REAL NUMBERS

DPP – 13
CLASS –10th
TOPIC – PROVING OF IRRATIONALITY

Q.1 Show that $2 - \sqrt{3}$ is an irrational number.

Q.2 Show that $3 + \sqrt{2}$ is an irrational number.

Q.3 Prove that $4 - 5\sqrt{2}$ is an irrational number.

Q.4 Show that $5 - 2\sqrt{3}$ is an irrational number.

Q.5 Prove that $2\sqrt{3} - 1$ is an irrational number.

Q.6 Prove that $2 - 3\sqrt{5}$ is an irrational number.

Q.7 Prove that $\sqrt{5} + \sqrt{3}$ is irrational.

Q.8 Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Q.9 Prove that for any prime positive integer p, $\sqrt{p}$ is an irrational number.

Q.10 If p and q are prime positive integers, prove that $\sqrt{p} + \sqrt{q}$ is an irrational number.

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TOPIC – PROVING OF IRRATIONALITY

SOL.1 Let's assume, on the contrary, that $2-\sqrt{3}$ is a rational number. Then, there exist coprime positive integers a and b such that

$2 - \sqrt{3} = a/b$

$\Rightarrow \sqrt{3} = 2 - a/b$

$\Rightarrow \sqrt{3} = (2b - a)/b$

$\Rightarrow \sqrt{3}$ is rational [$\because$ a and b are integers $\therefore$ $(2b - a)/b$ is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $2 - \sqrt{3}$ is an irrational number.

SOL.2 Let's assume, on the contrary, that $3 + \sqrt{2}$ is a rational number. Then, there exist coprime positive integers a and b such that

$3 + \sqrt{2} = a/b$

$\Rightarrow \sqrt{2} = a/b - 3$

$\Rightarrow \sqrt{2} = (a - 3b)/b$

$\Rightarrow \sqrt{2}$ is rational [$\because$ a and b are integers $\therefore$ $(a - 3b)/b$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.

Hence, $3 + \sqrt{2}$ is an irrational number.

SOL.3 Let's assume, on the contrary, that $4 - 5\sqrt{2}$ is a rational number. Then, there exist coprime positive integers a and b such that

$4 - 5\sqrt{2} = a/b$

$\Rightarrow 5\sqrt{2} = 4 - a/b$

$\Rightarrow \sqrt{2} = (4b - a)/(5b)$

$\Rightarrow \sqrt{2}$ is rational [$\because$ 5, a and b are integers $\therefore$ $(4b - a)/5b$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.

Hence, $4 - 5\sqrt{2}$ is an irrational number.

SOL.4 Let's assume, on the contrary, that $5 - 2\sqrt{3}$ is a rational number. Then, there exist coprime positive integers a and b such that

$5 - 2\sqrt{3} = a/b$

$\Rightarrow 2\sqrt{3} = 5 - a/b$

$\Rightarrow \sqrt{3} = (5b - a)/(2b)$

$\Rightarrow \sqrt{3}$ is rational [$\because$ 2, a and b are integers $\therefore$ (5b – a)/2b is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $5 - 2\sqrt{3}$ is an irrational number.

SOL.5 Let's assume, on the contrary, that $2\sqrt{3} - 1$ is a rational number. Then, there exist coprime positive integers a and b such that

$2\sqrt{3} - 1 = a/b$

$\Rightarrow 2\sqrt{3} = a/b + 1$

$\Rightarrow \sqrt{3} = (a + b)/(2b)$

$\Rightarrow \sqrt{3}$ is rational [$\because$ 2, a and b are integers $\therefore$ (a + b)/2b is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $2\sqrt{3} - 1$ is an irrational number.

SOL.6 Let's assume, on the contrary, that $2 - 3\sqrt{5}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$2 - 3\sqrt{5} = a/b$

$\Rightarrow 3\sqrt{5} = 2 - a/b$

$\Rightarrow \sqrt{5} = (2b - a)/(3b)$

$\Rightarrow \sqrt{5}$ is rational [$\because$ 3, a and b are integers $\therefore$ (2b – a)/3b is a rational number]

This contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect.

Hence, $2 - 3\sqrt{5}$ is an irrational number.

SOL.7 Let's assume, on the contrary, that $\sqrt{5} + \sqrt{3}$ is a rational number. Then, there exist coprime positive integers a and b such that

$\sqrt{5} + \sqrt{3} = a/b$

$\Rightarrow \sqrt{5} = (a/b) - \sqrt{3}$

$\Rightarrow (\sqrt{5})^2 = ((a/b) - \sqrt{3})^2$ [Squaring on both sides]

$\Rightarrow 5 = (a^2/b^2) + 3 - (2\sqrt{3}a/b)$

$\Rightarrow (a^2/b^2) - 2 = (2\sqrt{3}a/b)$

$\Rightarrow (a/b) - (2b/a) = 2\sqrt{3}$

$\Rightarrow (a^2 - 2b^2)/2ab = \sqrt{3}$

$\Rightarrow \sqrt{3}$ is rational [$\because$ a and b are integers $\therefore$ $(a^2 - 2b^2)/2ab$ is rational]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{5} + \sqrt{3}$ is an irrational number.

SOL.8 Let's assume, on the contrary, that $\sqrt{2} + \sqrt{3}$ is a rational number. Then, there exist coprime positive integers a and b such that

$\sqrt{2} + \sqrt{3} = a/b$

$\Rightarrow \sqrt{2} = (a/b) - \sqrt{3}$

$\Rightarrow (\sqrt{2})^2 = ((a/b) - \sqrt{3})^2$ [Squaring on both sides]

$\Rightarrow 2 = (a^2/b^2) + 3 - (2\sqrt{3}a/b)$

$\Rightarrow (a^2/b^2) + 1 = (2\sqrt{3}a/b)$

$\Rightarrow (a/b) + (b/a) = 2\sqrt{3}$

$\Rightarrow (a^2 + b^2)/2ab = \sqrt{3}$

$\Rightarrow \sqrt{3}$ is rational [$\because$ a and b are integers $\therefore$ $(a^2 + 2b^2)/2ab$ is rational]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{2} + \sqrt{3}$ is an irrational number.

SOL.9 Consider $\sqrt{p}$ as a rational number

Assume $\sqrt{p} = a/b$ where a and b are integers and $b \neq 0$

By squaring on both sides

$p = a^2/b^2$

$pb = a^2/b$

p and b are integers $pb = a^2/b$ will also be an integer

But we know that a^2/b is a rational number, so our supposition is wrong

Therefore, $\sqrt{p}$ is an irrational number.

SOL.10 Let's assume, on the contrary, that $\sqrt{p} + \sqrt{q}$ is a rational number. Then, there exist coprime positive integers a and b such that

$\sqrt{p} + \sqrt{q} = a/b$

$\Rightarrow \sqrt{p} = (a/b) - \sqrt{q}$

$\Rightarrow (\sqrt{p})^2 = ((a/b) - \sqrt{q})^2$ [Squaring on both sides]

$\Rightarrow p = (a^2/b^2) + q - (2\sqrt{q}\, a/b)$

$\Rightarrow (a^2/b^2) - (p+q) = (2\sqrt{q}\, a/b)$

$\Rightarrow (a/b) - ((p+q)b/a) = 2\sqrt{q}$

$\Rightarrow (a^2 - b^2(p+q))/2ab = \sqrt{q}$

$\Rightarrow \sqrt{q}$ is rational [$\because$ a and b are integers $\therefore$ $(a^2 - b^2(p+q))/2ab$ is rational]

This contradicts the fact that $\sqrt{q}$ is irrational. So, our assumption is incorrect.

Hence, $\sqrt{p} + \sqrt{q}$ is an irrational number.