

1. TOPIC “QUADRATIC EQUATIONS “

KEY POINTS:

1. The polynomial of degree two is called quadratic polynomial and equation corresponding to a

Quadratic polynomial $P(x)$ is called a quadratic equation in variable x .

Thus $p(x) = ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in \mathbb{R}$.

2. Zero of a quadratic polynomial:

The value of x for which the polynomial becomes zero is called zero of a polynomial.

Ex. 1 is zero of a polynomial $x^2 - 2x + 1$, because it became zero at $x = 1$.

3. Solution of a quadratic equation by factorisation method:

Roots of a quadratic equation $ax^2 + bx + c = 0$ can be found by factorization method or middle term split.

4. Discriminant method: The expression $b^2 - 4ac$ is called discriminant of the equation

$ax^2 + bx + c = 0$ and it is usually denoted by “D”. $D = b^2 - 4ac$

5. Nature of roots of $ax^2 + bx + c = 0$

i) If $D > 0$, then roots are real and unequal.

ii) $D = 0$, Then the equation has real and equal roots.

iii) $D < 0$, then the equation has no real roots.

iv) If $D > 0$, and D is a perfect square, then roots are rational and unequal.

v) If $D > 0$ and D is not a perfect square, then roots are irrational.

6. Roots of quadratic Equation:

Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$ and α and β are roots

$$\alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

7. Sum of Roots : $\alpha + \beta = \frac{-b}{a}$ Product of Roots : $\alpha \times \beta = \frac{c}{a}$

8. Forming quadratic equation ,when the roots α and β are given by :

$$X^2 - (\alpha + \beta)x + \alpha\beta = 0$$

9. Method of solving Word Problems:

10. Form the word problems into quadratic equations and solve them

(2 Marks Questions)

- Q.1 If $\frac{1}{2}$ is a root of the equation $x^2 + px - \frac{5}{4} = 0$, then find the value of p.
- Q.2 Check whether $x = -1$ is a solution of equation $4x^2 - 3x - 1 = 0$.
- Q.3 If $D > 0$, Then write the roots of a quadratic equation $ax^2 + bx + c = 0$
- Q.4 Find the Discriminant of $x^2 + 5x + 5 = 0$.
- Q.5 Find the sum of roots of a quadratic equation $x^2 + 4x - 32 = 0$
- Q.6 Find the product of the roots of the quadratic equation $2x^2 + 7x - 4 = 0$
- Q.7 Find the value of K for which the equation $9x^2 + 2Kx + 1 = 0$ have real roots.
- Q.8 Find the value of K if the equation $x^2 - 2(K+1)x + K^2 = 0$ has equal roots.
- Q.9 Represent the situation in the form of Quadratic equation:
“The product of Rohan’s age (in years) 5 years ago with his age 9 years later is 15.
- Q.10 Find the roots of $x^2 - 3x - 10 = 0$

(3 Marks each)

- Q.11 What is the nature of roots of the quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$?
- Q.12 Find the numerical difference of the roots of equation $x^2 - 7x - 18 = 0$
- Q.13 If the discriminant of the equation $6x^2 - bx + 2 = 0$ is 1, then find the value of b.
- Q.14 The product of two consecutive odd numbers is 483. Find the numbers.
- Q.15 Solve : $x - \frac{1}{x} = 3$ ($x \neq 0$)
- Q.16 The hypotenuse of right angled triangle is 6 meters. more than twice the shortest side.
If the third side is 2 meters. less than the hypotenuse, then find all the sides of the triangle.
- Q.17 The sum of the reciprocals of Anjali’s age 3 years ago and 5 years from now is $\frac{1}{3}$.
Find the present age of Anjali.
- Q.18 Check whether: $(x + 2)^3 = 2x(x^2 - 1)$ is a quadratic equation or not.
- Q.19 Solve for x : $\sqrt{2x + 9} + x = 13$.
- Q.20 Find the roots of quadratic equation $16x^2 - 24x - 1 = 0$ by using the quadratic formula

(4 Marks each)

Q.21 A passenger train takes 3 Hour less for a journey of 360 km. If its speed is increased by 10km/ h from its usual speed. Find its usual speed.

Q.22 The speed of boat in still water is 15 km/h. It can go 30 km upstream and return downstream to the original point in 4 hour and 30 minutes. Find the speed of stream.

Q.23 Two pipes running together can fill a small tank in $3\frac{1}{3}$ minutes. If one pipe takes 3 minutes more than the other to fill it, then find the time in which each pipe would fill the tank.

Q.24 Solve for:

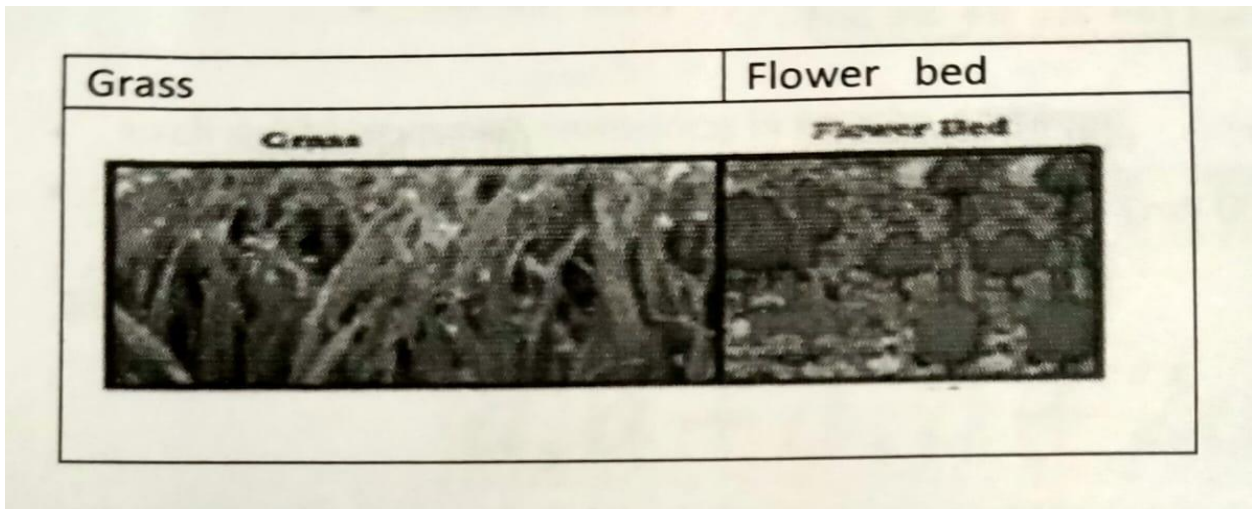
$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, \text{ where } x \neq 1, 2, 3$$

Q.25 If the equation $(1+m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$ has equal roots, then prove that

$$c^2 = a^2(1+m^2)$$

CASE –STUDY QUESTIONS (4 MARKS)

Q.26 Riya has a field with a flowerbed and grass land. The grassland is in the shape of rectangle while flowerbed is in the shape of square. The length of the grassland is found to be 3 meters more than twice the length of the flowerbed. Total area of the whole land is 1260 m².



- (i) If the length of the flowerbed is x meters, then what is the total length of the field? (2 Marks)
- (ii) What is the area of grassland? (2 Marks)

Q. 27 Nidhi and Riya are very close friends. Nidhi's parents have a Maruti Alto. Riya's parents have a Toyota. Both the families decided to go for a picnic to Somnath Temple in Gujarat by their own car. Nidhi's car travels x km/h, while Riya's car travels 5km/h more than Nidhi's car. Nidhi's car took 4 hours more than Riya's car in covering 400 km.



- (i) What will be the distance covered by Riya's car in two hours? How much time took Riya to travel 400 km? (2 Marks)
- (ii) Write the quadratic equation describe the speed of Nidhi's car. What is the speed of Nidhi's car? (2 Marks)

VALUE BASED QUESTIONS (4-MARKS EACH)

Q.28 If the price of petrol is increased by Rs.7 per litre, a person has to buy 1 litre less petrol for Rs. 1740. Find the original price of the petrol at that time.

- a) Why do you think the price of petrol is increasing day by day?
- b) What should we do to save petrol?

Q.29 Ramesh wants to design a rectangular park of perimeter 80 meters and area 400 m^2 . for jogging and walk for the people of colony. Is it possible to design the park? If so, find the length and breadth of the park. Which value of Ramesh is depicted here?

SOLUTIONS/ANSWER

(2 marks each)

1. PUT the value of $x=1/2$ in the given equation

$$\left(\frac{1}{2}\right)^2 + \frac{1}{2}p - \frac{5}{4} = 0$$

$$P=2$$

2. Check, if $x = -1$, then put the value in the given equation

$$4(-1)^2 - 3(-1) - 1 = 0,$$

$$4+3-1 \neq 0, \text{ NO}$$

3. IF $D > 0$, THEN, $\alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$, $\beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

4. compare the given equation to $ax^2 + bx + c = 0$ $a=1, b=5, c=5$

$$D = b^2 - 4ac, D = 5$$

5. Sum of roots = -4

6. Product of roots = -2

7. real roots, $D \geq 0$, $K \geq 3$ OR $K \leq -3$

8. Compare the given equation from $ax^2 + bx + c = 0$ $a=1, b=-2(k+1), c=k^2$

$$D = 0, \text{ then } k = -1/2$$

9. Let the Rahman's age be x years.

$$5 \text{ years ago} = (x-5), 9 \text{ years later} = (x+9)$$

$$\text{product of his age} = (x-5)(x+9) = 15, x^2 + 4x - 60 = 0$$

10. $x = (-2, 5)$

(3 marks each)

11. Compare to $ax^2 + bx + c = 0$, $a=2$, $b=-\sqrt{5}$, $c=1$

$$D = b^2 - 4ac, D = (-\sqrt{5})^2 - 4 \times 2 \times 1, D = 25 - 8 = 17, D > 0 \text{ Real and distinct (unequal)}$$

12. Solve the equation $x = 9$ and -2

$$\text{Numerical difference} = 9 - (-2) = 9 + 2 = 11$$

13. Compare to $ax^2 + bx + c = 0$ $a = 6, b = -b, c = 2$

$$D = b^2 - 4ac = 1$$

$$D = (-b)^2 - 4 \times 6 \times 2 = 1,$$

$$b^2 - 48 = 1, \quad b = 7, -7$$

14. Let the first odd number be x and consecutive odd number be $(x+2)$

$$x.(x+2) = 483, \quad x^2 + 2x - 483 = 0$$

$$x = 21, 23$$

15. $x - \frac{1}{x} = 3$, $\frac{x^2-1}{x} = 3$, $x^2 - 1 = 3x$, $x^2 - 3x - 1 = 0$, $x = \left(\frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}\right)$

16. Let the length of the shortest side be x meters.

According to question hypotenuse $= (2x+6)$

Third side $= (2x + 6 - 2) = 2x + 4$

By the Pythagoras theorem $(\text{Hypo})^2 = B^2 + P^2$

$$(2x+6)^2 = x^2 + (2x+4)^2$$

Then equation: $x^2 - 8x - 20 = 0$

$x = 10, -2$, but the length cannot be negative.

So $x = 10$ m.

Hypo $= 26$, $B = 10$, $P = 24$

17. Let the Anjali's age be x years

Anjali's age 3 years ago $= x - 3$

Anjali's age 5 years from now $= x + 5$

According to question $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$x = 7, -3$. but age cannot be negative. so $x = 7$ years

18. After solution we get $-x^3 + 6x^2 + 14x + 8 = 0$.

It is not in the form of $ax^2 + bx + c = 0$

So it is not a quadratic equation.

19. $\sqrt{2x+9} + x = 13$.

$\sqrt{2x+9} = 13 - x$. squaring both sides

$$(2x+9) = (13-x)^2$$

$$x^2 - 28x + 160 = 0$$

$x = 20, 8$ but $x=20$ does not satisfy the equation.

So $x = 8$

$$20. \quad x = \frac{3+\sqrt{10}}{4}, \frac{3-\sqrt{10}}{4}$$

(4 marks each)

21. Let the usual speed of the train = x km/h

Total distance = 360 km.

Time = Distance / speed.

So the time taken by the train = $\frac{300}{x}$ h

If the speed is increased by 10 km/h, then the new speed of the train = $(x+10)$ km/h

Time taken by the train = $\frac{300}{x+10}$

According to the question $\frac{300}{x} = \frac{300}{x+10} + 3$

$$\frac{300}{x} - \frac{300}{x+10} = 3$$

After solution $x = -40$ km and 30 km, but speed cannot be negative.

$x = 30$ km.

22. Let speed of the stream = x km/h

Given, speed of boat in still water = 15 km/h

Speed of boat upstream = $(15-x)$ km/h

Speed of boat downstream = $(15+x)$ km/h

According to the question

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2} \quad (\text{TIME} = \text{DISTANCE} / \text{SPEED})$$

$$x^2 - 225x + 200 = 0, x = 5, -5$$

But speed cannot be negative. So $x = 5$ km/h.

23. Let faster pipe takes x minutes to fill the tank.

Then. Slower pipe will take $(x+3)$ minutes to fill the tank.

Since, portion of the tank filled by the faster pipe in 1 minute = $1/x$

And portion of the tank filled by the slower pipe in 1 minute = $\frac{1}{x+3}$

IN $3\frac{1}{3}$ minutes = $(\frac{40}{13}$ minutes)

Both pipe will fill to tank together

$$\frac{40}{13} \left(\frac{1}{x} + \frac{1}{x+3} \right) = 1$$

After calculation we get $13x^2 - 41x - 120 = 0$

$x = 5$ or $x = \frac{-24}{13}$, but time cannot be negative.

So, $x = 5$ minutes.

Faster pipe takes 5 minutes and slower pipe takes $(x+3) = 8$ minutes to fill the tank.

24. Solution of the given equation

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$(x-1)(x-3) = 3$$

solution of $x = 0$ or $x = 4$

25. Compare the given equation to $Ax^2 + Bx + C = 0$

$$A(1+m^2), B=2mc \text{ and } C=(c^2 - a^2)$$

Since the given equation has equal roots.

So, $D = b^2 - 4ac = 0$, then prove it.

CASE –STUDY SOLUTIONS)

26. (i) $(3x+3)$ m.

(ii) $860m^2$.

27. (i) $2(x+5)$ km , 16 hours

(ii) (c) $x^2 + 5x - 500 = 0$, 20 km /h

SOLUTIONS OF VALUE BASED QUESTIONS

28. Let the original price of the petrol be Rs. x per litre.

The amount of petrol that can be purchased = $\frac{1740}{x}$

According to question

$$\frac{1740}{x} - \frac{1740}{x+2} = 1 ,$$

$$1740 (x+2 - x) = x(x+2)$$

$$x^2 + 2x - 3480 = 0$$

$$x = 58, (-60) \text{--rejected}$$

Original cost of petrol was Rs.58 per litres.

- a) Petrol is a natural resource which is depleting day by day. So due to more demand and less supply, its price is increasing.
- b) We should use more of public transport and substitute petrol with CNG or other renewable resources.

29. Let the Length = L and Breadth = B of the Park.

So, Area = L X B = 400 m², So L = 400/B

$$\text{PERIMETER} = 2 (L + B) = 80 \text{ m.}$$

So, L + B = 40, put the value of L = 400/B

$$\text{We get } B^2 - 40B + 400 = 0$$

$$B = 20 \text{ m, } L = 20 \text{ m.}$$

VALUE –Jogging and Morning walk are beneficial for our mental and physical health.