1. Centres of the three circles $x^2 + y^2 - 4x - 6y - 14 = 0$, $x^2 + y^2 + 2x + 4y - 5 = 0$ and $x^2 + y^2 - 10x - 16y + 7 = 0$

(A) are the vertices of a right triangle (B) the vertices of an isosceles triangle which is not regular

(C) vertices of a regular triangle (D) are collinear

2. $y - 1 = m_1(x - 3)$ and $y - 3 = m_2(x - 1)$ are two family of straight lines, at right angled to each other. The locus of their point of intersection is

(A) $x^2 + y^2 - 2x - 6y + 10 = 0$ (B) $x^2 + y^2 - 4x - 4y + 6 = 0$

(C) $x^2 + y^2 - 2x - 6y + 6 = 0$ (D) $x^2 + y^2 - 4x - 4y - 6 = 0$

3. Suppose that the equation of the circle having $(-3, 5)$ and $(5, -1)$ as end points of a diameter is $(x - a)^2 + (y - b)^2 = r^2$. Then $a + b + r$, $(r > 0)$ is

(A) 8 (B) 9 (C) 10 (D) 11

4. A circle of radius 5 has its centre on the negative x-axis and passes through the point (2, 3). The intercept made by the circle on the y-axis is

(A) 10 (B) $2\sqrt{21}$ (C) $2\sqrt{11}$ (D) imaginary y-intercept

5. The circle described on the line joining the point (0, 1), (a, b) as diameter cuts the x-axis in points whose abscissae are roots of the equation :

(A) $x^2 + ax + b = 0$ (B) $x^2 - ax + b = 0$ (C) $x^2 + ax - b = 0$ (D) $x^2 - ax - b = 0$

6. A straight line ℓ_1 with equation $x - 2y + 10 = 0$ meets the circle with equation $x^2 + y^2 = 100$ at B is the first quadrant. A line through B, perpendicular to ℓ_1 cuts the y-axis at P(0, t). The value of 't' is

(A) 12 (B) 15 (C) 20 (D) 25

7. The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 2x = 0$ is

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $\frac{3\sqrt{3}}{8}$ (D) None of these

8. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two if its vertices on the line joining the centres of the circles. The area of the rhombus is

(A) $8\sqrt{3}$ sq. units (B) $4\sqrt{3}$ sq. units (C) $16\sqrt{3}$ sq. units (D) None of these

9. The equation of a line inclined at an angle $\frac{\pi}{4}$ to the axis X, such that the two circles

$x^2 + y^2 = 4$, $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal lengths on it, is

(A) $2x - 2y - 3 = 0$ (B) $2x - 2y + 3 = 0$ (C) $x - y + 6 = 0$ (D) $x - y - 6 = 0$

10. (6, 0), (0, 6) and (7, 7) are the vertices of a triangle. The circle inscribed in the triangle has the equation

(A) $x^2 + y^2 - 9x + 9y + 36 = 0$ (B) $x^2 + y^2 - 9x - 9y + 36 = 0$

(C) $x^2 + y^2 + 9x - 9y + 36 = 0$ (D) $x^2 + y^2 - 9x - 9y - 36 = 0$

11. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd is equal to

(A) 4 (B) 1/4 (C) 1 (D) 16

12. Number of value(s) of A for which the system of equations $x^2 = y^2$ and $(x - A)^2 + y^2 = 1$ has exactly 3 solutions, is

(A) 1 (B) 2 (C) 3 (D) 4

13. The equation of the image of the circle $x^2 + y^2 + 16x - 24y + 183 = 0$ by the line mirror $4x + 7y + 13 = 0$ is

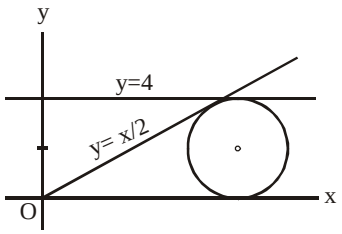
(A) $x^2 + y^2 + 32x - 4y + 235 = 0$ (B) $x^2 + y^2 + 32x + 4y - 235 = 0$

(C) $x^2 + y^2 + 32x - 4y - 235 = 0$ (D) $x^2 + y^2 + 32x + 4y + 235 = 0$

14. The x-coordinate of the centre of the circle in the first quadrant (see figure) tangent to the lines $y = \frac{1}{2}x$, $y = 4$ and the x-axis is

(A) $4 + 2\sqrt{5}$ (B) $4 + \frac{8\sqrt{5}}{5}$

(C) $2 + \frac{6\sqrt{5}}{5}$ (D) $8 + 2\sqrt{5}$



15. A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B be the area of the circle circumscribed about the triangle then the ratio $\frac{A}{B}$ is

(A) $\frac{9}{16}$ (B) $\frac{3}{4}$ (C) $\frac{27}{32}$ (D) $\frac{3\sqrt{6}}{8}$

16. $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$, represents :

(A) equation of a straight line, if θ is constant and r is variable

(B) equation of a circle, if r is constant and θ is a variable

(C) a straight line passing through a fixed point and having a known slope

(D) a circle with a known centre and a given radius.

17. If the equation of circle touching the y-axis at (0,3) and making an intercept of 8 unit on x-axis is $x^2 + y^2 + 2gx + 2fy + c = 0$, then $(g + f + c)$ can be-

(A) 1 (B) 7 (C) 11 (D) 14

18. Which of the following lines have the intercepts of equal lengths on the circle, $x^2 + y^2 - 2x + 4y = 0$?

(A) $3x - y = 0$ (B) $x + 3y = 0$

(C) $x + 3y + 10 = 0$ (D) $3x - y - 10 = 0$

19. In the xy plane, the segment with end points (3, 8) and (–5, 2) is the diameter of the circle. The point (k, 10) lies on the circle for

(A) no value of k (B) exactly one integral k

(C) exactly one non integral k (D) two real values of k

20. If $\frac{x^2 + y^2}{x + y} = 4$, then all possible values of $(x - y)$ is given by

(A) $[-2\sqrt{2}, 2\sqrt{2}]$ (B) $\{-4, 4\}$ (C) $[-4, 4]$ (D) $[-2, 2]$

21. The points A(a, 0), B(0, b), C(c, 0) and D(0, d) are such that ac = bd and a, b, c, d are all non-zero. Then the points

(A) form a parallelogram (B) do not lie on a circle

(C) form a trapezium (D) are concyclic

22. If the points $(\lambda, -\lambda)$ lies inside the circle $x^2 + y^2 - 4x + 2y - 8 = 0$, then find the range of λ.

23. The circle $x^2 + y^2 - 6x - 10y + c = 0$ does not touch or intersect the coordinate axes and the point (1, 4) is inside the circle. Find the set of the values of c.

24. Find the equation of the circle which passes through the points (1, –2) and (4, –3) and which has its centre on the straight line $3x + 4y = 0$.

25. If (4, 1) is an extremity of a diameter of the circle $x^2 + y^2 - 2x + 6y - 15 = 0$, find the co-ordinates of the other extremity of the diameter.

26. Find the radius of the circle $(x\cos\alpha + y\sin\alpha - a)^2 + (x\sin\alpha - y\cos\alpha - b)^2 = k^2$ and if α varies, find the locus of its centre.

27. Let A(–4, 0) and B(4, 0). Number of points C = (x, y) on the circle $x^2 + y^2 = 16$ such that the area of the triangle whose vertices are A, B and C is a positive integer, is

28. Find the equation to the circle which goes through the origin and cuts off intercepts equal to h and k from the positive parts of the axes.

29. Find the equation to the circle which touches the axis of x and passes through the two points (1, –2) and (3, –4)

30. (a) Find the shortest distance from the point M(–7, 2) to the circle $x^2 + y^2 - 10x - 14y - 151 = 0$.

(b) Find the co-ordinate of the point on the circle $x^2 + y^2 - 12x - 4y + 30 = 0$, which is farthest from the origin.

Answers

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1. (D) **2.** (B) **3.** (A) **4.** (B) **5.** (B) **6.** (C) **7.** (A) **8.** (A) **9.** (A) **10.** (B)
11. (C) **12.** (B) **13.** (D) **14.** (A) **15.** (C) **16.** (ABCD) **17.** (AC) **18.** (ABCD)
19. (B) **20.** (B) **21.** (D) **22.** (-1,4) **23.** (25,29) **24.** $(3x^2 + 3y^2 - 16x + 12y + 25 = 0)$
25. (–2,–7) **26.** r = k, $x^2 + y^2 = a^2 + b^2$ **27.** (62) **28.** $(x^2+y^2-4z-ky = 0)$ **29.** $(x^2 + y^2 - 6x + 4y + 9 = 0)$
30. (a) (2) (b) (9,3)a