QUADRATIC EQUATIONS
MAIN CONCEPTS AND RESULTS

**Quadratic equation : A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

** Roots of a quadratic equation : A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

** The roots of the quadratic equation $ax^2 + bx + c = 0$ are the same as the zeroes of the quadratic polynomial $ax^2 + bx + c$.

** Finding the roots of a quadratic equation by the method of factorisation : If we can factorise the quadratic polynomial $ax^2 + bx + c$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating to zero the linear factors of $ax^2 + bx + c$.

** Quadratic Formula : If $b^2 - 4ac \geq 0$, then the real roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**The expression $b^2 - 4ac$ is called the discriminant of the quadratic equation.

**Existence of roots of a quadratic equation: A quadratic equation ax2+bx+c=0 has

(i) two distinct real roots if $b^2 - 4ac > 0$

(ii) two equal real roots (i.e., coincident roots) if $b^2 - 4ac = 0$

(iii) no real roots if $b^2 - 4ac < 0$.

QUESTIONS FROM NCERT BOOKS

1. Check whether the following are quadratic equations :

(i) $(2x - 1)(x - 3) = (x + 5)(x - 1)$, (ii) $(x + 2)3 = 2x (x^2 - 1)$

2. Represent the following situation in the form of quadratic equation : The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

3. Represent the following situation in the form of quadratic equation : The product of two consecutive positive integers is 306. We need to find the integers.

4. Represent the following situation in the form of quadratic equation : Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

5. Represent the following situation in the form of quadratic equation : A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

6. Represent the following situation in the form of quadratic equation : John and Jivanti together have 45

marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

7. Represent the following situation in the form of quadratic equation : A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

8. Find the roots of the quadratic equation : $6x^2 - x - 2 = 0$.

9. Find the roots of the quadratic equation : $100\,x^2 - 20x + 1 = 0$.

10. Find the roots of the quadratic equation : $2x^2 - x + \frac{1}{8} = 0$.

11. Find the roots of the quadratic equation: $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$.

12. Find the roots of the quadratic equation : $3x^2 - 2\sqrt{6}\,x + 2 = 0$.

13. Find the roots of the quadratic equation : $x - \frac{1}{x} = 3,\ x \neq 0$.

14. Find the roots of the quadratic equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$.

15. Find the nature of the roots of the quadratic equation $2x^2 - 3x + 5 = 0$. If the real roots exist, find them.

16. Find the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$. If the real roots exist, find the

17. Find the values of k for each of the quadratic equation $kx(x - 2) + 6 = 0$, so that they have two equal roots.

18. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

19. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

20. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

21. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

22. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

23. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

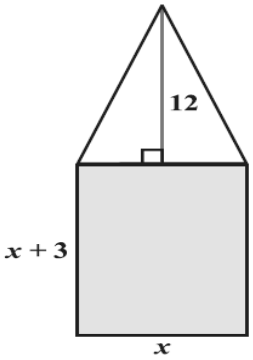
24. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than

the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

25. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

26. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

27. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.



28. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

ANSWERS

1. (i) YES (ii) NO

2. $2x^2 + x - 528 = 0$, where x is breadth (in metres) of the plot.

3. $x^2 + 32x - 273 = 0$, where x (in years) is the present age of Rohan.

4. $x^2 + x - 306 = 0$, where x is the smaller integer.

5. $x^2 - 8x - 1280 = 0$, where x (in km/h) is the speed of the train.

6. $x^2 - 45x + 324 = 0$, where x is the number of marbles John had.

7. $x^2 - 55x + 750 = 0$, where x is the number of toys produced.

8. $\frac{2}{3}, -\frac{1}{2}$ **9.** $\frac{1}{10}, \frac{1}{10}$ **10.** $\frac{1}{4}, \frac{1}{4}$

11. $-\frac{5}{\sqrt{2}}, -\sqrt{2}$ **12.** $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ **13.** $\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}$

14. 1, 2 **15.** Real roots do not exist **16.** Distinct roots; $\frac{3\pm\sqrt{3}}{2}$

17. k = 6 **18.** 5 cm and 12 cm

19. Number of articles = 6, Cost of each article = Rs 15

20. Marks in mathematics = 12, marks in English = 18;
or, Marks in mathematics = 13, marks in English = 17

21. 120 m, 90 m **22.** 18, 12 or 18, –12 **23.** 40 km/h

24. 15 hours, 25 hours

25. Speed of the passenger train = 33 km/h, speed of express train = 44 km/h

26. 18 m, 12 m **27.** Length = 7m and breadth = 4m. **28.** 6 km/h.

ADDITIONAL QUESTIONS

1. Solve: $6x^2 + 40 = 31x$.

2. Solve: $4\sqrt{3}\ x^2 + 5x - 2\sqrt{3} = 0$.

3. Solve the equation: $4x^2 - 4ax + (a^2 - b^2) = 0$.

4. Solve the equation: $\frac{1}{(x+4)} - \frac{1}{(x-7)} = \frac{11}{30},\ x \neq -4, 7$

5. Solve the equation: $x^2 - 2ax - (4b^2 - a^2) = 0$

6. Show that the equation $2x^2 + 5\sqrt{3}\ x + 6 = 0$ has real roots and solve it.

7. Using quadratic formula, solve for x: $abx^2 + (b^2 - ac)x - bc = 0$.

8. If the quadratic equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

9. If a and b are real and $a \neq b$ then show that the roots of the equation $(a - b)x2 + 5(a + b)x - 2(a - b) = 0$ are real and unequal.

10. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

11. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

12. The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

13. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

14. The total cost of a certain length of a piece of wire is ₹200. If the piece was 5 metres longer and each metre of wire costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

15. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

ANSWERS

1. $\frac{8}{3}, \frac{5}{2}$

2. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

3. $\frac{a+b}{2}, \frac{a-b}{2}$

4. 2, 1

5. x = (a – 2b) or x = (a + 2b)

6. $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

7. $\frac{c}{a}, -\frac{b}{a}$

11. 13 and 15.

12. 14 and 21.

13. (18 and 12) or (18 and –12).

14. ₹ 10 per m.

15. Sister's present age = 6 years and girl's present age = 12 years.