JEE Main Practice Test-9
Simple Harmonic Motion

Topic : SIMPLE HARMONIC MOTION **Time: 75Min** **Marking +4 –1**

Section - A : MCQs with Single Option Correct

1. If T_1 = time period of a simple pendulum of infinite length, T_2 = time period of simple harmonic motion of a body dropped in a tunnel dug along diameter of earth and T_3 = time period of circular motion of a satellite revolving near the surface of the earth, then:

(A) $T_1 > T_2 = T_3$ (B) $T_1 = T_2 = T_3$ (C) $T_1 = T_2 > T_3$ (D) $T_1 < T_2 < T_3$

2. A charged particle of mass m and having a charge Q is placed in an electric field E which varies with time as $E = E_0 \sin\omega t$. What is the amplitude of the SHM executed by the particle?

(A) $\frac{QE_0}{m\omega^2}$ (B) $\frac{1}{2}\frac{QE_0}{m\omega^2}$ (C) $\frac{2QE_0}{m\omega^2}$ (D) None of these

3. The amplitude of a particle acted upon by a force $F\cos\omega t$ along the x direction, is given by $x = \frac{A_0}{\sqrt{[a\omega^2 - b\omega + c]}}$ where a, b, c, are constants. For what value of ω does the resonance occur?

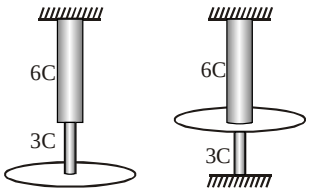
(A) $\omega = \frac{b}{2a}$ (B) $\omega = \frac{b}{a}$ (C) $\omega = c$ (D) $\omega = 0$

4. The potential energy of a particle of mass m is in a conservative field is given as $U(x) = (1/2)k_1x^2$ for $x > 0$ and $U(x) = k_1x^2$ for $x < 0$. When disturbed a little from the position $x = 0$, it will :

(A) not executes SHM (B) execute SHM with $T = 2\pi\sqrt{\frac{m}{3k_1}}$

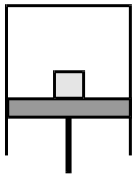
(C) execute SHM with $T^2 = 2\pi^2 m/(k_1^2)$ (D) execute SHM with $T = \frac{\pi\sqrt{m}}{\sqrt{k_1\left(1+\frac{1}{\sqrt{2}}\right)}}$

5. A disc of mass m and radius R is fastened with two rods of torsional constant $6C$ and $3C$ as shown. Time period of small oscillation of disc about it's axis in case-I and case-II are respectively T_1 and T_2. Value of $\frac{T_1^2}{T_2^2}$ is :



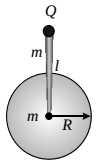
(A) $\frac{9}{4}$ (B) $\frac{2}{9}$ (C) $\frac{9}{2}$ (D) $\frac{3}{4}$

6. A block of mass m is resting on a piston as shown in figure which is moving vertically with a SHM of period 1 s. The minimum amplitude of motion at which the block and piston separate is ($g = \pi^2$):



(A) 0.25 m (B) 0.52 m (C) 2.5 m (D) 0.15 m

7. A meter stick of length l connected with a disc of same mass and radius $l/2$, which is joined at one end of stick and disc can freely rotates about its joint with the stick without friction. System swings in a vertical plane about a fixed horizontal axis passing through its one end, undergoes small oscillation of frequency 'f_0'. If disc is removed from the stick, then its new frequency of small oscillation will be



(A) $\sqrt{\frac{29}{6}}f_0$ (B) $\sqrt{\frac{4}{3}}f_0$

(C) $2f_0$ (D) $\sqrt{\frac{13}{3}}f_0$

8. A simple pendulum has time period T. charges are now fixed at the point of suspension of the pendulum and on the bob. If the pendulum continues to oscillate, its time period will now be

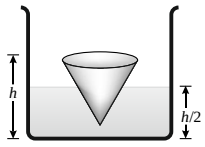
(A) greater than T

(B) Equal to T

(C) Less than T

(D) Either (a) or (c) depending on whether the charges attract or repel each other

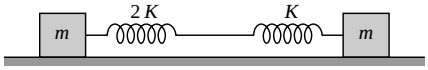
9. A solid cone of height h with heavy mass attached to it floats upside down in water as shown in the figure. In equilibrium, water is at a height $h/2$ in the vessel of large cross section. When displaced downward slightly and released, it performs simple harmonic motion. The frequency of these oscillations is :



(A) $\frac{1}{2\pi}\sqrt{\frac{6g}{h}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{g}{h}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3g}{h}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{8g}{h}}$

10. Two blocks are placed on a smooth surface connected with two springs as shown in figure. Find time period of oscillations of this system if blocks are slightly displaced toward each other and released:

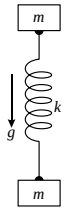


(A) $2\pi\sqrt{\frac{3m}{8k}}$ (B) $2\pi\sqrt{\frac{3m}{4k}}$ (C) $2\pi\sqrt{\frac{3m}{2k}}$ (D) None

11. Two particle performs simple harmonic motion with time period of 6 second and 7 second respectively. At $t = 0$ both were in their mean position and moving in the same direction. After what minimum non zero time they will be in phase again :

(A) 21 second (B) 42 second (C) 63 second (D) 84 second

12. A mass m is hung on an ideal massless spring. Another equal mass is connected to the other end of the spring. The whole system is at rest. At $t = 0$, m is released and the system falls freely under gravity. Assume that natural length of the spring is L_0, its initial stretched length is L and the acceleration due to gravity is g. What is distance between masses as function of time?



(A) $L_0 + (L - L_0)\cos\sqrt{\frac{2k}{m}}t$ (B) $L_0 + (L - L_0)\cos\sqrt{\frac{k}{m}}t$

(C) $L_0 - 2(L + 2L_0)\cos\sqrt{\frac{2k}{m}}t$ (D) $L_0 + (L - L_0)\sin\sqrt{\frac{2k}{m}}t$

13. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to :

(A) 100 s (B) 20 s (C) 10 s (D) 50 s

14. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}^{\text{th}}$ of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(A) $4T\sqrt{\frac{1}{15}}$ (B) $2T\sqrt{\frac{1}{10}}$ (C) $4T\sqrt{\frac{1}{14}}$ (D) $2T\sqrt{\frac{1}{14}}$

15. A particle is executing S.H.M. between $x = \pm A$ with $x = 0$ at mean position. The time taken to go from 0 to $\frac{\sqrt{3}}{2}A$ is T_1 and to go from $\frac{\sqrt{3}}{2}A$ to A is T_2, then

(A) $T_1 < T_2$ (B) $2T_1 < T_2$ (C) $T_1 = T_2$ (D) $T_1 = 2T_2$

16. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-01.1t}\cos(10\pi t + \phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to :

(A) 13 s (B) 7 s (C) 27 s (D) 4 s

17. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be :

(A) $T\sqrt{n}$ (B) $\frac{T}{\sqrt{n}}$ (C) nT (D) T

18. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$, then the ratio of $\frac{m}{M}$ is :

(A) $\frac{3}{5}$ (B) $\frac{25}{9}$ (C) $\frac{16}{9}$ (D) $\frac{5}{3}$

19. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is :

(A) -4 (B) 4 (C) $4\sqrt{2}$ (D) 8

20. A body executes simple harmonic motion. The potential energy (*PE*), the kinetic energy (*KE*) and total energy (*TE*) are measured as function of displacement *x*. Which of the following statements is true ?

(A) *KE* is maximum when $x = 0$ (B) *TE* is zero when $x = 0$

(C) *KE* is maximum when *x* is maximum (D) *PE* is maximum when $x = 0$

Section- B: INTEGER Answer Type Questions

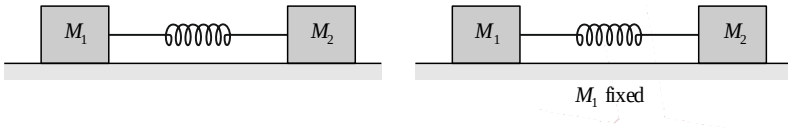
21. A body is executing simple harmonic motion. At a displacement *x* from mean position, its potential energy is $E_1 = 2$J and at a displacement *y* from mean position, its potential energy is $E_2 = 8$J. The potential energy *E* (J) at a displacement $(x + y)$ from mean position is :

22. Position vector of a particle is given as

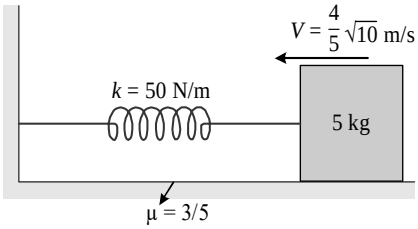
$\vec{r} = A\cos^2 \omega t\,\hat{i} + B\sin^2 \omega t\hat{j}$, $[A = 6 \text{ and } B = 8]$

Find the amplitude of motion :

23. Figures below shows two situations. In first the two masses are freely placed on a smooth surface and in second mass M_1 is kept fixed. In the two cases free oscillations of masses in first case, time period is T_1 and that of oscillations of mass M_2 is T_2. If $\frac{T_2}{T_1} = 3$, then calculate $\frac{M_2}{M_1}$.

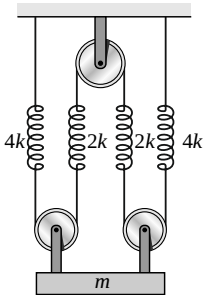


24. A spring block system is placed on a horizontal rough surface with friction coefficient 0.6 as shown in the figure. The block is given an initial velocity $\frac{4}{5}\sqrt{10}$ m/swhen the spring is in its natural length. The total distance travelled by the block before it finally comes to rest is $2/k$ meters. Find the value of *k* :

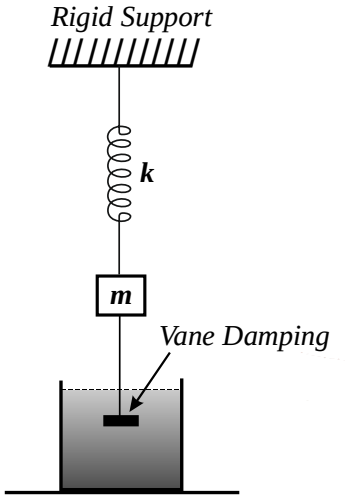


25. If the time period of a simple pendulum having length *L* is T_S and time period of a uniform rod having same length pivoted about one end and oscillating in vertical plane is T_R, then for small amplitude of oscillations, $\frac{T_s}{T_R}$ is $\sqrt{\frac{n}{2}}$. Find *n*.

26. A plank of mass *M* is connected with three massless pulleys and four massless springs as shown in the figure in vertical plane. Plank is slightly displaced from its mean position. Assuming plank remain in horizontal position throughout the motion, the frequency of oscillation of plank is $\frac{1}{2\pi}\sqrt{\frac{nK}{3m}}$. Find *n*.



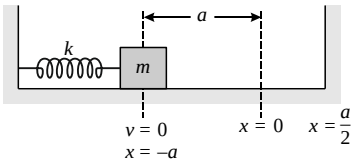
27. For the damped oscillator shown in figure, the mass m of the block is 200 g, k = 90 N m^{-1} and the damping constant b is 40 g s^{-1}. Calculate the time (sec) taken for its amplitude of vibrations to drop to half of its initial value.



28. The length of a simple pendulum, measured by an instrument of least count 4 mm, is found to be 40 cm. Its time period is measured by taking 200 oscillations using a watch of resolution 2 s. The time period is found to be 0.4 s. Find the percentage error in the determination of the acceleration due to gravity using this data.

29. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ where l is about 100 cm and is known to have 1 mm accuracy. The period is about 2 s. The time of 100 oscillations is measured by a stop watch of least count 0.1 s. If the percentage error in calculation of g is x% then calculate 10x.

30. A spring (spring constant k) having one end attached to rigid wall & other end attached to a block of mass m kept on a smooth surface as shown in figure. Initially spring is in its natural length at $x = 0$, now spring is compressed to $x = -a$ toward left as shown in figure and released. For collision of block with the wall, coefficient of restitution is 0.5. If velocity of block just after first collision is $a\sqrt{\frac{nk}{16\text{m}}}$. Find the value of n.



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ANSWER KEY

Section - A : MCQs with Single Option Correct

1. (B)	**2.** (A)	**3.** (A)	**4.** (A)
5. (C)	**6.** (A)	**7.** (B)	**8.** (B)
9. (A)	**10.** (B)	**11.** (B)	**12.** (A)
13. (B)	**14.** (A)	**15.** (D)	**16.** (B)
17. (B)	**18.** (C)	**19.** (C)	**20.** (A)

Section- B: INTEGER Answer Type Questions

21. [18]	**22.** [5]	**23.** [8]	**24.** [5]
25. [3]	**26.** [32]	**27.** [7]	**28.** [6]
29. [2]	**30.** [3]		

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