CHAPTER 1

Temperature and Thermal Expansion

LEVEL 1

Q. 1: In a temperature scale X ice point of water is assigned a value of $20° X$ and the boiling point of water is assigned a value of $220° X$. In another scale Y the ice point of water is assigned a value of $-20° Y$ and the boiling point is given a value of $380° Y$. At what temperature the numerical value of temperature on both the scales will be same?

Q. 2: The length of the mercury column in a mercury-in-glass thermometer is 5.0 cm at triple point of water. The length is 6.84 cm at the steam point. If length of the mercury column can be read with a precision of 0.01 cm, can this thermometer be used to distinguish between the ice point and the triple point of water?

Q. 3: What effect the following changes will makes to the range, sensitivity and responsiveness of a mercury in glass thermometer–

(a) Increase in size of the bulb.

(b) Increase in diameter of the capillary bore.

(c) Increase in length of the stem.

(d) Use of thicker glass for the bulb.

Q. 4: The focal length of a spherical mirror is given by $f = \frac{R}{2}$, where R is radius of curvature of the mirror. For a given spherical mirror made of steel the focal length is $f = 24.0$ cm. Find its new focal length if temperature increases by 50°C. Given $\alpha_{steel} = 1.2 \times 10^{-5}{}°C^{-1}$

Q. 5: A glass rod when measured using a metal scale at 30°C appears to be of length 100 cm. It is known that the scale was calibrated at 0°C. Find true length of the glass rod at–

(a) 30°C (b) 0°C

$\alpha_{glass} = 8 \times 10^{-6}{}°C^{-1}$ and $\alpha_{metal} = 26 \times 10^{-6}{}°C^{-1}$

Q. 6: A pendulum based clock keeps correct time in an aeroplane flying uniformly at a height h above the surface of the earth. The cabin temperature inside the plane is 10°C. The same pendulum keeps correct time on the surface of the earth when temperature is 30°C. Find the coefficient of linear expansions of the material of the pendulum. You can assume that $h << R$ (radius of the earth)

Q. 7: A liquid having coefficient of volume expansion γ_0 is filled in a cylindrical glass vessel. Glass has a coefficient of linear expansion of α_g. The liquid along with the container is heated to raise their temperature by ΔT. Mass of the container is negligible.

(a) Find relationship between α_g and γ_0 if it was found that the centre of mass of the system did not move due to heating.

(b) Find relationship between α_g and γ_0 if the fraction of volume of the container occupied by the liquid does not change due to heating.

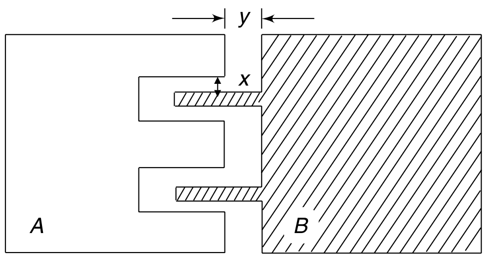
LEVEL 2

Q. 8: A water in glass thermometer has density of water marked on its stem [Density of water is the thermometric property in this case]. When this thermometer is dipped in liquid A the density of water read is 0.99995 $g\,cm^{-3}$. Thereafter it is dipped in liquid B and the reading remains unchanged. Maximum density of water is 1.00000 $g\,cm^{-3}$.

(a) Can we say that liquid A and B are necessarily in thermal equilibrium?

(b) If two liquids are mixed and the thermometer is inserted in the mixture, the height of water column in stem is found to change (i.e. reading is different from 0.99995 $g\,cm^{-3}$). Has the height increased or decreased?

Q. 9: Two metal plates A and B made of same material are placed on a table as shown in the figure. If the plates are heated uniformly, will the gap indicated by x and y in the figure increase or decrease?

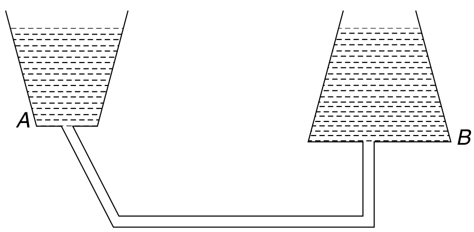


Q. 10: Containers A and B contain a liquid up to same height. They are connected by a tube (see figure).

(a) If the liquid in container A is heated, in which direction will the liquid flow through the tube.

(b) If the liquid in the container B is heated in which direction will the liquid flow through the tube?

Assume that the containers do not expand on heating.

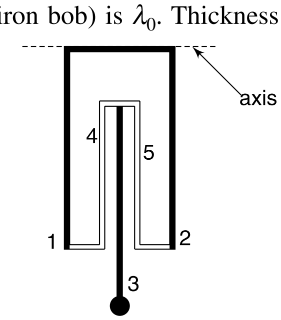


Q. 11: Height of mercury in a barometer is h_0 = 76. 0 cm at a temperature of θ_1 = 20°C. If the actual atmospheric pressure does not change, but the temperature of the air, and hence the temperature of the mercury and the tube rises to θ_2 = 35°C; what will be the height of mercury column in the barometer now? Coefficient of volume expansion of mercury and coefficient of linear expansion of glass are

$$\gamma_{Hg} = 1.8 \times 10^{-4}{}^\circ C^{-1};\ \alpha_g = 0.09 \times 10^{-4}{}^\circ C^{-1}$$

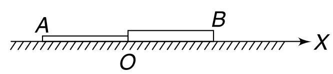
Q. 12: In the last problem if the scale for reading the height of mercury column is marked on the glass tube of the barometer, what reading will it show when temperature rises to θ_2 = 35°C?

Q. 13: Pendulum of a clock consists of very thin sticks of iron and an alloy. At room temperature the iron sticks 1 and 2 have length L_0 each. Length of each of the two alloy sticks 4 and 5 is ℓ_0 and the length of iron stick 3 (measured up to the centre of the iron bob) is λ_0. Thickness of connecting strips are negligible and mass of everything except the bob is negligible. The pendulum oscillates about the horizontal axis shown in the figure. It is desired that the time period of the pendulum should not change even if temperature of the room changes. Find the coefficient of linear expansion (α) of the alloy if the coefficient of linear expansion for iron is α_0.



Q. 14: Two samples of a liquid have volumes 400 cc and 220 cc and their temperature are 10°C and 110°C respectively. Find the final temperature and volume of the mixture if the two samples are mixed. Assume no heat exchange with the surroundings. Coefficient of volume expansion of the liquid is $\gamma = 10^{-3}{}^\circ C^{-1}$ and its specific heat capacity is a constant for the entire range of temperature.

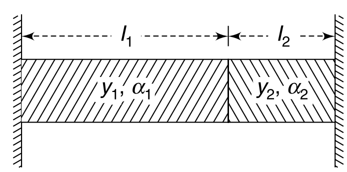
Q. 15: A composite bar has two segments of equal length L each. Both segments are made of same material but cross sectional area of segment OB is twice that of OA. The bar is kept on a smooth table with the joint at the origin of the co - ordinate system attached to the table. Temperature of the composite bar is uniformly raised by $\Delta\theta$. Calculate the x co-ordinate of the joint if coefficient of linear thermal expansion for the material is $\alpha{}^\circ C^{-1}$.



Q. 16: Two rods of different metals having the same area of cross section A, are placed between the two massive walls as shown in figure. The first rod has a length l_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for second rod are are l_2, α_2 and Y_2. The temperature of both the rods is now raised uniformly by T degrees.

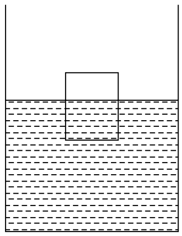
(a) Find the force with which the rods act on each other (at higher temperature) in terms of given quantities.

(b) Also find the length of the rods at higher temperature.



Q. 17: Two rods of equal cross-sections, one of copper and the other of steel, are joined to form a composite rod of length 2.0 m. At 30°C the length of the copper rod is 0.5 m. When the temperature is raised to 130°C the length of the composite rod increases to 2.002 m. If the composite rod is fixed between two rigid walls and is thus not allowed to expand, it is found that the lengths of the two component rods also do not change with the increase in temperature. Calculate the Young's modulus and the coefficient of linear expansion of steel. Given: Young's modulus of copper 1.3×10^{11} N/m^2, coefficient of linear expansion of copper = 1.6×10^{-5} per°C.

Q. 18: A beaker contains a liquid of volume V_0. A solid block of volume V floats in the liquid with 90% of its volume submerged in the liquid. The whole system is heated to raise its temperature by $\Delta\theta$. It is observed that the height of liquid in the beaker does not change and the solid in now floating with its entire volume submerged. Calculate $\Delta\theta$. It is given that coefficient of volume expansion of the solid and the glass (beaker) are γ_s and γ_g respectively.

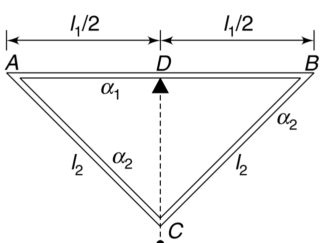


Q. 19: Assume that the coefficient of linear expansion of the material of a rod remains constant, equal to $\alpha°C^{-1}$ for a fairly large range of temperature. Length of the rod is L_0 at temperature θ_0.

(a) Find the length of the rod at a high temperature θ.

(b) Approximate the answer obtained in (a) to show that the length of the rod for small changes in temperature is given by $L = L_0[1 + \alpha(\theta - \theta_0)]$

Q. 20: In a compensated pendulum a triangular frame ABC is made using two different metals. AB of length ℓ_1 is made using a metal having coefficient of linear expansion α_1. BC and AC of length ℓ_2 each have coefficient of linear expansion α_2. A heavy bob is attached at C. Pendulum can oscillate about the pivot D. Find $\frac{\ell_2}{\ell_1}$ so that distance of bob from the pivot point D does not change with change in temperature.



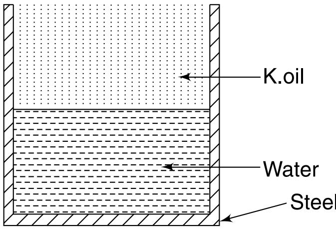
Q. 21: A thin uniform rod of mass M and length l is rotating about a frictionless axis passing through one of its ends and perpendicular to the rod. The rod is heated uniformly to increases its temperature by $\Delta\theta$. Calculate the percentage change in rotational kinetic energy of the rod. Explain why the answer is not zero. Take coefficient of linear expansion of the material of the rod to be α.

Q. 22: (a) A steel tank has internal volume V_0 (= 100 litre). It contains half water $\left(\text{volume} = \frac{V_0}{2}\right)$ and half kerosene oil at temperature $\theta_1 = 10°C$

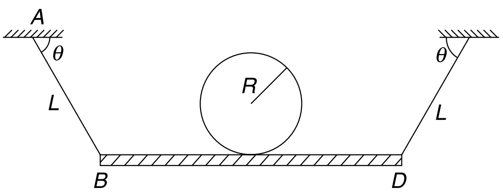
Calculate the mass of kerosene that flows out of the tank at temperature of $\theta_2 = 40°C$. Coefficient of cubical expansion for different substances are:

$\gamma_k = 10^{-3}\,°C^{-1}$; $\gamma_w = 2 \times 10^{-4}\,°C^{-1}$; $\gamma_{steel} = 1.2 \times 10^{-5}\,°C^{-1}$. Density of kerosene at 10°C is $\rho_1 = 0.8$ kg/litre

(b) In the last problem the height of water in the container at $\theta_1 = 10°C$ is $H_1 = 1.0$ m. Find the height of water at $\theta_2 = 40°C$.



Q. 23: A metal cylinder of radius R is placed on a wooden plank BD. The plank is kept horizontal suspended with the help of two identical string AB and CD each of length L. The temperature coefficient of linear expansion of the cylinder and the strings are α_1 and α_2 respectively. Angle θ shown in the figure is 30°. It was found that with change in temperature the centre of the cylinder did not move. Find the ratio $\frac{\alpha_1}{\alpha_2}$, if it is know that $L = 4R$. Assume that change in value of θ is negligible for small temperature changes.

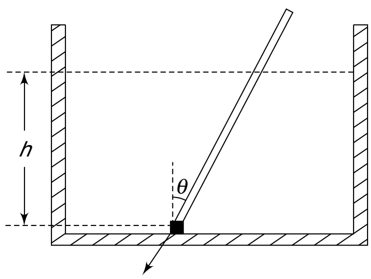


Q. 24: A vernier calliper has 10 divisions on vernier scale coinciding with 9 main scale divisions. It is made of a material whose coefficient of linear expansion is $\alpha = 10^{-3}\,°C^{-1}$. At 0°C each main scale division = 1mm. An object has a length of 10 cm at a temperature of 0°C and its material has coefficient of linear expansion equal to $\alpha_1 = 1 \times 10^{-4}\,°C^{-1}$. The length of this object is measured using the said vernier calliper when room temperature is 50°C.

(a) Find the reading on the main scale and the vernier scale

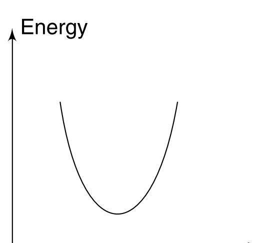
(b) The same object is measured (at 50°C) using a wooden scale whose least count is 1mm. Write the measured reading using this scale assuming it to be correct at all temperature.

Q. 25: A rectangular tank contains water to a height h. A metal rod is hinged to the bottom of the tank so that it can rotate freely in the vertical plane. The length of the rod is L and it remains at rest with a part of it lying above the water surface. In this position the rod makes an angle θ with the vertical. Assume that $y = \cos\theta$ and find fractional change in value of y when temperature of the system increases by a small value ΔT. Coefficient of linear expansion of material of rod and the tank are α_1 and α_2 respectively. Coefficient of volume expansion of water is γ. What is necessary condition for θ to increase?

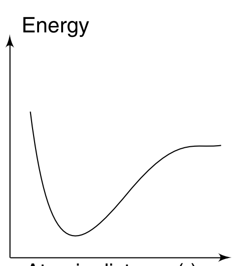


LEVEL 3

Q. 26: In the given figure graph B shows the variation of potential energy versus atomic separation (r) in a material. Argue qualitatively to show that if the potential energy graph was a symmetrical one as depicted in graph A, there would have been no thermal expansion on heating.

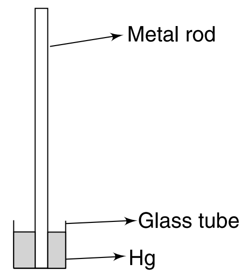


(A)



(B)

Q. 27: In design of a compensated pendulum, a light metal rod of length L_0 = 1.0 m is attached to a glass tube filled with mercury. Neglecting the mass of the glass tube as well, calculate the height of mercury column in the glass tube so that centre of mass of this system does not rise or fall with temperature Given : $\gamma_{Hg} = 1.82 \times 10^{-4}\,K^{-1}$; $\alpha_{glass} = 9 \times 10^{-6}\,K^{-1}$; $\alpha_{metal} = 1.2 \times 10^{-5}\,K^{-1}$

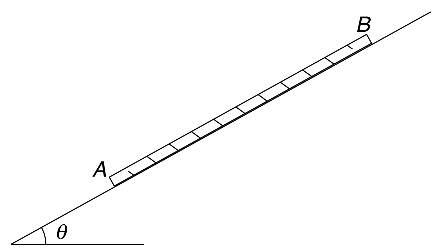


Q. 28: A uniform metal rod (AB) of mass m and length L is lying on a rough incline. The inclination of the incline and coefficient of friction between the rod and the incline is $\theta = 37°$ and $\mu = 1.0$ respectively. $\tan 37° = \frac{3}{4}$

(a) If temperature increases the rod expands. However, there is a point P on the rod which does not move. Find the distance of this point from the lower end of the rod.

(b) If the temperature falls the rod contracts. Once again there is a point Q which does not move. Find distance of Q from the lower end the rod.

(c) Will the repeated expansion and contraction cause the rod to slide down?



ANSWERS

1. 40° X = 40° Y

2. No

3.

Change made to thermometer	change in range	change in sensitivity	Change in responsiveness
Increase in size of bulb	shorter range (reach a lower maximum temperature)	More sensitive	Less responsiveness (longer response time)
Increase in diameter of capillary bore	Longer range	Less sensitive	No change
Increase in length of stem	Longer range	No change	No change
Use of thick glass in bulb	No change	No change	Less responsive

4. 24.0144 cm

5. (a) 100.078 cm (b) 100.054 cm

6. $\alpha = \dfrac{h}{10R}$

7. (a) $\gamma_0 = 2\alpha_g$ (b) $\gamma_0 = 3\alpha_g$

8. (a) No (b) decreased

9. x increases, y decreases

10. In both cases the liquid flows from B to A.

11. $h = h_0[1 + \gamma_{Hg}\Delta\theta] = 76.205$ cm

12. $\dfrac{h_0[1 + \gamma_{Hg}\Delta\theta]}{[1 + \alpha_g\Delta\theta]} \simeq 76.195$ cm

13. $\alpha = \dfrac{(L_0 + \lambda_0)\ \alpha_0}{\ell_0}$

14. 43.33°C; 620 cc

15. $-\dfrac{L\alpha\Delta\theta}{6}$

16. (a) $F = \dfrac{AT(l_1\alpha_1 + l_2\alpha_2)}{\left(\dfrac{l_1}{Y_1} + \dfrac{l_2}{Y_2}\right)}$

(b) $\left(l_1 + l_1\alpha_1 T - \dfrac{F}{A}\dfrac{l_1}{Y_1}\right), \left(l_2 + l_2\alpha_2 T - \dfrac{F}{A}\dfrac{l_2}{Y_2}\right)$

17. $Y_s = 2.6 \times 10^{11}$ N/m^2 $\alpha_s = 0.8 \times 10^{-5}$/°C

18. $\Delta\theta = \dfrac{0.1(V_0 - V)}{(0.9\ V_0 + V)\gamma_s - (V_0 + 0.9\ V)\ \gamma_g}$

19. $L = L_0 e^{\alpha(\theta - \theta_0)}$

20. $\dfrac{1}{2}\sqrt{\dfrac{\alpha_1}{\alpha_2}}$

21. $-200\,\alpha\Delta\theta\%$

22. (a) $\Delta m = \dfrac{V_0\rho_1}{2}\dfrac{(\gamma_k + \gamma_w - 2\gamma_s)}{(1 + \gamma_k\Delta\theta)}\ \Delta\theta;$

$= 1.37$ kg [where $\Delta\theta = \theta_2 - \theta_1$]

(b) $\dfrac{H_1[1 + \gamma_w\Delta\theta]}{\left[1 + \frac{2}{3}\gamma_s\Delta\theta\right]} = 1.0057$ m

23. $\dfrac{\alpha_1}{\alpha_2} = \dfrac{8}{1}$

24. (a) $MSR = 95$ mm; $VSR = 7$

(b) 100 ± 1 mm

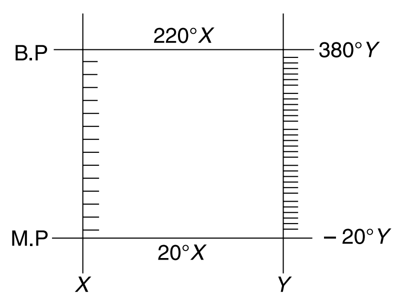
25. $\dfrac{1}{2}(\alpha_1 + \gamma - 4\alpha_2)\Delta T;\ 4\alpha_2 > \alpha_1 + \gamma$

27. $\dfrac{2L_0\,\alpha_{metal}}{\gamma_m - 2\alpha_g} = 0.146$ m

28. (a) $0.875\ L$ (b) $0.125\ L$

(c) Yes

SOLUTIONS



1. Change of 1° X = change of 2° Y

At a particular temperature, if we are x divisions away from 20° X and y divisions away from $-20°$ Y then–

$$20 + x = -20 + 2x \quad\Rightarrow\quad x = 40$$

2. The length Hg column increases linearly with temperature.

Steam point = 100°C

Triple point = 0.01°C

∴ Change in temperature of 99.99°C causes a change in length equal to 1.84 cm

∴ Change in temperature of 0.01°C will cause a change in length equal to

$$\frac{1.84}{99.99} \times 0.01 = 0.00018 \text{ cm}$$

Hence, the thermometer will fail to distinguish between the ice point and the triple point .

4.

$$R_0 = 2f = 48.0 \text{ cm}$$

$$R = R_0(1 + \alpha\Delta\theta) = 48.0\ [1 + 1.2 \times 10^{-5} \times 50]$$

$$= 48.0[1.0006] = 48.0288 \text{ cm}$$

∴ New focal length is $\quad f = \dfrac{48.0288}{2} = 24.0144$ cm

5. (i) Two markings on the metal scale at a separation of 100 cm at a temperature of 30°C, correspond to true length given by $\ell = 100(1 + \alpha_{\text{metal}}\Delta\theta)$

$$= 100\,(1 + 26 \times 10^{-6} \times 30) = 100.078 \text{ cm}$$

Hence, true length of glass at 30°C is 100.78 cm

(ii) If ℓ_0 = length of glass at 0°C, then

$$100.78 = \ell_0[1 + \alpha_g \times 30]$$

$$\ell_0 = \frac{100.78}{1 + 8 \times 10^{-6} \times 30} = 100.054 \text{ cm}$$

6. Acceleration due to gravity at height h is $g' \simeq g\left(1 - \dfrac{2h}{R}\right)$

$\therefore$ Time period at height h is

$$T' = 2\pi\sqrt{\frac{\ell}{g\left(1 - \frac{2h}{R}\right)}} \quad \text{where} \quad \ell = \text{length at } 10°\text{C}$$

Time period on the surface of the earth is $T = 2\pi\sqrt{\dfrac{\ell(1 + \alpha\Delta\theta)}{g}}$

Since $\qquad T' = T$

$$\left(1 - \frac{2h}{R}\right)^{-\frac{1}{2}} = (1 + \alpha\Delta\theta)^{\frac{1}{2}} \Rightarrow 1 + \frac{h}{R} = 1 + \frac{1}{2}\alpha\Delta\theta$$

$$\alpha = \frac{2h}{R\Delta\theta} = \frac{2h}{R\cdot(20)} = \frac{h}{10\cdot R}$$

7. (a) The COM will not move if the height of liquid column in the container does not change.

Let original volume of liquid, area of cross section of the container and height of liquid be V_0, A_0 and H_0 respectively.

$$H_0 = \frac{V_0}{A_0}$$

If temperature increases by $\Delta\theta$

$$V = V_0[1 + \gamma_0\Delta\theta] \quad \text{and} \quad A = A_0[1 + 2\alpha_g\Delta\theta]$$

$\therefore$ Height of liquid

$$H = \frac{V_0[1 + \gamma_0\Delta\theta]}{A_0[1 + 2\alpha_g\Delta\theta]} = \frac{H_0[1 + \gamma_0\Delta\theta]}{[1 + 2\alpha_g\Delta\theta]}$$

If $\qquad H = H_0$ then $1 + \gamma_0\Delta\theta = 1 + 2\alpha_g\Delta\theta$

$\Rightarrow \qquad \gamma_0 = 2\alpha_g$

(b) Let original value of the liquid and the container be $V_{\ell 0}$ and V_{C0}.

At increased temperature

$$V_\ell = V_{\ell 0}[1 + \gamma_0\Delta\theta] \quad \text{and} \quad V_C = V_{C0}[1 + 3\alpha_g\Delta\theta]$$

$$\frac{V_\ell}{V_C} = \frac{V_{\ell 0}[1 + \gamma_0\Delta\theta]}{V_{C0}[1 + 3\alpha_g\Delta\theta]}$$

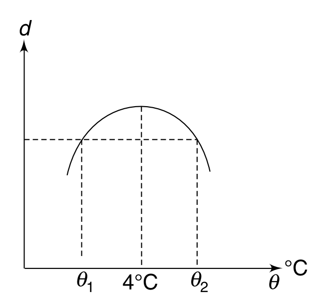
For $\dfrac{V_\ell}{V_C} = \dfrac{V_{\ell 0}}{V_{C0}}$ we must have

$$1 + \gamma_0\Delta\theta = 1 + 3\alpha_g\Delta\theta$$

$\Rightarrow \qquad \gamma_0 = 3\alpha_g$

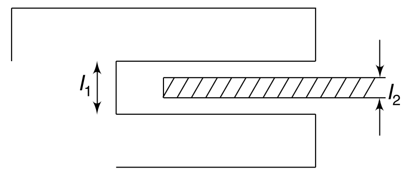
8. (a) The density of water changes with temperature as shown in the figure.

If is possible that the two liquids are at temperature θ_1 (< 4°C) and θ_2 (> 4°C) and therefore, the density of water is same.



(b) When liquids at θ_1 and θ_2 are mixed, the mixture will have a temperature between θ_1 and θ_2. It means density of water will increases and height of water column in the stem will decreases.

9. Change in length ℓ_1 due to increases in temperature $\Delta\ell_1 = \ell_1 \alpha \Delta\theta$



Change in length ℓ_2 is $\Delta\ell_2 = \ell_2 \alpha \Delta\theta$

Since $\ell_1 > \ell_2$ $\quad\therefore\quad \Delta\ell_1 > \Delta\ell_2$

$\therefore$ Gap indicated by x will increase.

It is trivial to understand that y will decrease.

10. Consider a cylindrical container containing a liquid. It is easy to see that weight of liquid divided by the area of the base of the container is pressure at the bottom. Since neither the area nor the weight of the liquid changes on heating, the pressure remains constant.

If the liquid is heated, its density decreases and volume increases but the pressure ($P = \rho gh$) at the bottom does not change.

When liquid in A is heated, the height change is less compared to the change in a cylindrical container but density change is identical in two case. Hence pressure at the bottom of A decreases and liquid flows from B to A.

When B is heated, height change of liquid is larger than the case of a cylindrical container and pressure at bottom of B increases. Hence the liquid flows from B to A.

11. $\theta_2 - \theta_1 = \Delta\theta = 15°C$

Let density of Hg at θ_1 be ρ_0

Then $$P_0 = \rho_0 g h_0 \qquad [P_0 = \text{atmospheric pressure}] \qquad ...(i)$$

Now density of Hg at temperature θ_2 is–

$$\rho = \frac{\rho_0}{1 + \gamma_{Hg}\Delta\theta} \qquad \therefore \quad P_0 = \frac{\rho_0 gh}{1 + \gamma_{Hg}\Delta\theta} \qquad ...(ii)$$

From (i) and (ii)

$$\frac{h}{1 + \gamma_{Hg}\Delta\theta} = h_0$$

$$\therefore \qquad h = h_0[1 + \gamma_{Hg}\Delta\theta]$$

$$= 76.0[1 + 1.8 \times 10^{-4} \times 15] = 76.0 \times 1.0027 = 76.205 \text{ cm}$$

12. Actual height of Hg column at changed temperature is

$$h = h_0[1 + \gamma_{Hg}\Delta\theta] \qquad ...(i)$$

But the glass scale also expands and the reading shown by it will be less than h given in (i). A reading of 1 cm on glass scale at θ_2 actually represents a length of

$$= 1 \text{ cm}[1 + \alpha_g \Delta\theta]$$

$\therefore$ A length h will be read (by the scale) as

$$\frac{1}{1[1 + \alpha_g \Delta\theta]} h = \frac{h_0[1 + \gamma_{Hg}\Delta\theta]}{1 + \alpha_g\Delta\theta}$$

$$= \frac{76.205}{1 + 0.09 \times 10^{-4} \times 15} = \frac{76.205}{1.000135} = 76.195 \text{ cm}$$

13. Length of the pendulum is $L = L_0 + \lambda_0 - \ell_0$

$$\Delta L = \Delta L_0 + \Delta\lambda_0 - \Delta\ell_0$$

$$\therefore \qquad 0 = L_0\alpha_0\Delta\theta + \lambda_0\alpha_0\Delta\theta - \ell_0\alpha\Delta\theta [\Delta\theta = \text{change in temperature}]$$

$$\Rightarrow \qquad \alpha = \frac{(L_0 + \lambda_0)\ \alpha_0}{\ell_0}$$

14. Mass of sample at 10°C is $m_1 = \rho_{10}V_{10} = \rho_{10}(400)$

Mass of sample at 110°C is $m_2 = \rho_{110}V_{110} = \rho_{110}(220)$

$$= \frac{\rho_{10}}{1 + \gamma \times 100}\cdot 220 = \frac{\rho_{10}}{1.1} \times 220 = \rho_{10}(200) = \frac{m_1}{2}$$

When mixed, let the final temperature be θ.

$$m_1 \cdot s \cdot (\theta - 10) = m_2 \cdot s \cdot (110 - \theta)$$

$$\Rightarrow \quad \theta - 10 = \frac{110 - \theta}{2} \quad \Rightarrow \quad \theta = \frac{130}{3} = 43.33°\text{C}$$

$$\text{Mass of mixture} = m_1 + \frac{m_1}{2} = \frac{3m_1}{2}$$

$\therefore$ Volume of $\frac{3m_1}{2}$ mass of liquid at 10°C will be $= \frac{3}{2} \times 400 = 600$ cc

At $\frac{130}{3}$ °C this volume will become

$$V = 600\left[1 + \left(\frac{130}{3} - 10\right)\gamma\right] = 600\left[1 + \frac{100}{3} \times 10^{-3}\right] = 600\left[\frac{3.1}{3}\right] = 620 \text{ cc}$$

15. Mass of $OA = m$ and Mass of $OB = 2$ m

Let the joint shift by x to left.

The COM of the composite rod will not move.

The COM of segment OA will move to left by $x + \frac{L\alpha\Delta\theta}{2}$

The COM of segment BO will move to right by $\frac{L\alpha\Delta\theta}{2} - x$

For COM of the composite rod to remain unmoved, we must have

$$m\left(x + \frac{L\alpha\Delta\theta}{2}\right) = 2m\left(\frac{L\alpha\Delta\theta}{2} - x\right)$$

$$\Rightarrow \quad 3x = \frac{L\alpha\Delta\theta}{2} \quad \Rightarrow \quad x = \frac{L\alpha\Delta\theta}{6}$$

$\therefore$ x co-ordinate of joint is $-\frac{L\alpha\Delta\theta}{6}$

16. (a) When the temperature is raised by T, then

Increase in length of first rod $= l_1\alpha_1 T$

and increase in length of second rod $= l_2\alpha_2 T$

$\therefore$ Total increase in length $l_1\alpha_1 T + l_2\alpha_2 T = T(l_1\alpha_1 + l_2\alpha_2)$ (i)

As the walls are rigid, the above increase will not be possible. This will be compensated by the force F producing decrease in the length of the rods.

$$\text{Decrease in length of first rod} = \frac{F \times l_1}{Y_1 \times A}$$

$$\text{Decrease in length of second rod} = \frac{F \times l_2}{Y_2 \times A}$$

$\therefore$ Total decrease in length due to $F = \frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)$ (ii)

From eq. (i) and (ii), we have

$$\frac{F}{A}\left(\frac{l_1}{Y_1}+\frac{l_1}{Y_1}\right) = T(l_1\alpha_1 + l_2\alpha_2)$$

Or, $$F = \frac{AT(l_1\alpha_1 + l_2\alpha_2)}{\left(\frac{l_1}{Y_1}+\frac{l_2}{Y_2}\right)} \quad \text{...(iii)}$$

(b) Length of the first rod = (original length) + (increase in length due to temp) – (decrease in length due to force F)

$$= \left(l_1 + l_1\alpha_1 T - \frac{F}{A}\frac{l_1}{Y_1}\right) \quad \text{...(iv)}$$

$$\text{Length of the second rod} = \left(l_2 + l_2\alpha_2 T - \frac{F}{A}\frac{l_2}{Y_2}\right) \quad \text{...(v)}$$

The total length will remain unaltered.

17. For copper rod

$$(l_T)_{cu} - (l_0)_{cu} = \alpha_{cu} \times (l_0)_{cu} \times (T_2 - T_1) = \alpha_{cu} \times 0.5 \times (130 - 30) = 50\alpha_{cu}$$

Similarly, for steel rod

$$(l_T)_s - (l_0)_s = \alpha_s \times 1.5 \times 100 = 150\alpha_s$$

$$\text{Total change in length} = 50\alpha_{cu} + 150\alpha_s = 0.002$$

Here $$\alpha_{cu} = 1.6 \times 10^{-5}/°C$$

$$\text{Solving for } \alpha_s \text{ gives } \alpha_s = 0.8 \times 10^{-5}/°C$$

According to the given question, there is no change in length of individual rod. So the length change due to stress is balanced by the length change due to thermal expansion.

Stress in steel rod = $Y_s \times$ strain = $Y_s(\Delta l/l_0)_s = Y_s \times \alpha_s \times \Delta T$

Similarly stress in copper rod = $Y_{cu} \times \alpha_{cu} \times \Delta T$

But, stress in steel rod = Stress in copper rod

$\therefore$ $$\frac{Y_s}{Y_{cu}} = \frac{\alpha_{cu}}{\alpha_s} \quad \text{Or,} \quad Y_s = Y_{cu}\left(\frac{\alpha_{cu}}{\alpha_s}\right)$$

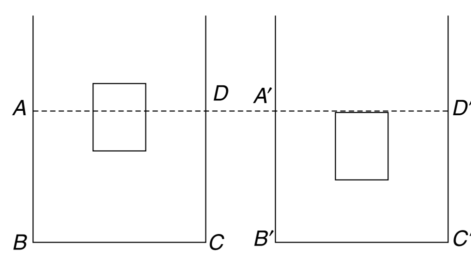
Putting the values we get $Y_s = 2.6 \times 10^{11}$ N/m^2.

18. Let ρ_ℓ and ρ_s be initial densities of the liquid and the solid respectively.

According to the problem $$\frac{\rho_s}{\rho_\ell} = 0.9 \quad \text{...(i)}$$

On increasing the temperature the two densities become equal.

$$\rho_s[1 + \gamma_s\Delta\theta] = \rho_\ell[1 + \gamma_\ell\Delta\theta] \Rightarrow 0.9(1 + \gamma_s\Delta\theta) = (1 + \gamma_l\Delta\theta)$$



V_0 = original volume of liquid

V = original volume of solid

Volume $ABCD = V_0 + 0.9$ V

Volume $A'B'C'D' = V_0(1 + \gamma_\ell\Delta\theta) + V(1 + \gamma_s\Delta\theta)$

The liquid level will not change if volume $ABCD$ of the container expands to be equal to volume $A'B'C'D'$

$$\Rightarrow (V_0 + 0.9V)(1 + \gamma_g\Delta\theta) = V_0(1 + \gamma_\ell\Delta\theta) + V(1 + \gamma_s\Delta\theta)$$

$$\Rightarrow (V_0 + 0.9V)(1 + \gamma_g\Delta\theta) = 0.9V_0(1 + \gamma_s\Delta\theta) + V(1 + \gamma_s\Delta\theta) \quad \text{[using (ii)]}$$

$$\Rightarrow (V_0 + 0.9V)(1 + \gamma_g\Delta\theta) = (0.9V_0 + V)(1 + \gamma_s\Delta\theta)$$

$$\Rightarrow (V_0 + 0.9V) - (0.9V_0 + V) = [(0.9V_0 + V)\gamma_s - (V_0 + 0.9V)\gamma_g]\Delta\theta$$

$$\Rightarrow \frac{0.1(V_0 - V)}{(0.9V_0 + V)\gamma_s - (V_0 + 0.9V)\gamma_s} = \Delta\theta$$

19. (a) Let the length of the rod be L at temperature θ. A small change in temperature by $d\theta$ will cause the length to change by $dL = \alpha L d\theta$

$$\Rightarrow \quad \int_{L_0}^{L} \frac{dL}{L} = \alpha \int_{\theta_0}^{\theta} d\theta$$

$$\Rightarrow \quad \ell n\left(\frac{L}{L_0}\right) = \alpha(\theta - \theta_0)$$

$$\Rightarrow \quad L = L_0 e^{\alpha(\theta - \theta_0)}$$

(b) It is known that $e^x = 1 + x + \frac{x^2}{2!} + \ldots$

When x is small: $e^x \simeq 1 + x$

$$L = L_0 e^{\alpha(\theta - \theta_0)} \qquad \therefore \quad L \simeq L_0[1 + \alpha(\theta - \theta_0)]$$

20.

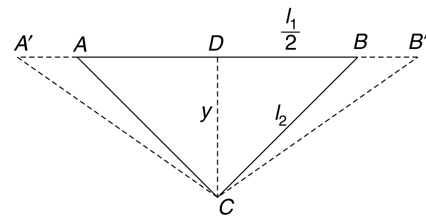
$$y^2 = \ell_2^2 - \frac{\ell_1^2}{4}$$

$$2y\Delta y = 2\ell_2\Delta\ell_2 - \frac{2}{4}\ell_1\Delta\ell_1$$

For $\quad \Delta y = 0$

$$4\ell_2\Delta\ell_2 = \ell_1\Delta\ell_1 \quad \Rightarrow \quad 4\ell_2^2\alpha_2\Delta T = \ell_1^2\alpha_1\Delta T$$

$$\frac{\ell_2}{\ell_1} = \frac{1}{2}\sqrt{\frac{\alpha_1}{\alpha_2}}$$



21. Moment of inertia of the rod about the rotation axis is $I_0 = \frac{1}{3}Ml_0^2$

When temperature rises, l changes and hence I change

$$I = \frac{1}{3}Ml^2$$

But $\quad l = l_0(1 + \alpha\Delta\theta)$

$$l^2 = l_0^2(1 + \alpha\Delta\theta)^2 \simeq l_0^2(1 + 2\alpha\Delta\theta)$$

$$\Rightarrow \quad I = I_0(1 + 2\alpha\Delta\theta)$$

Conservation of angular momentum gives $L = L_0$

Rotational KE before and after heating are

$$k_0 = \frac{1}{2}I_0\omega_0^2 = \frac{L_0^2}{2I_0}$$

And $\quad k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

$$\therefore \quad \frac{k}{k_0} = \frac{I_0}{I} \qquad \therefore \quad \frac{k}{k_0} = \frac{1}{1 + 2\alpha\Delta\theta}$$

$$\frac{k}{k_0} = [1 + 2\alpha\Delta\theta]^{-1} \simeq [1 - 2\alpha\Delta\theta]$$

$$\Rightarrow \quad \frac{k}{k_0} - 1 = -2\alpha\Delta\theta \qquad \Rightarrow \quad \frac{k - k_0}{k_0} = -2\alpha\Delta\theta$$

$$\Rightarrow \quad \frac{\Delta k}{k_0} \times 100 = -200\,\alpha\Delta\theta\%$$

22. (a) Volume of kerosene that spills out

$$\Delta V = \Delta V_k + \Delta V_w - \Delta V_s$$

$$= \frac{V_0}{2}\gamma_k\Delta\theta + \frac{V_0}{2}\gamma_w\Delta\theta - V_0\gamma_s\Delta\theta = \frac{V_0\Delta\theta}{2}(\gamma_k + \gamma_w - 2\gamma_s)$$

$$= \frac{100 \times 30}{2} [10^{-3} + 0.2 \times 10^{-3} - 2 \times 0.012 \times 10^{-3}]$$

$$= 1.5 [1.2 - 0.024] = 1.76 \text{ litre}$$

Density of kerosene at θ_2 is

$$\rho_2 = \frac{\rho_1}{1 + \gamma_k(\theta_2 - \theta_1)} = \frac{0.8}{1 + 10^{-3} \times 30} = \frac{0.8}{1.03} = 0.78 \text{ kg/litre}$$

$\therefore$ Mass of K.oil that flows out is

$$\Delta m = \Delta V \rho_2 = \frac{V_0 \rho_1}{2} \frac{(\gamma_k + \gamma_w - 2\gamma_s)}{1 + \gamma_k(\theta_2 - \theta_1)} (\theta_2 - \theta_1)$$

$$= 0.78 \times 1.76 = 1.37 \text{ kg}$$

(b) Volume of water at θ_2 is $V = \frac{V_0}{2} [1 + \gamma_w \Delta\theta]$ [where $\Delta\theta = \theta_2 - \theta_1$]

The area of cross section of the tank at θ_1 is $A_1 = \frac{V_0}{2H_1}$

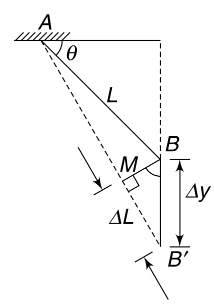
At θ_2 cross section will be $A_2 = A_1[1 + \beta\Delta\theta] = A_1\left[1 + \frac{2}{3}\gamma_s\Delta\theta\right] = \frac{V_0}{2H_1}\left[1 + \frac{2}{3}\gamma_s\Delta\theta\right]$

Height of water column is

$$H_2 = \frac{V}{A_2} = \frac{\frac{V_0}{2}[1 + \gamma_w\Delta\theta]}{\frac{V_0}{2H_1}\left[1 + \frac{2}{3}\gamma_s\Delta\theta\right]}$$

$$= H_1\left[\frac{1 + \gamma_w\Delta\theta}{1 + \frac{2}{3}\gamma_s\Delta\theta}\right] = \frac{1 + 2 \times 10^{-4} \times 30}{1 + \frac{2}{3} \times 1.2 \times 10^{-5} \times 30} = 1.0057 \text{ m}$$

23. Let change in temperature be ΔT



Length of a string changes by $\Delta L = L\alpha_2\Delta T$

The wooden plank descends by $\Delta y = \frac{\Delta L}{\sin\theta}$

$$\Delta y = 2\Delta L \quad \left[\because \sin\theta = \frac{1}{2}\right]$$

Change in radius of the ball: $\Delta R = R\alpha_1\Delta T$

The centre of the ball will not move if $\Delta y = \Delta R$

$$\Rightarrow \quad 2L\alpha_2\Delta T = R\alpha_1\Delta T \Rightarrow 8R\alpha_2 = R\alpha_1 \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{8}{1}$$

24. Length of object at 50°C is

$$L = 100[1 + 10^{-4} \times 50] \text{ mm} = 100.50 \text{ mm}$$

$$1\ MSD \text{ at } 50°\text{C} = 1[1 + 10^{-3} \times 50] = 1.050 \text{ mm}$$

$$1\ VSD \text{ at } 50°\text{C} = 0.9[1 + 10^{-3} \times 50] = 0.945 \text{ mm}$$

Least count $LC = 1\ MSD - 1\ VSD = 0.105$ mm

$$L = 95 \times MSD + 0.75 \text{ mm}$$

$$\therefore \quad MS \text{ reading} = 95$$

$$VS \text{ reading} = \left[\frac{0.75}{0.105}\right] = 7$$

(b) wooden scale reading = 100 ± 1 mm

25. In equilibrium the torque on the rod about the hinge is zero.

$$\therefore \quad Mg\,\frac{L}{2}\sin\theta = F_B\left(\frac{h\sec\theta}{2}\right)\sin\theta$$

Where buoyancy is $\quad F_B = A\,h\sec\theta\cdot\rho\cdot g$

A = area of cross section of rod, ρ = density of water, M = mass of the rod

$$\therefore \quad MgL = A\,(h\sec\theta)^2\rho g \quad \Rightarrow \quad \cos^2\theta = \frac{A\rho h_2}{ML}$$

$$\Rightarrow \quad \cos\theta = \left(\frac{A\rho}{ML}\right)^{1/2} h \quad \Rightarrow \quad y = \left(\frac{A\rho}{ML}\right)^{1/2}\frac{V}{A_0}$$

V = volume of water, A_0 = cross section of tank

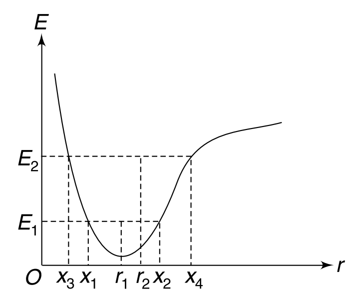
$$\therefore \quad \frac{\Delta y}{y} = \frac{1}{2}\frac{\Delta A}{A} + \frac{1}{2}\frac{\Delta\rho}{\rho} - \frac{1}{2}\frac{\Delta L}{L} + \frac{\Delta V}{V} - \frac{\Delta A_0}{A_0}$$

$$= \frac{1}{2}(2\alpha_1\Delta T) + \frac{1}{2}(-\gamma\Delta T) - \frac{1}{2}(\alpha_1\Delta T) + (\gamma\Delta T) - (2\alpha_2\Delta T)$$

$$= \frac{1}{2}(\alpha_1 + \gamma - 4\alpha_2)\,\Delta T$$

If θ increases y (= $\cos\theta$) will decreases. This is possible if $\Delta y < 0 \Rightarrow 4\alpha_2 > \alpha_1 + \gamma$

26. With rise in temperature the energy rises and atoms oscillate with higher amplitude.



If energy is E_1 at a temperature T_1, the inter atomic separation oscillates between x_1 to x_2 and the mean separation is r_1.

If temperature rises to T_2 the energy becomes higher at E_2. The atomic separation now oscillates between x_3 and x_4 with atoms spending more time at greater distances (due to reduced force as can be seen from the graph). Thus the average separation r_2 becomes higher than r_1 and the material expands.

27. Let m = mass of Hg; L_0 = 1.0 m = Length of metal rod

h_0 = height of Hg

A_0 = area of cross section of the tube.

Position of COM from top end of metal rod is $y^0_{\text{cm}} = L_0 - \dfrac{h_0}{2}$

When temperature increases by ΔT

$$y^T_{\text{cm}} = L^T - \frac{h^T}{2}$$

$$= L_0[1 + \alpha_{\text{metal}}\Delta T] - \frac{1}{2}\frac{[A_0h_0]\,[1 + \gamma_m\Delta T]}{A_0[1 + 2\alpha_g\Delta T]}$$

$$\simeq L_0[1 + \alpha_{\text{metal}}\Delta T] - \frac{h_0}{2}[1 + (\gamma_m - 2\alpha_g)\,\Delta T]$$

$$[\because \quad (1 + \gamma_m\Delta T)\,(1 + 2\alpha_g\Delta T)^{-1} \simeq (1 + \gamma_m\Delta T)\,(1 - 2\alpha_g\Delta T)$$

$$= 1 + \gamma_m\Delta T - 2\alpha_g\Delta T - 2\alpha_g\gamma_m\Delta T^2 \simeq 1 + (\gamma_m - 2\alpha_g)\,\Delta T]$$

$$y^T_{\text{cm}} = y^0_{\text{cm}} + L_0\alpha_{\text{metal}}\,\Delta T - \frac{h_0}{2}(\gamma_m - 2\alpha_g)\,\Delta T$$

For $\quad y^T_{\text{cm}} = y^0_{\text{cm}}$

$$L_0\alpha_{\text{metal}} = \frac{h_0}{2}(\gamma_m - 2\alpha_g)$$

$$\therefore \quad h_0 = \frac{2L_0\,\alpha_{\text{metal}}}{\gamma_m - 2\alpha_g} = 0.146 \text{ m}$$

28. (a) Let the required distance be ℓ_1. The lower part AP of the rod has mass $\frac{m}{L}\ \ell_1$ and the upper part BP has mass $\frac{m}{L}\ (L - \ell_1)$. When heated, the part AP will be moving down and friction on it (f_1) will be up the incline. The part BP will move up and friction on it will be down (say f_2).

For equilibrium of the entire rod

$$f_1 - f_2 = mg\sin\theta$$

$$\mu\frac{m}{L}\ell_1 g\cos\theta - \mu\frac{m}{L}(L - \ell_1)\ g\cos\theta = mg\sin\theta$$

$$\Rightarrow \qquad \frac{\mu}{L}(\ell_1 - L + \ell_1) = \tan\theta \quad \Rightarrow \quad 2\ell_1 = L + \frac{L}{\mu}\tan\theta$$

$$\Rightarrow \qquad \ell_1 = \frac{L}{2}\left[1 + \frac{\tan\theta}{\mu}\right] = 0.875\ \mathrm{L}$$

(b) In this case the friction on upper part will be up and that on the lower part will be down. Proceeding in similar way as above we can get

$$\ell_2 = \frac{L}{2}\left[1 - \frac{\tan\theta}{\mu}\right] = 0.125\ \mathrm{L}$$

(c) During expansion P is fixed but Q moves down. Then during contraction Q remains fixed. Therefore, in one cycle the point Q has moved down.