WAVES ON STRING

KEY CONCEPTS

INTRODUCTION OF WAVES

What is wave motion ?

- When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place)
- There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

  Ex. You (Kota) want to communicate your friend (Delhi)

  Write a letter | Use telephone

  1st option involves the concept of particle & the second choice involves the concept of wave.

  Ex. When you say "Namaste" to your friend no material particle is ejected from your lips to fall on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transferred to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

  Note :- In the above example air itself does not move.

A **wave** is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

  Few examples of waves :

  The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES

Wave classification According to

Necessity of medium	Propagation of energy	Dimension	Vibration of particle
(i) Elastic or mechanical wave (ii) Electro magnetic wave (E.M. wave)	(i) Progressive (ii) Sationary	(i) One dimensional (ii) Two dimensional (iii) Three dimentional	(i) Transverse (ii) Longitudinal

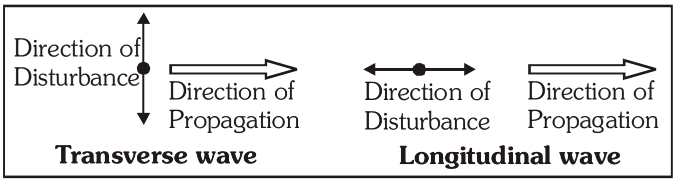
1. ***Based on medium necessity*** :- A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**.

   Example : Sound waves in water, seismic waves in earth's crust.

2. **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with fixed velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

3. ***Based on direction of propagation*** :- Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.

4. ***Based on the motion of particles of medium*** :



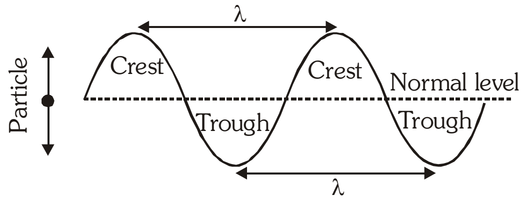
Waves are of two types on the basis of motion of particles of the medium.
(i) Longitudinal waves
(ii) Transverse waves

In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

TRANSVERSE WAVE MOTION

Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.



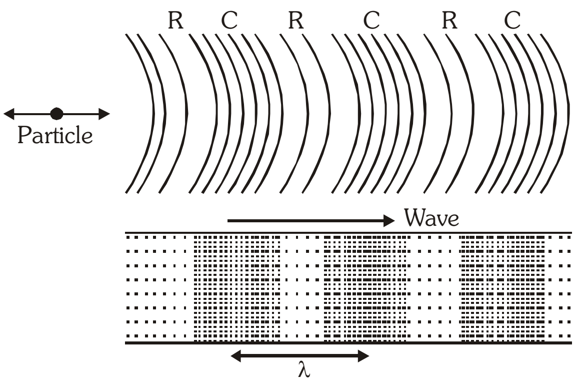
⇒ Mechanical transverse waves are generated only in solids & surface of liquid.

In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.

A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION



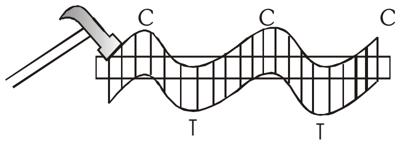
In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

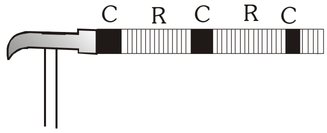
- The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation.
- In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



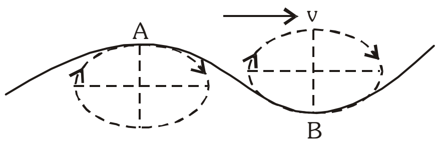
Transverse-wave
(A)



Longitudinal-wave
(B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds [$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane [Fig.]



Ripple

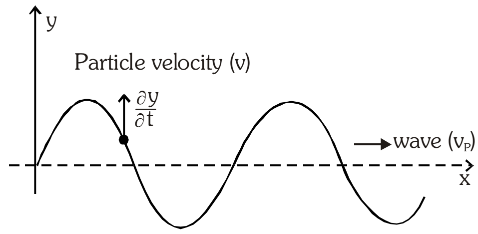
CHARACTERISTICS OF WAVE MOTION

Some of the important characteristics of wave motion are as follows :

- In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

SOME IMPORTANT TERMS CONNECTED WITH WAVE MOTION

- **Wavelength (λ) [length of one wave]**
  Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.
- **Frequency (n)** :Number of vibrations (Number of complete wavelengths) complete by a particle in one second.
- **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$
- **Phase :** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation y = A sin (ωt – kx); (ωt – kx) = phase.
- **Angular wave number (k) :** It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number ($\vec{\nu}$) :** It is defined as $\vec{\nu} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration :** In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae what we have read in SHM apply to the particles here also. For example, maximum particle velocity is $\pm A\omega$ at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between + Aω and – Aω) the wave velocity is constant for given characteristics of the medium.
- **Particle velocity in wave motion :**
  The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



- **Wave velocity :** The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.
- **Relation between particle velocity and wave velocity :**

Wave equation :- y = A sin (ωt - kx), Particle velocity $v = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx)$.

Wave velocity = $v_P = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}$, $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k\cos(\omega k - kx) = -\frac{1}{v_p}\frac{\partial y}{\partial t}$

$$\Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_P}\frac{\partial y}{\partial t}$$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

- **Differential equation of harmonic progressive waves :**

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v_P^2}\frac{\partial^2 y}{\partial t^2}$$

- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave :**

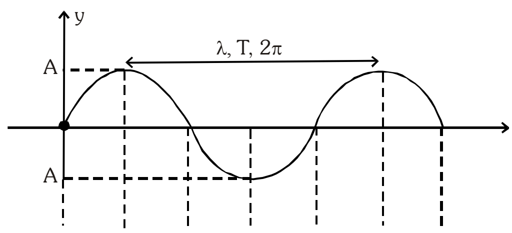
The acceleration of the particle is the second particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P = \frac{\partial^2 y(x,t)}{\partial t^2} = \omega^2 A \sin(kx - \omega t) = -\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, a_p = –w2 (displacement)

- **Relation between Phase difference, Path difference & Time difference**

Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3}{2}\lambda$
Time-period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$



$$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \Rightarrow \text{Path difference} = \left(\frac{\lambda}{2\pi}\right) \text{Phase difference}$$

Ex. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Sol. We know that for a wave v = f λ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60° = (\pi/180) \times 60 = (\pi/3)$ rad,

so path difference $\Delta x = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

THE GENERAL EQUATION OF WAVE MOTION

Some physical quantity (say y) is made to oscillate at one place and these oscillations of y propagate to other places. The y may be,

(i) displacement of particles from their mean position in case of transverse wave in a rope or longitudinal sound wave in a gas.

(ii) pressure difference (dP) or density difference ($d\rho$) in case of sound wave or

(iii) electric and magnetic fields in case of electromagnetic waves.

The oscillations of y may or may not be simple harmonic in nature. Consider one-dimensional wave travelling along x-axis. In this case y is a function of x and t. i.e. y = f(x, t) But only those function of x & t, represent a wave motion which satisfy the differential equation. $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ ...(i)

The general solution of this equation is of the form y (x, t) = f (ax ± bt) ...(ii)

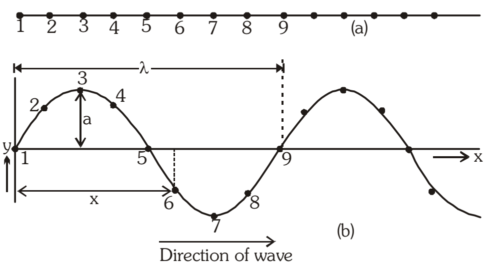
Thus, any function of x and t and which satisfies equation (i) or which can be written as equation (ii) represents a wave. The only condition is that it should be finite everywhere and at all times, Further, if these conditions are satisfied, then speed of wave (v) is given by $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{b}{a}$

Ex. Which of the following functions represent a travelling wave ?

(a) $(x - vt)^2$ (b) $\ell n(x + vt)$ (c) $e^{-(x-vt)^2}$ (d) $\frac{1}{x + vt}$

Sol. Although all the four functions are written in the form f (ax $\pm$ bt), only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Equation of a Plane Progressive Wave



If, on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'. Suppose, a simple harmonic progressive wave is propagating in a medium along the positive direction of the x-axis (from left to right). In fig. (a) are shown the equilibrium positions of the particles 1, 2, 3 ....

When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) are shown the instantaneous positions of these particles at a particular instant. The curve joining these positions represents the wave. Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then y = a sin ωt...(i)

where a is the amplitude of oscillation and ω = 2π n, where n is the frequency. As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time t – (x/v). The displacement of particle 1 at time t – (x/v) can be the particle 6, distant x from the origin (particle 1), at time t is given by

$y = a \sin \omega \left(t - \frac{x}{v}\right)$ But $\omega = 2\pi n$, $y = a \sin (\omega t - kx)$ $\left(k = \frac{\omega}{v}\right)$ ...(ii)

$y = a \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right]$ Also $k = \frac{2\pi}{\lambda}$ ...(iii) $y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$ ...(iv)

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equations, instead of minus sign there will be plus sign. For example, equation (iv) will be of the following form : $y = a \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$. If ϕ be the phase difference between the above wave travelling along the +x direction and an other wave, then the equation of that wave will be

$$y = a\sin\left\{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\pm\phi\right\}$$

Ex. The equation of a wave is, $y(x,t) = 0.05\sin\left[\frac{\pi}{2}(10x-40t)-\frac{\pi}{4}\right]m$

Find: (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Sol. : (a) The equation may be rewritten as, $y(x,t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right)m$

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A\sin(kx-\omega t+\phi)$ we have, wave number $k = \frac{2\pi}{\lambda} = 5\pi\,rad/m \quad \therefore \lambda = 0.4m$

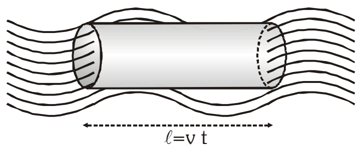
The angular frequency is, $\omega = 2\pi f = 20\pi\,rad/s \quad \therefore f = 10Hz$

The wave velocity is, $v = f\lambda = \frac{\omega}{k} = 4ms^{-1}$ in + x direction

(b) The particle velocity and acceleration are, $v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right)$ =2.22m/s

$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right)$ =140 m/s^2

INTENSITY OF WAVE



The amount of energy flowing per unit area and per unit time is called the intensity of wave. It is represented by I. Its units are J/m^{2}s or watt/metre^2. $I = 2\pi^2 f^2 A^2 \rho v$ i.e. $I \propto A^2$ and $I \propto A^2$. If P is the power of an isotropic point source, then intensity at a distance r is given by,

$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2} \text{ (for a point source)}$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r\ell} \text{ or } I \propto \frac{1}{r} \text{ (for a line source) As, } I \propto A^2$$

Therefore, $A \propto \frac{1}{r}$ (for a point source) and $A \propto \frac{1}{\sqrt{r}}$ (for a line source)

SUPERPOSITION PRINCIPLE

Two or more waves can propagate in the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle.

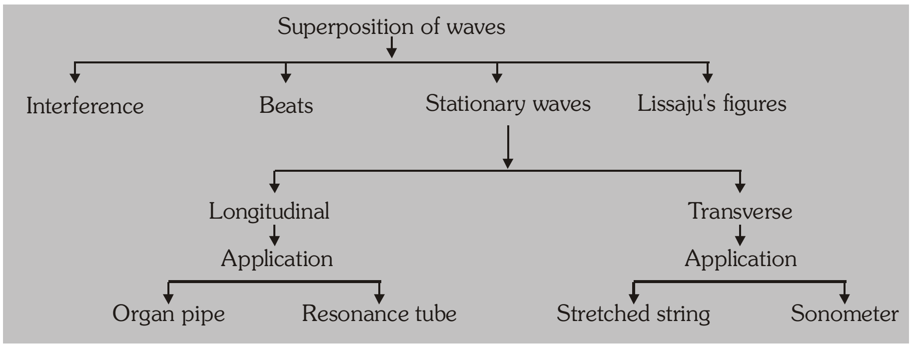
> The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \ldots$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

- **Interference :** Superposition of two waves having equal frequency and nearly equal amplitude.
- **Beats :** Superposition of two waves of nearly equal frequency in same direction.
- **Stationary waves :** Superposition of equal wave from opposite direction.
- **Lissajous' figure :** Superposition of perpendicular waves.



INTERFERENCE OF WAVES :

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves $y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition $y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

$$\text{where} \quad A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \ \text{and} \ \tan\delta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

As intensity $I \propto A^2$ so $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$

- **Constructive interference (maximum intensity) :**

  Phase difference $\phi = 2n\pi$ or path difference $= n\lambda$ where n = 0, 1, 2, 3, ...

  $$\Rightarrow A_{max} = A_1 + A_2 \quad \text{and} \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}$$

- **Destructive interference (minimum intensity) :**

  Phase difference $\phi = (2n+1)\pi$, or path difference $= (2n-1)\dfrac{\lambda}{2}$ where n = 0, 1, 2, 3, ...

  $\Rightarrow A_{min} = A_1 - A_2$ and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

KEY POINTS

- Maximum and minimum intensities in any interference wave form. $\dfrac{I_{Max}}{I_{Min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{a_1+a_2}{a_1-a_2}\right)^2$

- Average intensity of interference wave form :– $< I >$or $I_{av} = \dfrac{I_{max}+I_{min}}{2} = I_1 + I_2$

  if $a = a_1 = a_2$ and $I_1 = I_2 = I$ then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$
- Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or

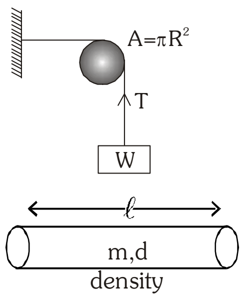
  Degree of visibility (Light Wave) $f = \dfrac{I_{max}-I_{min}}{I_{max}+I_{min}} \times 100$

  In condition of perfect interference degree of interference pattern is maximum $f_{max} = 1$ or 100%
- Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{max} = 4I$ and $I_{min} = 0$

  For perfect destructive interference we have a maximum contrast in interference wave form.

VELOCITY OF TRANSVERSE WAVE

Mass of per unit length $m = \dfrac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$, where d = Density of matter



Velocity of transverse wave in any wire $v = \sqrt{\dfrac{T}{m}}$ or $\sqrt{\dfrac{T}{\pi r^2 d}} = \sqrt{\dfrac{T}{Ad}}$ $\because \pi r^2 = A$

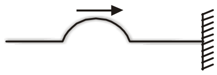
- If m is constant then, $v \propto \sqrt{T}$ it is called tension law.
- If tension is constant then $v \propto \sqrt{\dfrac{1}{m}}$ $\leftarrow$ it is called law of mass
- If T is constant & take wire of different radius for same material then $v \propto \dfrac{1}{r}$ $\leftarrow$ it is called law of radius
- If T is constant & take wire of same radius for different material. Then $v \propto \sqrt{\dfrac{1}{d}}$ $\leftarrow$ law of density

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall, the element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transmits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down, so the resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.

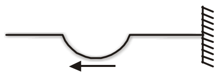
Equation of wave propagating in +ve x-axis

Incident wave $y_1 = a \sin(\omega t - kx)$



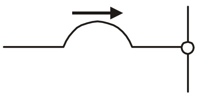
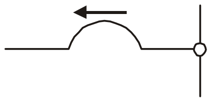
Reflected wave $y_2 = a \sin(\omega t + kx + \pi)$

or $y_2 = -a \sin(\omega t + kx)$



REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up. However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulse i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one. The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin(\omega t - kx)$  Reflected wave $y_2 = a \sin(\omega t + kx)$ 

STATIONARY WAVES

* **Definition :** The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave. Formation of stationary wave is possible only and only in bounded medium.

ANALYTICAL METHOD FOR STATIONARY WAVES

- **From rigid end** : We know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from rigid end $y_2 = a \sin(\omega t + kx + \pi) = -a \sin(\omega t + kx)$
  By principle of super position. $y = y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + kx) = -2a \sin kx \cos \omega t$
  This is equation of stationary wave reflected from rigid end

Amplitude $= 2a \sin kx$ **Velocity of particle** $v_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t$

Strain $\frac{dy}{dx} = -2ak \cos kx \cos \omega t$ **Elasticity** $E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$ **Change in pressure** $dp = E\frac{dy}{dx}$

- **Node** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A = 0,\ V_{pa} = 0$, strain $\rightarrow$ max **Change in pressure** $\rightarrow$ max

- **Antinode** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ .... $A \rightarrow$ max, $-V_{pa} \rightarrow$ max. strain $= 0$ **Change in pressure** $= 0$

- **From free end** : we know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from free end $y_2 = a \sin(\omega t + kx)$

By Principle of superposition $y = y_1 + y_2 = a \sin(\omega t - kx) + a \sin(\omega t + kx) = 2\ a \sin \omega t \cos kx$

Amplitude = $2a \cos kx$, **Velocity of particle** = $v_{Pa} = \frac{dy}{dt} = 2a\,\omega \cos \omega t \cos kx$

Strain $\frac{dy}{dx} = -2ak \sin \omega t \sin kx$ **Change in pressure** $dp = E\,\frac{dy}{dx}$

- **Antinode :** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A \rightarrow$ Max, $V_{pa} = \frac{dy}{dt} \rightarrow$ max. Strain = 0, dp = 0
- **Node :** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... $A = 0$, $V_{pa} = \frac{dy}{dt} = 0$, strain $\rightarrow$ max, $dp \rightarrow$ max

PROPERTIES OF STATIONARY WAVES

The stationary waves are formed due to the superposition of two identical simple harmonic waves travelling in opposite direction with the same speed.

Important characteristics of stationary waves are:–

(i) Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.

(ii) In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.

(iii) All the particles except at the nodes vibrate simple harmonically with the same period.

(iv) The distance between any two successive nodes or antinodes is $\lambda/2$.

(v) The amplitude of vibration gradually increases from zero to maximum value from node to antinode.

(vi) All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180°

(vii) All points of the medium pass through their mean position simultaneously twice in each period.

(viii) Velocity of the particles while crossing mean position varies from maximum at antinodes to zero at nodes.

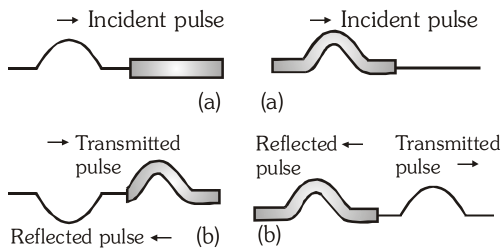
(ix) In a stationary wave the medium is splited into segments and each segment is vibrating up and down as a whole.

(x) In longitudinal stationary waves, condensation (compression) and refraction do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(xi) These waves do not transfer energy in the medium. Transmission of energy is not possible in a stationary wave.

TRANSMISSION OF WAVES

We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, same part of it is reflected and inverted and same part of it is transmitted to the heavier string.



As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected. When a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string. In the previous section, we found that the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

KEY POINTS

Phenomenon of reflection and transmission of waves obeys the laws of reflection and refraction.

The frequency of these wave remains constant i.e. does not change. $\omega_i = \omega_r = \omega_t = \omega$

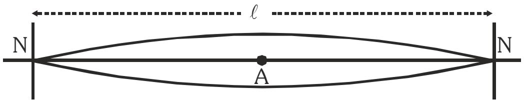
From rarer to denser medium $y_i = a_i \sin(\omega t - k_1x)$ $\quad y_r = -a_i \sin(\omega t + k_1x)$ $\quad y_t = a_t \sin(\omega t - k_2x)$

From denser to rarer medium $y_i = a_i \sin(\omega t - k_1x)$ $\quad y_r = a_i \sin(\omega t + k_1x)$ $\quad y_t = a_t \sin(\omega t - k_2x)$

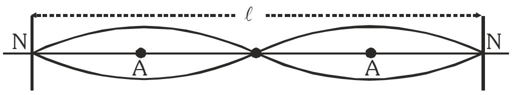
STATIONARY WAVE ARE OF TWO TYPES :

(i) Transverse st. wave (stretched string) **(ii)** Longitudinal st. wave (organ pipes)

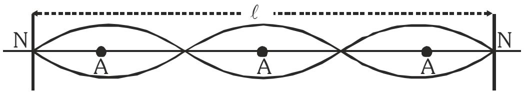
(i) Transverse Stationary wave

First Harmonic 

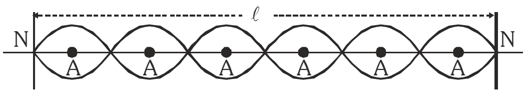
$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$

Second Harmonic 

$\frac{2\lambda}{2} = \ell \Rightarrow \lambda = \ell$

Third Harmonic 

$\frac{3\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{3}$

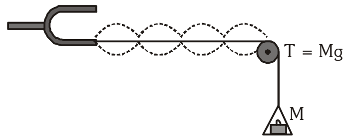
p th harmonic 

$\frac{p\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{p}$

- **Law of length :** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e, $n \propto \frac{1}{\ell}$ (T and m are constant)
- **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (ℓ and m are constant)

- **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{m}}$ (ℓ and T are constant)
- **Melde's experiment :** In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded. There are two arrangements to vibrate the tied fork with thread.

Transverse arrangement :

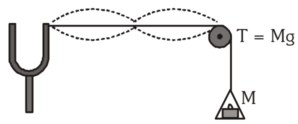


Case 1. In a vibrating string of fixed length, the product of number of loops and square root of tension are constant or $p\sqrt{T}$ = constant.

Case 2. When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3. If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread only makes node at the midpoint when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal arrangement :

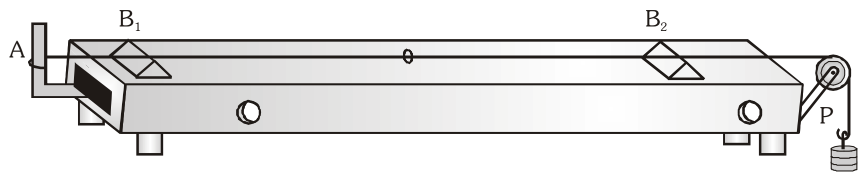


The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork.

Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

SONOMETER :

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ℓ, then the frequency of vibration is $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire. There are three laws of vibration of a wire.

COMPARISON OF PROGRESSIVE AND STATIONARY WAVES

	Progressive waves	Stationary waves
1.	These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3.	The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180°
4.	No particle of medium is Permanently at rest.	Particles at nodes are permanently at rest.
5.	All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6.	All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement

WORKED OUT EXAMPLES

Ex.1 A wave is propagating along x-axis. The displacement of particles of the medium in z-direction at t = 0 is given by: $z = \exp[-(x+2)^2]$, where 'x' is in meters. At t = 1s, the same wave disturbance is given by: $z = \exp[-(2-x)^2]$. Then, the wave propagation velocity is

(A) 4 m/s in + x direction (B) 4 m/s in –x direction

(C) 2 m/s in + x direction (D) 2 m/s in – x direction

Ans. (D)

Ex.2 A transverse wave is propagating along +x direction. At t = 2 sec, the particle at x = 4m is at y = 2 mm.With the passage of time its y coordinate increases and reaches to a maximum of 4 mm. The wave equation may be (using ω and k with their usual meanings)

(A) $y = 4\sin(\omega(t+2)+k(x-2)+\frac{\pi}{6})$ (B) $y = 4\sin(\omega(t+2)+k(x)+\frac{\pi}{6})$

(C) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{5\pi}{6})$ (D) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{\pi}{6})$

Ans. (D)

Ex.3 $Y(x,t) = 0.05 / [(4x + 2t)^2 + 5]$ represents a moving wave pulse, where x and y are in meters and t is in seconds. Then which statement(s) are **CORRECT**:

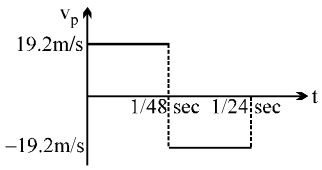
(A) Pulse is moving in – x direction (B) Wave speed is 0.5 m/s

(C) Maximum particle displacement is 1 cm (D) It is a symmetric pulse

Ans. **(A,B,C,D)**

Ex.4 A symmetrical triangular pulse of maximum height 0.4 m and total length 1 m is moving in the positive x-direction on a string on which the wave speed is 24 m/s. At t = 0 the pulse is entirely located between x = 0 and x = 1 m. Draw a graph of the transverse velocity of particle of string versus time at x = +1m.

Ans.



Ex.5 A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[IIT-JEE 2005]**

Ans. $y = (10\text{ cm}) \sin\left(30t \pm \frac{3}{2}x + f\right)$

Ex.6 A non-uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$ where x is the distance from one end of the wire and k is a constant.

(a) Show that $M = kL^2/2$

(b) Show that the time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$ where F (constant)is the tension in the wire.

Ex.7 One end of a long string of linear mass density 10^{-2} kg m^{-1} is connected to an electrically driven tuning fork of frequency 150 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves from this end have negligible amplitude. At t = 0, the left end (fork end) of the string is at x = 0 has a transverse displacement of 2.5 cm and is moving along positive y-direction. The amplitude of the wave is 5 cm. Write down the transverse displacement y (in cm) as function of x (in m) and t (in sec) that describes the wave on the string.

Ans. $y = 5\sin\left\{\pi(300t - x) + \frac{\pi}{6}\right\}$

Ex.8 If the tension in a stretched string fixed at both ends is changed by 20%, the fundamental frequency is found to increase by 15Hz. then the

(A) original frequency is 157 Hz

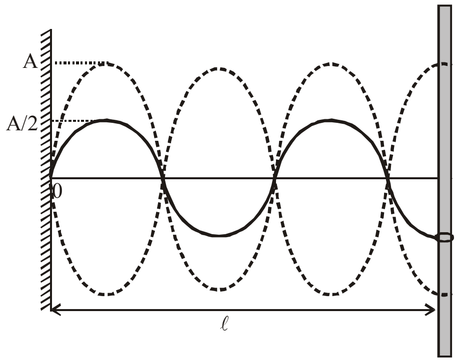
(B) velocity of propagation of the transverse wave along the string changes by 5%

(C) velocity of propagation of the transverse wave along the string changes by 10%.

(D) fundamental wave length on the string does not change.

Ans. **(A,C,D)**

Ex.9 Here given snap shot at $t = \frac{T}{12}$ of a standing wave. Then the equations of the wave will be when particles are moving towards their extreme and when particles are moving towards the mean position respectively $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$.



(A) $y = A \sin kx \sin \omega t$, $y = A \sin\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(B) $y = A \sin kx \cos \omega t$, $y = A \sin\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(C) $y = A \cos kx \cos \omega t$, $y = A \cos\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(D) $y = A \cos kx \sin \omega t$, $y = A \cos\left(\omega t + \frac{2\pi}{3}\right) \cos kx$

Ans. **(A)**

Ex.10 A standing wave of time period T is set up in a string clamped between two rigid supports. At t = 0 antinode is at its maximum displacement 2A.

(A) The energy density of a node is equal to energy density of an antinode for the first time at t = T/4.

(B) The energy density of node and antinode becomes equal after T/2 second.

(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

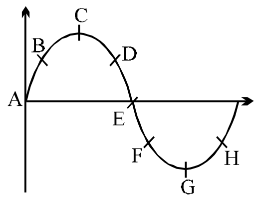
(D) The displacement of the particle at node is zero

Ans. **(C,D)**

EXERCISE (S-1)

Wave equation

1. A transverse wave is travelling along a string from left to right. The fig. represents the shape of the string (snap-shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity.



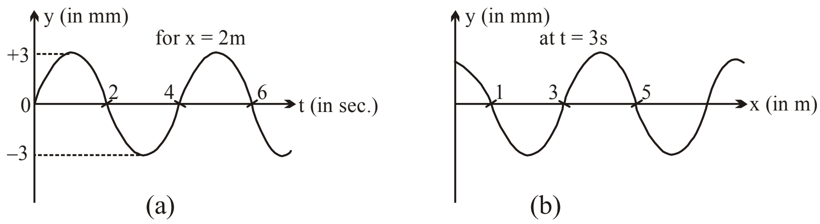
WA0001

2. A sinusoidal wave propagates along a string. In figure (a) and (b) 'y' represents displacement of particle from the mean position. 'x' & 't' have usual meanings. Find :

(a) wavelength, frequency and speed of the wave.

(b) maximum velocity and maximum acceleration of the particles

(c) the magnitude of slope of the string for x = 2 m at t = 4 sec.



WA0002

3. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and plate is allowed to fall freely. 8 complete oscillations are counted when the plate falls through 10 cm. What is the frequency of the tuning fork ?

WA0003

4. A long uniform string of mass density 0.1 kg/m is stretched with a force of 40 N. One end of the string (x = 0) is oscillated transversely (sinusoidally) with an amplitude of 0.02 m and a period of 0.1 sec, so that travelling waves in the +x direction are set up.

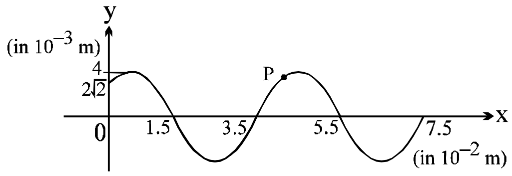
(a) What is the velocity of the waves?

(b) What is their wavelength?

(c) If at the driving end (x = 0) the displacement (y) at t = 0 is 0.01 m with dy/dt negative, what is the equation of the travelling waves?

WA0004

5. The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with x-axis at P is 6°.
(a) Find the direction in which the wave is moving
(b) the equation of the wave
(c) the total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm/m.



WA0005

Velocity of wave

6. The extension in a string, obeying Hooke's law is x. The speed of wave in the stretched string is v. If the extension in the string is increased to 1.5 x find the new speed of wave.

WA0006

7. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. Ignore gravity. Find the time required for a transverse wave to travel from one end of the rope to the other.

WA0007

8. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible. Find the speed of transverse waves in this wire at 10°C.
Given : Young modulus of copper = 1.3×10^{11} N/m^2.
Coefficient of linear expansion of copper = 1.7×10^{-5} °C^{-1}.
Density of copper = 9×10^3 kg/m^3. **[IIT-1979]**

WA0008

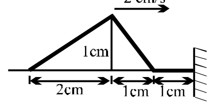
Energy of wave

9. A steel wire has a mass of 5g/m and is under tension 450 N. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength.

WA0009

Superposition of waves

10. The figure shown a triangular pulse on a rope at t = 0. It is approaching a fixed end at 2 cm/s
(a) Draw the pulse at t = 2 sec.
(b) The particle speed on the leading edge at the instant depicted is _____.



WA0010

11. A 40 cm long wire having a mass 3.2 gm and area of cross-section 1 mm^2 is stretched between the supports 40.05 cm apart. In its fundamental mode it vibrate with a frequency 1000/64 Hz. Find the young's modulus of the wire.

WA0011

12. A plane wave given by equation y = 0.04 sin (0.5πx – 100πt), where x and y are in meter and t in sec is incident normally on a boundary between two media beyond which wave speed becomes doubled. State boundary condition and find the equation of the reflected and transmitted waves. Take x = 0 as the boundary between two media.

WA0012

13. A string between x = 0 and x = l vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[IIT-JEE 2003(Scr)]**

WA0013

EXERCISE (S-2)

1. A long string under tension of 100 N has one end at x = 0. A sinusoidal wave is generated at x = 0 whose equation is given by $y = (0.01\text{ cm})\sin\left[\left(\frac{\pi x}{10}\text{ m}\right) - 50\pi t\,(\text{sec})\right]$

(i) Sketch the shape of the string at $t = \frac{1}{50}$ sec.

(ii) Find the average power transmitted by the wave.

(iii) Draw velocity time graph of particle at x = 5 m.

WA0014

2. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement?

WA0015

3. A uniform string of length L and total mass M is suspended vertically and a transverse pulse is given at the top end of it. At the same moment a body is released from rest and falls freely from the top of the string. How far from the bottom does the body pass the pulse.

WA0016

4. A sinusoidal wave is moving along the positive x-direction as shown in figure (i) and (ii).

(i) Write the complete expression for the wave y (x, t)

(ii) Find the possible values of x_0 for which figure (ii) refers.

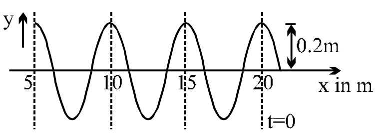


fig (i)

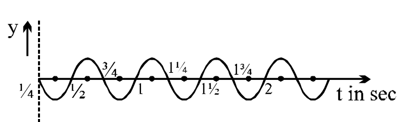


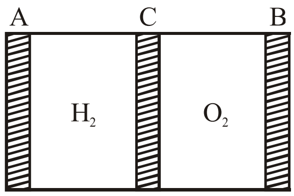
fig (ii)

WA0017

5. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6m apart along the line of propagation is ...., and the corresponding amplitude difference is ...... m. **[IIT-1997C]**

WA0018

6. AB is a cylinder of length 1m fitted a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibration for which diaphragm C is a not? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **[IIT-1978]**



WA0019

7. A steel wire 8×10^{-4} m in diameter is fixed to a support at one end and is wrapped round a cylindrical tuning peg 5 mm in diameter at the other end. The length of the wire between the peg and the support is 0.06 m. The wire is initially kept taut but without any tension. What will be the fundamental frequency of vibration of the wire if it is tightened by giving the peg a quarter of a turn?
Density of steel = 7800 kg/m^3,Y of steel = 20×10^{10} N/m^2.

WA0020

8. A steel of wire of length 25 cm is fixed at its ends to rigid walls. Young's modulus of steel = 200 GPa, coefficient of linear thermal expansion = 10^{-5} /°C. Initially, the wire is just taut at 20°C temperature. The density of steel = 8.0 g/cc. A tuning fork of frequency 200 Hz is touched to the wire, to execute oscillations. Simultaneously, the temperature is slowly lowered. At what temperature will resonance occur corresponding to the third overtone?

WA0021

9. A non-uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$ where x is the distance from one end of the wire and k is a constant.

(a) Show that $M = kL^2/2$

(b) Show that the time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$ where F (constant) is the tension in the wire.

WA0022

EXERCISE (O-1)

Wave equation

1. The function of x and t that does not represent a progressive wave is :-

(A) $y = 2\sin(4t - 3x)$ (B) $y = e^{(4+(4t-3x))}$ (C) $y = [4t - 3x]^{-1}$ (D) $y = [4t-3x]$

WA0023

2. At x = 0 particle oscillate by law $y = \frac{3}{2t^2+1}$. If wave is propagating along –ve x axis with velocity 2m/s. Find equation of wave

(A) $y = \frac{3}{2\left(t-\frac{x}{2}\right)^2+1}$ (B) $y = \frac{3}{2\left(t+\frac{x}{2}\right)^2+1}$ (C) $y = \frac{3}{2\left(t-\frac{z}{2}\right)^2+1}$ (D) $y = \frac{3}{2\left(t+\frac{z}{2}\right)^2+1}$

WA0024

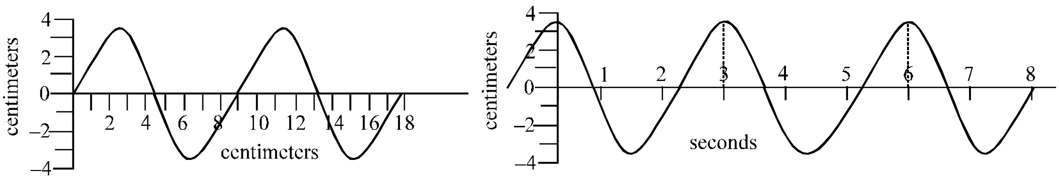
3. The shape of a wave propagating in the positive x or negative x- direction is given $y = \frac{1}{\sqrt{1+x^2}}$ at t = 0 and $y = \frac{1}{\sqrt{2-2x+x^2}}$ at t = 1s where x and y are in meters. The shape the wave disturbance does not change during propagation. Find the velocity of the wave.

(A) 1 m/s in positive x direction (B) 1 m/s in negative x direction

(C) $\frac{1}{2}$ m/s in positive x direction (D) $\frac{1}{2}$ m/s in negative x direction

WA0025

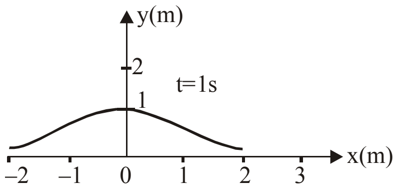
4. A transverse wave is travelling along a horizontal string. The first picture shows the shape of the string at an instant of time. This picture is superimposed on a coordinate system to help you make any necessary measurements. The second picture is a graph of the vertical displacement of *one* point along the string as a function of time. How far does this wave travel along the string in one second?

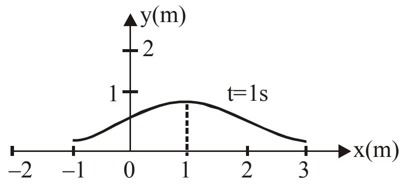


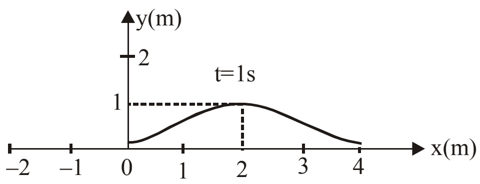
(A) 0.3 cm (B) 3.0 cm (C) 9.0 cm (D) 27 cm

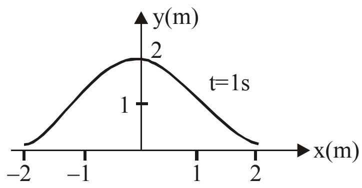
WA0026

5. A wave pulse is given by the equation $y = f(x, t) = A \exp(-B(x-vt)^2)$. Given $A = 1.0$m, $B = 1.0\ m^{-2}$ and $v = +2.0$ m/s, which of the following graph shows the correct wave profile at the instant $t = 1$ s?

(A) 

(B) 

(C) 

(D) 

WA0027

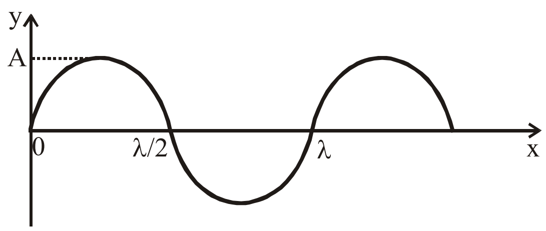
6. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment $t = T/6$ (T is the time period). Assume that the source was at mean position at $t = 0$. The wavelength of the running wave is

(A) 0.96 m (B) 0.48 m (C) 0.24 m (D) 0.12 m

WA0028

7. Here given snap shot of a progressive wave at $t = 0$ with time period = T. Then the equation of the wave if wave is going in +ve x-direction and if wave is going in –ve x-direction will be respectively.

$\left(\text{Here, } T = \dfrac{2\pi}{\omega}\right)$



(A) $y = A \sin(kx + \omega t)$, $y = A \sin(kx - \omega t)$ (B) $y = A \cos(kx + \omega t)$, $y = A \cos(kx - \omega t)$

(C) $y = A \sin(\omega t - kx)$, $y = A \sin(\omega t + kx)$ (D) $y = A \sin(kx - \omega t)$, $y = A \sin(kx + \omega t)$

WA0029

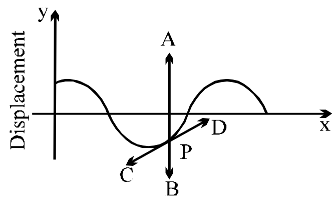
8. A sinusoidal wave travelling in the positive direction of x on a stretched string has amplitude 2.0 cm, wavelength 1 m and wave velocity 5.0 m/s. At $x = 0$ and $t = 0$, it is given that displacement $y = 0$ and $\dfrac{\partial y}{\partial t} < 0$. Express the wave function correctly in the form $y = f(x, t)$:-

(A) $y = (0.02\ \text{m}) \sin 2\pi(x-5t)$ (B) $y = (0.02\ \text{cm}) \cos 2\pi(x-5t)$

(C) $y = (0.02\ \text{m}) \sin 2\pi\left(x - 5t + \dfrac{1}{4}\right)$ (D) $y = (0.02\ \text{cm}) \cos 2\pi\left(x - 5t + \dfrac{1}{4}\right)$

WA0030

9. The figure below shows a snap photograph of a simple harmonic progressive wave, progressing in the negative X-axis, at a given instant. The direction of the velocity of the particle at the stage P on the figure is best represented by the arrow.



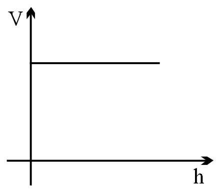
(A) $\vec{PA}$ (B) $\vec{PB}$

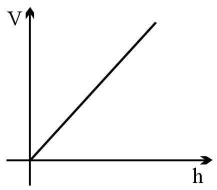
(C) $\vec{PC}$ (D) $\vec{PD}$

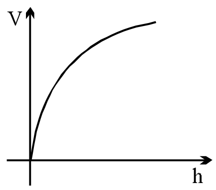
WA0031

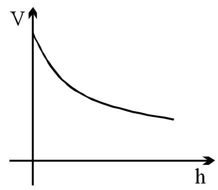
Velocity of wave

10. A uniform rope having some mass hanges vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A) 

(B) 

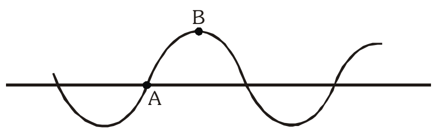
(C) 

(D) 

WA0032

Energy of wave

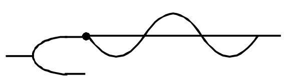
11. A progressive wave is travelling in a string as shown. Then which of the following statement about KE and potential energy of the elements A and B is true?



(A) For point A : kinetic energy is maximum and potential energy is min.
(B) For point B : kinetic energy is minimum and potential energy is min.
(C) For point A : kinetic energy is minimum and potential energy is max.
(D) For point B : kinetic energy is minimum and potential energy is max.

WA0033

12. The prong of a electrically operated tuning fork is connected to a long string of μ = 1 kg/m and tension 25N. The maximum velocity of the prong is 1 cm/s, then the average power needed to drive the prong is:

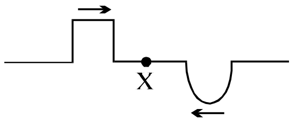


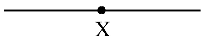
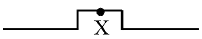
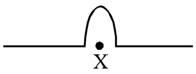
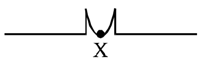
(A) 5×10^{-4} W (B) 2.5×10^{-4} W (C) 1×10^{-4} W (D) 10^{-3} W

WA0034

Superposition of waves

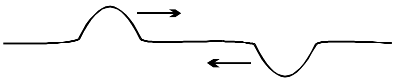
13. The diagram below shows two pulses traveling towards each other in a uniform medium with same speed. Pulses in the figure are at the same distance from X and has same height & width. Which diagram best represents the medium when the pulses meet at point X ?



(A) 
(B) 
(C) 
(D) 

WA0035

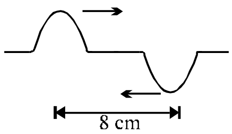
14. Two symmetric, identical pulses of opposite amplitude travel along a stretched string in opposite directions as shown in the figure below. Which one of the following statements most fully describes the situation ?



(A) There is an instant when the string is straight
(B) When the two pulses interfere completely, the energy of the wave is zero
(C) There is a point on the string that does not move up or down
(D) Both A and C
(E) Both A and B

WA0036

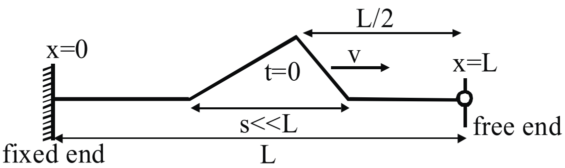
15. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be :- **[IIT-JEE 2001(Scr)]**

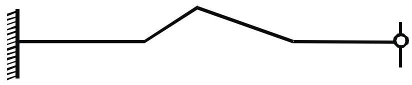


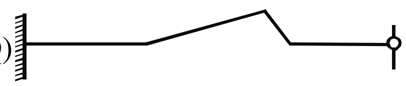
(A) zero
(B) purely kinetic
(C) purely potential
(D) partly kinetic and partly potential

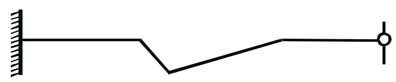
WA0037

16. A small pulse travelling with speed v in a string is shown at $t = 0$, moving towards free end. Which of these is not **CORRECTLY** matched.



(i) $t = \dfrac{L}{v}$ (P) 

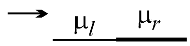
(ii) $t = \dfrac{2L}{v}$ (Q) 

(iii) $t = \dfrac{3L}{v}$ (R) 

(A) (i) (B) (ii) (C) (iii) (D) None of these

WA0038

17. A string consists of two parts attached at x = 0. The right part of the string (x > 0) has mass μ_r per unit length and the left part of the string (x < 0) has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string ?



(A) 1 (B) $\dfrac{2}{1+\sqrt{\mu_l/\mu_r}}$ (C) $\dfrac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$ (D) $\dfrac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

WA0039

18. A wave travels on a light string .The equation of the wave is Y= Asin(kx–wt+30°) .It is reflected from a heavy string tied to an end of the light string at x = 0.If 64% of the incident energy is reflected the equation of the reflected wave is

(A) $Y = 0.8A\sin(kx-wt+30^0+180^0)$
(B) $Y = 0.8A\sin(kx+wt+30^0+180^0)$
(C) $Y = 0.8A\sin(kx+wt-30^0)$
(D) $Y = 0.8A\sin(kx+wt+30^0)$

WA0040

19. A wave is represented by the equation y = 10 sin 2π(100t–0.02x) + 10 sin 2π(100t+0.02x). The maximum amplitude and loop length are respectively

(A) 20 units and 30 units
(B) 20 units and 25 units
(C) 30 units and 20 units
(D) 25 units and 20 units

WA0041

20. A wave represented by the equation y = A cos (kx – ωt) is superimposed with another wave to form a statioary wave such that the point x = 0 is a node. The equation of the other wave is:

(A) –A sin (kx + ωt) (B) – A cos (kx + ωt) (C) A sin (kx + ωt) (D) A cos (kx + ωt)

WA0042

21. Five waveforms moving with equal speeds on the x-axis

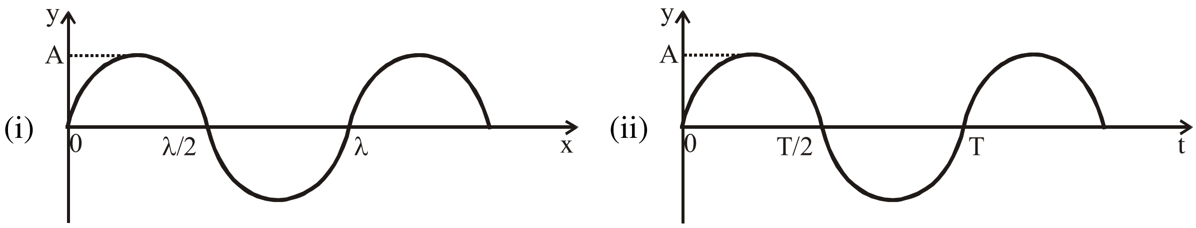
$y_1 = 8\sin(\omega t + kx)$; $y_2 = 6\sin(\omega t + \frac{\pi}{2} + kx)$; $y_3 = 4\sin(\omega t + \pi + kx)$; $y_4 = 2\sin(\omega t + \frac{3\pi}{2} + kx)$;

$y_5 = 4\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$ are superimposed on each other. The resulting wave is :

(A) $8\sqrt{2}\cos kx \sin(\omega t + \frac{\pi}{4})$
(B) $8\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$
(C) $8\sqrt{2}\sin kx \cos(\omega t + \frac{\pi}{4})$
(D) $8\sin(\omega t + kx)$

WA0043

22. Here given figure (i) shows snap shot at t = T/4 and figure (ii) shows motion of particle at x = λ/4. Then the possible equations of the wave will be $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$:-



(A) $y = A \sin\left(\omega t + kx - \frac{\pi}{2}\right)$

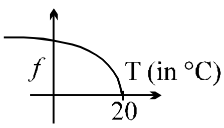
(B) $y = A \sin\left(\omega t - kx + \frac{\pi}{2}\right)$

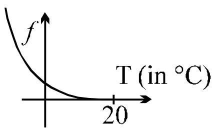
(C) Both $y = A \cos\left(\omega t + kx - \frac{\pi}{2}\right)$ and $y = A \cos\left(\omega t - kx + \frac{\pi}{2}\right)$

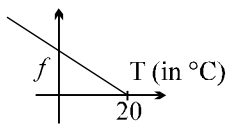
(D) Both $y = A \sin\left(\omega t + kx - \frac{\pi}{2}\right)$ and $y = A \sin\left(\omega t - kx + \frac{\pi}{2}\right)$

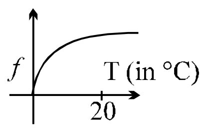
WA0044

23. A metal wire is clamped between two vertical walls. At 20 °C the unstrained length of the wire is exactly equal to the separation between walls. If the temperature of the wire is decreased the graph between fundamental frequency (*f*) and temperature (T) of the wire is

(A) 

(B) 

(C) 

(D) 

WA0045

24. What is the fractional change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

(A) 1/4 (B) 1/2 (C) 2/3 (D) 3/4

(E) 2/1

WA0046

25. A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be

(A) zero everywhere

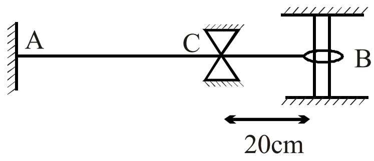
(B) dependent on the location along the string

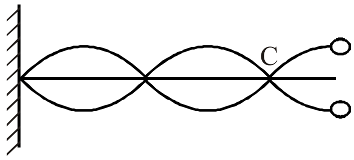
(C) non zero everywhere

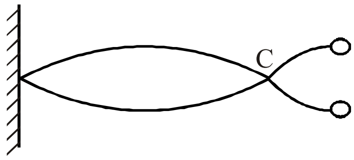
(D) non-zero and in the same direction everywhere

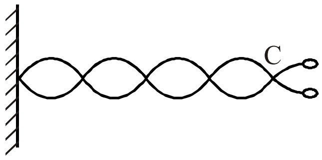
WA0047

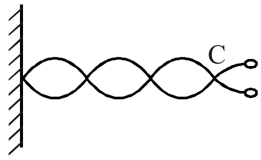
26. A 1m long wire having tension T is fixed at A and free at B. The point C, 20 cm from B is constrained to be stationary. What is shape of string for fundamental mode?



(A) 

(B) 

(C) 

(D) 

WA0048

27. The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then **[JEE 2001 (Scr)]**

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

WA0049

EXERCISE (O-2)

1. Consider a hypothetical wave pulse (at time t = 0) given by the following (y, x in meter)

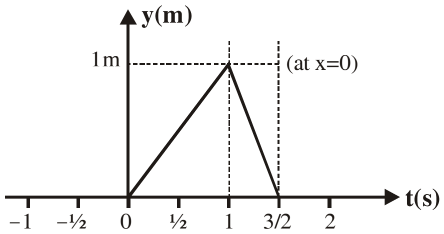
$y = 0, x < 0$

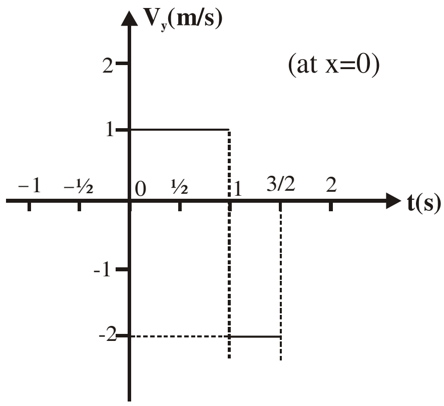
$y = x/2, 2 > x \geq 0$

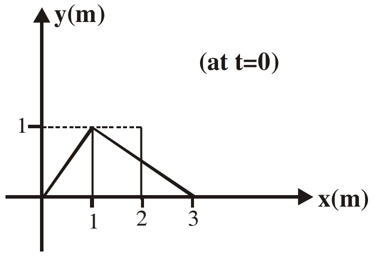
$y = 3 - x, 3 \geq x \geq 2$

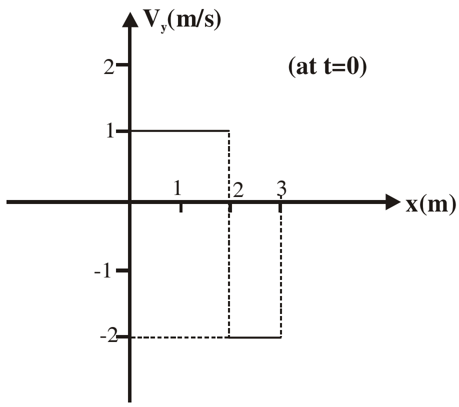
$y = 0, x > 3$

The pulse travels leftwards (negative x direction) at speed v = 2 m/s. Which of the following plots are correct? [V_y is the velocity of particle]

(A) 

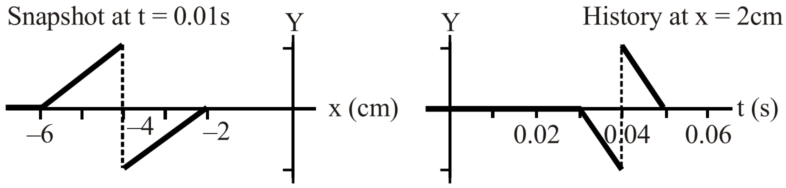
(B) 

(C) 

(D) 

WA0050

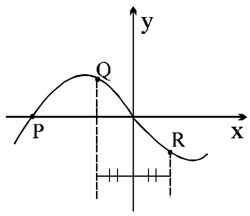
2. Figure shows a snapshot graph and a history graph for a wave pulse on a stretched string. They describe the same wave from two perspectives.



(A) the wave is travelling in positive x-direction
(B) the wave is travelling in negative x-direction
(C) the speed of the wave is 2 m/s.
(D) the peak is located at x = –6 cm at t = 0.

WA0051

3. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.



(A) v_P is upwards (B) $v_Q = -v_R$ (C) $|v_P| > |v_Q| = |v_R|$ (D) $v_Q = v_R$

WA0052

4. An string has resonant frequencies given by 1001 Hz and 2639 Hz.
(A) If the string is fixed at one end only, 910 Hz can be a resonance frequency.
(B) If the string is fixed at one end only, 1911 Hz can be a resonance frequency.
(C) If the string is fixed at both the ends, 364 Hz can be one of the resonant frequency.

WA0053

(D) 1001 Hz is definitely not the fundamental frequency of the string.

5. In a travelling one dimensional mechanical sinusoidal, wave
(A) potential energy and kinetic energy of an element become maximum simultaneously.
(B) all particles oscillate with the same frequency and the same amplitude
(C) all particles may come to rest simultaneously
(D) we can find two particles, in a length equal to half of a wavelength, which have the same non zero acceleration simultaneously.

WA0054

Paragraph for Question Nos. 6 to 8

A wave represented by equation y = 2(mm) sin [4π(sec^{-1})t–2π(m^{-1})x] is superimposed with another wave y = 2 (mm) sin [4π(sec^{-1})t+2π(m^{-1})x + π/3] on a tight string.

6. Phase difference between two particles which are located at x_1 = 1/7 and x_2 = 5/12 is :-

(A) 0 (B) $\frac{5\pi}{6}$ (C) π (D) $\frac{5\pi}{3}$

WA0055

7. Which of the following is not a location of antinode?

(A) $\frac{2}{3}$ (B) $\frac{11}{12}$ (C) $\frac{5}{12}$ (D) $\frac{17}{12}$

WA0055

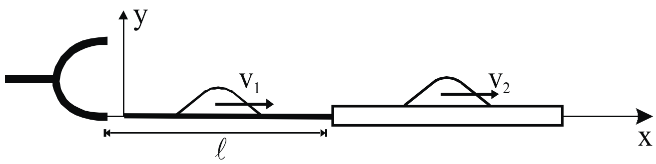
8. The location having maximum potential energy is

(A) 1/7 (B) 1/6 (C) 5/12 (D) 23/12

WA0055

Paragraph for Question Nos. 9 to 11

A harmonic oscillator at x = 0, oscillates with a frequency $\frac{\omega}{2\pi}$ and amplitude a. It is generating waves at end of a thin string in which velocity of wave is v_1 and which is connected to another heavier string in which velocity of wave is v_2 as shown, length of first string is ℓ.



9. If harmonic oscillator oscillates by an equation y = a sinωt. The equation of incident wave in first string is

(A) $y = a\sin\omega\left(t - \frac{x}{v_1}\right)$ (B) $y = a\sin\omega\left(t + \frac{x}{v_1}\right)$

(C) $y = a\sin\left[\omega\left(t - \frac{x}{v_1}\right) + \pi\right]$ (D) $y = a\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$

WA0056

10. Equation of transmitted wave in second string if its amplitude is a_t is

(A) $y = a_t \sin\omega\left(t - \frac{x}{v_2}\right)$ (B) $y = a_t \sin\omega\left(t - \frac{\ell}{v_1}\right)$

(C) $y = a_t \sin\omega\left(t - \frac{\ell}{v_1} - \frac{x-\ell}{v_2}\right)$ (D) $y = a_t \sin\omega\left(t - \frac{x}{v_2}\right)$

WA0056

11. Equation of reflected wave, if it is reflecting at the joint and amplitude of reflected wave is a_R

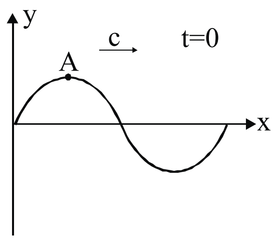
(A) $y = a_R \sin\omega\left(t - \frac{x}{v_2}\right)$ (B) $y = a_R \sin\left[\omega\left(t - \frac{\ell}{v_1} - \frac{\ell - x}{v_1}\right) + \pi\right]$

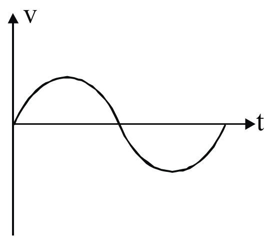
(C) $y = a_R \sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$ (D) $y = a_R \sin\left[\omega\left(t + \frac{2\ell + x}{v_1}\right) + \pi\right]$

WA0056

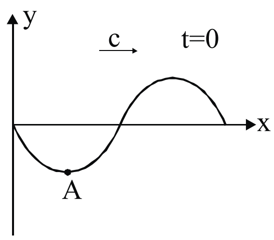
12. In column-I transverse waves travelling on a string at t = 0 is shown. Wave velocity is indicated by '*c*'. Column-II describes variation of different parameters for particle *A* or for all the particles.

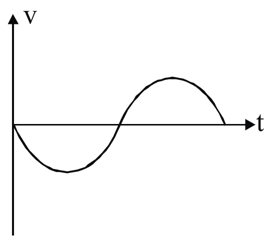
Column-I **Column-II**

(A)


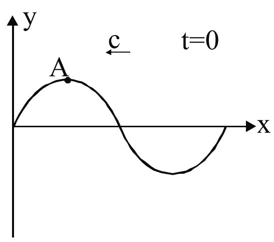
(P)


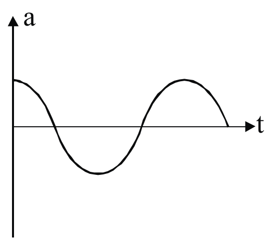
For particle A

(B)


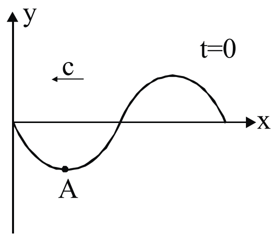
(Q)


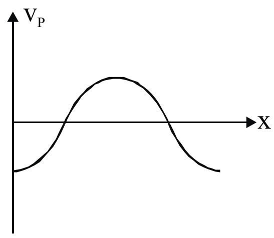
For particle A

(C)


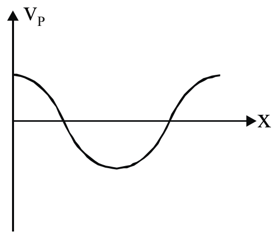
(R)


For particle A

(D) 

(S) 

At t = 0 for all the particles

(T) 

At t = 0 for all the particles

WA0057

13. In a string a standing wave is set up whose equation is given as $y = 2A \sin kx \cos \omega t$. The mass per unit length of the string is μ.

Column-I		Column-II
(A) at $t = 0$	(P)	Total energy per unit length at $x = 0$ is $2\mu A^2\omega^2$.
(B) at $t = \frac{T}{8}$	(Q)	Total energy per unit length at $x = \lambda/4$ is $2\mu A^2\omega^2$.
(C) at $t = \frac{T}{4}$	(R)	Total energy per unit length at $x = \lambda$ is $2\mu A^2\omega^2$.
(D) at $t = \frac{T}{2}$	(S)	power transmitted through a point at $x = \lambda$ is 0.
	(T)	power transmitted through a point at $x = \lambda/4$ is 0.

WA0058

EXERCISE-JM

1. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is : **[AIEEE - 2010]**

(1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N

WA0059

2. The transverse displacement y(x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$. This represents a :- **[AIEEE - 2011]**

(1) standing wave of frequency $\sqrt{b}$

(2) standing wave of frequency $\frac{1}{\sqrt{b}}$

(3) wave moving in +x directionwith speed $\sqrt{\frac{a}{b}}$

(4) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

WA0060

3. A travelling wave represented by y = A sin(ωt–kx) is superimposed on another wave represented by y = A sin (ωt + kx). The resultant is :- **[AIEEE-2011]**

(1) A standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0,1,2$

(2) A wave travelling along + x direction

(3) A wave travelling along –x direction

(4) A standing wave having nodes at $x = \frac{n\lambda}{2}$; n = 0, 1, 2

WA0061

4. A sonometer wire of length 1.5m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively ? **[JEE-Main-2013]**

(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz

WA0062

5. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :-
(take $g = 10\ ms^{-2}$) **[JEE-Main-2016]**

(1) $\sqrt{2}\,s$ (2) $2\pi\sqrt{2}\,s$ (3) 2s (4) $2\sqrt{2}\,s$

WA0063

6. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3\ kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ? **[JEE-Main-2018]**

(1) 2.5 kHz (2) 10 kHz (3) 7.5 kHz (4) 5 kHz

WA0064

EXERCISE (JA)

1. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01m) \sin [(62.8\ m^{-1})x]\cos [(628\ s^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) **[JEE-Advance-2013]**

(A) The number of nodes is 5.

(B) The length of the string is 0.25 m.

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.

(D) The fundamental frequency is 100 Hz.

WA0065

2. One end of a taut string of length 3m along the x–axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) :-

[JEE-Advance-2014]

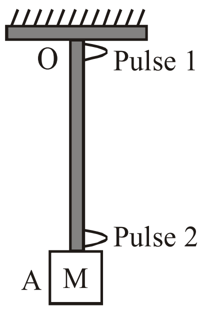
(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

WA0066

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are **correct** ?

[JEE-Advance-2017]



(A) The time $T_{AO} = T_{OA}$

(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(C) The wavelength of Pulse 1 becomes longer when it reaches point A

(D) The velocity of any pulse along the rope is independent of its frequency and wavelength.

WA0067

4. Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be, **[JEE-Advance-2019]**

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

(1) I→P, II→R, III→S, IV→Q
(2) I→P, II→Q, III→T, IV→S
(3) I→Q, II→S, III→R, IV→P
(4) I→Q, II→P, III→R, IV→T

WA0068

5. Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

[JEE-Advance-2019]

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

The length of the string 1,2,3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1,2,3 and 4 are vibrated at their 1^{st}, 3^{rd}, 5^{th} and 14^{th} harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be.

(1) I→P, II→Q, III→T, IV→U
(2) I→T, II→Q, III→R, IV→U
(3) I→P, II→Q, III→R, IV→T
(4) I→P, II→R, III→T, IV→U

WA0069

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) DEF, (b) ABH, (c) CG, (D) AE

2. Ans. (a) $\lambda = 4\text{m}$, $f = \frac{1}{4}$ Hz, 1 m/s, (b) $\frac{3\pi}{2}$ mm/s, $\frac{3\pi^2}{4}$ mm/s^2 , (c) $\frac{3\pi}{2}\times 10^{-3}$

3. Ans. $40\sqrt{2}$

4. Ans. (a) 20 m/s; (b) 2 m; (c) $y(x,t) = 0.02\sin(\pi x - 20\pi t + \pi/6)]$

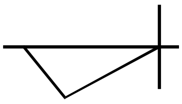
5. Ans. (a) negative x ; (b) $y = 4\times10^{-3} \sin 100\pi\left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2 \times 10^{-5}$ J

6. Ans. 1.22 v

7. Ans. $\frac{\pi}{\sqrt{2}\omega}$

8. Ans. 70 m/s

9. Ans. 10.8×10^4W

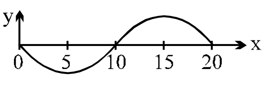
10. Ans. (a)  , (b) 2 cm/s

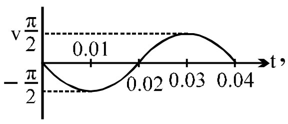
11. Ans. 1×10^9 Nm^2

12. Ans. $A_t = \frac{4}{3}A_i$, $A_r = \frac{1}{3}A_i$, $y_r = -\frac{0.04}{3}\sin(0.5\,\pi x + 100\,\pi t)$; $y_t = +\frac{0.16}{3}\sin(0.25\pi x - 100\,\pi t)$

13. Ans. $E = \frac{A^2\pi^2 T}{4l}$

EXERCISE (S-2)

1. Ans. (i) 

(ii) 25×10^{-6} W (iii) 

, solved

2. Ans. 300 Hz

3. Ans. L/9

4. Ans. $0.2\cos[2\pi/5(x - 10t)]$, (ii) $5n - (15/4)]$

5. Ans. π, 0

6. Ans. 1650 Hz

7. Ans. 10800 Hz

8. Ans. 17.5°

EXERCISE (O-1)

1. Ans. (C)	**2. Ans. (B)**	**3. Ans. (A)**	**4. Ans. (B)**	**5. Ans. (C)**	**6. Ans. (B)**
7. Ans. (D)	**8. Ans. (A)**	**9. Ans. (A)**	**10. Ans. (C)**	**11. Ans. (B)**	**12. Ans. (B)**
13. Ans. (D)	**14. Ans. (D)**	**15. Ans. (B)**	**16. Ans. (B)**	**17. Ans. (C)**	**18. Ans. (C)**
19. Ans. (B)	**20. Ans. (B)**	**21. Ans. (A)**	**22. Ans. (D)**	**23. Ans. (A)**	**24. Ans. (D)**
25. Ans. (B)	**26. Ans. (A)**	**27. Ans. (C)**			

EXERCISE (O-2)

1. Ans. (A,B,D) **2. Ans. (A,C,D)** **3. Ans. (C,D)** **4. Ans. (B,C,D)**

5. Ans. (A,B,D) **6. Ans. (C)** **7. Ans. (A)** **8. Ans. (B)** **9. Ans. (A)**

10. Ans. (C) **11. Ans. (B)**

12. Ans. (A) – (Q,S) ; (B) – (P,R,T) ; (C) – (Q,T) ; (D) – (P,R,S)

13. Ans. (A) – (P,R,S,T) ; (B) – (S,T) ; (C) – (Q,S,T) ; (D) – (P,R,S,T)

EXERCISE-JM

1. Ans. (1) **2. Ans. (4)** **3. Ans. (1)** **4. Ans. (2)** **5. Ans. (4)** **6. Ans. (4)**

EXERCISE (JA)

1. Ans. (B,C) **2. Ans. (A,C,D)** **3. Ans. (A,D)** **4. Ans. (1)** **5. Ans. (1)**