BSEH Practice Paper (March 2024)

CLASS: 12th (Senior Secondary) Code No. 835

Roll No. | | | | | | | | | | | SET: A

गणित

MATHEMATICS

[Hindi and English Medium]

ACADEMIC / OPEN

[Time allowed: 3 hours] [Maximum Marks: 80]

- *कृपया सुनिश्चित करें कि इस प्रश्न में मुद्रित पृष्ठ पेपर संख्या में 14 हैं और इसमें 38 प्रश्न हैं।*
  Please make sure that the printed pages in this question paper are 14 in number and it contains 38 questions.
- *प्रश्न पत्र के दायीं ओर दी गयी* ***कोड संख्या*** *को छात्र द्वारा उत्तरपुस्तिका- के पहले पृष्ठ पर लिखा जाना चाहिए।*
  The Code No. on the right side of the question paper should be written by the candidate on the front page of the answer-book.
- *किसी प्रश्न का उत्तर देना शुरू करने से पहले उसका क्रमांक लिखा जाना चाहिए।*
  Before beginning to answer a question, its Serial Number must be written.
- *अपनी उत्तर-पुस्तिका में खाली पृष्ठ/पृष्ठ न छोड़ें।*
  Don't *leave blank page/pages in your answer-book.*
- *उत्तर-पुस्तिका के अतिरिक्त कोई अतिरिक्त पत्रक नहीं दिया जायेगा। अत: आवयश्क्तानुसार ही लिखे और लिखे उत्तर को न काटे।*
  Except answer-book, no extra sheet will be given. Write to the point and do not strike the written answer.
- *परीक्षार्थी प्रश्नपत्र पर अपना रोल नंबर अवश्य लिखें।*
  Candidates must write their Roll Number on the question paper.
- *कृपया प्रश्नों का उत्तर देने से पहले, यह सुनिश्चित करें लें कि प्रश्न पत्र पूर्ण व सही हैं,* ***परीक्षा के उपरांत इस संबंध में कोई भी दावा स्वीकार नहीं किया जाएगा।***
  Before answering the questions, please ensure that you have been supplied the correct and complete question paper, ***no claim in this regard, will be entertained after examination.***

सामान्य निर्देश :

- इस प्रश्न- पत्र में कुल 38 प्रश्न हैं, जो कि पांच खंडों: अ, ब, स, द ल में बांटे गए हैं :

खंड अ : इस खंड में 1 से 20 तक कुल 20 प्रश्न हैं, प्रत्येक प्रश्न 1 अंक का है।

खंड ब : इस खंड में 21 से 25 तक कुल 05 प्रश्न हैं, प्रत्येक प्रश्न 2 अंक का है।

खंड स : इस खंड में 26 से 31 तक कुल 06 प्रश्न हैं, प्रत्येक प्रश्न 3 अंक का है।

खंड द : इस खंड में 32 से 35 तक कुल 04 प्रश्न हैं. प्रत्येक प्रश्न 5 अंक का है।

खंड ल : इस खंड में 36 से 38 तक कुल 03 केस आधारित प्रश्न हैं, प्रत्येक प्रश्न 4 अंक का है।

- सभी प्रश्न अनिवार्य हैं।
- कुछ प्रश्नों में आंतरिक चयन का विकल्प दिया गया है, उनमें से एक ही प्रश्न को चुनना है।
- कैलकुलेटर के प्रयोग की अनुमति नहीं है।

General Instructions:

- This question paper consists of 38 questions in total which are divided into five sections: A, B, C, D, E :
  Section A: This section consists of twenty questions from **1 to 20**. Each question carries **1 mark.**
  Section B: This section consists of five questions from **21 to 25**. Each question carries **2 marks.**
  Section C: This section consists of six questions from **26 to 31**. Each question carries **3 marks.**
  Section D: This section consists of four questions from **32 to 35**. Each question carries **5 marks.**
  Section E: This section consists of three case based questions from **36 to 38**. Each question carries **4 marks.**
- All questions are compulsory.
- There are some questions where **internal choice** has been provided. Choose only one of them.
- Use of calculator is **not** permitted.

खंड – अ

SECTION – A

इस खंड में प्रत्येक प्रशन 1 अंक का है।

*This section comprises questions of 1 mark each***.**

1. मान लीजिए कि समुच्चय **N** में R = {(a, b) : a = b - 2, b > 6} द्वारा दिया गया संबंध है। सही उत्तर का चयन करें।

   (A) $(2, 4) \in R$ (B) $(3, 8) \in R$

   (C) $(6, 8) \in R$ (D) $(8, 7) \in R$

   Let R be the relation in the set **N** given by R = {(a, b) : a = b -2, b > 6}. Choose the correct answer.

   (A) $(2, 4) \in R$ (B) $(3, 8) \in R$

   (C) $(6, 8) \in R$ (D) $(8, 7) \in R$

2. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ का मान:

   (A) $\frac{\pi}{3}$ है (B) $\frac{\pi}{6}$ है

   (C) 0 है (D) $2\sqrt{3}$ है

   $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is equal to:

   (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

   (C) 0 (D) $2\sqrt{3}$

3. यदि $A = \begin{bmatrix} \tan\theta & \cot\theta \\ -\cot\theta & \tan\theta \end{bmatrix}$, $0 < \theta < \frac{\pi}{2}$ तथा $A + A' = 2I$, तो θ का मान है:

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

   (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

If $A = \begin{bmatrix} \tan\theta & \cot\theta \\ -\cot\theta & \tan\theta \end{bmatrix}$, $0 < \theta < \frac{\pi}{2}$ and $A + A' = 2I$, then the value of θ is:

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

4. यदि एक आव्यूह सममित तथा विषम सममित दोनों ही है, तो:

(A). A एक विकर्ण आव्यूह है। (B) A एक शून्य आव्यूह है।

(C) A एक वर्ग आव्यूह है। (D) इनमें से कोई नहीं।

If a matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix (B) A is a zero matrix

(C) A is a square matrix (D) none of these

5. यदि एक त्रिभुज के शीर्ष (1, 0), (6, 0) और (4, 3) हैं, तो सारणिकों का प्रयोग द्वारा इस त्रिभूज का क्षेत्रफल है

(A) $\frac{37}{2}$ (B) $\frac{47}{2}$

(C) $\frac{15}{2}$ (D) इनमे से कोई नहीं

If the vertices of a triangle are (1, 0), (6, 0) and (4, 3), then by using determinants its area is

(A) $\frac{37}{2}$ (B) $\frac{47}{2}$

(C) $\frac{15}{2}$ (D) none of the above

6. यदि $y = x.\log x$, तो $\frac{d^2y}{dx^2}$ बराबर है:

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$

(C) $\frac{-1}{x^2}$ (D) $\frac{-1}{x}$

If y = x.logx , then $\frac{d^2y}{dx^2}$ is equal to:

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$

(C) $\frac{-1}{x^2}$ (D) $\frac{-1}{x}$

7. $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ का प्रतिअवकलज है:

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

The antiderivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals:

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

8. $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ बराबर है:

(A) $\frac{1}{x^2}e^x + C$ (B) $\frac{1}{x}e^x + C$

(C) $\frac{-1}{x}e^x + C$ (D) $\frac{-1}{x^2}e^{x^2} + C$

$\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ equals:

(A) $\frac{1}{x^2}e^x + C$ (B) $\frac{1}{x}e^x + C$

(C) $\frac{-1}{x}e^x + C$ (D) $\frac{-1}{x^2}e^{x^2} + C$

9. $\int_{-1}^{1} x^5\, dx$ का मान है:

(A) 1 (B) - 1

(C) 0 (D) 2

The value of $\int_{-1}^{1} x^5 \, dx$ is

(A) 1 (B) - 1

(C) 0 (D) 2

10. अवकल समीकरण $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ की कोटि है:

(A) 2 (B) 1

(C) 0 (D) परिभाषित नहीं

The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is :

(A) 2 (B) 1

(C) 0 (D) not defined

11. कोण सा प्रतिस्थापन $\frac{dx}{dy} = h(\frac{x}{y})$ के रूप वाले समघातीय अवकल समीकरण को हल कर सकता है ?

Which substitution can solve a homogeneous differential equation of the form $\frac{dx}{dy} = h(\frac{x}{y})$?

12. यदि फलन $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ बिंदु x = 0 पर संतत है, तो k का मान ज्ञात कीजिये ।

The function $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0, then find the value of k.

13. यदि एक रेखा के दिक्-अनुपात 2, -1, -2 है, तो इसकी दिक्-कोसाइन ज्ञात कीजिये।

If a line has the direction ratios 2, -1, -2, then what are its direction cosines?

14. P(A|B) ज्ञात कीजिये , यदि P(B) = 0.5 और P(A∩B) = 0.32

Compute P(A|B), if P(B) = 0.5 , P(A∩B) = 0.32 .

15. दो संरेख सदिशों का परिमाण सदैव समान होता है। (सत्य / असत्य)

Two collinear vectors are always equal in magnitude. (True / False)

16. दो घटनाओं A और B को परस्पर स्वतंत्र कहते हैं, यदि P(A'B') = [1-P(A)][1–P(B)] (सत्य / असत्य)

Two events will be independent, if P(A'B') = [1 – P(A)][1 – P(B)]. (True / False)

17. यदि पासों का एक जोड़ा उछाला जाता है तो प्रत्येक पासे पर सम अभाज्य संख्या प्राप्त करने की प्रायिकता _________है।

The probability of obtaining an even prime number on each die, when a pair of dice is rolled is.__________.

18. यदि $\vec{a} = 2\,\hat{\imath} + \hat{\jmath} + 3\hat{k}$ और $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$, तो $|\vec{a} \times \vec{b}|$ = ________.

If $\vec{a} = 2\,\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$, then $|\vec{a} \times \vec{b}|$ = ________.

प्रश्न संख्या 19 और 20 अभिकथन और तर्क आधारित प्रश्न हैं, जिनमें से प्रत्येक प्रश्न 1 अंक का है। दो कथन दिए गए हैं, एक को अभिकथन (A) और दूसरे को तर्क (R) अंकित किया गया है। इन प्रश्नो के सही उत्तर निचे दिए गए कोडो *(A), (B), (C)* और *(D)* में से चुनकर दीजिये।

(A) अभिकथन (A) और तर्क (R) दोनों सही है और तर्क (R), अभिकथन (A) की सही व्याख्या है।
(B) अभिकथन (A) और तर्क (R) दोनों सही है, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।
(C) अभिकथन (A) सही है तथा तर्क (R) गलत है।
(D) अभिकथन (A) गलत है तथा तर्क (R) सही है।

*Question number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labeled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A)
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

19. **अभिकथन (A):** यदि सम्मुच्य {1, 2, 3, 4, 5, 6} में R = { (a, b) : b = a +1 } द्वारा परिभाषित संबंध R है, तो R एक तुल्यता संबंध नहीं है।

तर्क (R) : एक संबंध को एक तुल्यता संबंध कहा जाता है यदि वह स्वतुल्य, सममित और संक्रामक हो।

Assertion (A): If R is the relation defined in set {1, 2, 3, 4, 5, 6} as R = { (a, b) : b = a +1 } then R is not an equivalence relation.

Reason (R): A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.

20. **अभिकथन (A):** रेखाएं $\vec{r} = \overrightarrow{a_1} + \lambda\, \overrightarrow{b_1}$ तथा $\vec{r} = \overrightarrow{a_2} + \mu\, \overrightarrow{b_2}$ परस्पर लंबवत है, जब $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = 0$ है।

तर्क (R) : रेखाओं $\vec{r} = \overrightarrow{a_1} + \lambda\, \overrightarrow{b_1}$ तथा $\vec{r} = \overrightarrow{a_2} + \mu\, \overrightarrow{b_2}$ के बीच का कोण $\cos\theta = \frac{\overrightarrow{b_1} . \overrightarrow{b_2}}{|\overrightarrow{b_1}| . |\overrightarrow{b_2}|}$ द्वारा प्रदत है।

Assertion (A): The lines are $\vec{r} = \overrightarrow{a_1} + \lambda\, \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\, \overrightarrow{b_2}$ are perpendicular, when $\overrightarrow{b_1} . \overrightarrow{b_2} = 0$.

Reason (R): The angle θ between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\, \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu \overrightarrow{b_2}$ is given by $\cos\theta = \frac{\overrightarrow{b_1} . \overrightarrow{b_2}}{|\overrightarrow{b_1}| . |\overrightarrow{b_2}|}$.

खंड– ब

SECTION – B

इस खंड में प्रत्येक प्रश्न 2 अंक का है।
This section comprises questions of 2 marks each.

21. मान लीजिये की L, किसी में स्थित समस्त रेखाओं का सम्मुचय है तथा $R = \{(L_1, L_2): L_1, L_2$ पर लंब है } समुच्चय L, में परिभाषित एक संबंध है। सिद्ध कीजिये की R, सममित है किन्तु यह न तो सवतुल्य है और न ही संक्रामक है।

Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2): L_1$ is perpendicular to $L_2\}$. Show that R is symmetric but neither reflexive nor transitive.

अथवा /OR

$\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$ का मान ज्ञात कीजिये।

Find the value of: $\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$

22. समीकरण $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ से a, b, c तथा d का मान ज्ञात कीजिये।

Find the value of a, b, c, and d from the equations:

$$\begin{bmatrix} a-b & 2a+d \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

23. k का मान ज्ञात कीजिये ताकि प्रदत फलन निर्दिष्ट बिंदु पर संतत हो

$$f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases} \qquad \text{at } x = 5.$$

Find the value of k so that the function is continuous at the indicated point

$$f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases} \qquad \text{at } x = 5.$$

24. सत्यापित कीजिए कि फलन y = x sin 3x, अवकल समीकरण $\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$ का हल है।

Verify that the function y = x sin 3x, is a solution of the differential equation
$\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$

अथवा /OR

अवकल समीकरण $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ का व्यापक हल ज्ञात कीजिये।

Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

25. दो गेंद एक बॉक्स से बिना प्रतिस्थापित किए निकाली जाती है। बॉक्स में 10 काली और 8 लाल गेदें हैं तो प्रायिकता ज्ञात कीजिए कि दोनों गेंदें लाल हों
Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that both balls are red.

खंड– स

SECTION – C

इस खंड में प्रत्येक प्रश्न 3 अंक का है।
This section comprises questions of 3 marks each.

26. मान लीजिये की A = **R** – {3} तथा **B =** R – {1} है। f(x) = $\left(\frac{x-2}{x-3}\right)$ द्वारा परिभाषित फलन f : A → B पर विचार कीजिये। क्या f एकैकी तथा आच्छादक है ? अपने उत्तर का औचित्य भी बतलाइय।

Let A = **R** – {3} and **B =** R – {1}. Consider the function f : A → B defined by f(x) = $\left(\frac{x-2}{x-3}\right)$.
Is f one one and onto? Justify your answer

अथवा /OR

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1-y^2}\right]$, |x| < 1, y > 0 तथा xy < 1

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1-y^2}\right]$, |x| < 1, y > 0 and xy < 1

27. X तथा Y ज्ञात कीजिये यदि 2X + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ और 3X + 2Y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Find X and Y, if $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

28. प्रदत फलन $y^x = x^y$ के लिए $\frac{dy}{dx}$ ज्ञात कीजिये।

Find $\frac{dy}{dx}$ of the function $y^x = x^y$

29. अंतराल ज्ञात कीजिये जिनमे $f(x) = 2x^3 - 3x^2 - 36x + 7$ द्वारा प्रदत्त फलन f वर्धमान या ह्रासमान है |
Find the intervals in which the function f is given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or strictly decreasing.

30. समाकलन कीजिये: $\int x^2 \log x \, dx$

Integrate: $\int x^2 \log x \, dx$

अथवा /OR

$\int_{-5}^{5} |x + 2| \, dx$ का मान ज्ञात कीजिये

Evaluate: $\int_{-5}^{5} |x + 2| \, dx$

31. एक समान्तर चतुर्भुज की संलग्न भुजाएं $2\hat{i} - 4\hat{j} + 5\hat{k}$ और $\hat{i} - 2\hat{j} - 3\hat{k}$ है। इसके विकर्ण के समान्तर एक मात्रक सदिश ज्ञात कीजिय।
The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal.

खंड– द

SECTION – D

इस खंड में प्रत्येक प्रश्न 5 अंक का है।
This section comprises questions of 5 marks each.

32. यदि $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ है तो A^{-1} ज्ञात कीजिये। A^{-1} का प्रयोग कर के निम्नलिखित समीकरणों को हल कीजिये।

$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

33. रेखाओं l_1 और l_2 के बीच की न्यूनतम दुरी ज्ञात कीजिये जिनके सदिश समीकरण है :

$$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$$

और $$\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$$

Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$$

and $$\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$$

अथवा /OR

बिंदु (1,2,−4) से जाने वाली और दोनो रेखाओं $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ और $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ पर लंब रेखा का सदिश समीकरण ज्ञात कीजिये|

Find the vector equation of the line passing through the point (1,2,−4) and perpendicular to the two lines :

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

34. वक्र $y^2 = x$, रेखाओं $x = 1$, $x = 4$ एवं x-अक्ष से घिरे क्षेत्र का प्रथम पाद में क्षेत्रफल ज्ञात कीजिए।
Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and x-axis in the first quadrant.

अथवा /OR

दीर्घवृत $\frac{x^2}{16} + \frac{y^2}{9} = 1$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

35. आलेखीये विधि से निम्न समस्या को हल कीजिये :
निम्न व्यवरोधो के अंतर्गत $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

$Z = 3x + 9y$ का न्यूनतम और अधिकतम मान ज्ञात कीजिये।

Solve the following problem graphically:
Minimise and Maximise $Z = 3x + 9y$
Subject to the constraints: $x + 3y \leq 60$
$x + y \geq 10$
$x \leq y$
$x \geq 0, y \geq 0$

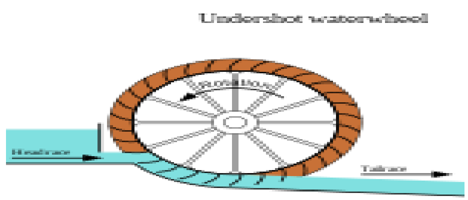
खंड– इ

SECTION – E

इस खंड में प्रत्येक प्रश्न 4 अंक का है।
This section comprises questions of 4 marks each.

Case Study – 1

36. एक नदी की ऊर्जा का अनुपात जो एक अंडरशॉट वॉटर व्हील से प्राप्त किया जा सकता है, वह है $E(x) = 2x^3 - 4x^2 + 2x$, इकाइयां जहां x नदी की गति के सापेक्ष वॉटर व्हील की गति है।
उपरोक्त जानकारी के आधार पर निम्नलिखित के उत्तर दीजिए:
(i) अंतराल [0, 1] में E(x) का अधिकतम मान ज्ञात कीजिए।
(ii) E(x) के अधिकतम मान के लिए वाटर व्हील की गति क्या है?
(iii)क्या आपका उत्तर मिल राइट के नियम से सहमत है कि पहिये की गति नदी की गति की लगभग एक तिहाई होनी चाहिए।



The proportion of a river's energy that can be obtained from an undershot water wheel is $E(x) = 2x^3 - 4x^2 + 2x$, units where x is the speed of the water wheel relative to the speed of the river.

Based on the above information answer the following:

(i) Find the maximum value of E(x) in the interval [0, 1].

(ii) What is the speed of water wheel for maximum value of E(x)?

(iii) Does your answer agree with Mill wrights rule that the speed of wheel should be about one-third of the speed of the river?

Case Study – 2

37. एक रैखिक अवकल समीकरण $\frac{dy}{dx}$ + Py = Q के रूप का हो, जहाँ P और Q, x के फलन हों, तो ऐसे समीकरण को रैखिक अवकल समीकरण कहते हैं। इसका समाधान y. (IF.) = $\int$ Q(I. F.) dx + c,

जहाँ I.F.(समाकलन गुणक) = $e^{\int Pdx}$

अब, मान लीजिए दिया गया समीकरण $xdy + ydx = x^3\,dx$ है

उपरोक्त जानकारी के आधार पर, निम्नलिखित प्रश्नों के उत्तर दें:

(i) क्रमशः P और Q के मान क्या हैं? (1)

(ii) I.F का मान क्या है? (1)

(iii) दिए गए समीकरण का हल ज्ञात कीजिए।. (2)

A linear differential equation is of the form $\frac{dy}{dx}$ + Py = Q, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by

y.(IF.) = $\int$ Q(IF.) dx +c, where I.F.(Integrating Factor) = $e^{\int Pdx}$

Now, suppose the given equation is $xdy + ydx = x^3\,dx$

Based on the above information, answer the following questions:

(i) What are the values of P and Q respectively? (1)

(ii) What is the value of I.F.? (1)

(iii) Find the Solution of given equation. (2)

Case Study – 3

38. रत्ना के पास दो डिब्बे I और II हैं। डिब्बे I में 3 लाल और 6 काली गेंदें हैं। बॉक्स II में 5 लाल और 5 काली गेंदें हैं। उसकी सहेली शिवानी यादृच्छया ढंग से दो बक्सों में से एक का चयन करती है और उसमें से एक गेंद निकालती है। शिवानी द्वारा खींची गई गेंद लाल पाई जाती है। माना E_1 , E_2 और A निम्नलिखित घटनाओं को दर्शाते हैं:

E_1 : बॉक्स I को शिवानी द्वारा चुना गया है

E_2 : बॉक्स II को शिवानी द्वारा चुना गया है

A : लाल गेंद शिवानी द्वारा खींची जाती है।

(i) $P(E_1)$ और $P(E_2)$ ज्ञात कीजिए। (1)

(ii) $P(A|E_1)$ और $P(A|E_2)$ ज्ञात कीजिए। (1)

(iii) $P(E | A)$ ज्ञात कीजिए। (2)

Ratna has two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 5 black balls. Her friend Shivani selects one of the two boxes randomly and draws a ball out of it. The ball drawn by Shivani is found to be red. Let E_1 , E_2 and A denote the following events:

E_1: Box I is selected by Shiavni

E_2: Box II is selected by Shiavni

A: Red ball is drawn by Shivani.

(i) Find $P(E_1)$ and $P(E_2)$ (1)

(ii) Find $P(A|E_1)$ and $P(A|E_2)$ (1)

(iii) Find $P(E_2 | A)$ (2)

BSEH Practice Paper (March 2024)

(2023-24)

Marking Scheme

Model Question Paper — SET-A

MATHEMATICS — **CODE: 835**

⇨ Important Instructions: ● All answers provided in the Marking scheme are SUGGESTIVE
● Examiners are requested to accept all possible alternative correct answer(s).

SECTION – A (1Mark × 20Q)

Q. No.	EXPECTED ANSWERS	Marks
Question 1.	Let R be the relation in the set **N** given by R = {(a, b) : a = b -2, b > 6}. Choose the correct answer.	
Solution:	**(C) (6, 8) ∈ R**	1
Question 2.	$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is equal to:	
Solution:	**(B)** $\boldsymbol{\frac{\pi}{6}}$	1
Question 3.	If A = $\begin{bmatrix} \tan\theta & \cot\theta \\ -\cot\theta & \tan\theta \end{bmatrix}$, $0 < \theta < \frac{\pi}{2}$ and A + A′ = 2I, then the value of θ is:	
Solution:	**(A)** $\boldsymbol{\frac{\pi}{4}}$	1
Question 4.	If a matrix A is both symmetric and skew symmetric, then	
Solution:	**(B) A is a zero matrix**	1
Question 5.	If the vertices of a triangle are (1, 0), (6, 0) and (4, 3), then by using determinants its area is	
Solution:	**(C)** $\boldsymbol{\frac{15}{2}}$	1
Question 6.	If y = x.logx , then $\frac{d^2y}{dx^2}$ is equal to:	
Solution:	**(A)** $\boldsymbol{\frac{1}{x}}$	1
Question 7.	The antiderivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals:	
Solution:	**(C)** $\boldsymbol{\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C}$	1
Question 8.	$\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ equals:	
Solution:	**(B)** $\boldsymbol{\frac{1}{x}e^x + C}$	1
Question 9.	The value of $\int_{-1}^{1} x^5\, dx$ is	
Solution:	(C) 0	1
Question 10.	The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is :	
Solution:	(A) 2	1
Question 11.	Which substitution can solve a homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$?	
Solution:	**Put x = vy**	1
Question 12.	The function $f(x) = \begin{cases} \sin x - \cos x & ,\text{if } x \neq 0 \\ k & ,\text{if } x = 0 \end{cases}$ is continuous at x = 0, then find the value of k.	
Solution:	$\lim_{x\to 0} f(x) = \lim_{x\to 0}(\sin x - \cos x)$ $= 0 - 1$ $= -1$ Since f(x) is continuous at x = 0 $\therefore \lim_{x\to 0} f(x) = f(0)$ ⇨ $\boldsymbol{-1 = k}$	1
Question 13.	If a line has the direction ratios 2, -1, -2, then what are its direction cosines?	
Solution:	$\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$	1

	⇨ $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$	
Question 14.	Compute P(A|B), if P(B) = 0.5 , P(A∩B) = 0.32 .	
Solution:	$P(A\|B) = \frac{P(A\cap B)}{P(B)}$ $= \frac{0.5}{0.32}$ **$P(A\|B) = \frac{25}{16}$**	1
Question 15.	Two collinear vectors are always equal in magnitude. (True / False)	
Solution:	**False**	1
Question 16.	Two events will be independent, if P(A'B') = [1 – P(A)][1 – P(B)]. (True / False)	
Solution:	**True**	1
Question 17.	The probability of obtaining an even prime number on each die, when a pair of dice is rolled is.___________.	
Solution:	**1/6**	1
Question 18.	If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$, then $\|\vec{a} \times \vec{b}\|$ = _________ .	
Solution:	$\sqrt{\mathbf{507}}$	
Question 19.	**Assertion (A):** If R is the relation defined in set {1, 2, 3, 4, 5, 6} as R = { (a, b) : b = a +1 } then R is not an equivalence relation. **Reason (R):** A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.	
Solution:	**(A).** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)	1
Question 20.	**Assertion (A):** The lines are $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are perpendicular, when $\vec{b_1}.\vec{b_2} = 0$. **Reason (R):** The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{\|\vec{b_1}\|.\|\vec{b_2}\|}$.	
Solution:	(A) . Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)	1
	SECTION – B (2Marks × 5Q)	
Question 21.	Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L_1, L_2): L_1 is perpendicular to L_2}. Show that R is symmetric but neither reflexive nor transitive.	
Solution:	R is not reflexive, as a line L_1 can't be perpendicular to itself, i.e., $(L_1, L_1) \notin R$. R is symmetric as $(L_1, L_2) \in R$ L_1 is perpendicular to L_2 ⇒ L_2 is perpendicular to L_1 ⇒ $(L_2, L_1) \in R$. $\forall L_1, L_2 \in L$ R is not transitive. Indeed, if L_1 is perpendicular to L_2 and L_2 is perpendicular to L_3, then L_1 can never be perpendicular to L_3. In fact, L_1 is parallel to L_3 i.e., $(L_1, L_2) \in R$, and $(L_2, L_3) \in R$ but $(L_1, L_3) \notin R$.	$\frac{1}{2}$ $\frac{1}{2}$ 1
OR Question 21.	Find the value of: $\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$	
Solution:	Let $\cos^{-1}(\frac{1}{2}) = x$. Then $\cos x = 1/2 = \cos(\pi/3)$ $\cos^{-1}(\frac{1}{2}) = \pi/3$ Let $\sin^{-1}(\frac{1}{2}) = y$. Then, $\sin y = 1/2 = \sin(\pi/6)$ $\sin^{-1}(\frac{1}{2}) = \pi/6$	$\frac{1}{2}$ $\frac{1}{2}$

	Now $\cos^{-1}(1/2) + 2\sin^{-1}(1/2) = \pi/3 + (2\pi)/6$ $= \pi/3 + \pi/3$ $= (2\pi)/3$	1
Question 22.	Find the value of a, b, c, and d from the equations: $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$	
Solution:	Equate the corresponding elements of the matrices: a - b = -1 ...(1) 2a + c = 5 ...(2) 2a - b = 0 ...(3) 3c + d = 13 ...(4) Equation (1) -Equation (3) - a = - 1 ⇒ a=1 Equation (1) ⇒ 1 - b = - 1 ⇒ b = 2 Equation (2) ⇒ 2(1) + c = 5 ⇒ c = 3 Equation (4) ⇒ 3(3) + d = 13 ⇒ d = 4 Therefore, a = 1, b = 2, c = 3 and d = 4	$\frac{1}{2}$ $\frac{1}{2}$ 1
Question 23.	Find the value of *k* so that the function is continuous at the indicated point $f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ at x = 5.	
Solution:	Given function is $f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ When x < 5, f(x) = kx + 1: A polynomial is continuous at each point x < 5 When x > 5, f(x)= 3x-5 : A polynomial is continuous at each point x > 5 Now f(5) = 5k + 1 $\lim_{x\to 5} f(x) = \lim_{h\to 0} f(5+h) = 3(5+h) - 5 = 15 + 3h - 5$ $= \lim_{h\to 0}(10 + 3h) = 10 + 3(0) = 10$ ….(1) $\lim_{x\to 5} f(x) = \lim_{h\to 0} f(5-h) = k(5-h) + 1$ $= \lim_{h\to 0} (5k - hk + 1) = 5k + 1$ ….(2) Since function is continuous, therefore, both the equations are equal, Equate both the equations and find the value of k, 10 = 5k + 1 5k = 9 k = 9/5	$\frac{1}{2}$ 1 $\frac{1}{2}$
Question 24.	Verify that the function y = x sin 3x, is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$	
Solution:	Given: y = x sin 3x Diff. w.r.t. ‘x’ , and we get	

	$\frac{dy}{dx} = \sin 3x + 3x \cos 3x$ …(1) Again differentiate (1) w.r.t. 'x', we get $\frac{d^2y}{dx^2} = 3 \cos 3x + 3 [\cos 3x + x (- \sin 3x). 3]$ On simplifying the above equation, we get $\frac{d^2y}{dx^2} = 6 \cos 3x - 9x \sin 3x$ …(2) Now, substitute (1) and (2) in the given differential equation, and we get the following: L.H.S = $\frac{d^2y}{dx^2} + 9y - 6 \cos 3x$ $= (6 \cos 3x - 9x \sin 3x) + 9(x \sin 3x) - 6 \cos 3x$ $= 6 \cos 3x - 9x \sin 3x + 9x \sin 3x - 6 \cos 3x$ = 0 = R.H.S As L.H.S = R.H.S, the given function is the solution of the corresponding differential equation.	$\frac{1}{2}$ $\frac{1}{2}$ 1
OR Question 24.	Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$	
Solution:	Since $1+ y^2 \neq 0$, therefore separating the variables, the given differential equation can be written as $\frac{dy}{1+y^2} = \frac{dy}{1+x^2}$ …..(1) Integrating both sides of equation (1), we get $\int\frac{dy}{1+y^2} = \int\frac{dy}{1+x^2}$ $\tan^{-1}y = \tan^{-1}x + C$ which is the general solution of equation (1)	$\frac{1}{2}$ $1\frac{1}{2}$
Question 25.	Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that both balls are red.	
Solution:	Total number of balls = 10 black balls + 8 red balls = 18 balls Probability of getting a red ball in the first draw = $\frac{8}{18} = \frac{4}{9}$ As the ball is replaced after the first throw, ∴ Probability of getting a red ball in the second draw = $\frac{8}{18} = \frac{4}{9}$ Since the two balls are drawn with replacement, the two draws are independent. P(both balls are red) = P(first ball is red) x P(second ball is red) Now, the probability of getting both balls red = $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$	$\frac{1}{2}$ $\frac{1}{2}$ 1

	SECTION – C (3Marks × 8Q)	
Question 26.	Let A = **R** – {3} and **B =** R – {1}. Consider the function f : A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one one and onto? Justify your answer.	
Solution:	A = R - {3} and B = R - {1} f : A → B defined by f(x) = (x - 2) / (x - 3) Let (x, y) ∈A then $f(x) = \frac{(x-2)}{(x-3)}$ and $f(y) = \frac{(y-2)}{(y-3)}$ For f(x) = f(y) $\frac{(x-2)}{(x-3)} = \frac{(y-2)}{(y-3)}$ (x - 2)(y - 3) = (y - 2)(x - 3) x y - 3x - 2y + 6 = xy - 3y - 2x + 6 - 3x - 2y = - 3y - 2x - 3x + 2x = - 3y + 2y - x = - y x = y Therefore, f is a one-one function. Again, $y = f(x) = \frac{(x-2)}{(x-3)}$ $y = \frac{(x-2)}{(x-3)}$ y(x – 3) = x - 2 xy - 3y = x - 2 x(y – 1) = 3y - 2 or $x = \frac{(3y-2)}{(y-1)}$ Now, $f\left(\frac{3y-2}{y-1}\right) =$ $\Rightarrow \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = y$ f(x) = y Therefore, f is onto function.	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
OR Question 26.	$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $\lvert x\rvert < 1$, $y > 0$ and $xy < 1$	
Solution:	Put x = tanθ and y = tanϕ, we have $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)\right]$ $= \tan\frac{1}{2}\left[\sin^{-1}\sin 2\theta + \cos^{-1}\cos 2\phi\right]$ $= \tan\frac{1}{2}\left[2\theta + 2\phi\right]$ $= \tan(\theta+\phi)$ $= \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$	$\frac{1}{2}$ $1\frac{1}{2}$ 1

	$= \frac{x+y}{1-xy}$	
Question 27.	Find X and Y, if 2X + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3X + 2Y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$	
Solution:	2X + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ...(1) 3X + 2Y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ ...(2) Multiply equation (1) by 2, 4X + 6Y = 2$\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ = $\begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$ ….(3) Multiply equation (2) by 3 9X + 6Y = 3$\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ = $\begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$ ….(4) Subtract equation (4) from (3) - 5X = $\begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$ - $\begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$ = $\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$ X= - 1/5$\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$ X = $\begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}$ Substitute this value of X in equation (1) 2$\begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}$ + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 4/5 & -24/5 \\ -22/5 & 6 \end{bmatrix}$ Y = 1/3 $\begin{bmatrix} 6/5 & 39/5 \\ 42/5 & -6 \end{bmatrix}$ Y = $\begin{bmatrix} 2/5 & -8/5 \\ 14/5 & -2 \end{bmatrix}$	1 1 1
Question 28.	Find $\frac{dy}{dx}$ of the function $y^x = x^y$	
Solution:	Given: $y^x = x^y$ $x^y = y^x$ Taking log on both sides $\log(x^y) = \log(y^x)$ y.log x = x.logy $\frac{d}{dx}$ (y.log x) = $\frac{d}{dx}$ (x.log y) $y.\frac{1}{x} + \log x.\frac{dy}{dx} = x.\frac{1}{y}.\frac{dy}{dx} + \log y.1$	**1**

	$(\log x - \frac{x}{y}).\frac{dy}{dx} = \log y - \frac{x}{y}$ $(\frac{(y\log(x) - x)}{y})\frac{dy}{dx} = \frac{(x\log(y) - y)}{x}$ $\frac{dy}{dx} = \frac{y(x\log(y) - y)}{x(y\log(x) - x)}$	$1\frac{1}{2}$ $\frac{1}{2}$
Question 29.	Find the intervals in which the function f is given by $f(x)= 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or strictly decreasing.	
Solution:	Given function: $f(x) = 2x^3 - 3x^2 - 36x + 7$ Diff. w.r.t. 'x' $f'(x) = 6x^2 - 6x + 36 = 6(x^2 - x - 6)$ $f'(x) = 6(x - 3)(x + 2)$,.......(1) Now for increasing or decreasing, $f'(x) = 0$ $6(x - 3)(x + 2) = 0$ $x - 3 = 0$ or $x + 2 = 0$ $x = 3$ or $x = -2$ Therefore, we have sub-intervals are $(-\infty,-2)$, $(-2, 3)$ and $(3, \infty)$ For interval $(-\infty,-2)$, picking $x = -3$, from equation (1), $f'(x) = (+ve)(-ve)(-ve) = (+ve) > 0$ Therefore, f is strictly increasing in $(-\infty,-2)$ For interval $(-2, 3)$, picking $x = 0$, from equation (1), $f'(x) = (+ve)(-ve)(+ve) = (-ve) < 0$ Therefore, f is strictly decreasing in $(-2, 3)$. For interval $(3, \infty)$, picking $x = 4$, from equation (1), $f'(x) = (+ve)(+ve)(+ve) = (+ve) > 0$ Therefore, is strictly decreasing in $(3, \infty)$. So, f is strictly increasing in $(-\infty, -2)$ and $(3, \infty)$. f is strictly decreasing in $(-2, 3)$.	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
Question 30.	Integrate: $\int x^2 \log x \, dx$	
Solution:	It is given that $I = \int x^2 .\log x \, dx$ Here by taking x as first function and x² as second function. Now integrating by parts we get $I = \log x \int x^2 \, dx - \int\{ \frac{d}{dx}(\log x) . \int x^2 \, dx\}.dx$ So we get $= \log(x).\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3} dx$ By multiplying the terms $= \frac{x^3.\log x}{3} - \int \frac{x^2}{3} dx$ It can be written as $= \frac{x^3.\log x}{3} - \frac{x^3}{9} + C$	$\frac{1}{2}$ 1 $1\frac{1}{2}$
OR Question 30.	Evaluate: $\int_{-5}^{5} \lvert x + 2 \rvert \, dx$	

Solution:	$I = \int_{-5}^{5} \|x+2\|\,dx$ We know $\|x+2\| = \begin{cases} -(x+2), & x \le -2 \\ (x+2), & x > -2 \end{cases}$ $I = \int_{-5}^{-2} \|x+2\|\,dx + \int_{-2}^{5} \|x+2\|\,dx$ $I = \int_{-5}^{-2} -(x+2)\,dx + \int_{-2}^{5} (x+2)\,dx$ $I = \left\| \frac{-(x+2)^2}{2} \right\|_{-5}^{-2} + \left\| \frac{(x+2)^2}{2} \right\|_{-2}^{5}$ $I = \left(\frac{-(0)^2}{2} - \frac{-(-3)^2}{2}\right) + \left(\frac{(7)^2}{2} - \frac{(0)^2}{2}\right)$ $I = \frac{9}{2} + \frac{49}{2}$ I = 29	$\frac{1}{2}$ $1\frac{1}{2}$ 1
Question 31.	The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to its diagonal.	
Solution:	Adjacent sides of a parallelogram are given as: $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ We know that, the diagonal of a parallelogram is given by $\vec{a} + \vec{b}$ $\vec{a} + \vec{b} = (2+1)\hat{\imath} + (-4-2)\hat{\jmath} + (5-3)\hat{k} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ $\|\vec{a} + \vec{b}\| = \sqrt{(3)^2 + (-6)^2 + (2)^2}$ Hence, the unit vector parallel to the diagonal is $\frac{\vec{a}+\vec{b}}{\|\vec{a}+\vec{b}\|} = \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{(3)^2 + (-6)^2 + (2)^2}}$ $= \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{9+36+4}}$ $= \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{7}$ $= \frac{3}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{2}{7}\hat{k}$	1 1 1
	SECTION – C (5Marks × 4Q)	
Question 32.	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations 2x – 3y + 5z = 11 3x + 2y – 4z = -5 x + y – 2z = -3	
Solution:	$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ |A| = 2(-4 + 4) + 3 (-6 + 4) + 5 (3 – 2) = 2(0) + 3(-2) + 5(1) = - 6 + 5 = -1 ≠ 0 ; Inverse of matrix exists.	1

	Find the inverse of matrix: Cofactors of matrix: $A_{11} = 0$, $A_{12} = 2$, $A_{13} = 1$ $A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$ $A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$ adj.A = $\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ So, $A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ Now, matrix of equations can be written as: AX=B $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ And, $X = A^{-1} B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ Therefore, x = 1, y = 2 and z =3.	$1\frac{1}{2}$ 1 $1\frac{1}{2}$
Question 33.	Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$	
Solution:	$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ ...(1) and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$ ...(2) Comparing (1) and (2) with $\vec{r} = \vec{a_1} + \lambda\, \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\, \vec{b_2}$ respectively, we get $\vec{a_1} = \hat{\imath} + \hat{\jmath}$, and $\vec{b_1} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ $\vec{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b_2} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$ Therefore $\vec{a_2} - \vec{a_1} = \hat{\imath} - \hat{k}$ and $\vec{b_1} \times \vec{b_2} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) \times (3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$	1 $\frac{1}{2}$

	$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{\imath} - \hat{\jmath} - 7\hat{k}$ $\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert = \sqrt{9 + 1 + 49} = \sqrt{59}$ Hence, the shortest distance between the given lines is given by $D = \dfrac{\lvert (\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) \rvert}{\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert} = \dfrac{\lvert 3 - 0 + 7 \rvert}{\sqrt{59}} = \dfrac{10}{\sqrt{59}}$	$1\frac{1}{2}$ 1 1
OR Question 33.	Find the vector equation of the line passing through the point (1,2,–4) and perpendicular to the two lines : $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$	
Solution:	The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda \vec{b}$ .It is given that, the line passes through (1, 2, -4) So, $\vec{a} = 1\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ Given lines are $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ It is also given that, line is perpendicular to both given lines. So we can say that the line is perpendicular to both parallel vectors of two given lines. We know that, $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$, so let $\vec{b}$ is cross product of parallel vectors of both lines i.e. $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$ where $\overrightarrow{b_1} = 3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}$ and $\overrightarrow{b_2} = 3\hat{\imath} - 8\hat{\jmath} - 5\hat{k}$ and Required Normal $\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ $= \hat{\imath}(80 - 56) - \hat{\jmath}(-15 - 21) + \hat{k}(24 + 48)$ $\vec{b} = 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$ Now, by substituting the value of $\vec{a}$ & $\vec{b}$ in the formula $\vec{r} = \vec{a} + \lambda \vec{b}$, we get $\vec{r} = (1\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(24\hat{\imath} + 36\hat{\jmath} + 72\hat{k})$	1 2 1 1
Question 34.	Find the area of the region bounded by the curve $y^2 = x$ and the lines x = 1, x = 4 and x-axis in the first quadrant.	
Solution:	Equation of the curve is $y^2 = x$. It is a rightward parabola having vertex at origin and symmetrical about x-axis. x = 1 and x = 4 are two straight lines parallel to y-axis. $y = \sqrt{x}$ ....(1) x = 1 and x = 4 Points of intersections of given curves At x = 1, $y = \sqrt{1} = \pm 1$ points are (1, 1) (1, -1) At x = 4, $y = \sqrt{4} = \pm 2$ points are (4, 2) (4, -2) ∴ points in first quadrant A(1, 1) B(4, 2) C(4, 0), D(1, 0) Make a rough hand sketch of given curves by taking some corresponding values	$1\frac{1}{2}$

of x and y.

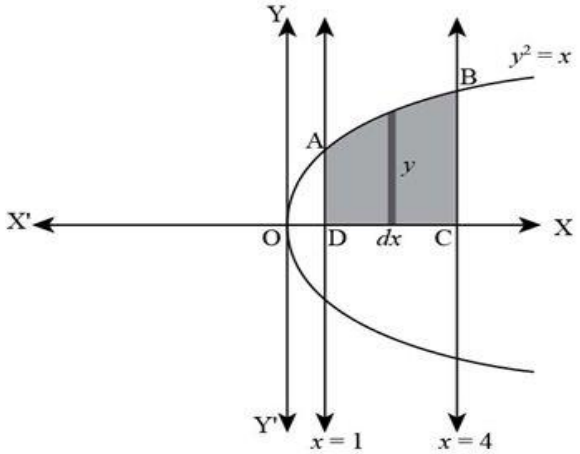


Figure (1)

Required area is shaded region ABCD: **1**

$\left|\int_1^4 y\,dx\right| = \left|\int_1^4 \sqrt{x}\,dx\right|$ [From equation (1)]

$= \left|\int_1^4 x^{1/2}\,dx\right|$

$\left|\frac{x^{3/2}}{3/2}\right|_1^4$ **$1\frac{1}{2}$**

$= \frac{2}{3}\left|(4^{3/2} - 1^{3/2})\right|$

$= \frac{2}{3}\left|(8 - 1)\right| = \frac{2}{3}(7) = \frac{14}{3}$ sq. units **1**

OR
Question 34. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Solution:

Here $\frac{x^2}{16} + \frac{y^2}{9} = 1$ …(1)

It is a horizontal ellipse having center at origin and is symmetrical about both axes (if we change y to -y or x to -x, equation remain same). **$\frac{1}{2}$**

Standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

By comparing, a = 4 and b = 3

From equation (1)

$\Rightarrow y^2 = \frac{9}{16}(16 - x^2)$

$\Rightarrow y = \frac{3}{4}\sqrt{16 - x^2}$ …..(2)

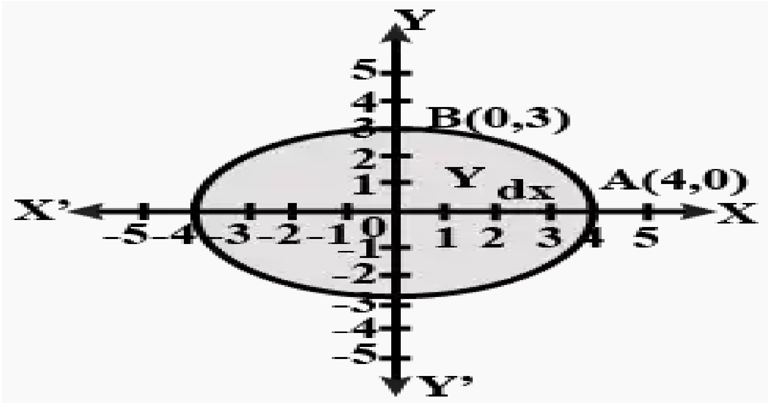
Points of Intersections of ellipse (1) with x-axis (y = 0)
Put y = 0 in equation (1), we have

$x^2/16 = 1$
$\Rightarrow x^2 = 16$
$\Rightarrow x = \pm 4$
Therefore, Intersections of ellipse(1) with x-axis are (0, 4) and (0, -4).

Now again, **1**
Points of Intersections of ellipse (1) with y-axis (x = 0)
Putting x = 0 in equation (1), $y^2/9 = 1$
$\Rightarrow y^2 = 9$
$\Rightarrow y = \pm 3$.

Therefore, Intersections of ellipse (1) with y-axis are (0, 3) and (0,-3).

for arc of ellipse in first quadrant.



1

Now,
Area of region bounded by ellipse (1)
Total shaded area = 4 x Area OAB of ellipse in first quadrant

$\frac{1}{2}$

$=4\left|\int_0^4 y.\,dx\right|$ [∵ at end B of arc AB of ellipse: x=0 and at end A of arc AB; x=4]

$=4\left|\int_0^4 \frac{3}{4}\sqrt{16-x^2}.\,dx\right| = 3\left|\int_0^4 \sqrt{4^2-x^2}.\,dx\right|$ 1

$=3\left|\frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right|_0^4$ $\left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$3[((4/2)\sqrt{16-16} + 8\sin^{-1}1) - (0+8\sin^{-1}0)] = 3[0+(8\pi/2)]$

$=3(4\pi) = 12\pi$ sq. units 1

Question 35.

Solve the following problem graphically:

Minimise and Maximise Z = 3x + 9y

Subject to the constraints: $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

Solution:

Z = 3x + 9y. (1)
$x + 3y \leq 60$ (2)
$x + y \geq 10$. (3)
$x \geq y$ (4)
$x \geq 0$, $y \geq 0$ (5)

First of all, let us graph the feasible region of the system of linear inequalities (2) to (5).
Let Z= 3x + 9y ....(1)
Converting inequalities to equalities

x + 3y = 60

X	0	60
Y	20	0

Points are (0, 20), (60,0)

Now put (0, 0) in inequation (2),

we find $0 \le 60$, which is true. $\frac{1}{2}$
Therefore area lies towards the origin from this line.

x + y=10

x	0	10
y	10	0

Points are (0, 10), (10, 0)

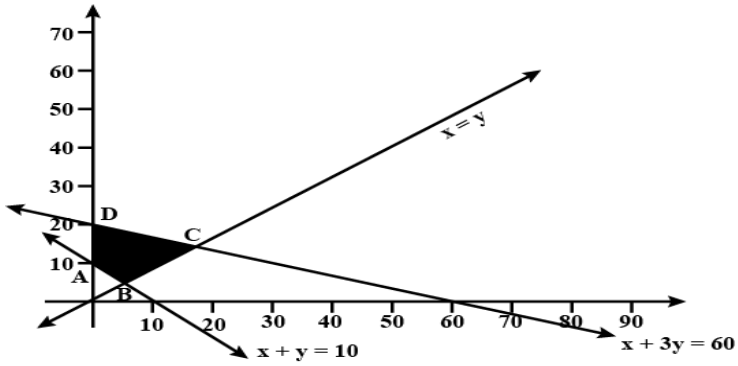
Now put (0, 0) in inequation (3),
we find $0 \ge 10$, which is False. $\frac{1}{2}$
Therefore area lies away from the origin from this line.

x – y = 0

X	0	10	20
y	0	10	20

Points are (0,0),(10,10),(20,20)
Now put (1, 0) in inequation (4),
we find $1 \ge 0$, which is false. $\frac{1}{2}$
Therefore area lies away from the (1, 0) from this line.

Plot the graph for the set of points



1

To find maximum and minimum
The feasible region ABCD is shown in the figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (0, 10), (5, 5), (15, 15) and (0, 20) respectively. $\frac{1}{2}$

Corner Point	Corresponding Value of Z = 3 x +9 y
A (0, 10)	90
B (5, 5)	**60←Minimum**
C (15, 15)	**180←Maximum**
D (0, 20)	**180 (**Multiple optimal solutions)

$1\frac{1}{2}$

We now find the minimum and maximum value of Z.
From the table, we find that the minimum value of Z is 60 at the point B (5, 5) of the feasible region.
The maximum value of Z on the feasible region occurs at the two corner points C (15, 15) and D (0, 20) and it is 180 in each case.

$\frac{1}{2}$

	SECTION – E (4Marks × 3Q)	
Question 36.	The proportion of a river's energy that can be obtained from an undershot water wheel is $E(x) = 2x^3 - 4x^2 + 2x$, units where x is the speed of the water wheel relative to the speed of the river. *Based on the above information answer the following :* (i) Find the maximum value of E(x) in the interval [0, 1]. (2) (ii) What is the speed of water wheel for maximum value of E(x)? (1) (iii) Does your answer agree with Mill wrights rule that the speed of wheel should be about one-third of the speed of the river? (1)	
Solution:	(i) We have, $E(x) = 2x^3 - 4x^2 + 2x$ …(1) Differentiating equation (1) w.r.t. x $E'(x) = 6x^2 - 8x + 2$ …(2) For maximum or minimum value of E(x), E'(x) = 0 we have $6x^2 - 8x + 2= 0$ $3x^2 - 4x + 1 = 0$ $(3x -1) (x – 1) = 0$ i.e. x = 1/3 , x = 1 Differentiating equation (2) w.r.t. x $E''(x) = 12x – 8$ Now , At x= 1 $E''(x) = 12(1) – 8 = 4 = +ve$ At x = 1/3 $E''(x) = 12(1/3) – 8 = -4 = -ve$ ⇒ E(x) has maximum value at x = 1/3 Maximum value $= E(1/3) = 2(1/3)^3 - 4(1/3)^2 + 2(1/3)$ $= 2/27 - 4/9 + 2/3 = 8/27$	1 1
	(ii) Speed for the Maximum value of E(x) is $\frac{1}{3}$ units.	1
	(iii) Yes	1
Question 37.	A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y.(IF.) = \int Q(IF.)\, dx + c$, where I.F.(Integrating Factor) $= e^{\int Pdx}$ Now, suppose the given equation is $xdy + ydx = x^3\, dx$ *Based on the above information, answer the following questions:* (i)What are the values of P and Q respectively? (1) (ii) What is the value of I.F.? (1) (iii) Find the Solution of given equation. (2)	

Solution:	(i) Given equation is $x.dy + y.dx = x^3\,dx$ Dividing on both side by dx, we have $$x\frac{dy}{dx} + y = x^3$$ $$\frac{dy}{dx} + \frac{1}{x}y = x^2$$ $\Rightarrow\ P = \frac{1}{x}\ ,\ Q = x^2$	1
	(ii) I.F.(Integrating Factor) = $e^{\int P dx}$ $= e^{\int \frac{1}{x} dx}$ $= e^{\log x}$ $= x$	1
	(iii) Solution of given equation is $y.(IF.) = \int Q(IF.)\,dx + c$ $y(x) = \int x^2(x)\,dx + c$ $xy = \int x^3 dx + c$ $xy = \frac{x^4}{4} + c$	2
Question 38.	Ratna has two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 5 black balls. Her friend Shivani selects one of the two boxes randomly and draws a ball out of it. The ball drawn by Shivani is found to be red. Let E_1 , E_2 and A denote the following events: E_1 : Box I is selected by Shiavni. E_2 : Box II is selected by Shiavni. A : Red ball is drawn by Shivani. (a) Find $P(E_1)$ and $P(E_2)$ (1) (b) Find $P(A\|E_1)$ and $P(A\| E_2)$ (1) (c) Find $P(E_2 \| A)$ (2)	
Solution:	(a) $P(E_1)$: Probability of selecting Box I by Shiavni = $\frac{1}{2}$ $P(E_1)$: Probability of selecting Box I by Shiavni = $\frac{1}{2}$	1
	(b) $P(A\|E_1)$ = Probability of selecting a red ball when box I has been already selected = $\frac{3}{9}$ $P(A\|E_2)$ = Probability of selecting a red ball when box II has been already selected = $\frac{5}{10}$	1
	(c) $P(E_2 \| A)$ = Probability that a red ball is drawn from the box II By Bayes' Theorem $P(E_2 \mid A) = \frac{P(E_2).P(A\mid E_2)}{P(E_1).P(A\mid E_1) + P(E_2).P(A\mid E_2)}$	

	$P(E_2 \mid A) = \frac{\frac{1}{2}.\frac{5}{10}}{\frac{1}{2}.\frac{3}{9}+\frac{1}{2}.\frac{5}{10}}$ $P(E_2 \mid A) = \frac{\frac{1}{4}}{\frac{1}{6}+\frac{1}{4}}$ $P(E_2 \mid A) = \frac{\frac{1}{4}}{\frac{4+6}{24}} = \frac{1}{4} \times \frac{24}{10} = \frac{3}{5}$	2