CHAPTER 4

QUADRATIC EQUATIONS

Syllabus

- Standard form of quadratic equation: $ax^2 + bx + c = 0$, where ($a \neq 0$). Solutions of the quadratic equations (only real roots) by factorization, by using quadratic formula. Relationship between discriminant and nature of roots.
- Situational problems based on quadratic equations related to day to day activities to be incorporated.

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Find the roots/solutions of quadratic equations	1 Q (1 M)		1 Q (1 M)	1 Q (3 M)		1 Q (1 M) 1 Q (3 M)
Word Problem based Questions	1 Q (3 M) 2 Q (4 M)		1 Q (4 M)	2 Q (4 M)	2 Q (3 M)	1 Q (3 M)
Discriminant and Nature of roots			1 Q (1 M)	1 Q (1 M) 1 Q (3 M)		

TOPIC - 2

Solutions of Quadratic Equations

Revision Notes

- A quadratic equation in variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.
- The values of x that satisfy an equation are called the solutions or roots or zeros of the equation.
- A real number α is said to be a solution/root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

- A quadratic equation can be solved by the following algebraic methods:
  (i) By factorization (splitting the middle term),
  (ii) Making perfect squares and
  (iii) Using quadratic formula.
- If $ax^2 + bx + c = 0$, where $a \neq 0$ can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Method for factorization of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.
  (i) Form the product a and c *i.e.*, "ac"
  (ii) Find a pair of numbers b_1 and b_2 whose product is "ac" and whose sum is "b" (if you can't find such number, it can't be factorized)
  (iii) Split the middle term using b_1 and b_2, that expresses the term bx as $b_1x \pm b_2x$. Now factorize, by grouping the pairs of terms.
- Roots of the quadratic equation can be found by equating each linear factor to zero. Since, product of two numbers is zero, then either or both of them are zero.
- Any quadratic equation can be converted in to the form $(x + a)^2 - b^2 = 0$ by adding and subtracting same terms. This method of finding the roots of quadratic equation is called the method of making the perfect square.
- Method of making the perfect square for quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.
  (i) Dividing throughout by a, we get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
  (ii) Multiplying and dividing the coefficient of x by 2

$$x^2 + 2\frac{b}{2a}x + \frac{c}{a} = 0$$

  (iii) Adding and subtracting $\frac{b^2}{4a^2}$

$$x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2$$

  If $(b^2 - 4ac) \geq 0$, then by taking square root:

$$\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The Old-Babylonians (400 BC) stated and solved problems involving quadratic equations.
- The Greek mathematician Euclid developed a geometrical approach for finding out roots, which are solutions of quadratic equations.
- In Vedic manuscripts, procedures are described for solving quadratic equations by geometric methods related to completing the square.
- Brahmagupta (C.E. 598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$.
- Sridharacharya (C.E. 1025) derived the quadratic formula for solving a quadratic equation by the method of completing the perfect square.
- An Arab mathematician Al-Khwarizmi (about C.E. 800) studied quadratic equations of different types.
- Abraham bar Hiyya Ha-nasi, in his book '*Liber Embadorum*' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.
- Golden ratio ϕ is the root of quadratic equation $x^2 - x - 1 = 0$.

Know the Formulae

- The real roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac > 0$.
- Roots of $ax^2 + bx + c = 0$, where $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $b^2 - 4ac = 0$
- **Quadratic identities:**
  (i) $(a + b)^2 = a^2 + 2ab + b^2$
  (ii) $(a - b)^2 = a^2 - 2ab + b^2$
  (iii) $a^2 - b^2 = (a + b)(a - b)$

Mnemonics

Concept: To Find the roots of quadratic equation, $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

A Negative **B**oy could not decided if he did or didn't want to go to a **Radical** party. The **B**oy was **Square** so he **missed out** on 4 **A**wesome **C**hicks.
This was all over by **2 a.**m.

Interpretation:

A negative boy = $(-b)$
Undecided mean he wanted to go or didn't want to go = $(+/-)$
To a radical party = $(\sqrt{\quad})$
Boy was square = (b^2)
Missed out = $(-)$
4 Awesome = $4a$
Chicks = c
All over = Divided by
2 a.m. = $2a$

How is it done on the GREENBOARD?

Q.1. Two water taps together can fill a tank in $2\frac{11}{12}$ hrs. The tap of smaller diameter takes 2 hours more than the larger one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution

Step I: Let time taken by tap of larger diameter be x hrs. Then, the time taken by tap of smaller diameter = $(x + 2)$ hrs.

Then, the part of the tank filled by the tank of larger diameter in 1 hour = $\frac{1}{x}$

and other tap in 1 hour = $\frac{1}{x+2}$

Step II: According to question,

$$\frac{1}{x}+\frac{1}{x+2}=\frac{12}{35}$$

$$\frac{x+2+x}{x(x+2)}=\frac{12}{35}$$

Step III: By cross multiplication,

$$35(2x+2)=12(x^2+2x)$$

$$\Rightarrow \quad 70x+70=12x^2+24x$$

$$\Rightarrow 12x^2-46x-70=0$$

$$\Rightarrow \quad 6x^2-23x-35=0$$

Step IV: Now factorizing by splitting the middle term.

$$6x^2-30x+7x-35=0$$

$$\Rightarrow \quad 6x(x-5)+7(x-5)=0$$

$$\Rightarrow \quad (x-5)(6x+7)=0$$

$$\Rightarrow \quad x=5 \text{ or } x=\frac{-7}{6}$$

Rejecting $x=\frac{-7}{6}$ (as time cannot be negative).

Therefore, larger tap takes 5 hrs and smaller tap takes 7 hrs.

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. Find the roots of the equation $x^2 + 7x + 10 = 0$.**

U [CBSE SQP, 2020-21]

Sol.
$$x^2+7x+10=0$$
$$x^2+5x+2x+10=0 \quad ½$$
$$(x+5)(x+2)=0$$
Either $x=-5$ or $x=-2$ ½

[CBSE SQP Marking Scheme, 2020-21]

Alternate Solution:

Given, $x^2+7x+10=0$

Comparing with $ax^2+bx+c=0$, we get

$a=1, b=7$ and $c=10$

$$\therefore \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}-\frac{1}{2}$$

$$\Rightarrow \quad x=\frac{-7\pm\sqrt{(7)^2-4\times1\times10}}{2\times1}$$

$$\Rightarrow \quad x=\frac{-7\pm\sqrt{9}}{2}$$

$$\Rightarrow \quad x=\frac{-7\pm3}{2}$$

$$\Rightarrow \quad x=\frac{-7+3}{2} \text{ or } \frac{-7-3}{2}$$

$$\Rightarrow \quad x=-2 \text{ or } -5$$

Hence, the roots of the given equation are -2 or -5. ½

AI **Q. 2. Find the roots of the quadratic equation $x^2 - 0.04 = 0$.**

A [CBSE OD Set-I, 2020]

Sol.
$$x^2-0.04=0$$
$$\Rightarrow \quad x^2=0.04$$
$$\Rightarrow \quad x=\pm\sqrt{0.04}$$
$$\Rightarrow \quad x=\pm 0.2. \quad 1$$

[CBSE SQP Marking Scheme, 2020]

Q. 3. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other, then find the value of k.

R + U [CBSE SQP, 2020]

Sol. 4. 1

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Let one root $=\alpha$

and the other root $=\frac{1}{\alpha}$

Product of roots $=\alpha\times\frac{1}{\alpha}$ ½

Given equation is

$$(k-1)x^2-10x+3=0$$

Product of roots $=\frac{3}{(k-1)}$

$$\because \quad \frac{3}{k-1}=\alpha\times\frac{1}{\alpha}$$

$$\frac{3}{k-1}=1$$

$$3=k-1$$

$$k=4. \quad ½$$

Q. 4. Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other. A [CBSE Delhi Set- I, II, III, 2019]

Sol. Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots $= 1$ ½

$\Rightarrow$ $\frac{k}{3} = 1 \Rightarrow k = 3$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given equation: $3x^2 - 10x + k = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 3, b = -10, c = k$ ½

Let one root be α so other root is $\frac{1}{\alpha}$

Now, product of roots $= \frac{c}{a}$

$\therefore$ $\alpha \times \frac{1}{\alpha} = \frac{k}{3}$

$\Rightarrow$ $k = 3$

Hence, value of k is 3. ½

Q. 5. Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$. A [CBSE Bord Term, 2019]

Topper Answer, 2019

Sol. $(x+5)^2 = 2(5x-3)$

$\Rightarrow x^2 + 25 + 10x = 10x - 6$

$\Rightarrow x^2 + 31 = 0.$

$a = 1,\ b = 0,\ c = 31$

Discriminant $= b^2 - 4ac$

$= 0^2 - 4 \times 1 \times 31$

$= 0 - 124$

$= -124$

1

Q. 6. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

C + U [CBSE Delhi & OD, 2018]

Sol. $x = 3$ is one root of the equation

$\therefore$ $9 - 6k - 6 = 0$ ½

$\Rightarrow$ $k = \frac{1}{2}$ [CBSE Marking Scheme, 2018] ½

Topper Answer, 2019

$x^2 - 2kx - 6 = 0.$ let α be other root.

Product $= \frac{c}{a} = \frac{-6}{1} = -6.$

$3 \times \alpha = -6$

$\alpha = -2.$

Sum $= \frac{-b}{a} = \frac{-(-2k)}{1} = 2k.$

$\Rightarrow 3 + (-2) = 2k$

$1 = 2k,\ k = \frac{1}{2}.$

$\boxed{\text{Value of } k \text{ is } 1/2}$

Q. 7. **Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is 2.**

U [CBSE SQP, 2018]

Sol. Try Yourself, Similar to Q. No. 6 (above) of Very Short Answer Type Questions.

Q. 8. **Find the positive root of $\sqrt{3x^2+6} = 9$.**

U [Board Term-II 2015, 2017]

Sol.
$$\sqrt{3x^2+6} = 9$$
$$3x^2 + 6 = 81$$
$$3x^2 = 81 - 6 = 75$$
$$x^2 = \frac{75}{3} = 25$$
$$\therefore \quad x = \pm 5$$
Hence, positive root $= 5$. **1**

[CBSE Marking Scheme, 2015]

AI **Q. 9.** **If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.** C + U

[Board Term-2 , 2015 & CBSE Delhi Set-I, II, III, 2017]

Sol. Putting $x = -\frac{1}{2}$ in $3x^2 + 2kx - 3 = 0$
$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$
$$\Rightarrow \quad k = \frac{3}{4} - 3$$
$$\Rightarrow \quad k = \frac{3-12}{4}$$
Hence, $k = \frac{-9}{4}$ **1**

[CBSE Marking Scheme, 2015]

Q. 10. **Find the roots of the quadratic equation $\sqrt{3}\,x^2 - 2x - \sqrt{3} = 0$.** U [Board Term-II 2015]

Sol. Given, $\sqrt{3}\,x^2 - 2x - \sqrt{3} = 0$
$$\Rightarrow \quad \sqrt{3}\,x^2 - 3x + x - \sqrt{3} = 0$$
$$\Rightarrow \quad \sqrt{3}\,x(x - \sqrt{3}) + 1(x - \sqrt{3}) = 0$$
$$\Rightarrow \quad (x - \sqrt{3})(\sqrt{3}\,x + 1) = 0$$
$$\therefore \quad x = \sqrt{3} \text{ or } \frac{-1}{\sqrt{3}}$$ **1**

Short Answer Type Questions-I

2 marks each

Q. 1. **Solve for x: $\sqrt{2x+9} + x = 13$** A [Board Term-2 OD Set II, 2016]

Topper Answer, 2016

Sol.
$$\sqrt{2x+9} + x = 13$$
$$\sqrt{2x+9} = 13 - x$$
$$2x + 9 = (13 - x)^2 \Rightarrow$$
$$2x + 9 = 169 + x^2 - 26x \Rightarrow \; x^2 + 169 - 26x - 9 - 2x = 0$$
$$\Rightarrow 169 + x^2 - 2x - 26 - 9$$
$$x^2 - 2x - 26 + 169 - 9 = 0$$
$$x^2 - 2x - 26 + 160 = 0$$
$$x^2 - 2x + 134 = 0$$
$$x^2 - 28x + 160 = 0$$
$$x^2 - 20x - 8x + 160 = 0$$
$$x(x - 20) - 8(x - 20) = 0$$
$$(x - 8)(x - 20) = 0$$
either $= \boxed{x = 8}$ or $\boxed{x = 20}$ **2**

Q. 2. **If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.** C + A [CBSE, Delhi Set I, II, III, 2016]

Sol. Substituting $x = \frac{2}{3}$ in $ax^2 + 7x + b = 0$
$$\frac{4}{9}a + \frac{14}{3} + b = 0$$
$$\Rightarrow \quad 4a + 42 + 9b = 0$$
$$\Rightarrow \quad 4a + 9b = -42 \qquad \text{...(i)} \; ½$$
again, substituting $x = -3$ in $ax^2 + 7x + b = 0$
$$9a - 21 + b = 0$$
$$\Rightarrow \quad 9a + b = 21 \qquad \text{...(ii)} \; ½$$
Solving (i) and (ii), we get
$$a = 3 \text{ and } b = -6$$ **1**

Q. 3. **Solve the following quadratic equation for x:**
$$4x^2 - 4a^2x + (a^4 - b^4) = 0$$
U [Delhi CBSE Term-II, 2015 (Set I, II, 2016)]

Sol. Given, $4x^2 - 4a^2x + (a^4 - b^4) = 0$

Comparing with $Ax^2 + Bx + C = 0$, we get

Here, $A = 4$, $B = -4a^2$ and $C = (a^4 - b^4)$

Since, $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Then, $x = \dfrac{-(-4a^2) \pm (\sqrt{(-4a^2)^2 - 4 \times 4 \times (a^4 - b^4)})}{2 \times 4}$

$\Rightarrow \quad = \dfrac{4a^2 \pm \sqrt{16a^4 - 16a^4 + 16b^4}}{8}$

$\Rightarrow \quad = \dfrac{4a^2 \pm \sqrt{16b^4}}{8}$ **1**

$\Rightarrow \quad x = \dfrac{4a^2 \pm 4b^2}{8} = \dfrac{a^2 \pm b^2}{2}$

$\therefore \quad x = \dfrac{a^2 + b^2}{2}$ or $x = \dfrac{a^2 - b^2}{2}$ **1**

[CBSE Marking Scheme, 2015]

Q. 4. A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

A [CBSE, Foreign Set I, 2016]

Sol. Let unit's digit and ten's digit of the two digit number be x and y respectively

$\therefore$ Number is $10y + x$

According to question,

$10y + x = 4(y + x)$ ½

$\Rightarrow \quad 10y + x = 4y + 4x$

$\Rightarrow \quad 10y - 4y = 4x - x$

$\Rightarrow \quad 6y = 3x$

$\Rightarrow \quad 2y = x$ ...(i)

Also, $10y + x = 3xy$ ...(ii)

$\Rightarrow \quad 10y + 2y = 3(2y)y$ [From eq (i)]

$\Rightarrow \quad 12y = 6y^2$

$\Rightarrow \quad 6y^2 - 12y = 0$

$\Rightarrow \quad 6y(y - 2) = 0$

$\Rightarrow \quad y = 0$ or $y = 2$ ½

Rejecting $y = 0$ as tens digit should not be zero for a two digit number.

$\Rightarrow \quad x = 4$

$\therefore$ Required number $= 10y + x$

$\Rightarrow \quad 10 \times 2 + 4 = 24.$ **1**

Q. 5. Find the roots of $x^2 - 4x - 8 = 0$ by the method of completing the square. U

Sol. On completing the square, $x^2 - 4x + 4 - 4 - 8 = 0$

$\Rightarrow \quad (x - 2)^2 - 8 - 4 = 0$ ½

$\Rightarrow \quad (x - 2)^2 - 12 = 0$

$\Rightarrow \quad (x - 2)^2 = 12$

$\Rightarrow \quad (x - 2)^2 = (2\sqrt{3})^2$ ½

$\Rightarrow \quad x - 2 = \pm 2\sqrt{3}$

$\Rightarrow \quad x = 2 \pm 2\sqrt{3}$ ½

$\therefore \quad x = 2 + 2\sqrt{3}$ or $2 - 2\sqrt{3}$ ½

[CBSE Marking Scheme, 2015]

Q. 6. In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir. The product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation. A [Board Term-2, 2015]

Sol. Let the number of wickets taken by Zahir be x.

Then, the number of wickets taken by Harbhajan $= 2x - 3$ ½

According to question, $x(2x - 3) = 20$ ½

$\Rightarrow \quad 2x^2 - 3x = 20$

$\therefore$ Required quadratic equation is,

$2x^2 - 3x - 20 = 0.$ **1**

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II — 3 marks each

AI **Q. 1. In a flight of 600 km, an aircraft was slowed due to bad wether. Its average speed for the trip was reduced by 200 km/h and time of flight increased by 30 minutes. Find the original duration of flight.**

C + A [CBSE Delhi Set-I & OD Set-I, 2020]

Sol. Let original speed of flight be x km/h, then according to question,

$$\frac{600}{x - 200} - \frac{600}{x} = 30 \text{ minutes} \quad ½$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad 600\left[\frac{1}{x - 200} - \frac{1}{x}\right] = \frac{30}{60}$$

$$\Rightarrow \quad \frac{x - x + 200}{x(x - 200)} = \frac{1}{2 \times 600}$$

$$\Rightarrow \quad \frac{200}{x^2 - 200x} = \frac{1}{1200}$$

$\Rightarrow \quad x^2 - 200x = 240000$

$\Rightarrow \quad x^2 - 200x - 240000 = 0$ ½

Here, $a = 1$, $b = -200$ and $c = -240000$

$$\therefore \quad x = \frac{200 \pm \sqrt{40000 + 960000}}{2 \times 1}$$

$$= \frac{200 \pm \sqrt{1000000}}{2}$$

$$= \frac{200 \pm 1000}{2}$$

$$= \frac{200+1000}{2}, \frac{200-1000}{2}$$

$$= 600, -400 \quad \textbf{1}$$

Since, speed cannot be negative, therefore

original speed = 600 km/h.

and original distance = 600 km

$$\therefore \quad \text{Time} = \frac{\text{original distance}}{\text{original speed}}$$

$$= \frac{600 \text{ km}}{600 \text{ km/hr.}} = 1 \text{ h}$$

Hence, the original duration of flight is 1 h. **1**

Q. 2. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

A [CBSE Delhi Set-II, 2020]

Sol. Let the speed of the train = x km/h

Total distance covered by the train = 480 km

$\therefore$ Time taken to cover the distance 480 km

$$= \frac{480}{x} \text{ h}$$

If the speed has increased 8 km/h, *i.e.*, $(x + 8)$ km/h

Then, time taken to cover the distance 480 km

$$= \frac{480}{x-8} \text{ h} \quad \textbf{1}$$

According to question,

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$\Rightarrow \quad 480\left[\frac{x-x+8}{x(x-8)}\right] = 3$$

$$\Rightarrow \quad \frac{8}{x^2-8x} = \frac{3}{480} = \frac{1}{160}$$

$$\Rightarrow \quad x^2 - 8x - 1280 = 0 \quad \textbf{1}$$

Compare with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -8$ and $c = -1280$

$$\therefore \quad x = \frac{8 \pm \sqrt{64 + 4 \times 1280}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{5184}}{2}$$

$$= \frac{8 \pm 72}{2}$$

$$= \frac{8+72}{2}, \frac{8-72}{2}$$

$$= \frac{80}{2}, \frac{-64}{2}$$

$$= 40, -32$$

Since, negative speed cannot be possible.

Hence, the original speed of the train = 40 km/h. **1**

AI **Q. 3. Solve for x: $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}, x \neq -4, 7.$**

U [CBSE Outside Delhi Set-I, 2020)]

Sol. Given, $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$

$$\Rightarrow \quad \frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{3}{x^2+11x+28} = \frac{11}{30}$$

$$\Rightarrow \quad 11x^2 + 121x + 308 = 90$$

$$\Rightarrow \quad 11x^2 + 121x + 218 = 0 \quad \textbf{1}$$

Comparing with $ax^2 + bx + c = 0$, we get $a = 11$, $b = 121$ and $c = 218$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \textbf{1}$$

$$= \frac{-121 \pm \sqrt{14641-9592}}{22}$$

$$\Rightarrow \quad x = \frac{-121 \pm \sqrt{5049}}{22}$$

$$= \frac{-121 \pm 71.06}{22}$$

$$\Rightarrow \quad x = \frac{-121+71.06}{22}, \frac{-121-71.06}{22}$$

$$\Rightarrow \quad x = \frac{-49.94}{22}, \frac{-192.06}{22}$$

$$\Rightarrow \quad x = -2.27, -8.73. \quad \textbf{1}$$

Q. 4. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train.

U [CBSE Outside Delhi Set-II, 2020]

Sol. Total distance of the journey = 600 km

Let speed of fast train = x km/h,

then speed of slow train = $(x - 10)$ km/h

According to question,

$$\frac{600}{x-10} - \frac{600}{x} = 3 \quad \textbf{1}$$

$$\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad 600\left[\frac{x-x+10}{(x-10)x}\right] = 3$$

$$\Rightarrow \quad \frac{6000}{x^2-10x} = 3 \quad \textbf{1}$$

$$\Rightarrow \quad x^2 - 10x - 2000 = 0$$

$$\Rightarrow \quad x^2 - 50x + 40x - 2000 = 0$$

$$\Rightarrow \quad x(x - 50) + 40(x - 50) = 0$$

$$\Rightarrow \quad (x - 50)(x + 40) = 0$$

Either, $x = 50$ or $x = -40$

$\because$ speed can not be negative.

So, the speed of fast train = 50 km/h,
and the speed of slow train = 50 – 10 = 40 km/h. **1**

COMMONLY MADE ERROR

- Some students do not know how to frame the equation. Some frame it correctly but fail to solve it.

ANSWERING TIP

- Emphasis on solving quadratic equation based on application problems is necessary.

Q. 5. Solve for x:

$$x^2 + 5x - (a^2 + a - 6) = 0$$

A [CBSE OD Set-II, 2019]
[Board Term-2 Foreign Set I, 2015]

Sol. Given,

$$x^2 + 5x - (a^2 + a - 6) = 0$$

Then, $$x = \frac{-5 \pm \sqrt{25 + 4(a^2 + a - 6)}}{2} \quad \mathbf{1}$$

$$\left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$= \frac{-5 \pm (2a+1)}{2} \quad \mathbf{1}$$

$$= \frac{2a-4}{2} \text{ or } \frac{-2a-6}{2}$$

Thus, $x = a - 2$ or $x = -(a + 3)$ ½ + ½

[CBSE Marking Scheme, 2019]

Alternate Solution:

$$x^2 + 5x - (a^2 + a - 6) = 0$$
$$\Rightarrow x^2 + 5x - (a^2 + 3a - 2a - 6) = 0 \quad ½$$
$$\Rightarrow x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0 \quad ½$$
$$\Rightarrow x^2 + 5x - (a + 3)(a - 2) = 0 \quad ½$$
$$\Rightarrow x^2 + [(a + 3) - (a - 2)]x - (a + 3)(a - 2) = 0$$
$$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$$
$$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0 \quad ½$$
$$\Rightarrow [x + (a + 3)][x - (a - 2)] = 0$$
$$\Rightarrow x = -(a + 3) \text{ or } x = a - 2$$

Hence, roots of given equation are $-(a + 3)$ and $a - 2$. **1**

Q. 6. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}$.

A [CBSE Compt. Set I, II, III, 2018]

Sol. Let two parts be x and $27 - x$.

$$\therefore \quad \frac{1}{x} + \frac{1}{27 - x} = \frac{3}{20}$$

$$\Rightarrow \quad \frac{27 - x + x}{x(27 - x)} = \frac{3}{20}$$

$$\Rightarrow \quad x^2 - 27x + 180 = 0$$
$$\Rightarrow \quad (x - 15)(x - 12) = 0$$
$$\Rightarrow \quad x = 12 \text{ or } 15.$$

$\therefore$ The two parts are 12 and 15 **3**

[CBSE Marking Scheme, 2018]

Q. 7. A plane left 30 minutes late then its scheduled time and in order to reach of destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

A [CBSE Delhi/OD Set-I, II, III, 2016, 2018]
[Board Term-II, 2015]

Sol. Let usual speed of the plane be x km/hr.

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x + 100} = \frac{30}{60} \quad \mathbf{1}$$

$$\Rightarrow \quad x^2 + 100x - 300000 = 0$$
$$\Rightarrow x^2 + 600x - 500x - 300000 = 0 \quad \mathbf{1}$$
$$\Rightarrow \quad (x + 600)(x - 500) = 0$$
$$x = -600 \text{ or } x = 500 \quad ½$$

(Rejecting negative value)

Speed of plane = 500 km/h ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

Let the speed of plane be x km/h
Time taken to cover 1500 km

$$(t_1) = \frac{\text{Distance}}{\text{speed}} = \frac{1500}{x}\text{h} \quad ½$$

Time taken to cover 1500 km when speed increased by 100 km/h

$$(t_2) = \frac{1500}{x + 100}\text{h} \quad ½$$

Given, $$t_1 - t_2 = 30 \text{ minutes} = \frac{1}{2}\text{h}$$

Then,

$$\frac{1500}{x} - \frac{1500}{x + 100} = \frac{1}{2} \quad \mathbf{1}$$

$$\frac{1500x + 150000 - 1500x}{x(x + 100)} = \frac{1}{2}$$

$$\frac{150000}{x^2 + 100x} = \frac{1}{2}$$

$$x^2 + 100x = 30,000$$
$$x^2 + 100x - 30,000 = 0$$
$$x^2 + 600x - 500x - 30,000 = 0$$
$$x(x + 600) - 500(x + 600) = 0$$
$$(x + 600)(x - 500) = 0$$

Either $x + 600 = 0$

$\Rightarrow \quad x = -600,$

but speed can not be negative.

or $x - 500 = 0$ ½

$\Rightarrow \quad x = 500$

$\therefore$ Speed of the plane = 500 km/h ½

OR

Topper Answer, 2018

Given distance is 1500 km.

Usual speed = s.

We know, speed = $\frac{\text{distance}}{\text{time}}$ → ~~dist~~ time = $\frac{\text{distance}}{\text{speed}}$.

→ From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$ [half an hour late]. (30 min = 0.5 hr).

$\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}$.

$\frac{1500}{100+s} = \frac{3000-s}{2s}$. Cross multiplying,

$3000s = 300000 - 100s + 3000s - s^2$.

$s^2 + 100s - 300000 = 0$.

$s^2 + 600s - 500s - 300000 = 0$.

$s(s+600) - 500(s+600) = 0$

$(s-500)(s+600) = 0$.

⇒ $s - 500 = 0$ or $s + 600 = 0$.

→ $s = 500$ km/h → $s = -600$ km/h.

⇒ $s = 500$ or -600 km/hr.

But speed cannot be negative.

⇒ The usual speed of the plane is 500 km/hr.

AI **Q. 8. Solve for x:**

$$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}\text{; where } x \neq 1, -2, 2$$

U [Board Term-2 Delhi Set II, 2016]

Sol. Here, $\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$ **1**

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

⇒ $5x^2 + 19x - 30 = 0$ **1**

⇒ $(5x-6)(x+5) = 0$

⇒ $x = -5$ or $\frac{6}{5}$ **1**

[CBSE Marking Scheme, 2016]

Detailed Solution:

$$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}$$

$$\frac{x+1}{x-1}-1+\frac{x-2}{x+2}-1 = 2-\frac{2x+3}{x-2}$$

$$\frac{x+1-x+1}{x-1}+\frac{x-2-x-2}{x+2} = \frac{2x-4-2x-3}{x-2}$$

$$\frac{2}{x-1}+\frac{-4}{x+2} = \frac{-7}{x-2}$$

$$\frac{2x+4-4x+4}{(x-1)(x+2)} = \frac{-7}{x-2}$$

$$\frac{-2x+8}{x^2+x-2} = \frac{-7}{x-2}$$

$$-2x^2 + 4x + 8x - 16 = -7x^2 - 7x + 14$$

$$5x^2 + 19x - 30 = 0$$

$$5x^2 + 25x - 6x - 30 = 0$$

$$5x(x+5) - 6(x+5) = 0$$

$$(x+5)(5x-6) = 0$$

if $x + 5 = 0 \Rightarrow x = -5$

if $5x - 6 = 0 \Rightarrow x = \frac{6}{5}$

AI **Q. 9. Solve the following quadratic equation for x:**

$$x^2 + \left(\frac{a}{a+b}+\frac{a+b}{a}\right)x + 1 = 0$$

U [Board Term-2 Delhi Set III, 2016]

Sol. Here, $x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$

⇒ $x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right) = 0$ **1**

⇒ $\left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right) = 0$ **1**

⇒ $x = \frac{-a}{a+b}$ or $\frac{-(a+b)}{a}$ **1**

[CBSE Marking Scheme, 2016]

Q. 10. Solve the following quadratic equation for x:

$$9x^2 - 9(a + b)x + 2a^2 + 5ab + 2b^2 = 0$$

U [Board Term-2 Foreign Set I, 2016]

Sol. Given,

$$9x^2 - 9(a + b)x + 2a^2 + 5ab + 2b^2 = 0$$

First, we solve,

$$2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$$

Here, $= 2a[a + 2b] + b[a + 2b]$

$= (a + 2b)(2a + b)$ **1**

Hence, the equation becomes

$$9x^2 - 9(a + b)x + (a + 2b)(2a + b) = 0$$

$\Rightarrow 9x^2 - 3[3a + 3b]x + (a + 2b)(2a + b) = 0$

$\Rightarrow 9x^2 - 3[(a + 2b) + (2a + b)]x + (a + 2b)(2a + b) = 0$

$\Rightarrow 9x^2 - 3(a + 2b)x - 3(2a + b)x + (a + 2b)(2a + b) = 0$

$\Rightarrow 3x[3x - (a + 2b)] - (2a + b)[3x - (a + 2b)] = 0$

$\Rightarrow [3x - (a + 2b)][3x - (2a + b)] = 0$ **1**

$\Rightarrow 3x - (a + 2b) = 0$ or $3x - (2a + b) = 0$

$\Rightarrow x = \dfrac{a+2b}{3}$ or $x = \dfrac{2a+b}{3}$ **½**

Hence, the roots $= \dfrac{a+2b}{3}, \dfrac{2a+b}{3}$ **½**

Q. 11. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

A [Board Term-2 O.D. Set III, 2016]

Sol. Let the three consecutive natural numbers be x, $x + 1$ and $x + 2$. **½**

$\therefore (x + 1)^2 = (x + 2)^2 - (x)^2 + 60$ **1**

$\Rightarrow x^2 + 2x + 1 = x^2 + 4x + 4 - x^2 + 60$

$\Rightarrow x^2 - 2x - 63 = 0$ **½**

$\Rightarrow x^2 - 9x + 7x - 63 = 0$

$\Rightarrow x(x - 9) + 7(x - 9) = 0$

$\Rightarrow (x - 9)(x + 7) = 0$

Thus, $x = 9$ or $x = -7$

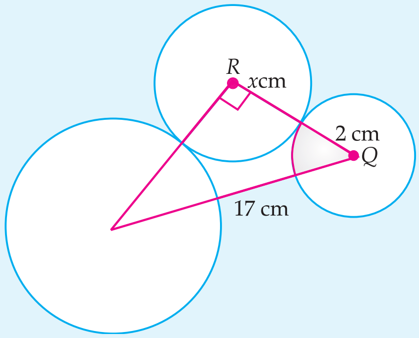
Rejecting -7, we get $x = 9$

Hence, three numbers are 9, 10 and 11. **1**

Q. 12. P & Q are centres of circles of radii 9 cm and 2 cm respectively. PQ = 17 cm. R is the centre of the circle of radius x cm which touches given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

A [Board Term-2, SQP, 2016]

Sol.



½

In right ΔPQR, by Pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

$\Rightarrow 17^2 = (x + 9)^2 + (x + 2)^2$ **½**

$289 = x^2 + 18x + 81 + x^2 + 4x + 4$

$\Rightarrow 2x^2 + 22x - 204 = 0$

$\Rightarrow x^2 + 11x - 102 = 0$ **½**

$\Rightarrow x^2 + 17x - 6x - 102 = 0$

$\Rightarrow x(x + 17) - 6(x + 17) = 0$ **½**

$(x - 6)(x + 17) = 0$

$\Rightarrow x = 6$ or $x - 17$ (x can't be negative) **½**

Thus, $x = 6$ cm

[CBSE Marking Scheme, 2016] **½**

Q. 13. Solve the quadratic equation $(x - 1)^2 - 5(x - 1) - 6 = 0$

A [Board Term-2, 2015]

Sol. Given, $(x - 1)^2 - 5(x - 1) - 6 = 0$

$\Rightarrow x^2 - 2x + 1 - 5x + 5 - 6 = 0$ **1**

$\Rightarrow x^2 - 7x + 6 - 6 = 0$

$\Rightarrow x^2 - 7x = 0$ **1**

$\Rightarrow x(x - 7) = 0$

$\therefore x = 0$ or 7 **1**

Long Answer Type Questions — 5 marks each

AI **Q. 1. Solve the following equation:**

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$

A [CBSE SQP, 2020]

Sol. $\dfrac{1}{x} - \dfrac{1}{x-2} = 3$ **1**

$\dfrac{x-2-x}{x(x-2)} = \dfrac{3}{1}$ **1**

$3x^2 - 6x = -2$

$3x^2 - 6x + 2 = 0$ **1**

$x = \dfrac{6 \pm \sqrt{12}}{6}$ **1**

$= \dfrac{3+\sqrt{3}}{3}, \dfrac{3-\sqrt{3}}{3}$ **1**

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Given, $\dfrac{1}{x} - \dfrac{1}{x-2} = 3$

$\Rightarrow \dfrac{x-2-x}{x(x-2)} = 3$ **1**

$$\Rightarrow \quad \frac{-2}{x^2 - 2x} = 3 \quad ½$$

$$\Rightarrow \quad 3x^2 - 6x = -2$$

$$\Rightarrow \quad 3x^2 - 6x + 2 = 0 \quad \mathbf{1}$$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 3, b = -6$ and $c = 2$ ½

Now, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6} \quad \mathbf{1}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{3 \pm \sqrt{3}}{3}$$

Hence, $$x = \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}. \quad \mathbf{1}$$

Q. 2. A train covers a distance of 360 km at a uniform speed. Had the speed been 5 km/h more, it would have taken 48 minutes less for the journey find the original speed of the train.

A [CBSE SQP, 2020]

Sol. Let the original speed of the train be x km/h

$$\therefore \quad \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} \quad \mathbf{2}$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0 \quad \mathbf{1}$$

$$\Rightarrow \quad (x + 50)(x - 45) = 0 \therefore x = 45 \quad \mathbf{1}$$

Hence original speed of the train = 45 km/h **1**

[CBSE SQP Marking Scheme, 2019]

Detailed Solution:

Let the original speed of the train be x km/h

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken to cover a distance of 360 km,

$$t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x} \text{ h} \quad \mathbf{1}$$

Time taken to cover a distance of 360 km, when speed is increased by 5 km/h

$$t_1 = \frac{360}{x+5} \text{ h} \quad \mathbf{1}$$

Given, $$t_1 = t_2 = 48 \text{ minutes}$$

$$= \frac{48}{60} = \frac{4}{5} \text{ h}$$

Then, $$\frac{360}{x} - \frac{360}{x+5} = \frac{4}{5} \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{360x + 1800 - 360x}{x(x+5)} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{1800}{x^2 + 5x} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{450}{x^2 + 5x} = \frac{1}{5}$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0 \quad \mathbf{1}$$

$$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow \quad x(x + 50) - 45(x + 50) = 0$$

$$\Rightarrow \quad (x + 50)(x - 45) = 0$$

If $$x + 50 = 0$$

$$\Rightarrow \quad x = -50,$$

but speed can not be negative and if $x - 45 = 0$

$$\Rightarrow \quad x = 45$$

Hence, the speed of the train = 45 km/h **1**

AI **Q. 3. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.**

A + U [CBSE Delhi Set-I, II, III, 2019, 2017]
[OD Set-III, Foreign Set-III, 2016]

Sol. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in $(x - 2)$ hrs.

Time taken by both the taps together $= \frac{15}{8}$ h

Therefore $$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15} \quad \mathbf{2}$$

$$\Rightarrow \quad 4x^2 - 23x + 15 = 0 \quad \mathbf{1}$$

$$\Rightarrow \quad (4x - 3)(x - 5) = 0$$

$$x \neq \frac{3}{4} \quad \therefore x = 5 \quad \mathbf{1}$$

Smaller and larger taps can fill the tank separately in 5 h and 3 h respectively. **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let the time taken by the smaller diameter tap $= x$ h.

$\therefore$ Time for larger diameter tap $= (x - 2)$ h.

$$\text{Total time taken} = 1\frac{7}{8} = \frac{15}{8} \text{ h.} \quad \mathbf{1}$$

Portion filled in one hour by smaller diameter tap

$$= \frac{1}{x}$$

and by larger diameter tap $= \dfrac{1}{x-2}$ **1**

According to the problem,

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{x-2} = \frac{8}{15} \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{x - 2 + x}{x(x-2)} = \frac{8}{15}$$

$$\Rightarrow \quad 15(2x - 2) = 8x(x - 2)$$

$$\Rightarrow \quad 30x - 30 = 8x^2 - 16x$$

$$\Rightarrow \quad 8x^2 - 46x + 30 = 0$$

$$\Rightarrow \quad 4x^2 - 23x + 15 = 0 \quad \mathbf{1}$$

$$\Rightarrow \quad 4x^2 - 20x - 3x + 30 = 0$$
$$\Rightarrow \quad 4x(x-5) - 3(x-5) = 0$$
$$\Rightarrow \quad (4x-3)(x-5) = 0$$
$$\Rightarrow \quad x = \frac{3}{4} \text{ or } x = 5$$

If $x = \frac{3}{4}$, then $x - 2 = \frac{3}{4} - 2 = \frac{-5}{4}$.

Since, time cannot be negative, we neglect $x = \frac{3}{4}$

Therefore, $x = 5$ and $x - 2 = 5 - 2 = 3$

Hence, time taken by larger diameter tap = 3 hours and time taken by smaller diameter tap = 5 hours. **1**

Q. 4. In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

C + A [CBSE Outside Delhi Set- 1, 2019]

Sol. Let marks in Hindi be x

Then marks in Eng $= 30 - x$ **1**

$\therefore \quad (x+2)(30-x-3) = 210$ **1**

$\Rightarrow \quad x^2 - 25x + 156 = 0$ or $(x-13)(x-12) = 0$ **1**

$\Rightarrow \quad x = 13$ or $x = 12$

$\therefore \quad 30 - 13 = 17$ or $30 - 12 = 18$ **1**

$\therefore$ Marks in Hindi & English are

(13, 17) or (12, 18) **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let the marks in Hindi be x

and the marks in English be y.

According to question,

$$x + y = 30$$
$$\Rightarrow \quad y = 30 - x \quad \text{...(i)}$$ **1**

If he had got 2 marks more in Hindi, then his marks would be $= x + 2$

and if he had 3 marks less in English, then his marks would be $= y - 3$ **1**

According to question,

$$(x+2)(y-3) = 210$$
$$\Rightarrow \quad (x+2)(30-x-3) = 210 \quad \text{[from eq. (i)]}$$ **1**
$$\Rightarrow \quad (x+2)(27-x) = 210$$
$$\Rightarrow \quad 27x - x^2 + 54 - 2x = 210$$
$$\Rightarrow \quad -x^2 + 25x - 156 = 0$$
$$\Rightarrow \quad x^2 - 25x + 156 = 0$$ **1**
$$\Rightarrow \quad x^2 - 13x - 12x + 156 = 0$$
$$\Rightarrow \quad x(x-13) - 12(x-13) = 0$$
$$\Rightarrow \quad (x-12)(x-13) = 0$$

$\Rightarrow$ Either $x = 12$ or $x = 13$

when $x = 12$,

then $\quad y = 30 - 12 = 18$

when $x = 13$,

then $\quad y = 30 - 13 = 17$

Hence, the marks in Hindi = 12 and marks in English = 18

or the marks in Hindi = 13 and marks in English = 17. **1**

AI **Q. 5. The total cost of a certain length of a piece of cloth is ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre ?**

C + A [CBSE OD Set-II, 2019]

Sol. Let total length ofcloth $= l$ m.

$$\therefore \quad \text{Rate per meter} = ₹\ \frac{200}{l}$$ **1**

$$\Rightarrow \quad (l+5)\left(\frac{200}{l} - 2\right) = 200$$ **1**

$$\Rightarrow \quad (l+5)(200-2l) = 200l$$
$$\Rightarrow \quad l^2 + 5l - 500 = 0$$ **1**
$$\Rightarrow \quad (l+25)(l-20) = 0$$
$$\Rightarrow \quad l = 20$$ **1**

$$\therefore \quad \text{Rate per meter} = ₹\left(\frac{200}{20}\right)$$
$$= ₹\ 10 \text{ per meter}$$ **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let length of the cloth $= x$ m

Cost of cloth per meter $= ₹\ y$

Given, $\quad x \times y = 200$

$$\Rightarrow \quad y = \frac{200}{x} \quad \text{...(i)}$$ **1**

According to given conditions,

1. If the piece were 5 m longer
2. Each meter of cloth cost ₹ 2 less

i.e.,
$$(x+5)(y-2) = 200$$
$$\Rightarrow \quad xy - 2x + 5y - 10 = 200$$
$$\Rightarrow \quad xy - 2x + 5y = 210$$ **1**
$$\Rightarrow \quad x\left(\frac{200}{x}\right) - 2x + 5\left(\frac{200}{x}\right) = 210$$ **1**

[from eq. (i)]

$$\Rightarrow \quad 200 - 2x + \frac{1000}{x} = 210$$
$$\Rightarrow \quad \frac{1000}{x} - 2x = 10$$
$$\Rightarrow \quad 1000 - 2x^2 = 10x$$
$$\Rightarrow \quad x^2 + 5x - 500 = 0$$ **1**
$$\Rightarrow \quad x^2 + 25x - 20x - 500 = 0$$
$$\Rightarrow \quad x(x+25) - 20(x+25) = 0$$
$$\Rightarrow \quad (x+25)(x-20) = 0$$
$$\Rightarrow \quad x = -25 \text{ or } x = 20$$
$$\therefore \quad x = 20 \quad (\text{neglecting } x = -25)$$
$$y = \frac{200}{x} = \frac{200}{20} = 10$$ **1**

[From eq. (i)]

Hence, length of the piece of cloth is 20 m and rate per meter is ₹ 10.

Q. 6. A shopkeeper buy certain number of books in ₹ 80. If he buy 4 more books then new cost price of each book is reduced by ₹ 1. Find the number of books initially he buy. [CBSE Delhi Board Term, 2019]

Topper Answer, 2019

Sol. Let the no. of books bought by the shopkeeper be 'n'.

Total money spent = Rs 80

$\therefore$ Cost of each book = Rs $\frac{80}{n}$

Now, given : He buys 4 more books, no. of books bought = $n+4$ (for same amount)

New cost of each book = Rs $\frac{80}{n+4}$

given, new cost of each book is Rs. 1 less than earlier.

$$\therefore \quad \frac{80}{n} - \frac{80}{n+4} = 1$$

$$\Rightarrow 80\left(\frac{1}{n} - \frac{1}{n+4}\right) = 1$$

$$\Rightarrow \frac{\cancel{n}+4-\cancel{n}}{n(n+4)} = \frac{1}{80} \Rightarrow 4\times 80 = n(n+4)$$

$$\Rightarrow n^2+4n-320 = 0$$

Using quadratic formula; $\Rightarrow n = \frac{-4 \pm \sqrt{16+4\times320}}{2}$

$$\cancel{\Rightarrow n = \frac{-4\pm\sqrt{676}}{2}}$$

$$\cancel{= \frac{-4\pm26}{2} \Rightarrow n = \frac{-30}{2} \text{ or } \frac{22}{2}}$$

$$\Rightarrow n = \frac{-4\pm\sqrt{1296}}{2}$$

$$\Rightarrow n = \frac{-4\pm36}{2}$$

$$\Rightarrow n = \frac{-40}{2} \text{ or } \frac{32}{2} \Rightarrow n = -20 \text{ or } 16$$

Since, no. of books is a whole no., it cannot be negative

$n=-20$ can be ignored.

$\therefore \boxed{n=16}$

No. of books bought by the shopkeeper = 16.

Q. 7. A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original average speed ?

C + A [CBSE Delhi Set, 2018]

Sol. Let the original average speed of train be x km/hr.

Therefore, $\frac{63}{x} + \frac{72}{x+6} = 3$ **1**

$x^2 - 39x - 126 = 0$ **1**

$(x - 42)(x + 3) = 0$ **1**

x is not equal to -3.

$\therefore \quad x = 42$ **1**

Thus, original speed of train is 42 km/h. **1**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Let the original speed of train be x km/h

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$ **1**

Total time to complete journey = 3 h

$$\frac{63}{x} + \frac{72}{x+6} = 3$$ **1**

$$\frac{63x + 378 + 72x}{x(x+6)} = 3$$

$$\frac{135x + 378}{x^2 + 6x} = 3$$

$$3x^2 + 18x = 135x + 378$$

$$3x^2 - 117x - 378 = 0$$

$$x^2 - 39x - 126 = 0$$ **1**

$$x^2 - 42x + 3x - 126 = 0$$

$$x(x - 42) + 3(x - 42) = 0$$

$$(x - 42)(x + 3) = 0$$ **1**

If $x + 3 = 0 \Rightarrow x = -3$, speed cannot be negative.

If $x - 42 = 0 \Rightarrow x = 42 \therefore$ Speed of train = 42 km/h **1**

Q. 8. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

C + A [CBSE Delhi/OD Set 2018, 2017]

[Board Term-II, O.D. Set-II, 2016]

Sol. Let the speed of stream be x km/h.

Then, the speed of boat upstream = $(18 - x)$ km/h

and speed of boat downstream = $(18 + x)$ km/h **1**

According to the question,

$$\Rightarrow \quad \frac{24}{18-x} - \frac{24}{18+x} = 1$$ **1**

$$\Rightarrow \quad \frac{24(18+x) - 24(18-x)}{18^2 - x^2} = 1$$

$$\Rightarrow \quad 432 + 24x - 432 + 24x = 324 - x^2$$

$$\Rightarrow \quad 48x = 324 - x^2$$

$$\Rightarrow \quad x^2 + 48x - 324 = 0$$ **1**

$$\Rightarrow \quad x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow \quad x(x + 54) - 6(x + 54) = 0$$

$$\Rightarrow \quad (x + 54)(x - 6) = 0$$

$$\Rightarrow \quad x + 54 = 0 \quad \text{or } x - 6 = 0$$

$$\Rightarrow \quad x = -54 \text{ or } \quad x = 6$$ **1**

Since, speed cannot be negative.

Hence, the speed of steam x = 6 km/h. **1**

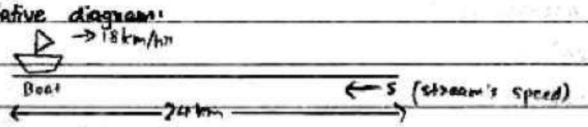
[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2018

Sol. 23) Representative diagram:

(Choice 1)



Given that:

Speed of boat = 18 km/hr in still water.

Speed of stream = 5 (variable, must find)

Distance upstream + back = 24 km.

Time upstream = $\frac{24}{18-5}$, Time downstream = $\frac{24}{18+5}$.

$\Rightarrow \frac{24}{18-s} = 1 + \frac{24}{18+s}.$

$\frac{24}{18-s} = \frac{18+s+24}{18+s}.$ (Cross-multiplying)

$24(18+s) = (42+s)(18-s)$

$432+24s = 756+18s-42s-s^2$

$\Rightarrow s^2+24s+24s+432-756 = 0.$

$s^2+48s+(-324) = 0.$

$s^2+54s-6s-324=0.$

$s(s+54)-6(s+54)=0.$

$(s-6)(s+54)=0.$

Now, either $s-6=0$ or $s+54=0.$

$\Rightarrow s=6$ $\quad\Rightarrow s=-54.$

So speed = 6 or −54 km/hr.

But speed cannot be negative.

$\Rightarrow$ $\boxed{\text{Speed of the stream is 6 km/hr.}}$ **5**

AI **Q. 9. Solve $\frac{1}{(a+b+x)} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x}$, where $a + b \neq 0$.**

A [CBSE SQP, 2018, 2016]

[Board Foreign set II, III 2017]

Sol. Given,

$$\frac{1}{a+b+x} = \frac{1}{a}+\frac{1}{b}+\frac{1}{x}$$

$$\Rightarrow \quad \frac{1}{a+b+x}-\frac{1}{x} = \frac{1}{a}+\frac{1}{b} \qquad 1$$

$$\Rightarrow \quad \frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab} \qquad 1$$

$$\Rightarrow \quad \frac{x-a-b-x}{x(a+b+x)} = \frac{a+b}{ab} \qquad 1$$

$$\Rightarrow \quad \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab} \qquad 1$$

$$\Rightarrow \quad x(a+b+x) = -ab$$

$$\Rightarrow \quad x^2 + (a+b)x + ab = 0$$

$$\Rightarrow \quad (x+a)(x+b) = 0$$

$$\Rightarrow \quad x = -a \text{ or } x = -b \qquad 1$$

[CBSE Marking Scheme, 2018]

COMMONLY MADE ERROR

- Candidates do error in simplifying this type of equations.

ANSWERING TIP

- Adequate practice is necessary for simplifying this type of quadratic equations.

AI **Q. 10. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it ?** A [Board OD Set III, 2017]

Topper Answer, 2017

Sol. 29. Let B complete a work in x days.

Then A takes $x-6$ days to complete it.

Together they complete it in 4 days.

$\therefore \frac{x + x-6}{x(x-6)} = \frac{1}{4}$

$4(2x-6) = x(x-6)$

$8x - 24 = x^2 - 6x$

$\therefore x^2 - 14x + 24 = 0$

$x^2 - 12x - 2x + 24 = 0$

$x(x-12) - 2(x-12) = 0$

$(x-2)(x-12) = 0$

$\therefore x = 2$ or 12.

$x = 2$ is not possible because then $x-6$ is (-4)

$\therefore x = 12$.

So, B takes 12 days to finish the work. **1**

Q. 11. In a rectangular part of dimensions 50 m× 40 m a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

Pond
Grass Strips

A [Board Foregin Set-I, III 2017]

Sol. Let width of grass strip be x m.

$\therefore$ Length of pond $= (50 - 2x)$ m

and Breadth of pond $= (40 - 2x)$ m **1**

And area of park – area of pond = area of grass strip

$\Rightarrow (50 \times 40) - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow 2000 - 2000 + 180x - 4x^2 = 1184$ **1**

$\Rightarrow x^2 - 45x + 296 = 0$

$\Rightarrow x^2 - 37x - 8x + 296 = 0$

$\Rightarrow x(x - 37) - 8(x - 37) = 0$

$\Rightarrow x = 8$ or 37 **1**

(37 is rejected, as it gives negatives values for length & breadth)

Thus, the length of pond $= 50 - 2 \times 8$

$= 34$ m **1**

and breadth of pond $= 40 - 2 \times 8$

$= 24$ m. **1**

Q. 12. In a class test Raveena got a total of 30 marks in English and Mathematics. Had she got 2 more marks in Mathematics and 3 marks less in English then the product of her marks obtained would have been 210. Find the individual marks obtained in two subjects. A [OD Compt. I, II, III, 2017]

Sol. Let marks in mathematics be x.

Then marks in English $= 30 - x$ **1**

$(x + 2)(30 - x - 3) = 210$ **1**

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x + 54 - x^2 - 2x = 210$

$\Rightarrow -x^2 + 25x = 210 - 54 = 156$

$\Rightarrow x^2 - 25x + 156 = 0$ **1**

$\Rightarrow x^2 - 13x - 12x + 156 = 0$

$\Rightarrow x(x - 13) - 12(x - 13) = 0$

$\Rightarrow (x - 13)(x - 12) = 0$

$\Rightarrow x = 12$ or $x = 13$

If $x = 12$, then marks in Mathematics = 12 **1**

and marks in English = 18

If $x = 13$, then marks in Mathematics = 13 **1**

and marks in English = 17.

Q. 13. Solve for x: $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

U [CBSE S.A.-2, 2016]

Sol. Given, $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{(x-5)} = y$ **1**

$\therefore y^2 + 5y - 24 = 0$ **1**

$\Rightarrow (y + 8)(y - 3) = 0$ **1**

$y = 3$ or -8

Putting $y = 3$ we get

$\frac{2x}{x-5} = 3$

$2x = 3x - 15$

$\Rightarrow x = 15$ **1**

Again, for $y = -8$, $\frac{2x}{x-5} = -8$

$2x = -8x + 40$

$10x = 40$

$x = 4$

Hence, $x = 15$ or 4 **1**

[CBSE Marking Scheme, 2016]

Q. 14. Find x in terms of a, b and c:

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c} \quad \text{where } x \neq a, b, c$$

C + U [Board Term-2 Delhi Set 1, 2016]

Sol. $a(x - b)(x - c) + b(x - a)(x - c) = 2c(x - a)(x - b)$ **2**

$\Rightarrow x^2(a + b - 2c) + x(-ab - ac - ab - bc + 2ac + 2bc) = 0$

$\Rightarrow x^2(a + b - 2c) + x(-2ab + ac + bc) = 0$ **2**

$\Rightarrow x = -\left(\frac{ac + bc - 2ab}{a + b - 2c}\right)$ and $x = 0$ **1**

[CBSE Marking Scheme, 2016]

Q. 15. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

A [Foreign Set III, 2016]

Sol. Let numerator be x.

Then, the fraction $= \frac{x}{2x+1}$ **1**

Again, $\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}$ **1**

$\Rightarrow \quad 21[x^2 + (2x+1)^2] = 58\,(2x^2 + x)$

$\Rightarrow \quad 11x^2 - 26x - 21 = 0$ **1**

$11x^2 - 33x + 7x - 21 = 0$

$(x-3)(11x+7) = 0$

$x = 3 \text{ or } -\frac{7}{11}$ **1**

(rejected negative value)

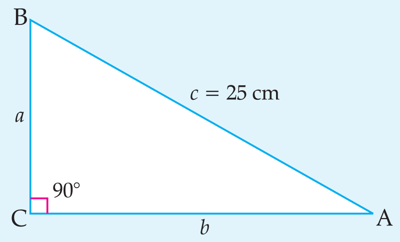
Hence, fraction $= \frac{3}{7}$ **1**

[CBSE Marking Scheme, 2016]

Q. 16. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

A [CBSE Delhi Set II, 2016]

Sol.



Here, $a + b + c = 60$ and $c = 25$

$a + b = 60 - c$

$a + b = 60 - 25 = 35$ **1**

Using Pythagoras theorem,

$a^2 + b^2 = 625$

Using identity,

$(a+b)^2 = a^2 + b^2 + 2ab$ **1**

$35^2 = 625 + 2ab$

or, $1225 - 625 = 2ab$

$\Rightarrow \quad ab = 300$ **1**

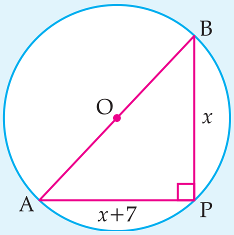
Hence, area of $\Delta ABC = \frac{1}{2}ab = 150 \text{ cm}^2$. **1**

[CBSE Marking Scheme, 2016]

Q. 17. A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected.

[Foreign Set I, II, 2016]

Sol.



1

Let P be the location of the pole such that its distance from gate B, x metres.

$\therefore \quad AP = x + 7$ **1**

AB is diameter, $\angle APB = 90°$ and $AB = 17$ m **½**

Using Pythagoras Theorem,

$\therefore \quad x^2 + (x+7)^2 = (17)^2$

$x^2 + x^2 + 14x - 240 = 0$

or $\quad x^2 + 7x - 120 = 0$ **1½**

$x = \frac{-7 \pm \sqrt{49 + 480}}{2}$

$x = \frac{-7 \pm 23}{2} = 8, -15$ (rejected)

$\therefore \quad x = 8$ m and $x + 7 = 15$ m **1**

Hence, distance between two gates = 8 m and 15 m.

[CBSE Marking Scheme, 2016]

Q. 18. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/h more than the speed while going, find the speed in km/h in each direction.

A [CBSE Delhi Set III, 2016]

Sol. Let the speed while going be x km/h.

$\therefore$ Speed while returning = $(x + 10)$ km/h

According to question,

$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2}$ **2**

$\Rightarrow \quad x^2 + 10x - 600 = 0$

$\Rightarrow \quad (x+30)(x-20) = 0$

$\Rightarrow \quad x = 20 \text{ or } -30$ **1**

Rejecting negative value,

$\therefore$ Speed while going = 20 km/h **1**

and speed while returning = 20 + 10 = 30 km/h **1**

[CBSE Marking Scheme, 2016]

AI **Q. 19. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.**

A [Board Term-2, Delhi, 2015 Set I, III]

Sol. Let the denominator be x, then numerators $= x - 3$

So, the fraction $= \dfrac{x-3}{x}$ **1**

By the given condition,

$$\text{new fraction} = \frac{x-3+2}{x+2}$$

$$= \frac{x-1}{x+2} \quad \mathbf{1}$$

Then, $\dfrac{x-3}{x} + \dfrac{x-1}{x+2} = \dfrac{29}{20}$

$\Rightarrow \quad 20[(x-3)(x+2) + x(x-1)] = 29(x^2 + 2x)$

$\Rightarrow \quad 20(x^2 - x - 6 + x^2 - x) = 29x^2 + 58x$ **1**

$\Rightarrow \quad 11x^2 - 98x - 120 = 0$

$\Rightarrow \quad 11x^2 - 110x + 12x - 120 = 0$

$(11x + 12)(x - 10) = 0$ or $x = 10$ **1**

$\therefore$ The fraction is $\dfrac{7}{10}$. **[CBSE Marking Scheme, 2015] 1**

Q. 20. The diagonal of a rectangular field is 16 metre more than the shorter side. If the longer side is 14 metre more than the shorter side, then find the length of the sides of the field.

C + A **[CBSE OD, Set I, II, III, 2015]**

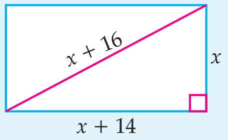
Sol. Let the length of shorter side be x m.

$\therefore$ Length of diagonal $= (x + 16)$ m **1**

and length of longer side $= (x + 14)$ m **1**

Using Pythagoras Theorem,

$x^2 + (x + 14)^2 = (x + 16)^2$ **1**



$\Rightarrow \quad x^2 - 4x - 60 = 0$

$\Rightarrow \quad x^2 + 6x - 10x - 60 = 0$

$\Rightarrow \quad x(x + 6) - 10(x + 6) = 0$

$x = -6$ or $x = 10$

$\Rightarrow \quad x = 10$ m **1**

$\therefore$ Length of sides are 10 m and 24 m. **1**

[CBSE Marking Scheme, 2015]

TOPIC - 2
Discriminant and Nature of Roots

Revision Notes

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant *i.e.*, Discriminant $D = b^2 - 4ac$
- Nature of roots of a quadratic equation:
  (i) If $b^2 - 4ac > 0$, the quadratic equation has two distinct real roots.
  (ii) If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  (iii) If $b^2 - 4ac < 0$, the quadratic equation has no real roots.

Know the Formulae

- Discriminant, D $= b^2 - 4ac$.

How is it done on the GREENBOARD?

Q.1. Find the value of k for which the equation $4x^2 + kx + 25 = 0$ has equal roots.

Solution:

Step 1: $4x^2 + kx + 25 = 0$

Comparing above equation with

$ax^2 + bx + c = 0$

$a = 4$, $b = k$ and $c = 25$

Step 2: Condition for equal roots is $D = 0$

i.e., $b^2 - 4ac = 0$

Step 3: Substituting the values of a, b and c in the above condition.

$(k^2) - 4(4)(25) = 0$

$\Rightarrow \quad k^2 - 400 = 0$

$\Rightarrow \quad k^2 - (20)^2 = 0$

$\Rightarrow \quad (k - 20)(k + 20) = 0$

$\Rightarrow \quad k = 20$ or -20

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. For what values of k, the given quadratic equation $9x^2 + 6kx + 4 = 0$ has equal roots ?**

C + A [CBSE SQP, 2020-21]

Sol. $9x^2 + 6kx + 4 = 0$

For equal roots $b^2 - 4ac = 0$

Discriminant $D = 0$ so,

$(6k)^2 - 4 \times 9 \times 4 = 0$ ½

$36k^2 = 144$

$k^2 = 4$

$k = \pm 2$ ½

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Given, $9x^2 + 6kx + 4 = 0.$

Comparing with $ax^2 + bx + c = 0$,

we get $a = 9, b = 6k, c = 4$ ½

Since, Discriminant, $D = b^2 - 4ac$

and for equal roots,

$b^2 - 4ac = 0$ $[\because D = 0]$

$\Rightarrow (6k)^2 - 4 \times 9 \times 4 = 0$

$\Rightarrow 36k^2 - 144 = 0$

$\Rightarrow 36k^2 = 144$

$\Rightarrow k^2 = 4$

$\Rightarrow k = \pm 2$ ½

AI **Q. 2. For what value(s) of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots ?**

A [CBSE SQP, 2020-21]

Sol. Given that,

$3ax^2 - 6x + 1 = 0$

For no real roots $b^2 - 4ac < 0$ ½

Discriminant $D < 0$ so,

$(-6)^2 - 4(3a)(1) < 0$

$12a > 36$

$a > 3$ ½

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Given, $3ax^2 - 6x + 1 = 0$

On Comparing with $ax^2 + bx + c = 0$,

we get $a = 3a, b = -6$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$= (-6)^2 - 4 \times 3a \times 1$

$= 36 - 12a$ ½

For condition of 'no real roots',

$b^2 - 4ac < 0$

$\Rightarrow 36 - 12a < 0$

$\Rightarrow 12a > 36$

$\Rightarrow a > 3.$ ½

AI **Q. 3. Find the values (s) of k for which the quadratic equation $x^2 + 2\sqrt{2}\ kx + 18 = 0$ has equal roots.**

C + A [CBSE SQP, 2020]

Sol. $D = (2\sqrt{2}k)^2 - 4(1)(18) = 0$ ½

$\Rightarrow k = \pm 3$ ½

[CBSE Marking Scheme, 2020]

Detailed Solution:

Since, $x^2 + 2\sqrt{2}\ kx + 18 = 0$ has equal roots

$D = 0$

$b^2 = 4ac$ ½

$(2\sqrt{2}k)^2 = 4 \times 1 \times 18$

$4 \times 2 \times k^2 = 72$

$k^2 = \dfrac{72}{8}$

$k^2 = 9 \Rightarrow k = \sqrt{9}$

$k = \pm 3.$ ½

Q. 4. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real ?

A [CBSE Delhi Set-I, II, III, 2019]

Sol. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow 16 - 4k \geq 0$ ½

$\Rightarrow k \leq 4$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given quadratic equation is $x^2 + 4x + k = 0.$

Comparing the given equation with $ax^2 + bx + c = 0$, we get $a = 1, b = 4, c = k$ ½

Since, given the equation has real roots

$\Rightarrow D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow 4^2 - 4 \times 1 \times k \geq 0$

$\Rightarrow 4k \leq 16$

$\Rightarrow k \leq 4$ ½

Q. 5. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0.$

A [CBSE OD, Set-I, II, III, 2019]

Sol. $2x^2 - 4x + 3 = 0 \Rightarrow D = 16 - 24 = -8$

$\therefore$ Equation has no real roots 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given: $2x^2 - 4x + 3 = 0$

On comparing above with $ax^2 + bx + c = 0$,

we get, $a = 2, b = -4, c = 3$ ½

We shall find $D = b^2 - 4ac$

So, $D = (-4)^2 - 4(2) \times (3)$

$= -8 < 0$ or (–ve)

Hence, the given equation has no real roots. ½

COMMONLY MADE ERROR

- Students often make mistakes in analyzing the nature of roots as they get confused with the conditions.

ANSWERING TIP

- Understand the different conditions for nature of roots.

Q. 6. Find the values (s) of k for which the equation $x^2 + 5kx + 16 = 0$ has real and equal roots.
U [CBSE SQP, 2018]

Sol. For roots to be real and equal, $b^2 - 4ac = 0$ ½

$\Rightarrow \quad (5k)^2 - 4 \times 1 \times 16 = 0$ ½

$$k = \pm\frac{8}{5}$$

[CBSE Marking Scheme, 2018]

Q. 7. Find the value(s) of k if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has real roots.
U [CBSE SQP 2017]

Sol. If Discriminant of quadratic equation is equal to zero, or more than zero, then roots are real.

Given, $\quad 3x^2 - k\sqrt{3}x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 3, b = -k\sqrt{3}$ and $c = 4$

Since, Discriminant, $D = b^2 - 4ac$ ½

and for real roots $b^2 - 4ac \geq 0$

$\Rightarrow \quad \left(-k\sqrt{3}\right)^2 - 4 \times 3 \times 4 \geq 0$

$\Rightarrow \quad 3k^2 - 48 \geq 0$

$\Rightarrow \quad k^2 - 16 \geq 0$

$\Rightarrow \quad (k-4)(k+4) \geq 0$

$\therefore \quad k \leq -4$ and $k \geq 4$. ½

Short Answer Type Questions-I

2 marks each

AI **Q. 1. For what value of k, the given quadratic equation $kx^2 - 6x - 1 = 0$ has no real roots ?**
[CBSE Delhi Board Term, 2019]

Topper Answer, 2019

Sol.

Given, quadratic equation $\Rightarrow kx^2 - 6x - 1 = 0$.
where $a = k$, $b = -6$, $c = -1$.

For no real roots (or imaginary roots), discriminant must be less than 0.

That is, $D < 0$

$\Rightarrow b^2 - 4ac < 0$

$\Rightarrow (-6)^2 - 4 \times k \times -1 < 0$

$\Rightarrow 36 + 4k < 0$

$\Rightarrow 4k < -36 \Rightarrow \boxed{k < -9}$

$\therefore k$ should be less than -9. ($k = -10, -11 \ldots$) 1

Q. 2. Find the value of k for which the roots of the quadratic equation $2x^2 + kx + 8 = 0$ will have the equal roots ?
C + A [Board Term-II OD Compt., 2017]

Sol. For equal roots, $D = 0$

$\therefore \quad b^2 - 4ac = 0$

$\Rightarrow \quad k^2 = 4 \times 2 \times 8$ 1

$k^2 = 64$

$k^2 = 64$

$k = \pm\sqrt{64}$

$\therefore \quad k = \pm 8$ 1

Q. 3. If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q. U [CBSE SQP, 2016]

Sol. Since, 2 is the root of $x^2 + kx + 12 = 0$

$$\Rightarrow \quad (2)^2 + 2k + 12 = 0$$
$$\Rightarrow \quad 2k + 16 = 0$$
$$k = -8 \qquad \tfrac{1}{2}$$

Putting, $k = -8$ in $x^2 + kx + q = 0$

$$\Rightarrow \quad x^2 - 8x + q = 0 \qquad \tfrac{1}{2}$$

For equal roots,

$$(-8)^2 - 4(1)q = 0 \qquad \tfrac{1}{2}$$
$$\Rightarrow \quad 64 - 4q = 0$$
$$\Rightarrow \quad 4q = 64$$
$$\Rightarrow \quad q = 16 \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2016]

Q. 4. Find k so that the quadratic equation $(k + 1)x^2 - 2(k + 1)x + 1 = 0$ has equal roots.

R [Board Term-2 2016]

[CBSE Board Term-2, Set- I, III, 2015]

Sol. Since, $(k + 1)x^2 - 2(k + 1)x + 1 = 0$

has equal roots.

$$D = 0$$
$$\Rightarrow \quad b^2 = 4ac \qquad 1$$
$$\Rightarrow \quad 4(k + 1)^2 = 4(k + 1)$$
$$\Rightarrow \quad k^2 + 2k + 1 = k + 1$$
$$\Rightarrow \quad k^2 + k = 0$$
$$\Rightarrow \quad k(k + 1) = 0$$
$$\Rightarrow \quad k = 0 \text{ or } -1 \qquad 1$$

$k = -1$ does not satisfy the equation

So, $\quad k = 0$

[CBSE Marking Scheme, 2016]

Short Answer Type Questions-II — 3 marks each

AI **Q. 1. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.** A

[CBSE OD Set-I, II, III, 2019]

$x^2 + px + 16 = 0$ have equal roots if

$$D = p^2 - 4(16)(1) = 0 \qquad 1$$
$$p^2 = 64 \Rightarrow p = \pm 8 \qquad \tfrac{1}{2}$$
$$\therefore \quad x^2 \pm 8x + 16 = 0 \Rightarrow (x \pm 4)^2 = 0 \qquad 1$$
$$x \pm 4 = 0$$

$\therefore$ Roots are $x = -4$ and $x = 4$ $\qquad$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given quadratic equation,

$$x^2 + px + 16 = 0 \qquad \text{...(i)}$$

If this equation has equal roots, then discriminant value is zero

i.e., $\quad D = b^2 - 4ac = 0 \qquad$ ...(ii)

Now, comparing the given quadratic equation with

$$ax^2 + bx + c = 0,$$

we get $a = 1$, $b = p$ and $c = 16$ $\qquad$ 1

$\therefore$ From eq (ii),

$$p^2 - 4 \times 1 \times 16 = 0$$
$$\Rightarrow \quad p^2 = 64$$
$$\Rightarrow \quad p = \pm 8$$

when $p = 8$, $\qquad$ ½

from eq (i), $\quad x^2 + 8x + 16 = 0$

$$\Rightarrow \quad x^2 + 4x + 4x + 16 = 0$$
$$\Rightarrow \quad x(x + 4) + 4(x + 4) = 0$$
$$\Rightarrow \quad (x + 4)(x + 4) = 0$$
$$\Rightarrow \quad x = -4, -4$$

Hence, roots are -4 and -4. $\qquad$ ½

when $p = -8$,

from eq. (i), $\quad x^2 - 8x + 16 = 0$

$$\Rightarrow \quad x^2 - 4x - 4x + 16 = 0$$
$$\Rightarrow \quad x(x - 4) - 4(x - 4) = 0$$
$$\Rightarrow \quad (x - 4)(x - 4) = 0$$
$$\Rightarrow \quad x = 4, 4$$

Hence, the required roots are either $-4, -4$ or $4, 4$ **1**

AI **Q. 2. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.**

A [Board OD Set III, 2017]

[Delhi Comptt. Set-I, OD Set III, 2017]

Topper Answer, 2017

Sol.

20 | $A = (a^2+b^2)$, $B = -2(ac+bd)$ $C = (c^2+d^2)$

as roots are equal,

$(a^2c^2 + 2abcd + b^2d^2) = A'(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$$2abcd = a^2d^2 + b^2c^2$$
$$0 = a^2d^2 - 2abcd + b^2c^2$$
$$0 = (ad - bc)^2$$
$$0 = ad - bc$$
$$ad = bc$$
$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$$
Hence, proved.

3

Q. 3. ***ad*** **≠ *bc*, then prove that the equation.** **$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.** U [CBSE OD Set-I 2017]

Sol. We have, $A = (a^2 + b^2)$, $B = 2(ac + bd)$ and $C = (c^2 + d^2)$

For no real roots, $D < 0$ ½

i.e., $D \Rightarrow b^2 - 4ac < 0$

$$b^2 - 4ac = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)] \quad \mathbf{1}$$
$$= 4[a^2c^2 + 2abcd + b^2d^2]$$
$$- 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$
$$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$
$$= -4[a^2d^2 + b^2c^2 - 2abcd] \quad \mathbf{1}$$
$$= -4(ad - bc]^2$$

Since, $ad \neq bc$

Therefore, $D < 0$

Hence, the equation has no real roots. ½

Q. 4. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find *k*.

C + A [O.D. Comptt. Set-II, III, 2017]

Sol. Given, 2 is a root of the equation, $3x^2 + px - 8 = 0$

Putting $x = 2$ in $3x^2 + px - 8 = 0$, we get

$$12 + 2p - 8 = 0$$
$$\Rightarrow \quad p = -2$$

Given, $4x^2 - 2px + k = 0$ has equal roots,

and $4x^2 + 4x + k = 0$ has equal roots. **1**

$$\therefore \quad D \Rightarrow b^2 - 4ac = 0$$
$$\Rightarrow \quad (4)^2 - 4(4)(k) = 0$$
$$\Rightarrow \quad 16 - 16k = 0 \quad \mathbf{1}$$
$$\Rightarrow \quad 16k = 16$$
$$\therefore \quad k = 1. \quad \mathbf{1}$$

Q. 5. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

U [OD, Set -II, 2016]

Topper Answer, 2016

Sol.

$(a-b)x^2 + (b-c)x + (c-a) = 0$ Soln

The root are equal, then $D = 0$

Comparing eq^n by $ax^2 + bx + c = 0$

$a = (a-b)$; $b = (b-c)$; $c = c-a$

$$D = b^2 - 4ac$$
$$= (b-c)^2 - 4\times(a-b)(c-a)$$

Here, $D = 0$

$$(b-c)^2 - 4(a-b)(c-a) = 0$$
$$b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$
$$b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$
$$4a^2 + b^2 + c^2 + 2bc - 4ab - 4ac = 0$$
$$\Rightarrow (-2a + b + c)^2 = 0 \quad [a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2]$$
$$-2a + b + c = 0$$
$$\boxed{b + c = 2a}$$

Hence proved

3

Long Answer Type Questions

5 marks each

Q. 1. Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it has, find them by the method of completing the square. Also, verify that roots obtained satisfy the given equation.

U [CBSE SQP, 2017]

Sol. Discriminant $D = b^2 - 4ac$.

Here, $a = 5, b = (-6)$ and $c = (-2)$

Then, $b^2 - 4ac = (-6)^2 - 4 \times 5 \times -2$

$= 36 + 40 = 76 > 0$ **1**

So the equation has real and two distinct roots.

Again, $5x^2 - 6x = 2$

(dividing both the sides by 5) **1**

$$x^2 - \frac{6}{5}x = \frac{2}{5}$$

On adding square of the half of coefficient of x

$$\Rightarrow \quad x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{2}{5} + \frac{9}{25}$$

$$\Rightarrow \quad x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$\Rightarrow \quad x = \frac{3+\sqrt{19}}{5} \text{ or } \frac{3-\sqrt{19}}{5}$$ **1**

Verification:

$$5\left[\frac{3+\sqrt{19}}{5}\right]^2 - 6\left[\frac{3+\sqrt{19}}{5}\right] - 2$$

$$= \frac{9+6\sqrt{19}+19}{5} - \left(\frac{18+6\sqrt{19}}{5}\right) - 2$$ **1**

$$= \frac{28+6\sqrt{19}}{5} - \frac{18+6\sqrt{19}}{5} - 2$$

$$= \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5}$$

$= 0$

Similarly,

$$5\left[\frac{3-\sqrt{19}}{5}\right]^2 - 6\left[\frac{3-\sqrt{19}}{5}\right] - 2 = 0$$

Hence Verified. **1**

Q. 2. If the roots of the quadratic equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

U [Board OD Set II, III 2017]

Sol. Here, $A = (c^2 - ab), B = -2(a^2 - bc), C = (b^2 - ac)$

For real equal roots $D \Rightarrow B^2 - 4AC = 0$ **1**

$\Rightarrow \quad [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$ **1**

$\Rightarrow 4(a^4 + b^2c^2 - 2a^2bc) - 4(b^2c^2 - c^3a - ab^3 + a^2bc) = 0$ **½**

$\Rightarrow \quad 4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + c^3a + ab^3 - a^2bc) = 0$ **½**

$\Rightarrow \quad 4[a^4 + ac^3 + ab^3 - 3a^2bc] = 0$

$\Rightarrow \quad a(a^3 + c^3 + b^3 - 3abc) = 0$ **1**

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$ **1**

Q. 3. The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between the areas of the two circles is 1078 sq. cm. Find the radius of the smaller circle.

U [Board Comptt. I, II, III, 2017]

Sol. We have $r_2 - r_1 = 7$ cm, $r_2 > r_1$ ...(i) **1**

and $\pi(r_2^2 - r_1^2) = 1078 \text{ cm}^2$ ...(ii)

$\Rightarrow \quad \pi(r_2 - r_1)(r_2 + r_1) = 1078$ **1**

$\Rightarrow \quad r_2 + r_1 = \frac{1078}{22} = 49$ ...(iii) **1**

On adding (i) and (iii) we get

$\Rightarrow \quad 2r_2 = 56$

$\Rightarrow \quad r_2 = 28$ cm

and $r_1 = 49 - 28 = 21$ **1**

Hence, radii of smaller circle is 21 cm **1**

[CBSE Marking Scheme, 2017]

Q. 4. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal. A [Foreign Set II, 2016]

Sol. For equal roots of $x^2 + 2px + mn = 0$, $4p^2 - 4mn = 0$

$\Rightarrow \quad p^2 = mn$ ...(i) **2**

For equal roots of

$x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0,$

$4(m + n)^2 - 4(m^2 + n^2 + 2p^2) = 0,$ **1**

$m^2 + n^2 + 2mn - m^2 - n^2 - 2(mn) = 0$ (From (i))

$\therefore$ If roots of $x^2 + 2px + mn = 0$ are equal, then those of $x^2 - 2a(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal. [CBSE Marking Scheme, 2016] **2**

AI **Q. 5. Find the positive values of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have the real roots.**

C + A [Foreign Set I-2016]

Sol. (i) For $x^2 + kx + 64 = 0$ to have real roots

$k^2 - 256 \geq 0$ ...(i) **1½**

(ii) For $x^2 - 8x + k = 0$ to have real roots

$64 - 4k \geq 0$ ...(ii) **1½**

For (i) and (ii) to hold simultaneously

$k = 16$ **2**

[CBSE Marking Scheme, 2016]

Q. 6. If (– 5) is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

U [CBSE Delhi Board, Set II, 2015]

Sol. Since, (–5) is a root of given quadratic equation

$$2x^2 + px - 15 = 0$$

$$\therefore \quad 2(-5)^2 + p(-5) - 15 = 0 \qquad 1$$

$$50 - 5p - 15 = 0$$

$$5p = 35 \Rightarrow p = 7 \qquad 1$$

And $\quad p(x^2 + x) + k = 0$ has equal roots

$$\Rightarrow \quad px^2 + px + k = 0$$

So, $\quad (b)^2 - 4ac = 0$

$$(p)^2 - 4p \times k = 0 \qquad 1$$

$$(7)^2 - 4 \times 7 \times k = 0$$

$$28k = 49$$

$$k = \frac{49}{28} = \frac{7}{4} \qquad 1$$

Hence, $p = 7$ and $k = \frac{7}{4}$. **1**

[CBSE Marking Scheme, 2015]

Q. 7. If the roots of the quadratic equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are equal. Then, show that $a = b = c$.

U [CBSE Delhi Board, Set II, 2015]

Sol. Given,

$$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$$

$$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$$

$$\Rightarrow \quad 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0 \qquad 1$$

$$\Rightarrow \quad 3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0 \qquad 1$$

For equal roots, $\quad B^2 - 4AC = 0$

$$\Rightarrow \quad \{-2(a + b + c)\}^2 = 4 \times 3(ab + bc + ca)$$

$$\Rightarrow \quad 4(a + b + c)^2 - 12(ab + bc + ca) = 0$$

$$\Rightarrow \quad (a + b + c)^2 - 3(ab + bc + ca) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$$

$$\Rightarrow \quad a^2 + b^2 + c^2 - ab - ac - bc = 0 \qquad 1$$

$$\Rightarrow \quad \frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc\right] = 0$$

$$\Rightarrow \frac{1}{2}\left[(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ac)\right] = 0$$

$$\Rightarrow \quad \frac{1}{2}\left[(a - b)^2 + (b - c)^2 + (c - a)^2\right] = 0 \qquad 1$$

$$\Rightarrow \quad (a - b)^2 + (b - c)^2 + (c - a)^2 = 0 \text{ if } a \neq b \neq c$$

$(a - b)^2 = 0$, $(b - c)^2 = 0$ and $(c - a)^2 = 0$

if $\quad (a - b)^2 = 0 \Rightarrow a = b$

$$(a - c)^2 = 0 \Rightarrow b = c$$

$$(c - a)^2 = 0 \Rightarrow c = a \qquad 1$$

$\therefore \quad a = b = c \qquad$ **Hence Proved.**

Visual Case Based Questions

4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

Q. 1. Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km. [CBSE QB, 2021]

(i) What will be the distance covered by Ajay's car in two hours ?

(a) $2(x + 5)$ km (b) $(x - 5)$ km

(c) $2(x + 10)$ km (d) $(2x + 5)$ km

Sol. Correct option: **(a).**

Explanation: Speed of Raj's car = x km/hr

Speed of Ajay's car = $(x + 5)$ km/hr

Distance covered by Ajay in 2 hours

$$= [(x + 5) \times 2] \text{ km}$$

$$= 2(x + 5) \text{ km.}$$

(ii) Which of the following quadratic equation describe the speed of Raj's car ?

(a) $x^2 - 5x - 500 = 0$ (b) $x^2 + 4x - 400 = 0$

(c) $x^2 + 5x - 500 = 0$ (d) $x^2 - 4x + 400 = 0$

Sol. Correct option: **(c).**

(iii) What is the speed of Raj's car ?

(a) 20 km/hour (b) 15 km/hour

(c) 25 km/hour (d) 10 km/hour

Sol. Correct option: **(a).**

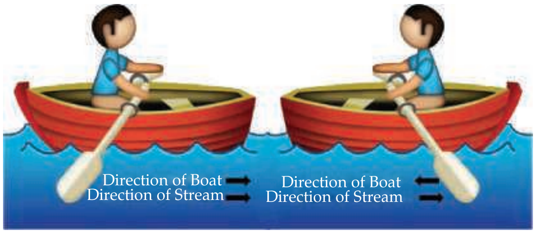
(iv) How much time took Ajay to travel 400 km ?

(a) 20 hour (b) 40 hour

(c) 25 hour (d) 16 hour

Sol. Correct option: **(d).**

Q. 2. The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream. [CBSE QB, 2021]



(i) Let speed of the stream be x km/hr. then speed of the motorboat in upstream will be

(a) 20 km/hr (b) $(20 + x)$ km/hr
(c) $(20 - x)$ km/hr (d) 2 km/hr

Sol. Correct option: **(c).**

Explanation: Speed of motorboat in upstream

= Speed of motorboat – Speed of stream

= $(20 - x)$ km/hr

(ii) What is the relation between speed ,distance and time ?

(a) speed = (distance)/time
(b) distance = (speed)/time
(c) time = speed x distance
(d) speed = distance x time

Sol. Correct option: **(b).**

(iii) Which is the correct quadratic equation for the speed of the current ?

(a) $x^2 + 30x - 200 = 0$ (b) $x^2 + 20x - 400 = 0$
(c) $x^2 + 30x - 400 = 0$ (d) $x^2 - 20x - 400 = 0$

Sol. Correct option: **(c).**

(iv) What is the speed of current ?

(a) 20 km/hour (b) 10 km/hour
(c) 15 km/hour (d) 25 km/hour

Sol. Correct option: **(b).**

(v) How much time boat took in downstream?

(a) 90 minute (b) 15 minute
(c) 30 minute (d) 45 minute

Sol. Correct option: **(c).**

AI **Q. 3. John and Jivanti are playing with the marbles. They together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124.** C

(i) If John had x number of marbles, then number of marbles Jivanti had:

(a) $x - 45$ (b) $45 - x$
(c) $45x$ (d) $x - 5$

Sol. Correct option: **(b).**

Explanation: If John had x number of marbles, then Jivanti had $(45 - x)$ marbles, because there are total 45 marbles. 1

(ii) Number of marbles left with Jivanti, when she lost 5 marbles:

(a) $x - 45$ (b) $40 - x$
(c) $45 - x$ (d) $x - 40$

Sol. Correct option: **(b).**

Explanation: Number of marbles left with Jivanti, when she lost 5 marbles $= (45 - x - 5)$

$= (40 - x)$ 1

(iii) The quadratic equation related to the given problem is:

(a) $x^2 - 45x + 324 = 0$ (b) $x^2 + 45x + 324 = 0$
(c) $x^2 - 45x - 324 = 0$ (d) $-x^2 - 45x + 324 = 0$

Sol. Correct option: **(a).**

Explanation: According to question,

$$(x - 5)(40 - x) = 124$$

$$\Rightarrow - x^2 - 200 + 40x + 5x - 124 = 0$$

$$\Rightarrow \quad x^2 - 45x + 324 = 0$$ 1

(iv) Number of marbles John had:

(a) 10 (b) 9
(c) 35 (d) 30

Sol. Correct option: **(b).**

Explanation:

$$x^2 - 45x + 324 = 0$$

$$\Rightarrow \quad x^2 - 9x - 36x + 324 = 0$$

$$\Rightarrow \quad x(x - 9) - 36(x - 9) = 0$$

$$\Rightarrow \quad (x - 9)(x - 36) = 0$$

Either $x = 9$ or $x = 36$.

Therefore, the number of marbles John had 9 or 36. 1

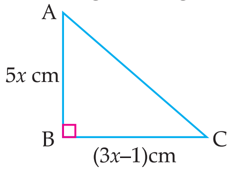
(v) If John had 36 marbles, then number of marbles Jivanti had:

(a) 10 (b) 9
(c) 36 (d) 35

Sol. Correct option: **(b).**

Explanation: If John had 36 marbles, then Jivanti had $(45 - 36) = 9$ marbles. 1

AI **Q. 4. There is a triangular playground as shown in the below figure. Many Children and people are playing and walking in the ground.**



As we see in the above figure of right angled triangle playground, the length of the sides are $5x$ cm and $(3x - 1)$ cm and area of the triangle is 60 cm^2. C + AE

(i) The value of x is:

(a) 8 (b) 3
(c) 4 (d) 5

Sol. Correct option: **(b).**

Explanation: Given, area of triangle = 60 cm^2

$$\Rightarrow \quad \frac{1}{2} \times AB \times BC = 0$$

$$\Rightarrow \quad (5x)(3x - 1) = 120$$

$$\Rightarrow \quad 3x^2 - x - 24 = 0$$

$$\Rightarrow \quad 3x^2 - 9x + 8x - 24 = 0$$
$$\Rightarrow \quad 3x(x-3) + 8(x-3) = 0$$
$$\Rightarrow \quad (x-3)(3x+8) = 0$$

Either $x = 3$ or $x = -\frac{8}{3}$

Since length can't be negative, then $x = 3$. **1**

(ii) The length of AB is:

(a) 8 cm **(b) 10 cm**

(c) 15 cm **(d) 17 cm**

Sol. Correct option: **(c)**.

Explanation:

$$\text{The length of AB} = 5x \text{ cm} = 5 \times 3 \text{ cm} = 15 \text{ cm}$$ **1**

(iii) The length of AC is:

(a) 17 cm **(b) 15 cm**

(c) 21 cm **(d) 20 cm**

Sol. Correct option: **(a)**.

Explanation:

$\because$ $AB = 15$ cm and

$BC = (3x - 1)$ cm

$= (3 \times 3 - 1)$ cm

$= 8$ cm ½

Now, in right angled ΔABC,

$AC^2 = AB^2 + BC^2$

(By using Pythagoras theorem)

$= (15)^2 + (8)^2 = 225 + 64$

$= 289 = (17)^2$

Hence, $AC = 17$ cm. ½

(iv) The perimeter of ΔABC is:

(a) 35 cm **(b) 45 cm**

(c) 30 cm **(d) 40 cm**

Sol. Correct option: **(d)**.

Explanation: Here, $AB = 15$ cm, $BC = 8$ cm and $AC = 17$ cm.

Then, the perimeter of $\Delta ABC = (AB + BC + CA)$ cm

$= (15 + 8 + 17)$ cm

$= 40$ cm. **1**

(v) The given problem is based on which mathematical concept ?

(a) AP

(b) Linear equation in one variable

(c) Quadratic Equations

(d) None of these

Sol. Correct option: **(c)**.

Explanation: The given problem is based on the concept of quadratic equations. **1**