

# Applications of Integrals



## TOPIC 1

**Curve & X-axis Between two Ordinates, Area of the Region Bounded by a Curve & Y-axis Between two Abscissa**



1. The area (in sq. units) of the region  $A = \{(x, y) : (x-1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$ , where  $[t]$  denotes the greatest integer function, is :  
[Sep. 05, 2020 (II)]
- (a)  $\frac{8}{3}\sqrt{2} - \frac{1}{2}$  (b)  $\frac{4}{3}\sqrt{2} + 1$   
(c)  $\frac{8}{3}\sqrt{2} - 1$  (d)  $\frac{4}{3}\sqrt{2} - \frac{1}{2}$
2. The area (in sq. units) of the region  $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\}$  is :  
[Sep. 03, 2020 (I)]
- (a)  $\frac{23}{16}$  (b)  $\frac{79}{24}$  (c)  $\frac{79}{16}$  (d)  $\frac{23}{6}$
3. Given:  $f(x) = \begin{cases} x & , 0 \leq x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \leq 1 \end{cases}$   
and  $g(x) = \left(x - \frac{1}{2}\right)^2$ ,  $x \in \mathbb{R}$ . Then the area (in sq. units) of the region bounded by the curves,  $y = f(x)$  and  $y = g(x)$  between the lines,  $2x = 1$  and  $2x = \sqrt{3}$ , is :  
[Jan. 9, 2020 (II)]
- (a)  $\frac{1}{3} + \frac{\sqrt{3}}{4}$  (b)  $\frac{\sqrt{3}}{4} - \frac{1}{3}$  (c)  $\frac{1}{2} - \frac{\sqrt{3}}{4}$  (d)  $\frac{1}{2} + \frac{\sqrt{3}}{4}$
4. The area (in sq. units) of the region  $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid 0 \leq x \leq 3, 0 \leq y \leq 4 - y \leq x^2 + 3x\}$  is :  
[April 8, 2019 (I)]
- (a)  $\frac{53}{6}$  (b) 8 (c)  $\frac{59}{6}$  (d)  $\frac{26}{3}$
5. The area of the region  $A = \{(x, y) : 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq 1\}$  in sq. units is :  
[Jan. 09, 2019 (II)]
- (a)  $\frac{2}{3}$  (b) 2 (c)  $\frac{4}{3}$  (d)  $\frac{1}{3}$
6. Let  $g(x) = \cos x^2$ ,  $f(x) = \sqrt{x}$ , and  $\alpha, \beta$  ( $\alpha < \beta$ ) be the roots of the quadratic equation  $18x^2 - 9\pi x + \pi^2 = 0$ . Then the area (in sq. units) bounded by the curve  $y = (g \circ f)(x)$  and the lines  $x = \alpha, x = \beta$  and  $y = 0$ , is : [2018]
- (a)  $\frac{1}{2}(\sqrt{3} + 1)$  (b)  $\frac{1}{2}(\sqrt{3} - \sqrt{2})$   
(c)  $\frac{1}{2}(\sqrt{2} - 1)$  (d)  $\frac{1}{2}(\sqrt{3} - 1)$
7. Let  $f: [-2, 3] \rightarrow [0, \infty)$  be a continuous function such that  $f(1-x) = f(x)$  for all  $x \in [-2, 3]$ . If  $R_1$  is the numerical value of the area of the region bounded by  $y = f(x)$ ,  $x = -2$ ,  $x = 3$  and the axis of  $x$  and  $R_2 = \int_{-2}^3 x f(x) dx$ , then : [Online April 25, 2013]
- (a)  $3R_1 = 2R_2$  (b)  $2R_1 = 3R_2$   
(c)  $R_1 = R_2$  (d)  $R_1 = 2R_2$
8. Let  $f(x)$  be a non-negative continuous function such that the area bounded by the curve  $y = f(x)$ ,  $x$ -axis and the ordinates  $x = \frac{\pi}{4}$  and  $x = \beta > \frac{\pi}{4}$  is  $\left(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2}\beta\right)$ . Then  $f\left(\frac{\pi}{2}\right)$  is [2005]

(a)  $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$       (b)  $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$

(c)  $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$       (d)  $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

9. The area enclosed between the curve  $y = \log_e(x + e)$  and the coordinate axes is [2005]

(a) 1      (b) 2      (c) 3      (d) 4

10. If  $y = f(x)$  makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of  $3/4$  square unit with the axes

then  $\int_0^2 xf'(x)dx$  is [2002]

(a)  $3/2$       (b) 1      (c)  $5/4$       (d)  $-3/4$

TOPIC 2

Different Cases of Area Bounded Between the Curves



11. The area (in sq. units) of the region  $A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$  is: [Sep. 06, 2020 (I)]

(a)  $\frac{1}{3}$       (b)  $\frac{7}{6}$       (c)  $\frac{1}{6}$       (d)  $\frac{5}{6}$

12. The area (in sq. units) of the region enclosed by the curves  $y = x^2 - 1$  and  $y = 1 - x^2$  is equal to: [Sep. 06, 2020 (II)]

(a)  $\frac{4}{3}$       (b)  $\frac{8}{3}$       (c)  $\frac{7}{2}$       (d)  $\frac{16}{3}$

13. Consider a region  $R = \{(x, y) \in \mathbf{R}^2 : x^2 \leq y \leq 2x\}$ . If a line  $y = \alpha$  divides the area of region  $R$  into two equal parts, then which of the following is true? [Sep. 02, 2020 (II)]

(a)  $\alpha^3 - 6\alpha^2 + 16 = 0$       (b)  $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$   
(c)  $3\alpha^2 - 8\alpha + 8 = 0$       (d)  $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

14. The area of the region, enclosed by the circle  $x^2 + y^2 = 2$  which is not common to the region bounded by the parabola  $y^2 = x$  and the straight line  $y = x$ , is:

[Jan. 7, 2020 (I)]

(a)  $(24\pi - 1)$       (b)  $(6\pi - 1)$   
(c)  $(12\pi - 1)$       (d)  $(12\pi - 1)/6$

15. The area (in sq. units) of the region  $\{(x, y) \in \mathbf{R}^2 : 4x^2 \leq y \leq 8x + 12\}$  is: [Jan. 7, 2020 (II)]

(a)  $\frac{125}{3}$       (b)  $\frac{128}{3}$       (c)  $\frac{124}{3}$       (d)  $\frac{127}{3}$

16. For  $a > 0$ , let the curves  $C_1: y^2 = ax$  and  $C_2: x^2 = ay$  intersect at origin  $O$  and a point  $P$ . Let the line  $x = b$  ( $0 < b < a$ ) intersect the chord  $OP$  and the x-axis at points  $Q$  and  $R$ , respectively. If the line  $x = b$  bisects the area bounded by

the curves,  $C_1$  and  $C_2$ , and the area of  $\Delta OQR = \frac{1}{2}$ , then

'a' satisfies the equation: [Jan. 8, 2020 (I)]

(a)  $x^6 - 6x^3 + 4 = 0$       (b)  $x^6 - 12x^3 + 4 = 0$   
(c)  $x^6 + 6x^3 - 4 = 0$       (d)  $x^6 - 12x^3 - 4 = 0$

17. The area (in sq. units) of the region  $\{(x, y) \in \mathbf{R}^2 : x^2 \leq y \leq |3 - 2x|\}$ , is: [Jan. 8, 2020 (II)]

(a)  $\frac{32}{3}$       (b)  $\frac{34}{3}$       (c)  $\frac{29}{3}$       (d)  $\frac{31}{3}$

18. If the area (in sq. units) of the region  $\{(x, y) : y^2 \leq 4x, x + y \leq 1, x \geq 0, y \geq 0\}$  is  $a\sqrt{2} + b$ , then  $a - b$  is equal to: [April 12, 2019 (I)]

(a)  $\frac{10}{3}$       (b) 6      (c)  $\frac{8}{3}$       (d)  $-\frac{2}{3}$

19. If the area (in sq. units) bounded by the parabola  $y^2 = 4\lambda x$  and the line  $y = \lambda x$ ,  $\lambda > 0$ , is  $\frac{1}{9}$ , then  $\lambda$  is equal to:

[April 12, 2019 (II)]

(a)  $2\sqrt{6}$       (b) 48      (c) 24      (d)  $4\sqrt{3}$

20. The region represented by  $|x - y| \leq 2$  and  $|x + y| \leq 2$  is bounded by a: [April 10, 2019 (I)]

(a) square of side length  $2\sqrt{2}$  units  
(b) rhombus of side length 2 units  
(c) square of area 16 sq. units  
(d) rhombus of area  $8\sqrt{2}$  sq. units

21. The area (in sq. units) of the region bounded by the curves  $y = 2^x$  and  $y = |x + 1|$ , in the first quadrant is:

[April 10, 2019 (II)]

(a)  $\log_e 2 + \frac{3}{2}$       (b)  $\frac{3}{2}$

(c)  $\frac{1}{2}$       (d)  $\frac{3}{2} - \frac{1}{\log_e 2}$

22. The area (in sq. units) of the region  $A = \{(x, y) : x^2 \leq y \leq x + 2\}$  is: [April 9, 2019 (I)]

(a)  $\frac{10}{3}$       (b)  $\frac{9}{2}$       (c)  $\frac{31}{6}$       (d)  $\frac{13}{6}$

23. The area (in sq. units) of the region

$A = \{(x, y) : \frac{y^2}{2} \leq x \leq y + 4\}$  is: [April 09, 2019 (II)]

(a)  $\frac{53}{3}$       (b) 30      (c) 16      (d) 18

24. Let  $S(\alpha) = \{(x, y) : y^2 \leq x, 0 \leq x \leq \alpha\}$  and  $A(\alpha)$  is area of the region  $S(\alpha)$ . If for a  $\lambda$ ,  $0 < \lambda < 4$ ,  $A(\lambda) : A(\alpha) = 2 : 5$ , then  $\lambda$  equals: [April 08, 2019 (II)]

(a)  $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$       (b)  $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

(c)  $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$       (d)  $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

25. The area (in sq. units) of the region bounded by the parabola,  $y = x^2 + 2$  and the lines,  $y = x + 1$ ,  $x = 0$  and  $x = 3$ , is :  
[Jan. 12, 2019 (I)]
- (a)  $\frac{15}{4}$  (b)  $\frac{21}{2}$  (c)  $\frac{17}{4}$  (d)  $\frac{15}{2}$
26. The area (in sq. units) of the region bounded by the curve  $x^2 = 4y$  and the straight line  $x = 4y - 2$  is :  
[Jan. 11, 2019 (I)]
- (a)  $\frac{5}{4}$  (b)  $\frac{9}{8}$  (c)  $\frac{7}{8}$  (d)  $\frac{3}{4}$
27. The area (in sq. units) in the first quadrant bounded by the parabola,  $y = x^2 + 1$ , the tangent to it at the point (2, 5) and the coordinate axes is :  
[Jan. 11, 2019 (II)]
- (a)  $\frac{8}{3}$  (b)  $\frac{37}{24}$  (c)  $\frac{187}{24}$  (d)  $\frac{14}{3}$
28. If the area enclosed between the curves  $y = kx^2$  and  $x = ky^2$ , ( $k > 0$ ), is 1 square unit. Then  $k$  is:  
[Jan. 10, 2019 (I)]
- (a)  $\frac{\sqrt{3}}{2}$  (b)  $\frac{1}{\sqrt{3}}$  (c)  $\sqrt{3}$  (d)  $\frac{2}{\sqrt{3}}$
29. The area (in sq. units) bounded by the parabola  $y = x^2 - 1$ , the tangent at the point (2, 3) to it and the  $y$ -axis is:  
[Jan. 9, 2019 (I)]
- (a)  $\frac{8}{3}$  (b)  $\frac{32}{3}$  (c)  $\frac{56}{3}$  (d)  $\frac{14}{3}$
30. If the area of the region bounded by the curves,  $y = x^2$ ,  $y = \frac{1}{x}$  and the lines  $y = 0$  and  $x = t$  ( $t > 1$ ) is 1 sq. unit, then  $t$  is equal to  
[Online April 16, 2018]
- (a)  $\frac{4}{3}$  (b)  $e^{2/3}$  (c)  $\frac{3}{2}$  (d)  $e^{3/2}$
31. The area (in sq. units) of the region  $\{x \in R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$ , is  
[Online April 15, 2018]
- (a)  $\frac{13}{3}$  (b)  $\frac{10}{3}$  (c)  $\frac{5}{3}$  (d)  $\frac{8}{3}$
32. The area (in sq. units) of the region  $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$  is :  
[2017]
- (a)  $\frac{5}{2}$  (b)  $\frac{59}{12}$  (c)  $\frac{3}{2}$  (d)  $\frac{7}{3}$
33. The area (in sq. units) of the smaller portion enclosed between the curves,  $x^2 + y^2 = 4$  and  $y^2 = 3x$ , is :  
[Online April 8, 2017]
- (a)  $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$  (b)  $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$
- (c)  $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$  (d)  $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$
34. The area (in sq. units) of the region  $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$  is :  
[2016]
- (a)  $\pi - \frac{4\sqrt{2}}{3}$  (b)  $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
- (c)  $\pi - \frac{4}{3}$  (d)  $\pi - \frac{8}{3}$
35. The area (in sq. units) of the region described by  $A = \{(x, y) | y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\}$  is:  
[Online April 9, 2016]
- (a)  $\frac{19}{6}$  (b)  $\frac{17}{6}$  (c)  $\frac{7}{2}$  (d)  $\frac{13}{6}$
36. The area (in sq. units) of the region described by  $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$  is  
[2015]
- (a)  $\frac{15}{64}$  (b)  $\frac{9}{32}$  (c)  $\frac{7}{32}$  (d)  $\frac{5}{64}$
37. The area (in square units) of the region bounded by the curves  $y + 2x^2 = 0$  and  $y + 3x^2 = 1$ , is equal to :  
[Online April 10, 2015]
- (a)  $\frac{3}{5}$  (b)  $\frac{1}{3}$  (c)  $\frac{4}{3}$  (d)  $\frac{3}{4}$
38. The area of the region described by  $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$  is :  
[2014]
- (a)  $\frac{\pi}{2} - \frac{2}{3}$  (b)  $\frac{\pi}{2} + \frac{2}{3}$  (c)  $\frac{\pi}{2} + \frac{4}{3}$  (d)  $\frac{\pi}{2} - \frac{4}{3}$
39. The area of the region above the  $x$ -axis bounded by the curve  $y = \tan x$ ,  $0 \leq x \leq \frac{\pi}{2}$  and the tangent to the curve at  $x = \frac{\pi}{4}$  is:  
[Online April 19, 2014]
- (a)  $\frac{1}{2} \left( \log 2 - \frac{1}{2} \right)$  (b)  $\frac{1}{2} \left( \log 2 + \frac{1}{2} \right)$
- (c)  $\frac{1}{2} (1 - \log 2)$  (d)  $\frac{1}{2} (1 + \log 2)$
40. Let  $A = \{(x, y) : y^2 \leq 4x, y - 2x \geq -4\}$ . The area (in square units) of the region  $A$  is:  
[Online April 9, 2014]
- (a) 8 (b) 9 (c) 10 (d) 11
41. The area (in square units) bounded by the curves  $y = \sqrt{x}$ ,  $2y - x + 3 = 0$ ,  $x$ -axis, and lying in the first quadrant is :  
[2013]
- (a) 9 (b) 36 (c) 18 (d)  $\frac{27}{4}$

42. The area under the curve  $y = |\cos x - \sin x|$ ,  $0 \leq x \leq \frac{\pi}{2}$ , and above  $x$ -axis is : **[Online April 23, 2013]**  
 (a)  $2\sqrt{2}$  (b)  $2\sqrt{2} - 2$   
 (c)  $2\sqrt{2} + 2$  (d) 0
43. The area of the region (in sq. units), in the first quadrant bounded by the parabola  $y = 9x^2$  and the lines  $x = 0$ ,  $y = 1$  and  $y = 4$ , is : **[Online April 22, 2013]**  
 (a)  $\frac{7}{9}$  (b)  $\frac{14}{3}$  (c)  $\frac{7}{3}$  (d)  $\frac{14}{9}$
44. The area bounded by the curve  $y = \ln(x)$  and the lines  $y = 0$ ,  $y = \ln(c)$  and  $x = 0$  is equal to : **[Online April 9, 2013]**  
 (a) 3 (b)  $3 \ln(c) - 2$   
 (c)  $3 \ln(c) + 2$  (d) 2
45. The area between the parabolas  $x^2 = \frac{y}{4}$  and  $x^2 = 9y$  and the straight line  $y = 2$  is : **[2012]**  
 (a)  $20\sqrt{2}$  (b)  $\frac{10\sqrt{2}}{3}$  (c)  $\frac{20\sqrt{2}}{3}$  (d)  $10\sqrt{2}$
46. The area bounded by the parabola  $y^2 = 4x$  and the line  $2x - 3y + 4 = 0$ , in square unit, is **[Online May 26, 2012]**  
 (a)  $\frac{2}{5}$  (b)  $\frac{1}{3}$  (c) 1 (d)  $\frac{1}{2}$
47. The area of the region bounded by the curve  $y = x^3$ , and the lines,  $y = 8$ , and  $x = 0$ , is **[Online May 19, 2012]**  
 (a) 8 (b) 12 (c) 10 (d) 16
48. If a straight line  $y - x = 2$  divides the region  $x^2 + y^2 \leq 4$  into two parts, then the ratio of the area of the smaller part to the area of the greater part is **[Online May 12, 2012]**  
 (a)  $3\pi - 8 : \pi + 8$  (b)  $\pi - 3 : 3\pi + 3$   
 (c)  $3\pi - 4 : \pi + 4$  (d)  $\pi - 2 : 3\pi + 2$
49. The area enclosed by the curves  $y = x^2$ ,  $y = x^3$ ,  $x = 0$  and  $x = p$ , where  $p > 1$ , is  $\frac{1}{6}$ . The  $p$  equals **[Online May 12, 2012]**  
 (a)  $\frac{8}{3}$  (b)  $\frac{16}{3}$  (c) 2 (d)  $\frac{4}{3}$
50. The parabola  $y^2 = x$  divides the circle  $x^2 + y^2 = 2$  into two parts whose areas are in the ratio **[Online May 7, 2012]**  
 (a)  $9\pi + 2 : 3\pi - 2$  (b)  $9\pi - 2 : 3\pi + 2$   
 (c)  $7\pi - 2 : 2\pi - 3$  (d)  $7\pi + 2 : 3\pi + 2$
51. The area bounded by the curves **[2011 RS]**  
 $y^2 = 4x$  and  $x^2 = 4y$  is:  
 (a)  $\frac{32}{3}$  sq units (b)  $\frac{16}{3}$  sq units  
 (c)  $\frac{8}{3}$  sq. units (d) 0 sq. units
52. The area of the region enclosed by the curves  $y = x$ ,  $x = e$ ,  $y = \frac{1}{x}$  and the positive  $x$ -axis is **[2011]**  
 (a) 1 square unit (b)  $\frac{3}{2}$  square units  
 (c)  $\frac{5}{2}$  square units (d)  $\frac{1}{2}$  square unit
53. The area bounded by the curves  $y = \cos x$  and  $y = \sin x$  between the ordinates  $x = 0$  and  $x = \frac{3\pi}{2}$  is **[2010]**  
 (a)  $4\sqrt{2} + 2$  (b)  $4\sqrt{2} - 1$   
 (c)  $4\sqrt{2} + 1$  (d)  $4\sqrt{2} - 2$
54. The area of the region bounded by the parabola  $(y - 2)^2 = x - 1$ , the tangent of the parabola at the point  $(2, 3)$  and the  $x$ -axis is: **[2009]**  
 (a) 6 (b) 9 (c) 12 (d) 3
55. The area of the plane region bounded by the curves  $x + 2y^2 = 0$  and  $x + 3y^2 = 1$  is equal to **[2008]**  
 (a)  $\frac{5}{3}$  (b)  $\frac{1}{3}$  (c)  $\frac{2}{3}$  (d)  $\frac{4}{3}$
56. The area enclosed between the curves  $y^2 = x$  and  $y = |x|$  is **[2007]**  
 (a)  $\frac{1}{6}$  (b)  $\frac{1}{3}$  (c)  $\frac{2}{3}$  (d) 1
57. The parabolas  $y^2 = 4x$  and  $x^2 = 4y$  divide the square region bounded by the lines  $x = 4$ ,  $y = 4$  and the coordinate axes. If  $S_1$ ,  $S_2$ ,  $S_3$  are respectively the areas of these parts numbered from top to bottom; then  $S_1 : S_2 : S_3$  is **[2005]**  
 (a) 1 : 2 : 1 (b) 1 : 2 : 3 (c) 2 : 1 : 2 (d) 1 : 1 : 1
58. The area of the region bounded by the curves  $y = |x - 2|$ ,  $x = 1$ ,  $x = 3$  and the  $x$ -axis is **[2004]**  
 (a) 4 (b) 2 (c) 3 (d) 1
59. The area of the region bounded by the curves  $y = |x - 1|$  and  $y = 3 - |x|$  is **[2003]**  
 (a) 6 sq. units (b) 2 sq. units  
 (c) 3 sq. units (d) 4 sq. units.
60. The area bounded by the curves  $y = \ln x$ ,  $y = \ln |x|$ ,  $y = |\ln x|$  and  $y = |\ln |x||$  is **[2002]**  
 (a) 4sq. units (b) 6 sq. units  
 (c) 10 sq. units (d) none of these

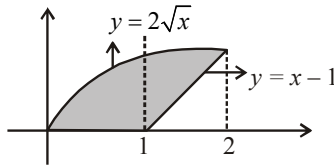


# Hints & Solutions



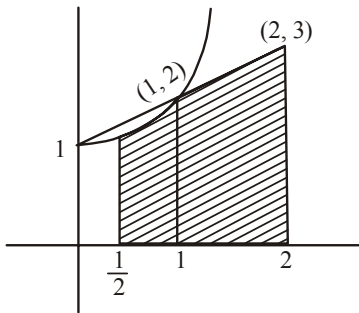
1. (a)  $[x] = 0$  when  $x \in [0, 1)$  and  $[x] = 1$  when  $x \in [1, 2)$

$$y = \begin{cases} 0 & 0 \leq x < 1 \\ x-1 & 1 \leq x < 2 \end{cases}$$



$$\begin{aligned} \therefore A &= \int_0^2 2\sqrt{x} \, dx - \frac{1}{2}(1)(1) \\ &= \frac{4x^{3/2}}{3} \Big|_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2} \end{aligned}$$

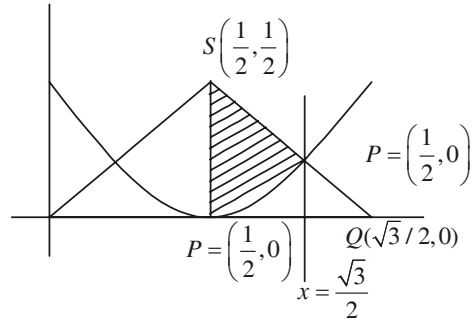
2. (b)



$$\begin{aligned} \text{Required area} &= \int_{\frac{1}{2}}^1 (x^2 + 1) \, dx + \int_1^2 (x + 1) \, dx \\ &= \left[ \frac{x^3}{3} + x \right]_{\frac{1}{2}}^1 + \left[ \frac{x^2}{2} + x \right]_1^2 \\ &= \left[ \frac{4}{3} - \frac{13}{24} \right] + \frac{5}{2} = \frac{79}{24} \end{aligned}$$

3. (b) Coordinates of  $P\left(\frac{1}{2}, 0\right)$ ,  $Q\left(\frac{\sqrt{3}}{2}, 0\right)$ ,  $R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$

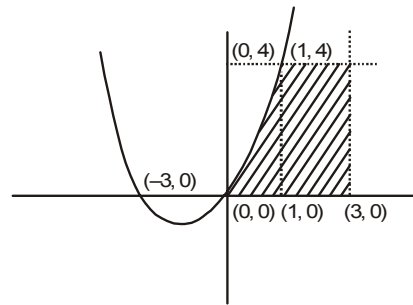
and  $S\left(\frac{1}{2}, \frac{1}{2}\right)$



Required area = Area of trapezium  $PQRS$

$$\begin{aligned} &= \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right) \, dx \\ &= \frac{1}{2} \left(\frac{\sqrt{3}-1}{2}\right) \left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3} \left(\left(x - \frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2} \\ &= \frac{\sqrt{3}}{4} - \frac{1}{3} \end{aligned}$$

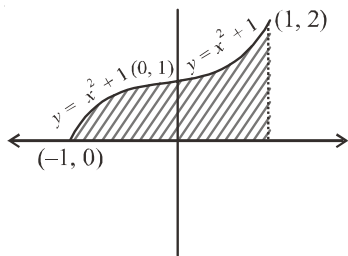
4. (c) Since, the relation  $y \leq x^2 + 3x$  represents the region below the parabola in the 1<sup>st</sup> quadrant



$$\begin{aligned} \therefore y &= 4 \\ \Rightarrow x^2 + 3x &= 4 \Rightarrow x = 1, -4 \\ \therefore \text{the required area} &= \text{area of shaded region} \end{aligned}$$

$$\begin{aligned} &= \int_0^1 (x^2 + 3x) \, dx + \int_1^3 4 \, dx = \left[ \frac{x^3}{3} + \frac{3x^2}{2} \right]_0^1 + [4x]_1^3 \\ &= \frac{1}{3} + \frac{3}{2} + 8 = \frac{59}{6} \end{aligned}$$

5. (b) Given  $A = \{(x, y) : 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq 1\}$



$\therefore$  Area of shaded region  
 $= \int_{-1}^0 (-x^2 + 1) dx + \int_0^1 (x^2 + 1) dx$   
 $= \left(-\frac{x^3}{3} + x\right)_{-1}^0 + \left(\frac{x^3}{3} + x\right)_0^1$   
 $= 0 - \left(\frac{1}{3} - 1\right) + \left(\frac{1}{3} + 1\right) - (0 + 0)$   
 $= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$  square units

6. (d) Here,  $18x^2 - 9\pi x + \pi^2 = 0$   
 $\Rightarrow (3x - \pi)(6x - \pi) = 0$

$\Rightarrow \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

Also,  $\text{gof}(x) = \cos x$

$\therefore$  Req. area  $= \int_{\pi/6}^{\pi/3} \cos x dx = \frac{\sqrt{3} - 1}{2}$

7. (d) We have

$R_2 = \int_{-2}^3 x f(x) dx = \int_{-2}^3 (1-x) f(1-x) dx$   
 $\left[ \text{Using } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$

$\Rightarrow R_2 = \int_{-2}^3 (1-x) f(x) dx$   
 $(\because f(x) = f(1-x) \text{ on } [-2, 3])$

$\therefore R_2 + R_2 = \int_{-2}^3 x f(x) dx + \int_{-2}^3 (1-x) f(x) dx$   
 $= \int_{-2}^3 f(x) dx = R_1$

$\Rightarrow 2R_2 = R_1$

8. (d) From given condition

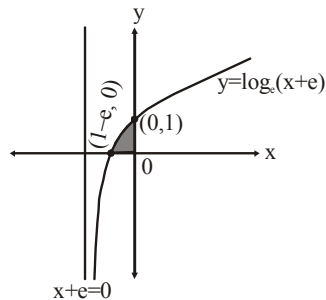
$\int_{\pi/4}^{\beta} f(x) dx = \beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta$

Differentiating w. r. t  $\beta$ , we get

$f(\beta) = \beta \cos \beta + \sin \beta - \frac{\pi}{4} \sin \beta + \sqrt{2}$

$f\left(\frac{\pi}{2}\right) = \beta \cdot 0 + \left(1 - \frac{\pi}{4}\right) \sin \frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$

9. (a)



Required area  $A = \int_{1-e}^0 y dx = \int_{1-e}^0 \log_e(x+e) dx$

put  $x+e = t \Rightarrow dx = dt$  also when  $x = 1-e, t = 1$  and when  $x = 0, t = e$

$\therefore A = \int_1^e \log_e t dt = [t \log_e t - t]_1^e$

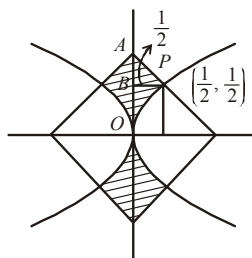
$e - e - 0 + 1 = 1$

Hence the required area is 1 square unit.

10. (d) Given that  $\int_0^2 f(x) dx = \frac{3}{4}$ ; Now,

$\int_0^2 x f'(x) dx = x \int_0^2 f'(x) dx - \int_0^2 f(x) dx$   
 $= [x f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$   
 $= 0 - \frac{3}{4} (\because f(2) = 0) = -\frac{3}{4}$

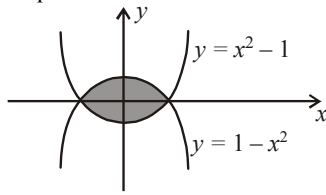
11. (d)



Required area  $= 4 \left[ \int_0^{\frac{1}{2}} 2y^2 dy + \frac{1}{2} \text{area}(\Delta PAB) \right]$

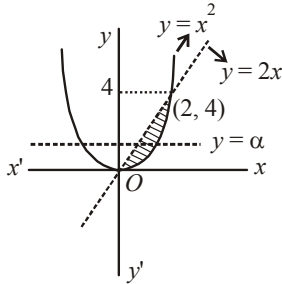
$= 4 \left[ \frac{2}{3} \left[ y^3 \right]_0^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right] = 4 \left[ \frac{2}{3} \times \frac{1}{8} + \frac{1}{8} \right]$   
 $= 4 \times \frac{5}{24} = \frac{5}{6}$

12. (b) Required area



$$\begin{aligned} \text{Area} &= 2 \int_0^1 \left( (1-x^2) - (x^2-1) \right) dx \\ &= 4 \int_0^1 (1-x^2) dx \\ &= 4 \left( x - \frac{x^3}{3} \right) \Big|_0^1 = 4 \left( 1 - \frac{1}{3} \right) = 4 \cdot \frac{2}{3} = \frac{8}{3} \text{ sq. units} \end{aligned}$$

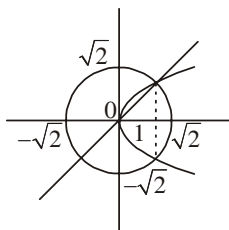
13. (b) Let  $y = x^2$  and  $y = 2x$



According to question

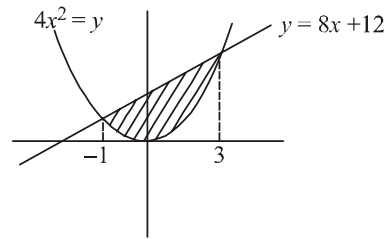
$$\begin{aligned} \therefore \int_0^\alpha \left( \sqrt{y} - \frac{y}{2} \right) dy &= \int_\alpha^4 \left( \sqrt{y} - \frac{y}{2} \right) dy \\ \Rightarrow \left[ \frac{y^{3/2}}{3/2} \right]_0^\alpha - \left[ \frac{y^2}{4} \right]_0^\alpha &= \left[ \frac{y^{3/2}}{3/2} \right]_\alpha^4 - \left[ \frac{y^2}{4} \right]_\alpha^4 \\ \Rightarrow \frac{2}{3} \alpha^{3/2} - \frac{\alpha^2}{4} &= \frac{2}{3} (8 - \alpha^{3/2}) - \frac{1}{4} (16 - \alpha^2) \\ \Rightarrow \frac{4}{3} \alpha^{3/2} - \frac{\alpha^2}{2} &= \frac{4}{3} \\ \Rightarrow 8\alpha^{3/2} - 3\alpha^2 &= 8 \\ \therefore 3\alpha^2 - 8\alpha^{3/2} + 8 &= 0 \end{aligned}$$

14. (d) Total area – enclosed area between line and parabola



$$\begin{aligned} &= 2\pi - \int_0^1 \sqrt{x} - x dx \\ &= 2\pi - \left( \frac{2x^{3/2}}{3} - \frac{x^2}{2} \right) \Big|_0^1 \\ &= 2\pi - \left( \frac{2}{3} - \frac{1}{2} \right) = 2\pi - \left( \frac{1}{6} \right) = \frac{12\pi - 1}{6} \end{aligned}$$

15. (b)



Given curves are

$$\begin{aligned} 4x^2 &= y & \dots(i) \\ y &= 8x + 12 & \dots(ii) \end{aligned}$$

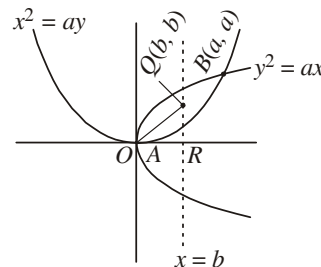
From eqns. (i) and (ii),

$$\begin{aligned} 4x^2 &= 8x + 12 \\ \Rightarrow x^2 - x - 3 &= 0 \\ \Rightarrow x^2 - 2x - 3 &= 0 \\ \Rightarrow x^2 - 3x + x - 3 &= 0 \\ \Rightarrow (x+1)(x-3) &= 0 \\ \Rightarrow x &= -1, 3 \end{aligned}$$

Required area bounded by curves is given by

$$\begin{aligned} A &= \int_{-1}^3 (8x + 12 - 4x^2) dx \\ &= \left[ \frac{8x^2}{2} + 12x - \frac{4x^3}{3} \right]_{-1}^3 \\ &= (4(9) + 36 - 36) - \left( 4 - 12 + \frac{4}{3} \right) \\ &= 36 + 8 - \frac{4}{3} = 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3} \end{aligned}$$

16. (b) Given eqns. are,  $x^2 = ay$  and  $y^2 = ax$



After solving, we get  $x = a, y = a$

Now, coordinates of B is  $(a, a)$  and A is  $(0, 0)$

Now, coordinates of Q is  $(b, b)$

$$\therefore \frac{1}{2}b^2 = \frac{1}{2} \Rightarrow b = 1$$

Area bounded by curves and  $x = 1$  is

$$\int_0^1 \left( \sqrt{ax^{1/2}} - \frac{x^2}{a} \right) dx = \frac{1}{2} \int_0^a \left( \sqrt{ax^{1/2}} - \frac{x^2}{a} \right) dx$$

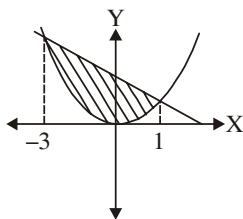
$$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$\Rightarrow 4a\sqrt{a} - 2 = a^3$$

$$\Rightarrow a^6 + 4a^3 + 4 = 16a^3$$

$$\Rightarrow a^6 - 12a^3 + 4 = 0$$

17. (a) Point of intersection of  $y = x^2$  and  $y = -2x + 3$  is obtained by  $x^2 + 2x - 3 = 0$



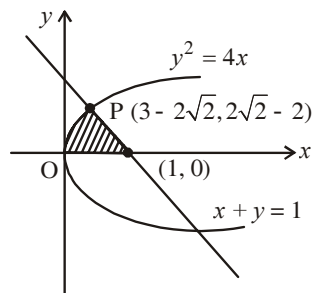
$$\Rightarrow x = -3, 1$$

$$\text{So, required area} = \int_{-3}^1 (\text{line} - \text{parabola}) dx$$

$$= \int_{-3}^1 (3 - 2x - x^2) dx = \left[ 3x - x^2 - \frac{x^3}{3} \right]_{-3}^1$$

$$= (3)4 - 2 \left( \frac{1^2 - 3^2}{2} \right) - \left( \frac{1^3 + 3^3}{3} \right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$$

18. (b) Consider  $y^2 = 4x$  and  $x + y = 1$



Substituting  $x = 1 - y$  in the equation of parabola,

$$y^2 = 4(1 - y) \Rightarrow y^2 + 4y - 4 = 0$$

$$\Rightarrow (y + 2)^2 = 8 \Rightarrow y + 2 = \pm 2\sqrt{2}$$

Hence, required area

$$= \int_0^{3-2\sqrt{2}} 2\sqrt{x} dx + \frac{1}{2} \times (2\sqrt{2} - 2) \times (2\sqrt{2} - 2)$$

$$= \left[ 2 \times \frac{2}{3} x^{3/2} \right]_0^{3-2\sqrt{2}} + \frac{1}{2} (8 + 4 - 8\sqrt{2})$$

$$= \frac{4}{3} \times (3 - 2\sqrt{2}) \sqrt{3 - 2\sqrt{2}} + 6 - 4\sqrt{2}$$

$$= \frac{4}{3} (3 - 2\sqrt{2})(\sqrt{2} - 1) + 6 - 4\sqrt{2}$$

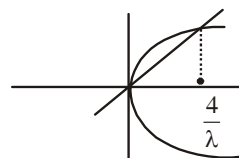
$$[\because (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}]$$

$$= \frac{4}{3} (3\sqrt{2} - 3 - 4 + 2\sqrt{2}) + 6 - 4\sqrt{2}$$

$$= -\frac{10}{3} + \frac{8}{3}\sqrt{2} = a\sqrt{2} + b$$

$$\therefore a = 8/3 \text{ and } b = -10/3 \Rightarrow a - b = \frac{10}{3} + \frac{8}{3} = 6$$

19. (c) Given parabola  $y^2 = 4\lambda x$  and the line  $y = \lambda x$



$$\text{Putting } y = \lambda x \text{ in } y^2 = 4\lambda x, \text{ we get } x = 0, \frac{4}{\lambda}$$

$$\therefore \text{required area} = \int_0^{\frac{4}{\lambda}} (2\sqrt{\lambda x} - \lambda x) dx$$

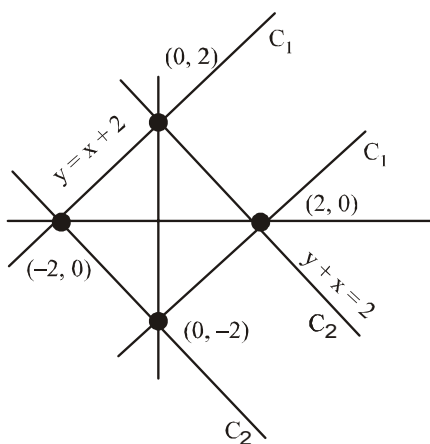
$$= \frac{2\sqrt{\lambda} \cdot x^{3/2}}{3/2} - \frac{\lambda x^2}{2} \Big|_0^{4/\lambda} = \frac{32}{3\lambda} - \frac{8}{\lambda}$$

$$= \frac{8}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$$

20. (a) Let,  $C_1: |y - x| \leq 2$

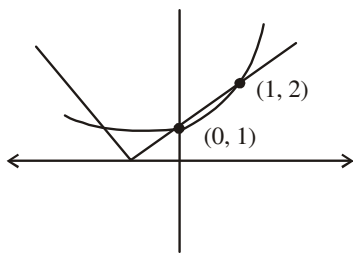
$$C_2: |y + x| \leq 2$$

By the diagram, region is square



Now, length of side =  $\sqrt{2^2 + 2^2} = 2\sqrt{2}$

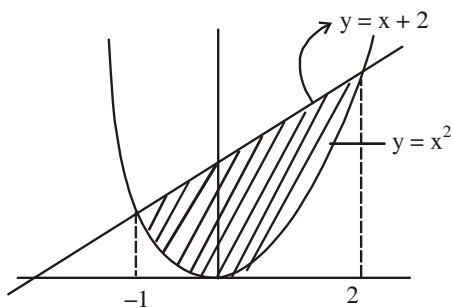
21. (d)



$$\text{Area} = \int_0^1 ((x+1) - 2^x) dx \quad (\because \text{Area} = \int y dx)$$

$$= \left[ \frac{x^2}{2} + x - \frac{2^x}{\ln 2} \right]_0^1 = \left( \frac{1}{2} + 1 - \frac{2}{\ln 2} \right) - \left( \frac{-1}{\ln 2} \right) = \frac{3}{2} - \frac{1}{\ln 2}$$

22. (b)

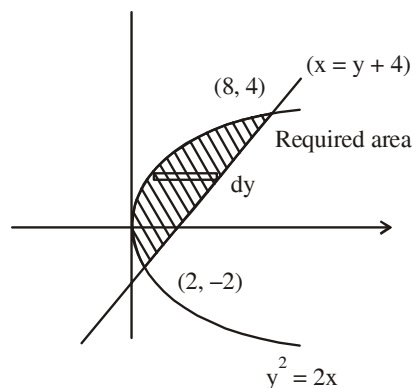


Required area is equal to the area under the curves  $y \geq x^2$  and  $y \leq x + 2$

$$\therefore \text{required area} = \int_{-1}^2 ((x+2) - x^2) dx$$

$$= \left( \frac{x^2}{2} + 2x - \frac{x^3}{3} \right)_{-1}^2 = \left( 2 + 4 - \frac{8}{3} \right) - \left( \frac{1}{2} - 2 + \frac{1}{3} \right) = \frac{9}{2}$$

23. (d)



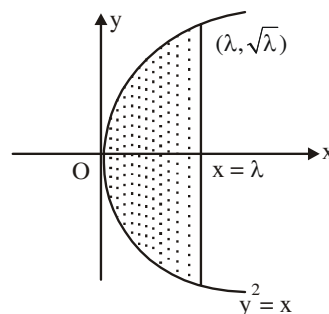
$$\text{Given region, } A = \left\{ (x, y) : \frac{y^2}{2} \leq x \leq y + 4 \right\}$$

$$\text{Hence, area} = \int_{-2}^4 x dy = \int_{-2}^4 \left( y + 4 - \frac{y^2}{2} \right) dy$$

$$= \left[ \frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^4 = \left( 8 + 16 - \frac{64}{6} \right) - \left( 2 - 8 + \frac{8}{6} \right)$$

$$= \left( 24 - \frac{32}{3} \right) - \left( -6 + \frac{4}{3} \right) = \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18$$

24. (d)



$$\text{Area of the region} = 2 \times \int_0^\lambda y dx = 2 \int_0^\lambda \sqrt{x} dx$$

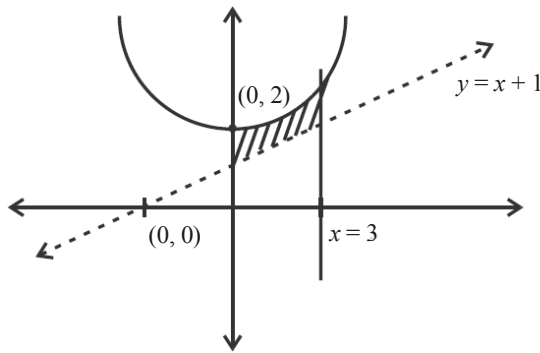
$$= 2 \times \frac{2}{3} \pi^{\frac{3}{2}}$$

$$A(\lambda) = 2 \times \frac{2}{3} (\lambda \times \sqrt{\lambda}) = \frac{4}{3} \lambda^{\frac{3}{2}}$$

$$\text{Given, } \frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{\frac{3}{2}}}{8} = \frac{2}{5}$$

$$\lambda = \left( \frac{16}{5} \right)^{\frac{2}{3}} = 4 \left( \frac{4}{25} \right)^{\frac{1}{3}}$$

25. (d)

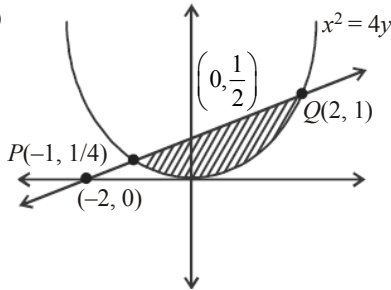


Area of the bounded region  $\int_0^3 [(x^2 + 2) - (x + 1)] dx$

$$= \left[ \frac{x^3}{3} - \frac{x^2}{2} + x \right]_0^3$$

$$= 9 - \frac{9}{2} + 3 = \frac{15}{2}$$

26. (b)



Let points of intersection of the curve and the line be P and Q

$$x^2 = 4\left(\frac{x+2}{4}\right)$$

$$x^2 - x - 2 = 0$$

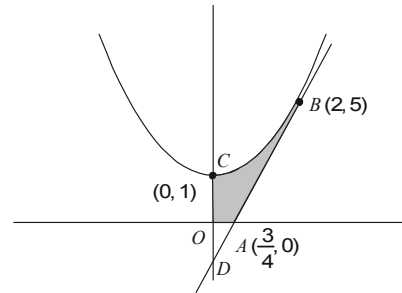
$$x = 2, -1$$

Point are (2, 1) and  $\left(-1, \frac{1}{4}\right)$

$$\text{Area} = \int_{-1}^2 \left[ \left(\frac{x+2}{4}\right) - \left(\frac{x^2}{4}\right) \right] dx = \left[ \frac{x^2}{8} + \frac{1}{2}x - \frac{x^3}{12} \right]_{-1}^2$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{3}\right) - \left(\frac{1}{8} - \frac{1}{2} + \frac{1}{12}\right) = \frac{9}{8}$$

27. (b)



The equation of parabola  $x^2 = y - 1$

The equation of tangent at (2, 5) to parabola is

$$y - 5 = \left(\frac{dy}{dx}\right)_{(2,5)} (x - 2)$$

$$y - 5 = 4(x - 2)$$

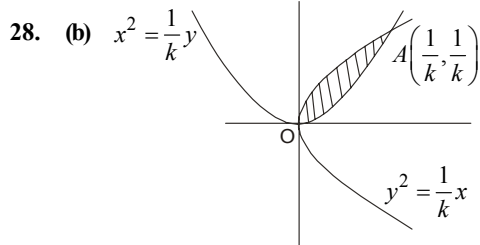
$$4x - y = 3$$

Then, the required area

$$= \int_0^2 \{(x^2 + 1) - (4x - 3)\} dx - \text{Area of } \triangle AOD$$

$$= \int_0^2 (x^2 - 4x + 4) dx - \frac{1}{2} \times \frac{3}{4} \times 3$$

$$= \left[ \frac{(x-2)^3}{3} \right]_0^2 - \frac{9}{8} = \frac{37}{24}$$



28. (b)

Two curves will intersect in the 1st quadrant at  $A\left(\frac{1}{R}, \frac{1}{R}\right)$

$\therefore$  area of shaded region = 1.

$$\therefore \int_0^{\frac{1}{k}} \left( \frac{\sqrt{x}}{\sqrt{k}} - kx^2 \right) dx = 1$$

$$\Rightarrow \left( \frac{1}{\sqrt{k}} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right)_{\frac{1}{k}} - \left( k \cdot \frac{x^3}{3} \right)_{\frac{1}{k}} = 1$$

$$\Rightarrow \frac{2}{3\sqrt{k}} \cdot \frac{1}{k^{\frac{3}{2}}} - \frac{k}{3k^3} = 1$$

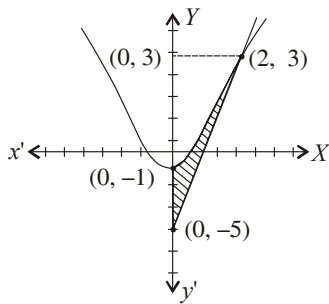
$$\Rightarrow \frac{2}{3k^2} - \frac{1}{3k^2} = 1$$

$$\Rightarrow 3k^2 = 1$$

$$\Rightarrow k = \pm \frac{1}{\sqrt{3}}$$

$$\therefore k = \frac{1}{\sqrt{3}} (\because k > 0)$$

29. (a)



$\therefore$  Curve is given as :

$$y = x^2 - 1$$

$$\Rightarrow \frac{dy}{dx} = 2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = 4$$

$\therefore$  equation of tangent at (2, 3)

$$(y - 3) = 4(x - 2)$$

$$\Rightarrow y = 4x - 5$$

$$\text{but } x = 0$$

$$\Rightarrow y = -5$$

Here the curve cuts Y-axis

$\therefore$  required area

$$= \frac{1}{4} \int_{-5}^3 (y+5) dy - \int_{-1}^3 \sqrt{y+1} dy$$

$$= \frac{1}{4} \left[ \frac{y^2}{2} + 5y \right]_{-5}^3 - \frac{2}{3} \left[ (y+1)^{3/2} \right]_{-1}^3$$

$$= \frac{1}{4} \left[ \frac{9}{2} + 15 - \frac{25}{2} + 25 \right]$$

$$= -\frac{2}{3} [4^{3/2} - 0]$$

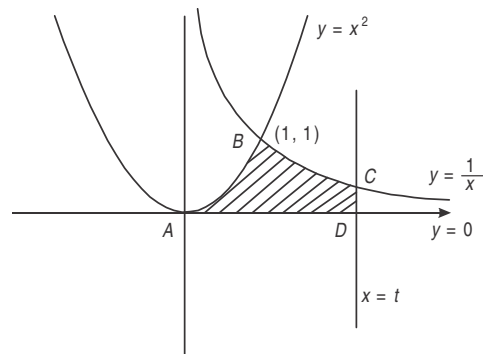
$$= \frac{32}{4} - \frac{16}{3} = \frac{8}{3} \text{ sq-units.}$$

30. (b) The intersection point of  $y = x^2$  and  $y = \frac{1}{x}$  is (1, 1)

Area bounded by the curves is the region ABCDA is given as:

$$\text{Area} = \int_0^1 x^2 dx + \int_1^t \frac{1}{x} dx$$

$$= \left[ \frac{x^3}{3} \right]_0^1 + [\ln(x)]_1^t = \frac{1}{3} + \ln(t)$$

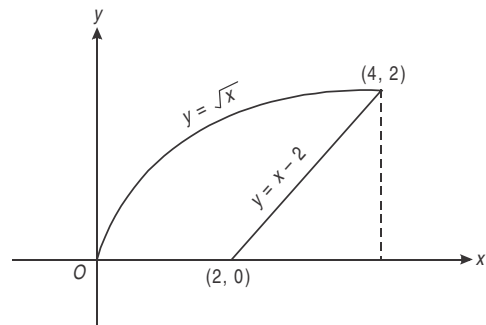


$\therefore$  area = 1

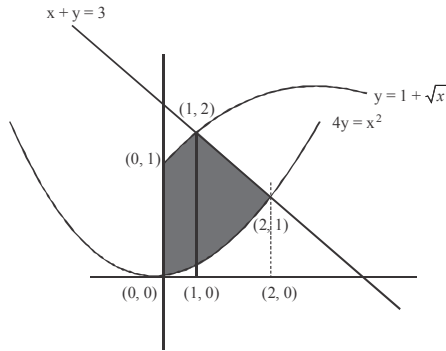
$$\Rightarrow \frac{1}{3} + \ln(t) = 1 \Rightarrow \ln(t) = \frac{2}{3} \Rightarrow t = e^{\frac{2}{3}}$$

31. (b) The intersection point of  $y = x - 2$  and  $y = \sqrt{x}$  is (4, 2). The required area

$$= \int_0^4 \sqrt{x} dx - \frac{1}{2} \times 2 \times 2 = \frac{16}{3} - 2 = \frac{10}{3}$$



32. (a)

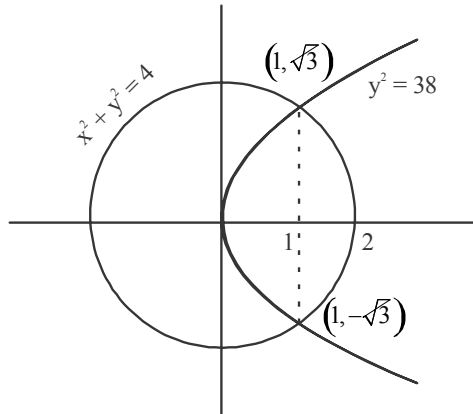


Area of shaded region

$$= \int_0^1 (1 + \sqrt{x}) dx + \int_1^2 (3 - x) dx - \int_0^2 \frac{x^2}{4} dx$$

$$= [x]_0^1 + \left[ \frac{3}{2}x + \frac{2}{3}x^{3/2} \right]_0^1 + [3x - \frac{x^2}{2}]_1^2 - \left[ \frac{x^3}{12} \right]_0^2 = \frac{5}{2} \text{ sq. units}$$

33. (d)



From the equations we get;

$$x^2 + 3x - 4 = 0$$

$$\Rightarrow (x+4)(x-1) = 0 \Rightarrow x = -4, x = 1$$

when  $x = 1, y = \sqrt{3}$

$$\text{Area} = \int_0^1 \left( \int_0^1 \sqrt{3} \cdot \sqrt{x} dx + \int_1^2 \sqrt{4-x^2} \cdot dx \right) \times 2$$

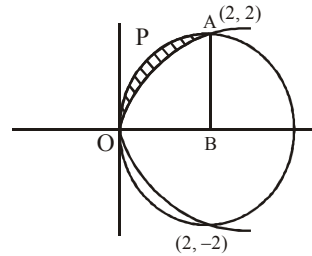
$$= \left( \sqrt{3} \left( \frac{x^{3/2}}{3/2} \right)_0^1 + \left( \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} \right)_1^2 \right) \times 2$$

$$= \left( \sqrt{3} \left( \frac{2}{3} \right) + \left\{ 2 \cdot \frac{\pi}{2} - \left( \frac{\sqrt{3}}{2} + \frac{\pi}{3} \right) \right\} \right) \times 2$$

$$= \left( \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} \right) \times 2$$

$$= \left( \frac{1}{2\sqrt{3}} + \frac{2\pi}{3} \right) \times 2 = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

34. (d)

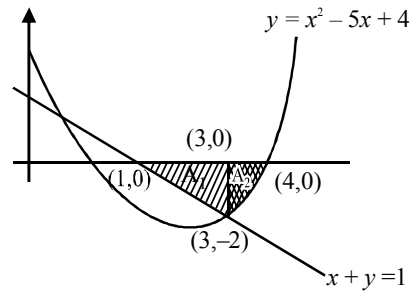


Points of intersection of the two curves are (0, 0), (2, 2) and (2, -2)

Area = Area (OPAB) – area under parabola (0 to 2)

$$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2} \sqrt{x} dx = \pi - \frac{8}{3}$$

35. (a)

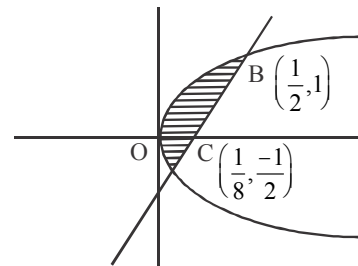


Required area =  $A_1 + A_2$

$$= \frac{1}{2} \times 2 \times 2 + \left| \int_3^4 (x^2 - 5x + 4) dx \right|$$

$$= 2 + \frac{7}{6} = \frac{19}{6} \text{ sq. units}$$

36. (b) Required area

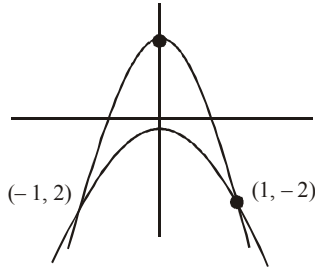


$$= \int_{-1/2}^1 \frac{y+1}{4} dy - \int_{-1/2}^1 \frac{y^2}{2} dy$$

$$= \frac{1}{4} \left[ \frac{y^2}{2} + y \right]_{-1/2}^1 - \frac{1}{2} \left[ \frac{y^3}{3} \right]_{-1/2}^1$$

$$= \frac{1}{4} \left[ \frac{3}{2} + \frac{3}{8} \right] - \frac{9}{48} = \frac{15}{32} - \frac{9}{48} = \frac{27}{96} = \frac{9}{32}$$

37. (c) Solving  
 $y + 2x^2 = 0$   
 $y + 3x^2 = 1$



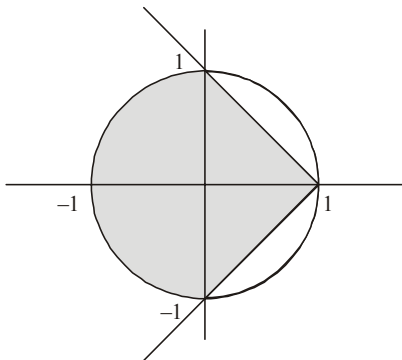
Point of intersection  $(1, -2)$  and  $(-1, -2)$

$$\text{Area} = 2 \int_0^1 \left( (1 - 3x^2) - (-2x^2) \right) dx$$

$$2 \int_0^1 (1 - x^2) dx = 2 \left[ x - \frac{x^3}{3} \right]_0^1 = \frac{4}{3}$$

$= 15 - 6 = 9$  sq units

38. (c) Given curves are  $x^2 + y^2 = 1$  and  $y^2 = 1 - x$ .  
 Intersecting points are  $x = 0, 1$



Area of shaded portion is the required area.

So, Required Area = Area of semi-circle  
 + Area bounded by parabola

$$= \frac{\pi r^2}{2} + 2 \int_0^1 \sqrt{1-x} dx = \frac{\pi}{2} + 2 \int_0^1 \sqrt{1-x} dx$$

( $\because$  radius of circle = 1)

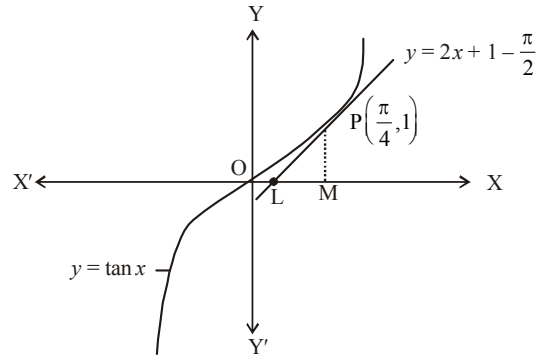
$$= \frac{\pi}{2} + 2 \left[ \frac{(1-x)^{3/2}}{-3/2} \right]_0^1 = \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3} \text{ sq. unit}$$

39. (a) The given curve is  $y = \tan x$  ... (1)

when  $x = \frac{\pi}{4}, y = 1$

Equation of tangent at P is

$$y - 1 = \left( \sec^2 \frac{\pi}{4} \right) \left( x - \frac{\pi}{4} \right)$$



or  $y = 2x + 1 - \frac{\pi}{2}$  ... (2)

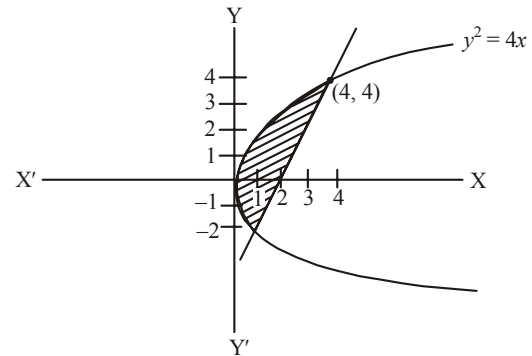
Area of shaded region  
 = area of OPMO - ar ( $\Delta$ PLM)

$$= \int_0^{\pi/4} \tan x dx - \frac{1}{2} (OM - OL) PM$$

$$= [\log \sec x]_0^{\pi/4} - \frac{1}{2} \left\{ \frac{\pi}{4} - \frac{\pi - 2}{4} \right\} \times 1$$

$$= \frac{1}{2} \left[ \log 2 - \frac{1}{2} \right] \text{ sq unit}$$

40. (b) Area of shaded portion



$$= \left| \int_2^4 \left( \frac{y+4}{2} \right) dy \right| - \left| \int_{-2}^4 \frac{y^2}{4} dy \right|$$

$$= \left| \frac{1}{2} \left[ \frac{y^2}{2} + 4y \right]_{-2}^4 \right| - \left| \frac{1}{4} \left[ \frac{y^3}{3} \right]_{-2}^4 \right|$$

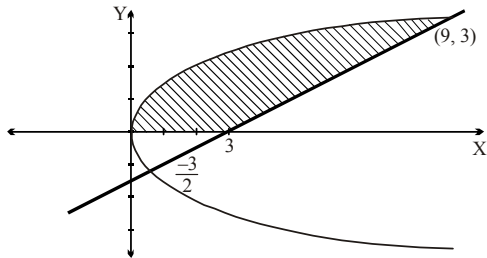
$$= \frac{1}{2} \left[ \{8 + 16\} - \{2 - 8\} \right] - \left| \frac{1}{4} \left\{ \frac{64}{3} + \frac{8}{3} \right\} \right| = 9$$

41. (a) Given curves are

$$y = \sqrt{x} \quad \dots(1)$$

$$\text{and } 2y - x + 3 = 0 \quad \dots(2)$$

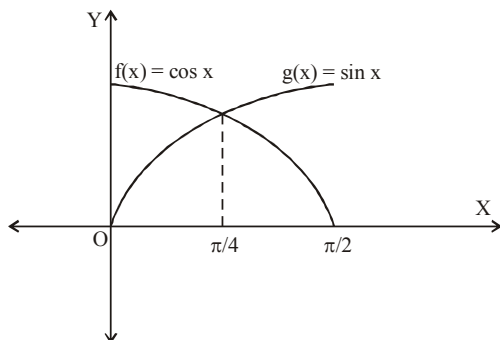
On solving both we get  $y = -1, 3$



$$\text{Required area} = \int_0^3 \left\{ (2y+3) - y^2 \right\} dy$$

$$= \left[ y^2 + 3y - \frac{y^3}{3} \right]_0^3 = 9.$$

42. (b)
- $y = |\cos x - \sin x|$



$$\text{Required area} = 2 \int_0^{\pi/4} (\cos x - \sin x) dx$$

$$= 2 [\sin x + \cos x]_0^{\pi/4}$$

$$= 2 \left[ \frac{2}{\sqrt{2}} - 1 \right] = (2\sqrt{2} - 2) \text{ sq. units}$$

43. (d) Required area =
- $\int_{y=1}^4 \sqrt{\frac{y}{9}} dy$

$$= \frac{1}{3} \int_{y=1}^4 y^{1/2} dy = \frac{1}{3} \times \frac{2}{3} (y^{3/2}) \Big|_1^4$$

$$= \frac{2}{9} [(4^{1/2})^3 - (1^{1/2})^3] = \frac{2}{9} [8 - 1]$$

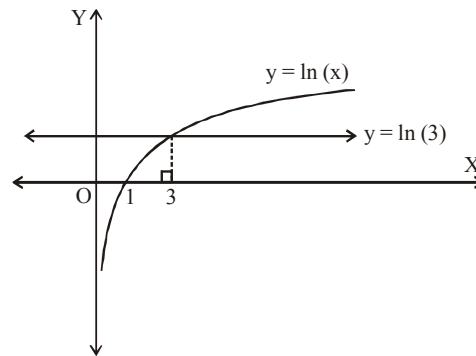
$$= \frac{2}{9} \times 7 = \frac{14}{9} \text{ sq. units.}$$

44. (d) To find the point of intersection of curves
- $y = \ln(x)$
- and
- $y = \ln(3)$
- , put
- $\ln(x) = \ln(3)$

$$\Rightarrow \ln(x) - \ln(3) = 0$$

$$\Rightarrow \ln(x) - \ln(3) = \ln(1)$$

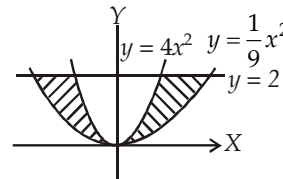
$$\Rightarrow \frac{x}{3} = 1, \Rightarrow x = 3$$



$$\text{Required area} = \int_0^3 \ln(3) dx - \int_1^3 \ln(x) dx$$

$$= [x \ln(3)]_0^3 - [x \ln(x) - x]_1^3 = 2$$

45. (c)



$$\text{Required area} = 2 \int_0^2 \left( \sqrt{9y} - \sqrt{\frac{y}{4}} \right) dy$$

$$= 2 \int_0^2 \left( 3\sqrt{y} - \frac{\sqrt{y}}{2} \right) dy = 2 \left[ \frac{2}{3} \times 3 \cdot y^{3/2} - \frac{1}{2} \times \frac{2}{3} \cdot y^{3/2} \right]_0^2$$

$$= 2 \left[ 2y^{3/2} - \frac{1}{3} y^{3/2} \right]_0^2 = 2 \times \left[ \frac{5}{3} y^{3/2} \right]_0^2$$

$$= 2 \cdot \frac{5}{3} \cdot 2\sqrt{2} = \frac{20\sqrt{2}}{3}$$

46. (b) Intersecting points are
- $x = 1, 4$

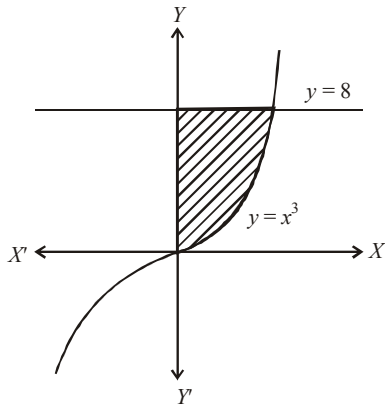
$$\therefore \text{Required area} = \int_1^4 \left[ 2\sqrt{x} - \left( \frac{2x+4}{3} \right) \right] dx$$

$$= \left[ \frac{2x^{3/2}}{3/2} \right]_1^4 - \left[ \frac{2x^2}{3 \times 2} \right]_1^4 - \left[ \frac{4}{3}x \right]_1^4$$

$$= \frac{4}{3} \left( 4^{3/2} - 1^{3/2} \right) - \frac{1}{3} (16 - 1) - \left[ \frac{4}{3} (4) - \frac{4}{3} \right]$$

$$= \frac{4}{3}(7) - 5 - 4 = \frac{28}{3} - 9 = \frac{28 - 27}{3} = \frac{1}{3}$$

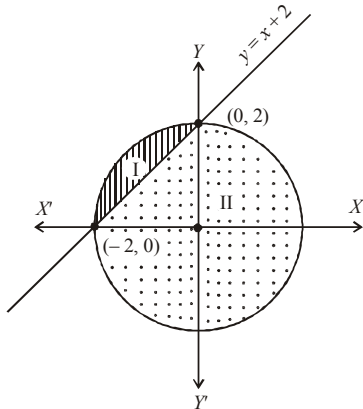
47. (b) Required Area =  $\int_{y=0}^8 y^{1/3} dy$



$$= \frac{y^{1/3+1}}{\frac{1}{3}+1} \Big|_0^8 = \frac{3}{4} \left( y^{4/3} \right) \Big|_0^8$$

$$= \frac{3}{4} \left[ (8)^{4/3} - 0 \right] = \frac{3}{4} [2^4] = \frac{3}{4} \times 16 = 12 \text{ sq. unit.}$$

48. (d) Let I be the smaller portion and II be the greater portion of the given figure then,



$$\text{Area of I} = \int_{-2}^0 \left[ \sqrt{4-x^2} - (x+2) \right] dx$$

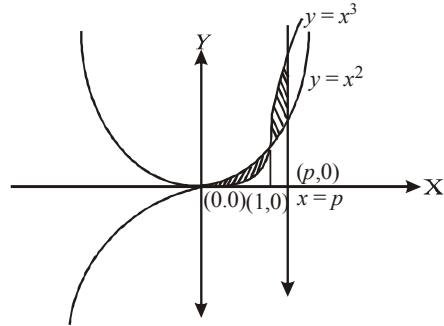
$$= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) \right]_{-2}^0 - \left[ \frac{x^2}{2} + 2x \right]_{-2}^0$$

$$= \left[ 2 \sin^{-1}(-1) \right] - \left[ -\frac{4}{2} + 4 \right] = 2 \times \frac{\pi}{2} - 2 = \pi - 2$$

Now, area of II = Area of circle - area of I.  
 $= 4\pi - (\pi - 2) = 3\pi + 2$

Hence, required ratio =  $\frac{\text{area of I}}{\text{area of II}} = \frac{\pi - 2}{3\pi + 2}$

49. (d) Given curves are  $y = x^2$  and  $y = x^3$   
 Also,  $x = 0$  and  $x = p, p > 1$   
 Now, intersecting point is (1, 1)



Required Area =  $\int_0^1 (x^2 - x^3) dx + \int_1^p (x^3 - x^2) dx$

$$\frac{1}{6} = \frac{x^3}{3} - \frac{x^4}{4} \Big|_0^1 + \frac{x^4}{4} - \frac{x^3}{3} \Big|_1^p$$

$$\Rightarrow \frac{1}{6} = \left( \frac{1}{3} - \frac{1}{4} \right) + \left( \frac{p^4}{4} - \frac{p^3}{3} - \frac{1}{4} + \frac{1}{3} \right)$$

$$\Rightarrow \frac{1}{6} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{3} = \frac{3p^4 - 4p^3}{12}$$

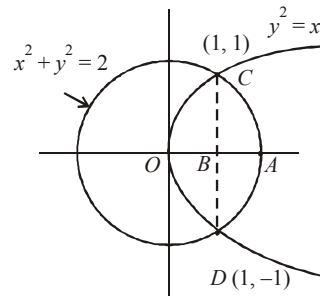
$$\Rightarrow \frac{p^3(3p-4)}{12} = 0 \Rightarrow p^3(3p-4) = 0$$

$$\Rightarrow p = 0 \text{ or } \frac{4}{3}$$

Since, it is given that  $p > 1$   
 $\therefore p$  can not be zero.

Hence,  $p = \frac{4}{3}$

50. (b)



Area of circle =  $\pi(\sqrt{2})^2 = 2\pi$

$$\text{Area of } OCADO = 2 \{ \text{Area of } OCAO \}$$

$$= 2 \{ \text{area of } OCB + \text{area of } BCA \}$$

$$= 2 \int_0^1 y_p dx + 2 \int_1^{\sqrt{2}} y_c dx$$

$$\text{where } y_p = \sqrt{x} \text{ and } y_c = \sqrt{2-x^2}$$

$$\therefore \text{ Required Area} = 2 \int_0^1 \sqrt{x} dx + 2 \int_1^{\sqrt{2}} \sqrt{2-x^2} dx$$

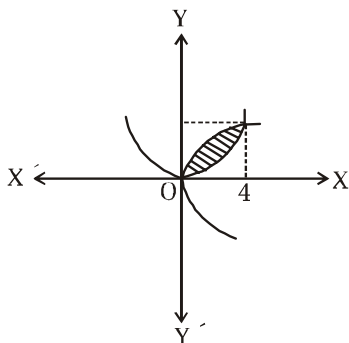
$$= 2 \left[ \frac{2}{3} \cdot 1 - 0 \right] + 2 \left[ \frac{x\sqrt{2-x^2}}{2} + \sin^{-1} \frac{x}{\sqrt{2}} \right]_1^{\sqrt{2}}$$

$$= \frac{4}{3} + 2 \left\{ \frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2} \right\} = \frac{4}{3} + 2 \left\{ \frac{\pi}{4} - \frac{1}{2} \right\} = \frac{3\pi+2}{6}$$

$$\text{Bigger area} = 2\pi - \frac{3\pi+2}{6} = \frac{9\pi-2}{6}$$

$$\therefore \text{ Required Ratio} = \frac{9\pi-2}{3\pi+2} \text{ i.e., } 9\pi-2 : 3\pi+2$$

51. (b)



$$\text{Required area} = \int_0^4 \left( 2\sqrt{x} - \frac{x^2}{4} \right) dx$$

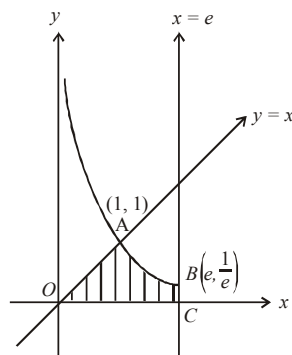
$$= \left[ 2 \left( \frac{x^{3/2}}{3/2} \right) - \frac{x^3}{12} \right]_0^4$$

$$= \frac{4}{3} \times 8 - \frac{64}{12} = \frac{32}{3} - 16 = \frac{16}{3} \text{ sq. units}$$

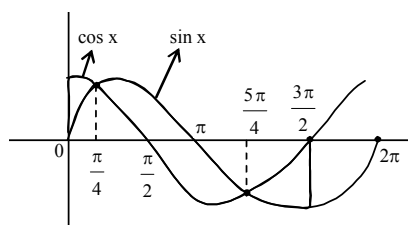
52. (b) Area of required region  $AOCBO$ 

$$= \int_0^1 x dx + \int_1^e \frac{1}{x} dx = \left[ \frac{x^2}{2} \right]_0^1 + [\log x]_1^e$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ sq. units}$$



53. (d)



Area above x-axis = Area below x-axis

 $\therefore$  Required area

$$= 2 \left[ \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} \sin x dx - \int_{\pi/4}^{\pi/2} \cos x dx \right]$$

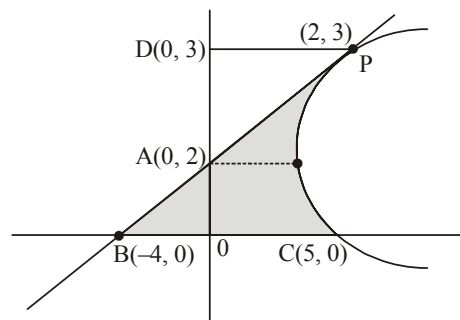
$$= 2 \left[ (\sin x + \cos x)_0^{\pi/4} + (-\cos x)_{\pi/4}^{\pi} - (\sin x)_{\pi/4}^{\pi/2} \right]$$

$$= 2 \left[ \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0+1) + \left( 1 + \frac{1}{\sqrt{2}} \right) - \left( 1 - \frac{1}{\sqrt{2}} \right) \right]$$

$$= 2 \left[ \sqrt{2} - 1 + 1 + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} \right]$$

$$= 2[\sqrt{2} + \sqrt{2} - 1] = 4\sqrt{2} - 2$$

54. (b)

For slope of tangents at  $(2, 3)$ 

$$(y-2)^2 = x-1$$

$$2(y-2)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)}$$

$$m = \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{1}{2(3-2)} = \frac{1}{2}$$

Equation of tangent

$$y - 3 = \frac{1}{2}(x - 2)$$

$$\Rightarrow x - 2y + 4 = 0 \quad \dots(i)$$

The given parabola is  $(y-2)^2 = x-1$  ... (ii)

vertex (1, 2) and it meets x-axis at (5, 0)

Then required area = Ar  $\Delta$ BOA + Ar (OCPD) - Ar ( $\Delta$ APD)

$$= \frac{1}{2} \times 4 \times 2 + \int_0^3 x dy - \frac{1}{2} \times 2 \times 1$$

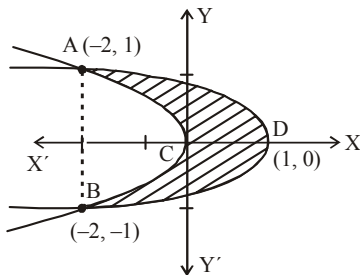
$$= 3 + \int_0^3 (y-2)^2 + 1 dy = 3 + \left[ \frac{(y-2)^3}{3} + y \right]_0^3$$

$$= 3 + \left[ \frac{1}{3} + 3 + \frac{8}{3} \right] = 3 + 6 = 9 \text{ sq. units}$$

55. (d) Given  $x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$

and  $x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$

On solving these two equations we get the points of intersection as (-2, 1), (-2, -1)



The required area is ACBDA, given by

$$A = 2 \left\{ \int_{-2}^1 \frac{1}{\sqrt{3}} \sqrt{1-x} dx - \frac{1}{\sqrt{2}} \int_{-2}^0 \sqrt{-x} dx \right\}$$

$$\Rightarrow 2 \left\{ \frac{1}{\sqrt{3}} \left[ \frac{2}{3}(1-x)^{3/2} \right]_{-2}^1 - \frac{1}{\sqrt{2}} \left[ \frac{2}{3}(-x)^{3/2} \right]_{-2}^0 \right\}$$

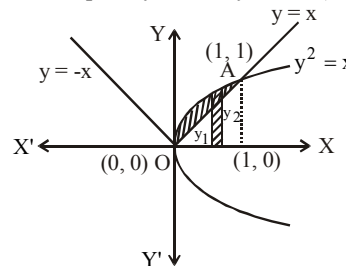
$$\Rightarrow 2 \left\{ \left[ -\frac{1}{\sqrt{3}} \times \frac{2}{3}(0-3^{3/2}) \right] - \left[ \frac{-1}{\sqrt{2}} \times \frac{2}{3}(0-2^{3/2}) \right] \right\}$$

$$\Rightarrow 2 \left\{ \frac{2}{\sqrt{3}\sqrt{3}} \times \sqrt{3}\sqrt{3} - \frac{1}{\sqrt{2}} \times \frac{2}{3} \cdot 2\sqrt{2} \right\}$$

$$\Rightarrow 2 \left\{ 2 - \frac{4}{3} \right\} = 2 \left\{ \frac{6-4}{3} \right\} = \frac{4}{3} \text{ sq. units}$$

56. (a) It is clear from the figure, area lies between  $y^2 = x$  and  $y=x$

Intersection point  $y = x$  and  $y^2 = x$  is (1, 1)

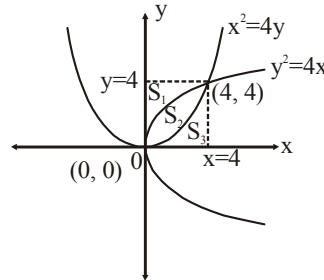


$$\therefore \text{Required area} = \int_0^1 (y_2 - y_1) dx$$

$$= \int_0^1 (\sqrt{x} - x) dx = \left[ \frac{x^{3/2}}{3/2} - \frac{x^2}{2} \right]_0^1$$

$$= \frac{2}{3} \left[ x^{3/2} \right]_0^1 - \frac{1}{2} \left[ x^2 \right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

57. (d) On solving, we get intersection points of  $x^2 = 4y$  and  $y^2 = 4x$  are (0, 0) and (4, 4).



By symmetry, we observe

$$S_1 = S_3 = \int_0^4 y dx$$

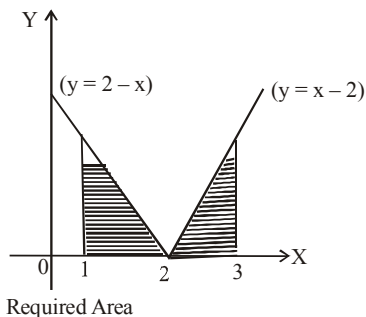
$$= \int_0^4 \frac{x^2}{4} dx = \left[ \frac{x^3}{12} \right]_0^4 = \frac{16}{3} \text{ sq. units}$$

$$\text{Also } S_2 = \int_0^4 \left( 2\sqrt{x} - \frac{x^2}{4} \right) dx = \left[ \frac{2x^{3/2}}{3/2} - \frac{x^3}{12} \right]_0^4$$

$$= \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3} \text{ sq. units}$$

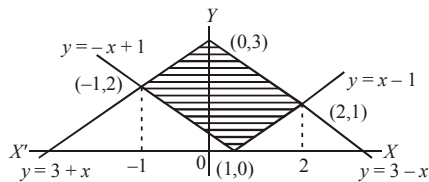
$$\therefore S_1 : S_2 : S_3 = 1 : 1 : 1$$

58. (d)



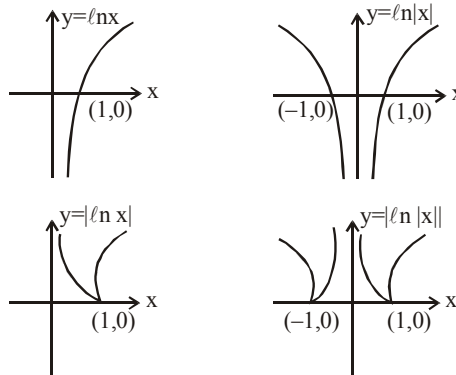
$$A = 2 \int_2^3 (x-2) dx = 2 \left[ \frac{x^2}{2} - 2x \right]_2^3 = 1$$

59. (d) Intersection point of  $y = x - 1$  and  $y = 3 - x$  is  $(2, 1)$  and eqns.  $y = -x + 1$  and  $y = 3 + x$  is  $(-1, 2)$

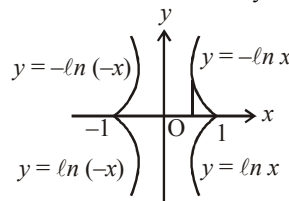


$$\begin{aligned} A &= \int_{-1}^0 \{(3+x) - (-x+1)\} dx + \\ & \int_0^1 \{(3-x) - (-x+1)\} dx + \int_1^2 \{(3-x) - (x-1)\} dx \\ &= \int_{-1}^0 (2+2x) dx + \int_0^1 2 dx + \int_1^2 (4-2x) dx \\ &= [2x+x^2]_{-1}^0 + [2x]_0^1 + [4x-x^2]_1^2 \\ &= 0 - (-2+1) + (2-0) + (8-4) - (4-1) \\ &= 1+2+4-3 = 4 \text{ sq. units} \end{aligned}$$

60. (a) Separate graph of each curve



[Note: Graph of  $y = |f(x)|$  can be obtained from the graph of the curve  $y = f(x)$  by drawing the mirror image of the portion of the graph below  $x$ -axis, with respect to  $x$ -axis. Hence the bounded area is as shown by combined all figure.



$$\begin{aligned} \text{Required area} &= 4 \int_0^1 (-\ln x) dx \\ &= -4 [x \ln x - x]_0^1 = 4 \text{ sq. units} \end{aligned}$$