SEQUENCE AND SERIES

1. DEFINITION

Sequence is a function whose domain is the set N of natural numbers.

Real Sequence : A sequence whose range is a subset of R is called a real sequence.

Series : If $a_1, a_2, a_3, a_4, ..., a_n, ...$ is a sequence, then the expression

$a_1 + a_2 + a_3 + a_4 + a_5 + ... + a_n + ...$ is a series.

A series is finite or infinite according as the number of terms in the corresponding sequence is finite or infinite.

Progressions : It is not necessary that the terms of a sequence always follow a certain pattern or they are described by some explicit formula for the n^{th} term. Those sequences whose terms follow certain patterns are called progressions.

1.1 An Arithmetic Progression (AP)

AP is a sequence whose terms increase or decrease by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then AP can be written as a, a + d, a + 2d, ... and n^{th} term of this AP can be written as $t_n = a + (n - 1)\, d$, where $d = a_n - a_{n-1}$.

The sum of the first n terms the AP is given by ;

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + \ell].$$

where ℓ is the last term.

NOTES :

Properties of Arithmetic Progression

(i) If each term of an A.P. is increased, decreased, multiplied or divided by the same non zero number, then the resulting sequence is also an AP.

(ii) 3 numbers in AP are $a - d, a, a + d$;

4 numbers in AP are $a - 3d, a - d, a + d, a + 3d$;

5 numbers in AP are $a - 2d, a - d, a, a + d, a + 2d$;

6 numbers in AP are $a - 5d, a - 3d, a - d, a + d, a + 3d; a + 5d$.

(iii) The common difference can be zero, positive or negative.

(iv) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms.

(v) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it.

$a_n = 1/2\,(a_{n-k} + a_{n+k}), k < n$.

For $k = 1$, $a_n = (1/2)\,(a_{n-1} + a_{n+1})$;

For $k = 2$, $a_n = (1/2)\,(a_{n-2} + a_{n+2})$ and so on.

(vi) $t_r = S_r - S_{r-1}$

(vii) If a, b, c are in AP $\Rightarrow 2b = a + c$.

(viii) A sequence is an AP, iff its n^{th} terms is of the form $An + B$ i.e., a linear expression in n. The common difference in such a case is A i.e., the coefficient of n.

(ix) A sequence is an AP if and only if the sum of its n terms is of the form $An^2 + Bn$, where A and B are constants independent of n.

1.2 Geometric Progression (GP)

GP is a sequence of numbers whose first term is non zero & each of the succeeding terms is equal to the proceeding terms multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the common ratio of the series & obtained by dividing any term by that which immediately proceeds it. Therefore a, ar, ar^2, ar^3, ar^4, ... is a GP with a as the first term & r as common ratio.

(i) n^{th} term $= a\, r^{n-1}$

(ii) Sum of the I^{st} n terms i.e. $S_n = \frac{a\left(r^n - 1\right)}{r-1}$, if $r \neq 1$.

(iii) Sum of an infinite GP when $|r| < 1$ when $n \to \infty$, $r^n \to 0$ if $|r| < 1$ therefore, $S_\infty = \frac{a}{1-r}\ \left(|r| < 1\right)$.

(iv) Any 3 consecutive terms of a GP can be taken as a/r, a, ar ;

any 4 consecutive terms of a GP can be taken as a/r^3, a/r, ar, ar^3 & so on.

NOTES :

Properties of Geometric Progressions

1. If all the terms of a GP be multiplied or divided by the same non–zero constant, then it remains a GP with the same common ratio.
2. The reciprocals of the terms of a given GP forms a GP.
3. If each term of a GP be raised to the same power, the resulting sequence also forms a G.P.
4. In a finite GP the product of the terms equidistant form the beginning and the end is always same and is equal to the product of the first and the last term.
5. Three non–zero numbers, a, b, c are in GP, if $b^2 = ac$.
6. If the terms of a given GP are chosen at regular intervals, then the new sequence so formed also forms a GP.
7. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ is a GP of non–zero non–negative terms, then $\log a_1, \log a_2, \ldots \log a_n, \ldots$ is an AP and vice versa.

1.3 HARMONIC PROGRESSION (HP)

A sequence is said to HP if the reciprocals of its terms are in AP.

If the sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ is an HP then $1/a_1, 1/a_2, \ldots\ldots, 1/a_n$ is an AP & converse. Here we do not have the formula for the sum of the n terms of an HP. For HP whose first terms is a & second term is b, then n^{th} term

is $t_n = \dfrac{ab}{b+(n-1)(a-b)}$

If a, b, c are in HP $\Rightarrow b = \dfrac{2ac}{a+c}$ or $\dfrac{a}{c} = \dfrac{a-b}{b-c}$.

2. MEANS

2.1 Arithmetic Mean

If three terms are in AP then the middle term is called the AM between the other two, so if a, b, c, are in AP, b is AM of a & c.

AM for any n positive numbers $a_1, a_2, \ldots\ldots, a_n$ is ;

$$A = \frac{a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n}{n}$$

2.2 n-Arithmetic Means between Two Numbers

If a, b are any two given numbers & a, $A_1, A_2, \ldots, A_n$, b are in AP then $A_1, A_2, \ldots A_n$ are n AM's between a & b.

$$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1}, \ldots\ldots, A_n = a + \frac{n(b-a)}{n+1}$$

$A_1 = a + d, A_2 = a + 2d, \ldots\ldots, A_n = a + nd$, where $d = \dfrac{b-a}{n+1}$

The arithmetic mean (AM) A of any two numbers a and b is given by the equation $(a + b)/2$. Plase note that the sequence a, A, b is in AP. If $a_1, a_2, \ldots\ldots, a_n$ are n numbers, the (AM)A, of these numbers is given by:

$$A = \frac{1}{n}\left(a_1 + a_2 + \ldots + a_n\right)$$

Inserting 'n' AMs between 'a' and 'b'

Suppose $A_1, A_2, A_3, \ldots\ldots, A_n$ be the n means between a and b. Thus, $a, A_1, A_2, \ldots\ldots, A_n, b$ is an AP and b is the $(n + 2)^{th}$ term.

Thus, $b = a(n+1)d \quad \Rightarrow d = \dfrac{b-a}{n+1}$.

Now,

$A_1 = a + d$

$A_2 = a + 2d$

$\vdots$

$A_n = a + nd$

$$\sum_{i=1}^{n} A_i = na + (1+2+3+.....+n)d = na = \left(\frac{n(n+1)}{2}\right)d$$

$$= na + \left(\frac{n(n+1)}{2}\right)\left(\frac{b-a}{n+1}\right)$$

$$= \frac{n}{2}(2a+b-a) = nA \text{ where, } A = \frac{a+b}{2}$$

NOTES :

Sum of n AM's inserted between a & b is equal to n times the single AM between a & b i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single AM between a & b.

2.3 Geometric Mean

If a, b, c are in GP, b is the GM between a & c. $b^2 = ac$, therefore $b = \sqrt{ac}$; $a > 0, c > 0$.

2.4 n-Geometric Means between a & b

If a, b are two given numbers & a, $G_1, G_2, ..., G_n$, b are in GP. Then $G_1, G_2, G_3, ..., G_n$ are n GMs between a & b.

$G_1 = a\,(b/a)^{1/n+1} = ar$, $G_2 = a\,(b/a)^{2/n+1} = ar^2$, ...,

G_n a $(b/a)^{n/n+1} = ar^n$ where $r = (b/a)^{1/n+1}$

To Insert 'n' GMs Between a and b : If a and b are two positive numbers and we have to insert n GMs, $G_1, G_2, ..., G_n$ between the two numbers 'a' and 'b' then $a, G_1, G_2, ..., G_n$, b will be in GP. The series consists of (n + 2) terms and the last term is b and the first term is

a. $b = ar^{n+2-1} \Rightarrow b = ar^{n+1} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$$\Rightarrow G_1 = ar, G_2 = ar^2 ....G_n = ar^n$$

Note : $\prod_{r=1}^{n} G_r = (G)^n$, where $G = \sqrt{ab}$ (GM between a and b)

NOTES :

The product of n GMs between a & b is equal to the nth power of the single GM between a & b i.e. $\pi_{r=1}^{n} G_r = (G)^n$ where G is the single GM between a & b.

2.5 Harmonic Mean

If a, b, c are in HP, b is the HM between a & c, then $b = 2ac/[a + c]$.

2.6 Arithmetic, Geometric and Harmonic means between two given numbers

Let A, G and H be arithmetic, geometric and harmonic means of two positive numbers a and b. Then,

$$A = \frac{a+b}{2}, \; G = \sqrt{ab} \text{ and } H = \frac{2ab}{a+b}$$

Relation between A.M. and GM

For any two non-negative number $A.M. \geq G.M.$

Proof . Let two non-negative numbers be $\sqrt{a}$ and $\sqrt{b}$.

Now, we can write $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0 \Rightarrow a - 2\sqrt{ab} + b \geq 0$

$$\Rightarrow a+b \geq 2\sqrt{ab} \Rightarrow \frac{a+b}{2} \geq \sqrt{ab} \Rightarrow A.M. \geq GM$$

Note : (i) Equality for AM, G.M. (i.e. A.M. = GM) exists when a = b.

(ii) Since $A.M. \geq GM; (AM)_{min} = GM; (GM)_{max} = AM$

These three means possess the following properties

1. $A \geq G \geq H$
2. A, G, H form a GP i.e., $G^2 = AH$.
3. The equation having a and b as its roots is $x^2 - 2Ax + G^2 = 0$
4. If A, G, H are arithmetic, geometric and harmonic means between three given numbers a, b and c, then the equation having a, b, c as its roots is

$$x^3 - 3Ax^2 + \frac{3G^3}{H}x - G^3 = 0.$$

NOTES :

If A and G be the AM and GM between two positive numbers, then the number are $A \pm \sqrt{A^2 - G^2}$.

3. SIGMA NOTATIONS

3.1 Theorems

(i) $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$

(ii) $\sum_{r=1}^{n} k\,a_r = k\sum_{r=1}^{n} a_r$

(iii) $\sum_{r=1}^{n} k = k + k + k$.......... n times = nk ; where k is a constant.

4. SUM TO n TERMS OF SOME SPECIAL SEQUENCES

4.1 Sum of first n natural numbers

$$\sum_{k=1}^{n} k = 1+2+3+.....+n = \frac{n(n+1)}{2}.$$

4.2 Sum of the squares of first n natural numbers

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+.....+n^2 = \frac{n(n+1)(2n+1)}{6}.$$

4.3 Sum of the higher powers of first n natural numbers

$$\sum_{k=1}^{n} k^3 = 1^3+2^3+........+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n} k\right)^2$$

$$\sum_{k=1}^{n} k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$$

4.4 Sum of first n odd numbers

$$\sum_{k=1}^{n}(2k-1) = 1+3+...+(2n-1) = n^2$$

4.5 Vn method :

This is method of resolving the nth term into partial fraction and summation by telescopic cancellation. First, find the n^{th} term of the series and try to create a denominator part in the numerator by using partial fraction whenever the series is in the form of fraction of T_n like the following:

$$T_n = \frac{2}{n^2-1}$$

Using the partial fraction, we can write the nth term as

$$T_n = \frac{1}{n-1} - \frac{1}{n+1}$$

Now, when we find the summation, there will be telescopic cancellation and thus we will get the sum of the given series.

4.6 Method of Difference :

If T_1, T_2, T_3, T_4, T_5.... is a sequence whose terms are sometimes in AP and sometimes in GP, then for such series we first compute their n^{th} term and then compute the sum to n terms using sigma notation.

5. ARITHMETICO-GEOMETRIC SERIES

A series each term of which is formed by multiplying the corresponding term of an AP & GP is called the Arithmetico-Geometric Series. e.g. $1 + 3x + 5x^2 + 7x^3 +$...............

Here, 1, 3, 5, ... are in AP & 1, x, x^2, x^3 .... are in GP.

5.1 Sum of n terms of an Arithmetico-Geometric Series

Let $S_n = a + (a + d)\,r + (a + 2\,d)\,r^2 + ... + [a + (n-1)\,d]\,r^{n-1}$

then $S_n = \dfrac{a}{1-r} + \dfrac{dr(1-r^{n-1})}{(1-r)^2} - \dfrac{[a+(n-1)d]r^n}{1-r}, r \neq 1.$

A series formed by multiplying the corresponding terms of AP and G.P. is called arithmetic geometric progression (AGP).

Let a = first term of AP, b = first term of GP, d = common difference and r = common ratio of GP. then

AP : a, a + d, a + 2d, a + 3d, ..., a + (n – 1) d

GP: $b, br, br^2, br^3, ..., br^{n-1}$

AGP :

$ab, (a+d)br, (a+2d)br^2 .... (a+(n-1)d)br^{n-1}$ (Standard appearance of AGP)

the general term (nth term) of an AGP is given as $T_n = [a+(n-1)d]br^{n-1}$

Series of AGP

To find the sum of n terms of an AGP, we suppose its sum as S_n and then multiply both the sides by the common ratio of the corresponding G.P. and then subtract as in the following way. Thus, we get a G.P. whose sum can be easily obtained.

$$S_n = ab + (a+d)br + (a+2d)br^2 + ... + (a+(n-1)d)br^{n-1} \quad ..(i)$$

$$rS_n = 0 + abr + (a+d)br^2 + ... + (a+(n-1)d)br^n \quad ...(ii)$$

After subtraction, we get

$$S_n(1-r) = ab + [dbr + dbr^2 + ... + up\ to\ (n-1)\ terms]$$
$$-[(a+(n-1)d)br^n]$$

$$S_n(1-r) = ab + \frac{dbr(1-r^{n-1})}{1-r} - (a+(n-1)d)br^n$$

$$S_n = \frac{ab}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - \frac{(a+(n-1)d)br^n}{1-r}$$. This is the sum of n terms of AGP

For an infinite AGP, $AGP, as\ n \to \infty, then\ r^n \to 0 (\because |r| < 1)$

$$\Rightarrow S_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}.$$

SOLVED EXAMPLES

Example - 1

Find

(i) 24^{th} term of the A.P. 5, 8, 11, 14...

(ii) 15^{th} term of the A.P. 21, 16, 11, 6,...

Sol. (i) 5, 8, 11, 14...

$\therefore$ $a = 5; d = 8 - 5 = 3; n = 24$

$\therefore$ $t_n = a + (n-1)\,d$

$\therefore$ $t_{24} = 5 + (24 - 1)\,3$

$= 5 + 23 \times 3$

$= 5 + 69$

$\therefore$ $t_{24} = 74$

(ii) 21, 16, 11, 6...

$a = 21, d = 16 - 21 = -5; n = 15$

$\therefore$ $t_{15} = 21 + (15 - 1)(-5)$

$= 21 + (14)(-5)$

$= 21 - 70$

$t_{15} = -49$

Example - 2

If for a sequence (t_n), $S_n = 4n^2 - 3n$, show that sequence is an A.P.

Sol. $S_n = 4n^2 - 3n$

$t_{n+1} = S_{n+1} - S_n$

$= [4\,(n + 1)^2 - 3\,(n + 1)] - [4n^2 - 3n]$

$= 4n^2 + 8n + 4 - 3n - 3 - 4n^2 + 3n$

$t_{n+1} = 8n + 1$

$t_n = 8(n - 1) + 1$

$= 8n - 8 + 1$

$t_{n+1} - t_n = 8n - 7$

$= (8n + 1) - (8n - 7)$

$= 8 =$ constant

Hence as the difference between two conseuctive terms is constant, it is A.P.

Example - 3

Find the value of n if 1 + 4 + 7 + 10 +... to n terms = 590

Sol. 1 + 4 + 7 + 10 + ... to n terms = 590, a = 1, d = 4 – 1 = 3

$\therefore$ $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$\therefore$ $590 = \frac{n}{2}[2(1) + (n-1)3]$

$\therefore$ $2 \times 590 = n\,[2 + 3n - 3]$

$\therefore$ $1180 = n\,[3n - 1]$

$\therefore$ $3n^2 - n - 1180 = 0$

$\therefore$ $3n^2 - 60n + 59n - 1180 = 0$

$\therefore$ $3n(n - 20) + 59\,(n - 20) = 0$

$\therefore$ $(3n + 59)\,(n - 20) = 0$

$\therefore$ $n = \frac{-59}{3}$ or $n = 20$

'n' can not be negative, $n \neq \frac{-59}{3}$

$\therefore$ $n = 20$

Example - 4

If for an A.P. $S_{16} = 784$, a = 4, find d

Sol. $S_{16} = 784, a = 4$

$\therefore$ $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$S_{16} = \frac{16}{2}[2(4) + (16-1)\,d]$

$784 = 8\,[8 + 15d]$

$784 = 64 + 120\,d$

$\therefore$ $720 = 120\,d$

$\therefore$ $d = \frac{720}{120}$

$\therefore$ $d = 6$

Example - 5

For the following G.P.'s find t_n

(i) $1, -4, 16, -64, \ldots$

(ii) $\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}, \frac{1}{9\sqrt{3}}$

Sol. (i) $1, -4, 16, -64, \ldots$

$a = 1$

$r = \frac{-4}{1} = -4$

$\therefore$ $t_n = a(r)^{n-1}$

$= 1\,(-4)^{n-1}$

$= (-4)^{n-1}$

(ii) $\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}, \frac{1}{9\sqrt{3}}, \ldots..$

$a = \sqrt{3}$

$r = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$

$\therefore$ $t_n = a\,(r^{n-1})$

$= \sqrt{3}\left(\frac{1}{3}\right)^{n-1}$

Example - 6

(i) Find three numbers in G.P. such that their sum is 35 and their product is 1000.

(ii) Find three numbers in G.P. such that their sum is 13/3 and the sum of their squares is $\frac{91}{9}$

Sol. (i) Let three number are $\frac{a}{r}$, a, ar

$\therefore$ $\frac{a}{r} \times a \times ar = 1000$

$a^3 = 1000$

$a = 10$ ... (1)

$a\left[\frac{1}{r} + 1 + r\right] = 35$

$\frac{1}{r} + 1 + r = \frac{35}{a} = \frac{35}{10}$

$2\,(1 + r + r^2) = 7r$

$2r^2 - 5r + 2 = 0$

$(r - 2)\,(2r - 1) = 0$

$r - 2 = 0$ or $2r - 1 = 0$

$r = 2$ or $2r = 1$

for $r = 2$, $\frac{a}{r} = \frac{10}{2} = 5$, $ar = 10 \times 2 = 20$

the number are 5, 10, 20

for $r = \frac{1}{2}, \frac{a}{r} = \frac{10}{1/2} = 20;\ ar = 10 \times \frac{1}{2} = 5$

the number are 20, 10, 5

(ii) Let $\frac{a}{r}$, a, ar be three numbers in G.P.

$\frac{a}{r} + a + ar = \frac{13}{3}$ ... (i)

$\frac{a^2}{r^2} + a^2 + a^2r^2 = \frac{91}{9}$ ... (ii)

Taking square of (i)

$\left(\frac{a}{r} + a + ar\right)^2 = \left(\frac{13}{3}\right)^2$

$\frac{a^2}{r^2} + a^2 + a^2r^2 + \frac{2a^2}{r} + 2a^2 + 2a^2r = \frac{169}{9}$

$\left(\frac{a^2}{r^2} + a^2 + a^2r^2\right) + 2a\left(\frac{a}{r} + a + ar\right) = \frac{169}{9}$

$\frac{91}{9} + 2a\left(\frac{13}{3}\right) = \frac{169}{9}$

$\frac{26a}{3} = \frac{169}{9} - \frac{91}{9}$

$$\frac{26a}{3} = \frac{26}{3}$$

$\Rightarrow$ $a = 1$

$$\frac{1}{r} + 1 + r = \frac{13}{3}$$

$$\frac{1 + r + r^2}{r} = \frac{13}{3}$$

$3 + 3r + 3r^2 = 13r$

$3r^2 - 10r + 3 = 0$

$(r - 3)(3r - 1) = 0$

$r = 3$ or $r = \frac{1}{3}$

for $r = 3$, $\frac{a}{r} = \frac{1}{3}$, $ar = 1 \times 3 = 3$

$\therefore$ three numbers are $\frac{1}{3}$, 1, 3

for $r = \frac{1}{3}$, $\frac{a}{r} = \frac{1}{\frac{1}{3}} = 3$; $ar = 1 \times \frac{1}{3} = \frac{1}{3}$ three numbers are 3, 1, $\frac{1}{3}$

Example - 7

If x, y and z are pth, qth and rth terms of a G.P. respectively then show that $x^{q-r} . y^{r-p} . z^{p-q} = 1$

Sol. Let A be the first term and R be the common ratio of the given G.P. Then,

$x = $ pth term $\Rightarrow x = AR^{(p-1)}$

$y = $ qth term $\Rightarrow y = AR^{(q-1)}$

and $z = $ rth term $\Rightarrow z = AR^{(r-1)}$

L.H.S.

$$= \{AR^{(p-1)}\}^{q-r} . \{AR^{(q-1)}\}^{r-p} . \{AR^{(r-1)}\}^{p-q}$$

$$= A^{(q-r)} R^{(p-1)(q-r)} A^{(r-p)} R^{(q-1)(r-p)} A^{(p-q)} R^{(r-1)(p-q)}$$

$$= A^{(q-r+r-p+p-q)} R^{(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)}$$

$$= A^{O} R^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$$

$$= A^{O} R^{O} = 1$$

$= $ R.H.S.

Example - 8

If for a squence, $t_n = \frac{2^{n-2}}{5^{n-3}}$, show that the sequence is a G.P. Find its first term and the common ratio.

Sol. $t_n = \frac{2^{n-2}}{5^{n-3}}$

$$= \frac{2^n . 2^{-2}}{5^n . 5^{-3}}$$

$$= \left(\frac{5^3}{2^2}\right)\left(\frac{2}{5}\right)^n$$

$$= \frac{125}{4}\left(\frac{2}{5}\right)^n$$

Let $t_{(n+1)} = \frac{125}{4}\left(\frac{2}{5}\right)^{n+1}$

$$\frac{t_{(n+1)}}{t_n} = \frac{\frac{125}{4}\left(\frac{2}{5}\right)^{n+1}}{\frac{125}{4}\left(\frac{2}{5}\right)^n} = \frac{2}{5} = \text{constant}$$

Hence sequence is in GP

$t_1 = a = \frac{125}{4} \times \frac{2}{5} = \frac{25}{2}$

Hence first term $= \frac{25}{2}$

and common ratio $= \frac{2}{5}$

Example - 9

For a G.P.

(i) If $a = 2, r = 3, S_n = 242$, find n.

(ii) If $S_3 = 125, S_6 = 152$. find r.

Sol. (i) $a = 2, r = 3, S_n = 242$

$$S_n = a\left[\frac{r^n - 1}{r - 1}\right]$$

$$242 = 2\left[\frac{3^n - 1}{3 - 1}\right]$$

$242 = 3^n - 1$

$243 = 3^n$

$3^5 = 3^n$

$n = 5$

(ii) $S_3 = 125, S_6 = 152,$

$$S_3 = a\left[\frac{r^3 - 1}{r - 1}\right] \text{ and } S_6 = a\left[\frac{r^6 - 1}{r - 1}\right]$$

$$\frac{S_6}{S_3} = \frac{a\left[\frac{r^6 - 1}{r - 1}\right]}{a\left[\frac{r^3 - 1}{r - 1}\right]} = \frac{r^6 - 1}{r^3 - 1}$$

$$\frac{152}{125} = \frac{r^6 - 1}{r^3 - 1}$$

By dividendo

$$\frac{152 - 125}{125} = \frac{r^6 - 1 - (r^3 - 1)}{r^3 - 1}$$

$$\frac{27}{125} = \frac{r^6 - 1 - r^3 + 1}{r^3 - 1} = \frac{r^3(r^3 - 1)}{(r^3 - 1)}$$

$$\left(\frac{3}{5}\right)^3 = r^3$$

$\therefore \quad r = \dfrac{3}{5}$

Example - 10

Find the sum to n terms.

(i) $0.9 + 0.99 + 0.999 + ...$

(ii) $0.5 + 0.55 + 0.555 + ...$

Sol. $S_n = [0.9 + 0.99 + 0.999 + ...]$

$= [(1–0.1) + (1 – 0.01) + (1–0.001)....]$

$= [(1+1+1+...) – (0.1 + 0.01 + 0.001 + ...)$

$= n –$ (sum of n terms in GP with $a = 0.1$ and $r = 0.1$)

$$S_n = n - \left[\frac{(0.1)(1 - 0.1^n)}{1 - 0.1}\right]$$

$$= n - \left[\frac{0.1}{0.9}\left(1 - \frac{1}{10^n}\right)\right]$$

$$= n - \frac{1}{9}\left[1 - \left(\frac{1}{10^n}\right)\right]$$

(ii) $S_n = 0.5 + 0.55 + 0.555 + ...$

$= 5\,(0.1 + 0.11 + 0.111 + ...)$

$$= \frac{5}{9}(0.9 + 0.99 + 0.999 + ...)$$

$= [0.9 + 0.99 + 0.999 + ...]$

$$= \frac{5}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001)..........]$$

$$= \frac{5}{9}[(1+1+1+...) - (0.1 + 0.01 + 0.001 + ...)]$$

$$= \frac{5}{9}\left[\begin{array}{c} n - \text{sum of n terms of GP with} \\ a = 0.1 \text{ and } r = 0.1 \end{array}\right]$$

$$= \frac{5}{9}\left\{n - \left[\frac{(0.1)(1 - 0.1^n)}{1 - 0.1}\right]\right\}$$

$$= \frac{5}{9}\left\{n - \left[\frac{0.1}{0.9}\left(1 - \frac{1}{10^n}\right)\right]\right\}$$

$$= \frac{5}{9}\left\{n - \frac{1}{9}\left[1 - \left(\frac{1}{10^n}\right)\right]\right\}$$

Example - 11

Determine whether the sum of infinity of the following G.P.s exist, in the case they exist then find the sum

(i) $1, 2, 4, 8, 16..........$

(ii) $1, \frac{3}{2}, \frac{9}{4}, \frac{27}{8}.........$

(iii) $5, \frac{-5}{2}, \frac{5}{4}, \frac{-5}{8}, \frac{5}{16}.........$

Sol. (i) $a = 1; r = 2$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$1\left[\frac{1-(2)^n}{1-2}\right] = 1\left[\frac{1-(2)^n}{-1}\right]$$

$S_n = [1-(2)^n]$

$r = 2 > 1$

$\therefore$ The sum of infinity does not exist.

(ii) $a = 1; r = \frac{3}{2}$

$$\therefore \quad S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$= 1\left[\frac{1-\left(\frac{3}{2}\right)^n}{1-\frac{3}{2}}\right]$$

$$= 1\left[\frac{1-\left(\frac{3}{2}\right)^n}{\frac{-1}{2}}\right]$$

$$S_n = -2\left[1-\left(\frac{3}{2}\right)^n\right]$$

$$\therefore \quad S_n = 2\left(\frac{3}{2}\right)^n - 2$$

$$\because \quad r = \frac{3}{2} > 1$$

Sum of infinity does not exist.

(iii) $a = 5,\ r = \frac{\frac{-5}{2}}{5} = \frac{-1}{2}$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

$$= 5\left[\frac{1-\left(\frac{-1}{2}\right)^n}{1-\left(\frac{-1}{2}\right)}\right]$$

$$= 5\left[\frac{1-\left(\frac{-1}{2}\right)^n}{\frac{3}{2}}\right]$$

$$\therefore \quad S_n = \frac{10}{3}\left[1-\left(\frac{-1}{2}\right)^n\right]$$

$$|r| = \left|\frac{-1}{2}\right| < 1$$

$$\therefore \quad S_n = \frac{10}{3}[1-0]\left\{\because \left(-\frac{1}{2}\right)^n \to 0\right\}$$

$$S_n = \frac{10}{3}$$

$\therefore$ Sum of infinity is $\frac{10}{3}$ of G.P.

Example - 12

For a sequence, if $S_n = 7(4^n - 1)$, find t_n and show that the sequence is a G.P.

Sol. $S_n = 7(4^n - 1)$

$t_n = S_n - S_{n-1} = 7[4^n - 1] - 7[4^{n-1} - 1]$

$$= 7(4^n) - 7 - 7\frac{(4)^n}{4} + 7$$

$$= 7 \times 4^n \left[1 - \frac{1}{4}\right]$$

$$= 7 \times 4^n \times \frac{3}{4} = 21[4^{n-1}]$$

$$r = \frac{t_{n+1}}{t_n} = \frac{21[4^{n+1-1}]}{21[4^{n-1}]} = \frac{4^n}{4^{n-1}} = 4$$

common ratio is constant. Hence the given sequence is GP.

Example - 13

Find S_n of the following arithmetic geometric sequence.

(i) $3, 6x, 9x^2, 12x^3, 15x^4$ ....

(ii) $1, 3x, 5x^2, 7x^3, 9x^4$ ....

Sol. (i) In the given sequence AP is

$3, 6, 9, 12$........

$\therefore$ $a = 3, d = 6 - 3 = 3$

nth term will be

$t_n = a + (n - 1)d$

$t_n = 3 + (n - 1)3$

$t_n = 3n$

And, G.P. is $1, x, x^2, x^3, x^4$, ...

$$\therefore \quad a = 1, r = \frac{x}{1} = x$$

$\therefore$ nth term will be

$t_n = ar^{n-1}$

$= (1)(x)^{n-1}$

$\therefore$ $t^n = x^{n-1}$

$S_n = 3 + 6x + 9x^2 + 12x^3 .... + 3(n-1).x^{n-2} + 3n.x^{n-1}$ ...... (i)

multiplying both the side x.

$x.S_n = 3x + 6x^2 + 9x^3 + 12x^4 ... + 3(n-1)x^{n-1} + 3n.x^n$ ... (ii)

Subtracting (ii) from (i)

$\therefore$ $S_n - xS_n = (3 + 6x + 9x^2 + 12x^3 ... + 3(n-1)x^{n-2} + 3(n)x^{n-1})$

$-(3x + 6x^2 + 9x^3 + 12x^4 + ... + 3(n-1)x^{n-1} + 3nx^n)$,

$\Rightarrow (1-x)S_n = 3 + 3x + 3x^2 + 9x^3 .. + 3x^{n-1} - 3nx^n$

$= 3 + 3x[1 + x + x^2 + x^3 .... + x^{n-2}] - 3nx^n$

$$= 3 + 3x\left[1 + \frac{x}{x-1}(x^{n-2} - 1)\right] - 3nx^n$$

$$= 3 + 3x\left[1 + \frac{x^{n-1}}{x-1} - \frac{x}{x-1}\right] - 3nx^n$$

$$(1-x)S_n = 3 + 3x + \frac{3x^n}{x-1} - \frac{3x^2}{x-1} - 3nx^n$$

$$S_n = \frac{3}{1-x}\left[(1+x) + \frac{x^n - x^2}{x-1} - nx^n\right]$$

$$\therefore \quad S_n = \frac{3}{1-x}\left[(1 + x - nx^n) + \frac{x^n - x^2}{x-1}\right]$$

(ii) $1, 3x, 5x^2, 7x^3, 9x^4$..........

In the given sequence A.P. will be

$1, 3, 5, 7, 9$..........

$\therefore$ $a = 1, d = 2$

$\therefore$ $t_n = a + (n-1)d$

$= 1 + 2n - 2$

$= 2n - 1$

In the given sequence GP will be

$\therefore$ $1, x, x^2, x^3$ ......

$\therefore$ $a = 1, r = x$

$\therefore$ $t_n = ar^{n-1}$

$t_n = x^{n-1}$

$\therefore$ $S_n = 1 + 3x + 5x^2 + 7x^3 .... + (2n-3).x^{n-2} + (2n-1)x^{n-1}$ ... (i)

multiplying both the side by x.

$\therefore$ $xS_n = x + 3x^2 + 5x^3 + 7x^4 .... + (2n-3)x^{n-1} + (2n-1)x^n$ ... (ii)

subtracting (ii) from (i)

$\therefore$ $S_n - xS_n = [(1 + 3x + 5x^2 + 7x^3 ...... + (2n-3).x^{n-2} + (2n-1)x^n)$

$-(x + 3x^2 + 5x^3 ..... + (2n-3)x^{n-1} + (2n-1)x^n]$

$(1-x)S_n = 1 + 2x + 2x^2 + 2x^3 ... + 2x^{n-1} - (2n-1)x^n$

$= 1 + [2(x + x^2 + x^3 + ... x^{n-1})] - (2n-1)x^n$

$$=1+\left[2.\frac{x}{x-1}\left\{x^{n-1}-1\right\}\right]-(2n-1)\,x^{n}$$

$$=1+\frac{2x^{n}}{x-1}-\frac{2x}{x-1}-(2n-1)\,x^{n}$$

$$=1+\frac{2x^{n}-2x}{x-1}-(2n-1)\,x^{n}$$

$$=1+\frac{2x(x^{n-1}-1)}{x-1}-(2n-1)\,x^{n}$$

$$\therefore \quad S_n=\frac{1}{1-x}\left[1-(2n-1)\,x^{n}+\frac{2x(x^{n-1}-1)}{(x-1)}\right]$$

Example - 14

Find the sum of $1 + (1 + x) + (1 + x + x^2) + ... + (1+x+x^2+...+x^{n-1})$

Sol. $S_n = 1+(1+x)+(1+x+x^2)+...+(1+x+x^2+...+x^{n-1})$

$T_r = 1+x+x^2+...\,x^{r-1}$

This is GP with first term 1 and common ratio 'x'

$$\therefore \quad T_r=\frac{1\left[1-x^{r}\right]}{1-x}$$

$$S_n=\sum_{r=1}^{n}\frac{1-x^{r}}{1-x}=\frac{1}{1-x}\sum_{r=1}^{n}1-\frac{1}{1-x}\sum_{r=1}^{n}x^{r}$$

$$=\frac{1}{1-x}(n)-\frac{1}{1-x}\left[\frac{x(1-x^{n})}{1-x}\right]$$

$$=\frac{n}{1-x}-\frac{x(1-x^{n})}{(1-x)^{2}}$$

Example - 15

Find the following sum

$1.2.3 + 2.\,3.4 + 3.4.5 + ... + n(n+1)\,(n+2)$

Sol. $Tr_1 = 1, 2, 3 = 1 + (r-1)\,1 = r$

$Tr_2 = 2, 3, 4 = 2 + (r-1)\,1 = r + 1$

$Tr_3 = 3, 4, 5 = 3 + (r-1)\,1 = r + 2$

$\therefore$ $1.2.3 + 2.3.4 + 3.4.5 + ...$ n terms

$$=\sum_{r=1}^{n}Tr_1\,.\,Tr_2.Tr_3=\sum_{r=1}^{n}(r\,(r+1)\,.\,(r+2))$$

$$=\sum_{r=1}^{n}r^{3}+\sum_{r=1}^{n}3r^{2}+\sum_{r=1}^{n}2r$$

$$=\frac{n^{2}(n+1)^{2}}{4}+\frac{3n(n+1)\,(2n+1)}{6}+\frac{2n(n+1)}{2}$$

$$=\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+(2n+1)+2\right]$$

$$=\frac{n(n+1)}{2}\left[\frac{n(n+1)+4n+6}{2}\right]$$

$$=\frac{n(n+1)}{2}\left[\frac{n^{2}+5n+6}{2}\right]$$

$$=\frac{n(n+1)\,(n+2)\,(n+3)}{4}$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Arithmetic Progression

1. nth term of the sequence

a, a + d, a + 2d, ... is

(a) n + nd (b) a + (n – 1) d

(c) a + (n + 1) d (d) none of these

2. Let T_r be the r^{th} term of an A.P., for r = 1, 2, 3,....... If for some positive integers m, n. We have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals :

(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$

(c) 1 (d) 0

3. Which of the following sequences is an A.P. with common difference 3 ?

(a) $a_n = 2n^2 + 3n, n \in N$ (b) $a_n = 3n + 5, n \in N$

(c) $a_n = 3n^2 + 1, n \in N$ (d) $a_n = 2n^2 + 3, n \in N$

4. If $a_1, a_2, a_3, ..., a_{n+1}$ are in A.P., then

$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} ...... \frac{1}{a_na_{n+1}}$, is

(a) $\frac{n-1}{a_1a_{n+1}}$ (b) $\frac{1}{a_1a_{n+1}}$

(c) $\frac{n+1}{a_1a_{n+1}}$ (d) $\frac{n}{a_1a_{n+1}}$

5. If log 2, log $(2^x - 1)$ and log $(2^x + 3)$ are in AP, then the value of x is given by

(a) 5/2 (b) $\log_2 5$

(c) $\log_3 5$ (d) $\log_5 3$

6. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. then which of the following is in A.P.

(a) a, b, c (b) a^2, b^2, c^2

(c) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ (d) none of these

Sum of terms in AP

7. Three numbers are in A.P, such that their sum is 18 and sum of their squares is 158. the greatest among them is

(a) 10 (b) 11

(c) 12 (d) None of these

8. If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then its common difference is

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 4

9. The sum of first ten terms of an AP is four times the sum of its first five terms, then ratio of first term and common difference is

(a) 2 (b) 1/2

(c) 4 (d) 1/4

10. The sum of all odd numbers of two digits is

(a) 2530 (b) 2475

(c) 4905 (d) none of these

11. Sum of first n odd natural numbers is

(a) 2n + 1 (b) n^2

(c) 2n – 1 (d) none of these

12. The sum of numbers lying between 10 and 200 which are divisible by 7 will be:

(a) 2800 (b) 2835

(c) 2870 (d) 2849

13. The sum of integers in between 1 and 100 which are divisible by 2 or 5 is

(a) 3100 (b) 3600

(c) 3050 (d) 3500

14. If $\frac{3+5+7+........+n \text{ terms}}{5+8+11+........+10 \text{ terms}} = 7$, then the value of n is

(a) 35 (b) 36

(c) 37 (d) 40

15. If for an A.P. $T_3 = 18$ and $T_7 = 30$ then S_{17} is equal to

(a) 612 (b) 622

(c) 306 (d) none of these

16. If $S_n = n P + \frac{n(n-1)}{2} Q$, where S_n denotes the sum of the first n terms of an A.P., then the common difference is

(a) $P + Q$ (b) $2P + 3Q$

(c) $2Q$ (d) Q

17. Let S_n denote the sum of first n terms of an A.P. If $S_{2n} = 3 S_n$ then the ratio S_{3n}/S_n is equal to

(a) 4 (b) 6

(c) 8 (d) 10

18. If a_1, a_2, a_3, ... is an A.P such that

$$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225,$$

then $a_1 + a_2 + a_3 + ... + a_{23} + a_{24}$ is equal to

(a) 909 (b) 75

(c) 750 (d) 900

19. The first, second and middle term of an AP are a, b, c respectively. Sum of all terms is

(a) $\frac{2(c-a)}{b-a}$ (b) $\frac{2c(c-a)}{b-a} + c$

(c) $\frac{2c(b-a)}{c-a}$ (d) $\frac{2b(c-a)}{b-a}$

20. The sum of the series

$a - (a + d) + (a + 2d) - (a + 3d) + ...$ upto $(2n + 1)$ terms is

(a) $-nd$ (b) $a + 2nd$

(c) $a + nd$ (d) $2nd$

21. The sum of first n (odd) terms of an A.P. whose middle term is m is

(a) mn (b) m^n

(c) n^m (d) none of these

22. If the sum of first *p* terms, first *q* terms and first r terms of an A.P. be *x*, *y* and *z* respectively, then

$$\frac{x}{p}(q-r) + \frac{y}{q}(r-p) + \frac{z}{r}(p-q) \text{ is}$$

(a) 0 (b) 2

(c) pqr (d) $\frac{8xyz}{pqr}$

23. If A_1, A_2 are two AM's between two numbers a and b, then $(2A_1 - A_2)(2A_2 - A_1)$ is equal to

(a) $a + b$ (b) $\frac{ab}{a+b}$

(c) ab (d) none of these

Geometric progression

24. If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$ then x, y, z are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

25. If x, $2x + 2$ and $3x + 3$ are first three terms of a G.P., then its 4th term is

(a) 27 (b) -27

(c) $-27/2$ (d) $27/2$

26. If first, second and eighth terms of a G.P. are respectively n^{-4}, n^n, n^{52}, then the value of n is

(a) 1 (b) 10

(c) 4 (d) none of these

27. If a_1, a_2, a_3 $(a_1 > 0)$ are three successive terms of a G.P. with common ratio r, the value of r for which $a_3 > 4a_2 - 3a_1$ holds is given by

(a) $1 < r < 3$ (b) $-3 < r < -1$

(c) $r > 3$ or $r < 1$ (d) None of these

28. If the first and the nth terms of a G.P. are a and b respectively and P is the product of the first n terms, then $P^2 =$

(a) ab (b) $(ab)^n$

(c) $(ab)^{n/2}$ (d) $(ab)^{2n}$

29. The fourth, seventh and tenth terms of a G.P. are p, q, r respectively, then

(a) $p^2 = q^2 + r^2$ (b) $q^2 = pr$

(c) $p^2 = qr$ (d) $pqr + pq + 1 = 0$

30. The product of first n (odd) terms of a G.P. whose middle term is m is

(a) mn (b) m^n

(c) n^m (d) none of these

31. Three numbers form an increasing GP. If the middle number is doubled, then the new numbers are in AP. The common ratio of the GP is

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $\sqrt{3}-2$ (d) $3+\sqrt{2}$

32. If a, b, c, d are in G.P. then $a^n + b^n$, $b^n + c^n$, $c^n + d^n$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

33. If a, b, c, d are in G.P., then $(a^3 + b^3)^{-1}$, $(b^3 + c^3)^{-1}$, $(c^3 + d^3)^{-1}$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

Sum of terms in GP

34. The sum of first n terms of the series.

$1-1+1-1+\ldots$ is

(a) 1 if n is odd and 0 when n is even

(b) –1

(c) $(-1)^n$ (d) ± 1

35. The n^{th} term of a GP is 128 and the sum of its n terms is 255. If its common ratio is 2 then its first term is

(a) 1 (b) 3

(c) 8 (d) none of these

36. In a G.P. of even numbers of terms, the sum of all terms is 5 times the sum of odd terms. The common ratio of the GP is

(a) $-\frac{4}{5}$ (b) $\frac{1}{5}$

(c) 4 (d) None of these

37. If $S = 1 + a + a^2 + \ldots\ldots$ to ∞ $(a < 1)$, then the value of a is

(a) $\frac{S}{S-1}$ (b) $\frac{S}{1-S}$

(c) $\frac{S-1}{S}$ (d) $\frac{1-S}{S}$

38. The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is 192. The common ratio of the original G.P. is

(a) 1/2 (b) 2/3

(c) 1/3 (d) –1/2

39. If the sum of an infinitely decreasing GP is 3, and the sum of the squares of its terms is 9/2, the sum of the cubes of the terms is

(a) 105/13 (b) 108/13

(c) 729/8 (d) none of these

40. If the sum of first two terms of an infinite GP is 1 and every term is twice the sum of all the successive terms, then its first term is

(a) 1/3 (b) 2/3

(c) 1/4 (d) 3/4

41. The value of $0.\overline{423}$ is

(a) $\frac{419}{999}$ (b) $\frac{423}{999}$

(c) $\frac{423}{100}$ (d) none

Geometric mean

42. If 4 GM's be inserted between 160 and 5, then third GM will be

(a) 8 (b) 118

(c) 20 (d) 40

43. If g_1, g_2 are two G.M's between two numbers a and b, then

$\frac{g_1^2}{g_2}+\frac{g_2^2}{g_1}$ is equal to

(a) a + b (b) ab

(c) $\frac{a+b}{ab}$ (d) none of these

44. If A_1, A_2 be two AM's and G_1, G_2 be two GM's between two numbers a and b, then $\frac{A_1+A_2}{G_1G_2}$ is equal to

(a) $\frac{a+b}{2ab}$ (b) $\frac{2ab}{a+b}$

(c) $\frac{a+b}{ab}$ (d) $\frac{ab}{a+b}$

Harmonic Progression

45. The fourth term of the sequence $3, \frac{3}{2}, 1, ...$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{2}{3}$ (d) none of these

46. Let the positive numbers a,b,c,d be in A.P. Then, abc, abd, acd, bcd are

(a) not in A.P./G.P./H.P. (b) in A.P.

(c) in G.P. (d) in H.P.

47. If $a_1, a_2, a_3, ..., a_n$ are in H.P. then

$$\frac{a_1}{a_2+a_3+...+a_n}, \frac{a_2}{a_1+a_3+...+a_n}, \frac{a_3}{a_1+a_2+a_4+...+a_n},$$

$....., \frac{a_n}{a_1+a_2+....+a_n}$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

Arithmetico-geometric Progression

48. The sum to n terms of the series

$1+2\left(1+\frac{1}{n}\right)+3\left(1+\frac{1}{n}\right)^2+....$ is given by

(a) n^2 (b) $n(n+1)$

(c) $n(1+1/n)^2$ (d) none of these

49. $1+2.2+3.2^2+4.2^3+....+100.2^{99}$ equals

(a) 99.2^{100} (b) 100.2^{100}

(c) $1+99.2^{100}$ (d) none of these

50. Sum of infinite terms of series $3+5.\frac{1}{4}+7.\frac{1}{4^2}+....$ is

(a) 33/4 (b) 11/4

(c) 44/9 (d) 44/8

51. The sum of the series

$1+3x+6x^2+10x^3+......\infty$ is (where $|x|<1$)

(a) $\frac{1}{(1-x)^2}$ (b) $\frac{1}{1-x}$

(c) $\frac{1}{(1+x)^2}$ (d) $\frac{1}{(1-x)^3}$

Summation of Series

52. Sum of n term of series $1.3+3.5+5.7+......$ is

(a) $\frac{1}{3}(n(n+1)(2n+1))+n$

(b) $\frac{2}{3}(n(n+1)(2n+1))-n$

(c) $\frac{2}{3}(n(n-1)(2n-1))-n$

(d) none of these

53. If $1+2+3+.....+n=45$, then $1^3+2^3+3^3+....+n^3$ is

(a) $(45)^2$ (b) $(45)^3$

(c) $(45)^2+45$ (d) none of these

54. The sum of series $1.3^2+2.5^2+3.7^2+...$ upto 20 terms is

(a) 188090 (b) 189080

(c) 199080 (d) None

55. Sum of the series $4+6+9+13+18+.........$ n terms, is

(a) $\frac{n}{6}(n^2+3n+20)$ (b) $n^2+3n+20$

(c) $\frac{n}{3}(n^2+3n+20)$ (d) None of these

56. Sum of the series $1+4+13+40+121+.......$ 16 terms, is

(a) $(3^{17}-35)/4$ (b) $3^{17}-35$

(c) $(3^{17}-33)/2$ (d) $(3^{17}-32)/4$

57. The sum to n terms of the series $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.....$ is

(a) $\frac{1}{2n+1}$ (b) $\frac{2n}{2n+1}$

(c) $\frac{n}{2n+1}$ (d) $\frac{2n}{n+1}$

58. If $t_n = \frac{1}{4}(n+2)(n+3)$ for $n = 1, 2, 3, \ldots\ldots\ldots$, then

$$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \ldots\ldots + \frac{1}{t_{2003}} =$$

(a) $\frac{4006}{3006}$ (b) $\frac{4003}{3007}$

(c) $\frac{4006}{3008}$ (d) $\frac{4006}{3009}$

59. The sum to n terms of the series

$$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots\ldots, \text{ is}$$

(a) $\frac{6n}{n+1}$ (b) $\frac{9n}{n+1}$

(c) $\frac{12n}{n+1}$ (d) $\frac{3n}{n+1}$

Numerical Value Type Questions

60. If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is

61. If a_n be the n^{th} term of an AP and if $a_7 = 15$, then the value of the common difference that would make $a_2a_7a_{12}$ greatest is

62. The 10th common term between the two arithmetic progressions 3, 7, 11, 15 ..... and 1, 6, 11, 16 .... is

63. If $(x + 1)$, $3x$ and $(4x + 2)$ are first three terms of an AP then its 5th term is

64. If first term of an AP is 5, last term is 45 and the sum of the 'n' terms is 400, then the number of terms are

65. The value of n, for which $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is A.M. between a and b is

66. Six arithmetic means are inserted between 1 and 9/2, the 4^{th} arithmetic mean is

67. n AM's are inserted between 2 and 38. If third AM is 14 then n is equal to

68. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be a GP such that $\frac{a_4}{a_6} = \frac{1}{4}$ and $a_2 + a_5 = 216$. Then integral value of a_1 is

69. The second, third and sixth terms of an A.P. are consecutive terms of a G.P. The common ratio of the G.P. is

70. If p^{th}, q^{th} and r^{th} terms of an A.P. are equal to corresponding terms of a G.P. and these terms are respectively x, y, z, then $x^{y-z} \cdot y^{z-x} \cdot z^{x-y}$ equals

71. If the sum of first 6 terms of a G.P. is nine times of the sum of its first three terms, then its common ratio is

72. The value of $9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} \ldots\ldots$ to ∞, is

73. If r^{th} term of a series is $(2r + 1)\, 2^{-r}$, then sum of its infinite terms is

74. If $3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \ldots\ldots\ldots$ to $\infty = 8$, then the value of d is

75. 9th term of the sequence 1, 1, 2, 3, 5, ... is

76. The sum of all numbers between 100 and 10,000 which are of the form n^3 $(n \in N)$ is equal to

77. Sum of the series $3 + 7 + 14 + 24 + 37 + \ldots$ 10 terms, is

78. The limiting value of the sum to n terms of the series

$$\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \ldots\ldots\ldots \text{ as } n \to \infty \text{ is}$$

79. If the value of

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \ldots\ldots\ldots \text{upto}\infty\right)^{\log_{(0,25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots\ldots \text{upto}\infty\right)} \text{ is ,}$$

then I^2 is equal to _____?

80. The mean of 10 numbers $7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots\ldots$ is ________?

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. The sum of first 9 terms of the series

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+......$ is **(2015)**

(a) 142 (b) 192

(c) 71 (d) 96

2. If m is the A.M. of two distinct real number l and n (l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4+2G_2^4+G_3^4$ equals. **(2015)**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$

(c) $4\,l^2mn$ (d) $4\,lm^2n$

3. The sum of the 3rd and the 4th term of a G.P. is 60 and the product of its first three terms is 1000. If the first term of this G.P. is positive, then its 7th term is :

(2015/Online Set–2)

(a) 7290 (b) 640

(c) 2430 (d) 320

4. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P, then the common ratio of this G.P. is : **(2016)**

(a) $\frac{4}{3}$ (b) 1

(c) $\frac{7}{4}$ (d) $\frac{8}{5}$

5. If the sum of the first ten terms of the series

$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)+......,$ is

$\frac{16}{5}m$, then m is equal to : **(2016)**

(a) 101 (b) 100

(c) 99 (d) 102

6. Let x, y, z be positive real numbers such that $x+y+z=12$ and $x^3y^4z^5=(0.1)(600)^3$. Then $x^3+y^3+z^3$ is equal to : **(2016/Online Set–1)**

(a) 270 (b) 258

(c) 342 (d) 216

7. Let $a_1,a_2,a_3,........,a_n$ be in A.P.

If $a_3+a_7+a_{11}+a_{15}=72$, then the sum of its first 17 terms is equal to : **(2016/Online Set–2)**

(a) 306 (b) 153

(c) 612 (d) 204

8. For any three positive real numbers a, b and c, $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$. Then: **(2017)**

(a) b, c and a are in G.P (b) b, c and a are in A. P

(c) a, b and c are in A.P (d) a, b and c are in G.P

9. If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to :

(2017/Online Set–1)

(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{3\sqrt{2}}{4}$

(c) $\frac{7\sqrt{3}}{12}$ (d) $\frac{5\sqrt{6}}{12}$

10. If the sum of the first n terms of the series $\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+......$ is $435\sqrt{3}$, then n equal :

(2017/Online Set–1)

(a) 18 (b) 15

(c) 13 (d) 29

11. If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is :

(2017/Online Set–2)

(a) 2 (b) $4^{\frac{1}{3}}$

(c) $4^{\frac{2}{3}}$ (d) 4

12. Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots\ldots + \frac{1+2+\ldots\ldots\ldots+n}{1^3+2^3+\ldots\ldots\ldots+n^3}$. If $100\,S_n = n$, then n is equal to : **(2017/Online Set–2)**

(a) 199 (b) 99

(c) 200 (d) 19

13. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to : **(2018)**

(a) 33 (b) 66

(c) 68 (d) 34

14. If $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 h_{10}$ equals : **(2018/Online Set–1)**

(a) 2560 (b) 2650

(c) 3200 (d) 1600

15. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval : **(2018/Online Set–1)**

(a) $(-\infty, -10]$ (b) $(-10, 0)$

(c) $(0, 10)$ (d) $[10, \infty)$

16. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is : **(2018/Online Set–2)**

(a) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$ (b) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$

(c) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4} - \frac{1}{\sqrt{2}}$

17. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \ge p$, is : **(2018/Online Set–2)**

(a) 9 (b) 7

(c) 11 (d) 5

18. Let $\frac{1}{x_1}, \frac{1}{x_2}, \ldots, \frac{1}{x_n}$ $(x_i \ne 0,\ for\ i = 1, 2, \ldots, n)$ be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to : **(2018/Online Set–3)**

(a) $\frac{1}{8}$ (b) 3

(c) $\frac{13}{8}$ (d) $\frac{13}{4}$

19. The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$, is **(2018/Online Set–3)**

(a) $38 + \frac{1}{2^{19}}$ (b) $38 + \frac{1}{2^{20}}$

(c) $39 + \frac{1}{2^{20}}$ (d) $39 + \frac{1}{2^{19}}$

20. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? **(8-04-2019/Shift-2)**

(a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (b) d, e, f are in A.P.

(c) d, e, f are in G.P. (d) are in G.P.

21. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to : **(8-04-2019/Shift-2)**

(a) $2 - \frac{3}{2^{17}}$ (b) $1 - \frac{11}{2^{20}}$

(c) $2 - \frac{11}{2^{19}}$ (d) $2 - \frac{21}{2^{20}}$

22. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3.....$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to: **(9-04-2019/Shift-1)**

(a) $(50, 50+46A)$ (b) $(50, 50+45A)$

(c) $(A, 50+45A)$ (d) $(A, 50+46A)$

23. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} terms is: **(9-04-2019/Shift-2)**

(a) -35 (b) 25

(c) -36 (d) -25

24. The sum of the series $1+2\times3+3\times5+4\times7+.....$ upto 11^{th} terms is: **(9-04-2019/Shift-2)**

(a) 915 (b) 946

(c) 945 (d) 916

25. If $a_1, a_2, a_3, ...$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to: **(10-04-2019/Shift-1)**

26. The sum $\frac{3\times1^3}{1^2} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + ...$ upto 10^{th} term, is: **(10-4-2019/Shift-1)**

27. The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + ... + \frac{1^3+2^3+3^3+....+15^3}{1+2+3+....+15} - \frac{1}{2}(1+2+3+....+15)$ is equal to : **(10-4-2019/Shift-2)**

(a) 620 (b) 1240

(c) 1860 (d) 660

28. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4^{th} terms of this A.P. is: **(10-04-2019/Shift-2)**

(a) $\frac{2}{3}a$ (b) 5a

(c) $\frac{7}{3}a$ (d) a

29. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to **(12-04-2019/Shift-1)**

(a) -260 (b) -410

(c) -320 (d) -380

30. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is ______. **(12-04-2019/Shift-2)**

(a) 200 (b) 280

(c) 120 (d) 150

31. If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then sum of all the integral values of x which don't satisfy the above equation is: **(9-01-2019/Shift-1)**

32. Let $a_1, a_2, ..., a_{30}$ be an A.P.,

$S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$

If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to: **(9-01-2019/Shift-1)**

33. The sum of the following series

$1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+....+5^2)}{11} + ...$ up to 15 terms, is: **(9-01-2019/Shift-2)**

(a) 7520 (b) 7510

(c) 7830 (d) 7820

34. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a GP., then $\frac{a}{c}$ is equal to **(9-01-2019/Shift-2)**

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{7}{13}$ (d) 4

35. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is: **(10-1-2019/Shift-1)**

(a) 1256 (b) 1465

(c) 1365 (d) 1356

36. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is : **(11-01-2019/Shift-1)**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{2}{9}$ (d) $\frac{4}{9}$

37. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals : **(11-01-2019/Shift-1)**

(a) 5^4 (b) $4(5^2)$

(c) 5^3 (d) $2(5^2)$

38. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression

$\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is : **(11-01-2019/Shift-2)**

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{m+n}{6mn}$

39. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45°)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to : **(11-01-2019)**

(a) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (b) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

(c) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$ (d) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

40. If 19th term of a non-zero A.P. is zero, then its (49th term):(29th term) is : **(11-01-2019/Shift-2)**

(a) 4 : 1 (b) 1 : 3

(c) 3 : 1 (d) 2 : 1

41. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an AP. Then the sum of the original three terms of the given G.P. is : **(12-01-2019/Shift-1)**

42. Let $S_k = \frac{1+2+3+...+k}{k}$. If $S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, Then A is equal to **(12-01-2019/Shift-1)**

43. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$ then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to **(12-01-2019/Shift-2)**

(a) 0 (b) -1

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

44. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^2 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225k then k is equal to _____. **(12-01-2019/Shift-2)**

(a) 108 (b) 27

(c) 54 (d) 9

45. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lies in : **(2-9-2020/Shift-1)**

(a) $(-\infty, -9] \cup [3, \infty)$ (b) $[-3, \infty)$

(c) $(-\infty, 9]$ (d) $(-\infty, -3] \cup [9, \infty)$

46. If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series

$(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + ...$ is: **(2-9-2020/Shift-1)**

(a) $\frac{x+y+xy}{(1-x)(1-y)}$ (b) $\frac{x+y-xy}{(1-x)(1-y)}$

(c) $\frac{x+y+xy}{(1+x)(1+y)}$ (d) $\frac{x+y-xy}{(1+x)(1+y)}$

47. If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, \ldots$ is $0\ (a_1 \neq 0)$ then the sum of the A.P., $a_1, a_3, a_5, \ldots, a_{23}$ is ka_1, where k is equal to : **(2-09-2020/Shift-2)**

(a) $-\frac{121}{10}$ (b) $-\frac{72}{5}$

(c) $\frac{72}{5}$ (d) $\frac{121}{10}$

48. Let S be the sum of the first 9 terms of the series :

$$\{x+ka\}+\{x^2+(k+2)a\}+\{x^3+(k+4)a\}$$
$$+\{x^4+(k+6)a\}+\ldots\ldots$$

where $a \neq 0$ and $a \neq 1$.

If $S=\frac{x^{10}-x+45a(x-1)}{x-1}$, then k is equal to :

(2-09-2020/Shift-2)

(a) 3 (b) –3

(c) 1 (d) –5

49. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is : **(3-09-2020/Shift-1)**

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{1}{7}$

50. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+\ldots\ldots\ to\ \infty\right)}$ is equal to ...

(3-9-2020/Shift-1)

51. If the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+\ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then :

(3-09-2020/Shift-2)

(a) $n=60$ (b) $n=41$

(c) n^{th} term is – 4 (d) nth term is $-4\frac{2}{5}$

52. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to ....

(3-09-2020/Shift-2)

53. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)$

$+\ldots\ldots+(1-20^2.19)=\alpha-220\beta$, then an ordered pair

(α, β) is equal to : **(4-09-2020/Shift-1)**

(a) (10,97) (b) (11,103)

(c) (11,97) (d) (10,103)

54. The minimum value of $2^{\sin x}+2^{\cos x}$ is :

(4-9-2020/Shift-2)

(a) $2^{1-\sqrt{2}}$ (b) $2^{1-\frac{1}{\sqrt{2}}}$

(c) $2^{-1+\sqrt{2}}$ (d) $2^{-1+\frac{1}{\sqrt{2}}}$

55. Let $a_1, a_2, \ldots, a_n$ be a given A.P. whose common difference is an integer and $S_n=a_1+a_2+\ldots+a_n$. If $a_1=1$, $a_n=300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to: **(4-9-2020/Shift-2)**

(a) (2480, 248) (b) (2480, 249)

(c) (2490, 249) (d) (2490, 248)

56. If $2^{10}+2^9\cdot3^1+2^8\cdot3^2+\ldots+2.3^9+3^{10}=S-2^{11}$, then S is equal to: **(5-09-2020/Shift-1)**

(a) 3^{11} (b) $\frac{3^{11}}{2}+2^{10}$

(c) 2.3^{11} (d) $3^{11}-2^{12}$

57. If $3^{2\sin 2\alpha-1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P. is:

(5-09-2020/Shift-1)

(a) 65 (b) 81

(c) 78 (d) 66

58. If the sum of the first 20 terms of the series $\log_{(7^{1/2})}x+\log_{(7^{1/3})}x+\log_{(7^{1/4})}x+\ldots$ is 460, then x is equal to: **(5-09-2020/Shift-2)**

(a) $7^{1/2}$ (b) 7^2

(c) e^2 (d) $7^{46/21}$

59. If the sum of the second, third and fourth terms of a positive term G.P is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P is: **(5-09-2020/Shift-2)**

(a) $\frac{2}{13}(3^{50}-1)$ (b) $\frac{1}{26}(3^{49}-1)$

(c) $\frac{1}{13}(3^{50}-1)$ (d) $\frac{1}{26}(3^{50}-1)$

60. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)=0$. Then: **(6-09-2020/Shift-1)**

(a) a, c, p are in G.P. (b) a, b, c, d are in G.P.

(c) a, b, c, d are in A.P. (d) a, c, p are in A.P.

61. If $f(x+y)=f(x)f(y)$ and

$\sum_{x=1}^{\infty} f(x)=2,\ x, y\in N,$ where N is the set of all natural number, then the value of $\frac{f(4)}{f(2)}$ is :

(6-09-2020/Shift-1)

(a) $\frac{2}{3}$ (b) $\frac{1}{9}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

62. The common difference of the A.P. $b_1, b_2, ...b_m$ is 2 more than the common difference of A.P. $a_1, a_2, ...a_n$ If $a_{40}=-159,\ a_{100}=-399$ and $b_{100}=a_{70}$, then b_1 is equal to : **(6-09-2020/Shift-2)**

(a) -127 (b) 81

(c) 127 (d) -81

63. Suppose that function $f:R\to R$ satisfies $f(x+y)=f(x)f(y)$ for all $x, y\in R$ and f(1) = 3. If

$\sum_{i=1}^{n} f(i)=363$, then n is equal to ....

(6-09-2020/Shift-2)

64. Five number are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$ then the greatest number amongst them is **(7-01-2020/Shift-1)**

(a) 16 (b) 27

(c) 7 (d) $\frac{21}{2}$

65. If the sum of the first 40 terms of the series,

$3+4+8+9+13+14+18+19+.....$ is (102)m, then m is equal to : **(7-01-2020/Shift-2)**

(a) 10 (b) 25

(c) 5 (d) 20

66. Let $a_1, a_2, a_3, ...$ be a G.P. such that $a_1<0$, $a_1+a_2=4$ and $a_3+a_4=16$. If $\sum_{i=1}^{9} a_i=4\lambda$, then λ is equal to:

(7-01-2020/Shift-2)

(a) 171 (b) $\frac{511}{3}$

(c) -171 (d) -513

67. Let $f:R\to R$ be such that for all

$x\in R, \left(2^{1+x}+2^{1-x}\right), f(x)$ and $\left(3^{x}+3^{-x}\right)$ are in A.P.,

then the minimum value of $f(x)$ is:

(8-01-2020/Shift-1)

(a) 0 (b) 4

(c) 3 (d) 2

68. The sum $\sum_{k=1}^{20}(1+2+3+....+k)$ is ______.

(8-01-2020/Shift-1)

69. If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is:

(8-01-2020/Shift-2)

(a) $50\frac{1}{4}$ (b) 100

(c) 50 (d) $100\frac{1}{2}$

70. The sum, $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to ________.

(8-01-2020/Shift-2)

71. The product $2^{\frac{1}{4}}\times 4^{\frac{1}{16}}\times 8^{\frac{1}{48}}\times 16^{\frac{1}{128}}$.....to ∞ is equal to:

(9-01-2020/Shift-1)

(a) $2^{\frac{1}{4}}$ (b) 2

(c) $2^{\frac{1}{2}}$ (d) 1

72. If $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta$ and $y=\sum_{n=0}^{\infty}\cos^{2n}\theta$, where $0<\theta<\frac{\pi}{4}$, then: **(9-1-2020/Shift-2)**

(a) $y(1+x)=1$ (b) $x(1-y)=1$

(c) $y(1-x)=1$ (d) $x(1+y)=1$

73. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100}a_{2n+1}=200$ and $\sum_{n=1}^{100}a_{2n}=100$ then $\sum_{n=1}^{200}a_n$ is equal to: **(9-1-2020/Shift-2)**

(a) 300 (b) 175

(c) 225 (d) 150

74. The number of terms common to the A.P.'s 3,7,11,…407 and 2,9,16,…709 is _______. **(9-1-2020/Shift-2)**

75. If sum of the first 21 terms of the series $\log_{9^{1/2}}x+\log_{9^{1/3}}x+\log_{9^{1/4}}x+$........, where $x>0$ is 504, then x is equal to? **(20-07-2021/Shift-2)**

(a) 81 (b) 243

(c) 7 (d) 9

76. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1=1, a_2=1$ and $a_{n+2}=2a_{n+1}+a_n$ for all $n\geq 1$. Then the value of

$47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to _______? **(20-07-2021/Shift-2)**

77. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n}=3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is ?

(25-07-2021/Shift-1)

(a) 4 (b) 6

(c) 8 (d) 2

78. If $\log_3 2, \log_3(2^x-5), \log_3\left(2^x-\frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to_____.

(27-07-2021/Shift-1)

79. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x-2y|$ is equal to: **(27-07-2021/Shift-2)**

(a) 0 (b) 3

(c) 4 (d) 1

80. The sum of all the elements in the set $\{n\in\{1,2,.....,100\}\mid \text{H.F.C of n and 2040 is 1}\}$ is equal to ________. **(22-07-2021/Shift-2)**

81. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10}=530, S_5=140$, then $S_{20}-S_6$ is equal to: **(22-07-2021/Shift-2)**

(a) 1852 (b) 1842

(c) 1872 (d) 1862

82. If $[x]$ be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to: **(25-07-2021/Shift-2)**

(a) –2 (b) 4

(c) 2 (d) 0

83. Let

$S_n = 1\cdot(n-1)+2\cdot(n-2)+3\cdot(n-3)+.....+(n-1)\cdot 1, n \geq 4$

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!}-\frac{1}{(n-2)!}\right)$ is equal to

(01-09-2021/Shift-2)

(a) $\frac{e-1}{3}$ (b) $\frac{e-2}{6}$

(c) $\frac{e}{6}$ (d) $\frac{e}{3}$

84. Let $a_1, a_2, ..., a_{21}$ be an AP such that $\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to

(01-09-2021/Shift-2)

(a) 72 (b) 57

(c) 36 (d) 48

85. $\lim_{x\to 2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to

(26-08-2021/Shift-2)

(a) $\frac{7}{36}$ (b) $\frac{5}{24}$

(c) $\frac{1}{5}$ (d) $\frac{9}{44}$

86. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is ________. **(26-08-2021/Shift-2)**

87. Let $a_1, a_2 .... a_{10}$ be an AP with common difference -3 and $b_1, b_2 .... b_{10}$ be a GP with common ration 2. Let $c_k = a_k + b_r, k = 1, 2, ..., 10$. If $C_2 = 12$ and $C_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to ________. **(26-08-2021/Shift-2)**

88. If the sum of an infinite GP $a, ar, ar^2, ar^3, ...$ is 15 and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, ...$ is: **(26-08-2021/Shift-1)**

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$

(c) $\frac{25}{2}$ (d) $\frac{9}{2}$

89. The sum of the series

$\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+.......+\frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is:

(26-08-2021/Shift-1)

(a) $1-\frac{2^{101}}{4^{101}-1}$ (b) $1+\frac{2^{101}}{4^{101}-1}$

(c) $1-\frac{2^{100}}{4^{100}-1}$ (d) $1+\frac{2^{100}}{4^{100}-1}$

90. If $0 < x < 1$ and $y = \frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+...$, then the value of e^{1+y} at $x = \frac{1}{2}$ is: **(27-08-2021/Shift-2)**

(a) $\frac{1}{2}e^2$ (b) $2e$

(c) $2e^2$ (d) $\frac{1}{2}\sqrt{e}$

91. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to? **(31-08-2021/Shift-1)**

(a) $7+3\sqrt{3}$ (b) $7-\sqrt{3}$

(c) $7-7\sqrt{3}$ (d) $7+\sqrt{3}$

92. The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2}+\frac{5}{2^2 \times 3^2}+\frac{7}{3^2 \times 4^2}+....$ is? **(31-08-2021/Shift-1)**

(a) $\frac{120}{121}$ (b) 1

(c) $\frac{143}{144}$ (d) $\frac{99}{100}$

93. Let $a_1, a_2, a_3, ...$ be an A.P. If

$\frac{a_1+a_2+...+a_{10}}{a_1+a_2+...+a_p}=\frac{100}{p^2}, p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to:

(31-08-2021/Shift-2)

(a) $\frac{121}{100}$ (b) $\frac{100}{121}$

(c) $\frac{19}{21}$ (d) $\frac{21}{19}$

94. If $S=\frac{7}{5}+\frac{9}{5^2}+\frac{13}{5^3}+\frac{19}{5^4}+...$, then $160S$ is equal to ___

(31-08-2021/Shift-2)

95. If for $x, y \in R$, $x>0$,

$y=(\log_{10} x)+\left(\log_{10} x^{\frac{1}{3}}\right)+\left(\log_{10} x^{\frac{1}{9}}\right)+...$ upto ∞ terms

and $\frac{2+4+6+...+2y}{3+6+9+...+3y}=\frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to : **(27-08-2021/Shift-1)**

(a) $(10^6, 6)$ (b) $(10^6, 9)$

(c) $(10^2, 3)$ (d) $(10^4, 6)$

96. If $0<x<1$, then $\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+...$, is equal to:

(27-08-2021/Shift-1)

(a) $x\left(\frac{1+x}{1-x}\right)+\log_e(1-x)$

(b) $x\left(\frac{1-x}{1+x}\right)+\log_e(1-x)$

(c) $\frac{1-x}{1+x}+\log_e(1-x)$

(d) $\frac{1+x}{1-x}+\log_e(1-x)$

97. Let

$S_n(x)=\log_{a^{\frac{1}{2}}} x+\log_{a^{\frac{1}{3}}} x+\log_{a^{\frac{1}{6}}} x+\log_{a^{\frac{1}{11}}} x+\log_{a^{\frac{1}{18}}} x$

$+\log_{a^{\frac{1}{27}}} x+......$ upto n-terms where $a>1$.

If $S_{24}(x)=1093$ and $S_{12}(2x)=265$, then the value of a is equal to ___. **(16-03-2021/Shift-2)**

98. Let $\frac{1}{16}, a$ and b in G.P. $\frac{1}{a}, \frac{1}{b}, 6$ are in A.P., where $a, b>0$. Then $72(a+b)$ is equal to _______.

(16-03-2021/Shift-2)

99. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to:

(16-03-2021/Shift-1)

(a) $\tan^{-1}\left(\frac{3}{2}\right)$ (b) $\tan^{-1}(3)$

(c) $\frac{\pi}{2}$ (d) $\cot^{-1}\left(\frac{3}{2}\right)$

100. Let $[x]$ denote greatest integer less than or equal to x. If for $n \in N$, $\left(1-x+x^3\right)^n = \sum_{j=0}^{3n} a_j x^j$, then

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$$ is equal to: **(16-03-2021/Shift-1)**

(a) 2^{n-1} (b) n

(c) 1 (d) 2

101. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _______. **(16-03-2021/Shift-1)**

102. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to : **(18-03-2021/Shift-2)**

(a) 3000 (b) 5000

(c) 7000 (d) 1000

103. If $\sum_{r=1}^{10} r!\,(r^3 + 6r^2 + 2r + 5) = \alpha\,(11!)$, then the value of α is equal to _______. **(18-03-2021/Shift-2)**

104. The value $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\ldots\ldots \infty}}}}$ is equal to :

(18-03-2021/Shift-1)

(a) $2+\sqrt{3}$ (b) $4+\sqrt{3}$

(c) $3+2\sqrt{3}$ (d) $1.5+\sqrt{3}$

105. $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \ldots\ldots + \frac{1}{(201)^2-1}$ is equal to:

(18-03-2021/Shift-1)

(a) $\frac{101}{408}$ (b) $\frac{101}{404}$

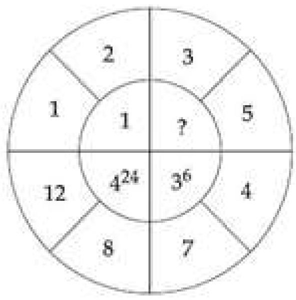
(c) $\frac{99}{400}$ (d) $\frac{25}{101}$

106. If α, β are natural numbers such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through (α, β) and origin is **(18-03-2021/Shift-1)**

(a) 540 (b) 510

(c) 550 (d) 530

107. The missing value in the following figure is ....



(18-03-2021/Shift-1)

108. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α, is ____. **(24-02-2021/Shift-2)**

109. The minimum of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to: **(25-02-2021/Shift-2)**

(a) $a+1$ (b) $2\sqrt{a}$

(c) $2a$ (d) $a + \frac{1}{a}$

110. If $0<\theta,\phi<\frac{\pi}{2}, x=\sum_{n=0}^{\infty}\cos^{2n}\theta, y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ and

$z=\sum_{n=0}^{\infty}\cos^{2n}\theta\cdot\sin^{2n}\phi$ then: **(25-02-2021/Shift-1)**

(a) $xyz=4$ (b) $xy+yz+zx=z$

(c) $xy+z=(x+y)z$ (d) $xy-z=(x+y)z$

111. Let $A_1, A_2, A_3, \ldots$ be squares such that for each $n\geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is _____. **(25-02-2021/Shift-1)**

112. The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to **(26-02-2021/Shift-2)**

(a) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$ (b) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(c) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$ (d) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$

113. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2-9x+5=0$, then $p+q$ is equal to _______. **(26-02-2021/Shift-2)**

114. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{th}, 6^{th}$ and 8^{th} terms is equal to : **(26-02-2021/Shift-1)**

(a) 26 (b) 30

(c) 32 (d) 35

115. The sum of the infinite series

$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots\ldots$ is equal to

(26-02-2021/Shift-1)

(a) $\frac{13}{4}$ (b) $\frac{15}{4}$

(c) $\frac{9}{4}$ (d) $\frac{11}{4}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. If the sum to n terms of a series be $5n^2 + 2n$, then second term is

(a) 15 (b) 17

(c) 10 (d) 5

2. If the sum of the 10 terms of an A.P. is 4 times to the sum of its 5 terms, then the ratio of first term and common difference is

(a) 1 : 2 (b) 2 : 1

(c) 2 : 3 (d) 3 : 2

3. In a G.P. if the $(m+n)^{th}$ term be p and $(m-n)^{th}$ term be q then the m^{th} term is

(a) $\sqrt{pq}$ (b) $\sqrt{p/q}$

(c) $\sqrt{q/p}$ (d) $\sqrt{p/q}$

4. The least value of n for which the sum $1 + 3 + 3^2 + ...$ to n terms is greater than 7000 is

(a) 7 (b) 9

(c) 11 (d) 13

5. A G.P. consist of even number of terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in the even places is S_2, then the common ratio of the G.P. is

(a) $\frac{S_1}{S_2}$ (b) $\frac{S_2}{S_1}$

(c) $\frac{2S_1}{S_2}$ (d) $\frac{S_2}{2S_1}$

6. The value of $9^{1/3} . 9^{1/9} . 9^{1/27}$ ...... will be

(a) 3^2 (b) 3^3

(c) 3 (d) 3^∞

7. If the third term of a G.P. is 4, then the product of its first 5 terms is

(a) 4^3 (b) 4^4

(c) 4^5 (d) None

8. Six arithmetic means are inserted between 1 and 9/2, the 4^{th} arithmetic mean is

(a) 2 (b) 1

(c) 3 (d) 4

9. If one G.M., g and two A.M.'s p and q are inserted between two number a and b, then $\frac{(2p-q)(p-2q)}{g^2} =$

(a) 1 (b) –1

(c) 2 (d) –3

10. If a, b and c are positive real numbers, then the least value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is

(a) 9 (b) 3

(c) 10/3 (d) none of these

11. If a, b and c are positive real numbers then $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$ is greater than or equal to

(a) 3 (b) 6

(c) 27 (d) none of these

12. The sum of the series $1^3 + 3^3 + 5^3 + ...$ to 20 terms is

(a) 319600 (b) 321760

(c) 306000 (d) 347500

13. If a, 4, b are in AP; a, 2, b are in G.P., then a, 1, b are in

(a) HP (b) AP

(c) GP (d) none of these

14. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. where $a_i > 0 \ \forall i$, then

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} =$$

(a) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ (b) $\frac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$

(c) $\frac{n-1}{\sqrt{a_1}-\sqrt{a_n}}$ (d) $\frac{n+1}{\sqrt{a_1}-\sqrt{a_n}}$

15. The first and last term of an A.P. are a and l respectively. If s be the sum of all terms of the A.P., then common difference is

(a) $\frac{\ell^2 - a^2}{2s - (\ell + a)}$ (b) $\frac{\ell^2 - a^2}{2s - (\ell - a)}$

(c) $\frac{\ell^2 + a^2}{2s + (\ell + a)}$ (d) $\frac{\ell^2 + a^2}{2s - (\ell + a)}$

16. Given p A.P.'s, each of which consists of n terms. If their first terms are 1, 2, 3, ..., p and common differences are 1, 3, 5, ..., 2p – 1 respectively, then sum of the terms of all the progressions is

(a) $\frac{1}{2}np\,(np+1)$ (b) $\frac{1}{2}n\,(p+1)$

(c) np (n + 1) (d) none of these

17. If the sum of m consecutive odd integers is m^4, then the first integer is

(a) $m^3 + m + 1$ (b) $m^3 + m - 1$

(c) $m^3 - m - 1$ (d) $m^3 - m + 1$

18. If α, β be roots of $x^2 - 3x + a = 0$ and γ, δ are the roots of $x^2 - 12x + b = 0$ and α, β, γ, δ (in order) form an increasing G.P., then

(a) a = 3, b = 12 (b) a = 12, b = 13

(c) a = 2, b = 32 (d) a = 4, b = 16

19. The sum of the first 10 terms of $\frac{3}{2} + \frac{5}{4} + \frac{9}{8} + \frac{17}{16} + \ldots$ is

(a) $10 - 2^{-10}$ (b) $9 - 2^{-10}$

(c) $11 - 2^{-10}$ (d) none of these

20. The sum of an infinite G.P. series is 3. A series which is formed by squares of its terms have the sum also 3. First series will be

(a) $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16} \ldots$ (b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$ (d) $1, -\frac{1}{3}, \frac{1}{3^2}, -\frac{1}{3^3}, \ldots$

21. The sum of the series 5.05 + 1.212 + 0.29088 + ... ∞ is

(a) 6.93378 (b) 6.87342

(c) 6.74384 (d) 6.64474

22. If a, b, c are in H.P., then the value of $\frac{b+a}{b-a} + \frac{b+c}{b-c}$ is

(a) 0 (b) 1

(c) 2 (d) 3

23. The harmonic mean of roots of the equation

$(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + (8 + 2\sqrt{5}) = 0$ is

(a) 2 (b) 4

(c) 6 (d) 8

24. The harmonic mean between two numbers is $14\frac{2}{5}$ and the geometric mean is 24. The greatest number between them is :

(a) 72 (b) 36

(c) 18 (d) 60

25. Let x be the arithmetic mean and y, z be the two geometric means between any two positive number. Then value of $\frac{y^3 + z^3}{xyz}$ is

(a) 2 (b) 3

(c) 1/2 (d) 3/2

26. If $x_1^2 + x_2^2 + x_3^2 + \ldots. + x_{50}^2 = 50$ and $\frac{1}{x_1^2\, x_2^2 \ldots x_{50}^2} = A$ then

(a) $A_{minimum} = 1$ (b) $A_{maximum} = 1$

(c) $A_{minimum} = 50$ (d) $A_{maximum} = 50$

27. A series whose nth term is $\frac{n}{x} + y$, the sum of r terms will be

(a) $\frac{r(r+1)}{2x} + ry$ (b) $\frac{r(r-1)}{2x}$

(c) $\frac{r}{2x}(r-1) - ry$ (d) $\frac{r\,(r+1)}{2y} - rx$

28. The sum of series $1.3^2 + 2.5^2 + 3.7^2 + \ldots$ upto 20 terms is

(a) 188090 (b) 189080

(c) 199080 (d) None

29. $1^2 - 2^2 + 3^2 - 4^2 + \ldots$ to 21 terms =

(a) 210 (b) 231

(c) –210 (d) – 231

30. $1 + 3 + 7 + 15 + 31 + \ldots$ to n terms =

(a) $2^{n+1} - n$ (b) $2^{n+1} - n - 2$

(c) $2^n - n - 2$ (d) None

31. If nth term of a series is $\frac{1}{(n+1)(n+3)}$, then sum of infinite terms of the series

(a) 3/2 (b) 1/2

(c) 5/2 (d) 5/12

32. Let $\sum_{r=1}^{n} r^4 = f(n)$, then $\sum_{r=1}^{n} (2r-1)^4$ is equal to

(a) $f(2n) - 16f(n)$ (b) $f(2n) - 7f(n)$

(c) $f(2n-1) - 8(f(n)$ (d) none of these

33. The sum of the n terms of the series $1 + (1+3) + (1+3+5) \ldots$

(a) n^2 (b) $\left[\frac{n(n+1)}{2}\right]^2$

(c) $\left[\frac{n(n+1)(2n+1)}{6}\right]$ (d) none of these

34. Consider the sequence 1, 2, 2, 3, 3, 3 ... where n occurs n times. The number that occurs as 2007^{th} term is

(a) 61 (b) 62

(c) 63 (d) 64

35. If p, q, r are in A.P., then pth, qth and rth terms of any G.P. are in

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

36. If $ln(x+z) + ln(x-2y+z) = 2\, ln(x-z)$, then x, y, z are in

(a) A.P (b) G.P

(c) H.P (d) none of these

37. If a, b, c are three unequal numbers such that a, b, c are in A.P. and b – a, c – b, a are in G.P., then a : b : c =

(a) 2 : 3 : 5 (b) 1 : 2 : 4

(c) 1 : 3 : 5 (d) 1 : 2 : 3

38. If a, b, c are 3 positive numbers in A.P. and a^2, b^2, c^2 are in H.P., then

(a) a = b = c (b) 2b = 3a + c

(c) $b^2 = \left(\frac{ac}{8}\right)^{1/2}$ (d) None

39. The sum of three consecutive terms in G.P. is 14. If 1 is added to the first and the second term and 1 subtracted from the third, the resulting new terms are in A.P. Then the lowest of the original terms is

(a) 1 (b) 2

(c) 4 (d) 8

40. If $5x - y, 2x + y, x + 2y$ are in A.P. and $(x-1)^2, (xy+1), (y+1)^2$ are in G.P., $x \neq 0$, then $x + y =$

(a) $\frac{3}{4}$ (b) 3

(c) –5 (d) none of these

41. Four distinct integers a, b, c, d are in A.P. If $a^2 + b^2 + c^2 = d$, then a + b + c + d =

(a) 1 (b) 0

(c) –1 (d) none of these

42. The sum of n terms of the following series $1 + (1 + x) + (1 + x + x^2) + \ldots$ will be

(a) $\frac{1-x^n}{1-x}$ (b) $\frac{x(1-x^n)}{1-x}$

(c) $\frac{n(1-x) - x(1-x^n)}{(1-x)^2}$ (d) none

43. If the sum of n terms of G.P. is S, product is P and sum of their inverses is R, then $P^2 =$

(a) R/S (b) S/R

(c) $(R/S)^n$ (d) $(S/R)^n$

44. If x = 111 ... 1 (20 digits), y = 333...3 (10 digits) and z = 222...2 (10 digits), then $\frac{x - y^2}{z} =$

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 3

45. The largest positive term of the H.P., whose first two terms are $\frac{2}{5}$ and $\frac{12}{23}$ is

(a) $\frac{13}{2}$ (b) 6

(c) $\frac{15}{2}$ (d) 8

46. If a, b, c are in H.P., then which one of the following is true

(a) $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{b}$ (b) $\frac{ac}{a+c}=b$

(c) $\frac{b+a}{b-a}+\frac{b+c}{b-c}=1$ (d) None

47. $\sum_{n=1}^{\infty}\frac{n}{4n^4+1}$ equals to:

(a) 0 (b) 1

(c) ∞ (d) 1/4

48. If a, b, c are in A.P., then $\frac{a}{bc},\frac{1}{c},\frac{2}{b}$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) None

Objective Questions II [One or more than one correct option]

49. If the first two terms of a progression are $\log_2 256$ and $\log_3 81$ respectively, then which of the following statements are true :

(a) If third term is $\log_4 16$, then the terms are in G.P.

(b) If third term is $2\log_6 1$, then the terms are in A.P.

(c) If third term is $\frac{2}{3}\log_2 16$, then the terms are in H.P.

(d) If the third term is $\log_2 8$, then terms are in A.P.

50. If the first and the (2n – 1) th term of an AP, GP and HP are equal and their nth terms are a, b and c respectively, then which of the following may be correct.

(a) $a=b=c$ (b) $a\ge b\ge c$

(c) $a+c=b$ (d) $ac-b^2=0$

Numerical Value Type Questions

51. Two consecutive numbers from 1, 2, 3 .... n are removed. The arithmetic mean of remaining n – 2 numbers is $\frac{105}{4}$. Then n must be

52. The value of n for which

$1.2^1+2.2^2+3.2^3+.......+n.2^n=2^{n+10}+2$, is

53. If $S_n=\frac{1}{2n}-\frac{1}{n+1}+\frac{1}{2(n+2)}$ then $\sum_{n=1}^{15}S_n=\frac{135}{k}$, then the numerical quantity k must be

Assertion & Reason

Use the following codes to answer the questions

(A) If both ASSERTION and REASON are correct and reason is the correct explanation of ASSERTION.

(B) If both ASSERTION and REASON are true but and REASON is not the correct explanation of ASSERTION.

(C) If ASSERTION is true but REASON is false.

(D) If ASSERTION is false but REASON is true.

(E) If ASSERTION and REASON are both false.

54. **Assertion :** There exists an A.P. whose three terms are $\sqrt{2},\sqrt{3},\sqrt{5}$.

Reason : There exists distinct real numbers p, q, r satisfying

$\sqrt{2}=A+(p-1)\,d$, $\sqrt{3}=A+(q-1)\,d$,

$\sqrt{5}=A+(r-1)\,d$.

(a) A (b) B (c) C

(d) D (e) E

55. **Assertion :** If all terms of a series with positive terms are smaller than 10^{-5}, then the sum of the series upto infinity will be finite.

Reason : If $S_n<\frac{n}{10^5}$ then $\lim_{n\to\infty}S_n$ is finite.

(a) A (b) B (c) C

(d) D (e) E

56. **Assertion :** If three positive numbers in G.P. represent sides of a triangle, then the common ratio of the G.P. must lie between $\frac{\sqrt{5}-1}{2}$ and $\frac{\sqrt{5}+1}{2}$.

Reason : Three positive real numbers can form sides of a triangle if sum of any two is greater than the third.

(a) A (b) B (c) C

(d) D (e) E

57. **Assertion :** The sum of an infinite A.G.P.

$a+(a+d)\,x+(a+2d)\,x^2+(a+3d)\,x^3+..........$, where $|x|<1$ always exist.

Reason : The sum of the infinite series

$a+ar+ar^2+.........$ converges if $|r|<1$.

(a) A (b) B (c) C

(d) D (e) E

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

58. Column I consist of some terms where a,b,c are in HP and Column II consist of name of corresponding progression formed by terms in column I.

Column - I	Column - II
(I) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$	**(P)** HP
(II) $\frac{1}{b-a}, \frac{1}{b}, \frac{1}{b-c}$	**(Q)** GP
(III) $a-\frac{b}{2}, \frac{b}{2}, c-\frac{b}{2}$	**(R)** AP
(IV) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$	

Which of the Following is Incorrect:

(a) I – P (b) II – Q

(c) III – Q (d) IV – P

59. Column I consist of progression which roots of equation $ax^3+bx^2+cx+d=0$ form and column II consist of relation between a,b,c,d

Column - I	Column - II
(I) AP	**(P)** $b^3d=ac^3$
(II) GP	**(Q)** $27ad^3=9bcd^2-2c^3d$
(II) HP	**(R)** $2b^3-9abc+27a^2d=0$

Which of the Following is Incorrect:

(a) I – R (b) II – P

(c) III – Q (d) I – Q

Text

60. If the equation $x^4-4x^3+ax^2+bx+1=0$ has four positive roots, then find a and b.

61. If a, b, c are different positive numbers prove that $a^4+b^4+c^4>abc\,(a+b+c)$.

62. If x, y, z are positive real numbers satisfying the equation $x^2+9y^2+25z^2=3xy+15\,yz+5zx$ then find the progression of x, y and z.

63. Show that $\frac{1^4}{1.3}+\frac{2^4}{3.5}+\frac{3^4}{5.7}+....+\frac{n^4}{(2n-1)(2n+1)}$

$$=\frac{n\left(4n^2+6n+5\right)}{48}+\frac{n}{16(2n+1)}.$$

64. Find the sum of the series $\frac{1}{1.3}+\frac{2}{1.3.5}+\frac{3}{1.3.5.7}+...$ n terms

65. A sequence of real numbers $a_1, a_2, a_3, ..., a_n$ is such that $a_1=0, |a_2|=|a_1+1|, |a_3|=|a_2+1|, ..., |a_n|=|a_{n-1}+1|$.

Prove that $\frac{1}{n}\left(\sum_{i=1}^{n}a_i\right)\geq-\frac{1}{2}$.

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. If a, b, c and d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation : **(2000)**

(a) $0 \le M \le 1$ (b) $1 \le M \le 2$

(c) $2 \le M \le 3$ (d) $3 \le M \le 4$

2. Consider an infinite geometric series with first term *a* and common ratio *r*. If its sum is 4 and the second term is 3/4, then : **(2000)**

(a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = 3/8$

(c) $a = 3/2, r = 1/2$ (d) $a = 3, r = 1/4$

3. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$ If $\alpha, \beta, \gamma, \delta$ are in G. P., then the integer values of p and q respectively are : **(2001)**

(a) $-2, -32$ (b) $-2, 3$

(c) $-6, 3$ (d) $-6, -32$

4. If the sum of the first 2n terms of the A.P. 2, 5, 8, ... is equal to the sum of the first *n* terms of the A. P. 57, 59, 61 .... then n equals : **(2001)**

(a) 10 (b) 12

(c) 11 (d) 13

5. Let the positive numbers a, b, c, d be in A. P. Then abc, abd, acd, bcd are : **(2001)**

(a) in H.P. (b) in A. P.

(c) in G. P. (d) none of these

6. If $a_1, a_2 \ldots\ldots\ldots a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \ldots\ldots + a_{n-1} + 2a_n$ is : **(2002)**

(a) $n(2c)^{1/n}$ (b) $(n + 1)c^{1/n}$

(c) $2nc^{1/n}$ (d) $(n + 1)(2c)^{1/n}$

7. Suppose a, b, c are in A. P. and a^2, b^2, c^2 are in G. P. If $a < b < c$ and $a + b + c = 3/2$, then the value of *a* is : **(2002)**

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$

(c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

8. If $\alpha \in \left(0, \frac{\pi}{2}\right)$ then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to : **(2003)**

(a) $2 \tan \alpha$ (b) 1

(c) 2 (d) $\sec^2 \alpha$

9. An infinite G.P. has first term 'x' and sum 5, then x belongs to : **(2004)**

(a) $x < -10$ (b) $-10 < x < 0$

(c) $0 < x < 10$ (d) $x > 10$

10. α, β are roots of $ax^2 + bx + c = 0$, $a \ne 0$ and $\Delta = b^2 - 4ac$. If $\alpha + \beta$, $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ are in G. P., then : **(2005)**

(a) $\Delta \ne 0$ (b) $b\Delta = 0$

(c) $c\,\Delta = 0$ (d) $bc \ne 0$

11. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is : **(2009)**

(a) $\frac{n(4n^2 - 1)c^2}{6}$ (b) $\frac{n(4n^2 + 1)c^2}{3}$

(c) $\frac{n(4n^2 - 1)c^2}{3}$ (d) $\frac{n(4n^2 + 1)c^2}{6}$

12. Let $a_1, a_2, a_3, \ldots$ be in a harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **(2012)**

(a) 22 (b) 23

(c) 24 (d) 25

13. Let $b_i > 1$ for $i = 1, 2, \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \ldots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots + a_{51}$, then **(2016)**

(a) $s > t$ and $a_{101} > b_{101}$ (b) $s > t$ and $a_{101} < b_{101}$

(c) $s < t$ and $a_{101} > b_{101}$ (d) $s < t$ and $a_{101} < b_{101}$

Objective Questions II [One or more than one correct option]

14. Let $S_n = \sum_{1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then, S_n can take value(s) **(2013)**

(a) 1056 (b) 1088

(c) 1120 (d) 1332

Numerical Value Type Questions

15. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$

$B_n = 1 - A_n$. Find a least odd natural number n_0, so that $B_n > A_n, \forall\, n \geq n_0$. **(2006)**

16. Let S_k, k = 1, 2, ... 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of

$\frac{100^2}{100!} + \sum_{k=2}^{100} |(k^2 - 3k + 1)|\, S_k$ is... **(2010)**

17. Let $a_1, a_2, a_3, ..., a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4,..., 11. If $\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + ... + a_{11}}{11}$ is equal to ..... **(2010)**

18. Let $a_1, a_2, a_3, ..., a_{100}$ be a non constant arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i,\ 1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is ...... **(2011)**

19. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is ... **(2011)**

20. A pack contains n card numbered from 1 to n. Two consecutive numbered card are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smallest of the numbers on the removed cards is k, then k – 20 is equal to **(2013)**

21. Let a,b,c, be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is **(2014)**

22. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is. **(2015)**

23. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ? **(2017)**

24. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ________. **(2018)**

25. Let AP (a, d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If

$AP(1,3) \cap AP(2,5) \cap AP(3,7) = AP(a,d)$

then a + d equals........ **(2019)**

26. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is .... **(2020)**

27. Let $a_1, a_2, a_3, ...$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, ...$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$2(a_1 + a_2 + ... + a_n) = b_1 + b_2 + ... + b_n$

Holds for some positive integer n, is .... **(2020)**

Assertion & Reason

(A) **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

(B) **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

(C) **If ASSERTION is true, REASON is false.**

(D) **If ASSERTION is false, REASON is true.**

28. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Assertion : The numbers b_1, b_2, b_3, b_4 are neither in AP nor in G. P.

Reason : The numbers b_1, b_2, b_3, b_4 are in HP. **(2008)**

(a) A (b) B

(c) C (d) D

Using the following passage, solve Q.29 to Q.31

Passage – 1

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$ **(2007)**

29. The sum $V_1 + V_2 + \ldots + V_n$ is :

(a) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (b) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(c) $\frac{1}{2}n(2n^2 - n + 1)$ (d) $\frac{1}{3}(2n^3 - 2n + 3)$

30. T_r is always :

(a) an odd number (b) an even number

(c) a prime number (d) a composite number

31. Which one of the following is a correct statement ?

(a) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5

(b) $Q_1, Q_2, Q_3 \ldots$ are in A.P. with common difference 6

(c) $Q_1, Q_2, Q_3 \ldots$ are in A.P. with common difference 11

(d) $Q_1 = Q_2 = Q_3 = \ldots$

Using the following passage, solve Q.32 to Q.34

Passage – 2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively. **(2007)**

32. Which one of the following statements is correct ?

(a) $G_1 > G_2 > G_3 > \ldots$

(b) $G_1 < G_2 < G_3 < \ldots$

(c) $G_1 = G_2 = G_3 = \ldots$

(d) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$

33. Which of the following statements is correct ?

(a) $A_1 > A_2 > \ldots$

(b) $A_1 < A_2 < A_3 < \ldots$

(c) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$

(d) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

34. Which of the following statements is correct ?

(a) $H_1 > H_2 > H_3 > \ldots$

(b) $H_1 < H_2 < H_3 < \ldots$

(c) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$

(d) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

Text

35. Let $a_1, a_2, \ldots$ be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1, a_2, \ldots, a_n$. Find an expression for the geometric mean of $G_1, G_2, \ldots, G_n$ in terms of $A_1, A_2, \ldots, A_n, H_1, H_2, \ldots, H_n$. **(2001)**

Answer Key

CHAPTER - 3 | SEQUENCE AND SERIES

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)
6. (c)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (a)
16. (d)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (c)
21. (a)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (c)
26. (c)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (b)
31. (b)	**32.** (b)	**33.** (b)	**34.** (a)	**35.** (a)
36. (c)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (d)
41. (b)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (a)
46. (d)	**47.** (c)	**48.** (a)	**49.** (c)	**50.** (c)
51. (d)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (a)
56. (a)	**57.** (c)	**58.** (d)	**59.** (a)	**60.** (89)
61. (0)	**62.** (191)	**63.** (24)	**64.** (16)	**65.** (1)
66. (3)	**67.** (8)	**68.** (12)	**69.** (3)	**70.** (1)
71. (2)	**72.** (3)	**73.** (5)	**74.** (9)	**75.** (34)
76. (53261)		**77.** (570)	**78.** (1)	**79.** (3.00)
80. (398.00)				

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (d)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (a)
6. (d)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (b)
11. (a)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (c)
16. (c)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (a)
21. (c)	**22.** (d)	**23.** (d)	**24.** (b)	
25. (76.00)	**26.** (660.00)		**27.** (a)	**28.** (d)
29. (c)	**30.** (a)	**31.** 3.00	**32.** (52.00)	**33.** (d)
34. (d)	**35.** (d)	**36.** (b)	**37.** (a)	**38.** (c)
39. (c)	**40.** (c)	**41.** (28.00)	**42.** (303.00)	
43. (d)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (b)
48. (b)	**49.** (a)	**50.** (4.00)	**51.** (c)	
52. (39.00)		**53.** (b)	**54.** (b)	**55.** (d)
56. (a)	**57.** (d)	**58.** (b)	**59.** (d)	**60.** (b)
61. (d)	**62.** (d)	**63.** (5.00)	**64.** (a)	**65.** (d)
66. (c)	**67.** (c)	**68.** (1540.00)		**69.** (d)
70. (504.00)		**71.** (c)	**72.** (c)	**73.** (d)
74. (14.00)	**75.** (a)	**76.** (7.00)	**77.** (b)	**78.** (3.00)
79. (a)	**80.** (1251.00)		**81.** (d)	**82.** (b)
83. (a)	**84.** (a)	**85.** (d)	**86.** (7744.00)	
87. (2021.00)		**88.** (a)	**89.** (a)	**90.** (a)
91. (d)	**92.** (a)	**93.** (d)	**94.** (305.00)	
95. (b)	**96.** (a)	**97.** (16.00)	**98.** (14.00)	**99.** (d)
100. (c)	**101.** (3.00)	**102.** (a)	**103.** (160.00)	
104. (d)	**105.** (d)	**106.** (c)	**107.** (4.00)	
108. (3.00)	**109.** (b)	**110.** (c)	**111.** (9.00)	**112.** (c)
113. (10.00)	**114.** (d)	**115.** (a)		

CHAPTER -3 | SEQUENCE AND SERIES

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (b)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (b)
6. (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (a)
11. (a)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (a)
16. (a)	**17.** (d)	**18.** (c)	**19.** (c)	**20.** (a)
21. (d)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (a)
26. (a)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (b)
31. (d)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (b)
36. (c)	**37.** (d)	**38.** (a)	**39.** (b)	**40.** (a)
41. (d)	**42.** (c)	**43.** (d)	**44.** (a)	**45.** (b)
46. (d)	**47.** (d)	**48.** (d)	**49.** (a,b,c)	
50. (a,b,d)	**51.** (50)	**52.** (513)	**53.** (544)	**54.** (d)
55. (e)	**56.** (a)	**57.** (a)	**58.** (b)	**59.** (d)
60. (a=6, b = -4)		**62.** (Harmonic progression)		

64. $= \frac{1}{2}\left[1 - \frac{1}{1.3.5...(2n+1)}\right]$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (a)
6. (a)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (c)
11. (c)	**12.** (d)	**13.** (b)	**14.** (a,d)	**15.** (7)
16. (3)	**17.** (0)	**18.** (9)	**19.** (8)	**20.** (5)
21. (4)	**22.** (9)	**23.** (6)	**24.** (3748)	
25. (157.00)		**26.** (8.00)	**27.** (1.00)	**28.** (c)
29. (b)	**30.** (d)	**31.** (b)	**32.** (c)	**33.** (a)

34. (b) **35.** $G_m = (A_1 A_2 ....A_n H_1 H_2 ....H_n)^{\frac{1}{2n}}$