ELECTROMAGNETIC WAVES

Chapter 15

REVIEW OF BASIC CONCEPTS

1. Displacement Current

We know that a magnetic field changing with time gives rise to an emf (due to change in magnetic flux) and hence an electric field. Is the converse also true?

James Clerk Maxwell (1831–1879) argued that this was indeed true, i.e. a time varying electric field gives rise to a magnetic field. From Ampere's circuital law, this magnetic field will give rise to a current. Maxwell recognised that this current cannot be the conventional conduction current because it can exist even in a vacuum. He called this current the *displacement current* which exists in addition to the conduction current.

2. Maxwell's Equations

Maxwell modified Ampere's circuital law by including the displacement current and formulated a set of equations involving electric and magnetic fields, their sources and charge and current densities. These equations are known as Maxwell's equations, which are as follows:

1. $\oint \mathbf{E}\cdot\mathbf{dA} = \dfrac{Q}{\varepsilon_0}$ (Gauss's law for electrostatics)

2. $\oint \mathbf{B}\cdot\mathbf{dA} = 0$ (Gauss's law for magnetism)

3. $\oint \boldsymbol{E}\cdot\boldsymbol{dl} = -\dfrac{d\Phi_B}{dt}$ (Faraday's law)

4. $\oint \boldsymbol{B}\cdot\boldsymbol{dl} = \mu_0 i_c + \mu_0\varepsilon_0\dfrac{d\Phi_E}{dt}$ (Ampere–Maxwell law)

$$= \mu_0(i_c + i_d)$$

where i_c = conduction current and i_d = displacement current. The other symbols have their usual meaning.

EXAMPLE 1 A parallel plate capacitor is charged by connecting its plates to a battery. Show that the displacement current between the plates during the charging process is equal to conduction current in the connecting wires.

SOLUTION When a capacitor is connected to a battery, it takes a finite time to be fully charged to the voltage of the battery. During the charging process, a current called displacement current i_d flows between the capacitor plates which is given by

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Consider a closed Gaussian surface enclosing the plates. According to Gauss's law, the electric flux through the surface is given by

$$\Phi_E = \frac{q}{\varepsilon_0}$$

where q is the charge at the positive plate. Hence

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{dq}{dt}$$

But $\dfrac{dq}{dt}$ is the rate at which the charge is carried to the positive plate through the connecting wires. Hence $i_d = i_c$. This also follows from the principle of continuity of current.

EXAMPLE 2 A parallel plate air capacitor of capacitance 1 nF is charged by connecting the plates to a battery of voltage 2 V through a series resistance of 500 Ω. At a certain instant of time, the current in the circuit is 0.5 A during the charging process. What is the displacement current between the capacitor plates at that instant?

SOLUTION From continuity of current, the displacement current = conduction current = 0.5 A.

EXAMPLE 3 A parallal plate capacitor of capacitance 100 μF is connected across a 50 Hz variable voltage source. The displacement current between the plates at an instant when the voltage of source is 200 V is

(a) π ampere (b) 2π ampere

(c) 3π ampere (d) 4π ampere

SOLUTION Capacitative reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= \frac{100}{\pi} \text{ ohm}$$

Current through the capacitor is

$$i_d = \frac{200}{100/\pi} = 2\pi \text{ ampere}$$

EXAMPLE 4 At a certain instant of time, the potential difference between the plates of a capacitor of capacitance 2 μF is charging at a rate of 2×10^6 V s^{-1}. The displacement current at that instant is

(a) 1 A (b) 2 A

(c) 3 A (d) 4 A

SOLUTION $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$

$$\Phi_E = EA = \frac{VA}{d}$$

$$\therefore \quad i_d = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right) = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$$

$$= C\frac{dV}{dt}$$

$$= 2 \times 10^{-6} \times 2 \times 10^6$$

$$= 4 \text{ A}$$

EXAMPLE 5 A parallel plate capacitor with square plates, each of side 20 cm has a capacitance of 1 μF. At time t = 0, it is connected for charging to a 2 V battery through a series resistance of 1 kΩ. The charge on the capacitor varies with time t (in second) as

$$q(t) = q_0\,[1 - e^{-t/\tau}]$$

where $q_0 = CV$ and $\tau = CR$ is the time constant of the CR circuit. Calculate the displacement current between the plates at $t = 10^{-3}$ s.

SOLUTION $q = CV = \dfrac{\varepsilon_0 AV}{d} = \varepsilon_0 AE \;(\because E = \dfrac{V}{d})$

$$\therefore \quad E = \frac{q}{\varepsilon_0 A}$$

A = area of plate = 20 cm × 20 cm

$= 400 \text{ cm}^2 = 4 \times 10^{-2} \text{ m}^2$

Electric flux through the capacitor is

$$\Phi_E = E \times A = \frac{q}{\varepsilon_0}$$

where $\quad q_0 = CV = 10^{-6} \times 2 = 2 \times 10^{-6}$ C

Also $\quad \tau = CR = 10^{-6} \times 10^3 = 10^{-3}$ s

$$\therefore \quad \Phi_E = \frac{q}{\varepsilon_0} = \frac{q_0\left[1 - e^{-t/\tau}\right]}{\varepsilon_0}$$

Displacement current $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$

$$= \varepsilon_0 \frac{d}{dt}\left[\frac{q_0(1 - e^{-t/\tau})}{\varepsilon_0}\right]$$

$$= \frac{q_0}{\tau} e^{-t/\tau}$$

$$= \frac{2 \times 10^{-6}}{10^{-3}} e^{-t/10^{-3}}$$

At $t = 10^{-3}$ s,

$$i_d = 2 \times 10^{-3} e^{-1} = \frac{2 \times 10^{-3}}{e} = \frac{2 \times 10^{-3}}{2.718}$$

$$= 7.36 \times 10^{-4} \text{ A}$$

3. History of Electromagnetic Waves

Ampere's law states that a time-varying electric field at any point is a source of magnetic field. Faraday's law states just the reverse namely, that a time-varying magnetic field is a source of electric field. These laws led Maxwell to conclude that space and time varying electric and magnetic fields produce an electromagnetic disturbance which can travel even in particle-free space. This disturbance is called the electromagnetic wave. Thus, *electromagnetic waves are those waves in which electric and magnetic fields vary sinusoidally in space and time.*

In 1865, Maxwell predicted the existence of electromagnetic waves. His theory further predicted that electromagnetic waves of all frequencies (and hence all wavelengths) should propagate with the speed of light. This theory was first experimentally verified by a German physicist, Heinrich Hertz in 1888. He used a simple electric oscillator and was able to pick up its radiation of electromagnetic waves on a radio receiver some distance away. In 1899, an Italian engineer Guglielmo Marconi succeeded in transmitting electromagnetic waves across the English Channel and 1901 across the Atlantic Ocean.

4. Transverse Nature of Electromagnetic Waves

In an electromagnetic wave, the electric and magnetic fields are mutually perpendicular to each other and each field is perpendicular to the direction of propagation of the wave.

5. Velocity of Electromagnetic Waves

Maxwell's theory predicted that electromagnetic waves of all frequencies (and hence all wavelengths) propagate in vacuum with a speed given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{15.1}$$

where μ_0 is the magnetic permeability and ε_0 the electric permittivity of vacuum. Now, for vacuum, we know that $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1} and $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1}m^{-2}, Substituting these values in Eq. (15.1) we have

$$c = \frac{1}{\left[\left(4\pi \times 10^{-7}\right)\left(8.85 \times 10^{-12}\right)\right]^{1/2}}$$

$$= 3.0 \times 10^8 \text{ ms}^{-1}$$

which is the speed of light in vacuum measured experimentally. The excellent agreement between the experimentally measured speed of light (c = 2.997924 × 10^8 ms^{-1}) to such a high degree of accuracy and the value based on the experimental measurements of ε_0 and μ_0 gave the first quantitative proof of the fact that light is an electromagnetic wave. The emergence of the speed of light from purely electromagnetic considerations is the crowning achievement of Maxwell's electromagnetic theory.

In a material medium, the speed of electromagnetic waves is given by

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

where μ is the magnetic permeability and ε the electric permittivity of the medium.

6. Production of Electromagnetic Waves

Consider a charge at rest. It has an electric field in the region around it but no magnetic field. If it is given an impulse so that it begins to move, it produces electric and magnetic fields. If it is moving with a constant velocity (i.e. if the current is not changing with time) the magnetic field will not change with time, so it cannot produce an electromagnetic wave. But if the charge is somehow accelerated, the magnetic and electric fields will change with space and time; it then produces an electromagnetic wave. Thus *an accelerated charge emits an electromagnetic wave.*

In an L-C circuit, the charge oscillates between the capacitor plates. An oscillating charge has a non-zero acceleration; hence it will emit an electromagnetic wave whose frequency is the same that of the oscillating charge.

An electron circulating round its nucleus in a stable orbit does not emit an electromagnetic wave, although it is accelerating; it does so only when it falls to a lower orbit.

When fast-moving electrons hit a metal target, electromagnetic waves (X-rays) are produced.

7. Energy Density of Electromagnetic Field

Just as an oscillating pendulum has energy associated with it, oscillating electric and magnetic fields also have electric and magnetic energies associated with them. The average energy densities of electric and magnetic fields of an electromagnetic wave are respectively given by

$$u_e = \frac{1}{4} \varepsilon_0 E_0^2 \tag{15.2}$$

and

$$u_m = \frac{B_0^2}{4\mu_0} \tag{15.3}$$

where E_0 and B_0 are the amplitudes of the electric and magnetic fields respectively. E_0 and B_0 are related as

$$B_0 = \frac{E_0}{c} = \sqrt{\mu_0 \varepsilon_0}\ E_0 \tag{15.4}$$

Using (15.4) in (15.3), we find that

$$u_m = \frac{1}{4} \varepsilon_0 E_0^2 = u_e$$

i.e. in an electromagnetic wave, the average energy densities of electric and magnetic fields are equal. The total average energy density of the electromagnetic field is

$$u = u_e + u_m = \frac{1}{2} \varepsilon_0 E_0^2$$

$$= \frac{B_0^2}{2\mu_0} \tag{15.5}$$

8. Characteristics of Electromagnetic Waves

1. Electromagnetic waves are producted by an accelerating charge.
2. Electromagnetic waves can propagate even in vacuum.
3. They travel in vacuum with a speed of 3 × 10^8 ms^{-1}, the speed of light.
4. Electromagnetic waves are transverse in nature, i.e. the oscillating electric and magnetic field vectors are perpendicular to the direction of propagation of the wave and are also perpendicular to each other.

5. In an electromagnetic wave, the total energy of the electromagnetic field is shared equally between the electric and magnetic fields.
6. Electromagnetic waves of all frequencies exhibit the phenomena of interference, diffraction and polarization.

9. Spectrum of Electromagnetic Radiation

Beginning with the remarkable demonstration by Hertz of the existence of long wavelength electromagnetic waves, scientists began looking for electromagnetic waves of wavelengths much shorter than the visible light. In 1898 Rontgen discovered X-rays which are electromagnetic waves of wavelength about 10^{-10} m. The wavelength of visible light is in the range of 4×10^{-7} to 8×10^{-7} m. Radiowaves, X-rays and visible light are all electromagnetic waves and travel with the same speed $c = 3.0 \times 10^8$ ms^{-1} in free space. They differ in wavelength (and hence in frequency, $\nu = c/\lambda$) only, which means that the sources that produce them and their detectors are different. Table 15.1 shows the frequency range, wavelength range, the names and the sources of the known electromagnetic radiations. The spectrum of electromagnetic radiation has no upper or lower limits and all the regions overlap.

Notice that visible light is only a very small part of the total electromagnetic spectrum. We see that electromagnetic waves have a very wide of wavelengths (and hence of frequencies). Although they are identical in nature, their interaction with matter or their physiological action on living bodies depends on their frequency. Infrared rays are thermal radiations which produce heat, X-rays and gamma rays are highly penetrating, to mention only a few of the effects.

10. Uses of Electromagnetic Spectrum

The different regions of the total electromagnetic spectrum have been put to the following uses:

1. Radiowaves are used in radar and radio broadcasting.
2. Microwaves are used in long distance wireless communications via satellites.
3. Infrared, visible and ultraviolet traditions are used to know the structure of molecules.
4. Diffraction of X-rays by crystals gives the details of the structure of crystals.
5. The bones are opaque to X-rays but flesh is transparent. X-ray pictures of a human body are used in medical diagnosis of fractures and cracks of bones.
6. The γ-rays are used in the study of the structure of the nuclei of atoms.

EXAMPLE 6 A charged particle oscillates about its equilibrium position with a frequency of 10^{10} Hz. What is the frequency of the electromagnetic waves produced by the oscillating charge?

SOLUTION The frequency of the electromagnetic waves is the same as that of the oscillating charge, i.e. 10^{10} Hz.

EXAMPLE 7 A plane electromagnetic wave of frequency 60 MHz travels in vacuum along the positive x-direction. The electric field **E** at a particular space point x and an instant of time t is 9.6 $\hat{\mathbf{j}}$ Vm^{-1}. Find the magnitude and direction of magnetic field **B** at this point at time t.

SOLUTION $$B = \frac{E}{c} = \frac{9.6}{3 \times 10^8} = 3.2 \times 10^{-8}\ \text{T}$$

In an electromagnetic wave, the **E** and **B** fields oscillate in mutually perpendicular direction, each perpendicular to the direction of propagation of the wave. Since **E** is along $+y$ direction and the wave propagated along $+x$ direction, the direction of **B** will be along $+z$ direction. Thus

$$\mathbf{B} = 3.2 \times 10^{-8}\ \hat{\mathbf{k}}\ \text{tesla}$$

EXAMPLE 8 In a plane electromagnetic wave, the electric field (in Vm^{-1}) is given by

Table 15.1 *The electromagnetic spectrum*

Name	*Frequency range (Hz)*	*Wavelength range (m)*	*Source*
Radiowaves	10^4 to 10^8	0.1 to 600	Oscillating electric circuits
Microwaves	10^9 to 10^{12}	10^{-3} to 0.3	Oscillating current in special vacuum tubes
Infrared	10^{11} to 5×10^{14}	10^{-6} to 5×10^{-3}	Outer electrons in atoms and molecules
Visible light	4×10^{14} to 7×10^{14}	4×10^{-7} to 8×10^{-7}	Outer electrons in atoms
Ultraviolet	10^{15} to 10^{17}	1.5×10^{-7} to 3.5×10^{-7}	Outer electrons in atoms
X-rays	10^{18} to 10^{20}	10^{-11} to 10^{-8}	Inner electrons in atoms and sudden deceleration of high energy free electrons
Gamma rays	10^{19} to 10^{24}	10^{-16} to 10^{-13}	Nuclei of atoms and sudden deceleration of high energy free electrons

$$E_z = 6.0 \sin [2\pi (2.0 \times 10^{10}\, t + 500\, x)]$$

where x is in metre and t in second

(a) What is the direction of propagation of the wave?

(b) What is the rms value of electric field?

(c) Find the wavelength and frequency of the wave.

(d) Write the expression for the magnetic field.

SOLUTION

(a) The wave travels along the negative x direction.

(b) The electric field for a wave travelling along negative x direction is given by

$$E_y = E_0 \sin(\omega t + kx) \qquad (1)$$

where E_0 = peak value of E_y,

$$\omega = 2\pi\nu\,; \qquad (\nu = \text{frequency})$$

and $$k = \frac{2\pi}{\lambda}\,; \qquad (\lambda = \text{wavelength})$$

Comparing Eq. (1) with the given equation, we get

$$E_0 = 6.0 \text{ Vm}^{-1}$$

$$\therefore \quad E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{6.0}{1.414} = 4.2 \text{ V m}^{-1}$$

(c) Also $$\omega = 2\pi \times (2.0 \times 10^{10}) \text{ rad s}^{-1}$$

$$\therefore \quad \nu = \frac{\omega}{2\pi} = 2.0 \times 10^{10} \text{ Hz}$$

And $$k = 2\pi \times 500 \text{ m}^{-1}$$

$$\therefore \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi \times 500} = 2 \times 10^{-3} \text{ m} = 2.0 \text{ mm}$$

(d) Peak value of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{6.0}{3 \times 10^8} = 2.0 \times 10^{-8} \text{ T}$$

In an electromagnetic wave, the phase ($\omega t + kx$) is the same for both **E** and **B** fields. Also **B** must be perpendicular to both **E** and the direction of propagation, i.e. **B** is along the z direction. Hence

$$B_z = B_0 \sin(\omega t + kx)$$

or $$B_z = 2.0 \times 10^{-8} \sin [2\pi(2 \times 10^{10}\, t + 500\, x)] \text{ tesla}$$

EXAMPLE 9 Light of intensity 1500 W m^{-2} falls at normal incidence for 1 minute on a plane square surface of side 10 cm. Find the average force exerted on the surface during this time if surface is a

(a) perfect absorber and (b) perfect reflector of light.

SOLUTION Intensity is defined as the amount of electromagnetic energy falling per second per unit area of a surface held normally to the light.

Given $I = 1500$ Js^{-1} m^{-2}, $A = 10$ cm $\times$ 10 cm $= 10^{-2}$ m^2 and $t = 1$ minute = 60 s. Now

$$I = \frac{E}{At}$$

$$\Rightarrow \quad E = IAt = 1500 \times 10^{-2} \times 60 = 900 \text{ J}$$

Light consists of photons of energy $E = mc^2$

or $$mc = \frac{E}{c}$$

or $p = \dfrac{E}{c}$ is the momentum of incident photons, which is

$$p_i = \frac{E}{c} = \frac{900}{3 \times 10^8} = 3 \times 10^{-6} \text{ kg ms}^{-1}$$

(a) Since the surface is a perfect absorber, the final momentum $p_f = 0$. The magnitude of change in momentum is $|\Delta p| = p_i$

Average force exerted on the surface is

$$F = \frac{|\Delta p|}{t} = \frac{3 \times 10^{-6}}{60} = 5 \times 10^{-8} \text{ N}$$

(b) For a perfect reflector, $p_f = -p_i$. Therefore, the magnitude of change in momentum is $|\Delta p| = 2p_i$. Hence $F = 1.0 \times 10^{-7}$ N

EXAMPLE 10 The earth receives energy from the sun at the rate of 1400 Wm^{-2}. Calculate the peak values of the electric and magnetic fields on the earth due to the solar radiation.

SOLUTION Energy received per unit area per second is $U = 1400$ Wm^{-2}. Since this energy is equally shared by the electric and magnetic fields,

$$U = U_e + U_m = \varepsilon_0 E_{\text{rms}}^2 c$$

Thus $$E_{\text{rms}} = \left(\frac{U}{\varepsilon_0 c}\right)^{1/2} = \left[\frac{1400}{(8.85 \times 10^{-12}) \times (3 \times 10^8)}\right]^{1/2} = 7.26 \times 10^2 \text{ Vm}^{-1}$$

$$\therefore \quad \text{Peak value } E_0 = \sqrt{2}\, E_{\text{rms}} = \sqrt{2} \times 7.26 \times 10^2 = 1.0\,3 \times 10^3 \text{ Vm}^{-1}$$

Peak value of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{1.03 \times 10^3}{3 \times 10^8} = 3.4 \times 10^{-6} \text{ T}$$

EXAMPLE 11 A 200 W electric bulb emits radiation which falls normally on a squre surface of side 5 m. Assuming that only 5% of the incident intensity falls on the surface, calculate the peak values of the electric and magnetic fields.

SOLUTION

$$\text{Intensity} = \frac{\text{power}}{\text{area}} = \frac{200\text{ W}}{(5\times5)\text{ m}^2} = 8\text{ Wm}^{-2}$$

Useful intensity I = 5% of 8 Wm^{-2} = $\frac{5}{100}\times 8 = 0.4$ Wm^{-2}

$$\text{Average intensity} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$0.4 = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{4\pi\varepsilon_0}{8\pi}E_0^2 c$$

$$= \frac{E_0^2\times 3.0\times10^8}{(9\times10^9)\times 8\pi}$$

$$= \frac{E_0^2}{24\pi}$$

$$\therefore \quad E_0 = \sqrt{0.4\times 24\pi} = 1.74\text{ V m}^{-1}$$

$$B_0 = \frac{E_0}{c} = \frac{1.74}{3\times10^8} = 5.8\times10^{-9}\text{ T}$$

EXAMPLE 12 The electric field in a plane electromagnetic wave is given by

$$E = 100\sin(\omega t - kx)$$

where E is in NC^{-1}, t in s and x in m. Find the average electromagnetic energy contained in a cylinder of radius 2 cm and length 50 cm lying along the x-axis.

SOLUTION Peak value of E is $E_0 = 100$ N C^{-1} Volume of cylinder is $V = \pi r^2 l$. Average energy density is

$$u = \frac{1}{2}\varepsilon_0 E_0^2$$

∴ Energy contained in the cylinder is

$$U = uV$$

$$= \pi r^2 l\times\frac{1}{2}\varepsilon_0 E_0^2$$

$$= 3.14\times(2\times10^{-2})^2\times0.5\times\frac{1}{2}\times8.85\times10^{-12}\times(100)^2$$

$$= 2.8\times10^{-11}\text{ J}$$

EXAMPLE 13 The magnetic field in a plane electromagnetic wave travelling along the x-axis is given by

$$B = 2\times10^{-4}\sin(\omega t - kx)$$

where B is in tesla, t in s and x in m. Calculate the peak values of electric and magnetic forces acting on a particle of charge 5 μC moving with a velocity of 4×10^5 ms^{-1} along the y-axis.

SOLUTION $B_0 = 2\times10^{-4}$ T

$$E_0 = cB_0 = (3\times10^8)\times(2\times10^{-4})$$
$$= 6\times10^4\text{ NC}^{-1}$$

Maximum electric force = qE_0

$$= (5\times10^{-6})\times(6\times10^4) = 0.3\text{ N}$$

Maximum magnetic force = $q v B_0$

$$= (5\times10^{-6})\times(4\times10^5)\times(2\times10^{-4})$$
$$= 4.0\times10^{-4}\text{ N}$$

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. Which of the following electromagnetic waves has the longest wavelength?
   (a) Radiowaves (b) Infrared radiation
   (c) Microwaves (d) X-rays
2. Which of the following electromagnetic waves has the highest frequency?
   (a) Radiowaves (b) X-rays
   (c) γ-rays (d) Microwaves
3. Which of the following electromagnetic waves is used in telecommunication?

(a) Radiowaves (b) Visible radiation
(c) Ultraviolet rays (d) Microwaves

4. Which of the following electromagnetic waves emitted by the sun is responsible for heating the earth's atmosphere due to greenhouse effect?
(a) Visible light (b) Infra-red radiation
(c) Ultraviolet rays (d) γ-rays

5. The speed of electromagnetic waves in vacuum is given by
(a) $\frac{1}{\mu_0 \varepsilon_0}$ (b) $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
(c) $\mu_0 \, \varepsilon_0$ (d) $\sqrt{\mu_0 \varepsilon_0}$

6. In an electromagnetic wave travelling in air, the amplitudes E_0 and B_0 of the electric and magnetic fields are related as (here c is the speed of the wave in air)
(a) $E_0 = cB_0$ (b) $E_0 = \frac{B_0}{c}$
(c) $E_0 = c^2 \, B_0$ (d) $E_0 = B_0$

7. If v_x, v_m and v_r respectively are the speeds of X-rays, microwaves and radiowaves in air, then
(a) $v_x > v_m > v_r$ (b) $v_x < v_m < v_r$
(c) $v_x = v_m > v_r$ (d) $v_x = v_m = v_r$

8. When a plane electromagnetic wave travels in vacuum, the average electric energy density is given by (here E_0 is the amplitude of the electric field of the wave)
(a) $\frac{1}{4} \, \varepsilon_0 \, E_0^2$ (b) $\frac{1}{2} \, \varepsilon_0 \, E_0^2$
(c) $2 \, \varepsilon_0 \, E_0^2$ (d) $4 \, \varepsilon_0 \, E_0^2$

9. When a plane electromagnetic wave travels in a dielectric medium, its electric and magnetic energy densities u_e and u_m are related as
(a) $u_e = cu_m$ (b) $u_e = \frac{u_m}{c}$
(c) $u_e = u_m$ (d) none of these

10. The amplitude of the electric field of a plane electromagnetic wave in air is 6.0×10^{-4} Vm^{-1}. The amplitude of the magnetic field will be
(a) 1.8×10^5 T (b) 5.0×10^3 T
(c) 2.0×10^{-4} T (d) 2.0×10^{-12} T

11. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz. What is the wavelength of the wave?
(a) 1.0 cm (b) 1.5 cm
(c) 2.0 cm (d) 3.0 cm

12. In Q.11 if the peak value of the electric field is 60 Vm^{-1}, the average energy density (in Jm^{-3}) of the magnetic field of the wave will be (given $\mu_0 = 4\pi \times 10^{-7}$ Fm^{-1})
(a) $2\pi \times 10^{-7}$ (b) $4\pi \times 10^{-7}$
(c) $\frac{1}{2\pi} \times 10^{-7}$ (d) $\frac{1}{4\pi} \times 10^{-7}$

13. In Q.12, the total average energy density (in Jm^{-3}) of the electromagnetic field of the wave is
(a) $2\pi \times 10^{-7}$ (b) $4\pi \times 10^{-7}$
(c) $\frac{1}{2\pi} \times 10^{-7}$ (d) $\frac{1}{4\pi} \times 10^{-7}$

14. An electromagnetic wave is produced by oscillating electric and magnetic fields **E** and **B**. Choose the only *incorrect* statement from the following.
(a) **E** is perpendicular to **B**.
(b) **E** is perpendicular to the direction of propagation of the wave
(c) **B** is perpendicular to the direction of propagation of the wave
(d) **E** is parallel to **B**.

15. Which of the following pairs of space and time varying **E** $\left(= \hat{\mathbf{i}}\, E_x + \hat{\mathbf{j}}\, E_y + \hat{\mathbf{k}}\, E_z\right)$ and **B** = $\left(= \hat{\mathbf{i}}\, B_x + \hat{\mathbf{j}}\, B_y + \hat{\mathbf{k}}\, E_z\right)$ would generate a plane electromagnetic wave travelling in the z-direction?
(a) E_x, B_z (b) E_y, B_z
(c) E_z, B_x (d) E_x, B_y

16. Which of the following statements is false? Electromagnetic waves
(a) are transverse
(b) travel in free space at the speed of light
(c) travel with the same speed in all media
(d) are produced by an accelerating charge

Level B

17. Displacement current was first postulated by
(a) Ampere (b) Maxwell
(c) Hertz (d) Marconi

18. The SI unit of displacement current is
(a) A (b) Am^{-1}
(c) Am^{-2} (d) Am^{-3}

19. The displacement current flows in the dielectric of a capacitor when the potential difference between its plates
(a) is changing with time
(b) is changing with distance

(c) has assumed a constant value
(d) becomes zero

20. The potential difference between the plates of a parallel plate capacitor is changing at the rate of 10^6 Vs^{-1}. If the capacitance is 2 μF, the displacement current in the dielectric of the capacitor will be
(a) 1 A (b) 2 A
(c) 3 A (d) 4 A

21. If V is the accelerating potential in an X-ray tube, the minimum wavelength of the emitted X-ray will be
(a) $\frac{hc}{eV}$ (b) $\frac{hV}{ec}$
(c) $\frac{cV}{eh}$ (d) $\frac{eV}{hc}$

22. If the frequency of an X-ray is increased by 20%, its energy
(a) increases by 10% (b) increases by 20%
(c) decreases by 10% (d) decreases by 20%

23. If μ_r and $\in_r$ are the relative permeability and relative permittivity of a medium, its refractive index is given by
(a) $n = \frac{1}{\sqrt{\mu_r \in_r}}$ (b) $n = \sqrt{\mu_r \in_r}$
(c) $n = \frac{1}{\mu_r \in_r}$ (d) $n = \mu_r \in_r$

24. The earth receives energy from the sun at the rate of 1400 Wm^{-2}. The total power incident on a roof of dimensions (16 m × 10 m) is
(a) 8.75 W (b) 2.80×10^3 W
(c) 0.11 W (d) 2.24×10^5 W

25. In Q. 24 above, the force exerted by the solar radiation on the roof is nearly equal to
(a) 7.5×10^{-4} N (b) 7.5×10^{-3} N
(c) 7.5×10^{-2} N (d) 0.75 N

26. The peak value of the electric field of a radiowave reaching the receiver antenna is 3×10^{-3} Vm^{-1}. The peak value of the magnetic field will be
(a) 10^{-9} T (b) 10^{-10} T
(c) 10^{-11} T (d) 10^{-12} T

27. An electromagnetic wave travelling in a medium is given by
$$E = E_0 \sin(\omega t - kx)$$
The speed of the wave in the medium is given by
(a) $v = \omega k$ (b) $v = \frac{k}{\omega}$
(c) $v = \frac{\omega}{k}$ (d) $v = \sqrt{\frac{\omega}{k}}$

28. A plane electromagnetic wave travelling in vacuum is described by
$$E = E_0 \sin(\omega t - kx)$$
and
$$B = B_0 \sin(\omega t - kx)$$
Which of the following relations is correct?
(a) $\omega B_0 = kE_0$ (b) $kB_0 = \omega E_0$
(c) $E_0 B = \omega k$ (d) $E_0 B_0 = \frac{\omega}{k}$

29. A 10 watt LED lamp converts 95% of the power into electromagnetic waves. The peak value of the electric field at a distance of 2 m from the lamp is nearly equal to
(a) 10 Vm^{-1} (b) 12 Vm^{-1}
(c) 10^2 Vm^{-1} (d) 1.2×10^2 Vm^{-1}

30. A plane electromagnetic wave travelling in vacuum is given by
$$E = E_0 \sin(\omega t - kx)$$
where $E_0 = 100$ NC^{-1}. The average energy density of the wave is nearly equal to
(a) 1.1×10^{-5} Jm^{-3} (b) 2.2×10^{-6} Jm^{-3}
(c) 8.8×10^{-7} Jm^{-3} (d) 4.4×10^{-8} Jm^{-3}

31. A plane electromagnetic ware travelling in a non-magnetic medium is given by
$$E = (9 \times 10^8\ NC^{-1}) \sin[(9 \times 10^8\ \text{rad s}^{-1})t - (6\ m^{-1})x]$$
where x is in metre and t is in second.
The dielectric constant of the medium is
(a) 5 (b) 4
(c) 3 (d) 2

Answers

Level A

1. (a)	2. (c)	3. (d)	4. (b)
5. (b)	6. (a)	7. (d)	8. (a)
9. (c)	10. (d)	11. (b)	12. (d)
13. (c)	14. (d)	15. (d)	16. (c)

Level B

17. (b)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)	23. (b)	24. (d)
25. (a)	26. (c)	27. (c)	28. (a)
29. (b)	30. (d)	31. (b)	

Solutions

Level A

1. The correct choice is (a).
2. The correct choice is (c).
3. The correct choice is (d).
4. The correct choice is (b).
5. The correct choice is (b).
6. The correct choice is (a).
7. Electromagnetic waves of all frequencies travel with the same speed in air. Hence the correct choice is (d).
8. The correct choice is (a).
9. When an electromagnetic wave travels in a dielectric medium, its total energy density is divided equally between its electric and magnetic fields. Hence the correct choice is (c).
10. $B_0 = \frac{E_0}{c} = \frac{6.0\times 10^{-4}}{3\times 10^8} = 2.0 \times 10^{-12}$ T which is choice (d).
11. $\lambda = \frac{c}{\nu} = \frac{3\times 10^8}{2.0\times 10^{10}} = 1.5 \times 10^{-2}$ m = 1.5 cm, which is choice (b)
12. $B_0 = \frac{E_0}{c} = \frac{60}{3\times 10^8} = 2.0 \times 10^{-7}$ T

    $$u_m = \frac{B_0^2}{4\mu_0} = \frac{\left(2.0\times 10^{-7}\right)^2}{4\times 4\pi\times 10^{-7}} = \frac{1}{4\pi}\times 10^{-7}\ \text{Jm}^{-3}$$

    Hence the correct choice is (d).
13. $u = u_e + u_m = 2u_m$ (since $u_e = u_m$). Hence

    $$u = 2\times\frac{1}{4\pi}\times 10^{-7} = \frac{1}{2\pi}\times 10^{-7}\ \text{Jm}^{-3},$$

    which is choice (c).
14. The correct choice is (d).
15. The correct choice is (d).
16. The refractive index (n) of a medium is defined as

    $$n = \frac{c}{v}$$

    where c is the speed of the electromagnetic wave in free space and v that in the medium. Thus, $v = c/n$. Since the value of n is different for different media, the value of v will also be different for different media. Hence choice (c) is false.

Level B

17. The correct choice is (b).
18. The correct choice is (a).
19. The displacement current I_d is given by

    $$I_d = \varepsilon_0 \frac{d\Phi}{d\Phi}$$

    where Φ is the electric flux. For a parallel plate capacitor

    $$\Phi = EA$$

    where A is the area of the plate and E is the electric field between the plates. Now, $E = V/d$ where d is the distance between the plates. Hence

    $$I_d = \varepsilon_0 \frac{d}{dt}(AE) = \varepsilon_0 A\frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$$

    Thus I_d exists as long as the potential difference V is changing with time t. Hence the correct choice is (a).
20. We have seen above that the displacement current is given by

    $$I_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$$

    $$= C\frac{dV}{dt} \qquad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

    $$= (2\times 10^{-6})\times 10^6 = 2\ \text{A}$$

    Hence the correct choice is (b).
21. $E = h\nu = \frac{hc}{\lambda}$. Also $E = eV$. Equating them, we get

    $$\frac{hc}{\lambda} = eV \quad\Rightarrow\quad \lambda = \frac{hc}{eV}$$

    So the correct choice is (a).
22. $E = h\nu$. So $E \propto \nu$. Hence if ν is increased by 20%, E will also increase by 20%. So the correct choice is (b).
23. $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

    $$v = \frac{1}{\sqrt{\mu \epsilon}}$$

    $$n = \frac{c}{v} = \sqrt{\frac{\mu}{\mu_0}\frac{\epsilon}{\epsilon_0}} = \sqrt{\mu_r \epsilon_r}$$

    The correct choice is (b).

24. P = (1400 Wm^{-2}) × (16 × 10) m^2 = 2.24 × 10^5 W, which is choice (d).

25. $P = Fc \Rightarrow F = \dfrac{P}{c} = \dfrac{2.24\times10^5}{3\times10^8} \simeq 7.5\times10^{-4}$ N

So the correct choice is (*a*).

26. $B_0 = \dfrac{E_0}{c} = \dfrac{3\times10^{-3}}{3\times10^8} = 10^{-11}$ T, which is choice (c).

27. $\omega = 2\pi\nu$ and $k = \dfrac{2\pi}{\lambda}$. Therefore

$$v = \nu\lambda = \frac{2\pi\nu\times\lambda}{2\pi} = \frac{\omega}{k}$$

So the correct ch-oice is (c).

28. $B_0 = \dfrac{E_0}{c} = \dfrac{E_0}{\omega/k} \Rightarrow \omega B_0 = k E_0$, which is choice (a).

29. 95% of 10 W = 9.5 W. So P = 9.5 W

Intensity of the wave at a distance r from the lamp is

$$I = \frac{P}{4\pi r^2}$$

If u is the average energy density of the wave, then

$$I = uc = \frac{1}{2}\epsilon_0 E_0^2 c$$

$$\therefore \quad \frac{P}{4\pi r^2} = \frac{1}{2}\epsilon_0 E_0^2 c$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{P}{2\pi r^2 \epsilon_0 c}}$$

$$= \sqrt{\frac{9.5}{2\times3.14\times(2)^2\times(8.85\times10^{-12})\times3\times10^8}}$$

$$= 11.9 \simeq 12 \text{ Vm}^{-1}$$

So the correct choice is (b).

30. $\langle u\rangle = \dfrac{1}{2}\epsilon_0 E_0^2$

$$= \frac{1}{2}\times(8.85\times10^{-12})\times(100)^2$$

$$= 4.425 \times 10^{-8} \text{ Jm}^{-3}$$

So the correct choice is (d).

31. $v = \dfrac{\omega}{k} = \dfrac{9\times10^8}{6} = 1.5\times10^8$ ms^{-1}

Refractive index $n = \dfrac{c}{v} = \dfrac{3\times10^8}{1.5\times10^8} = 2$

Also $n = \sqrt{\mu_r \epsilon_r}$. For a non-magnetic medium μ_r = 1.

Therefore

$$n = \sqrt{\epsilon_r} \Rightarrow \epsilon_r = n^2 = (2)^2 = 4$$

$\therefore$ Dielectric constant $k = \epsilon_r = 4$, which is choice (b).

SECTION 2 Multiple Choice Questions Based on Passage

Questions 1 to 5 are based on the following passage.

Passage I

Electric and magnetic fields exhibit a wave-like behaviour. When electric and magnetic fields vary in space and time, they produce an electromagnetic wave. An accelerated charge produces electromagnetic waves. An oscillating charge has oscillating electric and magnetic fields around it and hence it produces electromagnetic waves. Electrons falling from a higher to a lower energy orbit in an atom radiate electromagnetic waves. The motion of electrons in an antenna radiates electromagnetic waves. Electromagnetic waves are transverse in nature. They do not require a material medium for their propagation. The speed of an electromagnetic wave is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ is the magnetic permeability and ε is the electrical permittivity of the medium. Radiowaves, microwaves, infrared radiations, visible light, ultraviolet rays, X-rays and gamma rays are all electromagnetic waves. They have a very wide range of wavelengths and hence of frequencies. Although they are identical in nature, their method of production and their interaction with matter are different.

1. Which of the following electromagnetic waves has the longest wavelength?

(a) Radiowaves (b) Infrared radiation
(c) Microwaves (d) X-rays

2. Which of the following electromagnetic waves has the highest frequency?
(a) Radiowaves (b) Visible light
(c) Ultraviolet rays (d) Microwaves

3. Which of the following waves are transverse in nature?
(a) Light emitted from a sodium lamp
(b) Sound waves travelling in air
(c) X-rays from an X-ray machine
(d) Microwaves used in a radar.

4. Which of the following statements is false? Electromagnetic waves
(a) are transverse.
(b) travel in free space at the same speed.
(c) travel in all media at the same speed.
(d) are produced by an accelerating charge.

5. An electromagnetic wave is produced by oscillating electric and magnetic fields **E** and **B**. Choose the only incorrect statement from the following.
(a) **E** is perpendicular to **B**.
(b) **E** is perpendicular to the direction of propagation of the wave.
(c) **B** is perpendicular to the direction of propagation of the wave.
(d) **E** is parallel to **B**.

Answers

1. (a) 2. (c) 3. (a), (c), (d)
4. (c) 5. (d)

3 SECTION Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The fact that electromagnetic waves are transverse by nature is established by
(a) polarisation (b) interference
(c) reflection (4) diffraction **[2002]**

2. Infrared radiations are detected by
(a) spectrometer (b) pyrometer
(c) manometer (d) photometer **[2002]**

3. The dimensions of $\dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ are the same as those of
(a) velocity (b) $(\text{velocity})^2$
(c) $\dfrac{1}{(\text{velocity})}$ (d) $\dfrac{1}{(\text{velocity})^2}$ **[2003]**

4. Which of the following radiations has the least wavelength?
(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays **[2003]**

5. The earth emits radiations in the infrared region of the spectrum. The spectrum is correctly given by
(a) Rayleigh-Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law **[2003]**

6. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium of relative permittivity $\varepsilon_r = 4.0$. Then
(a) wavelength is doubled and frequency remains unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength and frequency both remain unchanged. **[2004]**

7. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead, which will reduce the intensity to $I/2$ is
(a) 6 mm (b) 9 mm
(c) 18 mm (d) 12 mm **[2005]**

8. The rms value of the electric field of the light coming from the Sun is 720 N/C. The average total energy density of the electromagnetic wave is
(a) 81.35×10^{-12} J/m^3 (b) 3.3×10^{-3} J/m^3
(c) 4.58×10^{-6} J/m^3 (d) 6.37×10^{-9} J/m^3
[2006]

9. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

(a) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ (b) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

(c) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$ (d) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

[2012]

10. The magnetic field in a travelling electromagnetic wave has a peak value of 20nT. the peak value of electric field strength is:

(a) 6 V/m (b) 9 V/m

(c) 12 V/m (d) 3 V/m **[2013]**

11. Which of the following pairs of space and time varying **E** $(= \hat{i}E_x + \hat{\mathbf{j}}E_y + \hat{\mathbf{k}}E_z)$ and **B** $(= \hat{i}B_x + \hat{\mathbf{j}}B_y + \hat{\mathbf{k}}B_z)$ would generate a plane electromagnetic wave travelling in the z-direction?

(a) E_x, B_z (b) E_y, B_z

(c) E_z, B_x (d) E_x, B_y **[2014]**

Answers

1. (a)	2. (b)	3. (b)	4. (a)
5. (a)	6. (c)	7. (d)	8. (c)
9. (b)	10. (a)	11. (d)	

Solutions

1. The phenomena of interference, reflection and diffraction are exhibited by both transverse and longitudinal waves. But only transverse waves can be polarised. Hence the correct choice is (a).
2. The correct choice is (b).
3. $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, where c = velocity of light. Hence the correct choice is (b).
4. γ-rays have the highest frequency and hence the shortest wavelength.
5. The correct choice is (a).
6. When a wave travels from one medium into another, its frequency never changes.

For vacuum: $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = v\lambda_0$ (1)

For dielectric medium $v = \frac{1}{\sqrt{\mu\varepsilon}} = v\lambda$ (2)

Dividing (2) by (1) we get

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{\mu_0 \varepsilon_0}{\mu\varepsilon}}$$

For non-magnetic media $\mu = \mu_0$. Hence

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{\varepsilon_0}{\varepsilon}} = \sqrt{\frac{1}{\varepsilon_r}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore \lambda = \frac{\lambda_0}{2}$$

Thus the correct choice is (c).

7. The intensity of radiation of intensity I when it passes through a thickness x of an absorbing medium is given by

$$I' = I e^{-\mu x}$$

where μ is a constant. When $x_1 = 36$ m, $I' = \frac{I}{8}$. Therefore,

$$\frac{I}{8} = I e^{-\mu x}$$

$$\Rightarrow \quad \frac{1}{8} = e^{-\mu x_1}$$

$$\text{or} \quad e^{\mu x_1} = 8$$

$$\Rightarrow \quad \mu x_1 = \ln(8) = 2.08$$

$$\Rightarrow \quad \mu = \frac{2.08}{x_1}$$

Let x_2 be the thickness of lead for I to reduce to $I_2 = \frac{I}{2}$

Then

$$\frac{I}{2} = Ie^{-\mu x_2}$$

$$\Rightarrow \quad \mu x_2 = \ln(2) = 0.693$$

$$\Rightarrow \frac{2.08}{x_1} \times x_2 = 0.693$$

$$\Rightarrow \quad x_2 = \frac{0.693 \times x_1}{2.08}$$

$$= \frac{0.693 \times 36\,\text{mm}}{2.08} = 11.99 \text{ mm} \simeq 12 \text{ mm}$$

8. The total energy density of an electromagnetic wave is

$$u = u_e + u_m$$

where $u_e = \frac{1}{2}\varepsilon_0 E^2$ and $u_m = \frac{B^2}{2\mu_0}$.

The electric field of the wave is given by

$$E = E_0 \sin \omega t$$

where E_0 is the peak value of E and ω is the angular frequency. The average energy density associated with electric field is

$$<u_e> = \frac{1}{2}\varepsilon_0 E^2{}_0 <\sin^2 \omega t>$$

where $$<\sin^2 \omega t> = \frac{1}{T}\int_0^T \sin^2 \ (\omega t)\ dt = \frac{1}{2}$$

Hence $$<u_e> = \frac{1}{4}\varepsilon_0 E^2{}_0$$

Also $$<u_m> = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\ (E/c)^2 = \frac{1}{2}\varepsilon_0 E^2$$

$$\left(\because\ c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

Hence $$<u_m> = \frac{1}{4}\varepsilon_0 E^2{}_0$$

Total energy density $<u> = <u_e> + <u_m> = \frac{1}{4}\varepsilon_0 E^2{}_0 + \frac{1}{4}\varepsilon_0 E^2{}_0$

$$= \frac{1}{2}\varepsilon_0 E^2$$

$$= \varepsilon_0 E^2{}_{\text{rms}} \qquad \left(\because\ E_{\text{rem}} = \frac{E_0}{\sqrt{2}}\right)$$

$$= (8.85 \times 10^{-12}) \times (720)^2$$

$$= 4.58 \times 10^{-6}\ \text{J m}^{-3}$$

9. The direction of polarisation vaector $\vec{X}$ is parallel to the plane containing the electric field $\vec{E}$. The propagation vector $\vec{k}$ is perpendicular to $(\vec{E} \times \vec{B})$. So the correct choice is (b).

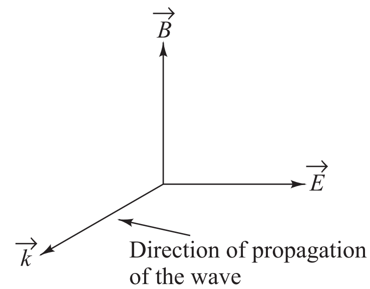

Fig. 15.1

10. $E_0 = cB_0$

$= (3\times10^8) \times (20\times10^{-9})$

$= 6\ \text{Vm}^{-1}$

11. The correct choice is (d).