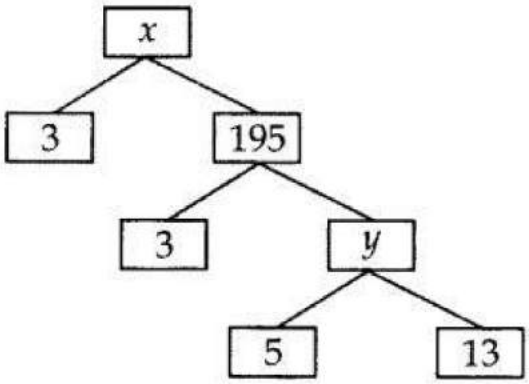
1. REAL NUMBERS

1. What is the least number that is divisible by all the numbers from 1 to 10?
2. Complete the following factor tree and find the composite number x and y .



3. Write the decimal representation of the rational number 8/27.
4. If HCF of a and b is 12 and product of these numbers is 1800. Then what is LCM of these numbers?
5. 5. If a is an odd number, b is not divisible by 3 and LCM of a and b is P, what is the LCM of 3a and 2b?
6. A rational number in its decimal expansion is 1.7351. What can you say about the prime factors of q when this number is expressed in the form p/q? Give reason.
7. Find LCM of numbers whose prime factorisation are expressible as 3×5^2 and $3^2 \times 7^2$.
8. Find the LCM of 96 and 360 by using fundamental theorem of arithmetic.
9. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.
10. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. 198 and 144.
11. If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ and $y = p^3q$, what can you say about their LCM and HCF. Is LCM a multiple of HCF? Explain.
12. Find the largest number which divides 70 and 125 leaving remainder 5 and 8 respectively.
13. In a school, there are two Sections A and B of class X. There are 48 students in Section A and 60 students in Section B. Determine the least number of books required for the library of the school so that the books can be distributed equally among all students of each Section.
14. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.
15. Prove that √7 is an irrational number
16. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11, and 15 respectively.
17. Using prime factorisation method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.
18. Amita, Sneha, and Raghav start preparing cards for all persons of an old age home. In order to complete one card, they take 10, 16 and 20 minutes respectively. If all of them started together, after what time will they start preparing a new card together?
19. Dudhnath has two vessels containing 720 ml and 405 ml of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.
20. There are 104 students in class X and 96 students in class IX in a school. In a house examination, the students are to be evenly seated in parallel rows such that no two adjacent rows are of the same class.

(a) Find the maximum number of parallel rows of each class for the seating arrangement.
(b) Also, find the number of students of class IX and also of class X in a row.
(c) What is the objective of the school administration behind such an arrangement?

Answer

1. Required number = LCM of 1, 2, 3, ... 10 = 2520.

2. $y = 5 \times 13 = 65$

a. $x = 3 \times 195 = 585$

3. Decimal representation of number 8/27 = 0.296

4. Product of two numbers = Product of their LCM and HCF

$\Rightarrow 1800 = 12 \times LCM$

$\Rightarrow LCM = 1800/12 = 150.$

5. 6P

6. As 1.7351 is a terminating decimal number, so q must be of the form $2^m 5^n$, where in, n are natural numbers.

7. $LCM (3 \times 5^2, 3^2 \times 7^2) = 3^2 \times 5^2 \times 7^2 = 9 \times 25 \times 49 = 11025$

8. $96 = 2^5 \times 3$

$360 = 2^3 \times 3^2 \times 5$

$LCM = 2^5 \times 3^2 \times 5 = 32 \times 9 \times 5 = 1440$

9. . $\therefore$ LCM $(198,144) 2^4 \times 3^2 \times 11 = 1584$

$HCF(198, 144) = 2 \times 3^2 = 18$

Now, LCM (198, 144) × HCF (198, 144) = 1584 × 18 = 28512

and product of 198 and 144 = 28512

Thus, product of LCM (198, 144) and HCF (198, 144)

= Product of 198 and 144.

10. Let HCF of the numbers be x then according to question LCM of the number will be 14x

And $x + 14x = 600 \Rightarrow 15x = 600 \Rightarrow x = 40$

Then HCF = 40 and LCM = 14 × 40 = 560

$\because$ LCM × HCF = Product of the numbers

560 × 40 = 280 × Second number Second number = 560×40/280 = 80

Then other number is 80.

11. $x = p^2q^3$ and $y = p^3q$

$LCM = p^3q^3$

$HCF = p^2q$ .....(i)

Now, $LCM = p^3q^3$

$\Rightarrow LCM = pq^2 (p^2q)$

$\Rightarrow LCM = pq^2 (HCF)$

Yes, LCM is a multiple of HCF.

12. It is given that on dividing 70 by the required number, there is a remainder 5.

This means that 70 – 5 = 65 is exactly divisible by the required number.

Similarly, 125 – 8 = 117 is also exactly divisible by the required number.

$65 = 5 \times 13$

$117 = 3^2 \times 13$

HCF = 13

Required number = 13

13. Since the books are to be distributed equally among the students of Section A and Section B. therefore, the number of books must be a multiple of 48 as well as 60.

Hence, required number of books is the LCM of 48 and 60.

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$LCM = 2^4 \times 3 \times 5 = 16 \times 15 = 240$

Hence, required number of books is 240.

14. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.

We know that an odd positive integer n is of the form (4q + 1) or (4q + 3) for some integer q.

Case – I When n = (4q + 1)

In this case $n^2 - 1 = (4q + 1)^2 1 = 16q^2 + 8q = 8q(2q + 1)$

which is clearly divisible by 8.

Case – II When n = (4q + 3)

In this case, we have

$n2^2 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8(2q^2 + 3q + 1)$

which is clearly divisible by 8.

Hence $(n^2 - 1)$ is divisible by 8.

15. Let us assume, to the contrary, that √7 is a rational number.

Then, there exist co-prime positive integers and such that

$\sqrt{7} = a/b$, $b \neq 0$

So, $a = \sqrt{7}\, b$

Squaring both sides, we have

$a^2 = 7b^2$ ..... (i)

$\Rightarrow$ 7 divides $a^2 \Rightarrow$ 7 divides a

So, we can write

a = 7c (where c is an integer)

Putting the value of a = 7c in (i), we have

$49c^2 = 7b^2$ $7^2 = b^2$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that √7 is rational is wrong.

Hence, we conclude that √7 is an irrational number.

16. 398 – 7 = 391, 436 – 11 = 425, 542 – 15 = 527

HCF of 391, 425, 527 = 17

17. Given numbers = 30, 72, 432.

$30 = 2 \times 3 \times 5$; $72 = 2^3 \times 3^2$ and $432 = 2^4 \times 3^3$

So, HCF (30, 72, 432) = $2^1 \times 3^1 = 2 \times 3 = 6$

Again,

LCM (30, 72, 432) = $2^4 \times 3^3 \times 5^1 = 2160$

HCF × LCM = 6 × 2160 = 12960

Product of numbers = 30 × 72 × 432 = 933120 .

Therefore, HCF × LCM ≠ Product of the numbers.

18. To find the earliest (least) time, they will start preparing a new card together, we find the LCM of 10, 16 and 20.

$10 = 2 \times 5$

$16 = 2^4$

$20 = 2^2 \times 5$

LCM $= 2^4 \times 5 = 16 \times 5 = 80$ minutes

They will start preparing a new card together after 80 minutes.

19. 1st vessel = 720 ml; 2nd vessel = 405 ml

We find the HCF of 720 and 405 to find the maximum quantity of milk to be filled in one glass.

$405 = 3^4 \times 5$

$720 = 2^4 \times 3^2 \times 5$

HCF $= 3^2 \times 5 = 45$ ml = Capacity of glass

No. of glasses filled from 1st vessel = 720/45 = 16

No. of glasses filled from 2nd vessel = 405/45 = 9

Total number of glasses = 25

20. $104 = 2^3 \times 13$

$96 = 2^5 \times 3$

HCF $= 2^3 = 8$

(a) Number of rows of students of class X = 104/8 = 13

Number maximum of rows class IX = 96/8 = 12

Total number of rows = 13 + 12 = 25

(b) No. of students of class IX in a row = 8

No. of students of class X in a row = 8

(c) The objective of school administration behind such an arrangement is fair and clean examination, so that no student can take help from any other student of his/her class.