Q1: NTA Test 01 (Single Choice)

$\int \frac{\ln(x+1)-\ln x}{x(x+1)}dx$ is equal to (where C is an arbitarary constant)

(A) $-\frac{1}{2}\left[\ln\left(\frac{x+1}{x}\right)\right]^2 + C$

(B) $C - \left[\{\ln(x+1)\}^2 - (\ln x)^2\right]$

(C) $-\ln\left[\ln\left(\frac{x+1}{x}\right)\right] + C$

(D) $-\ln\left(\frac{x+1}{x}\right) + C$

Q2: NTA Test 02 (Single Choice)

The integral $\int x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx;\ (x > 0)$ is equal to

(A) $-x + \left(1+x^2\right)\cot^{-1}x + c$

(B) $x - \left(1+x^2\right)\cot^{-1}x + c$

(C) $x - \left(1+x^2\right)\tan^{-1}x + c$

(D) $-x + \left(1+x^2\right)\tan^{-1}x + c$

Q3: NTA Test 03 (Single Choice)

$\int \frac{\sin^4 x}{\cos^8 x}\,dx$ is equal to (where C is an arbitrary constant)

(A) $\frac{(1+\tan^5 x)}{5} + \frac{\tan^5 x}{7} + C$

(B) $\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$

(C) $\frac{\tan^7 x}{5} + \frac{\tan^5 x}{7} + C$

(D) None of these

Q4: NTA Test 04 (Single Choice)

If m is any natural number, then the value of the integral $\int\left(x^{3m} + x^{2m} + x^m\right)\left(2x^{2m} + 3x^m + 6\right)^{(1/m)}dx$ is (where, C is an arbitrary constant)

(A) $\frac{1}{6(m+1)}\left\{2x^{3m} + 3x^{2m} + 6x^m\right\}^{(1/m)+1} + C$

(B) $\frac{1}{6m}\left\{2x^{3m} + 3x^{2m} + 6x^m\right\}^{(1/m)+1} + C$

(C) $\frac{1}{6m}\left\{2x^{3m} + 3x^{2m} + 6x^m\right\}^{1/m} + C$

(D) None of the above

Q5: NTA Test 05 (Single Choice)

$\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x}dx$ is equal to (where C is an arbitrary constant)

(A) $\frac{1}{2}\sin 2x + C$

(B) $-\frac{1}{2}\sin 2x + C$

(C) $-\frac{1}{2}\sin x + C$

(D) $-\sin^2 x + C$

Q6: NTA Test 06 (Numerical)

If $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = a\sqrt{x} + b\left(\sqrt[3]{x}\right) + c\left(\sqrt[6]{x}\right) + d\ \ln\left(\sqrt[6]{x}+1\right) + e$, e being an arbitrary constant, then the value of $20a + b + c + d$ is

Q7: NTA Test 07 (Single Choice)

If $I = \int\left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$, then I equals to (where, C is an arbitrary constant)

(A) $\sqrt{2}\sin^{-1}(\sin x + \cos x) + C$

(B) $\sqrt{2}\cos^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + C$

(D) $\sqrt{2}\cos^{-1}(\sin x + \cos x) + C$

Q8: NTA Test 08 (Numerical)

The graph of the antiderivative of $f(x) = xe^{\frac{x}{2}}$ passes through $(0, 3)$, then the value of $g(2) - f(0)$ is

Q9: NTA Test 09 (Single Choice)

If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\log\left|\sin x - 2\cos x\right| + k$, then the value of a is

(A) 1

(B) 2

(C) -1

(D) -2

Q10: NTA Test 10 (Numerical)

Let $f(x)=\frac{9x}{25}+c, c>0$. If the curve $y=f^{-1}(x)$ passes through $\left(\frac{1}{4},-\frac{5}{4}\right)$ and $g(x)$ is the antiderivative of $f^{-1}(x)$ such that $g(0)=\frac{5}{2}$, then the value of $[g(1)]$ is, (where $[.]$ represents the greatest integer function)

Q11: NTA Test 12 (Single Choice)

$\int \frac{ln\left(\frac{x-1}{x+1}\right)}{x^2-1}dx$ is equal to

(A) $\frac{1}{2}\left(ln\left(\frac{x-1}{x+1}\right)\right)^2+C$

(B) $\frac{1}{2}\left(ln\left(\frac{x+1}{x-1}\right)\right)^2+C$

(C) $\frac{1}{4}\left(ln\left(\frac{x-1}{x+1}\right)\right)^2+C$

(D) $\frac{1}{4}\left(ln\left(\frac{x+1}{x-1}\right)\right)+C$

Q12: NTA Test 13 (Single Choice)

The value of $\int \frac{dx}{x\left(x^n+1\right)}$ is equal to

(A) $\frac{1}{n}\log_e\left(\frac{x^n}{x^n+1}\right)+c$

(B) $\frac{1}{n}\log_e\left(\frac{x^n+1}{x^n}\right)+c$

(C) $\log_e\left(\frac{x^n}{x^n+1}\right)+c$

(D) None of these

Q13: NTA Test 14 (Numerical)

The value of $\int(\sin 101x).\sin^{99}xdx$ is $\frac{\sin(100x)\sin^{100}x}{k+5}$, then $\frac{k}{19}$ is

Q14: NTA Test 15 (Numerical)

If $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}=k\left(\frac{x-1}{x+2}\right)^{1/4}+c$, then the number of divisors of 30k is

Q15: NTA Test 17 (Numerical)

If $\int \frac{x^{pq-p-1}}{(x^p+1)^q}dx=\frac{2(1+x^{-p})^{1-q}}{\lambda p(q-1)}+c \quad (p,q\in N-\{1\})$, then the value of λ is (here, c is an arbitrary constant)

Q16: NTA Test 18 (Single Choice)

The value of the integral $\int e^{3\sin^{-1}x}\left(\frac{1}{\sqrt{1-x^2}}+e^{3\cos^{-1}x}\right)dx$ is equal to

(where, c is an arbitrary constant)

(A) $\frac{e^{3\sqrt{\sin^{-1}x}}}{3}+xe^{\frac{3\pi}{2}}+c$

(B) $e^{\sqrt{\sin^{-1}x}}+e^{\pi/2}+c$

(C) $\frac{e^{3\sin^{-1}x}}{3}+xe^{\frac{3\pi}{2}}+c$

(D) $e^{\frac{\pi}{2}}+e^{x\left(\frac{\pi}{2}\right)}+c$

Q17: NTA Test 19 (Single Choice)

The value of the integral $\int\left(\frac{\sin x}{x}\right)^6\left(\frac{x\cos x-\sin x}{x^2}\right)dx$ is

(where, c is an arbitrary constant)

(A) $\frac{\sin x}{x}+c$

(B) $\frac{\sin x}{x^2}+c$

(C) $\frac{\sin^7 x}{x^7}+c$

(D) $\frac{\sin^7 x}{7x^7}+c$

Q18: NTA Test 20 (Single Choice)

If the integral $I=\int x^{\sin x}\left(\cos x\cdot\ln x+\frac{\sin x}{x}\right)dx=\left(f(x)\right)^{g(x)}+c \ (\forall x>0)$, then the range of $y=g(x)$ is (where, c is an arbitrary constant)

(A) $[-1,1]$

(B) $[0,1]$

(C) $[0,1)$

(D) $(-1,1)$

Q19: NTA Test 21 (Single Choice)

If the value of integral $\int \frac{dx}{\left(x+\sqrt{x^2-1}\right)^2} = ax^3 - x + b\left(x^2-1\right)^{\frac{1}{b}} + C$, (where, C is the constant of integration), then $a \times b$ is equal to

(A) 1

(B) $\frac{4}{9}$

(C) 2

(D) $\frac{9}{4}$

Q20: NTA Test 22 (Single Choice)

The value of the integral $\int \frac{x^2-4x\sqrt{x}+6x-4\sqrt{x}+1}{x-2\sqrt{x}+1} dx$ is equal to

(A) $\frac{x^{\frac{3}{2}}}{2} + x + c$

(B) $\frac{x^2}{2} - \frac{4}{3}x^{\frac{3}{2}} + x + c$

(C) $x^{\frac{3}{2}} + \frac{x}{2} + c$

(D) $\frac{2}{3}x^{\frac{3}{2}} + c$

Q21: NTA Test 23 (Numerical)

If $f(x) = \sin x,\ g(x) = \cos x$ and $h(x) = \cos(\cos x)$, then the integral $\mathrm{I} = \int f(g(x)).\, f(x).\, h(x) dx$ simplifies to $-\lambda \sin^2(\cos x) + C$ (where, C is the constant of integration). The value of λ is equal to

Q22: NTA Test 24 (Single Choice)

The value of the integral $\int \frac{\sin^2 x \sec^2 x + 2\sqrt{1-x^2}\tan x \sin^{-1} x}{\sqrt{1-x^2}(1+\tan^2 x)} dx$ is (where, C is the constant of integration)

(A) $\left(\sin^{-1} x\right)\left(\cos^2 x\right) + C$

(B) $\left(\sin^{-1} x\right)\left(\sin^2 x\right) + C$

(C) $\left(\cos^{-1} x\right)\left(\sin^2 x\right) + C$

(D) $-\sin^{-1} x\left(\sin^2 x\right) + C$

Q23: NTA Test 25 (Single Choice)

If a function $f: R \rightarrow R$ is defined as $f(x) = \int \frac{x^8+4}{x^4-2x^2+2} dx$ and $f(0) = 1$, then which of the following is correct?

(A) $f(x)$ is an even function

(B) $f(x)$ is an onto function

(C) $f(x)$ is an odd function

(D) $f(x)$ is many one function

Q24: NTA Test 26 (Single Choice)

The value of $\int \frac{(x-4)}{x^2\sqrt{x-2}} dx$ is equal to (where, C is the constant of integration)

(A) $2x\sqrt{x-2} + C$

(B) $-\frac{2}{x}\sqrt{x-2} + C$

(C) $\frac{\sqrt{x-2}}{x} + C$

(D) $\frac{x}{\sqrt{x-2}} + C$

Q25: NTA Test 27 (Single Choice)

The value of $\int \frac{x\,dx}{(x+3)\sqrt{x+1}}$ is (where, c is the constant of integration)

(A) $2\sqrt{x+1} + 3\ \tan^{-1}\sqrt{x+1} + c$

(B) $2\sqrt{x+1} + 3\sqrt{2}\ \tan^{-1}\sqrt{\frac{x+1}{2}} + c$

(C) $2\sqrt{x+1} - 3\sqrt{2}\ \tan^{-1}\sqrt{\frac{x+1}{2}} + c$

(D)
$2\sqrt{x+1} - 3\ \tan^{-1}\sqrt{x+1} + c$

Q26: NTA Test 28 (Single Choice)

If $\int e^{\sin\theta}\left(\sin\theta + \sec^2\theta\right) d\theta$ is equal to $f(\theta) + C$ (where, C is the constant of integration) and $f(0) = 0$, then the value of $f\left(\frac{\pi}{4}\right)$ is

(A) $e^{\sqrt{2}}$

(B) $e^{\frac{1}{\sqrt{2}}}$

(C) e^2

(D) $e^{\frac{1}{2}}$

Q27: NTA Test 29 (Numerical)

If $f(x)$ is the antiderivative of $(1+2\tan x(\tan x+\sec x))^{\frac{1}{2}}$ and $f\left(\frac{\pi}{6}\right)=\log 2$, then the value of $f(0)$ is

Q28: NTA Test 30 (Numerical)

The integral $\mathrm{I}=\int\left(\sin\left(x^2\right)+2x^2\cos\left(x^2\right)\right)dx=xH(x)+C$, (where C is the constant of integration). If the range of $H(x)$ is $[a,b]$, then the value of $a+2b$ is equal to

Q29: NTA Test 31 (Numerical)

Let $\int e^x\cdot x^2dx=f(x)e^x+C$ (where, C is the constant of integration). The range of $f(x)$ as $x\in R$ is $[a,\infty)$. The value of $\frac{a}{4}$ is

Q30: NTA Test 32 (Single Choice)

The value of $\int\frac{e^{\sqrt{x}}}{\sqrt{x}\left(1+e^{2\sqrt{x}}\right)}dx$ is equal to (where, C is the constant of integration)

(A) $\tan^{-1}\left(2e^{\sqrt{x}}\right)+C$

(B) $\ln\left(\frac{1+e^{\sqrt{x}}}{1-e^{\sqrt{x}}}\right)+C$

(C) $2\tan^{-1}\left(e^{\sqrt{x}}\right)+C$

(D) $\left(\tan^{-1}x\right)e^{\sqrt{x}}+C$

Q31: NTA Test 33 (Single Choice)

The integral $\mathrm{I}=\int 2^{\left(2^x+x\right)}\mathrm{dx}=\lambda.\left(2^{2^x}\right)+\mathrm{C}$ (where, C is the constant of integration). Then the value of $\sqrt{\lambda}$ is equal to

(A) $\frac{1}{\ln 4}$

(B) $\frac{1}{(\ln 2)^2}$

(C) $\frac{1}{\ln 2}$

(D) $\frac{1}{(\ln 4)^2}$

Q32: NTA Test 34 (Single Choice)

The value of $\int\frac{\cos x}{\sin^2x(\sin x+\cos x)}dx$ is equal to

(A) $\log\left|\frac{1+\tan x}{\tan x}\right|-\cot x+C$

(B) $\log\left|\frac{\tan x}{1+\tan x}\right|+C$

(C) $\log\left|\frac{\tan x}{1+\tan x}\right|-\tan x+C$

(D) $\log\left|\frac{\tan x}{1+\tan x}\right|+\cot x+C$

Q33: NTA Test 35 (Single Choice)

If the integral $\int\frac{\mathrm{x}^4+\mathrm{x}^2+1}{\mathrm{x}^2-\mathrm{x}+1}\mathrm{dx}=\mathrm{f}(\mathrm{x})+\mathrm{C}$, (where C is the constant of integration and$\mathrm{x}\in R$), then the minimum value of $\mathrm{f}'(\mathrm{x})$ is

(A) 1

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) 2

Q34: NTA Test 36 (Single Choice)

If $\mathrm{I_n}=\int(\ln x)^{\mathrm{n}}\mathrm{dx}$, then $\mathrm{I}_{10}+10\mathrm{I}_9$ is equal to (where C is the constant of integration)

(A) $x(\ln x)^{10}+C$

(B) $10(\ln x)^9+C$

(C) $9(\ln x)^{10}+C$

(D) $x(\ln x)^9+C$

Q35: NTA Test 37 (Single Choice)

The value of the integral $\int\mathrm{x}^{\frac{1}{3}}(1-\sqrt{\mathrm{x}})^3\mathrm{dx}$ is equal to (where c is the constant of integration)

(A) $6\left(\frac{\mathrm{x}^{\frac{4}{3}}}{8}+\frac{3}{11}\mathrm{x}^{\frac{11}{6}}+\frac{3}{14}\mathrm{x}^{\frac{7}{3}}+\frac{1}{17}\mathrm{x}^{\frac{17}{6}}\right)+\mathrm{c}$

(B) $6\left(\frac{\mathrm{x}^{\frac{4}{3}}}{8}-\frac{3}{11}\mathrm{x}^{\frac{11}{6}}+\frac{3}{14}\mathrm{x}^{\frac{7}{3}}-\frac{1}{17}\mathrm{x}^{\frac{17}{6}}\right)+\mathrm{c}$

(C) $2\left(\frac{\mathrm{x}^{\frac{4}{3}}}{8}-\frac{3}{11}\mathrm{x}^{\frac{4}{6}}-\frac{3}{14}\mathrm{x}^{\frac{7}{3}}-\frac{1}{17}\mathrm{x}^{\frac{17}{6}}\right)+\mathrm{c}$

(D) $2\left(\frac{\mathrm{x}^4}{8}-\frac{3}{11}\mathrm{x}^{11}-\frac{3}{14}\mathrm{x}^7-\frac{1}{17}\mathrm{x}^{17}\right)+\mathrm{c}$

Q36: NTA Test 38 (Single Choice)

The value of the integral $\mathrm{I}=\int e^x(\sin x+\cos x)dx$ is equal to $e^x\cdot f(x)+C$, C being the constant of integration. Then the maximum value of

$y=f\left(x^{2}\right),\ \forall x\in R$ is equal to

(A) 0

(B) -1

(C) 1

(D) $\frac{1}{2}$

Q37: NTA Test 39 (Single Choice)

If $\mathrm{I}=\int\frac{\tan^{-1}\left(e^{x}\right)}{e^{x}+e^{-x}}dx=\frac{\left[\tan^{-1}(f(x))\right]^{2}}{2}+C$ (where C is the constant of integration), then the range of $y=f\left(x\right)\ \forall x\in R$ is

(A) $(-\infty,\infty)$

(B) $[0,\infty)$

(C) $(0,\infty)$

(D) $(-\infty,0)$

Q38: NTA Test 40 (Single Choice)

The value of the integral $\int\frac{\operatorname{cosec}^{2}x-2019}{\cos^{2019}x}\ dx$ is equal to (where C is the constant of integration)

(A) $\frac{\cot x}{(\cos x)^{2019}}+C$

(B) $\frac{-\cot x}{(\cos x)^{2019}}+C$

(C) $\cot x(\cos x)^{2019}+C$

(D) $-\cot x(\cos x)^{2019}+C$

Q39: NTA Test 41 (Single Choice)

Let $A_{n}=\int\tan^{n}x dx,\ \forall n\in N$. If $A_{10}+A_{12}=\frac{\tan^{m}x}{m}+\lambda$ (where λ is an arbitrary constant), then the value of m is equal to

(A) 10

(B) 11

(C) 12

(D) 13

Q40: NTA Test 42 (Single Choice)

The value of the integral $\int e^{x^{2}+\frac{1}{x}}\left(2x^{2}-\frac{1}{x}+1\right)dx$ is equal to (where C is the constant of integration)

(A) $e^{x^{2}+\frac{1}{x}}+C$

(B) $x^{2}e^{x^{2}+\frac{1}{x}}+C$

(C) $xe^{x^{2}+\frac{1}{x}}+C$

(D) $x\cdot e^{x^{2}}+C$

Q41: NTA Test 43 (Single Choice)

If $\int\frac{dx}{\sqrt{e^{x}-1}}=2\tan^{-1}\left(f\left(x\right)\right)+C$, (where $x>0$ and C is the constant of integration) then the range of $f\left(x\right)$ is

(A) $(0,\infty)$

(B) $[0,\infty)$

(C) $[1,\infty)$

(D) $(1,\infty)$

Q42: NTA Test 44 (Single Choice)

Let $f\left(n,x\right)=\int n\cos\left(nx\right)dx$, with $f\left(n,0\right)=0$. If the expression $\sum_{x=1}^{89}f\left(1,x\right)$ simplifies to $\frac{\sin a\sin b}{\sin c}$, then the value of $\frac{b}{ac}$ is (where $a>b$)

(A) 45

(B) 89

(C) $\frac{89}{45}$

(D) $\frac{45}{89}$

Q43: NTA Test 45 (Numerical)

If $\int\frac{\cos^{4}x dx}{\sin^{3}x(\sin^{5}x+\cos^{5}x)^{\frac{3}{5}}}=-\frac{1}{K}\left(1+\cot^{P}x\right)^{\frac{K}{P}}+C$, then the value of $K+P$ is equal to (where C is the constant of integration)

Q44: NTA Test 46 (Single Choice)

If $\int\frac{dx}{x^{2}+x}=\ln\left|f\left(x\right)\right|+C$ (where C is the constant of integration), then the range of $y=f\left(x\right),\ \forall x\in R-\{-1,0\}$ is

(A) $R-\{1\}$

(B) $R-\{0\}$

(C) $R-\{0,1\}$

(D) $R-\{0,-1\}$

Q45: NTA Test 47 (Single Choice)

If $I = \int \frac{dx}{x^3(x^8+1)^{3/4}} = \frac{\lambda(1+x^8)^{\frac{1}{4}}}{x^2} + c$ (where c is the constant of integration), then the value of λ is equal to

(A) 2

(B) $\frac{1}{2}$

(C) -2

(D) $-\frac{1}{2}$

Q46: NTA Test 48 (Single Choice)

The value of $\int \frac{(\tan^{-1}(\sin x+1)) \cos x}{(3+2 \sin x-\cos^2 x)} dx$ is (where c is the constant of integration)

(A) $\tan^{-1}(\sin x) + c$

(B) $(\tan^{-1}(\sin x))^2 + c$

(C) $\frac{(\tan^{-1}(\sin x+1))^2}{2} + c$

(D) $\frac{(\tan^{-1}(\sin x))^2}{2} + c$

Answer Keys

Q1: (A)	**Q2:** (D)	**Q3:** (B)
Q4: (A)	**Q5:** (B)	**Q6:** 37
Q7: (C)	**Q8:** 7	**Q9:** (B)
Q10: 2	**Q11:** (C)	**Q12:** (A)
Q13: 5	**Q14:** 8	**Q15:** 2
Q16: (C)	**Q17:** (D)	**Q18:** (A)
Q19: (B)	**Q20:** (B)	**Q21:** 0.50
Q22: (B)	**Q23:** (B)	**Q24:** (B)
Q25: (C)	**Q26:** (B)	**Q27:** 0
Q28: 1	**Q29:** 0.25	**Q30:** (C)
Q31: (C)	**Q32:** (A)	**Q33:** (C)
Q34: (A)	**Q35:** (B)	**Q36:** (C)
Q37: (C)	**Q38:** (B)	**Q39:** (B)
Q40: (C)	**Q41:** (A)	**Q42:** (C)
Q43: 7	**Q44:** (C)	**Q45:** (D)
Q46: (C)		

Solutions

Q1: (A) $-\frac{1}{2}\left[\ln\left(\frac{x+1}{x}\right)\right]^2 + C$

Put $\ln(x+1) - \ln x = t$

$\Rightarrow \frac{1}{x+1} - \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{x-(x+1)}{x(x+1)} = \frac{dt}{dx}$

$\Rightarrow \frac{-dx}{x(x+1)} = dt \Rightarrow \frac{dx}{x(x+1)} = -dt$

so question becomes

$-\int t\, dt = -\frac{t^2}{2} + C$

Q2: (D) $-x + \left(1 + x^2\right) \tan^{-1} x + c$

$I = \int x \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right) dx; \quad (x > 0)$

Let, $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$\cos^{-1} \left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1} \cos 2\theta$

$= 2\theta$

$= 2\tan^{-1} x$

$I = \int (\tan\theta) 2\theta \cdot \sec^2\theta\, d\theta$

$= 2\left[\theta \int \tan\theta \sec^2\theta d\theta - \int \left(\int \tan\theta \sec^2\theta d\theta\right) \left(\frac{d}{d\theta}\theta\right) d\theta\right]$

$= 2\left[\theta \frac{\tan^2\theta}{2} - \int \frac{\tan^2\theta}{2} 1 d\theta\right]$

$= \frac{2}{2}\left[\theta \tan^2\theta - \int \left(\sec^2\theta - 1\right) d\theta\right]$

$= \theta \tan^2\theta - \tan\theta + \theta + c$

$= x^2 \tan^{-1} x - x + \tan^{-1} x + c$

$= -x + \left(1 + x^2\right) \tan^{-1} x + c$

Q3: (B) $\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$

$\int \frac{\frac{\sin^4 x}{\cos^4 x}}{\frac{\cos^8 x}{\cos^4 x}} . dx = \int \tan^4 x . \sec^4 x\, dx$

$= \int \tan^4 x \left(1 + \tan^2 x\right) \sec^2 x\, dx$

$= \int \tan^4 x \left(1 + \tan^2 x\right) d(\tan x) = \int t^4 \left(1 + t^2\right)\ dt = \frac{t^5}{5} + \frac{t^7}{7} + C$

Q4: (A) $\frac{1}{6(m+1)} \left\{2x^{3m} + 3x^{2m} + 6x^m\right\}^{(1/m)+1} + C$

Put, $I = \int \left(x^{3m} + x^{2m} + x^m\right) \left(\frac{\left(2x^{3m} + 3x^{2m} + 6x^m\right)^{1/m}}{x}\right) dx$

$= \int \left(x^{3m-1} + x^{2m-1} + x^{m-1}\right) \left(2x^{3m} + 3x^{2m} + 6x^m\right)^{1/m} dx$

$= \frac{1}{6m} \int \left(2x^{3m} + 3x^{2m} + 6x^{m}\right)^{1/m} \left(6\,mx^{3m-1} + 6\,mx^{2m-1} + 6\,mx^{m-1}\right) dx$

Now, let $\left(2x^{3m} + 3x^{2m} + 6x^{m}\right) = t \Rightarrow \left(6\,mx^{3m-1} + 6\,mx^{2m-1} + 6\,mx^{m-1}\right)dx = dt$

$\Rightarrow I = \frac{1}{6m} \int t^{1/m}\,dt \;= \frac{1}{6m}\left(\frac{t^{(1/m)+1}}{(1/m)+1}\right)$

$= \frac{1}{6(m+1)}\left(2x^{3m} + 3x^{2m} + 6x^{m}\right)^{(1/m)+1} + C$

Q5: (B) $-\frac{1}{2}\sin 2x + C$

$\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x} dx$

$= \int \frac{\left(\sin^4 x - \cos^4 x\right)\left(\sin^4 x + \cos^4 x\right)}{1-2\sin^2 x\cos^2 x} dx$

$= \int \frac{\left(\sin^2 x - \cos^2 x\right)\left(\sin^4 x + \cos^4 x\right)}{1-2\sin^2 x\cos^2 x} dx$

$= \int -\cos 2x\,dx = -\frac{1}{2}\sin 2x + C$

Q6: 37

Let $x = u^6,\ dx = 6u^5 du$

$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = \int \frac{6u^5 du}{u^3+u^2} = 6\int \frac{u^3}{u+1} du$

$= 6\int \left(u^2 - u + 1 - \frac{1}{u+1}\right) du$

$= 2u^3 - 3u^2 + 6u - 6\ln(u+1) + e$

$= 2\sqrt{x} - 3\left(\sqrt[3]{x}\right) + 6\left(\sqrt[6]{x}\right) - 6\ln\left(\sqrt[6]{x}+1\right) + e$

$\therefore a = 2,\ b = -3,\ c = 6,\ d = -6$

$\therefore 20a + b + c + d = 37$

Q7: (C) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + C$

$I = \int \frac{\sin x + \cos x}{\sqrt{\cos x \sin x}} dx$

Put $\sin x - \cos x = t$, so that $(\sin x + \cos x)dx = dt$ and $1 - 2\sin x\cos x = t^2$

$\therefore\ I = \sqrt{2}\int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1}(t) + C$

$= \sqrt{2}\sin^{-1}(\sin x - \cos x) + C$

Q8: 7

Let $g(x)$ represent the antiderivative of $f(x)$

Therefore, $g(x) = \int xe^{\frac{x}{2}} dx$

$= x\left(2e^{\frac{x}{2}}\right) - \int (1)\cdot 2e^{\frac{x}{2}} dx$ ...[integration by parts]

$= 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + c$

The graph of $y = g(x)$ passes through $(0, 3)$

$\Rightarrow 3 = 0 - 4e^0 + c$

$\Rightarrow c = 7$

Hence, $g(x) = 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + 7$

$\Rightarrow g(x) = 2\left(x-2\right)e^{\frac{x}{2}} + 7$
$\Rightarrow g(2) = 7,\ f(0) = 0$
$\Rightarrow g(2) - f(0) = 7$

Q9: (B) 2

$\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln\left|\sin x - 2\cos x\right| + k$

Differentiating both sides, we get

$\frac{5\tan x}{\tan x - 2} = 1 + \frac{a\left(\cos x + 2\sin x\right)}{\sin x - 2\cos x}$

$\Rightarrow \frac{5\sin x}{\sin x - 2\cos x} = \frac{\sin x\left(1+2a\right) + \cos x\left(a-2\right)}{\sin x - 2\cos x}$

$\Rightarrow a = 2$

Q10: 2

$y = f(x) = \frac{9x}{25} + c_1$
$\Rightarrow x = \frac{25}{9}(y - c_1) = f^{-1}(y)$
$\Rightarrow f^{-1}(x) = \frac{25}{9}(x - c_1)$
The curve $y = f^{-1}(x)$ passes through $\left(\frac{1}{4}, -\frac{5}{9}\right)$
$\Rightarrow -\frac{5}{9} = \frac{25}{9}\left(\frac{1}{4} - c_1\right)$
$\Rightarrow c_1 = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$
$\Rightarrow f^{-1}(x) = \frac{25}{9}\left(x - \frac{9}{20}\right) = \frac{25x}{9} - \frac{5}{4}$
$g(x) = \int f^{-1}(x)dx = \int\left(\frac{25x}{9} - \frac{5}{4}\right)dx$
$= \frac{25}{9}\left(\frac{x^2}{2}\right) - \frac{5x}{4} + c_2$
$g(0) = \frac{5}{2} \Rightarrow c_2 = \frac{5}{2}$
$\Rightarrow g(x) = \frac{25}{9}\left(\frac{x^2}{2}\right) - \frac{5x}{4} + \frac{5}{2}$
$\Rightarrow g(1) = \frac{25}{18} - \frac{5}{4} + \frac{5}{2} = \frac{50-45+90}{36} = \frac{95}{36}$
$\Rightarrow [g(1)] = 2$

Q11: (C) $\frac{1}{4}\left(ln\left(\frac{x-1}{x+1}\right)\right)^2 + C$

$I = \int \frac{ln\left(\frac{x-1}{x+1}\right)}{x^2-1}dx,$
Let $t = ln\left(\frac{x-1}{x+1}\right)$

$\Rightarrow \frac{dt}{dx} = \frac{x+1}{x-1}\left\{\frac{x+1-(x-1)}{(x+1)^2}\right\} = \frac{2}{\left(x^2-1\right)}$

$\Rightarrow \frac{dx}{x^2-1} = \frac{dt}{2}$

$\Rightarrow I = \frac{1}{2}\int tdt = \frac{1}{4}t^2 + C = \frac{1}{4}\left(ln\left(\frac{x-1}{x+1}\right)\right)^2 + C$

Q12: (A) $\frac{1}{n}\log_e\left(\frac{x^n}{x^n+1}\right) + c$

I= $\int \frac{dx}{\left(1+\frac{1}{x^n}\right) \times x^{n+1}}$

put $1+\frac{1}{x^n}=t$

$-\frac{n}{x^{n+1}} . dx=dt$

$I=\int \frac{-\frac{dt}{n}}{t}=-\frac{1}{n} \ln |t|+c$

$I=-\frac{1}{n} \ln \left(1+\frac{1}{x^n}\right)+c$

$I=\frac{1}{n} \ln \left(\frac{x^n}{x^n+1}\right)+c$

Q13: 5

$I=\int(\sin 100x \cdot \cos x+\cos 100x \cdot \sin x) \sin^{99} x \cdot dx$
$I=\int sin\, 100x \cdot \cos x . \sin^{99} x dx+\int \cos 100x \cdot \sin^{100} x \cdot dx\ III$
$I=\frac{\sin(100x) \sin^{100} x}{100}-\frac{100}{100} \int \cos(100x) \sin^{100} x dx+\int \cos(100x) . \sin^{100} x \cdot dx$
$I=\frac{\sin(100x) \cdot \sin^{100} x}{100}+C$

Q14: 8

$\mathrm{I}=\int \frac{\mathrm{dx}}{\left(\frac{\mathrm{x}-1}{\mathrm{x}+2}\right)^{\frac{3}{4}}(\mathrm{x}+2)^2}$ put $\frac{\mathrm{x}-1}{\mathrm{x}+2}=\mathrm{t}$

On solving, we get,

$\mathrm{I}=\frac{4}{3}\left(\frac{\mathrm{x}-1}{\mathrm{x}+2}\right)^{\frac{1}{4}}+\mathrm{c}$

$\mathrm{k}=\frac{4}{3} \Rightarrow 30\mathrm{k}=40=2^3 \times 5^1$

Hence, total divisors are $4 \times 2=8$

Q15: 2

$I=\int \frac{x^{-p-1}}{(1+x^{-p})^q} dx$

Let, $1+x^{-p}=t$

$=\frac{-1}{p} \int \frac{dt}{t^q}$

$=\frac{t^{-q+1}}{p(1-q)}+c$

$\therefore \frac{2}{\lambda}=1$

Q16: (C) $\frac{e^{3\sin^{-1} x}}{3}+x e^{\frac{3\pi}{2}}+c$

$\int \frac{e^{3\sin^{-1} x}}{\sqrt{1-x^2}} dx+\int e^{3\left(\sin^{-1} x+\cos^{-1} x\right)} dx$
$=\int e^{3t} dt+\int e^{\frac{3\pi}{2}} dx$ (Put $\sin^{-1} x=t$)
$=\frac{e^{3t}}{3}+e^{\frac{3\pi}{2}} \cdot x+c$

Q17: (D) $\frac{\sin^7 x}{7x^7}+c$

Put $\frac{\sin x}{x}=t.$
Then, $I=\int t^6 dt=\frac{t^7}{7}+c$

Q18: (A) $[-1,1]$

Let $x^{\sin x}=t$; then,

$I=\int dt=t+c=x^{\sin x}+c$

Hence, range of $\sin x\in[-1,1]$

Q19: (B) $\frac{4}{9}$
$\int\left(x-\sqrt{x^2-1}\right)^2 dx=\int x^2+\left(x^2-1\right)-2x\sqrt{x^2-1}dx$
$=\frac{2x^3}{3}-x+\frac{2}{3}\left(x^2-1\right)^{\frac{3}{2}}+C\Rightarrow a=b=\frac{2}{3}$

Q20: (B) $\frac{x^2}{2}-\frac{4}{3}x^{\frac{3}{2}}+x+c$
$\mathrm{I}=\int\frac{(\sqrt{x}-1)^4}{(\sqrt{x}-1)^2}dx=\int(\sqrt{x}-1)^2 dx=\int(x-2\sqrt{x}+1)dx$
$=\frac{x^2}{2}-\frac{4}{3}x^{\frac{3}{2}}+x+c$

Q21: 0.50

Given integral, $\mathrm{I}=\int\sin(\cos x).\sin x.\cos(\cos x)dx$
Let, $\sin(\cos x)=t\Rightarrow-\cos(\cos x).\sin x dx=dt$
$\therefore\mathrm{I}=\int-t dt=-\frac{t^2}{2}+C$
$=C-\frac{\sin^2(\cos x)}{2}$

Q22: (B) $\left(\sin^{-1}x\right)\left(\sin^2 x\right)+C$
$\int\left(\frac{\sin^2 x}{\sqrt{1-x^2}}+\frac{2\tan x}{(1+\tan^2 x)}\sin^{-1}x\right)dx$
$=\int\left(\frac{\sin^2 x}{\sqrt{1-x^2}}+(\sin 2x)\left(\sin^{-1}x\right)\right)dx$
$=\left(\sin^2 x\right)\left(\sin^{-1}x\right)+C$

Q23: (B) $f(x)$ **is an onto function**
$f(x)=\int\frac{(x^8+4+4x^4)-(4x^4)}{x^4-2x^2+2}dx$
$=\int\left(\frac{(x^4+2)^2-(2x^2)^2}{x^4-2x^2+2}\right)dx$
$=\int\frac{(x^4+2x^2+2)(x^4-2x^2+2)}{(x^4-2x^2+2)}dx$
$f(x)=\frac{x^5}{5}+\frac{2x^3}{3}+2x+C$
$f(0)=0+0+0+C=1\Rightarrow C=1$
$f(x)=\frac{x^5}{5}+\frac{2x^3}{3}+2x+1$
Range of $f(x)$ is R, so $f(x)$ is an onto function
$f(-x)\neq f(x)$, so not even
$f(-x)\neq-f(x)$, so not odd
$f^{`}(x)>0\forall x\in R$, so $f(x)$ is one-one

Q24: (B) $-\frac{2}{x}\sqrt{x-2}+C$

$\int \frac{x-4}{x^2\sqrt{x-2}}dx$
$\int \frac{\left(\frac{1}{x^2}-\frac{4}{x^3}\right)}{\sqrt{\frac{1}{x}-\frac{2}{x^2}}}dx$
Let $\frac{1}{x}-\frac{2}{x^2}=t$
$\left(-\frac{1}{x^2}+\frac{4}{x^3}\right)dx=dt$
$\int \frac{-dt}{\sqrt{t}}=-2\sqrt{t}+C$
$=-2\sqrt{\frac{1}{x}-\frac{2}{x^2}}+C$
$=-\frac{2}{x}\sqrt{x-2}+C$

Q25: (C) $2\sqrt{x+1}-3\sqrt{2}\ \tan^{-1}\sqrt{\frac{x+1}{2}}+c$

Substitute $\sqrt{x+1}=t$, we get,

$2\int \frac{t^2-1}{t^2+2}\ dt=2t-6\int \frac{1}{t^2+2}\,dt$

$=2t-3\sqrt{2}\tan^{-1}\frac{t}{\sqrt{2}}+c$

$=2\sqrt{x+1}-3\sqrt{2}\tan^{-1}\sqrt{\frac{x+1}{2}}+c$

Q26: (B) $e^{\frac{1}{\sqrt{2}}}$
$\int e^{\sin\theta}\sin\theta d\theta+\int e^{\sin\theta}\sec^2\theta d\theta$
$\int e^{\sin\theta}\sin\theta d\theta+\left(\tan\theta e^{\sin\theta}\right)-\int \tan\theta e^{\sin\theta}\cos\theta d\theta$
$=\int e^{\sin\theta}\sin\theta d\theta+(\tan\theta)e^{\sin\theta}-\int e^{\sin\theta}\sin\theta d\theta=e^{\sin\theta}(\tan\theta)+C$
$=e^{\sin\theta}\tan\theta+C$
$f(\theta)=e^{\sin\theta}\tan\theta$
$f\left(\frac{\pi}{4}\right)=e^{\frac{1}{\sqrt{2}}}$

Q27: 0

$f(x)=\int(1+2\tan x(\tan x+\sec x))^{\frac{1}{2}}dx$
$=\int\left(1+2\tan^2 x+2\tan x\sec x\right)^{\frac{1}{2}}dx$
$=\int\left(1+\tan^2 x+\tan^2 x+2\tan x\sec x\right)^{\frac{1}{2}}dx$
$=\int\left(\sec^2 x+\tan^2 x+2\tan x\sec x\right)^{\frac{1}{2}}dx$
$=\int\left((\sec x+\tan x)^2\right)^{\frac{1}{2}}dx$
$=\log(\sec x+\tan x)+\log\sec x+c$
$=\log\sec x(\sec x+\tan x)+c$
$f\left(\frac{\pi}{6}\right)=\log 2$
$\Rightarrow \log\frac{2}{\sqrt{3}}\left(\frac{2}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)+c=\log 2$
$\Rightarrow c=0$
$f(x)=\log(\sec x(\sec x+\tan x))$
$\Rightarrow f(0)=\log((1)(1+0))=0$

Q28: 1

Given integral $\mathrm{I}=\int\left(1\cdot\sin\left(x^2\right)+x\cdot 2x\cos\left(x^2\right)\right)dx$
$\left(\because \int\left(f'g+f\cdot g'\right)dx=\int(fg)'dx=f\cdot g+C\right)$
$=x\cdot\sin\left(x^2\right)+C$

$\therefore\ H(x)=\sin\left(x^2\right)$, whose range is $[-1,1]$
$\therefore\ a=-1,\ b=1 \Rightarrow a+2b=1$

Q29: 0.25

As we know, $\int e^x\left(f(x)+f'(x)\right)dx=e^x\cdot f(x)+C,$
Thus, $\int e^x\cdot x^2dx=\int e^x\left(x^2+2x-2x-2+2\right)dx$
$=\int\left[e^x\left(x^2-2x+2\right)+e^x\left(2x-2\right)\right]dx$
$=e^x\left(x^2-2x+2\right)+C$
i.e. $f(x)=x^2-2x+2=(x-1)^2+1$
Hence, range is $[1,\infty)\Rightarrow a=1$
$\therefore \frac{a}{4}=0.25$

Q30: (C) $2\tan^{-1}\left(e^{\sqrt{x}}\right)+C$

Let, $I=\int\frac{e^{\sqrt{x}}}{\sqrt{x}\left(1+e^{2\sqrt{x}}\right)}dx$
Substituting $e^{\sqrt{x}}=t$
$\frac{e^{\sqrt{x}}}{2\sqrt{x}}dx=dt$
$I=2\int\frac{dt}{1+t^2}$
$=2\left(\tan^{-1}t\right)+C$
$I=2\tan^{-1}\left(e^{\sqrt{x}}\right)+C$

Q31: (C) $\frac{1}{\ln 2}$

Given, $\mathrm{I}=\int 2^{2^x}.2^x\mathrm{dx}$
Let, $2^{2^x}=\mathrm{t}\Rightarrow 2^{2^x}.2^x.(\ln 2)^2\mathrm{dx}=\mathrm{dt}$
$\Rightarrow 2^{2^x}.2^x\mathrm{dx}=\frac{\mathrm{dt}}{(\ln 2)^2}$
Thus, $\mathrm{I}=\int\frac{\mathrm{dt}}{(\ln 2)^2}=\frac{\mathrm{t}}{(\ln 2)^2}+\mathrm{C}$
$=\frac{2^{2^x}}{(\ln 2)^2}+\mathrm{C}$
$\therefore\lambda=\frac{1}{(\ln 2)^2}\Rightarrow\sqrt{\lambda}=\frac{1}{\ln 2}$

Q32: (A) $\log\left|\frac{1+\tan x}{\tan x}\right|-\cot x+C$

Let, $I=\int\frac{\cos x}{\sin^2 x(\sin x+\cos x)}dx$

On dividing numerator and denominator by $\cos^3 x$, and substituting $\tan x=t,\ \sec^2 x dx=dt$, we get,

$I=\int\frac{dt}{t^2(1+t)}$

$=\int\left[-\frac{1}{t}+\frac{1}{t^2}+\frac{1}{1+t}\right]dt$ (using partial fraction)

$\Rightarrow I=-\int\frac{1}{t}dt+\int t^{-2}dt+\int\frac{1}{1+t}dt$

$=-\log|t|-\frac{1}{t}+\log|1+t|+C$

$=-\frac{1}{\tan x}+\log\left|\frac{1+\tan x}{\tan x}\right|+C$

$=-\cot x+\log\left|\frac{1+\tan x}{\tan x}\right|+C$

Q33: (C) $\frac{3}{4}$

As $x^4+x^2+1=\left(x^2+1\right)^2-x^2=\left(x^2+1+x\right)\left(x^2+1-x\right)$
Thus, $\int\frac{x^4+x^2+1}{x^2-x+1}dx=\int\left(x^2+x+1\right)dx$
i.e., $f'(x)=x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\geq\frac{3}{4}$

Q34: (A) $x(\ln x)^{10}+C$

$I_n=\int(\ln x)^n dx$
$I_n=x(\ln x)^n-\int\frac{xn(\ln x)^{n-1}}{x}dx$
$I_n=x(\ln x)^n-nI_{n-1}$
$I_n+nI_{n-1}=x(\ln x)^n+C$
$I_{10}+10I_9=x(\ln x)^{10}+C$

Q35: (B) $6\left(\frac{x^{\frac{4}{3}}}{8}-\frac{3}{11}x^{\frac{11}{6}}+\frac{3}{14}x^{\frac{7}{3}}-\frac{1}{17}x^{\frac{17}{6}}\right)+c$

Let, $I=\int x^{\frac{1}{3}}(1-\sqrt{x})^3dx$
Let, $x=t^6\ \Rightarrow dx=6t^5dt$
$I=\int t^2\left(1-t^3\right)^3 6t^5dt$
$I=6\int t^7\left(1-3t^3+3t^6-t^9\right)dt$
$=6\int\left(t^7-3t^{10}+3t^{13}-t^{16}\right)dt$
$I=6\left(\frac{t^8}{8}-\frac{3t^{11}}{11}+\frac{3t^{14}}{14}-\frac{t^{17}}{17}\right)+c$
$I=6\left(\frac{x^{\frac{4}{3}}}{8}-\frac{3}{11}x^{\frac{11}{6}}+\frac{3}{14}x^{\frac{7}{3}}-\frac{1}{17}x^{\frac{17}{6}}\right)+c$

Q36: (C) 1

As we know,

$\int e^x\left(f(x)+f'(x)\right)dx=e^x\cdot f(x)+C$
Thus, $\int e^x(\sin x+\cos x)dx=e^x\cdot\sin x+C$
i.e. $f(x)=\sin x$
Hence, $f\left(x^2\right)=\sin\left(x^2\right)$: which has the maximum value of '1'

Q37: (C) $(0,\infty)$

Given integral $=\int\frac{\tan^{-1}(e^x)}{1+e^{2x}}e^x dx$
Let, $\tan^{-1}\left(e^x\right)=t$
$\Rightarrow\frac{1}{1+e^{2x}}\cdot e^x dx=dt$
$\Rightarrow I=\int t dt=\frac{t^2}{2}+C$
$=\frac{\left[\tan^{-1}(e^x)\right]^2}{2}+C$
Hence, $f(x)=e^x\in(0,\infty)$

Q38: (B) $\frac{-\cot x}{(\cos x)^{2019}}+C$

$\int\operatorname{cosec}^2x(\cos x)^{-2019}dx-\int\frac{2019}{(\cos x)^{2019}}dx$
$=(-\cot x)(\cos x)^{-2019}+\int(\cot x)(2019)(\cos x)^{-2020}\sin x dx-\int(2019)(\cos x)^{-2019}dx$
$=\frac{-\cot x}{(\cos x)^{2019}}+C$

Q39: (B) 11

$A_{10}+A_{12}=\int \tan^{10} x d x+\int \tan^{12} x d x$

$=\int\left(\tan^{10} x+\tan^{12} x\right) d x$

$=\int \tan^{10} x\left(1+\tan^2 x\right) d x$

$=\int \tan^{10} x \cdot \sec^2 x d x$

Let, $\tan x=t \Rightarrow \sec^2 x d x=d t$

$\therefore A_{10}+A_{12}=\int \mathrm{t}^{10} d t=\frac{t^{11}}{11}+\lambda$

$=\frac{\tan^{11} x+\lambda}{11}$

Q40: (C) $x e^{x^2+\frac{1}{x}}+C$

$\mathrm{I}=\int \underbrace{e^{x^2+\frac{1}{x}}}_{\mathrm{I}} \cdot \underbrace{1}_{\mathrm{II}} d x+\int e^{\left(x^2+\frac{1}{x}\right)}\left(2 x-\frac{1}{x^2}\right) \cdot x d x$

Using integration by parts, we get,

$\mathrm{I}=e^{x^2+\frac{1}{x}} \cdot x-\int e^{\left(x^2+\frac{1}{x}\right)}\left(2 x-\frac{1}{x^2}\right) \cdot x d x+\int e^{\left(x^2+\frac{1}{x}\right)}\left(2 x-\frac{1}{x^2}\right) \cdot x d x$

$=x e^{x^2+\frac{1}{x}}+C$

Q41: (A) $(0, \infty)$

Le,t $e^x-1=t^2$

$\Rightarrow e^x \cdot d x=2 t d t$

$\therefore \mathrm{I}=\int \frac{2 t d t}{t\left(e^x\right)}$

$=\int \frac{2 d t}{t^2+1}$

$=2 \tan^{-1} t+C$

$=2 \tan^{-1}\left(\sqrt{e^x-1}\right)+C$

$\therefore f(x)=\sqrt{e^x-1}$

For $x \in(0, \infty) ; e^x \in(1, \infty)$

$\Rightarrow \sqrt{e^x-1} \in(0, \infty)$

Q42: (C) $\frac{89}{45}$

$f(n, x)=\frac{n \sin (n x)}{n}+C$

As $f(n, 0)=C=0$

$\Rightarrow f(n, x)=\sin (n x)$

Thus, $\sum_{x=1}^{89} f(1, x)=\sin 1+\sin 2+\ldots . .+\sin 89$

$=\frac{\sin \left(\frac{1+89}{2}\right) \sin \frac{89 \times 1}{2}}{\sin \left(\frac{1}{2}\right)}$

$=\frac{\sin (45) \sin \left(\frac{89}{2}\right)}{\sin \left(\frac{1}{2}\right)}$

Q43: 7

Let, $I=\int \frac{\cos^4 x d x}{\sin^6 x\left(1+\cot^5 x\right)^{\frac{3}{5}}}=\int \frac{\sec^2 x d x}{\tan^6 x\left(1+\left(\frac{1}{\tan x}\right)^5\right)^{\frac{3}{5}}}$

Substituting, $1+\frac{1}{(\tan x)^5}=t$ and $-\frac{5}{\left(\tan^6 x\right)}\left(\sec^2 x\right) d x=d t$

$I=-\frac{1}{5} \int \frac{d t}{t^{3 / 5}}=\left(\frac{-1}{5}\right) \frac{t^{\frac{2}{5}}}{\left(\frac{2}{5}\right)}+C$

$=-\frac{1}{2} \int\left(1+\cot^5 x\right)^{\frac{2}{5}}+C$

Hence, $K=2, P=5$

Q44: (C) $R-\{0,1\}$

$\mathrm{I}=\int \frac{dx}{x(x+1)}=\int \frac{dx}{x}-\int \frac{dx}{x+1}$
$\Rightarrow \mathrm{I}=\ln |x|-\ln |x+1|+C$
$=\ln \left|\frac{x}{x+1}\right|+C$
i.e. $y=f(x)=\frac{x}{x+1}$
$\Rightarrow yx+y=x$
$\Rightarrow x=\frac{y}{1-y}$
$\Rightarrow y \in R-\{0,1\}$ (as $x \neq 0$)

Q45: (D) $-\frac{1}{2}$

$\mathrm{I}=\int \frac{x^{-9}}{\left(1+x^{-8}\right)^{\frac{3}{4}}} dx$
Let, $1+x^{-8}=t$
$-8x^{-9} dx=dt$
$\mathrm{I}=\int \frac{-dt}{8t^{3/4}}$
$=-\frac{1}{8} \int t^{-3/4} dt$
$=-\frac{1}{8} \frac{t^{1/4}}{\frac{1}{4}}+c$
$=-\frac{\left(1+x^{-8}\right)^{1/4}}{2}+c$
$=\frac{-\left(1+x^{8}\right)^{1/4}}{2x^{2}}+c$
$\Rightarrow \lambda=-\frac{1}{2}$

Q46: (C) $\frac{\left(\tan^{-1}(\sin x+1)\right)^{2}}{2}+c$

Let, $\mathrm{I}=\int \frac{\tan^{-1}(\sin x+1) \cos x}{3+2 \sin x-\left(1-\sin^{2} x\right)} dx$

Substituting, $1+\sin x=t \Rightarrow \cos x dx=dt$

$\mathrm{I}=\int \frac{\tan^{-1}(t) dt}{2+(t-1)^{2}+2(t-1)}$

$\mathrm{I}=\int \frac{\tan^{-1}(t) dt}{1+t^{2}}$

$\mathrm{I}=\frac{\left(\tan^{-1}(t)\right)^{2}}{2}+c$

$\mathrm{I}=\frac{\left(\tan^{-1}(\sin x+1)\right)^{2}}{2}+c$