# MODEL QUESTION PAPER : 2020-21

## MATHEMATICS

**Time Allowed: 3 Hours** **Maximum Marks: 80**

**General Instructions:**

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

**Part – A:**

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

**Part – B:**

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section–III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

**Part – A**

**Section I**

**All questions are compulsory. In case of internal choices attempt any one.**

**1.** If R = {(x, y) : x + 2y = 8} is a relation on N, write the range of R.

**OR**

State whether the relation R = {(1, 1), (2, 2), (3, 3), (1,2), (2, 3), (1,3)} defined on A = {1,2,3} is reflexive, symmetric or transitive.

**2.** Prove that the greatest integer function $f : R \to R$ given by f(x) = [x] is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

**3.** Let $f : R \to R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}$ $\forall$ $x \in R$. Then, find the range of f.

**OR**

Let R be a relation in P(X), where X is a non-empty set, given by ARB if and only if $A \subset B$, where A and B are subsets in P(X). Is R an equivalence relation on P(X) ? Justify your answer.

**4.** If $\begin{bmatrix} x - y & z \\ 2x - y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of x + y.

**5.** If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.

**OR**

If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.

6. If $A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$, write the cofactors of elements of $3^{rd}$ row. Hence, find its determinant.

7. Evaluate : $\int_{-1}^{1} |x \cos \pi x| \, dx$

8. Find the area bounded by $x^2 + y^2 = 4$ in the first quadrant using integrals.

9. Write the sum of degree and order of the differential equation :

$$x^3 \left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$$

**OR**

Verify that y = A cos x - B sin x is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

10. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

11. Find the value of 'p' for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

12. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$ making an angle of $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{4}$ with y-axis and an acute angle $\theta$ with z-axis.

13. If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{4} = \frac{2z-6}{4}$, then write its vector form.

14. Find the distance between the parallel planes $2x - 2y - z = -3$ and $4x - 4y - 2z + 5 = 0$.

15. Write the equation of plane whose intercepts on the co-ordinate axes are –4, 2, 3.

16. Three persons A, B and C fire at a target in turn, starting with A. Their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. Find the probability of two hits.

## Section-II

**Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18. Each question carries 1 mark**

17. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers so as to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. And the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others.

Based on the above information, answer the following questions :

(i) Represent the given word problem using the system of linear equations, with the help of matrix method :

(a) $x + y + z = 12$, $2x + 3y + 3z = 33$, $x - 2y + z = 0$

(b) $x + y + z = 12$, $3x + 2y + 3z = 33$, $x - 2y + z = 0$

(c) $x + y + z = 12$, $3x + 3y + 2z = 33$, $x + 2y + z = 0$

(d) $x + y + z = 12$, $2x + 3y + 3z = 33$, $x - 2y - z = 0$

(ii) Write the coefficient matrix, variable matrix and constant matrix; represented by A, X and B; using matrix method :

(a) $A=\begin{bmatrix}1&1&1\\3&3&2\\1&2&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}12\\0\\33\end{bmatrix}$

(b) $A=\begin{bmatrix}1&1&1\\3&2&3\\1&-2&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}12\\33\\0\end{bmatrix}$

(c) $A=\begin{bmatrix}1&1&1\\2&3&3\\1&-2&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}12\\33\\0\end{bmatrix}$

(d) $A=\begin{bmatrix}1&1&1\\2&3&3\\1&-2&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}33\\12\\0\end{bmatrix}$

(iii) Write the values of co-factors of first column?

(a) $A_{11} = 9, A_{21} = 0, A_{31} = -3$
(b) $A_{11} = 9, A_{21} = -3, A_{31} = 0$
(c) $A_{11} = -7, A_{21} = 3, A_{31} = 1$
(d) $A_{11} = 1, A_{21} = 0, A_{31} = -1$

(iv) The inverse of co-efficient matrix will be represented as :

(a) $A^{-1}=\frac{1}{3}\begin{bmatrix}9&-3&0\\1&0&-1\\-7&3&1\end{bmatrix}$

(b) $A^{-1}=\frac{1}{3}\begin{bmatrix}1&0&-1\\-7&3&1\\9&-3&0\end{bmatrix}$

(c) $A^{-1}=\frac{1}{3}\begin{bmatrix}9&-3&0\\-7&3&1\\1&0&-1\end{bmatrix}$

(d) $A^{-1}=\frac{1}{4}\begin{bmatrix}9&-3&0\\1&0&-1\\-7&3&1\end{bmatrix}$

(v) Find the number of awardees of each category :

(a) x = 3, y = 5, z = 4
(b) x = 5, y = 3, z = 4
(c) x = 4, y = 3, z = 5
(d) x = 3, y = 4, z = 5

**18.** In a shop X, 30 tins of pure ghee and 40 tins of adulterated ghee which look alike, are keep for sale; while in shop Y, similar 50 tins of pure ghee and 60 tins of adulterated ghee are there. One tin of ghee is purchased from one of the randomly selected shops and is found to be adulterated. .

Based on the above information answer the following :

(i) The conditional probability that the ghee is adulterated, given that ghee is purchased from shop Y, is :

(a) $\frac{6}{11}$ (b) $\frac{4}{7}$ (c) $\frac{4}{11}$ (d) $\frac{6}{7}$

(ii) The probability that ghee is purchased from shop X and is adulterated is :

(a) $\frac{3}{11}$ (b) $\frac{2}{7}$ (c) $\frac{2}{11}$ (d) $\frac{3}{7}$

(iii) The total probability of getting adulterated ghee when purchased from shop is :

(a) $\frac{45}{77}$ (b) $\frac{41}{77}$ (c) $\frac{47}{77}$ (d) $\frac{43}{77}$

(iv) The probability that the ghee is purchased from shop Y; given that it is found to be adulterated is:

(a) $\frac{22}{43}$ (b) $\frac{21}{43}$ (c) $\frac{23}{43}$ (d) $\frac{20}{43}$

(v) Let A be the event of getting an adulterated ghee and $E_1$, $E_2$ be the events of purchasing ghee from shop X and shop Y, respectively. Then the value of $\sum_{i=1}^{2} P(E_i / A)$ is :

(a) 0 (b) $\frac{40}{43}$ (c) 1 (d) $\frac{42}{43}$

## Part - B

## Section-III

**All questions are compulsory. In case of internal choices attempt any one.**

**19.** Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

**20.** If the value of determinant $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$ is zero, then find the value of 'a'.

**OR**

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then find the value of $A^2 - 5A + 7I$.

**21.** Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & ; \quad x < 0 \\ a & ; \quad x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ; \quad x > 0 \end{cases}$

Find the value of 'a' for which f is continuous at x = 0?

**22.** Find the local minimum value of $f(x) = x^2 + 4x + 5$

**23.** If $f(x) = \int_0^x t\sin t \, dt$, then write the value of $f'(x)$

**OR**

Evaluate : $\int_e^{e^2} \frac{dx}{x \log x}$

**24.** Find the area of the region bounded by the line $2y = -x + 8$, x-axis and the lines $x = 2$ and $x = 4$ using integration.

**25.** Solve the following differential equation :

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

**26.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

**27.** Find the equation of the line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 1, 1)

**28.** Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that the die shows a number greater than 3 given that ' there is at least one head'.

## Section IV

**All questions are compulsory. In case of internal choices attempt any one.**

**29.** If A = {1, 2, 3, ……… 9} and R be the relation in $A \times A$ defined by (a, b) R (c, d) if $a + d = b + c$ for a, b, c, d $\in$ A, prove that R is an equivalence relation. Then, find the equivalence class [(2, 5)]

**30.** Differentiate with respect to x :

$$\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$$

**31.** If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$

**OR**

If $(ax + b)\, e^{y/x} = x$, then show that $x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2$

**32.** Find the equation of tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $\left(\sqrt{2}a, b\right)$

**33.** Evaluate : $\int_0^{\pi/4}\left(\frac{\sin x + \cos x}{3 + \sin 2x}\right)dx$

**34.** Find the area of the region bounded by the curves $x^2 + y^2 = 9$, $x + 2y = 3$ and y-axis in the $2^{nd}$ quadrant.

**OR**

Find the area of the ellipse $4x^2 + 9y^2 = 36$ using integration.

**35.** Solve the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

## Section V

**All questions are compulsory. In case of internal choices attempt any one.**

**36.** Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

**OR**

An isosceles triangle of vertical angle $2\theta$ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$.

**37.** From the point P(1, 2, 4), perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the equation, the length and the co-ordinates of the foot of the perpendicular.

**OR**

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} - 2\hat{j} + \hat{k}) = 0$.

**38.** Determine graphically the minimum value of the objective function :

$$\mathbf{Z} = 5x + 4y$$

subject to the constraints

$$x + 2y \geq 50;\ 2x + y \geq 40;\ x + y \leq 35;\ x, y \geq 0$$

**OR**

Determine graphically the maximum value of the objective function :

$$Z = 12x + 16y$$

subject to the constraints :

$$x + y \leq 1200;\ x \geq 2y;\ x - 3y \leq 600;\ x, y \geq 0$$

# MATHEMATICS (SOLUTIONS)

**1.** Given; $R = \{(x, y) : x + 2y = 8\} \ \forall \ x, y \in N$

$\Rightarrow \quad R = \{(6, 1), (4, 2), (2, 3)\}$

Range of R is $\{1, 2, 3\}$

**OR**

Given that $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ and $A = \{1, 2, 3\}$

$\because$ $(1, 1), (2, 2), (3, 3) \in R$ ; Hence, R is reflexive.

Now; $(1, 2) \in R$ but $(2, 1) \in R$; Hence, R is not symmetric

and $(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$

Hence, R is transitive

**2.** $f : R \to R$, given by $f(x) = [x]$

It is seen that $f(1.2) = [1.2] = 1$, $f(1.9) = [1.9] = 1$

$\therefore \quad f(1.2) = f(1.9)$, but $1.2 \quad 1.9$

$\Rightarrow$ f is not one-one

Now consider, $0.7 \in R$

It is known that $f(x) = [x]$ is always an integer. Thus, there does not exists any element $x \in R$ such that $f(x) = 0.7$.

$\therefore$ f is not onto.

**3.** Let $y = \dfrac{1}{2 - \cos x} \ \forall \ x \in R$

$\Rightarrow \quad 2y - y\cos x = 1$

$\Rightarrow \quad \cos x = 2 - \dfrac{1}{y}$

Now, we know that $-1 \le \cos x \le 1$

$$\Rightarrow \quad -1 \le 2 - \frac{1}{y} \le 1 \quad \text{or} \quad -3 \le -\frac{1}{y} \le -1$$

$$\Rightarrow \quad 1 \le \frac{1}{y} \le 3 \quad \Rightarrow \quad \frac{1}{3} \le y \le 1 \quad \text{So, range is } \left[\frac{1}{3}, 1\right]$$

**OR**

Let ARB $\Rightarrow A \subset B$

Then, it is not necessary that B is a subset of A

i.e., $B \not\subset A$

$\Rightarrow$ B is not related to A

$\therefore$ R is not symmetric and hence R is not an equivalence relation

e.g., Let $X = \{1, 2, 3\}$

$P(X) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

Clearly, $\{2\} \subset \{1, 2\}$ but $\{1, 2\} \not\subset \{2\}$

$\therefore$ R is not symmetric

**4.** Given $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

$\Rightarrow \quad x - y = -1$

and $\quad 2x - y = 0$

Solving, we get x = 1 and y = 2

$x + y = 1 + 2 = 3$

**5.** Given, $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$\Rightarrow \quad 12x + 14 = 32 - 42$

$\Rightarrow \quad x = -2$

**OR**

$$\begin{aligned} 7A - (I + A)^3 &= 7A - (I^3 + A^3 + 3A^2I + 3I^2A) \\ &= 7A - (I + A.A^2 + 3A + 3A) \\ &= 7A - (I + A.A + 6A) \\ &= 7A - (I + A^2 + 6A) \\ &= 7A - (I + A + 6A) \\ &= -I \end{aligned}$$

**6.** Given, $A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$

Cofactors of elements of 3rd row are

$$C_{31} = \begin{vmatrix} 3 & -2 \\ -5 & 6 \end{vmatrix} = 18 - 10 = 8$$

$$C_{32} = \begin{vmatrix} 1 & -2 \\ 4 & 6 \end{vmatrix} = -(6 + 8) = -14$$

$$C_{33} = \begin{vmatrix} 1 & 3 \\ 4 & -5 \end{vmatrix} = -5 - 12 = -17$$

$\Rightarrow \quad |A| = a_{31}.C_{31} + a_{32}.C_{32} + a_{33}.C_{33}$

$= 3(8) + 5(-14) + 2(-17) = 24 - 70 - 34 = -80$

**7.** Let $I = \int_{-1}^{1} |x \cos \pi x| \, dx$

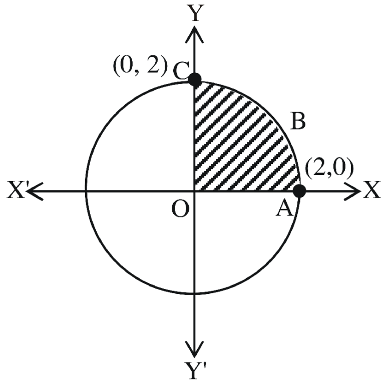
$$I = 2\int_0^1 |x\cos \pi x| \, dx = 2\int_0^{1/2} (x\cos\pi x)dx - 2\int_{1/2}^{1} (x\cos\pi x)\,dx$$

$$= 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^{1/2} - 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{1/2}^{1}$$

$$= 2\left[\frac{1}{2\pi}-\frac{1}{\pi^2}\right]-2\left[\frac{-1}{\pi^2}+\frac{-1}{2\pi}\right] = \frac{2}{\pi}$$

**8.** Given, $x^2 + y^2 = 4 \Rightarrow y = \sqrt{4-x^2}$

Shaded area = Area OABCO




$$= \int_0^2 y\,dx$$

$$= \int_0^2 \sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \left(2\cdot\frac{\pi}{2}\right)-0 = \pi \text{ sq. units}$$

**9.** Degree = 2 and Order = 2

$\therefore$ Sum = 4

**OR**

Given; y = A cos x - B sin x

$$\Rightarrow \quad \frac{dy}{dx} = -A\sin x - B\cos x$$

$$\therefore \quad \frac{d^2y}{dx^2} = -A\cos x + B\sin x$$

$$\text{or} \quad \frac{d^2y}{dx^2} = -y \;\Rightarrow\; \frac{d^2y}{dx^2}+y=0 \; \frac{d^2y}{dx^2}+y=0$$

**10.** Let $\vec{a} = 2\hat{i}-3\hat{j}+6\hat{k}$

$\Rightarrow$ Unit vector along given vector = $\frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

$\therefore$ Vector of magnitude 21 units is = $21\left(\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}\right) = 6\hat{i}-9\hat{j}+18\hat{k}$

**11.** Let $\vec{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b} = \hat{i}-2p\hat{j}+3\hat{k}$

Since $\vec{a}\,||\,\vec{b}$ $\quad \therefore \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

$$\Rightarrow p = \frac{-1}{3}$$

**12.** We know that $\vec{a} = |\vec{a}|(\ell\hat{i} + m\hat{j} + n\hat{k})$

Given ; $\ell = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$m = \cos\frac{\pi}{2} = 0$

and $n = \cos\theta$

Now, $\ell^2 + m^2 + n^2 = 1$

$\frac{1}{2} + 0 + \cos^2\theta = 1$

$\Rightarrow$ ($\theta$ is acute)

$\therefore \quad \vec{a} = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{i} + 0\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right)$

$\Rightarrow \quad \vec{a} = 5(\hat{i} + \hat{k})$

**13.** Given $\frac{3-x}{5} = \frac{y+4}{4} = \frac{2z-6}{4}$

$\Rightarrow \quad \frac{x-3}{-5} = \frac{y+4}{4} = \frac{z-3}{2}$

Vector equation is $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 4\hat{j} + 2\hat{k})$

**14.** Given, $2x - 2y - z = -3$

and $4x - 4y - 2z + 5 = 0 \Rightarrow 2x - 2y - z = \frac{-5}{2}$

Distance between the parallel planes is given as:

$$d = \left|\frac{d_2 - d_1}{\sqrt{A^2 + B^2 + C^2}}\right| = \left|\frac{-\frac{5}{2} + 3}{\sqrt{4+4+1}}\right| = \frac{1}{6} \text{ units}$$

**15.** Equation of plane in intercept form is given as:

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\therefore \quad \frac{x}{-4} + \frac{y}{2} + \frac{z}{3} = 1$

$\Rightarrow \quad -3x + 6y + 4z = 1$

**16.** Given, P(A hits) = 0.4, P(B hits) = 0.3,P(C hits) = 0.2

$\therefore$ P (A does not hit) = 1 – 0.4

= 0.6

Also, P(B does not hit) = 0.7    and P(C does not hit) = 0.8

$\Rightarrow$ $P(2 \text{ hits}) = P(\bar{A} \cap B \cap C) + P(A \cap \bar{B} \cap C) + P(A \cap B \cap \bar{C})$

$= 0.6 \times 0.3 \times 0.2 + 0.4 \times 0.7 \times 0.2 + 0.4 \times 0.3 \times 0.8$

$= 0.188$

**17.** x = awarded members for honesty

y = awarded members for helping (Co-operation)

z = awarded member for supervision

According to the given eqestion

$x + y + z = 12$ ……….(1)

$3(y + z) + 2x = 33$

$\Rightarrow$ $2x + 3y + 3z = 33$ ……….(2)

and $x + z = 2y$

$\Rightarrow$ $x - 2y + z = 0$ ……….(3)

All the above three equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$\Rightarrow$ A . X = B ……...(4)

Here, $|A| = 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$

$= 9 + 1 - 7 = 3 \neq 0$

Here, A is non-singular and so its inverse exists. Now,

$A_{11} = 9,\ A_{12} = 1,\ A_{13} = -7$

$A_{21} = -3,\ A_{22} = 0,\ A_{23} = 3$

$A_{31} = 0,\ A_{32} = -1,\ A_{33} = 1$

$A_{11} = 9,\ A_{21} = -3,\ A_{31} = 0$ ……..(5)

$$\text{Now, adj } A = \left[A_{ij}\right]^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \quad \text{……..(6)}$$

$$\text{So,} \quad X = A^{-1}.B = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} . \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 108 - 99 + 0 \\ 12 + 0 + 0 \\ -84 + 99 + 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

or $x = 3, y = 4, z = 5$ ……..(7)

(i) **(a)** $x + y + z = 12$ (from (1))

$2x + 3y + 3z = 33$ (from (2))

$x - 2y + z = 0$ (from (3))

(ii) **(c)** $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 33 \\ 12 \\ 0 \end{bmatrix}$ (from (4))

(iii) **(b)** $A_{11} = 9, A_{21} = -3, A_{31} = 0$ (from (5))

(iv) **(a)** $A^{-1} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$ (from (6))

(v) **(d)** $x = 3, y = 4, z = 5$ (from (7))

**18.** Let us define the events as:

$E_1$ = ghee purchased from shop X,

$E_2$ = ghee purchased from shop Y,

and A = getting adulterated ghee

$\Rightarrow$ $P(E_1) = P(E_2) = \frac{1}{2}$

and $P(A/E_1) = \frac{40}{70} = \frac{4}{7}$

$P(A/E_2) = \frac{60}{110} = \frac{6}{11}$ …….(1)

Now; $P(E_1).(A/E_1) = \frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$ …….(2)

and $P(E_2).P(A/E_2) = \frac{1}{2} \times \frac{6}{11} = \frac{3}{11}$

$\Rightarrow$ $P(E_1).\ P(A/E_1) + P(E_2).\ P(A/E_2) = \frac{2}{7} + \frac{3}{11} = \frac{22+21}{77} = \frac{43}{77}$ …….(3)

Now; $P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$

$= \frac{3/11}{43/77} = \frac{3}{11} \times \frac{77}{43} = \frac{21}{43}$ …….(4)

and $P(E_1/A)$

$= \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)} = \frac{2/7}{43/77} = \frac{2}{7} \times \frac{77}{43} = \frac{22}{43}$ …….(5)

From equations (4) and (5), we have;

$$P(E_1/A) + P(E_2/A)$$

$$= \frac{22}{43} + \frac{21}{43} = \frac{43}{43} = 1 \qquad \text{.......(6)}$$

(i) **(a)** Required Probability

$$= P(A/E_2) = \frac{6}{11} \qquad \text{(from (1))}$$

(ii) **(b)** Required Probability

$$= P(E_1).P(A/E_1) = \frac{2}{7} \qquad \text{(from (2))}$$

(iii) **(d)** Required Probability

$$= P(E_1).P(A/E_1) + P(E_2).\ P(A/E_2) = \frac{43}{77} \qquad \text{(from (3))}$$

(iv) **(b)** Required Probability

$$= P(E_2/A) = \frac{21}{43} \qquad \text{(from (4))}$$

(v) **(c)** We have;

$$\sum_{i=1}^{2} P\left(E_i / A\right) = P(E_1/A) + P(E_2/A) = 1 \qquad \text{(from (6))}$$

**19.** Let $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \theta$

$\Rightarrow$ $\sin\theta = \sin\left(\frac{5\pi}{3}\right)$

or $\sin\theta = \sin\left(2\pi - \frac{\pi}{3}\right)$

$\Rightarrow$ $\sin\theta = -\sin\frac{\pi}{3}$ $[\because \sin(2\pi - \theta) = -\sin\theta]$

or $\sin\theta = \sin\left(-\frac{\pi}{3}\right)$

$\Rightarrow$ $\theta = -\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

**20.** Given, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = 0$

$\Rightarrow$ $(1 + a)[(1 + a)^2 - 1] - 1[1 + a - 1] + 1[1 - (1 + a)] = 0$

$\Rightarrow \quad (1 + a)\,[a^2 + 2a] - 1\,[a] + 1\,[-a] = 0$

$\Rightarrow \quad 3a^2 + a^3 = 0$

Solving, we get a = –3 or a = 0

**OR**

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \quad A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**21.** At x = 0, f(0) = a …..(1)

LHL = $\lim\limits_{x \to a} f(x) = \lim\limits_{h \to 0} f(-h)$

$$= \lim_{h \to 0} \frac{1 - \cos 4(-h)}{(-h)^2} = \lim_{h \to 0} \frac{1 - \cos 4h}{h^2} = \lim_{h \to 0} \frac{2\sin^2 2h}{h^2} \times \frac{4}{4} = \lim_{h \to 0} 8\left(\frac{\sin 2h}{2}\right)^2 = 8 \quad \text{…..(2)}$$

Since f(x) is continuous function, from (1) and (2); a = 8

**22.** Given, $f(x) = x^2 + 4x + 5$

$\Rightarrow \quad f'(x) = 2x + 4$

For maxima or minima, f '(x) = 0

$\Rightarrow \quad 2x + 4 = 0$

or $\quad x = -2$

Also f '(x) = 2 > 0 $\quad \therefore$ f(x) is minimum at x = –2

$\Rightarrow \quad f(-2) = (-2)^2 + 4(-2) + 5 = 1$

**23.** Given ; $f(x) = \int_0^x t \sin t \, dt = \int_0^x \cos t \, dt$

$\Rightarrow \quad f(x) = \left[-t\cos t + \sin t\right]_0^x$

or $\quad f(x) = -x\cos x + \sin x$

$\Rightarrow \quad f(x) = -(x.(-\sin x) + \cos x.1) + \cos x = x \sin x$

**OR**

$$I = \int_e^{e^2} \frac{dx}{x \log x}$$

Put logx = t $\quad x = e^2, \quad t = 2$

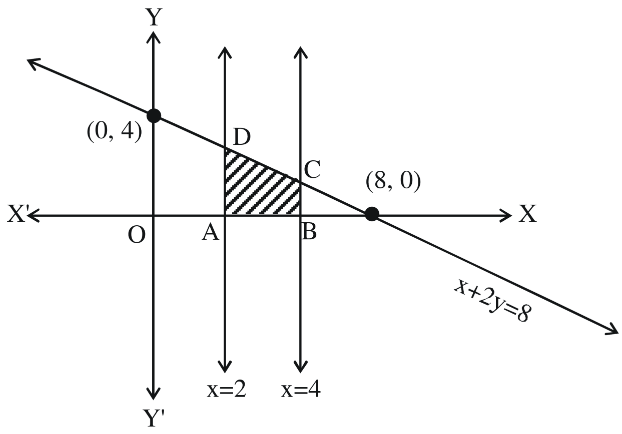
$\Rightarrow \frac{1}{x}dx = dt \quad x = e, \quad t = 1$

$$I = \int_1^2 \frac{1}{t}\,dt = \left[\log t\right]_1^2 = \log 2$$

**24.** Given, $2y = -x + 8$

$\Rightarrow \quad y = \dfrac{-x}{2} + 4$

Now, Shaded area = Area ABCDA




$$= \int_2^4 y\, dx$$

$$= \int_2^4 \left(\frac{-x}{2} + 4\right) dx$$

$$= \left[\frac{-x^2}{4} + 4x\right]_2^4 = (-4 + 16) - (-1 + 8) = 12 - 7 = 5 \text{ sq. units}$$

**25.** Given, $\cos^2 x \dfrac{dy}{dx} + y = \tan x$

$$\Rightarrow \frac{dy}{dx} + \sec^2 x . y = \sec^2 x \tan x$$

This is of the form $\dfrac{dy}{dx} + Py = Q$

where $P = \sec^2 x$, $Q = \sec^2 x \tan x$

$\therefore \quad$ I.F. $= e^{\int P dx} = e^{\int \sec^2 x\, dx} = e^{\tan x}$

Its solution is given by

$$y.\, IF = \int Q.\, IF\, dx + C$$

$$\Rightarrow \quad y.e^{\tan x} = \int \sec^2 x \tan x \cdot e^{\tan x}\, dx + C$$

$$\therefore \quad y.e^{\tan x} = \tan x \,.\, e^{\tan x} - e^{\tan x} + C$$

**26.** Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$$\Rightarrow \quad 2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$$

$$= \hat{i} - 2\hat{j} + 2\hat{k} = \vec{d} \text{ (let)}$$

$\therefore$ Vector parallel to $\vec{d}$ and having magnitude 6 units

$$= 6\left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1 + 4 + 4}}\right)$$

$$= 2(\hat{i} - 2\hat{j} + 2\hat{k})$$

**27.** Line through the point A(4, 3, 2) and B(1, –1, 0) is :

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \quad \frac{x - 4}{-3} = \frac{y - 3}{-4} = \frac{z - 2}{-2}$$

Similarly, line passing through C(1, 2, –1) ad D(2, 1, 1) is :

$$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{2}$$

Direction ratios of line perpendicular to AB and CD is :

$$\frac{a}{(-4)(2)-(-1)(-2)}=\frac{b}{(1)(-2)-(-3)(2)}=\frac{c}{(-3)(-1)-(1)(-4)}$$

$\Rightarrow$ $$\frac{a}{-10}=\frac{b}{4}=\frac{c}{7}$$

$\therefore$ Equation of line passing through (1, –1, 1) and having direction ratios –10, 4, 7 is :

$$\frac{x-1}{-10}=\frac{y+1}{4}=\frac{z-1}{7}$$

**28.** The sample space of the experiment is :

S = {(T, H), (T, T), (H, 1), (H, 2), (H, 3),(H, 4), (H, 5), (H, 6)}

Let A be the event that die shows number greater than 3 and B be the event that there is atleast 1 head

$\therefore$ A = {(H, 4), (H, 5), (H, 6)}

and B = {(T, H), (H, 1), (H, 2), (H, 3),(H, 4), (H, 5), (H, 6)}

$\Rightarrow$ $A \cap B$ = {(H, 4), (H, 5), (H, 6)}

$$P(A \cap B) = \frac{1}{12}\times 3=\frac{1}{4}$$

and $$P(B) = \frac{1}{4}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$$

$$=\frac{3}{4}$$

$\Rightarrow$ $$P(A/B) = \frac{P(A\cap B)}{P(B)}=\frac{1/4}{3/4}=\frac{1}{3}$$

**29.** Consider (a, b) R (a, b) where $(a, b) \in A \times A$

$\therefore$ a + b = b + a

Hence, R is reflexive relation

Now consider (a, b) R (c, d) given by (a, b), $(c, d) \in A \times A$. Then,

a + d = b + c $\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b)

$\therefore$ R is symmetric relation

Let (a, b) R (c, d) and (c, d) R (e, f)

Where (a, b) (c, d), $(e, f) \in A \times A$

$\Rightarrow$ a + d = b + c …. (1) $\Rightarrow$ (1) + (2) gives:

and c + f = d + e ….. (2)

a + f = b + e

$\Rightarrow$ (a, b) R (e, f)

R is transitive relation

Hence; R is an equivalence relation

Now,[(2, 5)] = {(1, 4), (2,5), (3, 6), (4, 7), (5, 8), (6, 9)} [$\because$ Let (x,y) R (2,5) $\Rightarrow$ x+5 = y +2 or x+3 = y]

**30.** Let $y = \sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

$= \sin^{-1}\left(\frac{2.6^x}{1+(6^x)^2}\right)$

Put $6^x = \tan\theta$, $\theta = \tan^{-1} 6^x$

$\Rightarrow \quad y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\therefore \quad y = \sin^{-1}(\sin 2\theta)$ or $y = 2\theta$

$\Rightarrow \quad y = 2\tan^{-1} 6^x$

or $\quad \frac{dy}{dx} = \frac{2}{1+36^x} \times 6^x \log_e 6$

**31.** $x = a\cos^3\theta$; $y = a\sin^3\theta$

$\Rightarrow \quad \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$ ….(1)

and $\quad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$ ….(2)

(2) ÷ (1) gives:

$$\frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{d\theta}\left[-\tan\theta\right].\frac{d\theta}{dx} = -\sec^2\theta.\frac{d\theta}{dx}$

$\therefore \quad \frac{d^2y}{dx^2} = \frac{-\sec^2\theta}{-3a\cos^2\theta.\sin\theta}$ [from (1)]

or $\quad \frac{d^2y}{dx^2} = \frac{1}{3a\cos^4\theta\sin\theta}$

At $\theta = \frac{\pi}{6}$; $\frac{d^2y}{dx^2} = \frac{1}{3a\times\left(\frac{\sqrt{3}}{2}\right)^4\times\frac{1}{2}} = \frac{32}{27a}$

**OR**

$(ax + b)e^{y/x} = x$

$\Rightarrow \quad e^{y/x} = \frac{x}{ax+b}$ …..(1)

Taking log both sides;

$$\frac{y}{x} = \log x - \log(ax+b)$$

Differentiating w.r.t. x; we have:

$$\frac{x\frac{dy}{dx}-y}{x^2}=\frac{1}{x}-\frac{a}{ax+b}$$

$\Rightarrow$ $$x\frac{dy}{dx}-y=\frac{bx}{ax+b}$$

From (1);

$$x\frac{dy}{dx}-y=b\cdot e^{y/x} \qquad \text{.....(2)}$$

Differentiating w.r.t. x again;

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{be^{y/x}\left(x\frac{dy}{dx}-y\right)}{x^2}$$

From eq. (2), we have:

$$x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$$

**32.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Differentiating w.r.t x;

$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$ or $\frac{dy}{dx}=\frac{b^2x}{a^2y}$ ....(1)

$\Rightarrow$ Slope of tangent at $(\sqrt{2}a,b)=\frac{\sqrt{2}\,b}{a}$ (from (1))

$\therefore$ Slope of normal at $(\sqrt{2}a,b)=\frac{-a}{\sqrt{2}\,b}$

$\Rightarrow$ Equation of tangent is :

$$y-b=\frac{\sqrt{2}b}{a}(x-\sqrt{2}a)$$

or $\sqrt{2}bx-ay=ab$

and Equation of normal is :

$$y-b=\frac{-a}{\sqrt{2}b}(x-\sqrt{2}a)$$

$$ax+\sqrt{2}by=\sqrt{2}(a^2+b^2)$$

**33.** Let $\quad I = \int_0^{\pi/4} \left( \frac{\sin x + \cos x}{3 + \sin 2x} \right) dx$

$\Rightarrow \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2} dx$

Put sinx – cosx = t $\quad$ x = 0, $\quad$ t = –1

(cosx + sinx)dx = dt $\quad$ $x = \frac{\pi}{4}$, $\quad$ t = 0

$\therefore \quad I = \int_{-1}^{0} \frac{1}{4 - t^2} dt$

or $\quad I = \left[ \frac{1}{4} \log \frac{2+t}{2-t} \right]_{-1}^{0}$

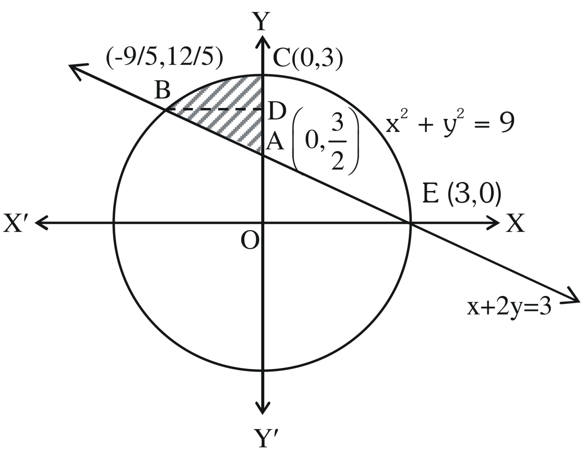
$\Rightarrow \quad I = \frac{1}{4} \left[ \log 1 - \log \frac{1}{3} \right]$

or $\quad I = \frac{1}{4} \log 3$

**34.** Given ; $x^2 + y^2 = 9$

and $x + 2y = 3$

Their point of intersection is $\left( -\frac{9}{5}, \frac{12}{5} \right)$




Required shaded area = Area (ABDA) + Area (BDCB)

$$= \int_{3/2}^{12/5} x\,dy + \int_{12/5}^{3} x\ dy$$

$$= \int_{3/2}^{12/5} (3-2y)\,dy + \int_{12/5}^{3} \sqrt{9-y^2}\,dy$$

$$= \left[3y - y^2\right]_{3/2}^{12/5} + \left[\frac{y}{2}\sqrt{9-y^2} + \frac{9}{2}\sin^{-1}\frac{y}{3}\right]_{12/5}^{3}$$

$$= \left[\left(\frac{36}{5} - \frac{144}{25}\right) - \left(\frac{9}{2} - \frac{9}{4}\right)\right] + \left[\frac{9}{2}.\frac{\pi}{2} - \left(\frac{6}{5}.\frac{9}{5} + \frac{9}{2}\sin^{-1}\frac{4}{5}\right)\right]$$

$$= \frac{-81}{100} + \frac{9\pi}{4} - \frac{54}{25} - \frac{9}{2}\sin^{-1}\frac{4}{5}$$

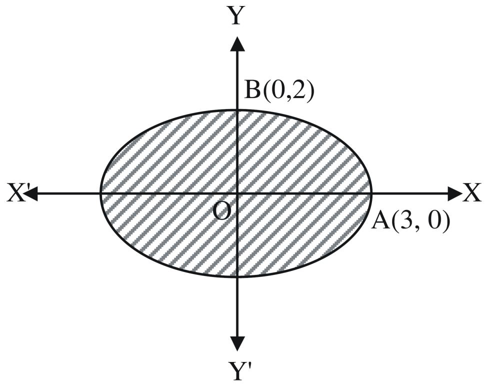
$$= \left(\frac{9\pi}{4} - \frac{9}{2}\sin^{-1}\frac{4}{5} - \frac{297}{100}\right) \text{ sq. units}$$

**OR**

Given $4x^2 + 9y^2 = 36$

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Area bounded by ellipse = 4 × Area (OABO)




$$= 4 \times \int_0^3 y\,dx$$

$$= 4 \times \int_0^3 \frac{2}{3}\sqrt{9-x^2}\,dx$$

$$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= \frac{8}{3}\left[\frac{9}{2}.\frac{\pi}{2}\right]$$

$= 6\pi$ sq. units

**35.** The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right) + x}{x\cos\left(\frac{y}{x}\right)} \qquad \text{.......(1)}$$

Put $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From(1);

$$v + x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v}$$

$$\therefore\ x\frac{dv}{dx} = \frac{1}{\cos v}$$

or $\quad \cos v dv = \frac{dx}{x}$

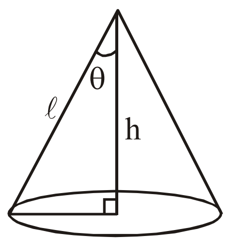
Integrating both sides; we have :

$$\int \cos v\ dv = \int \frac{1}{x}\ dx$$

$\Rightarrow \quad \sin v = \log|x| + C$ or $\sin\frac{y}{x} = \log|x| + C$

**36.** Let radius of cone = r
height of cone = h
slant height of cone = $\ell$
and semi-vertical angle = $\alpha$

Volume of cone $V = \frac{1}{3}\pi r^2 h = \frac{\pi\ell^3}{3}\sin^2\theta\cos\theta$ [$\because$ $h = \ell\cos\theta$, $r = \ell\sin\theta$]




$$\frac{dV}{d\theta} = \frac{\pi\ell^3}{3}\left[2\sin\theta\cos^2\theta - \sin^3\theta\right]$$

for maxima and minima $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = \frac{\pi\ell^3}{3}\left[2\sin\theta\cos^2\theta - \sin^3\theta\right] = 0$$

$\Rightarrow \quad \sin\theta\left[2\cos^2\theta - (1 - \cos^2\theta)\right] = 0$

$\Rightarrow \quad 3\cos^2 - 1 = 0 \qquad [\because \sin\theta \neq 0]$

$$\cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{3}}$$

$$\frac{d^2V}{d\theta^2} = \frac{\pi\ell^3}{3}\left[2\cos^3\theta - 7\sin^2\theta\cos\theta\right]$$

$\frac{d^2V}{d\theta^2}$ is negative at $\theta = \cos^{-1}\frac{1}{\sqrt{3}}$

Hence V is maximum at $\theta = \cos^{-1}\frac{1}{\sqrt{3}}$

**OR**

Let ABC be an isosceles triangle inscribed in the circle with radius a such that

AB = AC.

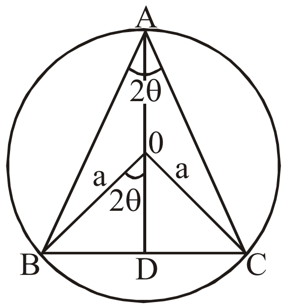
AD = AO + OD = a + $a\cos^2\theta$ and BC = 2BD = $2a\sin^2\theta$ (see figure)

Therefore, area of the triangle ABC i.e. $\Delta = \frac{1}{2}BC.AD$

$$= \frac{1}{2}2a\sin 2\theta.(a + a\cos 2\theta)$$

$$= a^2\sin^2\theta\ (1 + \cos^2\theta)$$

$$\Rightarrow \qquad \Delta = a^2\sin 2\theta + \frac{1}{2}a^2\sin 4\theta$$




Therefore, $\frac{d\Delta}{d\theta} = 2a^2\cos 2\theta + 2a^2\cos 4\theta$

$$= 2a^2(\cos 2\theta + \cos 4\theta)$$

$\frac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$

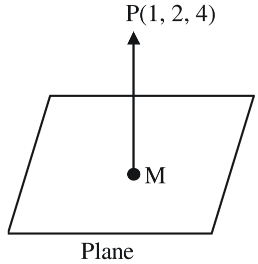
Therefore, $2\theta = \pi - 4\theta \Rightarrow \theta = \frac{\pi}{6}$

$$\frac{d^2\Delta}{d\theta^2} = 2a^2(-2\sin 2\theta - 4\sin 4\theta) < 0 \left(\text{at } \theta = \frac{\pi}{6}\right)$$

Therefore, Area of triangle is maximum when $\theta = \frac{\pi}{6}$.

**37.** Given equation of plane is :

$2x + y - 2z + 3 = 0$ ......(1)




Direction ratios of normal to the plane are 2, 1, –2

∴ Equation of line PM is :

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = \lambda(\text{say})$$

∴ Co-ordinates of M = $(2\lambda + 1, \lambda + 2, -2\lambda + 4)$

Since M lies on plane (1); we have :

∴ $2(2\lambda + 1) + (\lambda + 2) - 2(-2\lambda + 4) + 3 = 0$

$\Rightarrow \quad \lambda = \frac{1}{9}$

$\therefore \quad$ Foot of perpendicular $= \left(\frac{2}{9}+1, \frac{1}{9}+2, \frac{-2}{9}+4\right) = \left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

$\Rightarrow \quad$ Length of perpendicular from (1, 2, 4) i.e.

$$PM = \left|\frac{2(1)+2-2(4)+3}{\sqrt{4+1+4}}\right| = \frac{1}{3} \text{ units}$$

**OR**

Any point on the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ is x = 2 + 3λ, y = –4 + 4λ, z = 2 + 2λ

Equation of plane is

$$\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$\Rightarrow \quad x - 2y + z = 0$

Point lies on plane

$\Rightarrow \quad (2 + 3\lambda) - 2(-4 + 4\lambda) + (2 + 2\lambda) = 0$

$\Rightarrow \quad \lambda = 4$

$\Rightarrow \quad$ Point of intersection is (14, 12, 10)

Distance of point (2, 12, 5) from (14, 12, 10) is

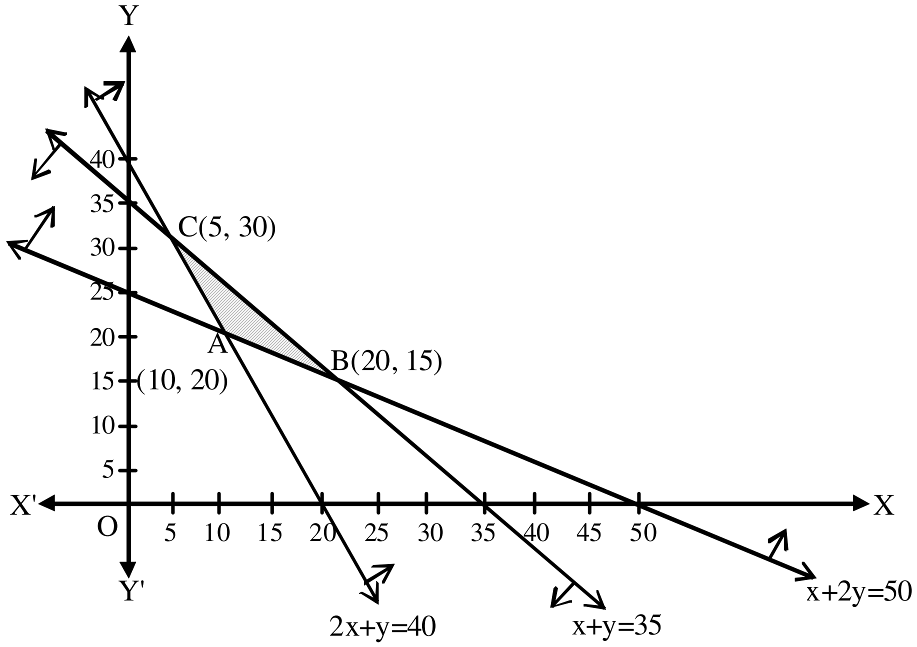
$$= \sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2} = 13 \text{ units}$$

**38.** Minimize $Z = 5x + 4y$

subject to constraints :

$$x + 2y \geq 50 \; ; \; 2x + y \geq 40 \; ; \; x + y \leq 35 \; ; \; x, y \geq 0$$

Plot the straight lines on the graph as shown :



Corner points of bounded feasible region are A(10, 20), B(20, 15), C(5, 30)

| Corner points | $Z = 5x + 4y$ |
|---|---|
| A(10, 20) | 130 |
| B(20, 15) | 160 |
| C(5, 30) | 145 |

Minimum value of Z is 130.

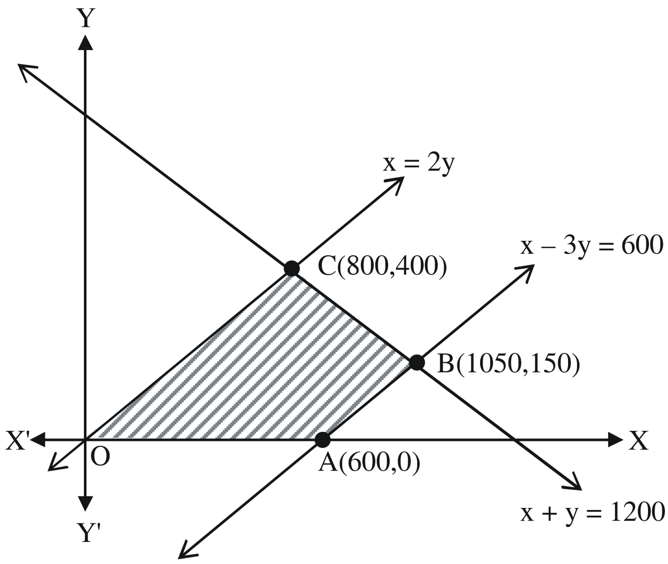
**OR**

Given objective function : $Z = 12x + 16y$

subject to constraints :

$x + y \leq 1200$ ; $x \geq 2y$ ; $x - 3y \leq 600$ ; $x, y \geq 0$

Plot the straight lines in graph as shown




Corner points of bounded feasible region are A(600, 0), B(1050, 150), C(800, 400), O(0, 0)

| Corner points | $Z = 12x + 16y$ |
|---|---|
| A(600,10) | 7360 |
| B(1050, 150) | 15000 |
| C(800, 400) | 16000 |
| O(0, 0) | 0 |

Maximum value of Z is 16000