where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 1\{(3\times1\text{-}0\times5)\} = 3$

Similarly,

A_{12} = -12, A_{13} = 6, A_{21} = 1, A_{22} = 5, A_{23} = 2, A_{31} = -11, A_{32} = -1, A_{33} = 5.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{vmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{vmatrix}$$

Q. 3

Verify A (adj A) = (adj A) A = |A|

$$\begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix}$$

Answer:

Adjoint of the matrix A = $[a_{ij}]n \times n$ is defined as the transpose of the matrix $[A_{ij}]n \times n$ where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, A_{11} = -6, A_{12} = 4, A_{21} = -3, A_{22} = 2.

$$\therefore \text{Adj A} = \begin{vmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{vmatrix}$$

$$= \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\text{So LHS} = \text{A(Adj A)} = \begin{bmatrix} 2 & 3 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Also Adj A(A) = $\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Determinant of A = |A| = 2(-6) - (3) (-4) = 0

So RHS = |A|I = 0

Hence A (Adj A) = Adj A(A) = |A|I = 0 {hence proved}

Q. 4

Verify A (adj A) = (adj A) A = |A|

$$\begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

Answer:

Adjoint of the matrix A = $[a_{ij}]n \times n$ is defined as the transpose of the matrix $[A_{ij}]n \times n$ where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 0$, $A_{12} = -11$, $A_{13} = 0$, $A_{21} = 3$, $A_{22} = 1$, $A_{23} = -1$, $A_{31} = 2$, $A_{32} = 8$, $A_{33} = 3$.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

So, LHS = A (Adj A)

$$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Also Adj A(A) = $\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Determinant of A = |A| = 11

So RHS = |A|I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$.

Hence A (Adj A) = Adj A(A) = |A|I = $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ {hence proved}

Q. 5

Find the inverse of each of the matrices (if it exists)

$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Answer:

We know that $A^{-1} = \frac{1}{|A|}$ Adj A

Adjoint of the matrix A = [aij]n × n is defined as the transpose of the matrix [Aij]n × n where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 3$, $A_{12} = -4$, $A_{21} = 2$, $A_{22} = 2$.

∴ Adj A = $\begin{vmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{vmatrix}$

$= \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

And |A| = 2(3) - (-2) (4) = 14

So $A^{-1} = \frac{1}{|A|}$ Adj A $= \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & \frac{2}{14} \\ -\frac{4}{14} & \frac{2}{14} \end{bmatrix} = \begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$.

Q. 6

Find the inverse of each of the matrices (if it exists)

$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Answer:

We know that $A^{-1} = \frac{1}{|A|}$ Adj A

Adjoint of the matrix $A = [a_{ij}]n \times n$ is defined as the transpose of the matrix $[A_{ij}]n \times n$ where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 2$, $A_{12} = 3$, $A_{21} = -5$, $A_{22} = -1$.

∴ Adj A $= \begin{vmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{vmatrix}$

$= \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

And |A| = -1(2) - (-3) (5) = 13

So $A^{-1} = \frac{1}{|A|}$ Adj A $= \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} \frac{2}{13} & \frac{-5}{13} \\ \frac{3}{13} & \frac{-1}{13} \end{bmatrix}$

Q. 7

Find the inverse of each of the matrices (if it exists)

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Answer:

Adjoint of the matrix A = $[a_{ij}]n \times n$ is defined as the transpose of the matrix $[A_{ij}]n \times n$ where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, $A_{11} = 10$, $A_{12} = 0$, $A_{13} = 0$, $A_{21} = -10$, $A_{22} = 5$, $A_{23} = 0$, $A_{31} = 2$, $A_{32} = -4$, $A_{33} = 2$.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

And |A| = 10.

$$A^{-1} = \frac{1}{|A|}\text{Adj A} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{10}{10} & -\frac{10}{10} & \frac{2}{10} \\ 0 & \frac{5}{10} & -\frac{4}{10} \\ 0 & 0 & \frac{2}{10} \end{bmatrix}$$

$$A^{-1} = \begin{vmatrix} 1 & -1 & \frac{1}{5} \\ 0 & \frac{1}{2} & -\frac{2}{5} \\ 0 & 0 & \frac{1}{5} \end{vmatrix}$$

Q. 8

Find the inverse of each of the matrices (if it exists)

$$\begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$$

Answer:

Adjoint of the matrix $A = [a_{ij}]n \times n$ is defined as the transpose of the matrix $[A_{ij}]n \times n$ where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, $A_{11} = -3$, $A_{12} = 3$, $A_{13} = -9$, $A_{21} = 0$, $A_{22} = -1$, $A_{23} = -2$, $A_{31} = 0$, $A_{32} = 0$, $A_{33} = 3$.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

And |A| = -3.

$$A^{-1} = \frac{1}{|A|}\text{Adj A} = \frac{1}{-3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{-3}{-3} & 0 & 0 \\ \frac{3}{-3} & \frac{-1}{-3} & 0 \\ \frac{-9}{-3} & \frac{-2}{-2} & \frac{3}{-3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{3} & 0 \\ 3 & \frac{2}{3} & -1 \end{bmatrix}$$

Q. 9

Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Answer:

Adjoint of the matrix $A = [a_{ij}]n \times n$ is defined as the transpose of the matrix $[Aij]n \times n$ where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, A_{11} = -1, A_{12} = -4, A_{13} = 1, A_{21} = 5, A_{22} = 23, A_{23} = -11, A_{31} = 3, A_{32} = 12, A_{33} = -6.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

And |A| = -3.

$$A^{-1} = \frac{1}{|A|} \text{Adj A} = \frac{1}{-3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & 11 & -6 \end{bmatrix} = \begin{bmatrix} \frac{-1}{-3} & \frac{5}{-3} & \frac{3}{-3} \\ \frac{-4}{13} & \frac{23}{-3} & \frac{12}{-3} \\ \frac{1}{-3} & \frac{-11}{-3} & \frac{-6}{-3} \end{bmatrix}$$

Q. 10

Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Answer:

Adjoint of the matrix A = $[a_{ij}]$n × n is defined as the transpose of the matrix [Aij]n × n where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, A_{11} = 2, A_{12} = -9, A_{13} = -6, A_{21} = 0, A_{22} = -2, A_{23} = -1, A_{31} = -1, A_{32} = 3, A_{33} = 2.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

And |A| = -1.

$$A^{-1} = \frac{1}{|A|} \text{Adj A} = \frac{1}{-1} \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Q. 11

Find the inverse of each of the matrices (if it exists)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Answer:

Adjoint of the matrix A = $[a_{ij}]$n × n is defined as the transpose of the matrix $[A_{ij}]$n × n where Aij is the co-factor of the element aij.

Let's find the cofactors for all the positions first-

Here, A_{11} = -1, A_{12} = 0, A_{13} = 0, A_{21} = 0, A_{22} = -cosα, A_{23} = -sinα, A_{31} = 0, A_{32} = -sinα, A_{33} = cosα.

$$\therefore \text{Adj A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

And |A| = 1.

$A^{-1} = \frac{1}{|A|}$ Adj A

$$= \frac{1}{-1}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Q. 12

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1} A^{-1}$.

Answer:

We have $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$ = (61)(67)-(47)(87) = -2

Here determinant of matrix = $|AB| \neq 0$ hence $(AB)^{-1}$ exists.

$(AB)^{-1} = \frac{1}{|AB|}$ Adj (AB) = $-\frac{1}{2}\begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix} = \begin{bmatrix} \frac{61}{-2} & \frac{-47}{-2} \\ \frac{-87}{-2} & \frac{67}{-2} \end{bmatrix}$

{∴ Adj (AB) = $\begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix}$}

So, $(AB)^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{47}{2} \\ \frac{87}{2} & -\frac{67}{2} \end{bmatrix}$

Also |A| = 1 ≠ 0 and |B| = -2 ≠ 0.

∴ A^{-1} and B^{-1} will also exist and are given by-

(A)-1 = $\frac{1}{|A|}$ Adj A $= \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$(B)^{-1} = \frac{1}{|B|}$ Adj A = - $\frac{1}{2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

And hence,

$(B)^{-1} (A)^{-1} = -\frac{1}{2}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix} = \begin{bmatrix} \frac{61}{-2} & \frac{-47}{-2} \\ \frac{-87}{-2} & \frac{67}{-2} \end{bmatrix}$

{Hence proved}

Q. 13

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Answer:

We have $A^2 = A.^A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$.

So $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

Hence $A^2 - 5A + 7I = 0$

∴ A.A – 5A = -7I

Now post multiply with A^{-1}

So $A.A.A^{-1}$ - 5A. A^{-1} = -7I. A^{-1}

→ A.I – 5I = -7I. A^{-1} {since $A.A^{-1} = I$}

A – 5I = $-7A^{-1}$ {since X.I = X}

→ $A^{-1} = \frac{5I-A}{7} = \frac{1}{7}\left(5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right) = -\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\rightarrow A^{-1} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

Q. 14

For the matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Answer:

We have $A^2 = A.A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix}$

Since $A^2 + aA + bI = \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix} + a\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

So $A^2 + aA + bI = \begin{bmatrix} 10 + 3a + b & 5 + a \\ 5 + a & 5 + 2a + b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence 10+3a+b = 0 ...(i)

5+a = 0 ...(ii)

5+2a+b = 0 ...(iii)

From (ii) a = -5

Putting a in (iii) we get b = 5

So a = -5 and b = 5 satisfy the equation.

Q. 15

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

Answer:

Here $A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

And hence $A^3 = A.\ A^2 =$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 5A + 11\ I =$

$$\begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} = 0$$

Thus, $A^3 - 6A^2 + 5A + 11\ I = 0$

Now, $A^3 - 6A^2 + 5A + 11\ I = 0$,

→ (A.A.A)- 6 (A.A) +5A = -11I

Post-multiply with A-1 on both sides-

→ $(A.A.A.A^{-1})$- 6 $(A.A.A^{-1})$ +5A.A^{-1} = -11I. A^{-1}

→ (A.A.I) – 6(A.I) + 5I = -11I. A-1 {since A.A^{-1} = I}

→ (A.A) – 6A +5I = -11A^{-1} {since X.I = X}

Q. 16

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}

Answer:

Here $A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

And hence $A^3 = A.\,A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} =$
$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & -22 \end{bmatrix}$

$\therefore A^3 - 6A^2 + 9A - 4I$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} -$
$4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = 0$

Thus, $A^3 - 6A^2 + 9A - 4I = 0$

Now, $A^3 - 6A^2 + 9A - 4I = 0$,

$\rightarrow$ (A.A.A)- 6 (A.A) +9A = 4I

Post-multiply with A^{-1} on both sides-

$\rightarrow (A.A.A.A^{-1}) - 6\,(A.A.A^{-1}) + 9A.A^{-1} = 4I.\,A^{-1}$

$\rightarrow (A.A.I) - 6(A.I) + 9I = 4I.\,A^{-1}$ {since $A.A^{-1} = I$}

$\rightarrow (A.A) - 6A + 9I = 4A^{-1}$ {since X.I = X}

$\rightarrow A^{-1} = \frac{1}{4}(A^2 - 6A + 9I)$

$$= \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Hence $A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

Q. 17

Let A be a non-singular square matrix of order 3 × 3. Then |adj A| is equal to

A. |A |

B. $|A|^2$

C. $|A|^3$

D. 3|A|

Answer:

For a square matrix of order n × n,

We know that $|Adj\ A| = |A|^{n-1}$

So, $|Adj\ A| = |A|\ (3^{-1}) = |A|^2$

Q. 18

If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

A. det (A)

B. $\frac{1}{\det(A)}$

C. 1

D. 0

Answer:

$(A)^{-1} = \frac{1}{|A|}$ Adj A

SO, $|(A)^{-1}| = \left|\frac{1}{|A|} Adj\ (A)\right| = \frac{1}{|A|^n}$ |Adj (A) $= \frac{1}{|A|^n}$ $|A|^{n-1} = \frac{1}{|A|^1}$

{since adj(A) is of order n and $|Adj(A)| = |A|^{n-1}$}

Alternative-

We know that $AA^{-1} = I$

So $|A||A^{-1}| = |I| = 1$

Hence $|(A)^{-1}| = \frac{1}{|A|}$