

Minor Test - 3

PART-A: MATHEMATICS

SECTION-I

1. $\frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ca} + \frac{1}{1+\log_c ab}$ is equal to.

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Ans. (B)

Sol.

$$\begin{aligned} & \frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ca} + \frac{1}{1+\log_c ab} \\ &= \frac{\log a}{\log a + \log b + \log c} + \frac{\log b}{\log a + \log b + \log c} \\ & \quad + \frac{\log c}{\log a + \log b + \log c} \\ &= 1 \end{aligned}$$

2. If $A = \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$, then A is equal to.

- (A) 2
- (B) 3
- (C) 5
- (D) 7

Ans. (C)

Sol.

$$\begin{aligned} A &= \log_2 \log_2 \log_4 (4)^4 + 2\log_{2^{1/2}} 2 \\ &= \log_2 \log_2 4 + 2\left(\frac{1}{1/2}\right) \\ &= \log_2 \log_2 2^2 + 4 \\ &= \log_2 2 + 4 \\ &= 1 + 4 = 5 \end{aligned}$$

3. The value of $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2}$ is.

- (A) 3
- (B) 0
- (C) 2
- (D) 1

Ans. (A)

Sol. Let $\log_2 12 = a$, then

$$\begin{aligned} \frac{1}{\log_{96} 2} &= \log_2 96 = \log_2 2^3 \times 12 = 3 + a \\ \log_2 24 &= 1 + a \\ \Rightarrow \log_2 192 &= \log_2 (16 \times 12) = 4 + a \\ \text{and } \frac{1}{\log_{12} 2} &= \log_2 12 = a \end{aligned}$$

Therefore, the given expression is:

$$(1 + a)(3 + a) - (4 + a)a = 3$$

4. If $\log_{30} 3 = c$, $\log_{30} 5 = d$ then the value of $\log_{30} 8$.

- (A) $2(1 - c - d)$
- (B) $3(1 + c + d)$
- (C) $3(1 + c - d)$
- (D) $3(1 - c - d)$

Ans. (D)

Sol.

$$\begin{aligned}\log_{30} 8 &= 3\log_{30} 2 = 3\log_{30} (30/15) \\ &= 3(\log_{30} 30 - \log_{30} 3 - \log_{30} 5) \\ &= 3(1 - c - d)\end{aligned}$$

5. The value of $(\log_3 11)(\log_{11} 13)(\log_{13} 15)(\log_{15} 27)(\log_{27} 81)$ is.

- (A) 2
- (B) 1
- (C) 4
- (D) 3

Ans. (C)

Sol.
$$= \frac{\log_3 11}{\log_3 11} \times \frac{\log_3 13}{\log_3 11} \times \frac{\log_3 15}{\log_3 11} \times \frac{\log_3 27}{\log_3 15} \times \frac{\log_3 81}{\log_3 27}$$

$$\log_3 81 = \log_3 3^4 = 4.$$

6. The value of $(\log_b a)(\log_c b)(\log_a c)$ is.

- (A) 0
- (B) $\log abc$
- (C) 1
- (D) 10

Ans. (C)

Sol.
$$\frac{\log_{10} a}{\log_{10} b} \times \frac{\log_{10} b}{\log_{10} c} \times \frac{\log_{10} c}{\log_{10} a} = 1$$

7. Solution set of the inequality.

$$\log_3 (x+2)(x+4) + \log_{1/3} (x+2) < \frac{1}{2} \log_{\sqrt{3}} 7 \text{ is.}$$

- (A) $(-2, -1)$
- (B) $(-2, 3)$
- (C) $(-1, 3)$
- (D) $(3, \infty)$

Ans. (B)

Sol. $(x+2)(x+4) > 0, x+2 > 0$

$$\Rightarrow x > -2.$$

Now it can be written as

$$\begin{aligned}\log_3 (x+2)(x+4) - \log_3 (x+2) &< \frac{(\log 7)/2}{(\log 3)/2} \\ \Rightarrow \log_3 (x+4) &< \log_3 7 \\ \Rightarrow x+4 &< 7 \text{ or } x < 3\end{aligned}$$

8. The solution of the equation $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$ is.

- (A) 1
- (B) 3
- (C) 4
- (D) 5

Ans. (C)

Sol. It is clear that $x \geq 0$ and $x \geq -5$

i.e, $x \geq 0$

$$\because \log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$$

$$\Rightarrow \log_5 (\sqrt{x+5} + \sqrt{x}) = 1$$

$$\Rightarrow \sqrt{x+5} + \sqrt{x} = 5$$

$$\text{or } \sqrt{x+5} = 5 - \sqrt{x}$$

$$\text{or } (x+5) = (5 - \sqrt{x})^2$$

$$\Rightarrow x+5 = 25 + x - 10\sqrt{x}$$

$$\text{or } 10\sqrt{x} = 20$$

$$\Rightarrow \sqrt{x} = 2$$

$$\therefore x = 4$$

9. The solution set of the inequality $\log_{10}(x^2 - 16) \leq \log_{10}(4x - 11)$ is.

- (A) $(4, \infty)$
- (B) $(4, 5]$
- (C) $\left(\frac{11}{4}, \infty\right)$
- (D) $\left(\frac{11}{4}, 5\right)$

Ans. (B)

Sol.

$$x^2 - 16 \leq 4x - 11$$

$$\Rightarrow x^2 - 4x - 5 \leq 0$$

$$\Rightarrow (x-5)(x+1) \leq 0$$

$$\Rightarrow -1 \leq x \leq 5 \dots (i)$$

$$\text{Also } x^2 - 16 > 0 \Rightarrow x < -4 \text{ or } x > 4 \dots (ii)$$

$$\text{and } 4x - 11 > 0 \Rightarrow x > \frac{11}{4} \dots (iii)$$

From Eqs. (i), (ii) and (iii), we have $x \in (4, 5]$.

10. Solution set of the equation is.

$$\log(8 - 10x - 12x^2) = 3\log(2x - 1)$$

$$(A) \{1\}$$

$$(B) \{3, 2\}$$

$$(C) \{5\}$$

$$(D) \phi$$

Ans. (D)

Sol. $2x - 1 > 0, 8 - 10x - 12x^2 > 0$ and $8 - 10x - 12x^2 = (2x - 1)^3$

$$\Leftrightarrow (2x - 1)(4x^2 + 2x + 9) = 0.$$

No solution as $x > 1/2$.

11. Sum of all the solution(s) of the equation $\log_{10} (x) + \log_{10} (x + 2) - \log_{10} (5x + 4) = 0$ is:

- (A) -1
- (B) 3
- (C) 4
- (D) 5

Ans. (C)

Sol. $\log_{10} x + \log_{10} (x + 2) - \log_{10} (5x + 4) = 0$

$$\log_{10} x(x + 2) - \log_{10} (5x + 4) > 0$$

$$\log_{10} \frac{x(x+2)}{(5x+4)} = 0$$

$$= \log_{10} 1$$

$$\frac{x(x+2)}{5x+4} = 1$$

$$x^2 + 2x = 5x + 4$$

$$x^2 - 3x - 4 = 0$$

$$x^2 - 4x + x - 4 = 0$$

$$x(x - 4) + (x - 4) = 0$$

$$(x + 1)(x - 4) = 0$$

$$x = -1, x = 4$$

$$x = -1 \text{ rejected}$$

12. The domain of $f(x) = \frac{\log_2 (x+3)}{x^2+3x+2}$ is.

- (A) $\mathbb{R} - \{-1, -2\}$
- (B) $(-2, \infty)$
- (C) $\mathbb{R} - \{-1, -2, -3\}$
- (D) $(-3, \infty) - \{-1, -2\}$

Ans. (D)

Sol.

$$x^2 + 3x + 2 \neq 0$$

$$(x + 1)(x + 2) \neq 0$$

$$x \neq -1, -2$$

$$\text{also } x + 3 > 0$$

$$x > -3$$

$$\text{so } x \in (-3, \infty) - \{-1, -2\}$$

13. If $\sqrt{\log_2 x} - 0.5 = \log_2 \sqrt{x}$, then x equals.

- (A) odd integer
- (B) prime number
- (C) composite number
- (D) irrational

Ans. (B)

Sol.

$$\begin{aligned}
\sqrt{\log_2 x} - 0.5 &= \log_2 \sqrt{x} \\
\text{or } \sqrt{\log_2 x} - 0.5 &= 0.5 \log_2 x \\
\Rightarrow y - 0.5 &= 0.5y^2 \\
\Rightarrow y^2 - 2y + 1 &= 0 \Rightarrow y = 1 \\
\Rightarrow \log_2 x &= 1 \Rightarrow x = 2
\end{aligned}$$

14. Domain of $\sqrt{4x - x^2}$ is

- (A) $\mathbb{R} - [0, 4]$
- (B) $\mathbb{R} - (0, 4)$
- (C) $(0, 4)$
- (D) $[0, 4]$

Ans. (D)

Sol. $4x - x^2 \geq 0$

$$x^2 - 4x \leq 0$$

$$x(x - 4) \leq 0$$

$$x \in [0, 4]$$

15. Range of function $f(x) = x + |x|$.

- (A) $[0, \infty)$
- (B) $\mathbb{R}$
- (C) $\mathbb{R}^-$
- (D) $[-2, \infty)$

Ans. (A)

Sol. $|x| = x, x \geq 0$

$$= -x, x \leq 0$$

$$f(x) = 2x; x \geq 0$$

$$f(x) = 0; x \leq 0$$

$$\text{Range } [0, \infty).$$

16. Domain of $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

- (A) $[-2, 1)$
- (B) $[-2, 0) \cup (0, 1)$
- (C) $(1, \infty)$
- (D) $\mathbb{R}$

Ans. (B)

Sol. $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

$$x + 2 \geq 0 \Rightarrow x \geq -2$$

$$1 - x > 0 \Rightarrow x < 1$$

$$x \neq 0 \text{ as } \log_{10} 1 = 0$$

$$\text{final so ln is } x \in [-2, 0) \cup (0, 1)$$

17. Solution set of $\frac{1}{2}\log_3(x+1) - \log_9(1-x) = \log_9(2x+3)$ is

(A) $\left\{\frac{1}{2}(\sqrt{5}-1)\right\}$

(B) $\left\{\frac{1}{2}(\sqrt{5}+1)\right\}$

(C) $\left\{\frac{1}{2}, \frac{1}{3}\right\}$

(D) ϕ

Ans. (A)

Sol. $x+1 > 0, 1-x > 0, 2x+3 > 0 \Rightarrow -3/2 < x < 1$

Then equation becomes

$$\frac{x+1}{1-x} = 2x+3 \Rightarrow x = \frac{1}{2}(\sqrt{5}-1)$$

18. If $\log_{10} x = y$ then $\log_{1000} x^2$ is equal to.

(A) y^2

(B) $2y$

(C) $\frac{3y}{2}$

(D) $\frac{2y}{3}$

Ans. (D)

Sol.

$$\begin{aligned}\log_{1000} x^2 &= \frac{\log_{10} x^2}{\log_{10} 10^3} = \frac{2\log_{10} x}{3} \\ &= \frac{2y}{3}\end{aligned}$$

19. If $\log_{10} 2 = 0.30103$ and $\log_{10} 3 = 0.47712$ then number of digits in $3^{12} \times 2^8$ is.

(A) 7

(B) 8

(C) 9

(D) 10

Ans. (C)

Sol. $y = 3^{12} \times 2^8$

$$\Rightarrow \log_{10} y = 12\log_{10} 3 + 8\log_{10} 2$$

$$= 12 \times 0.47712 + 8 \times 0.30103$$

$$= 5.72544 + 2.40824$$

$$= 8.13368$$

$$\text{Number of digits in } y = 8 + 1$$

$$= 9.$$

20. The number of solutions of $2^{\log_6(-4x)} = \log_7 2401$ is.

(A) 0

(B) 1

(C) 2

(D) infinite

Ans. (B)

Sol. $x < 0$ and $\log_6(-4x) = \log_2(\log_7 7^4) = 2$

$$\Rightarrow -4x = 6^2 = 36 \Rightarrow x = -9$$

SECTION-II

21. If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to.

Ans. 10

Sol.

$$\begin{aligned} (4)^{\log_9 3} + (9)^{\log_2 4} &= (10)^{\log_x 83} \\ \Rightarrow (4)^{1/2} + 9^2 &= (10)^{\log_x 83} \\ \Rightarrow (83)^1 &= (83)^{\log_x 10} \\ \therefore 1 &= \log_x 10 \Rightarrow x = 10 \end{aligned}$$

22. The number of solutions of $\log_4 (x - 1) = \log_2 (x - 3)$ is.

Ans. 1

Sol.

$$\begin{aligned} \log_4 (x - 1) &= \log_2 (x - 3) \\ \therefore x - 1 &> 0 \text{ and } x - 3 > 0 \\ \Rightarrow x &> 3 \\ \text{From Eq. (i),} \\ \Rightarrow \log_{2^2} (x - 1) &= \log_2 (x - 3) \\ \Rightarrow \frac{1}{2} \log_2 (x - 1) &= \log_2 (x - 3) \\ \Rightarrow \log_2 (x - 1) &= 2 \log_2 (x - 3) \\ \Rightarrow \log_2 (x - 1) &= \log_2 (x - 3)^2 \\ \text{or } x - 1 &= (x - 3)^2 \\ \Rightarrow x^2 - 7x + 10 &= 0 \\ \therefore x &= 2, 5 \\ \therefore x &= 5 \\ (\because x > 3) \end{aligned}$$

23. The number of solutions of $2 \log_e 2x = \log_e (7x - 2 - 2x^2)$ is.

Ans. 2

Sol. This equation is equivalent to the system.

$$\begin{aligned} &\begin{cases} 2x > 0 \\ (2x)^2 = 7x - 2 - 2x^2 \end{cases} \\ \Rightarrow &\begin{cases} x > 0 \\ 6x^2 - 7x + 2 = 0 \end{cases} \\ \Rightarrow &\begin{cases} x > 0 \\ (x - 1/2)(x - 2/3) = 0 \end{cases} \\ \Rightarrow &\begin{cases} x = \frac{1}{2} \\ x = \frac{2}{3} \end{cases} \end{aligned}$$

$\therefore$ Number of solutions = 2.

24. The value of $6 + \log_{3/2} \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots \right)$ is.

Ans. 4

Sol. Let $\sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}}}}} = y$

So, $4 - \frac{1}{3\sqrt{2}}y = y^2$

or $y^2 + \frac{1}{3\sqrt{2}}y - 4 = 0$

or $y = \frac{8}{3\sqrt{2}}$

So, the required value of expression

$$= 6 + \log_{3/2} \left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}} \right)$$

$$= 6 + \log_{\frac{3}{2}} \frac{4}{9} = 6 - 2 = 4.$$

25. Characteristic of the logarithm of 2008 to the base 2 is.

Ans. 10

Sol. We have to find characteristic of $\log_2 2008$.

We know that $\log_2 1024 = 10$ and $\log_2 2048 = 11$, therefore $10 < \log_2 2008 < 11$

Hence, it has characteristic is 10 .

26. The value of x satisfying $\log_2 (3x - 2) = \log_{1/2} x$ is.

Ans. 1

Sol. $\log_2 (3x - 2) = \log_{1/2} x = \frac{\log_2 x}{\log_2 2^{-1}} = \log_2 x^{-1}$

$\Rightarrow 3x - 2 = x^{-1} \Rightarrow 3x^2 - 2x = 1 \Rightarrow x = 1$ or $x = -1/3$.

But $\log_2 (3x - 2)$ and $\log_{1/2} x$ are meaningful if $x > 2/3$.

Hence $x = 1$.

27. The value of x which satisfies the equation

$\log_2 (x^2 - 3) - \log_2 (6x - 10) + 1 = 0$ is.

Ans. 2

Sol. $x^2 - 3 > 0, 6x - 10 > 0 \Rightarrow x > \sqrt{3}$

Also $\log_2 \left(\frac{x^2 - 3}{6x - 10} \right) = -1 \Rightarrow \frac{x^2 - 3}{6x - 10} = \frac{1}{2}$

$\Rightarrow x^2 - 3 = 3x - 5$

$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

Thus, $x = 2$.

28. Let (x_0, y_0) be the solution of the following equations:

$(2x)^{\ln 2} = (3y)^{\ln 3};$

$3^{\ln x} = 2^{\ln y}$

Then $2x_0$ is

Ans. 1

Sol. $(2x)^{\ln 2} = (3y)^{\ln 3} \dots\dots(i)$

$3^{\ln x} = 2^{\ln y} \dots\dots(ii)$

In Eq. (ii) taking log on both sides, we get

$$\Rightarrow (\log x)(\log 3) = (\log y)\log 2$$

$$\Rightarrow \log y = \frac{(\log x)(\log 3)}{\log 2} \dots \dots \text{(iii)}$$

In Eq. (i), taking log on both sides, we get

$$(\log 2)\{\log 2 + \log x\} = \log 3\{\log 3 + \log y\}$$

$$(\log 2)^2 + (\log 2)(\log x) = (\log 3)^2 + \frac{(\log 3)^2(\log x)}{\log 2} \text{ [from (iii)]}$$

$$\text{or } (\log 2)^2 - (\log 3)^2 = \frac{(\log 3)^2 - (\log 2)^2}{\log 2} (\log x)$$

$$\text{or } -\log 2 = \log x$$

$$\Rightarrow x = \frac{1}{2} \Rightarrow x_0 = \frac{1}{2}$$

29. The reciprocal of $\frac{2}{\log_4 (2000)^6} + \frac{3}{\log_5 (2000)^6}$ is.

Ans. 6

Sol. Let $N = 2\log_x 4 + 3\log_x 5$; where $x = (2000)^6$

$$= \log_x 4^2 + \log_x 5^3$$

$$= \log_x 4^2 \cdot 5^3 = \log_{(2000)^6} (2000) = \frac{1}{6}$$

Hence, the reciprocal of given value is 6.

30. If $f(x) = 2x^2 + 3, x \in \mathbb{R}$ then minimum value of $f(x)$

Ans. 3

Sol. Range of $f(x)$ is $[3, \infty)$ for $x \in \mathbb{R}$

PART-B: PHYSICS

SECTION-I

31. A body starts from rest and moves with constant acceleration. The ratio of distance covered by the body in n^{th} second to that covered in n second is

(A) $\frac{2n-1}{2n^2}$

(B) $\frac{2n-1}{n^2}$

(C) $\frac{n^2}{2n-1}$

(D) 1: n

Ans. (B)

Sol. As body starts from rest, therefore $u = 0$

Distance covered in n^{th} second is given by

$$S_n = u + \frac{a}{2}(2n-1)$$

$$S_n = \frac{a}{2}(2n-1)$$

Total distance covered in n seconds

$$S = un + \frac{1}{2}an^2 = \frac{1}{2}an^2$$

$$\text{So, } \frac{S_n}{S} = \frac{2n-1}{n^2}$$

32. A particle is moving in a straight line with initial velocity u and uniform acceleration f . If the sum of the distances travelled in t^{th} and $(t + 1)^{\text{th}}$ seconds is 100 cm, then its velocity after t seconds, in cm/s, is

- (A) 20
- (B) 30
- (C) 50
- (D) 80

Ans. (C)

Sol. Step 1. Given data

The sum of the distance travelled in t^{th} and $(t + 1)^{\text{th}}$ seconds is 100 cm.

We have to find the velocity.

Step 2. Concept used.

The distance travelled in n^{th} second is,

$$S_n = u + \frac{1}{2}(2n - 1)a \quad \dots (1)$$

Here, S is the distance travelled time, u is the initial velocity, and a is acceleration of the body in motion.

Step 3. Calculate the velocity.

The distance travelled in t^{th} and $(t + 1)^{\text{th}}$ second is 100 cm.

Now we will find the sum of distances.

By using the expression (1), the distance travelled in t^{th} is,

$$S_t = u + \frac{1}{2}(2t - 1)a \quad \dots (2)$$

And the distance travelled in $(t + 1)^{\text{th}}$ is,

$$\begin{aligned} S_{t+1} &= u + \frac{1}{2}(2(t + 1) - 1)a \\ &= u + \frac{1}{2}(2t + 1)a \quad \dots (3) \end{aligned}$$

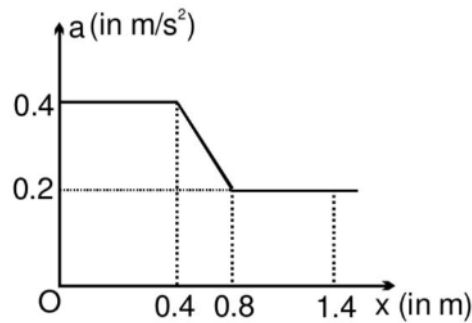
According to the given, the total distance travelled in t^{th} and $(t + 1)^{\text{th}}$ second is 100 cm, so, we have to add expression (1) and (2), we get,

$$S_t + S_{t+1} = 100$$

Substitute the values of expression (1) and (2) in above expression, we get,

$$\begin{aligned} u + \frac{1}{2}(2t - 1)a + u + \frac{1}{2}(2t + 1)a &= 100 \\ 2(u + at) &= 100 \\ (u + at) &= \frac{100}{2} \\ (u + at) &= 50 \quad \dots (4) \end{aligned}$$

- 33.** The acceleration of a particle which moves along the positive x-axis varies with its position as shown. If the velocity of the particle is 0.8 m/s at $x = 0$, the velocity of the particle at $x = 1.4$ is (in m/s)



- (A) 1.6
(B) 1.2
(C) 1.4
(D) none

Ans. (B)

Sol. Area = $0.4 \times 0.2 + 0.4 \times 0.2 + 0.4 \times 0.2$

$$+ \frac{1}{2} \times 0.4 \times 0.2 + 0.6 \times 0.2$$

$$\text{Area} = 0.4 \left[\int a \cdot dx = \int v dv \right]$$

$$\text{Now, } v_f^2 - v_i^2 = 2 (\text{Area})$$

$$v_f^2 = 0.8 + (0.8)^2$$

$$V_f = 1.2 \text{ m/s}$$

- 34.** A particle moves along x - axis. It's velocity is a function of time according to relation $V = (3t^2 - 18t + 24)$ m/s assume at $t = 0$ particle is at origin.

Distance travelled by particle in 0 to 3 second time interval is:

- (A) 18 m
(B) 20 m
(C) 22 m
(D) 24 m

Ans. (C)

Sol. $V = (3t^2 - 18t + 24)$ m/s

$$V = 3(t - 2)(t - 4)$$

$$s = \left| \int_0^2 V dt \right| + \left| \int_2^3 V dt \right|$$

$$= \left| \int_0^2 (3t^2 - 18t + 24) dt \right| + \left| \int_2^3 (3t^2 - 18t + 24) dt \right|$$

$$= |20| + |-2| = 22 \text{ m}$$

35. A ball is thrown vertically upwards from the ground. It crosses a point at the height of 25 m twice at an interval of 4 s. The ball was thrown with the velocity of:

- (A) 20 m/s
- (B) 25 m/s
- (C) 30 m/s
- (D) 35 m/s

Ans. (C)

Sol. After reaching the maximum height, ball falls for 2 seconds to reach the 25 m height.

$$\text{Distance it has fallen is } h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

$$\text{Maximum height} = 25 + 20 = 45 \text{ m}$$

$$\text{Velocity at the bottom or the initial velocity is found as, } v^2 - u^2 = 2(-g)h$$

Since final velocity $v = 0$ and acceleration due to gravity will be negative because ball is thrown vertically upward)

$$u^2 = 2 \times 10 \times 45 = 900$$

$$u = \sqrt{900}$$

$$u = 30 \text{ m/s}$$

36. Pick the incorrect statement

- (A) Average speed of a particle in a given time interval is never less than the magnitude of the average velocity.
- (B) It is possible to have a situation in which $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ but $\frac{d}{dt}|\vec{v}| = 0$.
- (C) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval.
- (D) The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval. (Infinite acceleration are not allowed)

Ans. (D)

Sol. Conceptual

37. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top, both stones reach the bottom of building simultaneously. The height of the building is: [Take $g = 10 \text{ m/s}^2$]

- (A) 45 m
- (B) 35 m
- (C) 25 m
- (D) 50 m

Ans. (A)

Sol. Step 1: Given data assumptions.

$$\text{Height covered by the first stone} = 5 \text{ m}$$

$$\text{Height covered by the second stone} = 25 \text{ m}$$

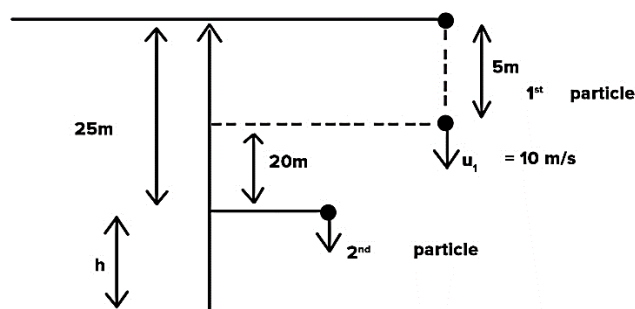
$$\text{Acceleration due to gravity, } g = 10 \text{ m/s}^2$$

Step 2: Finding The height of the building.

Formula Used:

$$S = ut + \frac{1}{2}gt^2$$

Where, S is distance, u initial velocity, and t is time Since,



The velocity of the first stone at 5m below top,

$$u_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$$

Now,

Apply a formula for the first stone which is shown in the figure we get,

$$20 + h = u_1 t + \frac{1}{2} g t^2$$

$$\Rightarrow 20 + h = 10t + \frac{1}{2} g t^2 \quad \dots (i)$$

Apply a formula for the second stone which is shown in the figure we get,

$$h = ut + \frac{1}{2} g t^2$$

$$\Rightarrow h = \frac{1}{2} g t^2 \quad [\because u = 0] \quad \dots (ii)$$

Substitute equation (ii) in (i) we get.

$$20 + \frac{1}{2} g t^2 = 10t + \frac{1}{2} g t^2$$

$$\Rightarrow t = 2 \text{ s} \quad \dots (iii)$$

Now, again substitute equation (iii) in equation (ii) we get.

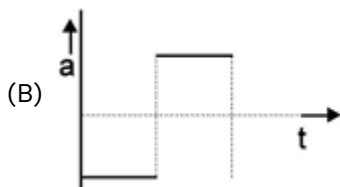
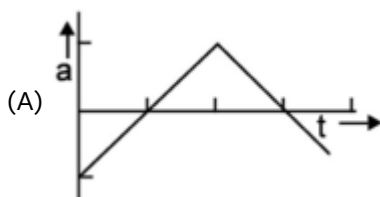
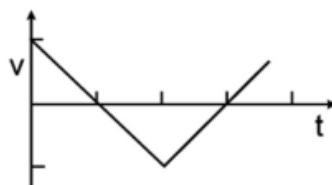
$$h = \frac{1}{2} \times 10 \times 2^2$$

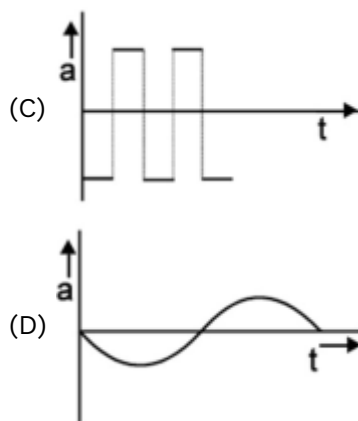
$$\Rightarrow h = 20 \text{ m}$$

Hence, the height of the building from the drawn figure = $h + 25$

$$= 20 + 25 = 45 \text{ m}$$

- 38.** The graph shown in the figure shows the velocity v versus time t of a body. Which of the graphs shown in figure represents the corresponding acceleration versus time graphs?





Ans. (B)

Sol. $a = \text{Slope of } v - t \text{ curve}$

39. A body slides on a smooth inclined plane. If height of inclined plane is 'h' and length is 'l' and angle of inclination is θ then time taken for travelling from upper point to lower point is

(A) $\sin \theta \sqrt{\frac{2h}{g}}$

(B) $\sqrt{\frac{2l}{g}}$

(C) $\frac{1}{\sin \theta} \sqrt{\frac{2h}{g}}$

(D) $\sqrt{\frac{2h}{g}}$

Ans. (C)

Sol. The length of incline plane is l.

The distance travelled by the particle = l

And the acceleration is sin component of g is $a = g \sin \theta$

By using newtons equation of motion:

$$S = ut + \frac{1}{2}at^2$$

Here $u = 0$ because particle start from rest.

$$S = l = \frac{1}{2} g \sin \theta t^2 \quad (\text{equation 1})$$

For right angle triangle:

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypoteneous}} = \frac{\text{height}}{\text{incline lenght}} = \frac{h}{l}$$

$$\text{So, } l = \frac{h}{\sin \theta} \quad (\text{equation 2})$$

By solving equation 1 and 2:

$$\frac{h}{\sin \theta} = \frac{1}{2} g \sin \theta t^2$$

$$t = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}}$$

40. Two trains, each travelling with a speed of 37.5 kmh^{-1} , are approaching each other on the same straight track. A bird that can fly at 60 kmh^{-1} flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train, it flies back to the first and so on. Total distance covered by the bird is

- (A) 90 km
(B) 54 km
(C) 72 km
(D) 36 km

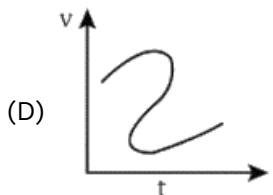
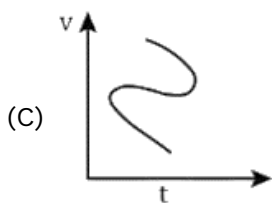
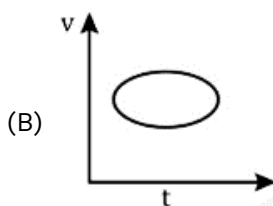
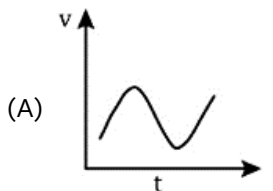
Ans. (C)

Sol. Relative speed of trains = $37.5 + 37.5 = 75 \text{ kmh}^{-1}$

$$\text{Time taken by the trains to meet} = \frac{90}{75} = \frac{6}{5} \text{ h}$$

$$\text{Since speed of bird} = 60 \text{ kmh}^{-1}, \text{ distance travelled by the bird} = 60 \times \frac{6}{5} = 72 \text{ km}$$

41. Which of the following velocity time graphs shows a realistic situation for a body in motion?



Ans. (A)

Sol. We know that any physical body cannot have two different magnitudes of velocity at the same instant. So if you draw a line parallel to velocity line in the graph and if it crosses more than one point, we can say that that graph is unrealistic.

For the graphs b, c and d, the graphs turns out to be unrealistic. Hence option a is the only realistic graph here.

42. The relation between time t and position x is $t = ax^2 + bx$ where a and b are constants. The acceleration is

- (A) $2av^2$
(B) $-2av^3$

- (C) $2bv^3$
 (D) $-2av^2$

Ans. (B)

Sol. $t = ax^2 + bx$

Differentiate w.r.t time

$$1 = 2ax \frac{dx}{dt} + b \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{2ax + b}$$

$$V = \frac{1}{2ax + b} \quad \dots (1)$$

$$a = \frac{dV}{dt} = \frac{-\left(2a \frac{dx}{dt}\right)}{(2ax + b)^2}$$

$$a = \frac{-2av}{(2ax + b)^2} \quad \dots (2)$$

Substituting $\frac{1}{2ax+b} = V$ in (2)

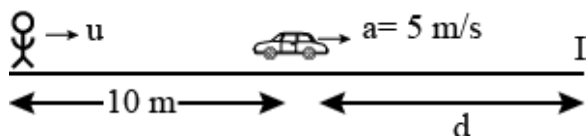
we will get, $a = -2av^3$

- 43.** A passenger is standing 10 m away from a bus. The bus begins to move with constant acceleration of 5 m/s^2 away from passenger. With what minimum speed must passenger run to catch the bus.

- (A) 12 m/s
 (B) 10 m/s
 (C) $5\sqrt{2}$ m/s
 (D) 8 m/s

Ans. (B)

Sol. Let man catches bus after time t , and let the bus travels a distance d and achieves a speed v .



Then $v = 0 + 5t$ [For bus]

$v = \frac{10+d}{t}$ [For man] and $d = \frac{1}{2} \times 5t^2$ [For bus]

Solving all we get is $d = 10 \text{ m}$ & $v = 10 \text{ m/s}$ & $t = 2 \text{ s}$

- 44.** A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. Simultaneously, a bullet of mass 10 g is fired from the foot of the cliff upward with a velocity of 100 ms^{-1} . At what time will the bullet and the block meet ?

- (A) 4 s
 (B) 3 s
 (C) 2 s
 (D) 1 s

Ans. (D)

Sol. Height of the tower $= s_1 + s_2 = 100 \text{ m}$.

Where s_1 and s_2 are the displacements covered by wooden block and bullet respectively before the collision.

For the block:

$$s_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times t^2 = 4.9t^2$$

For the bullet:

$$s_2 = ut - \frac{1}{2}gt^2$$

$$= 100t - \frac{1}{2} \times 9.8 \times t^2 = 100t - 4.9t^2$$

Since, $s_1 + s_2 = 100$ m

$$4.9t^2 + 100t - 4.9t^2 = 100$$

$$\Rightarrow 100t = 100$$

$$\Rightarrow t = 1\text{s}$$

- 45.** An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v . The velocity with which middle point of the train passes the signal post is:

(A) $\frac{u+v}{2}$

(B) $\frac{v-u}{2}$

(C) $\sqrt{\frac{v^2+u^2}{2}}$

(D) $\sqrt{\frac{v^2-u^2}{2}}$

Ans. (C)

Sol. Let the acceleration of the train is " a "

and the length of the train is " l "

The initial velocity is " u "

and final velocity is " v "

The third equation of motion, it is written as;

$$v^2 - u^2 = 2al \quad \dots (1)$$

When the train is in the middle point, the length of the train becomes $\frac{l}{2}$, therefore third equation of the motion becomes,

$$v'^2 - u^2 = 2a \frac{l}{2}$$

$$v'^2 - u^2 = al \quad \dots (2)$$

Now, by solving equations (1) and (2) we have;

$$v^2 - u^2 = 2al$$

$$\Rightarrow l = \frac{v^2 - u^2}{2a}$$

On putting this value in equation (2) we have;

$$v'^2 - u^2 = a \frac{v^2 - u^2}{2a}$$

$$\Rightarrow v'^2 - u^2 = \frac{v^2 - u^2}{2}$$

$$\Rightarrow v'^2 = \frac{v^2 + u^2}{2}$$

$$\Rightarrow v' = \sqrt{\frac{v^2 + u^2}{2}}$$

46. A particle is dropped from rest from a large height. Assume acceleration due to gravity to be constant throughout the motion. The time taken by it to fall through successive distances of 1 m each will be

- (A) In the ratio of the difference in the square roots of the integers, i.e., $\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), (\sqrt{4} - \sqrt{3}), \dots$
 (B) In the ratio of the reciprocals of the square roots of the integers, i.e., $\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \dots$
 (C) All equal, being equal to $\sqrt{2/g}$ second
 (D) In the ratio of the square roots of the integers 1, 2, 3,

Ans. (A)

Sol. For first 1 m

$$1 = \frac{1}{2} \times g \times t^2$$

$$t_1 = \sqrt{\frac{2}{g}}$$

For first 1 m

$$2 = \frac{1}{2} \times g \times t^2$$

$$t_1 = \sqrt{2} \times \sqrt{\frac{2}{g}}$$

for second 1 m

$$t_2 = t - t_1$$

$$= \sqrt{\frac{2}{g}} (\sqrt{2} - \sqrt{1})$$

similarly,

$$t_3 = \sqrt{\frac{2}{g}} (\sqrt{3} - \sqrt{2})$$

47. The position x of a particle varies with time as $x = at^2 - bt^3$. The acceleration of a particle is zero at time t equal to

- (A) $\frac{2a}{3b}$
 (B) $\frac{a}{b}$
 (C) $\frac{a}{3b}$
 (D) 0

Ans. (C)

Sol. Step 1: Given:

Acceleration at time t , $a = 0$.

Displacement equation, $x = at^2 - bt^3$

Step 2: Write an expression of acceleration:

$$a = \frac{d^2x}{dt^2}$$

Step 3: Differentiate two times x with respect to t to get the acceleration:

$$\frac{dx}{dt} = 2at - 3bt^2$$

$$\frac{d^2x}{dt^2} = 2a - 6bt$$

Step 4: Substitute 0 for $\frac{d^2x}{dt^2}$ and T for t in above acceleration expression:

$$2a - 6bt = 0$$

$$t = \frac{2a}{6b}$$

$$t = \frac{a}{3b}$$

- 48.** A parachutist drops first freely from an aeroplane for 10 s and then parachute opens out. Now he descends with a net retardation of 2.5 m/s^2 . If he falls out of the plane at a height of 2495 m and $g = 10 \text{ m/s}^2$, his velocity on reaching the ground will be

- (A) 5 m/s
(B) 10 m/s
(C) 15 m/s
(D) 20 m/s

Ans. (A)

Sol. The velocity v acquired by the parachutist after 10 s.

$$V = u + gt = 0 + 10 \times 10 = 100 \text{ ms}$$

$$\text{Then, } s_1 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times 10^2 = 500 \text{ m}$$

The distance travelled by the parachutist under retardation, $s_2 = 2495 - 500 = 1995 \text{ m}$.

Then

$$v_g^2 - v^2 = 2a s_2$$

$$\text{or } v_g^2 - (100)^2 = 2 \times (-2.5) \times 1995$$

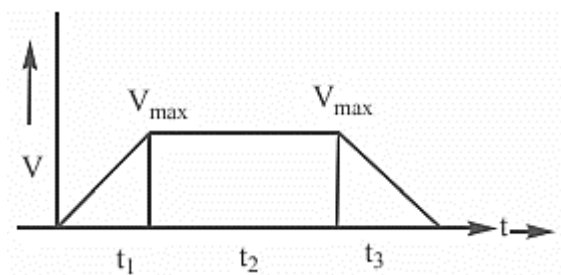
$$\text{or } v_g = 5 \text{ m/s}$$

- 49.** A body starts from rest and travels a distance S with uniform acceleration, then moves uniformly a distance $2S$ and finally comes to rest after moving further $5S$ under uniform retardation. The ratio of the average velocity to maximum velocity is

- (A) $2/5$
(B) $3/5$
(C) $4/7$
(D) $5/7$

Ans. (C)

Sol. Area of the $(V - t)$ curve represents displacement.



$$S = \frac{1}{2}v_{\max}t_1 \Rightarrow t_1 = \frac{2S}{v_{\max}}$$

$$2S = v_{\max}t_2 \Rightarrow t_2 = \frac{2S}{v_{\max}}$$

$$5S = \frac{1}{2}v_{\max}t_3 \Rightarrow t_3 = \frac{10S}{v_{\max}}$$

$$V_{\text{avg}} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{S + 2S + 5S}{\frac{2S}{v_{\max}} + \frac{2S}{v_{\max}} + \frac{10S}{v_{\max}}} = \frac{8S}{\left(\frac{14S}{v_{\max}}\right)} = v_{\max} \frac{4}{7}$$

$$\text{The ratio of the average velocity to maximum velocity} = \frac{v_{\text{avg}}}{v_{\max}} = \frac{v_{\max} \frac{4}{7}}{v_{\max}} = \frac{4}{7}$$

50. The location of a particle has changed. What can we say about the displacement and the distance covered by the particle?

- (A) Neither can be zero
- (B) One may be zero
- (C) Both may be zero
- (D) One is +ve, other is -ve

Ans. (A)

Sol. When location of a particle has changed, it must have covered some distance and undergone some displacement.

As, initial and final position of body are different, so distance and displacement, both can neither be zero.

SECTION-II

51. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time interval with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is

Ans. 4

Sol. Given,

The particle is covering half of the distance with 3 m/s

Other half distance in two equal time intervals with speed of 4.5 m/s and 7.5 m/s

Let the total distance traveled by the particle is S

So,

Time taken by the particle for the first half distance (t_1)

$$t_1 = \frac{\frac{S}{2}}{3} = \frac{S}{6}$$

According to the question second half distance is traveled in two equal time intervals

So,

Let time interval be t

We can write,

$$\frac{S}{2} = 4.5 \times t + 7.5t \Rightarrow 12t = \frac{S}{2}$$

$$\text{Or, } t = \frac{S}{24}$$

Now,

$$\text{Total time } t = t_1 + t + t = \frac{S}{6} + \frac{S}{24} + \frac{S}{24} = \frac{S}{4}$$

We know,

$$\text{Average speed} = \frac{\text{Total distance traveled}}{\text{Total time taken}}$$

$$\text{So, Average} = \frac{S}{t} = \frac{S}{\frac{S}{4}} = 4 \text{ m/s}$$

Hence, the average speed is 4 m/s

- 52.** A train of 150 length is going toward north direction at a speed of 10 ms^{-1} . A parrot flies at a speed of 5 ms^{-1} towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to

Ans. 10

Sol. Relative velocity of the parrot w.r.t. the train

$$[10 - (-5)] = (10 - (-5))\text{m/s} = 15 \text{ m/s}$$

Time taken by the parrot to cross the train

$$= \frac{150}{1.5} = 10 \text{ s.}$$

- 53.** A man Walks up a stationary escalator in 90 seconds when this man stands on a moving escalator he goes up and 60 seconds the time taken by the man to walk up the moving escalator is

Ans. 36

Sol. Distance traveled = D

$$\text{speed of escalator} = D/60 \text{ units}$$

$$\text{speed of the person} = D/90 \text{ units}$$

$$\text{total speed} = D/60 + D/90 = D/36 \text{ units}$$

$$\text{Time if walks on moving escalator} = D/[D/36] = 36\text{sec.}$$

- 54.** Drops of water falls from the roof of a building 18 m high in regular intervals of time. When the first drop reaching the ground, at the same instant fourth drop starts to fall. What is the distance between the second and third drop?

Ans. 6

Sol.

Diagram illustrating the positions of four drops falling from a height of 18 m at regular intervals of time t :

- 1st drop: $t = 0$, distance fallen S_{3t}
- 2nd drop: $t = t$, distance fallen S_{2t}
- 3rd drop: $t = 2t$, distance fallen S_t
- 4th drop: $t = 3t$

The distance between the second and third drop is labeled d .

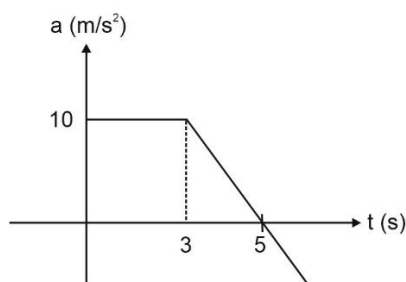
$$\frac{S_{2t}}{S_{3t}} = \frac{4}{9} \quad \frac{S_t}{S_{3t}} = \frac{1}{9}$$

$$\frac{S_{2t}}{18} = \frac{4}{9} \quad \frac{S_t}{18} = \frac{1}{9}$$

$$S_{2t} = 8 \text{ m} \quad S_t = 2 \text{ m}$$

$$d = S_{2t} = S_t = 6 \text{ m}$$

- 55.** The acceleration- time graph of a particle moving along a straight line is as shown in figure. At what time the particle acquires its initial velocity?



Ans. 9

Sol. We know that under area under $a - t$ graph represents change in velocity.

Let A_1 and A_2 be the areas of upper and lower triangle respectively.

Particle will acquire the initial velocity again when,

$$A_1 = 10 \times 3 + \frac{1}{2} \times 10 \times 2 = 40$$

$$A_2 = \frac{1}{2} \times (t_0 - 5) \times 5(t_0 - 5)$$

Equating A_1 & A_2 we get

$$t_0 = 9 \text{ s}$$

Hence particle will acquire its initial velocity at 9 s

- 56.** A body is started from rest with acceleration 2 m/s^2 till it attains the maximum velocity then retard to rest with 3 m/s^2 . If total time taken is 10 second then maximum speed attained is

Ans. 12

Sol. Given that,

$$\text{Acceleration } a = 2 \text{ m/s}^2$$

$$\text{Retardation } a = -3 \text{ m/s}^2$$

$$\text{Time } t = 10 \text{ s}$$

Now, from equation of motion

The maximum velocity attained be v at t

$$v = u + at$$

$$v = 0 + 2 \times t$$

$$v = 2tm/s. \dots (I)$$

Now, again it reaches velocity 0 after 10 s with retardation $v = u + at$

$$v = u + at$$

$$0 = v - 3(10 - t)$$

Now from equation (I)

$$2t - 3(10 - t) = 0$$

$$2t - 30 + 3t = 0$$

$$5t = 30$$

$$t = 6 \text{ s}$$

Now, put the value of t in equation (I)

$$v = 2t$$

$$v = 2 \times 6$$

$$v = 12 \text{ m/s}$$

Hence, the maximum speed is 12 m/s

- 57.** A drunkard is walking along a straight road. He takes 5 steps forward and 3 steps backward and so on. Each step is 1 m long and takes 1 s. There is a pit on the road 11 m away from the starting point. Find the difference of Distance travelled and displacement travelled till he is about to fall in the pit.

Ans. 18

Sol. Distance = 29 m, Displacement = 11 m

- 58.** A steel ball is dropped from the roof of a building. A man standing in front of a 1 m high window in the building notes that the ball takes 0.1 s to fall from the top to bottom of the window. The ball continues to fall and strikes the ground. On striking the ground, the ball gets rebounded with the same speed with which it hits the ground. If the ball reappears at the bottom of the window 2s after passing the bottom of the window on the way down, find the height of the building.

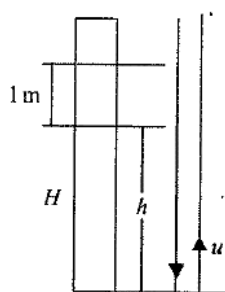
Ans. 21

Sol. The height covered by the ball is given by:

$$h = u \times 1 - \frac{1}{2}g(1)^2$$

$$h + 1 = u \times 1.1 - \frac{1}{2}g(1.1)^2 \Rightarrow u = 20.5 \text{ ms}^{-1}$$

Now, the height covered by the ball finally is:



$$H = \frac{u^2}{2g} = \frac{(20.5)^2}{2 \times 10} = 21 \text{ m}$$

- 59.** Position of particle varies with time as $x = t^2 - 4t + 8$. Find average speed for the time interval $t = 0$ to $t = 4$ sec.

Ans. 2

Sol. $V = 2t - 4$

$V = 0$ at $t = 2$ sec.

$x = 8$ at $t = 0$

$x = 4$ at $t = 2$

$x = 8$ at $t = 4$

Distance = $|4 - 8| + |8 - 4| = 8 \text{ m}$

$$V_{avg} = \frac{\text{Distance}}{\text{time}} = \frac{8}{4} = 2 \text{ m/s}$$

- 60.** Two trains one of length 100 m and another of length 125 m, are moving in mutually opposite directions along parallel lines, meet each other, each with speed 10 m/s. If their accelerations are 0.3 m/s^2 and 0.2 m/s^2 respectively, then the time they take to pass each other will be

Ans. 10

Sol. Relative velocity of one train w.r.t other = $10 + 10 = 20 \text{ m/s}$.

Relative acceleration = $0.3 + 0.2 = 0.5 \text{ m/s}^2$.

$$\text{As, } s = s_1 + s_2 = 100 + 125 = 225$$

$$\Rightarrow 225 = 20t + \frac{1}{2} \times 0.5 \times t^2 \Rightarrow 0.5t^2 + 40t - 450 = 0$$

$$\Rightarrow t = \frac{40 \pm \sqrt{1600 + 4 \times (0.5) \times 450}}{1} = -40 \pm 50$$

$\therefore t = 10 \text{ sec}$ (Taking +ve value).

PART-C: CHEMISTRY

SECTION-I

- 61.** If an ion is represented as ${}_{11}^{23}\text{Na}^+$ then sum of electron, proton and neutron will be.

- (A) 11
(B) 23
(C) 33
(D) 32

Ans. (C)

Sol. ${}_{11}^{23}\text{Na}^+$

11 = Proton

12 = Neutron

$$e^- = 11 - 1$$

$$= 10$$

- 62.** Uncertainty in the position of an electron (mass = $9.1 \times 10^{-31} \text{ kg}$) moving with a velocity of 300 m/s accurate up to 0.001% will be

- (A) $5.76 \times 10^{-3} \text{ m}$
(B) $1.92 \times 10^{-2} \text{ m}$
(C) $3.84 \times 10^{-3} \text{ m}$
(D) $19.2 \times 10^{-4} \text{ m}$

Ans. (B)

Sol.

$$v = 300 \text{ m/s}$$

$$\Delta v = \frac{0.001}{100} \times 300 \text{ m/s}$$

$$\Delta v = 0.003$$

$$\Delta x \cdot \Delta v = \frac{h}{4\pi m}$$

$$\Delta x = \frac{6.634 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.003}$$

$$\Delta x = 19.2 \times 10^{-3} \text{ m}$$

- 63.** The ratio of Bohr's radius of first orbit in hydrogen to the radius of first orbit in deuterium will be :
(A) 1:1
(B) 1:2
(C) 2:1
(D) 4:1

Ans. (A)

Sol. $R = 0.529 \times \frac{n^2}{Z^2}$

$$\frac{(R_{H^1})}{(R_{H^2})} = \frac{(1)^2 / (1)^2}{(1)^2 / (1)^2}$$
$$= 1:1$$

- 64.** Which of the following matter waves will have the shortest wavelength, if travelling with same kinetic energy?
(A) Electron
(B) Alpha particle
(C) Neutron
(D) Proton

Ans. (B)

Sol. $\lambda \propto \frac{1}{\sqrt{m}}$

- 65.** After filling the 4d-orbitals, electron will enter in:
(A) 5p
(B) 4p
(C) 4s
(D) 4f

Ans. (A)

Sol. After filling the 4 d-orbital, an electron will enter in 5p orbital.

- 66.** The ratio of the energy of first orbit of Na^{+10} and H-atom would be:
(A) 121:1
(B) 1:4
(C) 11:1
(D) 22:1

Ans. (A)

Sol. Total energy $\propto Z^2$

$$\therefore \frac{E_{\text{Na}^{+10}}}{E_{\text{H}}} = \frac{(11)^2}{1} = \frac{121}{1}$$

- 67.** The ratio of the energy of a photon of wavelength 3000 Å to that of a photon of wavelength 6000 Å is:
(A) $\frac{1}{2}$
(B) 2
(C) 3
(D) $\frac{1}{3}$
-

Ans. (B)

Sol. $\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = 2$

68. Four electrons in an atom have the sets of quantum numbers as given below. Which electron is at the highest energy level?

(A) $n = 4, l = 1, m_l = -1, m_s = -1/2$

(B) $n = 4, l = 0, m_l = 0, m_s = +1/2$

(C) $n = 3, l = 0, m_l = 0, m_s = -1/2$

(D) $n = 3, l = 2, m_l = 0, m_s = +1/2$

Ans. (A)

Sol. More value of $(n + l)$

69. Choose the correct statement(s).

(A) The orbital wave function or ψ for an electron has no physical meaning

(B) Square of wave function (ψ^2) at a point gives the probability density of the electron at that point.

(C) Boundary surface diagrams of constant probability density for different orbitals give a fairly good representation of the shapes of the orbitals.

(D) All of the above

Ans. (D)

Sol. Conceptual

70. Light of wavelength λ falls on metal having work function hc/λ_0 . Photoelectric effect will take place only if:

(A) $\lambda \geq \lambda_0$

(B) $\lambda \geq 2\lambda_0$

(C) $\lambda \leq \lambda_0$

(D) None of these

Ans. (C)

Sol. For photoelectric effect to take place,

Energy of one photon $\geq$ work function

$$\frac{hc}{\lambda} \geq \frac{hc}{\lambda_0}$$

This immediately implies that photoelectric effect will take place only if $\lambda \leq \lambda_0$

71. Out of first 100 elements, number of elements having electrons in $3d$ -orbitals are:

(A) 10

(B) 80

(C) 100

(D) 60

Ans. (B)

Sol. From atomic no. 1 to 20 $3d$ is not present so in remaining 80 element $3d$ will present.

- 72.** The wavelength of a spectral line for an electronic transition is inversely related to:
- (A) number of electrons undergoing transition
 - (B) the nuclear charge of the atom
 - (C) the velocity of an electron undergoing transition
 - (D) the difference in the energy levels involved in the transition

Ans. (D)

Sol. The wavelength of a spectral line for an electronic transition is inversely proportional to the difference in the energy levels involved in the transition.

$$\Delta E = E_2 - E_1 = \frac{hc}{\lambda}$$

$$\lambda \propto \frac{1}{\Delta E}$$

- 73.** Photon of which light has maximum energy?

- (A) Red
- (B) Blue
- (C) Violet
- (D) Green

Ans. (C)

Sol. $E \propto \frac{1}{\lambda}$; Violet light has lowest wavelength.

- 74.** The magnetic moment of Ni^{x+} ion ($Z = 28$) is about 2.82 BM. The value of x is:

- (A) 2
- (B) 4
- (C) 1
- (D) 3

Ans. (A)

Sol. $\mu = 2.82 = \sqrt{n(n+2)}$

So unpaired $\bar{e} = 2$



1↓	1↓	1↓	1	1
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- 75.** According to Bohr's atomic theory, which of the following is correct?

- (A) Potential energy of electron $\propto \frac{Z^2}{n^6}$
- (B) The product of velocity of electron and principle quantum number (n) $\propto Z^2$
- (C) Coulombic force of attraction on the electron $\propto \frac{Z^2}{n^2}$
- (D) Frequency of revolution of electron in an orbit $\propto \frac{Z^2}{n^3}$

Ans. (D)

Sol. $t = \frac{d}{v} = \frac{2\pi r}{v} = \frac{n^3}{Z^2}$

$$\therefore \text{frequency } \nu = \frac{1}{t} \propto \frac{Z^2}{n^3}$$

76. A photon of energy $h\nu$ is absorbed by a free electron of a metal having work-function $\phi < h\nu$. Choose the correct option(s).

- (A) Number of electrons coming out depends on magnitude of ν
- (B) The electron is sure to come out with a kinetic energy $h\nu - \phi$
- (C) Either the electron does not come out or it comes out with a kinetic energy $h\nu - \phi$
- (D) It may come out with a kinetic energy less than and equal to $h\nu - \phi$

Ans. (D)

Sol. When a photon's energy exceeds the work function (ϕ), the electron absorbs the extra energy as kinetic energy. Consequently, the highest kinetic energy is

$$KE_{\max} = h\nu - \phi.$$

77. An electron, a proton and an alpha particle have kinetic energies of $16E, 4E$ and E respectively. What is the quantitative order of their de Broglie wavelengths?

- (A) $\lambda_e > \lambda_p = \lambda_\alpha$
- (B) $\lambda_p = \lambda_\alpha = \lambda_e$
- (C) $\lambda_p > \lambda_e > \lambda_\alpha$
- (D) $\lambda_e > \lambda_\alpha > \lambda_p$

Ans. (A)

Sol. $\lambda \propto \frac{1}{\sqrt{m \cdot (KE)}} \Rightarrow \lambda_e > \lambda_p = \lambda_\alpha$

78. Which orbital is represented by the complete wave function, ψ_{420} ?

- (A) $4s$
- (B) $4p$
- (C) $4d$
- (D) $4f$

Ans. (C)

Sol. $\psi_{n,l,m}$ $n = 4, l = 2$, So, $4d$

79. The first emission line of Balmer series for H-spectrum has the wave no. equal to :

- (A) $\frac{9R_H}{400} \text{ cm}^{-1}$
- (B) $\frac{7R_H}{144} \text{ cm}^{-1}$
- (C) $\frac{5R_H}{36} \text{ cm}^{-1}$
- (D) $\frac{3R_H}{4} \text{ cm}^{-1}$

Ans. (C)

Sol. $\vec{\nu} = \frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{3^2} \right]; n_1 = 2$ for Balmer series and $n_2 = 3$ for first line or H_α line of Balmer series.

80. The possible correct set of quantum numbers for the unpaired electron of Cl atom is:

- (A) $2, 0, 0, +\frac{1}{2}$
- (B) $2, 1, -1, +\frac{1}{2}$
- (C) $3, 0, \pm\frac{1}{2}$
- (D) $3, 1, 1, \pm\frac{1}{2}$

Ans. (D)

Sol. Cl: $1s^2 2s^2 2p^6 3s^2 3p^5$

Unpaired e^- is in $3p$

$$\text{So } n = 3l = 1m = +1s = \pm \frac{1}{2}$$

SECTION-II

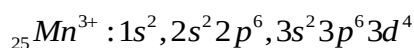
81. An ion Mn^{a+} has the magnetic moment equal to 4.9 BM. What is the value of a ?

Ans. 3

Sol. $\therefore$ Magnetic moment $= \sqrt{n(n+2)} = 4.9$

$\therefore n = 4$ (Where n is no. of unpaired electron)

Thus Mn^{a+} ion has four unpaired electron



$$\therefore a = 3$$

82. $\frac{h}{\pi}$ is the angular momentum of the electron in the $n = \dots\dots\dots$ orbit of He^+ .

Ans. 2

Sol. $L = \frac{nh}{2\pi} = \frac{h}{\pi} \Rightarrow x = 2$

83. Visible spectrum contains light of following colours "Violet - Indigo - Blue - Green - Yellow - Orange Red" (VIBGYOR).

Its frequency ranges from violet (7.5×10^{14} Hz) to red (4×10^{14} Hz). Find out the maximum wavelength (in Å) in this range.

Ans. 7500

Sol. $\lambda_{\max} = \frac{3 \times 10^8}{4 \times 10^{14}} = 7.5 \times 10^{-7} \text{ m} = 7500 \text{ Å}$

84. Number of unpaired electron in Ground state electronic configuration of ${}_{24}\text{Cr}$.

Ans. 6

Sol. Cr [Ar] $3d^5 4s^1$

85. Wavelength of electron waves in two Bohr orbits is in ratio 3:5. The ratio of kinetic energy of electron is $25:x$, hence x is:

Ans. 9

Sol. $\frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2} = \frac{3}{5}; \frac{KE_1}{KE_2} = \frac{n_2^2}{n_1^2}$

86. The circumference of the second orbit of an atom or ion having single electron, is 4 nm. The de Broglie wavelength of electron (in nm) revolving in this orbit is:

Ans. 2

Sol. Circumference $= n\lambda = 4 \text{ nm} \Rightarrow \lambda = 2 \text{ nm}$

87. Find the number of spectral lines in Paschen series emitted by atomic H, when electron is excited from ground state to 7th energy level returns back.

Ans. 4

Sol. $n = 7$ to $n = 3$; $\Delta n = 4$

88. Find the total number of degenerate orbitals in $\Psi_{4,2,0}$ of H-atom.

Ans. 5

Sol. 4d has 5 degenerate orbitals.

89. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is:

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Ans. 4

Sol. $E_{\text{incident}} = \frac{1240}{300} = 4.1\text{eV}$

So Li, Na, K, Mg will show.

90. The de Broglie wavelength of an electron in the 4th Bohr orbit is $X\pi a_0$. Find the value of X.

Ans. 8

Sol. According to Bohr's model

$$r_n = \frac{n^2}{Z} \times a_0 \quad (a_0 = 1^{\text{st}} \text{ Bohr radius})$$

$$\therefore 2\pi r = n\lambda \quad (\text{using de-Broglie relation})$$

$$\Rightarrow 2\pi \times \frac{4^2}{1} \times a_0 = 4\lambda$$

$$\Rightarrow \lambda = 8\pi a_0$$

Answer Key

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	A	D	C	C	B	C	B	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	D	B	D	A	B	A	D	C	B
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	10	1	2	4	10	1	2	1	6	3
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	B	C	B	C	C	D	A	B	C	C
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	A	B	B	D	C	A	C	A	C	A
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	4	10	36	6	9	12	18	21	2	10
Que.	61	62	63	64	65	66	67	68	69	70
Ans.	C	B	A	B	A	A	B	A	D	C
Que.	71	72	73	74	75	76	77	78	79	80
Ans.	B	D	C	A	D	D	A	C	C	D
Que.	81	82	83	84	85	86	87	88	89	90
Ans.	3	2	7500	6	9	2	4	5	4	8