EXERCISE 5.2 (NCERT)

Differentiate the functions with respect to x in Exercise 1 to 8

Q No 1 $\sin(x^2+5)$

Sol. Applying Chain Rule.

$$\frac{d}{dx}\left(\sin(x^2+5)\right) = \cos(x^2+5)\cdot\frac{d}{dx}(x^2+5)$$

$$= \cos(x^2+5)\cdot(2x+0) = 2x\cos(x^2+5)$$

Q No 2 $\cos(\sin x)$

Sol. $$\frac{d}{dx}\left[\cos(\sin x)\right] = -\sin(\sin x)\frac{d}{dx}(\sin x)$$

$$= -\sin(\sin x)\cdot\cos x = -\cos x\cdot\sin(\sin x)$$

Q No 3 $\sin(ax+b)$

Sol $$\frac{d}{dx}\left(\sin(ax+b)\right) = \cos(ax+b)\cdot\frac{d}{dx}(ax+b)$$

$$= \cos(ax+b)\cdot a = a\cos(ax+b)$$

Q No 4 $\sec(\tan(\sqrt{x}))$

Sol. $$\frac{d}{dx}\left[\sec(\tan(\sqrt{x}))\right] = \sec(\tan(\sqrt{x}))\cdot\tan(\tan(\sqrt{x}))\,\frac{d}{dx}\left[\tan\sqrt{x}\right]$$

$$= \sec(\tan(\sqrt{x}))\cdot\tan(\tan(\sqrt{x}))\cdot\sec^2(\sqrt{x})\,\frac{d}{dx}(\sqrt{x})$$

$$= \sec(\tan(\sqrt{x}))\cdot\tan(\tan(\sqrt{x}))\cdot\sec^2(\sqrt{x})\cdot\frac{1}{2}x^{\frac{1}{2}-1}$$

$$= \frac{\sec^2(\sqrt{x})\cdot\sec(\tan(\sqrt{x}))\cdot\tan(\tan(\sqrt{x}))}{2\sqrt{x}}$$

Q No 5 $\frac{d}{dx}\left(\frac{\sin(ax+b)}{\cos(cx+d)}\right) =$

Sol. $$\frac{\cos(cx+d)\cdot\frac{d}{dx}\sin(ax+b) - \sin(ax+b)\frac{d}{dx}\cos(cx+d)}{\left[\cos(cx+d)\right]^2} \quad \left|\ \frac{u}{v}\text{ form.}\right.$$

$$= \frac{\cos(cx+d)\cdot\cos(ax+b)\,(a+0) + \sin(ax+b)\sin(cx+d)(c+0)}{\cos^2(cx+d)}$$

$$= \frac{a\cos(cx+d)\cos(ax+b) + c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$$

Q No 6 $\cos x^3 \cdot \sin^2(x^5)$

Sol. $\frac{d}{dx}\left[\cos x^3 \cdot [\sin(x^5)]^2\right] = \cos x^3 \frac{d}{dx}[\sin(x^5)]^2 + \sin^2(x^5)\frac{d}{dx}\cos x^3$ | uv form

$= \cos x^3 \cdot 2(\sin(x^5))^1 \frac{d}{dx}(\sin(x^5)) + \sin^2 x^5\left[-\sin x^3 \cdot \frac{d}{dx}(x^3)\right]$

$= \cos x^3 \cdot 2\sin x^5 \cdot \cos x^5 \cdot \frac{d}{dx}(x^5) + \sin^2 x^5\left[-\sin x^3 \cdot 3x^2\right]$

$= \cos x^3 \cdot \sin(2x^5) \cdot 5x^4 - 3x^2 \sin^2 x^5 \sin(x^3)$ $\left[\because 2\sin\theta\cos\theta = \sin 2\theta\right]$

$= 5x^4 \cos(x^3)\sin(2x^5) - 3x^2\sin^2(x^5)\cdot\sin(x^3)$

Q No 7 $2\sqrt{\cot(x^2)}$

Sol. $\frac{d}{dx}\left(2\sqrt{\cot(x^2)}\right) = \frac{d}{dx}\left[2(\cot(x^2))^{1/2}\right]$

$= 2\frac{1}{2}\{\cot(x^2)\}^{1/2-1} \cdot \frac{d}{dx}\cot(x^2)$

$= \frac{1}{\sqrt{\cot(x^2)}}\left(-\operatorname{cosec}^2(x^2)\cdot\frac{d}{dx}(x^2)\right)$

$= \frac{-\operatorname{cosec}^2(x^2)\cdot 2x}{\sqrt{\cot(x^2)}} = \frac{-2x\operatorname{cosec}^2(x^2)}{\sqrt{\cot(x^2)}}$

Q No. 8: $\cos(\sqrt{x})$

Sol. $\frac{d}{dx}(\cos(\sqrt{x})) = -\sin(\sqrt{x})\cdot\frac{d}{dx}(\sqrt{x}) = -\sin(\sqrt{x})\cdot\frac{d}{dx}(x^{1/2})$

$= -\sin(\sqrt{x})\cdot\frac{1}{2}(x)^{1/2-1} = -\frac{\sin\sqrt{x}}{2\sqrt{x}}$.

Q No. 9 Prove that the function f given by $f(x) = |x-1|$ $x \in R$ is not differentiable at $x=1$

Sol Given $f(x) = |x-1| = \begin{cases} x-1 \ ; & x-1 \geq 0 \text{ ie } x \geq 1 \\ -(x-1); & x-1 < 0 \text{ ie } x < 1 \end{cases}$

Now. LHD ie. $Lf'(1) = \underset{h\to 0^-}{Lt}\frac{f(1+h)-f(1)}{h} = \underset{h\to 0}{Lt}\frac{-(1+h-1)-0}{h} = -1$

and RHD ie $Rf'(1) = \underset{h\to 0^+}{Lt}\frac{f(1+h)-f(1)}{h} = \underset{h\to 0^+}{Lt}\frac{1+h-1-0}{h} = 1$ $\left[\because f(1) = |1-1| = 0\right]$

$\therefore Lf'(1) \neq Rf'(1)$

$\therefore f$ is not differentiable at $x=1$.

Q No 10: Prove that the greatest integer function defined by $f(x) = [x]$, $0<x<3$ is not differentiable at $x=1$ and 2

Sol: At $x=1$

Let $f(x) = [x]$

$$\underset{x\to 1^-}{Lt} f(x) = \underset{x\to 1^-}{Lt} [x] \qquad [\text{Put } x = 1-h,\ h>0 \text{ so that } h\to 0 \text{ as } x\to 1^-]$$

$$= \underset{h\to 0}{Lt} [1-h] = \underset{h\to 0}{Lt}\ 0 = 0 \qquad [\because [1-h] = 0$$

$$\underset{x\to 1^+}{Lt} f(x) = \underset{x\to 1^+}{Lt} [x] \qquad [\text{Put } x = 1+h,\ h>0 \text{ so that } h\to 0 \text{ as } x\to 1^+$$

$$= \underset{x\to 1^+}{Lt} [x] = \underset{h\to 0}{Lt} [1+h]$$

$$= \underset{h\to 0}{Lt}\ 1 = 1 \qquad [\because [1+h] = 1]$$

$$\therefore \underset{x\to 1^-}{Lt} f(x) \neq \underset{x\to 1^+}{Lt} f(x)$$

$\therefore$ f is not continuous at $x=1$

$\Rightarrow$ f is not differentiable at $x=1$.

Similarly we can prove that f is not differentiable at $x=2$