UNIT – IV

Electromagnetic Induction and Alternating Currents

CHAPTER 6

ELECTRO-MAGNETIC INDUCTION

Syllabus

- *Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law.*
- *Eddy currents, Self and mutual induction.*

Chapter Analysis

List of Topics	2016		2017		2018
	D	OD	D	OD	D/OD
Magnetic flux and faraday's laws			1 Q (2 marks)	1 Q (1 mark)	
Eddy currents, self and Mutual induction and Generation	1 Q (3 marks)	1 Q (5 marks)	1 Q (3 marks)	1 Q (2 marks) 2 Q (3 marks)	

TOPIC-1
Magnetic Flux and Faraday's Laws

Revision Notes

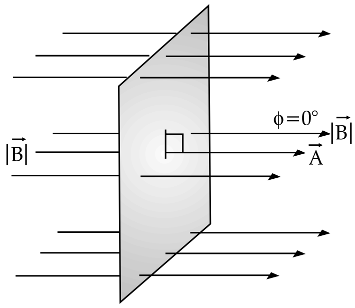
(a) Concept Notes

Electromagnetic induction

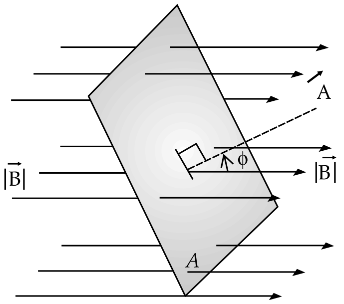
- Electromagnetic induction is the process of generating electric current with a magnetic field.
- It takes place whenever a magnetic field is changing or electric conductor moves relative to one another when they are in each others magnetic field.
- The current produced by electromagnetic induction is more when the magnet or coil moves faster. When magnet or coil moves back and forth repeatedly, then alternating current is produced.

Magnetic Flux :

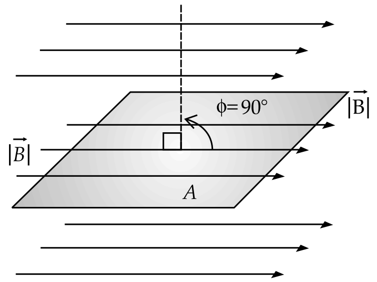
- Magnetic flux through an enclosed area is the amount of field lines cutting through a surface area A defined by unit area vector.
- The units for magnetic flux is weber, where, 1 Wb=1 T/m^2.
- Magnetic flux ϕ_B is related to number of field lines passing through a given area.
- If magnetic field is changing, then changing magnetic flux will be $\phi_B = NBA \cos \theta$, where angle θ is the angle between magnetic field and normal to the plane.



$\vec{B}$ parallel to A (ϕ= 0°) magnetic flux ϕ_B = BA.



$\vec{B}$ at an angle ϕ with the perpendicular to A: magnetic flux ϕ_B = BA cos θ



$\vec{B}$ perpendicular to A (ϕ=90°): magnetic flux ϕ_B = 0.

Magnetic flux density

- The change in magnetic flux per unit change in area is called magnetic flux density.
- Magnetic flux is given by : $d\phi = \vec{B} \cdot \vec{dA}$.

  For $\vec{B}$ parallel to $\vec{dA}$, we have

$$d\phi = B(dA)\cos 0° = B(dA)$$

Therefore,

$$B = \frac{d\phi}{dA} \quad \text{(i)}$$

i.e., **magnetic induction** is equal to the magnetic flux density. In other words, the magnetic field may be measured in terms of magnetic flux density. From equation (i), we find :

Unit of $B = \frac{\text{Unit of } d\phi}{\text{Unit of } dA}$

Or $T = \frac{\text{Wb}}{\text{m}^2}$

i.e., Tesla = weber per square metre.

Faraday's Laws of Electromagnetic Induction

- The induced emf in a closed loop due to a change in magnetic flux through the loop is known as Faraday's law.
- **Faraday's First Law** of Electromagnetic Induction states that whenever a conductor is placed in varying magnetic field an emf is induced which is known as induced emf and if the conductor circuit is closed, current is also induced which is called induced current.
- **Faraday's Second Law** of Electromagnetic Induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is nothing but the product of number of turns in the coil and flux associated with the coil.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

ϕ_B is magnetic flux through the circuit as $\phi_B = \int \vec{B}.d\vec{A}$

With N loops of similar area in a circuit and ϕ_B being the flux through a loop, then emf is induced in every loop making Faraday law as

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

where, ε = Induced emf [V], N = number of turns in the coil

$\Delta\phi$= change in the magnetic flux [Wb], Δt = change in time [s]

The negative sign means that ε opposes its cause.

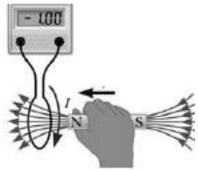
- If there is no change in magnetic flux, no induced emf is induced.

Induced emf and current

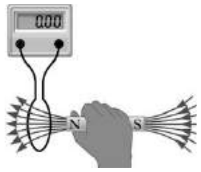
- A changing magnetic flux induces an electric field which induces a current in the circuit
- A wire moving in the field induces a current which acts same as current created by a battery
- Changing magnetic flux and induced electric field are related to induced emf as per Faraday's law.
- The induced EMF is related to the magnetic field as $E = B.l.v\sin\theta$

Induced Current

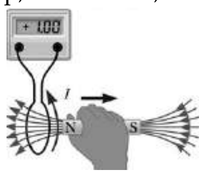
- When a conductor moves across flux lines, magnetic forces on the free electrons induce an electric current.
- When a magnet is moved towards a loop of wire connected to ammeter, in such case, ammeter shows current induced in the loop.



- When a magnet is held stationary, there will be no induced current in the loop, if also the magnet is inside the loop.



- When a magnet is moved away from the loop, in such case, ammeter shows opposite current induced in the loop.



Motional Emf

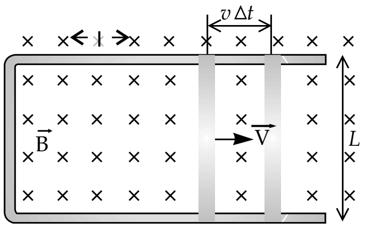
- The relationship between an induced emf ε in a wire moving at a constant speed v through a magnetic field B is given by :

$$\phi_B = Blx$$

$$\varepsilon = \frac{-d\phi_B}{dt} = \frac{-d}{dt}(Blx)$$

$$= -Bl\frac{dx}{dt}$$

$$= Blv \qquad \left(\frac{dx}{dt} = -v\right)$$



- An induced emf from Faraday's law is created from a motional emf that opposes the change in flux.
- Magnetic and electric forces on charges in a rod moving perpendicular to magnetic field is given as :

At equilibrium

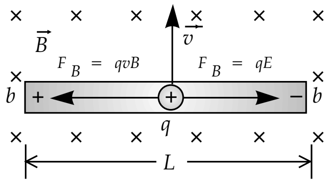
$$F_E = F_B$$

$$qE = qvB$$

$$E = vB$$

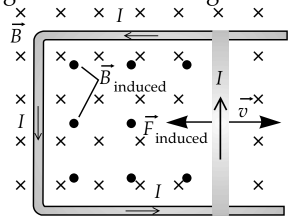
$$\frac{V}{l} = vB \qquad \left[\text{Here, } E = \frac{V}{l}\right]$$

$$V = Bvl$$



Lenz's Law

- Lenz's law is used to determine the directions of induced magnetic fields, currents, and *emfs*.
- The direction of an induced emf always opposes the change in magnetic flux that causes the emf.
- It explains about the negative sign in Faraday's flux rule, $\varepsilon = -\frac{d\phi_B}{dt}$ showing that the polarity of induced emf tends to produce a current that opposes the change in magnetic flux that produced it.
- As per conservation of energy, induced emf opposes the cause which produces it making mechanical work to continue with the process which gets converted into electrical energy.
- Slide wire containing induced current, magnetic field and magnetic force :



Electric Generators and Back Emf

- Electric generator rotates a coil in a magnetic field inducing an emf which is given as a function of time $\varepsilon = NBA\omega \sin(\omega t)$.
  where, A = area of N-turn coil rotated at constant angular velocity ω in uniform magnetic field $\vec{B}$.
- The peak emf of a generator is, $\varepsilon_0 = NBA\omega$
- Any rotating coil produces an induced emf. In motors, it is known as back emf as it opposes the emf input to the motor.

Know the Terms

- **Electric generator :** Device for converting mechanical work into electric energy that induces an emf by rotating a coil in magnetic field.
- **Induced electric field :** Field created due to changing magnetic flux with time.
- **Induced emf :** It is a short-lived voltage generated by a conductor or coil moving in a magnetic field.
- **Magnetic damping :** It is a drag which is produced by eddy currents.
- **Magnetic flux :** It is the amount of magnetic field lines that is measured through a given area.
- **Motional induced *emf* :** Voltage produced by movement of conducting wire in a magnetic field.
- **Peak *emf* :** It is the *emf* produced by a generator.

Know the Formulae

- Magnetic flux : $\phi_m = \int \vec{B}.\vec{dn}$
- Faraday's law : $\varepsilon = -N\frac{d\phi_m}{dt}$
- Motional induced emf : $\varepsilon = Blv$
- Motional emf around a circuit : $\varepsilon = \oint E.dl = -\frac{d\phi_m}{dt}$
- *emf* produced by an electric generator $\varepsilon = NBA \sin \omega t$

Objective Type Questions **(1 mark each)**

Q. 1. A square of side L meters lies in the x-y plane in a region where the magnetic field is given by $\vec{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ Tesla, where B_0 is constant. The magnitude of flux passing through the square is

(a) $2B_0L^2$Wb. **(b) $3B_0L^2$Wb.**

(c) $4B_0L^2$Wb. **(d) $\sqrt{29}B_0L^2Wb$**

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : Magnetic flux is defined as the total number of magnetic lines of force passing normally through an area placed in a magnetic field and is equal to the magnetic flux linked with that area.

Square lies in x-y plane in $\vec{B}$ so $\vec{A} = L^2\hat{k}$

$Q = B.A$

$= B_0(2\hat{i} + 3\hat{j} + 4\hat{k}).(L^2\hat{k})$

$= B_0[2 \times \hat{i}\hat{k} + 3 \times \hat{j}\hat{k} + 4 \times \hat{k}.\hat{k}]$

$= B_0L^2[0 + 0 + 4]$

$= 4B_0L^2Wb.$

Q. 2. A loop, made of straight edges has six corners at $A(0, 0, 0)$, $B(L, 0, 0)$ $C(L, L, 0)$, $D(0, L, 0)$, $E(0, L, L)$ and $F(0, 0, L)$. A magnetic field $\vec{B} = B_0(\hat{i}+\hat{k})$ Tesla is present in the region. The flux passing through the loop *ABCDEFA* (in that order) is

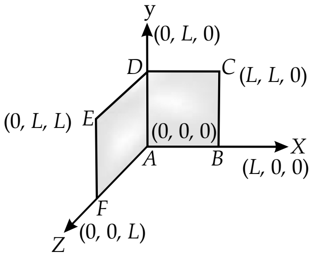
(a) B_0L^2 Wb. **(b) $2B_0L^2$ Wb.**

(c) $\sqrt{2}B_0L^2$ Wb. **(d) $4B_0L^2$ Wb.**

[NCERT Exemplar]

Ans. Correct option : (b)

Explanation : The loop can be considered in two planes :



(i) Plane of *ABCDA* is in *X-Y* plane. So its vector $\vec{A}$ is in *Z*-direction. Hence,

$$A_1 = |A|\hat{k} = L^2\hat{k}$$

(ii) Plane of *DEFAD* is in *Y-Z* plane

So $A_2 = |A|\hat{i} = L^2\hat{i}$

$\therefore \quad A = A_1 + A_2 = L^2(\hat{i}+\hat{k})$

$B = B_0(\hat{i}+\hat{k})$

So, $Q = B.A$

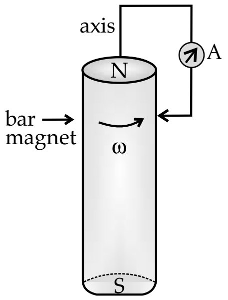
$= B_0(\hat{i}+\hat{k}).L^2(\hat{i}+\hat{k})$

$= B_0L^2 \quad [\hat{i}.\hat{i}+\hat{i}.\hat{k}+\hat{k}.\hat{i}+\hat{k}.\hat{k}]$

$= B_0L^2[1+0+0+1] \quad (\because \cos 90° = 0)$

$= 2B_0L^2 Wb$

Q. 3. A cylindrical bar magnet is rotated about its axis in the figure. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then



(a) a direct current flows in the ammeter *A*.

(b) no current flows through the ammeter *A*.

(c) an alternating sinusoidal current flows through the ammeter *A* with a time period $2\pi/\omega$.

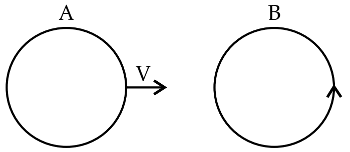
(d) a time varying non-sinusoidal current flows through the ammeter *A*. **[NCERT Exemplar]**

Ans. Correct option : (b)

Explanation : The phenomenon of electromagnetic induction is used in this problem. Whenever the number of magnetic lines of force (magnetic flux) passing through a circuit changes (or a moving conductor cuts the magnetic flux) an emf is produced in the circuit (or emf induces across the ends of the conductor) is called induced emf. The induced emf persists only as long as there is a change or cutting of flux.

When cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence, no current flows through the ammeter *A*. Hence the ammeter shows no deflection.

Q. 4. There are two coils *A* and *B* as shown in figure. A current starts flowing in *B* as shown, when *A* is moved towards *B* and stops when *A* stops moving. The current in *A* is counter clockwise. *B* is kept stationary when *A* moves. We can infer that



(a) there is a constant current in the clockwise direction in *A*.

(b) there is a varying current in *A*.

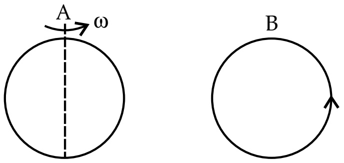
(c) there is no current in *A*.

(d) there is a constant current in the counter clockwise direction in *A*. **[NCERT Exemplar]**

Ans. Correct option : (d)

Explanation : When coil *A* moves towards coil *B* with constant velocity, so rate of change of magnetic flux due to coil *B* in coil *A* will be constant that gives constant current in coil *A* in same direction as in coil *B* by Lenz's law.

Q. 5. Same as problem 4 except the coil *A* is made to rotate about a vertical axis figure. No current flows in B if *A* is at rest. The current in coil *A*, when the current in B (at $t = 0$) is counter-clockwise and the coil *A* is as shown at this instant, $t = 0$, is



(a) constant current clockwise.

(b) varying current clockwise.

(c) varying current counter-clockwise.

(d) constant current counter-clockwise.

[NCERT Exemplar]

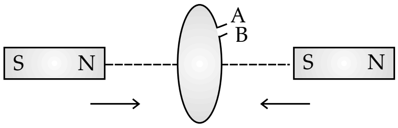
Ans. Correct option : (a)

Explanation : In this case the direction of the induced electromotive force/induced current is determined by the Lenz's law. According to the Lenz's law the direction of induced emf or current in a circuit is such as to oppose the cause that produces it. This law is based upon law of conservation of energy.

When the current in coil *B* (at $t = 0$) is counter-clockwise and the coil *A* is considered above it. The counter-clockwise flow of the current in coil *B* is equivalent to North Pole of magnet and magnetic field lines are emanating upward to coil *A*. When coil *A* starts rotating at $t = 0$, the current in coil A is constant along clockwise direction by Lenz's rule.

Very Short Answer Type Questions (1 mark each)

AI Q. 1. In the figure given, mark the polarity of plates *A* and *B* of a capacitor when the magnets are quickly moved towards the coil.



U **[O.D. Comptt. III 2017]**

Ans. Accept both the answers :

A : +ve; B; – ve **1**

or A : –ve; B; + ve **1**

[CBSE Marking Scheme 2017]

Detailed Answer :

From the figure, both magnets move towards the coil. Flux where induced current is in anticlockwise direction, if observed from left side and its direction clockwise as observed from right side. Hence, the direction of induced current is in clockwise direction which shows that plate *A* of the capacitor is at higher potential as compared to plate *B* of the capacitor, so plate *B* will be negative while plate *A* is positive.

Q. 2. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease ? Explain. U **[SQP II 2017]**

Ans. On inserting the iron core, the magnetic flux linked with the solenoid will increase and an induced e.m.f. will be produced. According to Lenz's law, the induced e.m.f. will act in a manner so as to oppose the cause, which has produced it thereby opposing an increase in magnetic flux which happens due to decrease in current.

OR

As per Lenz's law, direction of induced emf/ current in a circuit will oppose the cause which produces it. If an iron core gets inserted in a solenoid having current, it will increase the magnetic field as a result of magnetization of iron core which increases the flux. Hence, emf induced in the coil should oppose the flux increase which induces the current in the coil in the direction opposing the increasing magnetic field by decreasing in current. **1**

[CBSE Marking Scheme 2017]

Commonly Made Error

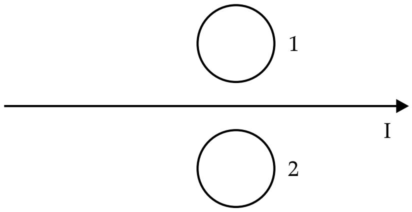
- Students make probable errors while applying the applications of Lenz' law in questions.

Answering Tip

- Student should remember while answering the questions that current should be marked in such a way to oppose the increase or decrease of flux.

Q. 3. What are the directions of induced currents in metal rings 1 and 2 when current I in the wire is increasing steadily ?

U **[O.D. Set I, II, III 2017]**

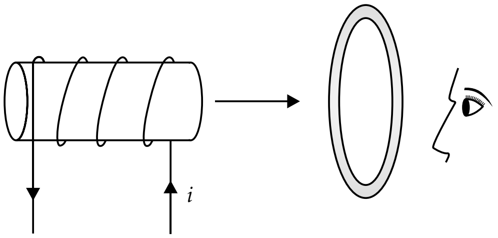


Ans. When current is increasing, magnetic flux linked with the two coils also increases. The $\vec{B}$ due to the current element in 2 is into the plane and 1 is out of the plane. Since flux increases direction of induced current is oppo. such that the $\vec{B}$ due to it is opposite to the original flux. So the induced current in the loop 1 is in clockwise direction and 2 is in anticlock-wise direction

[Topper's Answer 2017]

Q. 4. Figure shows a current carrying solenoid moving towards a conducting loop. Find the direction of the current induced in the loop.

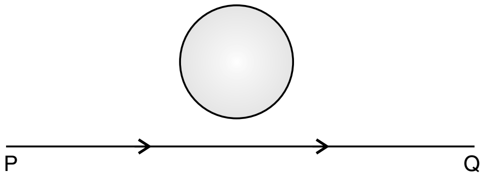
U **[Delhi I, II, III 2015]**



Ans. From an observer, the direction of current in the solenoid is anti-clockwise. On displacing it towards the loop, current in the loop will get induced in a direction in order to oppose the approach of solenoid, hence direction of induced current as seen by the observer will be clockwise. **1**

[CBSE Marking Scheme 2015]

AI **Q. 5. A conducting loop is held above a current carrying wire '*PQ*' as shown in the figure. Depict the direction of the current induced in the loop when the current in the wire *PQ* is constantly increasing.** U **[OD. I, II, III 2014]**



Ans. It is observed that an increasing magnetic field in the loop due to wire *PQ* is perpendicular to the plane, so the direction of an induced current in the loop will be such that it tends to produce an inward magnetic field making the current induced in the loop in the clockwise direction. **1**

[CBSE Marking Scheme 2014]

Q. 6. Two spherical bobs, one metallic and the other of glass, of the same size are allowed to fall freely from the same height above the ground. Which of the two would reach earlier and why ?

U **[Delhi I, II, III 2014]**

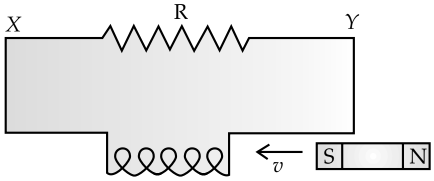
Ans. Glass, ½

In glass, there is no effect of electromagnetic induction, due to the presence of the Earth's magnetic field, unlike in the case of metallic bob.½

[CBSE Marking Scheme 2014]

Q. 7. A magnet is moving towards a coil with a uniform speed *v* as shown in the figure. State the direction of the induced current in the resistor *R*.

U **[CBSE SQP 2013]**



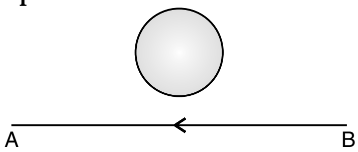
Ans. From *X* to *Y*. **1**

[CBSE Marking Scheme 2013]

Detailed Answer :

As seen from the side of the approaching magnet the current inside the coil should be clockwise so that the induced magnetic pole on the right side of the coil will be 'south' which tends to repel the approaching magnet, hence magnet's field inside the coil points from right to left which induces clockwise current to flow from left to right.

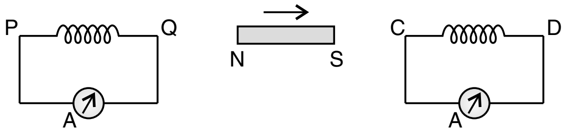
Q. 8. The electric current flowing in a wire in the direction from *B* to *A* is decreasing. Find out the direction of the induced current in the metallic loop kept above the wire as shown.



C **[O.D. I, II, III 2014]**

Ans. Clockwise. **[CBSE Marking Scheme 2014] 1**

AI **Q. 9. A bar magnet is moved in the direction indicated by the arrow between two coils *PQ* and *CD*. Predict the directions of induced current in each coil.**



C **[O.D. I, II, III 2012]**

Ans. Clockwise in both coils or the direction of current is from P → Q and C → D. ½ + ½

[CBSE Marking Scheme 2012]

Short Answer Type Questions-I

(2 marks each)

Q. 1. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop ? Justify.

R **[Delhi I, II, III 2017]**

Ans. No, ½

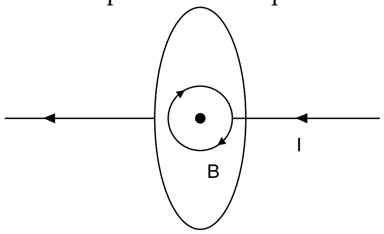
As the magnetic field due to current carrying wire will be in the plane of the circular loop, so magnetic flux will remain zero. ½

Alternatively,
[Magnetic flux does not change with the change of current.] **1**

[CBSE Marking Scheme 2017]

Detailed Answer :

No, there will not be an induced emf occur in the loop as magnetic flux doesn't change linked with the circular loop because magnetic field lines are parallel to the plane of the loop.



As, induced emf (e) is proportional to the rate of change of magnetic flux (ϕ_B)

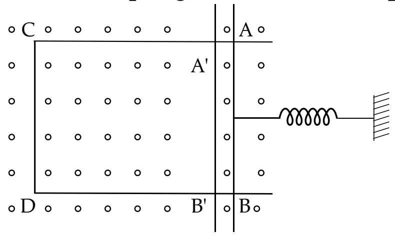
and $\phi_B = B \cdot A = BA\cos\theta$

Here, $B \perp A \Rightarrow \phi_B = BA\cos 90° = 0$

So, induced emf = 0

Hence, a change in current of wire will not create any emf in the loop.

AI **Q. 2. A rectangular frame of wire is placed in a uniform magnetic field directed outwards, normal to the paper. *AB* is connected to a spring which is stretched to *A'B'* and then released at time $t = 0$. Explain qualitatively how induced e.m.f. in the coil would vary with time. (Neglect damping of oscillations of spring)** A **[CBSE Comptt. I 2018]**



Ans. SHM nature of oscillation of the wire AB ½

Expression for instantaneous magnetic flux ½

Expression for instantaneous induced emf ½

Qualitative explanation ½

The wire *AB* would oscillate in a simple harmonic way ½

We can write

$$x = -a\cos\omega t$$

as $x = -a$ at $t = 0$

Therefore Instantaneous magnetic Flux ½

$$\phi(t) = Blx \quad (l = AB)$$

Instantaneous induced emf

$$e(t) = -\frac{d\phi}{dt} = aBl\omega\sin\omega t \quad ½$$

The induced emf, therefore varies with time sinusoidally. ½

[CBSE Marking Scheme 2018]

Alternate Answer :

Arm *AB* executes SHM under the influence of restoring force developed in the spring, consequently and induced *emf* is produced across the ends of moving arm *AB* which varies sinusoidally.)

(Given full credit for the above part if the student explains qualitatively without using mathematical equations)

Q. 3. A wire in the form of a tightly wound solenoid is connected to a *dc* source and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease ? Explain. U **[SQP I 2017] [NCERT Exemplar]**

Ans. Lenz's law allows the emf to induce in spirals that opposes and lead to decrease in magnetic flux by increasing the current. This is because when the coil is stretched and air gaps are created between successive elements of the spiral coil, magnetic flux will leak through the gaps. As magnetic flux decreases, current increases to oppose the decrements of flux. **2**

Q. 4. State Lenz's Law. A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends ? Justify your answer. U **[Delhi I, II, III 2013]**

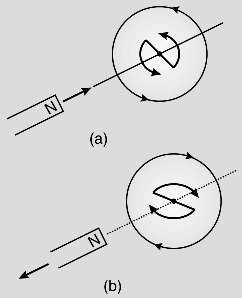
Ans. The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it. **1**

Yes, as the magnetic flux due to vertical component of Earth's magnetic field keeps on changing as the metallic rod falls down. ½ + ½

[CBSE Marking Scheme 2013]

AI **Q. 5. Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it.** A **[Delhi I, II, III 2014]**

Ans.



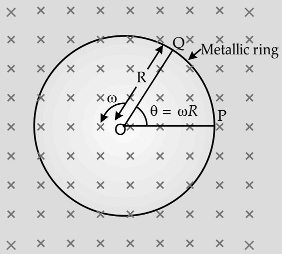
1

When a bar magnet is brought close to the coil (fig a), the approaching North pole of the bar magnet increases the magnetic flux linked to it. This produces an induced emf which produces (or tends to produce, if the coil is open) an induced current in the anticlockwise direction. The face of the coil, facing the approaching magnet, then has the same polarity as that of the approaching pole of the magnet. The induced current, therefore, is seen to oppose the change of magnetic flux that produces it. **1**

[CBSE Marking Scheme 2014]

Q. 6. A metallic rod of length '*L*' is rotated with angular frequency of 'ω' with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius *R*, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field *B* parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring. U **[Delhi I, II, III 2012]**

Ans.



1

The magnitude of the emf, generated across a length dr of the rod, as it moves at right angles to the magnetic field, is given by

$$d\varepsilon = Bvdr$$

Therefore,

$$\varepsilon = \oint d\varepsilon = \int_0^R Bvdr = \int_0^R B\omega r dr = \frac{B\omega R^2}{2}$$ 1

[CBSE Marking Scheme 2012]

Short Answer Type Questions-II

(3 marks each)

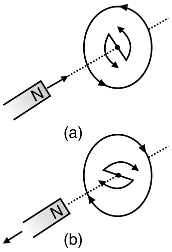
Q. 1. State Lenz's law. Explain, by giving examples that Lenz's law is a consequence of conservation of energy. U **[Delhi Comptt I, II, III 2017]**

Ans. Statement of Lenz's Law 1
Explanation (with example) 2
[CBSE Marking Scheme 2017]

Detailed Answer :

Try yourself similar to Q. 4 of short Answer Type Questions-I 1

Explanation :



1

When the north pole of a bar magnet is pushed towards the close coil, the magnetic flux through coil increases and the current is induced in the coil in such a direction that it opposes the increase in flux. This is possible when the induced current in the coil is in the anticlockwise direction. Opposite will happen when the north pole is moved away from the coil. ½

In either case, it is the work done against the force of magnetic repulsion/attraction that gets 'converted' into the induced emf. ½

Q. 2. A metallic ring of mass m and radius l is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z-component of magnetic field is $B_z = B_0 (1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with a velocity v, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m, B, λ and acceleration due to gravity g. U **[CBSE SQP, 2016]**

Ans. Rate of change of flux $= \frac{d\phi}{dt} = (\pi l^2) B_0 \lambda \frac{dz}{dt}$

$$= IR$$ ½

$$I = (\pi l^2 \lambda) B_0 \frac{v}{R}$$ ½

Energy lost per second $= I^2 R = (\pi l^2 \lambda)^2 B_0^2 \frac{v^2}{R}$ ½

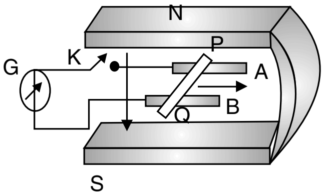
Rate of change in P.E. $= mg \frac{dz}{dt} = mgv$ ½

$$mgv = (\pi l^2 \lambda)^2 B_0^2 \frac{v^2}{R}$$ ½

$$v = \frac{mgR}{(\pi l^2 \lambda)^2 B_0^2}$$ ½

[CBSE Marking Scheme 2016]

AI **Q. 3. Figure shows a metallic rod PQ of length l, resting on the smooth horizontal rails AB positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Assume the magnetic field to be uniform. Given the resistance of the closed loop containing the rod is R.**



(i) Suppose K is open and the rod is moved with a speed v in the direction shown. Find the polarity and magnitude of induced emf.

(ii) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to motion of the rod. Explain.

(iii) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular ?

U **[CBSE SQP 2017-18] [NCERT Exemplar]**

Ans. (i) $|\varepsilon| = Bvl$

P is positive end

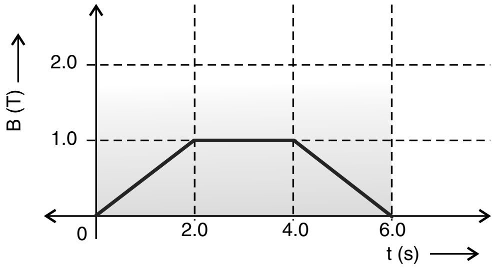
Q is negative end 1

(ii) Magnetic force gets cancelled by electric force that generates due to extra charge of opposite sign at rod ends. 1

(iii) Induced emf is zero as motion of rod not cutting field lines **1**

[CBSE Marking Scheme 2017]

Q. 4. The magnetic field through a single loop of wire, 12 cm in radius and 8.5 Ω resistance, changes with time as shown in the figure. The magnetic field is perpendicular to the plane of the loop. Plot induced current as a function of time.



A **[CBSE SQP 2015]**

Ans.

$$\varepsilon = -\frac{d\phi}{dt}$$

$$= -\pi R^2 \times \frac{dB}{dt}$$

$$= -\frac{22}{7}\times(0.12)^2\times\frac{1}{2}$$

$$\varepsilon = -0.023 \text{ V},$$

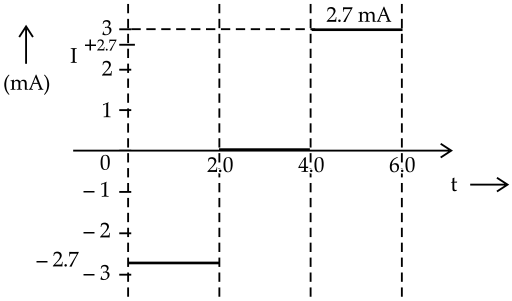
$$I = \frac{\varepsilon}{R}$$

$= -2.7$ mA for $0<t<2$s. **1**

Similarly :

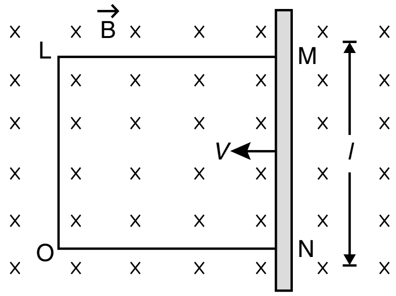
	$0<t<2\,s$	$2<t<4$ s	$4<t<6$ s
ε (V)	–0.023	0	+0.023
I (mA)	–2.7	0	+2.7

1



1

AI Q. 5. A rectangular conductor *LMNO* is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor.



When the arm *MN* of length of 20 cm is moved towards left with a velocity of 10 ms^{-1}, calculate the emf induced in the arm. Given the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance) find the value of the current in the arm. A **[O.D. I, II, III 2013]**

Ans. Let *ON* be at some point *x*.

The emf induced in the loop = ε

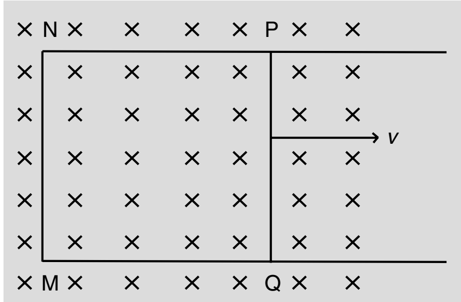
$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d(Blx)}{dt} = Blv \quad \textbf{1}$$

$$= 0.5 \times 0.2 \times 10 = 1 \text{ V} \quad \textbf{1}$$

∴ Current in the arm,

$$I = \frac{\varepsilon}{R} = \frac{1}{5} = 0.2 \text{ A} \quad \textbf{1}$$

Q. 6. A rectangular loop *PQMN* with movable arm *PQ* of length 10 cm and resistance 2 Ω is placed in a uniform magnetic field of 0.1 Tesla perpendicular to the plane of the loop as shown in the figure. The resistances of the arms *MN*, *NP* and *MQ* are negligible. Calculate the (i) emf induced in the arm *PQ* and (ii) current induced in the loop when arm *PQ* is moved with velocity 20 m/s. C



Ans. Try yourself Similar Q. 5 in SAT Question-II

Q. 7. A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the earth's magnetic field. The earth's magnetic field at the place is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased ?

C **[CBSE O.D. I, II, III 2013]**

Ans. ∵ $H = B\cos\theta = 0.4\cos 60°$

$= 0.2\text{ G} = 0.2 \times 10^{-4}\text{ T}$

This component is parallel to the plane of the wheel. Thus, the emf induced is

$$\varepsilon = \frac{1}{2}Hl^2\omega, \quad \text{where, } \omega = 2\pi f$$

$$= \frac{1}{2}Hl^2(2\pi f)$$

∴

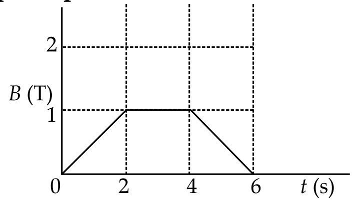
$$\varepsilon = \frac{1}{2}\times 0.2\times 10^{-4}\times(0.5)^2 \times \frac{2\times 3.14\times 120}{60}$$

$\varepsilon = 3.14 \times 10^{-5}$ V **3**

Long Answer Type Questions (5 marks each)

Q. 1. (i) State Faraday's laws of electromagnetic induction.

(ii) The magnetic field through a circular loop of wire 12 cm in radius and 8·5 Ω resistance, changes with time as shown in the figure. The magnetic field is perpendicular to the plane of the loop. Calculate the induced current in the loop and plot it as a function of time.



(iii) Show that Lenz's law is a consequence of conservation of energy. R **[Foreign I, II, III 2017]**

Ans. (i) Faraday's Laws of Electromagnetic Induction :

Faraday's First Law of Electromagnetic Induction states that whenever a conductor is placed in varying magnetic field emf is induced which is known as induced emf. If the conductor circuit is closed current is also induced which is called induced current.

Faraday's Second Law of Electromagnetic Induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is nothing but the product of number of turns in the coil and flux associated with the coil.

$$\varepsilon = \frac{-d\phi_B}{dt}$$

ε_B is magnetic flux through the circuit as $\phi_B = \int \vec{B}.d\vec{A}$

With N loops of similar area in a circuit and ϕ_B being the flux through a loop, then emf is induced in every loop making Faraday law as

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

where, ε = Induced emf [V],
N = number of turns in the coil
$\Delta\phi$ = change in the magnetic flux [Wb],
Δt = change in time [s] 2

- The negative sign means that ε opposes its cause.

(ii) Try yourself similar Q. 4 of Short Answer Type Questions-II 2

(iii) Try yourself similar Q. 5 of Short Answer Type Questions-II 1

[CBSE Marking Scheme 2017]

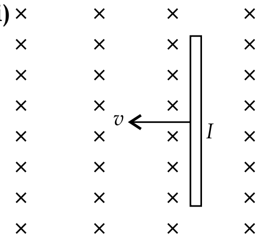
Q. 2. (i) A metallic rod of length *l* is moved perpendicular to its length with velocity *v* in a magnetic field *B* acting perpendicular to the plane in which rod moves. Derive the expression for the induced emf.

(ii) A wheel with 15 metallic spokes each 60 cm long, is rotated at 360 rev/min in a plane normal to the horizontal component of earth's magnetic field. The angle of dip at that place is 60°. If the emf induced between rim of the wheel and the axle is 400 mV, calculate the horizontal component of earth's magnetic field at the place. How will the induced emf change, if the number of spokes is increased ? U **[O.D. Comptt. I, II, III 2017]**

Ans. (i) Derivation of induced emf 2½
(ii) Numerical 2½
[CBSE Marking Scheme 2017]

Detailed Answer :

(i)



½

$$\phi_B = Blx \quad ½$$

$$\varepsilon = \frac{-d\phi_B}{dt} \quad ½$$

$$= -Bl\frac{dx}{dt} \quad ½$$

$$= Blv \quad ½$$

(ii)

$$\omega = 360 \times \frac{2\pi}{60} = 12\pi \text{ rad/s} \quad ½$$

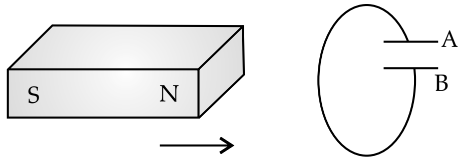
$$\varepsilon = \frac{1}{2}B_H l^2\omega \quad ½$$

$$\therefore \quad 400 \times 10^{-3} = \frac{1}{2}.B_H \times (60\times10^{-2})^2 \times 12\pi \quad ½$$

$$\therefore \quad B_H = \frac{5}{27\pi} = 0.06 \text{ T} \quad ½$$

No change in emf if no. of spokes is increased. ½

AI **Q. 3. (i) State Lenz's law. Use it to predict the polarity of the capacitor in the situation given below :**



R **[O.D. I, II, III 2014]**

(ii) A jet plane is travelling towards west at a speed of 1800 km/h.

(a) Estimate voltage difference developed between the ends of the wing having a span of 25 m if the earth's magnetic field at the location has a magnitude of 5 × 10^{-4} T and dip angle is 30°.

(b) How will the voltage developed be affected if the jet changes its direction from west to north ? **[O.D. I, II, III 2015]**

Ans. **(i)** Try Yourself, Similar Q. 4 of SAT questions-I ½

In the above situation, north pole is approaching the magnet, so induced current in the face of the loop seen from left side will flow in such a way that it will behave like north pole, hence south pole gets developed in the loop as seen from right hand side of the loop. In such case the flow of induced current is clockwise, so A has positive polarity while *B* has negative polarity. ½

(ii) (a) $V = Blv$ ½ + ½

Here, B = vertical component of Earth's magnetic field ½

$$B = (5\times10^{-4} \sin 30°)\text{ T}$$
$$= 2.5\times10^{-4}\text{ T}$$

$$\therefore \quad V = \left[2.5\times10^{-4}\times25\times\frac{1800\times10^{3}}{60\times60}\right]\text{V} \quad ½$$

$$= 3.125\text{ V} \quad ½$$

(b) Now B' = horizontal component of Earth's magnetic field ½

$$B' = B\cos 30° = \frac{B\sqrt{3}}{2} \quad ½$$

$$V' = B'lV \quad ½$$

$$\therefore \quad V' = \sqrt{3}V = 1.732 \times 3.125\text{ volt}$$
$$\cong 5.4\text{ volt} \quad ½$$

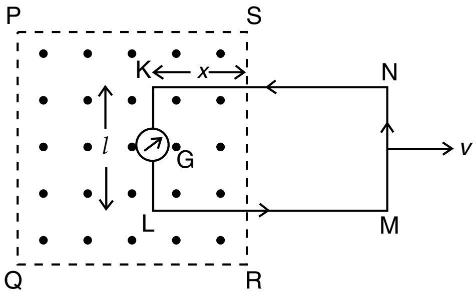
AI **Q. 4. State the law which relates to generation of induced emf in a conductor being moved in a magnetic field.**

Apply this law to obtain an expression for the induced emf when one 'rod' of a rectangular conductor is free to move in a uniform, time independent and 'normal' magnetic field.

Apply the concept of the Lorentz (magnetic) force acting on a moving charge to justify the expression obtained. R A **[CBSE SQP 2014]**

Ans. Faraday's law of electromagnetic induction : It states that the induced emf is equal to the rate of change of magnetic flux.

In region *PQRS* as shown in figure, a uniform magnetic field *B* which is perpendicular to the plane of paper is directed outwards as shown by dotted lines. In this, a rectangular loop of wire *KLMN* is held partially in magnetic field inside the plane of paper with $KL = l$. If x is the portion on the loop inside the field at instant t, then on moving loop with velocity v in plane of paper, the loop area linked with magnetic field changes. With this, an induced emf is set up in the wire which is observed through deflection in galvanometer *G* which is connected in the loop. 1



½

To calculate the emf induced, suppose in a small time Δt, the loop is moved out of magnetic field through a small distance Δx.

∴ Decrease in area of the loop

$$= -l\Delta x \quad ½$$

Decrease in magnetic flux linked with the loop

$$d\phi = -Bl\Delta x$$

As induced emf, $\varepsilon = -d\phi/dt$

$$\therefore \quad \varepsilon = \frac{Bl\Delta x}{\Delta t} = Blv \quad ½$$

i.e., $\boxed{\varepsilon = Blv}$ ½

We see that it is possible to show the motional emf expression using Lorentz force that acts on the free charge carriers of conductor *KL*. If an arbitrary charge q inside conductor *KL*, moves with speed v in magnetic field *B*, then as per Lorentz force, the charge will be qvB in magnitude on. Here all charges will have similar force resulting in magnitude and direction irrespective of their position in rod *KL*.

The work done in moving the charge from *K* to *L* is, $W = qvB \times l$

Since emf is the work done per unit charge,

$$\varepsilon = \frac{W}{q}$$

$$\varepsilon = Blv$$

This equation gives emf induced across the rod *KL*. **2**

Q. 5. A metallic rod of length l and resistance R is rotated with a frequency ν, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere.

(i) Derive the expression for the induced emf and the current in the rod.

(ii) Due to the presence of the current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

(iii) Hence obtain the expression for the power required to rotate the rod.

C **[O.D. Comptt. I, II, III 2014]**

Ans. (i) In one revolution

Change of area, $dA = \pi l^2$

$\therefore$ change of magnetic flux

$d\phi = \vec{B} \cdot \vec{dA} = B.dA\cos 0°$

$= B\pi l^2$ ½

(a) Induced emf, $\varepsilon = B\pi l^2/T = B\pi l^2 \nu$ ½

(b) Induced current in the rod, $I = \dfrac{\varepsilon}{R} = \dfrac{\pi\nu B l^2}{R}$ **1**

(ii) Force acting on the rod, $F = IlB$ ½

$= \dfrac{\pi\nu B^2 l^3}{R}$ ½

The external force required to rotate the rod opposes the Lorentz force acting on the rod/ external force acts in the direction opposite the Lorentz force. ½

(iii) Power required to rotate the rod ½

Power = Force × velocity

$P = F \times v$ ½

$P = \dfrac{\pi\nu B^2 l^3}{R} \times v$ ½

[CBSE Marking Scheme, 2014]

Detailed Answer :

(i) In a revolution, change in Area $= \pi l^2$

Given, Magnetic field $= B$

$\because \quad d\phi = \vec{B} \cdot \vec{dA}$

or $\quad d\phi = B \cdot \pi l^2 \cos 0°$

or $\quad d\phi = B\pi l^2$

Now, induced *emf*

$$\varepsilon = \frac{d\phi}{dt}$$

or $$\varepsilon = \frac{B\pi l^2}{T}$$

and $$\because \frac{1}{T} = \nu$$

$$\varepsilon = B\pi l^2 \nu\delta$$ 1

$$\text{induced current} = \frac{\varepsilon}{R}$$

$$= \frac{B\pi l^2 \nu}{R}$$ 1

(ii) $$F = IlB$$

$$= \frac{B\pi l^2 \nu}{R} lB$$

$$= \frac{B^2 l^3 \pi\nu}{R}$$ 1

According to law, the external force acts in the direction opposite toe lorentz force.

(iii) Power = Force × velocity

$$= \frac{B^2 l^3 \pi\nu}{R} \times v$$

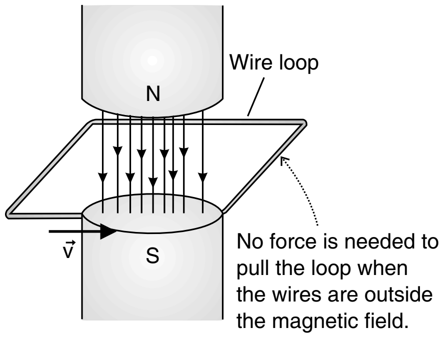
$$= \frac{\pi\nu B^2 l^3 \nu}{R}$$ 2

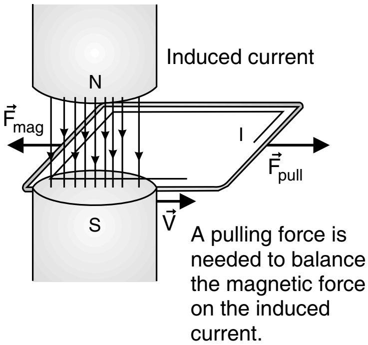
TOPIC-2
Eddy Currents, Self and Mutual Induction and AC Generator

Revision Notes

Eddy Currents

- Current loops induced in moving conductors are called eddy currents. They can create significant drag, called as magnetic damping.
- Eddy currents give rise to magnetic fields that oppose any external change in the magnetic field.
- Mathematically, $i = \dfrac{e}{R}$
- Eddy currents are induced electric currents that flow in a circular path





- Eddy currents flowing in a material will generate their own secondary magnetic field that opposes the coil's primary magnetic field.

Mutual Induction

- The production of induced emf in a circuit, when the current in the neighbouring circuit changes is called **mutual induction.**
  When the circuit of the primary coil is closed or opened, deflection is produced in the galvanometer of the secondary coil. This is due to the mutual induction.
- The mutual induction between two coils depends on the following factors :
  - The number of turns of primary and secondary coils.
  - The shape, size or geometry of the two coils. *i.e.*, the area of cross-section and the length of the coils.

Coefficient of mutual induction :

- Suppose, the instantaneous current in the primary coil is I. Let the magnetic flux linked with the secondary coil be ϕ. It is found that the magnetic flux is proportional to the current. *i.e.*,

$$\phi \propto I \quad \text{or} \quad \phi = MI \qquad \text{...(i)}$$

where, M is the constant of proportionality. It is called coefficient of mutual induction.
The induced emf ε in the secondary coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt} \qquad \text{...(ii)}$$

The negative sign is in accordance with the Lenz's law *i.e., the induced emf in the secondary coil opposes the variation of current in the primary coil.*
From the equation (ii), we find

$$M = \frac{\varepsilon}{(dI/dt)}$$

Therefore, Unit of $M = \frac{\text{V}}{\text{As}^{-1}} = \text{VA}^{-1}\text{s}$

If n_1, n_2 be the number of turns in primary and secondary coils permit length and r be their radius, then coefficient of mutual inductance is given as

$$M = \pi_0 x_1 x_2 \pi r^2 l$$

Self-Induction :

- The production of induced emf in a circuit, when the current in the same circuit changes is known as **self-induction**.
  Suppose the instantaneous current in the circuit is I and if the magnetic flux linked with the solenoid is ϕ, then it is found that :

$$\phi \propto \text{I} \quad \text{or} \quad \phi = LI \qquad \text{...(i)}$$

where, L is the constant of proportionality. It is called **coefficient of self-induction.**
The induced emf ε in the coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \qquad \text{...(ii)}$$

The negative sign is in accordance with the Lenz's law *i.e., the induced emf opposes the variation of current in the coil.*
From the equation (ii), we find :

$$L = \varepsilon / (dI/dt) \qquad \text{...(iii)}$$

Then, the coefficient of self-induction is the ratio of induced emf in the circuit to the rate of change of the current in the circuit.
Unit of L : The unit of self-induction is also called henry (symbol H).
From equation (ii), we find that if $dI/dt = 1\ \text{AS}^{-1}$ and $\varepsilon = 1$ V,
then $L = 1\ \text{H} \Rightarrow$ Unit of $H = \text{VA}^{-1}\text{s}$

- If a rod of length l moves perpendicular to a magnetic field B with a velocity v, then the induced emf produced across it, is given by

$$\varepsilon = vBl$$

In general, we have, $\varepsilon = \vec{B}.(\vec{v} \times \vec{l})$

- If a metallic rod of length l rotates about one of its ends in a plane perpendicular to the magnetic field, then the induced emf produced across its ends is given by

$$\varepsilon = \frac{B\omega l^2}{2} = \frac{B2\pi f l^2}{2} = BAf$$

Here, ω = angular velocity of rotation, $A = \pi l^2$ = area of circle and f = frequency of rotation.

- Inductance in the electrical circuit is equivalent to the inertia (mass) in mechanics.
- When a bar magnet is dropped into a coil, the electromagnetic induction in the coil opposes its motion, so the magnet falls with acceleration less than that due to gravity.
- The inductance of a coil depends on the following factors :
  - area of cross-section,
  - number of turns
  - permeability of the core.

- Unit of induction, $H = \frac{Wb}{A} = \frac{Vs}{A} = \Omega.s$
- The inductance of a circular coil is given by :

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \frac{\mu}{4\pi}.\frac{(2\pi NI)}{rI} \times AN \qquad \left[\because B = \frac{\mu}{4\pi}.\frac{2\pi NI}{r}\right]$$

$$L = \frac{\mu N^2}{2r}A = \frac{\mu N^2}{2r} \times \pi r^2$$

or

$$L = \frac{\mu N^2 \pi r}{2}$$

- The inductance of a solenoid of length l is given by

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \left(\frac{\mu NI}{l}\right)\frac{AN}{I} \qquad \left[\because B = \frac{\mu NI}{l}\right]$$

or

$$L = \frac{\mu N^2 A}{l} = \mu n^2 Al = \mu n^2 V \qquad \left[\because n = \frac{N}{l}\right]$$

Here, $n = N/l$ = number of turns per unit length and $V = Al$ = volume of the solenoid.

- If two coils of inductance L_1 and L_2 are coupled together, then their mutual inductance is given by

$$M = k\sqrt{L_1 L_2}$$

where, k is called the coupling constant.

- The value of k lies between 0 and 1.
  For perfectly coupled coils, $k = 1$, it means that the magnetic flux of primary coil is completely linked with the secondary coil.
- Eddy currents do not cause sparking.
- If a current I is set up in a coil of inductance L, then the magnetic field energy stored in it is given by

$$U_m = \frac{1}{2}LI^2$$

AC Generator

- It converts mechanical energy into electrical energy.
- It is based on the principle of mutual induction. It has mainly three components :
  - **Rotator Coil :** It can rotate about an axis on a shaft.
  - **Stator Coil :** It provides magnetic field.
  - **Commutator :** It is pair of slip rings and carbon brushes. It will facilitate flow of current between moving coil and stationary circuit.

$$\phi_B = B.A\cos\theta \text{ or } BA\cos\omega t \qquad (\theta = \omega t)$$

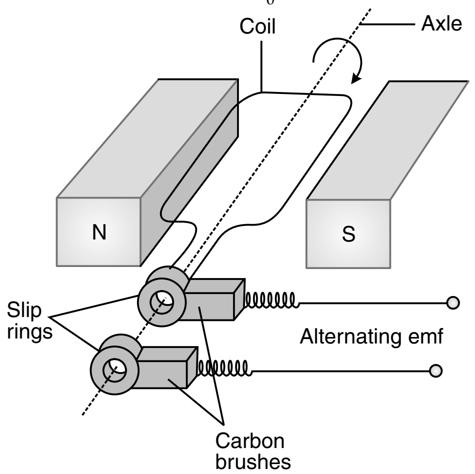
$$\varepsilon = -N\frac{d\phi_B}{dt}$$

$$\varepsilon = -N\frac{d(BA\cos\omega t)}{dt}$$

$$\varepsilon = NBA\omega \sin\omega t$$

Maximum value of emf, $\varepsilon_0 = NBA\,\omega$ (when $\sin\omega t = 1$)

$\therefore$ $\varepsilon = \varepsilon_0 \sin\omega t$



Know the Terms

- **Back emf :** emf generated by a running motor due to coil that turns in a magnetic field which opposes the voltage that powers the motor.
- **Inductor :** It is a device used to store electrical energy in a from of magnetic field when electric current flows.
- emf produced by an electric generator : $\varepsilon = NBA\omega\sin(\omega t)$

Know the Formulae

- For Self Inductor $\varepsilon = \frac{d\phi}{dt} = -L\frac{dI}{dt}$
- For Mutual Inductor $\varepsilon = \frac{d\phi}{dt} = -M\frac{dI}{dt}$
- The inductance in series is given by $L_s = L_1 + L_2 + L_3 + ...$
- The inductance in parallel is given by $\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ...$
- Mutual Inductance of two coils is given by

$$M = \frac{\mu_0\mu_r N_P N_S A_P}{I_P} = \frac{\mu_0\mu_r N_P N_S A_S}{I_P}$$

where, μ_0 is the permeability of fee space ($4\pi \times 10^{-7}$)
μ_r is the relative permeability of the soft iron core
N_S is number of turns is secondary coil.
N_P is number of turns is primary coil.
A_P is the cross-sectional area of primary coil in m^2.
A_S is the cross-sectional area of secondary will in m^2.
I is the coil current.

- For *A.C.* Generator $\varepsilon = \varepsilon_0\sin\omega t$ or $\varepsilon = \varepsilon_0 \sin 2\pi\nu t$

Objective Type Question (1 mark)

Q. 1. The self-inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as
(a) l and A increase.
(b) l decreases and A increases.
(c) l increases and A decreases.
(d) both l and A decrease. [NCERT Exemplar]

Ans. Correct option : (b)
Explanation : As we know that,

$$L = \mu_r\mu_0\frac{N^2A}{l}$$

As L is constant for a coil,

$$L \propto A \text{ and } L \propto \frac{1}{l}$$

As μ_r and N are constant here so, to increase L for a coil, area A must be increased and l must be decreased.

Important point : The self and mutual inductance of capacitance and resistance depend on the geometry of the devices as well as permittivity/permeability of the medium.

Very Short Answer Type Questions (1 mark each)

Q. 1. An iron-cored solenoid has self-inductance 2.8 H. When the core is removed, the self inductance becomes 2 mH. What is the relative permeability of the core used ? U [Delhi Comptt I, II, III 2017]

Ans. Relative permeability

$$\mu_r = \frac{L}{L_0}$$

$$\mu_r = \frac{2{\cdot}8}{2{\cdot}0\times10^{-3}} \quad ½$$

$$= 1400 \quad ½$$

[CBSE Marking Scheme 2017]

AI **Q. 2. A metallic piece gets hot when surrounded by a coil carrying high frequency alternating current. Why ? C [Delhi Comptt. I, II, III 2014]**

Ans. Due to the heating effect of eddy currents set up in the metallic piece. **[CBSE Marking Scheme 2014] 1**

Short Answer Type Questions-I (2 marks each)

AI **Q. 1. A light bulb and a solenoid are connected in series across an *ac* source of voltage. Explain, how the glow of the light bulb will be affected when an iron rod is inserted in the solenoid.**

U **[Foreign I, II, III 2017]**

Ans. Effect on brightness ½

Explanation ½

Brightness decreases ½

Explanation : Self inductance of solenoid increases; this increases the impedance of the circuit and hence current decreases. ½

(Even student just writes self inductance increases, award this 1 mark.)

[CBSE Marking Scheme 2017]

Detailed Answer :

(i) Brightness will decrease when an iron rod is inserted in the solenoid.

(ii) When an iron rod is inserted in the solenoid with a velocity, the iron rod will cut the magnetic field lines of an inductor, so as per Kirchhoff's and Faraday's Laws, cutting of magnetic field will tend to induce a current inside the inductor which opposes the direction of it's cause. So when the current is being induced by the moving rod, it opposes the flow of existing current in the circuit, causing the bulb's brightness to go down as there is less current passing through it.

AI **Q. 2. Define self-inductance of a coil. Show that magnetic energy required to build up the current *I* in a coil of self-inductance *L* is given by $\frac{1}{2}LI^2$.**

R **[Delhi I, II, III 2014]**

Ans. Self-Inductance is the property by which an opposing induced emf is produced in a coil due to a change in current, or magnetic flux, linked with the coil.

OR

Self-inductance of a coil is numerically equal to the flux linked with the coil when the current through the coil is 1 A.

OR

Self-inductance of a coil is equal to the induced emf developed in the coil when the rate of change of current in the coil is one ampere per second. **1**

Energy stored in an inductor :

Consider a source of emf connected to an inductor *L*. As the current starts growing, the opposing induced emf is given by

$$\varepsilon = -L\frac{di}{dt} \quad ½$$

If the source of emf flows a current *i* through the inductor for a small time *dt*, then the amount of work done by the source, is given by

$$dW = |\varepsilon|\,i\,dt$$

$$= Li\frac{di}{dt}\,dt$$

$$= Li\,di$$

Hence the total amount of work done (by the source of emf) when the current increases from its initial value ($i = 0$) to its final value (I) is given by

$$W = \int_0^I Li\,di = L\int_0^I i\,di = L\left[\frac{i^2}{2}\right]_0^I = \frac{1}{2}LI^2 \quad ½$$

This work done gets stored in the inductor in the form of magnetic energy.

$$\therefore \quad U = \frac{1}{2}LI^2$$

[CBSE Marking Scheme 2014]

Q. 3. Starting from the expression for the energy $W = \frac{1}{2}LI^2$, stored in a solenoid of self-inductance *L* to build up the current *I*, obtain the expression for the magnetic energy in terms of the magnetic field *B*, area *A* and length *l* of the solenoid having *n* number of turns per unit length. Hence show that the energy density is given by $B^2/2\pi_0$.

A **[Delhi Comptt. I, II, III 2013]**

Ans. Given :

Energy $W = \frac{1}{2}LI^2$

A solenoid having magnetic field *B*, area *A*, & length *l* and having *n* numbers of turns per unit length. ½

Self-inductance of the solenoid is given by :

$$L = \pi_0 n^2 lA$$

$$\therefore \quad W = \frac{1}{2}\pi_0 n^2 lAI^2 \quad \text{...(i)}$$

But, $B = \mu_0 nI$

$$\therefore \quad B^2 = \mu_0^2 n^2 I^2 \quad \text{...(ii)}$$

$$V = Al \text{ (volume)} \quad \text{...(iii)} ½$$

From equation (i), (ii), (iii), we have

$$\Rightarrow \quad W = \frac{1}{2\mu_0}B^2V$$

$$\text{Energy density} = \frac{W}{V} = \frac{B^2}{2\mu_0} \quad \mathbf{1}$$

Short Answer Type Questions-II (3 marks each)

AI Q. 1. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. R **[O.D. I, II, III 2017, 2014]**

Ans. Definition of mutual inductance **1**

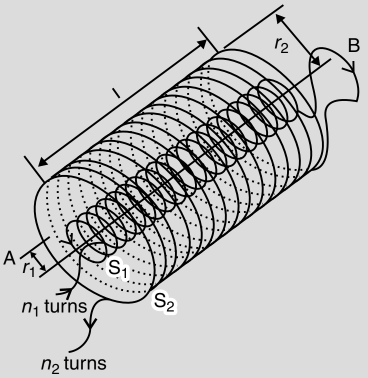
Derivation of mutual inductance for two long solenoids **2**

(i) Mutual inductance, is numerically equal to the induced emf in the secondary coil when the current in the primary coil changes by unity.

Alternatively,

Mutual inductance in numerically equal to the magnetic flux linked with one coil/secondary coil when unit current flows through the other coil/ primary coil. **1**

(ii)



½

Let a current I_2, flow in the secondary coil

$$\therefore \quad B_2 = \frac{\mu_0 N_2 i_2}{l} \qquad ½$$

$\therefore$ Flux linked with the primary coil

$$= \frac{\mu_0 N_2 N_1 A_1 i_2}{l}$$

$$= M_{12} i_2 \qquad ½$$

Hence,
$$M_{12} = \frac{\mu_0 N_2 N_1 A_2}{l}$$

$$= \mu_0 n_2 n_1 A_1 l$$

$$\left(n_1 = \frac{N_1}{l}; n_2 = \frac{N_2}{l}\right) \qquad ½$$

[CBSE Marking Scheme 2017]

Q. 2. Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor *L* connected across a source of emf. R **[O.D. I,II,III 2017]**

Ans. Try yourself, Similar to Q. 2, Short Answer Type Questions-I **3**

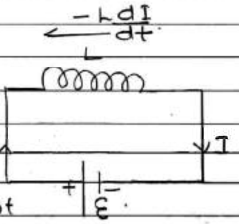
[CBSE Marking Scheme 2017]

OR

Self inductance of a coil

When a

$\varepsilon = -L\frac{dI}{dt}$



Self inductance of a coil or coefficient of self inductance is defined as the emf induced across a coil when the current in the coil is changing at the rate of 1 A/s.

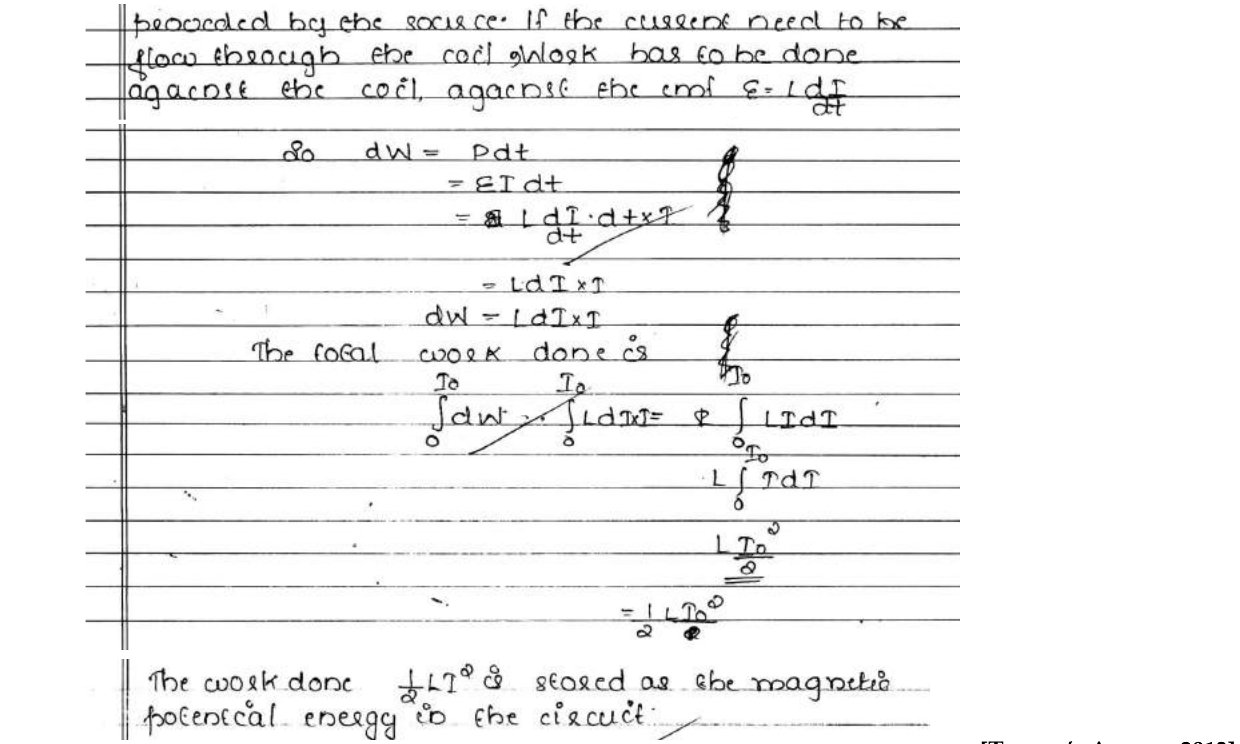
i.e. $|\varepsilon| = L\frac{dI}{dt}$

$L = \frac{|\varepsilon|}{dI/dt}$ when $\frac{dI}{dt} = 1$ A/s

$L = \varepsilon$

S.I unit is Henry.

Consider the coil of inductance L. A back emf $|\varepsilon| = L\frac{dI}{dt}$ is set up in the coil against the current


provided by the source. If the current need to be flow through the coil work has to be done against the coil, against the emf $\varepsilon = L\frac{dI}{dt}$

So $dW = P\,dt$
$= \varepsilon I\,dt$
$= L\frac{dI}{dt}\cdot dt \times I$
$= L\,dI \times I$
$dW = L\,dI \times I$
The total work done is
$\int_0^{I_0} dW = \int_0^{I_0} L\,dI\,I = \int_0^{I_0} LI\,dI$
$L\int_0^{I_0} I\,dI$
$\frac{LI_0^2}{2}$
$= \frac{1}{2}LI_0^2$

The work done $\frac{1}{2}LI^2$ is stored as the magnetic potential energy in the circuit.

[Topper's Answer 2013]

Q. 3. (i) Define the term 'self-inductance' and write its S.I. unit.

(ii) Obtain the expression for the mutual inductance of two long co-axial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2 and n_1 and n_2, number of turns per unit length, when a current I is set up in the outer solenoid S_2.
R [Delhi I, II, III 2017]

Ans. (i) Try yourself, Similar to Q. 2 (a), Short Answer Type Questions-I.
The S.I. unit of self-inductance is Henry (H). **1½**

(ii) Try yourself, Similar to Q. 1 (ii), Short Answer Type Questions-II **1½**

Q. 4. The current through two inductors of self-inductance 15 mH and 25 mH is increasing with time at the same rate. Draw graphs showing the variation of the

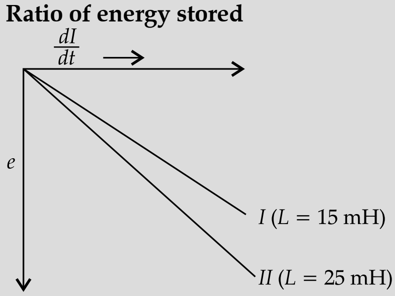
(i) emf induced with the rate of change of current

(ii) energy stored in each inductor with the current flowing through it.

Compare the energy stored in the coils, if the powers dissipated in the coils are same.
U [O.D. Comptt. II 2017]

Ans. Graph of emf ½
Graph of energy stored ½
Ratio of energy stored 2

(i)



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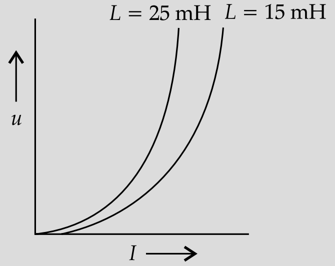
(ii)



$$\frac{U_1}{U_2} = \frac{\frac{1}{2}L_1 i_1^2}{\frac{1}{2}L_2 i_2^2}$$ ½

But $\varepsilon_1 i_1 = \varepsilon_2 i_2$ ½
($\because$ power dissipated is same)

$$\therefore \quad \frac{i_1}{i_2} = \frac{\varepsilon_2}{\varepsilon_2} = \frac{L_2}{L_1}$$ ½

$$\left(\because \frac{dI}{dt} \text{ is same and } \varepsilon = -L\frac{dI}{dt}\right)$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{L_1}{L_2}\left(\frac{L_2}{L_1}\right)^2$$

$$\frac{L_2}{L_1} = \frac{25}{15} = 1{\cdot}67$$ ½

[CBSE Marking Scheme 2017]

AI Q. 5. The current through two inductors of self-inductance 12 mH and 30 mH is increasing with time at the same rate. Draw graphs showing the variation of the

(i) emf induced with the rate of change of current in each inductor

(ii) energy stored in each inductor with the current flowing through it.
Compare the energy stored in the coils, if the powers dissipated in the coils are same.
R [O.D. Comptt. I, III 2017]

Ans. Try yourself, Similar to Q. 4 Short Answer Type Questions-II. **3**

Q. 6. (i) Define mutual inductance.
(ii) A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil ? R A [Delhi I, II, III 2016]

Ans. (i) Magnetic flux, linked with the secondary coil due to the unit current flowing in the primary coil,

$$\phi_2 = MI_1$$ **1**

[Alternatively,
Induced emf associated with the secondary coil, for a unit rate of change of current in the primary coil.

$$e_2 = -M\frac{dI_1}{dt}]$$ **1**

[Also accept the definition of Mutual Induction, as per the Hindi translation of the questions]
[*i.e.*, the phenomenon of production of induced emf in one coil due to change in current in neighbouring coil]

(ii) Change of flux linkage

$$d\phi = MdI$$ **1**
$$= 1.5 \times (20 - 0) \text{ W}$$
$$= 30 \text{ weber}$$

[CBSE Marking Scheme, 2016]

Detailed Answer :

(i) Try yourself, Similar to Q. 1 (a), Short Answer Type Questions-II

(ii) Given :
Mutual inductance of a pair of coils, $\mu = 1.5$ H
Initial current, $I_1 = 0$ A
Final current $I_2 = 20$ A
Change in current will be :

$$\Delta I = I_2 - I_1$$
$$20 - 0 = 20\,A$$

and we know,

$$\Delta\phi = M\Delta I,$$

where,
$\Delta\phi$ is change in magnetic flux

$$\Delta\phi = 1{\cdot}5 \times 20$$
$$= 30 \text{ Wb}$$

Hence, change in the flux linkage will be 30 Wb.

AI Q. 7. Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance *L* to build up a current *I* through it. R [O.D. I, II, III 2014]

Ans. Try yourself, Similar to Q. 2, Short Answer Type Questions-I **3**

Q. 8. (i) Define self-inductance of a coil and hence write the definition of 'Henry'.
(ii) Write any two factors each on which the following depends :
(a) Self-inductance of a coil.
(b) Mutual inductance of a pair of coils.
R U [Foreign 2016]

Ans. (i) The self inductance, *L*, of a coil equals the magnetic flux linked with it, when a unit current flows through it. ½

One henry is the self inductance of a coil which the magnetic flux, linked with it, due to a current of 1 A, flowing in it, equals one weber. ½

[**NOTE :** Also accept these two definitions based on $\varepsilon = -L\frac{dI}{dt}$]

(ii) (a) Self inductance of a coil depends on ½ + ½
(a) Its geometry (area and length of a coil)
(b) Number of turns
(c) Number of turns in each coil.
(d) Nature of medium in the intervening space.
(Any two)

(b) Mutual inductance of a given pair of coils depends on ½ + ½
(a) Their geometries
(b) Their distance of separation
(c) Number of turns in each coil.
(d) Nature of medium in the intervening space.
(Any two)

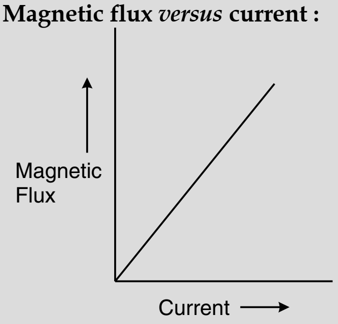
[CBSE Marking Scheme 2016]

Q. 9. Define the term 'mutual inductance' between the two coils. Obtain the expression for mutual inductance of a pair of long co-axial solenoids each of length *l* and radii r_1 and r_2 ($r_2 >> r_1$). Total number of turns in the two solenoids are N_1 and N_2 respectively. U [O.D. I, II, III 2014]

Ans. Try yourself, Similar to Q. 1, Short Answer Type Questions-II

Q. 10. The current flowing through an inductor of self-inductance *L* is continuously increasing. Plot a graph showing the variation of
(i) Magnetic flux *versus* the current
(ii) Induced emf *versus* *dI*/*dt*
(iii) Magnetic potential energy stored *versus* the current. U [Delhi I, II, III 2014]

Ans. (i) Magnetic flux *versus* current :



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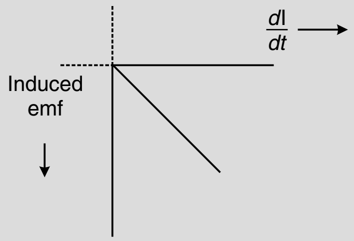
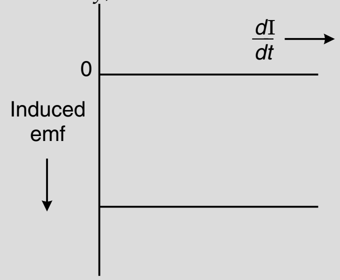
(ii) Induced emf *versus* $\frac{dI}{dt}$

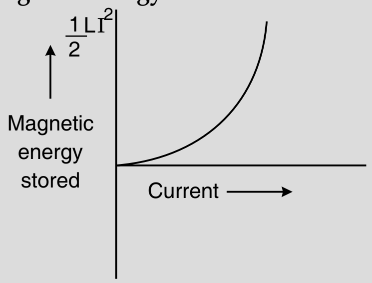


Alternatively,



When I is increasing at constant rate. 1

(iii) Magnetic energy stored *versus* the current :



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[CBSE Marking Scheme 2014]

Q. 11. Derive the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid carrying a steady current I. How does this magnetic energy per unit volume compare with the electrostatic energy density stored in a parallel plate capacitor ? U

Ans. Rate of work done,

$$\frac{dW}{dt} = |\varepsilon| i$$

$$dW = \left(LI\frac{dI}{dt}\right)dt \quad ½$$

$$dW = LI\,dI$$

Total amount of work done,

$$\int dW = \int LIdI$$

$$W = \frac{1}{2}LI^2 \quad ½$$

For the solenoid :

Inductance, $L = \mu_0 n^2 Al$; also $B = \mu_0 nI$ ½

$$\therefore \quad W = U_B = \frac{1}{2}LI^2 \quad ½$$

$$= \frac{1}{2}(\mu_0 n^2 Al)\left(\frac{B}{\mu_0 n}\right)^2$$

$$= \frac{B^2 Al}{2\mu_0} \quad ½$$

Magnetic energy per unit volume $= \frac{B^2}{2\mu_0}$

Also, Electrostatic Energy Stored Per Unit Volume

$$= \frac{1}{2}\varepsilon_0 E^2 \quad ½$$

Q. 12. The currents flowing in the two coils of self-inductance L_1 = 16 mH and L_2 = 12 mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of (i) induced voltages, (ii) the currents (iii) the energies stored in the two coils at a given instant.

A **[Foreign 2014]**

Ans. (i) Induced emf (voltage) in a coil,

$$\varepsilon = -L\frac{dI}{dt} \quad ½$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{L_1\frac{dI}{dt}}{L_2\frac{dI}{dt}} = \frac{L_1}{L_2} = \frac{4}{3} \quad ½$$

(ii) Power supplied $P = \varepsilon I$ ½

As power is same for both coils

$$\varepsilon_1 I_1 = \varepsilon_2 I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{3}{4} \quad ½$$

(iii) Energy stored in a coil,

$$U = \frac{1}{2}LI^2 \quad ½$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2}L_1 I_1^2}{\frac{1}{2}L_2 I_2^2} = \frac{L_1 I_1^2}{L_2 I_2^2} = \frac{3}{4} \quad ½$$

[CBSE Marking Scheme 2014]

Long Answer Type Questions

(5 marks each)

AI **Q. 1. (a) State the principle of an *ac* generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A, rotating with a constant angular speed 'ω' in a magnetic field $\vec{B}$, directed perpendicular to the axis of rotation.**

(b) An aeroplane is flying horizontally from west to east with a velocity of 900 km/hour. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of the Earth's magnetic field is 5 × 10^{-4} T and the angle of dip is 30°. U **[CBSE SQP 2018]**

Ans. (a) Principle of *ac* generator ½

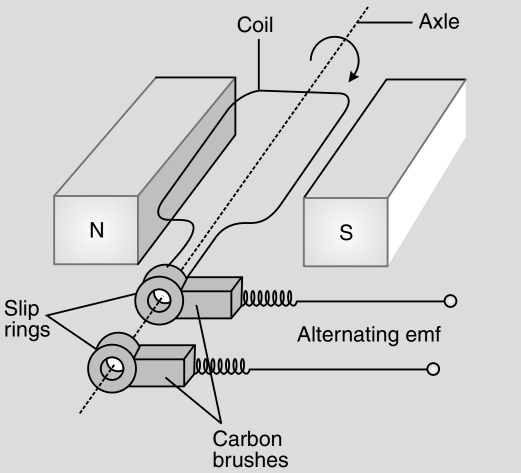
working ½

mark Labelled diagram 1

Derivation of the expression for induced emf 1½

(b) Calculation of potential difference 1½

(a)



The *AC* Generator works on the principle of electromagnetic induction. ½

When the magnetic flux through a coil changes, an emf is induced in it. ½

As the coil rotates in magnetic field the effective area of the loop, (*i.e.*, A cos θ) exposed to the magnetic field keeps on changing, hence magnetic flux changes and an emf is induced. ½

∴ The induced emf, $e = -N\dfrac{d\phi}{dt}$ ½

$$= -NBA\frac{d}{dt}(\cos \omega t)$$

$$e = NBA\omega\sin \omega t$$ ½

(b) Potential difference developed between the ends of the wings $e = Blv$ ½

Given : Velocity, v = 900 km/hour = 250 m/s

Wing span, l = 20 m

Vertical component of Earth's magnetic field

$B_v = B_H\tan\delta$

$= 5\times10^{-4}$ (tan 30°) tesla ½

∴ Potential difference,

$$\varepsilon = 5\times10^{-4}(\tan 30°)\times20\times250$$

$$= \frac{5\times20\times250\times10^{-4}}{\sqrt{3}}$$

= 1.44 volt ½

[CBSE Marking Scheme 2018]

Answering Tips

- While answering the question related to ac generator always remember to make a proper and labelled diagram following with explanation of principle, construction and working of generator.

Detailed Answer :

(a) A.C. generator :

A.C. generator is a device which converts mechanical energy into electric energy.

Principle : It works on the principle of electromagnetic induction. ½

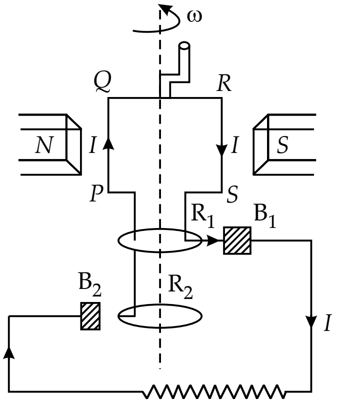
Working : It consists of

(i) Armature coil of large number of turns of copper wire wound over soft iron core soft iron core is used to increase magnetic flux.

(ii) Field magnets are used to apply magnetic field in which armature coil is rotated with its axis perpendicular to field lines.

(iii) Slip rings used to provide movable contacts of armature coil with external circuit containing load.

(iv) Brushes are the metallic pieces used to pass an electric current from armature coil to the external circuit containing load. ½



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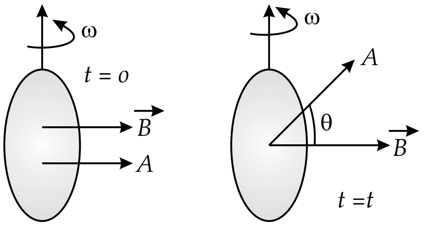
Theory :

Consider a coil *PQRS* free to rotate in a uniform magnetic field $\vec{B}$. The initial flux through the coil is maximum *i.e.*, $\phi = BA$ but as the coil is rotating with angular velocity ω, at any instant 't' the flux is given by

$$\phi = BA\cos\theta = BA\cos\omega t$$



½

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by,

$$\varepsilon = \frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\ \omega t) = \omega BA\sin\ \omega t$$ ½

If the coil has *N* turns, then the total induced emf will be

$$\varepsilon = NBA\omega\sin t.$$ ½

Thus, the induced emf varies sinusoidally with time '*t*'. The value of induced emf is maximum when $\sin\omega t = 1$ or $\sin\omega t = 90°$, *i.e.*, when the plane of the coil parallel to the field $\vec{B}$. Denoting this maximum value by ε_0, we have

$$\varepsilon_0 = NBA\omega$$

$\varepsilon = \varepsilon_0\sin\omega t = \sin 2\pi ft$ where *f* is the frequency of rotation of the coil.

We consider the following special cases :

(i) When $\omega t = 0°$, the plane of the coil is perpendicular to *B*, $\sin\omega t = \sin 0°$ so that $\varepsilon = 0$

(ii) When $\omega t = \pi/2$, the plane of coil is parallel to field *B*, $\sin\omega t = \sin\pi/2 = 1$, so that $\varepsilon = \varepsilon_0$

(iii) When $\omega t = \pi$, the plane of the coil is again perpendicular to B, $\sin \omega t = \sin \pi = 0$ so that $\varepsilon = 0$

(iv) When $\omega t = \frac{3\pi}{2}$, the plane of the coil is again parallel to B, $\sin \omega t - \sin \frac{3\pi}{2} = -1$ so that $\varepsilon = -\varepsilon_0$

(v) When $\omega = 2\pi$, the plane of the coil again becomes perpendicular to B after completing one rotation, $\sin \omega t = \sin 2\pi = 0$ so that $\varepsilon = 0$.

As the coil continues to rotate in the same sense the same cycle of changes repeats again and again. Such an emf is called sinusoidal or alternating emf. Both the magnitude and direction of this emf changes regularly with time.

- The fact that an induced emf is set up in a coil when rotated a magnetic field forms the basic principle of a dynamo or a generator.
- The electric current produced varies sinusoidally with time, so is known as 'alternating current' and hence the generator is known as 'A.C. generator'.

(b) As

$$\frac{B_V}{B_H} = \tan \delta$$

$$B_V = B_H \tan \delta \qquad \tfrac{1}{2}$$

$$= 5 \times 10^{-4} \tan 30°$$

$$= \frac{5 \times 10^{-4}}{\sqrt{3}} \qquad \tfrac{1}{2}$$

$$e = Blv$$

$$= \frac{5 \times 10^{-4}}{\sqrt{3}} \times 20 \times 250$$

$$= 1.44 \text{ V} \qquad \tfrac{1}{2}$$

Q. 2. Define 'self-inductance' of a coil. Obtain an expression for self inductance of a long solenoid of cross sectional area A, length L having n turns for unit length. Prove that self inductance is the analogue of mass in mechanics. R **[SQP II 2017]**

Ans. Try yourself, **Definition :** Similar to Q. 2, Short Answer Type Questions-I **5**

[CBSE Marking Scheme 2017]

Expression :

Consider a long solenoid of length l, area of cross-section A with N number of closely wound turns. If I is the amount of current flowing through the solenoid, then magnetic field B inside the solenoid will be :

$$B = \frac{\mu_0 NI}{l} \qquad \tfrac{1}{2}$$

Now magnetic flux through each turn of solenoid is :

$$\phi = BA = \frac{\mu_0 N^2 AI}{l} \qquad \tfrac{1}{2}$$

Since $\phi = LI$

$$LI = \frac{\mu_0 N^2 AI}{l} \qquad \tfrac{1}{2}$$

Hence, $L = \frac{\mu_0 N^2 A}{l}$ ½

If a coil of wire with few turns around metal core carries a charge is passed through. The current will create a magnetic field that runs through the center of the coil pointing downward. If the current is stopped suddenly, then the magnitude of magnetic field tends to be zero.

It is known that changing magnetic fields will influence the charges in loops of wire. If magnitude of magnetic field in the coil approaches to zero, it induces a voltage in the coil which creates the magnetic field. So as per Lenz's law, voltage induced by changing magnetic field gives rise to a current which counteract the changes.

Mathematically, voltage across the inductor, the loop of coil, is

$$V_{\text{Ind}} = L\frac{dI}{dt}, \qquad 1$$

The above expression looks similar to expression of force :

$$F = m\frac{dv}{dt}$$

Now,

$$E_{\text{Ind}} = \frac{1}{2}LI^2 \qquad 1$$

$$E_{\text{Kinetic}} = \frac{1}{2}mv^2$$

From the above description, there appears an analogy between mechanical motion and flow of electricity. Here, self-inductance L and mass m, both provide inertia that will resists in changing the current I or velocity v suddenly. **1**

AI **Q. 3. Define mutual inductance of a coil. On what factors it depends ?** R **[SQP II 2017]**
Two concentric circular coils, one of small radius r_1 and the other of large radius r_2, such that $r_1 << r_2$, are placed co-axially with centers coinciding. Obtain the mutual inductance of the arrangement.

Ans. Try yourself, **Definition :** Similar to Q. 2, Short Answer Type Questions-II. **2**

Try yourself, **Factors :** Similar to Q. 8 (ii) b, Short Answer Type Questions-II. **1**

Expression :

If a current I_2 flows through the outer circular coil of radius r_2, the magnetic field at the centre of the coil will be,

$$B_2 = \frac{\mu_0 I_2}{2r_2} \qquad \tfrac{1}{2}$$

As $r_1 << r_2$, so field B_2 will be considered to be constant over the entire cross-sectional area of inner coil with radius r_1. Hence, magnetic flux linked with the smaller coil will be :

$$\phi_1 = B_2 A_1$$

$$= \frac{\mu_0 I_2}{2r_2} \times \pi r_1^2 \qquad \tfrac{1}{2}$$

From the definition :

$$\phi_1 = M_{12} I_2$$

Hence, mutual inductance will be,

$$M_{12} = \frac{\phi_1}{I_2} = \frac{\mu_0 \pi r_1^2}{2r_2} \qquad \tfrac{1}{2}$$

But $M_{12} = M_{21} = M$

$\therefore$ Mutual inductance, $M = \oint E.dl = -\frac{d\phi}{dt}$ ½

Q. 4. (i) Explain the meaning of the term mutual inductance. Consider two concentric circular coils, one of radius r_1 and the other of radius r_2 ($r_1 < r_2$) placed co-axially with centres coinciding with each other. Obtain the expression for the mutual inductance of the arrangement.

(ii) A rectangular coil of area A, having number of turns N is rotated at 'f' revolutions per second in a uniform magnetic field B, the field being perpendicular to the coil. Prove that the maximum emf induced in the coil is $2\pi fNBA$.

U [O.D. I, II, III 2016]

Ans. (i) Mutual Inductance is the property of a pair of coils due to which an emf induced in one of the coils due to the change in the current in the other coil. ½

Mathematically, $\varepsilon_2 = \frac{M di_1}{dt}$ ½

$\therefore \quad M = -\frac{e_2}{di_1/dt}$ ½

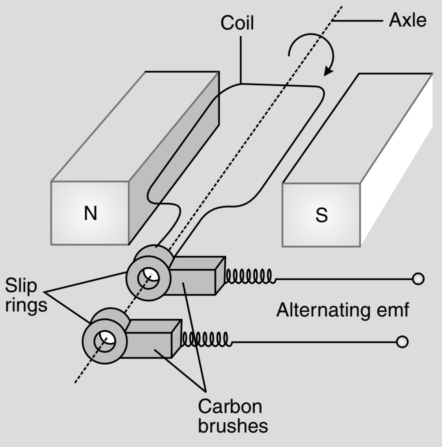
Let a current I_2 flows through the outer circular coil. Then

$$B_2 = \frac{\mu I_2}{2r_2}$$ ½

$\therefore \quad \phi_1 = \pi r^2 B_2 = \frac{\mu \pi r_1^2}{2r_2} I_2 = M_{12} I_2$

Thus $M_{12} = \frac{\mu \pi r_1^2}{2r_2} = M_{21}$ ½

(ii)



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Flux at any time 't'.

$\phi_B = BA\cos\theta = BA\cos\omega t$ ½

From Faraday's Law, induced emf

$$e = -N\frac{d\phi_B}{dt} = NBA\frac{d}{dt}(\cos\omega t)$$ ½

Thus the instantaneous value of emf is

$$e = NBA\omega \sin\omega t$$

For maximum value of emf $\sin\omega t = \pm 1$

i.e., $e_0 = NBA\omega = 2\pi fNBA$ ½

[CBSE Marking Scheme 2016]

Q. 5. (i) Define mutual inductance and write its S.I. unit.

(ii) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.

(iii) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for the emf induced the coil C_1 due to a change in the current through the coil C_2.

R U A [Delhi I, II, III 2015]

Ans. (i) (ii) Try yourself, Similar to Q. 2, Short Answer Type Questions-II 2

(iii) Let a magnetic flux be (ϕ_1) linked with coil C_1 due to current (I_2) in coil C_2.

We have :

$$\phi_1 \propto I_2$$

$\Rightarrow \quad \phi_1 = MI_2$

$\therefore \quad \frac{d\phi_1}{dt} = M\frac{dI_2}{dt}$ ½

$\Rightarrow \quad e = -M\frac{dI_2}{dt}$ ½

[CBSE Marking Scheme 2015]

Q. 6. Define mutual inductance of a pair of coils and write on which factors does it depend.

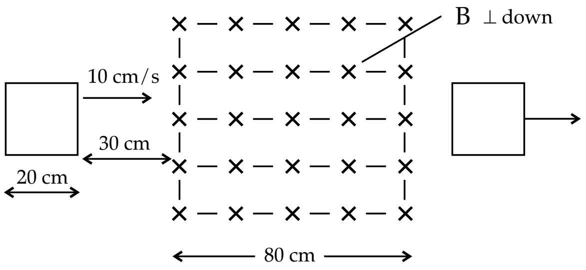
A square loop of side 20 cm is initially kept 30 cm away from a region of uniform magnetic field of 0.1 T as shown in the figure. It is then moved towards the right with a velocity of 10 cm s^{-1} till it goes out of the field.

Plot a graph showing the variation of

(i) magnetic flux (ϕ) through the loop with time (τ).

(ii) induced emf (ε) in the loop with time t.

(iii) induced current in the loop if it has resistance of 0.1 Ω.



R A [O.D. I, II, III 2015]

Ans. Mutual Inductance : The mutual inductance, of a pair of coils, equals the magnetic flux linked with one of them due to a unit current in the other. 1

Alternatively, The mutual inductance, of a pair of coils, equals the emf induced in one of them when the rate of change of current in the other is unity.

Factors affecting the mutual inductance of a pair of coils :

(i) The sizes of the two coils

(ii) The shape of the two coils

(iii) The distance of separation between the two coils

(iv) The nature of the medium between the two coils

(v) The relative orientation of the two coils.

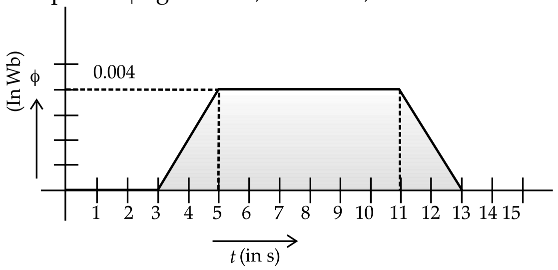
[Any two] 1

From $t = 0$ to $t = 3$ s $\left(= \frac{30 \text{ cm}}{10 \text{ cm/s}}\right)$, the flux through the coil is zero.

From $t = 3$ s to $t = 5$ s, the flux through the coil increases from 0 to $\left[0.1 \times \left(\frac{20}{100}\right)^2\right]$ Wb. *i.e.* 0.004 Wb.

From $t = 5$ s to $t = 11$ s, the flux remains constant at the value 0.004 Wb.
From $t = 11$ s to $t = 13$ s, the flux decreases through the will from 0.004 Wb to 0 Wb.

(i) The plot of ϕ against t is, therefore, as shown :



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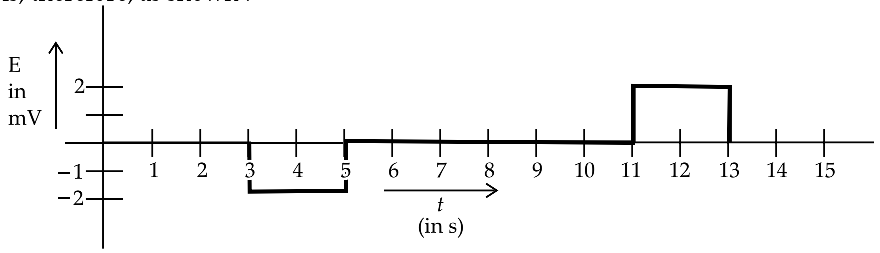
(ii) $\varepsilon = -\frac{d\phi}{dt}$

The plot of ε against t is, therefore, as shown :



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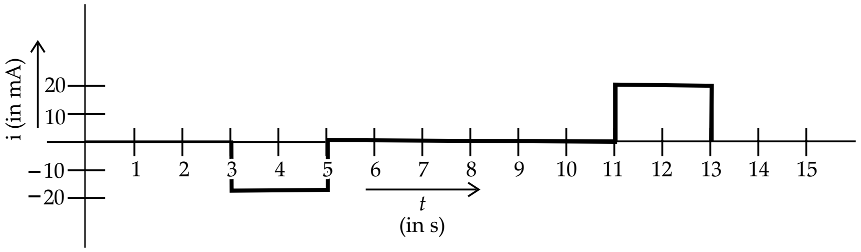
(iii) $i = \frac{\varepsilon}{R} = \frac{2 \times 10^{-3}}{0.1\Omega} = 20$ mA



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