JEE MAIN 2024

Sample Paper - 2

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

- All questions are compulsory.
- There are three parts and each part carries 30 questions where the first 20 questions are MCQs and the next 10 questions are numerical.
- Section-A within each part is compulsory. Attempt any 5 questions from section-B within each part.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer. However, there is no negative marking for Section-B (Numerical Questions)

PHYSICS (Section-A)

1. If force [F], acceleration [A] and time [T] are chosen as the fundamental physical quantities. Find the dimensions of energy. **[4]**

a) $[F][A][T^{-1}]$ b) $[F][A^{-1}][T]$

c) [F][A][T] d) $[F][A][T^2]$

2. A body travelling along a straight line traversed one-third of the total distance with a velocity v_1. The remaining part of the distance was covered with a velocity v_2 for half the time and with velocity v_3 for the other half of time. The mean velocity averaged over the whole time of motion is: **[4]**

a) $\frac{3v_1(v_2+v_3)}{2v_1+v_2+v_3}$ b) $\frac{v_1(v_2+v_3)}{v_1+v_2+v_3}$

c) $\frac{3v_1(v_2+v_3)}{4v_1+v_2+v_3}$ d) $\frac{v_1(v_2+v_3)}{4v_1+v_2+v_3}$

3. Four bodies A, B, C, and D are projected with equal speeds having angles of projection 15°, 30°, 45°, and 60° with the horizontal respectively. The body having the shortest range is: **[4]**

a) C b) D

c) B d) A

4. A body is projected upwards with a kinetic energy of 100 J. Taking the friction of air into account, when it returns to the earth, its kinetic energy will be: **[4]**

a) >1000 J b) 100 J

c) < 100 J d) > 100 J

5. A spring gun of spring constant 90 N/cm is compressed 12 cm by a ball of mass 16 g. If the trigger is pulled, the velocity of the ball is: **[4]**

a) $40\ ms^{-1}$

b) $90\ ms^{-1}$

c) $50\ ms^{-1}$

d) $9\ ms^{-1}$

6. A disc is rotating with an angular velocity ω_0. A constant retarding torque is applied on it to stop the disc. The angular velocity becomes $\left(\frac{\omega_0}{2}\right)$ after n rotations. How many more rotations will it make before coming to rest? **[4]**

a) 2n

b) n

c) $\frac{n}{3}$

d) $\frac{n}{2}$

7. A small cylinder of 2 cm diameter is connected to a large cylinder of 20 cm diameter and incompressible fluid is filled in the cylinders. If a force of 40 N is applied to the piston of the small cylinder, then the force exerted on the piston of the large cylinder will be: **[4]**

a) 400 N

b) 8,000 N

c) 4,000 N

d) 800 N

8. An aluminum sphere is dipped into the water. Which of the following is true? **[4]**

a) Buoyancy in water at 0^oC will be same as that in water at 4^oC.

b) Buoyancy may be more or less in water at 4^oC depending on the radius of the sphere.

c) Buoyancy will be less in water at 0^oC than that in water at 4^oC.

d) Buoyancy will be more in water at 0^oC than that in water at 4^oC.

9. 540 calories of heat convert 1 cubic centimetre of water at 100°C into 1671 cm^3 of steam at 100°C. Then, the work done against atmospheric pressure is near: **[4]**

a) 540 cal

b) Zero cal

c) 500 cal

d) 40 cal

10. A simple pendulum of length 1 m at sea level oscillates with time period T_1 s. If pendulum is displaced upwards by $y = 3kt^2 + 5t + 1$, time period changes to T_2 s. If k is a constant equal to 2 m/s^2, then difference in time period will be, ($g = 10\ m/s^2$) **[4]**

a) 0.82

b) 0.78

c) 0.29

d) 0.16

11. A charge Q is placed at the corner of a cube. The electric flux through all the six faces of the cube is: **[4]**

a) $\frac{Q}{\varepsilon_0}$

b) $\frac{Q}{3\varepsilon_0}$

c) $\frac{Q}{8\varepsilon_0}$

d) $\frac{Q}{6\varepsilon_0}$

12. A current loop consists of two identical semicircular parts each of radius R, one lying in the x, y-plane and the other in x, z-plane. If the current in the loop is i. The resultant magnetic field due to the two semicircular parts at their common centre is: **[4]**

a) $\frac{\mu_0 i}{2\sqrt{2}R}$

b) $\frac{\mu_0 i}{4R}$

c) $\frac{\mu_0 i}{\sqrt{2}R}$

d) $\frac{\mu_0 i}{2R}$

13. The susceptibility of a paramagnetic material at 300 K is 1.4×10^{-5}. The material is cooled and at a particular temperature, its susceptibility increased to 2.1×10^{-5}. What is the change in temperature of the material? **[4]**

a) 200 K

b) 100 K

c) 300 K

d) 400 K

14. A metallic ring is attached to the wall room. When the north pole of a magnet is brought near the ring, the induced current in the ring is: **[4]**

a) in anticlockwise direction

b) zero

c) in clockwise direction

d) infinite

15. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance X_L = 250 $\pi\Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be: (Take π^2 = 10) **[4]**

a) 25 μF

b) 4 μF

c) 40 μF

d) 400 μF

16. In an electromagnetic wave, the electric and magnetising fields are 100 V/m and 0.265 A/m. The maximum energy flow is: **[4]**

a) 46.7 W/m^2

b) 765 W/m^2

c) 36.5 W/m^2

d) 26.5 W/m^2

17. An electron accelerated under a potential difference V volt has a certain wavelength λ. Mass of proton is some 2000 times of the mass of the electron. If the proton has to have the same wavelength λ, then it will have to be accelerated under a potential difference of: **[4]**

a) $\frac{V}{2000}$ volt

b) V volt

c) $\sqrt{2000}$ V volt

d) 2000 V volt

18. When an α-particle of mass m moving with velocity v bombards on heavy nucleus of charge Ze its distance of closest approach from the nucleus depends on m as: **[4]**

a) $\frac{1}{\sqrt{m}}$

b) $\frac{1}{m^2}$

c) m

d) $\frac{1}{m}$

19. $^{238}_{92}\mathrm{A} \rightarrow {}^{234}_{90}\mathrm{B} + {}^{4}_{2}\mathrm{D} + \mathrm{Q}$ **[4]**

In the given nuclear reaction, the approximate amount of energy released will be:
[Given, mass of $^{238}_{92}\mathrm{A} = 238.05079 \times 931.5\mathrm{MeV/c^2}$,
mass of $^{234}_{90}\mathrm{B} = 234.04363 \times 931.5\mathrm{MeV/c^2}$
mass of $^{4}_{2}\mathrm{D} = 4.00260 \times 931.5\mathrm{MeV/c^2}$]

a) 3.82 MeV

b) 4.25 MeV

c) 5.9 MeV

d) 2.12MeV

20. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is: **[4]**

a) 200Ω

b) 300Ω

c) 50Ω

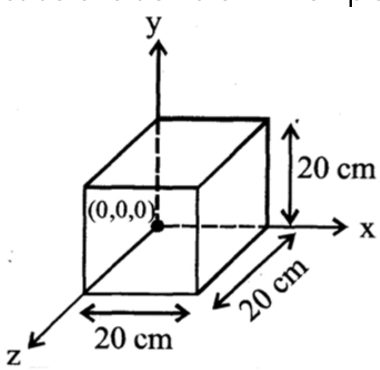
d) 100Ω

PHYSICS (Section-B)

Attempt any 5 questions

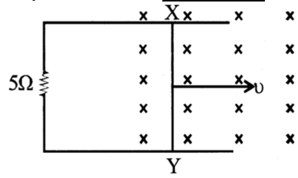
21. A cylindrical wire of radius 0.5 mm and conductivity 5 $\times$ 10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ________ **[4]**

22. Expression for an electric field is given by $\vec{E} = 4000x^2\,\hat{i}\,\frac{V}{m}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is ________ V cm. **[4]**



23. A 1m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the circuit is 5 Ω , the force needed to move the rod in direction, as indicated, with a constant speed of 4 **[4]**

m/s will be _______ 10^{-3} N.



24. Acceleration due to gravity on moon is $\frac{1}{6}$th of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_e) and moon (ρ_m) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$, then radius of moon R_m is kR_e. What will be the value of k? **[4]**

25. At a given point of time the value of displacement of a simple harmonic oscillator is given as y = A cos (30^o). If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant is 1.0×10^x Nm^{-1}. The value of x is _______. **[4]**

26. If the ratio of magnifications produced by a simple microscope in near point adjustment and far point adjustment is 6/5, then the focal length of the lens is _______ cm. **[4]**

27. A current I = 3.36(1 + 2t) $\times 10^{-2}$ A increases at a steady rate in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and is placed at a distance of 1 m from the wire. The resistance of the loop is 8.4×10^{-4} Ω. The magnitude of the induced current in the loop is (n $\times 10^{-11}$ A), then n is equal to? **[4]**

28. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at a quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement. **[4]**

29. The temperature of a thin uniform circular disc, of one metre diameter is increased by 10^oC. The percentage increase in moment of inertia of the disc about an axis passing through its centre and perpendicular to the circular face is _______. (linear coefficient of expansion = 11 $\times 10^{-6}/^oC$) **[4]**

30. The breaking stress of a material is 10^6 N /m^2. If the density of the material is 3×10^3 kg/m^3, what should be the length of the material in metres so that it breaks by its own weight? **[4]**

CHEMISTRY (Section-A)

31. Correct set of four quantum numbers for the valence (outermost) electron of rubidium (Z = 37) is: **[4]**

a) 5, 0, 0, $+\frac{1}{2}$
b) 6, 0, 0, $+\frac{1}{2}$
c) 5, 1, 1, $+\frac{1}{2}$
d) 5, 1, 0, $+\frac{1}{2}$

32. Two elements A and B have values of electronegativity respectively as 4 and 1.2, then the compound A — B will be: **[4]**

a) both predominantely ionic and %
b) % covalent character less than 50%

covalent character less than 50%

c) predominantely covalent

d) predominantely ionic

33. Which of the molecules listed have an sp^3 hybridized central atom? **[4]**

i. PCl_3

ii. $COCl_2$

iii. SF_4

a) A only

b) A, B and C

c) A and C only

d) B and C only

34. C_p - C_v = _______. **[4]**

a) 8.314 J mol^{-1} deg^{-1}

b) 2 cal mol^{-1} deg^{-1}

c) both 2 cal mol^{-1} deg^{-1} and 8.314 J mol^{-1} deg^{-1}

d) 8.314 cal mol^{-1} deg^{-1}

35. What is the pH of the solution on mixing 100 mL of 0.1 M NH_4OH and 100 mL of 0.1 M $(NH_4)_3$ PO_4? K_b for NH_4OH = 1.8×10^{-5} **[4]**

a) 9.5728

b) 9.2553

c) 9.2718

d) 8.7782

36. CrO_5 reacts with H_2SO_4 to give $Cr_2(SO_4)_3$, H_2O and O_2. Moles of O_2 liberated by 1 mole of ClO_5 in this reaction are: **[4]**

a) 1.75

b) 4.5

c) 1.25

d) 2.5

37. Moissan boron is _______ form of boron. **[4]**

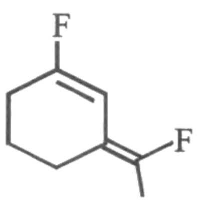
a) inert crystalline

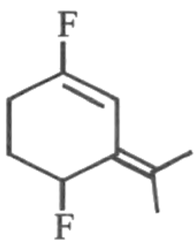
b) inert amorphous

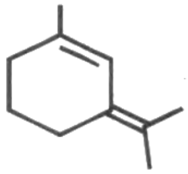
c) reactive amorphous

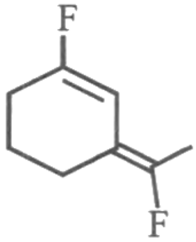
d) reactive crystalline

38. The most polar compound among the following is: **[4]**

a) 

b) 

c) 

d) 

39. Which is the most suitable reagent among the following to distinguish compound (3) from the rest of the compound? **[4]**

i. $CH_3 — C = C — CH_3$

ii. $CH_3 — CH_2 — CH_2 — CH_3$

iii. $CH_3CH_2 — C = CH$

iv. $CH_3 — CH = CH_2$

a) Br_2 in CCl_4

b) $\frac{AgNO_3}{NH_4OH}$

c) Alk. $KMnO_4$

d) Br_2 in CH_3COOH

40. K_H value for Ar(g), CO_2(g), HCHO(g) and CH_4(g) are 40.39, 1.67, 1.83×10^{-5} and 0.413 respectively. **[4]**
Arrange these gases in the order of their increasing solubility:

a) HCHO < CO_2 < CH_4 < Ar

b) Ar < CO_2 < CH_4 < HCHO

c) Ar < CH_4 < CO_2 < HCHO

d) HCHO < CH_4 < CO_2 < Ar

41. Boiling point of a 2% aqueous solution of a non-volatile solute A is equal to the boiling point of 8% aqueous solution of a non-volatile solute B. The relation between molecular weights of A and B is **[4]**

a) $M_A = 4M_B$

b) $M_B = 4M_A$

c) $M_A = 8M_B$

d) $M_B = 8M_A$

42. Using the data given below and find out which of the following is the strongest oxidising agent. **[4]**
$E^{\ominus}_{Cr_2O_7^{2-}/Cr^{3+}}$ = 1.33 V $E^{\ominus}_{Cl_2/Cl^-}$ = 1.36 V
$E^{\ominus}_{MnO_4^-/Mn^{2+}}$ = 1.51 V $E^{\ominus}_{Cr^{3+}/Cr}$ = -0.74 V

a) Mn^{2+}

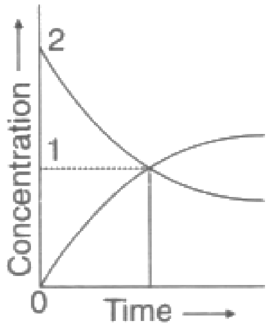
b) Cl^-

c) MnO_4^-

d) Cr^{3+}

43. The accompanying figure depicts the change in concentration of species X and Y for the reaction X $\rightarrow$ Y, as a function of time. The point of intersection of the two curves **[4]**

represents:



a) $t_{1/2}$

b) $t_{2/3}$

c) $t_{3/4}$

d) data insufficient to predict

44. Atomic number of Cr and Fe are Respectively 24 and 26, which of the following is paramagnetic with the spin of an electron? **[4]**

a) $[Cr(NH_3)_6]^{3+}$

b) $[Fe(CO)_5]$

c) $[Cr(CO)_6]$

d) $[Fe(CN)_6]^{4-}$

45. Which of the following represents calcium chlorite? **[4]**

a) $Ca(ClO_4)_2$

b) $Ca(ClO_3)_2$

c) $Ca(ClO_2)_2$

d) $CaClO_2$

46. Among $[Ni(CN)_4]^{4-}$, $[Ni(PPh_3)_3Br]$ and $[Ni(dmg)_2]$ species, the hybridisation states of the Ni-atoms are respectively: **[4]**

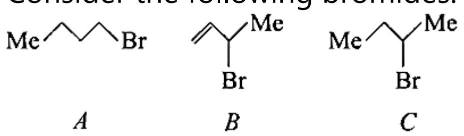
a) sp^3, sp^3, dsp^2

b) dsp^2, dsp^2, sp^3

c) sp^3, dsp^2, dsp^2

d) dsp^2, sp^3, dsp^2

47. Consider the following bromides: **[4]**



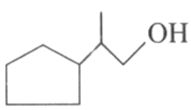
The correct order of S_N1 reactivity is:

a) B > C > A

b) B > A > C

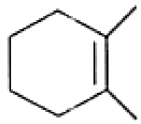
c) A > B > C

d) C > B > A

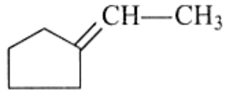
48. 

on dehydration gives: **[4]**

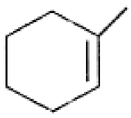
a)



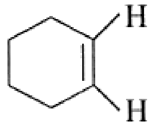
b)



c)



d)



49. The compound most suitable for the preparation of cyanohydrin is **[4]**

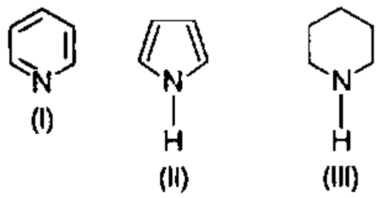
a) $C_6H_5NH_2$

b) C_2H_5COOH

c) C_2H_5-C_2H_5

d) $C_2H_5COC_2H_5$

50. Arrange the following amines in the decreasing order of basicity: **[4]**



a) III > II > I

b) III > I > II

c) I > II > III

d) I > III > II

CHEMISTRY (Section-B)

Attempt any 5 questions

51. The number of spherical nodes in 4p orbital is/are _______. **[4]**

52. In a chemical reaction, A + 2B $\overset{K}{\rightleftharpoons}$ 2C + D, the initial concentration of B was 1.5 times of the concentration of A, but the equilibrium concentrations of A and B were found to be equal. The equilibrium constant (K) for the aforesaid chemical reaction is ________. **[4]**

53. Ratio of σ-bonds and π-bonds in benzene molecule is: **[4]**

54. Total number of acidic oxides among **[4]**
N_2O_3, NO_2, N_2O, $C1_2O_7$, SO_2, CO, CaO, Na_2O and NO is _______.

55. Consider the following metal complexes: **[4]**

$[Co(NH_3)_6]^{3+}[CoCl(NH_3)_5]^{2+}$

$[Co(CN)_6]^{3-}[Co(NH_3)_5\,(H_2O)]^{3+}$

The spin-only magnetic moment value of the complex that absorbs light with shortest wavelength is _______ B.M. (Nearest integer)

56. The number of statement(s) correct from the following for copper (at no. 29) is/are _______. **[4]**

A. Cu (II) complexes are always paramagnetic

B. Cu (I) complexes are generally colourless

C. Cu (I) is easily oxidized

D. In Fehling solution, the active reagent has Cu (I)

57. A 100 mL solution was made by adding 1.43 g of $Na_2CO_3 \cdot xH_2O$. The normality of the solution is 0.1 N. The value of x is _______. (The atomic mass of Na is 23 g/mol) **[4]**

58. The abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5% respectively. The atomic mass of Fe is _______. **[4]**

59. Total number of planes in CH_4 which contain 3 atoms in a plane with carbon, is: **[4]**

60. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27^oC. The work done is 3 kJ mol^{-1}. The final temperature of the gas is K (Nearest integer). Given C_V= 20 J $mol^{-1}K^{-1}$. **[4]**

MATHEMATICS (Section-A)

61. The number of bijective functions f: {1, 3, 5, 7, 99} $\rightarrow$ {2, 4, 6, 8... 100}, such that f(3) $\geq$ f(9) $\geq$ f(15) $\geq$ f(21) $\geq$...$\geq$ f(99) is **[4]**

a) $^{50}P_{17}$

b) 33! $\times$ 17!

c) $\frac{50!}{2}$

d) $^{50}P_{33}$

62. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0(x>0)$ is equal to **[4]**

a) 12

b) 9

c) 4

d) 10

63. The total number of functions, f:{1, 2, 3, 4} $\rightarrow$ {1, 2, 3, 4, 5, 6} such that f(1) + f(2) = f(3), is equal to: **[4]**

a) 108

b) 90

c) 60

d) 126

64. The sum of the last 30 coefficients in the expansion of $(1 + x)^{59}$ is **[4]**

a) 2^{59}

b) 2^{56}

c) 2^{57}

d) 2^{58}

65. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then: **[4]**

a) a, b, c, d are in A.P.

b) a, b, c, d are in G.P.

c) a, c, p are in A.P.

d) a, c, p are in G.P.

66. If $f(x) = (\sqrt{4-x^2} - 3)^2 + (\sqrt{4-x^2} + 1)^3$, then the maximum value of f(x), is : **[4]**

a) 40
b) 25
c) 36
d) 28

67. A wire of length 20 m is to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then (πl_1): l_2 is equal to: **[4]**

a) 3 : 1
b) 6 : 1
c) 4 : 1
d) 1 : 6

68. $\int \frac{x^4}{(x-1)(x^2+1)} dx$ = **[4]**

a) $\frac{x(x+2)}{2} + \frac{\log|x-1|}{2} - \frac{\log|x^2+1|}{4} - \frac{\tan^{-1} x}{2} + c$
b) $\frac{x(x-2)}{2} - \frac{\log|x+1|}{2} - \frac{\log|x^2-1|}{4} + \frac{\tan^{-1} x}{2} + c$
c) $\frac{x(x+2)}{2} + \frac{\log|x-1|}{2} + \frac{\log|x^2+1|}{4} + \frac{\tan^{-1} x}{2} + c$
d) $\frac{x(x+2)}{2} + \frac{\log|x-1|}{2} + \frac{\log|x^2+1|}{4} - \frac{\tan^{-1} x}{2} + c$

69. If length of the tangent drawn from each and every point on the curve y = $\sqrt{\lambda - x^2}$ to the circle $x^2 + y^2 = 36$ is 8 units, then λ is : **[4]**

a) 30
b) 50
c) 100
d) 90

70. A stick of length l rests against the floor and a wall of a room. If the stick begins to slide on the floor, then the locus of its middle point is **[4]**

a) a straight line
b) an ellipse
c) a parabola
d) a circle

71. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{\sqrt{2}}, -2\right)$, then the equation of the tangent to it at B is: **[4]**

a) 2x - y - 24 = 0
b) x - 2y + 8 = 0
c) 2x + y - 24 = 0
d) x + 2y + 8 = 0

72. Solution of the differential equation $\left(e^{x^2} + e^{v^2}\right) y\frac{dy}{dx} + e^{x^2}$ $(xy^2 - x) = 0$, is : **[4]**

a) $e^{y^2}\left(y^2 - 1\right) + e^{x^2} = C$
b) $e^{y^2}\left(x^2 - 1\right) + e^{x^2} = C$

c) $e^{x^2}(y-1)+e^{y^2}=C$

d) $e^{x^2}\left(y^2-1\right)+e^{y^2}=C$

73. The angle between a line with direction ratios proportional to (2, 3, 6) and a line joining (1, -2, 4) to (-1, 1, -2) is: **[4]**

a) $\cos^{-1}\left(\frac{31}{49}\right)$

b) $\cos^{-1}\left(\frac{21}{49}\right)$

c) $\cos^{-1}\left(\frac{3}{7}\right)$

d) $\cos^{-1}\left(\frac{6}{7}\right)$

74. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR}=\frac{RB}{BP}=\frac{PC}{CQ}=\frac{1}{2}$. Then $\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)}$ is equal to **[4]**

a) $\frac{5}{2}$

b) 3

c) 2

d) 4

75. If the arithmetic mean of the numbers x_1, x_2, x_3, ... x_n is $\bar{x}$. Then the arithmetic mean of numbers ax_1 + b, ax_2 + b, ax_3 + b, ... ax_n + b, Where a, b are two constants would be **[4]**

a) n$a\bar{x}$ + nb

b) $\bar{x}$

c) $a\bar{x}$ + b

d) $a\bar{x}$

76. Numbers 1 to 100 are written on slips of papers and are kept in a box. A draws one slip randomly and replaces it. B then draws a slip randomly. What is the probability that B draws a bigger number? **[4]**

a) $\frac{99}{20000}$

b) $\frac{99}{500}$

c) $\frac{49}{500}$

d) $\frac{99}{200}$

77. The maximum value of $(\cos\alpha_1)\cdot(\cos\alpha_2)\ldots\ldots(\cos\alpha_n)$, under the restrictions $0\leq\alpha_1,\alpha_2,\ldots,\alpha_n\leq\frac{\pi}{2}$ and $\cot\alpha_1\cdot\cot\alpha_2\ldots.\cot\alpha_n$ = 1 is : **[4]**

a) $\frac{1}{2^n}$

b) 1

c) $\frac{1}{2n}$

d) $\frac{1}{2^{n/2}}$

78. Find the equation of axis of the given hyperbola $\frac{x^2}{3}-\frac{y^2}{2}=1$ which is equally inclined to the axes? **[4]**

a) y = x + 2

b) y = x + 1

c) y = x - 2

d) y = x - 1

79. The set $(\mathrm{A}\cup\mathrm{B}\cup\mathrm{C})\cap(A\cap B'\cap C')'\cap C'$ is equal to **[4]**

a) B' ∩ C'

b) A ∩ C

c) A ∩ C' d) B ∩ C'

80. If S = $\left\{x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0\right\}$ then $\sum_{x \in S}$ tan $\left(\frac{\pi}{3} + x\right)$ is equal to **[4]**

a) - 2 - $\sqrt{3}$ b) -2 + $\sqrt{3}$

c) - 4 - $2\sqrt{3}$ d) 4 + $2\sqrt{3}$

MATHEMATICS (Section-B)

Attempt any 5 questions

81. Let f : [0, ∞) → R be a continuous, strictly increasing function such that $f^3(x) = \int_0^x t f^2(t) dt$. **[4]**

If a normal is drawn to the curve y = f(x) with gradient $\frac{-1}{2}$, then find the intercept made by it on the y-axis.

82. Let g(x) is the only invertible function from R ⟶ R which satisfy the equation $g^3(x) - (x^3 + 2) g^2(x) + (2x^3 + 1) g(x) - x^3 = 0$. Find the value of $g'(8) \cdot (g^{-1})'(8)$. **[4]**

83. Let $\vec{\mathbf{a}} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\overrightarrow{\mathbf{c}}$ is a vector such that $\vec{a} \cdot \vec{c} = 11, \vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to **[4]**

84. If the area of the region {(x, y): $|x^2 - 2| \le y \le x$} is A, then 6A + $16\sqrt{2}$ is equal to ________. **[4]**

85. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to ________. **[4]**

86. Find the minimum number of tosses of a pair of dice so that the probability of getting the sum of the digits on the dice equal to 7 on at least one toss is greater than 0.95. ($\log_{10}2$ = 0.3010; $\log_{10}3$ = 0.4771) **[4]**

87. If x = cos 2θ - 2 $\cos^2 2\theta$ + 3 $\cos^3 2\theta$ - 4 $\cos4^2\theta$ + ... ∞ **[4]**
and y - cos 2θ + 2 $\cos^2 2\theta$ + 3 $\cos^3 2\theta$ + 4 $\cos^4 2\theta$ +...... ∞
where $\theta \in \left(0, \frac{\pi}{4}\right)$, then find least integral value of $\left(\frac{1}{x} + \frac{1}{y}\right)$.

88. If tan A · tan B = $\frac{1}{2}$. Then (5 - 3 cos 2A) (5 - 3 cos 2B) is equal to: **[4]**

89. The number of elements in the set {A = $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$: a, b, d ∈ {-1, 0, 1} and $\{I - A)^3 = I - A^3$}, where I is 2 × 2 identity matrix, is: **[4]**

90. Find the sum of all integral values of a where a ∈ [-10, 10] such that the graph of the function f(x) = $||x - 2| - a|$ - 3 has exactly three x-intercepts. **[4]**

JEE MAIN 2024

Sample Paper - 2

Solution

PHYSICS (Section-A)

1.

(d) $[F][A][T^2]$

Explanation: Given,

Choosen fundamental quantities are,

Force = F

Acceleration = A

Time = T

Dimensions of energy = ?

$\mathrm{E} \propto \mathrm{F}^a\ \mathrm{A}^b\ \mathrm{T}^c$

$\Rightarrow [E] = [F^a]\left[A^b\right][T^c]$

Now,

$\Rightarrow [\mathrm{E}] = \left[\mathrm{ML}^2\ \mathrm{T}^{-2}\right]$

$[\mathrm{A}] = \left[\mathrm{M}^{\circ}\mathrm{LT}^{-2}\right]$

$[\mathrm{T}] = \left[\mathrm{M}^{\circ}\mathrm{L}^{\circ}T^1\right]$

Substituting the values,

$\left[\mathrm{ML}^2\ \mathrm{T}^{-2}\right] = \left[\mathrm{MLT}^{-2}\right]^a[T]^b[T]^c$

$\left[\mathrm{ML}^2\ \mathrm{T}^{-2}\right] = \left[\mathrm{M}^a\ \mathrm{L}^{a+b}\ \mathrm{T}^{-2a-2b+c}\right]$

Now, comparing the exponents,

a = 1 ...(i)

a + b = 2 ...(ii)

-2a - 2b + c = -2 ...(iii)

From equation (i), (ii) and (iii)

b = 2 - 1

b = 1

And C = -2 + 2a + 2b

= -2 + 2 + 2

C = +2

Hence, $[E] = [FAT^2]$

2.

(c) $\frac{3v_1(v_2+v_3)}{4v_1+v_2+v_3}$

Explanation: Let S be the total distance travelled by the body. If t_1 is the time taken to cover first one-third distance, then

$t_1 = \frac{S/3}{v_1} = \frac{S}{3v_1}$

Let t_2 be the time for each of the remaining two journeys. Then

$\frac{2S}{3} = v_2t_2 + v_3t_2$

or $t_2 = \frac{2S}{3(v_2+v_3)}$

$\therefore$ Average velocity = $\frac{\text{total displacement}}{\text{total time}} = \frac{S}{t_1+2t_2}$

$= \frac{S}{\frac{S}{3v_1}+\frac{4S}{3(v_2+v_3)}} = \frac{3v_1(v_2+v_3)}{v_2+v_3+4v_1}$

3.

(d) A

Explanation: From the formula of range for a given initial velocity,

Range, R = $\frac{u^2 \sin 2\theta}{\mathrm{g}}$

For a given initial velocity,

R $\propto$ sin 2θ

4.

(c) < 100 J

Explanation: Kinetic energy = $\frac{1}{2}\mathrm{mv}^2$

When the body returns to the earth its velocity will decrease due to friction of air i.e. some energy is lost in overcoming the friction. Thus, now its kinetic energy is less than 100J.

5.

(b) 90 ms^{-1}

Explanation: The kinetic energy of ball = potential energy of spring

i.e. $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\therefore 16 \times 10^{-3} \times v^2$ = $\frac{90}{10^{-2}} \times \left(12 \times 10^{-2}\right)^2$

or v^2 = $\frac{90 \times 144 \times 10^{-4}}{10^{-2} \times 16 \times 10^{-3}}$

or v = 90 ms^{-1}

6.

(c) $\frac{n}{3}$

Explanation: Retarding torque is constant. Therefore, angular retardation say α will also be constant.

Applying, $\omega^2 = \omega_0^2 - 2\alpha\theta$

we get; $\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha\theta_1$ ...(i)

and $0 = \left(\frac{\omega_0}{2}\right)^2 - 2\alpha\theta_2$ ...(ii)

Solving equations (i) and (ii), we get;

$\theta_2 = \frac{\theta_1}{3}$

Therefore, the disc will make $\frac{n}{3}$ more rotations before coming to rest.

7.

(c) 4,000 N

Explanation: According to Pascal's law, the pressure of a fluid is transmitted in equal directions, i.e.,

$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\therefore$ F_2 = $\left(\frac{F_1}{A_1}\right) \times A_2$

= $\frac{40}{\pi\frac{(2)^2}{4}} \times \pi\frac{(20)^2}{4}$ ...($\because$ A = $\pi\frac{d^2}{4}$)

= 4000 N

8.

(c) Buoyancy will be less in water at 0°C than that in water at 4°C.

Explanation: As buoyancy at 0°C, F_b = Vρ_0g and buoyancy at 4°C, F'_b = Vρ_4g

$\frac{F_b}{F_b'} = \frac{\rho_0}{\rho_4}$ < 1 (Density of water at 4°C is maximum)

So, F_b < F'_b

9.

(d) 40 cal

Explanation: Amount of heat given = 540 calories

Change in volume ΔV = 1670 c.c

Atmospheric pressure P = 1.01 × 10^6 dyne/cm^2

Work done against atmospheric pressure W = PΔV = $\frac{1.01 \times 10^6 \times 1670}{4.2 \times 10^7} \approx$ 40 cal

10.

(b) 0.78

Explanation: Vertical displacement

y = $3kt^2$ + 5t + 1

Differentiating above equation w.r.t. time,

$\frac{dy}{dt}$ = 6kt + 5

Differentiating again with respect to time,

$\frac{d^2y}{dt^2}$ = 6k + 5

i.e., a = 6k + 5

a = 6 × 2 + 5 ...(∵ k = 2 m/s^2)

= 17 m/s^2

Total effective acceleration after displacement is given by,

g' = g + a = 10 + 17 = 27 m/s^2

T_1 = $2\pi\sqrt{\frac{l}{g}}$; T_2 = $2\pi\sqrt{\frac{l}{g'}}$

∴ Difference in time period,

T_1 - T_2 = $2\pi\sqrt{l}\left(\frac{1}{\sqrt{g}} - \frac{1}{\sqrt{g'}}\right) = 2\pi\left(\frac{1}{\sqrt{10}} - \frac{1}{\sqrt{27}}\right)$

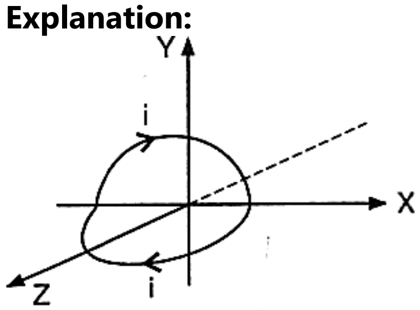
= 0.78

11. **(a)** $\frac{Q}{\varepsilon_0}$

Explanation: According to the Gauss' theorem, the electric flux through a close surface in $\frac{Q}{\varepsilon_0}$.

12. **(a)** $\frac{\mu_0 i}{2\sqrt{2}R}$

Explanation:



The loop mentioned in the question must look like one as shown in the figure.

The magnetic field at the centre due to semicircular loop lying in the X-Y plane, $B_{xy} = \frac{1}{2}\left(\frac{\mu_0 i}{2R}\right)$ negative Z-direction.

Similarly, field due to loop in X-Z plane $B_{xz} = \frac{1}{2}\left(\frac{\mu_0 i}{2R}\right)$ in negative Y-direction.

∴ The magnitude of the resultant magnetic field,

$B = \sqrt{B_{xy}^2 + B_{xz}^2} = \sqrt{\left(\frac{\mu_0 i}{4R}\right)^2 + \left(\frac{\mu_{(L} i}{4R}\right)^2}$

$= \frac{\mu_0 i}{4R}\sqrt{2} = \frac{\mu_0 i}{2\sqrt{2}R}$

13.
(b) 100 K
Explanation: As per Curie law,
$\chi \propto \frac{1}{T}$
$\therefore \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$
$\therefore \frac{2.1 \times 10^{-5}}{1.4 \times 10^{-5}} = \frac{300}{T_2}$
$\frac{3}{2} = \frac{300}{T_2}$
$\therefore T_2 = \frac{600}{3}$
= 200 K
∴ Change in temperature
= T_1 - T_2
= 300 - 200
= 100 K
14. **(a)** in anticlockwise direction
Explanation: As it is seen from the magnet side induced current is will be anticlockwise.
15.
(b) 4 μF
Explanation: Power will be maximum at resonance
At resonance X_L = X_C ⇒ Lω = $\frac{1}{C\omega}$
⇒ 250 π = $\frac{1}{2\pi(50)C}$ ⇒ C = 4 × 10^{-6}
16.
(d) 26.5 W/m^2
Explanation: Here, the amplitude of the electric field, E_0 = 100 V/m; the amplitude of the magnetic field, H_0 = 0.265 A/m. We know that the maximum rate of energy flow,
S = E_0 × H_0 = 100 × 0.265 = 26.5 W/m^2
17. **(a)** $\frac{V}{2000}$ volt
Explanation: $\lambda_e = \lambda_p$
or $\frac{h}{\sqrt{2m_eQ_eV}} = \frac{h}{\sqrt{2m_pQ_pV_p}}$
∴ $m_eQ_eV = m_pQ_pV_p$
$\therefore V_p = \left(\frac{m_e}{m_p}\right)\left(\frac{Q_e}{Q_p}\right)V = \left(\frac{1}{2000}\right)(1)V = \frac{V}{2000}$ volt
18.
(d) $\frac{1}{m}$
Explanation: As we know that,
$(K.E.)_{initial} = (P.E)_{closest\ approach}$
$\frac{1}{2}mv^2 = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$
$\Rightarrow r_0 \propto \frac{1}{m}$
19.
(b) 4.25 MeV
Explanation: For the given nuclear reaction, amount of energy released Q = (m_A - m_B - m_D) × 931.5 MeV = (238.05079 - 234.04363 - 4.00260) × 931.5 or, Q = 4.25 MeV
20.
(d) 100Ω

Explanation: According to the question, when the diode is forward biased,
V_{diode} = 0.5 V

Safe limit of current, I = 10 mA = 10^{-2}A
R_{min} = ?
Voltage through resistance
V_R = 1.5 - 0.5 = 1 volt
iR = 1 (= V_R)
$\therefore R_{min} = \frac{1}{i} = \frac{1}{10^{-2}} = 100\Omega$

PHYSICS (Section-B)

21. 5.0
Explanation:
Given: Conductivity of wire, $\sigma = 5 \times 10^7$ S/m
Radius of wire, r = 0.5 mm = 5×10^{-4} m
Electric field, E = 10×10^{-3} V/m
J = $\sigma E = 10 \times 10^{-3} \times 5 \times 10^7 \Rightarrow J = 5 \times 10^5$
Since, $J = \frac{i}{A} \Rightarrow \frac{i}{A} = 5 \times 10^5$
$\Rightarrow i = 5 \times 10^5 \times \pi r^2 = 5 \times 10^5 \times \pi \times (5 \times 10^{-4})^2$
$= 125\pi \times 10^{-3}$ A
$\therefore$ i = 125π mA $\therefore$ i = $5^3\pi$ mA $\therefore$ x = 5

22. 640.0
Explanation:
Flux through surface x = 0, y = 0, y = 20 cm, z = 0 and z = 20 cm
will be zero as for these surface $\vec{E} \perp \vec{A}$
Flux through surface (x = 0.2 m) = $4000 \times 0.2^2 \times 0.2^2$ = 6.4 Vm = 640 Vcm

23. 18.0
Explanation:
Magnetic field ℓF = B ($\because \varepsilon$ = iR)
$= \left(\frac{\varepsilon}{R}\right)\ell B = \left(\frac{vB\ell}{R}\right)\ell B = \frac{vB^2\ell^2}{R} = \frac{4}{5} \times \left(\frac{15}{100}\right)^2 \times 1^2$
$= \frac{4}{5} \times \frac{225}{10^4} = 18 \times 10^{-3}$ N ($\because \varepsilon = vB\ell$)

24. 0.28
Explanation:
g = $\frac{4}{3}\pi G\rho R$
g $\propto \rho$R
$\therefore \frac{g_e}{g_m} = \frac{\rho_e}{\rho_m} \times \frac{R_e}{R_m}$
$\therefore \frac{6}{1} = \frac{5}{3} \times \frac{R_e}{R_m}$
$\therefore R_m = \frac{5}{18} R_e$
$\therefore k = \frac{5}{18}$ = 0.28

25. 4.0
Explanation:
General equation for displacement is x = A sin (ωt + ϕ)
Comparing with given equation, y = A cos (30°)
$\omega t + \phi = 30^\circ$

$\therefore$ x = 40 × $\frac{\sqrt{3}}{2}$ ⇒ 20$\sqrt{3}$ cm and A = 40 cm

Kinetic energy, K .E = $\frac{1}{2}k\left(A^2 - x^2\right)$ = 200

200 = $\frac{1}{2}$k $\left(\frac{1600-1200}{100\times100}\right)$

⇒ 400 × 100 × 100 = k × 400 ⇒ k = 10^4

$\therefore$ x = 4

26. 5

Explanation:

Given: $\frac{6}{5} = \frac{\left(1+\frac{D}{f}\right)}{\frac{D}{f}}$

or $\frac{6}{5} = \left(\frac{f}{D} + 1\right)$

or $\frac{f}{D} = \frac{6}{5} - 1 = \frac{1}{5}$ or f = $\frac{D}{5}$ = 5 cm.

27. 5

Explanation:

I = 3.36[1 + 2t] × 10^{-2}A, R = 1m, radius of loop x = 10^{-3}m

The magnetic field at the location of loop at a distance r = 1m from wire

$B = \frac{\mu_0 I}{2\pi r}$ perpendicular to the plane of loop

The magnetic flux linked with the loop

$\phi = BA\cos 0^\circ = BA = B\pi x^2 = \frac{\mu_0 I}{2\pi r}\left(\pi x^2\right)$

Induced emf, $e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I}{2\pi r}\pi x^2\right)$

$= \frac{\mu_0}{2r}x^2 \cdot \frac{dI}{dt}$

Current induced, $I = \frac{e}{R} = \frac{\mu_0 x^2}{2rR}\frac{dI}{dt}$

$= \frac{\mu_0 x^2}{2rR}\frac{d}{dt}\left[3.36(1+2t) \times 10^{-2}\right]$

$= \frac{\mu_0 x^2}{2rR}\left[3.36 \times 2 \times 10^{-2}\right]$

Here r = 1m, x = 10^{-3} m, R = 8.4 × 10^{-4} Ω

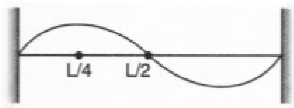
$I = \frac{4\pi\times10^{-7}\times\left(10^{-3}\right)^2\times2\times3.36\times10^{-2}}{2\times1\times8.4\times10^{-4}}$

$= 16\pi \times 10^{-12}\text{A} = 5.0 \times 10^{-11}\text{A}$

28. 300

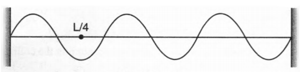
Explanation:

Point of maximum displacement mean antinode. The situation is shown in figure.



For initial situation, $\frac{2\lambda}{2}$ = L

f = $\frac{v}{\lambda}$ = $\frac{v}{L}$ = 100 Hz



For final situation, $\frac{6\lambda}{2}$ = L

f' = $\frac{v}{\lambda}$ = $\frac{3v}{L}$ = 300 Hz

29. 0.022

Explanation:

$I \propto r^2 \Rightarrow \Delta l = 2r(\Delta r)$

or $\frac{\Delta I}{I} = \frac{2r(\Delta r)}{r^2} = \frac{2(\Delta r)}{r}$

but $\frac{\Delta r}{r} = (\alpha)(\Delta t)$

$\therefore \frac{\Delta I}{I} \times 100 = 2(\alpha)(\Delta t)100$ = $2\left(11 \times 10^{-6}\right)(10)(100)$ = 0.022.

30. 33.3

Explanation:

Stress = $\frac{\text{weight}}{\text{area}} = \frac{mg}{A} = \frac{Vdg}{A} = \frac{ALdg}{A}$ = Ldg

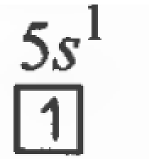
$\therefore$ L = $\frac{\text{breaking stress}}{dg}$ = $\frac{10^6}{3\times10^3\times10}$ = 33.3 m.

CHEMISTRY (Section-A)

31. **(a)** 5, 0, 0, $+\frac{1}{2}$

Explanation: $_{37}$Rb = 2, (2, 6)(2, 6, 10)(2, 6)(1) = $1s^2$, $2s^2 2p^6$, $3s^2 3p^6 3d^{10}$, $4s^2 4p^6$, $5s^1$

Valence electron is $5s^1$



$\therefore$ n = 5, l = 0, m = 0, s = +$\frac{1}{2}$(or $-\frac{1}{2}$)

32. **(a)** both predominantely ionic and % covalent character less than 50%

Explanation: both predominantely ionic and % covalent character less than 50%

33. **(a)** A only

Explanation: PCl_3 has sp^3 hybridised.

34.

(c) both 2 cal mol^{-1} deg^{-1} and 8.314 J mol^{-1} deg^{-1}

Explanation: C_p - C_v = R = 2 cal mol^{-1} deg^{-1}

= 8.314 J mol^{-1} deg^{-1}

35.

(d) 8.7782

Explanation: For basic buffer mixture

pOH = pK_a + log $\frac{[\text{Conjugate acid}]}{[\text{Base}]}$

[base] = [NH_4OH] = $\frac{100\times0.1}{200}$ = 0.05

[conjugate acid] = $\left[NH_4^+\right] = \frac{100\times0.1\times3}{200}$ = 0.15

$(NH_4)_3$ PO_4 gives three ions of NH_4^+ (conjugate acid)

$\therefore$ pOH = -log 1.8 $\times$ 10^{-5} + log $\frac{0.15}{0.05}$

= 4.7447 + 0.4771 = 5.2218

$\therefore$ pH = 14 - 5.2218 = 8.7782

36. **(a)** 1.75

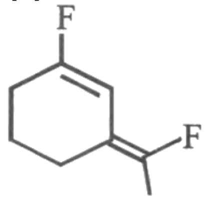
Explanation: 1.75

37.

(c) reactive amorphous

Explanation: reactive amorphous

38. **(a)**



Explanation: Among the substituents attached to the given compounds, fluorine has maximum electronegativity, so it will push the electron pair towards itself. In this option, the compound has the two F groups along the same direction, thus net dipole moment will increase in this direction, and therefore it will exhibit maximum polarity. Hence the compound in this option has maximum polarity.

39.

(b) $\frac{AgNO_3}{NH_4OH}$

Explanation: $\frac{AgNO_3}{NH_4OH}$

40.

(b) Ar < CO_2 < CH_4 < HCHO

Explanation: The unit of K_H should have been reported. However, HCHO being polar will be more soluble. CH_4 is more soluble than CO_2.

41.

(b) $M_B = 4M_A$

Explanation: $\Delta_A T_b = \frac{2}{M_A \times 100} \times 1000 \times K_b$

$\Delta_B T_b = \frac{8}{M_B \times 100} \times 1000 \times K_b$

Given, $\Delta_A T_b = \Delta_B T_b$

$\Rightarrow \frac{2}{M_A} = \frac{8}{M_B} \Rightarrow M_B = 4M_A$

42.

(c) MnO_4^-

Explanation: MnO_4^-

43. **(a)** $t_{1/2}$

Explanation: The inter-section point indicate that half of the reactant X is converted into Y.

44. **(a)** $[Cr (NH_3)_6]^{3+}$

Explanation: Odd electrons, ions, and molecules are paramagnetic.

In $Cr(CO)_6$ molecule 12 electrons are contributed by CO group and it contains no odd electron.

Cr $\rightarrow$ $3d^5 4s^1$

$Fe(CO)_5$ molecule also does not contain an odd electron.

Fe $\rightarrow$ $3d^6 4s^2$

In $[Fe(CN)_6]^{4-}$ ion Fe(+II) $\rightarrow$ $3d^6 4s^0$

$\therefore$ No odd electrons.

In $[Cr(NH_3)_6]^{3+}$ ion Cr(+III) $\rightarrow$ $3d^3 4s^0$

This ion contains an odd electron so it is paramagnetic.

45.

(c) $Ca(ClO_2)_2$

Explanation: Since the valency of calcium is 2 and a chlorite ion is ClO_2^-, therefore calcium chlorite is $Ca(ClO_2)_2$.

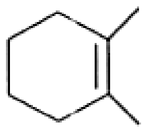
46. **(a)** sp^3, sp^3, dsp^2

Explanation: sp^3, sp^3, dsp^2

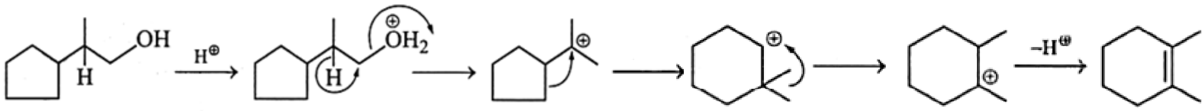
47. **(a)** B > C > A

Explanation: Substrate which form more stable cation they leads to major product.

48. **(a)**



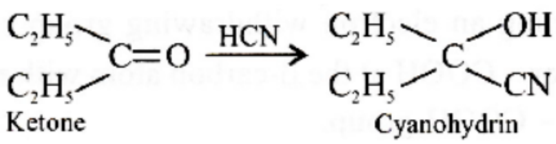
Explanation:



49.

(d) $C_2H_5COC_2H_5$

Explanation: Ketone reacts with HCN to form an addition product, known as cyanohydrin compound.

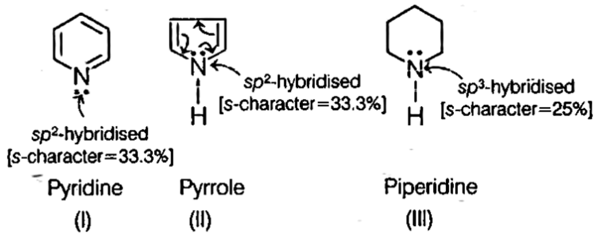


50.

(b) III > I > II

Explanation:

The % of 5-character in the given amines are as follows:



Therefore, piperidine (III) having minimum % s-character is most basic. Among the rest, pyridine (I) and pyrrole (II) the lone pair of electrons of N in pyrrole (II) is involved in delocalisation and follows (4n + 2) n aromatic (n = 1) system. So, the N-atom of pyrrole (II) will show least basicity. Thus, the order of basicity is as follows:

(III) > (I) > (II)

CHEMISTRY (Section-B)

51. 2

Explanation:

The number of spherical or radial nodes in an orbital = (n - l - 1)

4p orbital (n = 4, l = 1) has 2 spherical nodes.

52. 4

Explanation:

For the given chemical reaction,

$$\begin{array}{lcccc} & A & + \; 2B & \rightleftharpoons & 2C \; + \; D \\ \text{At, } t=0 & a_0 & 1.5a_0 & 0 & 0 \\ t=t_{eq} & a_0 - x & 1.5a_0 - 2x & 2x & x \end{array}$$ [x = degree of dissociation]

Given, at equilibrium.

[A] = [B]

$a_0 - x = 1.5a_0 - 2x$

$x = 0.5a_0$

$\therefore$ [A] = $a_0 - x = a_0 - 0.5a_0 = 0.5a_0$

[B] = $1.5a_0 - 2x = 1.5a_0 - 2 \times 0.5a_0 = 0.5a_0$

[C] - 2x = $2 \times 0.5a_0 = a_0$

[D] = x = $0.5a_0$

Now, $K = \frac{[C]^2[D]}{[A][B]^2}$

Now, substituting the values in above equation, we get

$K = \frac{(a_0)^2 \times (0.5a_0)}{(0.5a_0) \times (0.5a_0)} = 4$

53. 4

Explanation:

12σ bonds, 3π bonds

54. 4

Explanation:

Acidic oxides are N_2O_3, NO_2, Cl_2O_7, SO_2

Basic oxides are CaO, Na_2O

Neutral Oxides are CO, NO, N_2O

55. 2.0

Explanation:

The complex that absorbs the light with shortest wavelength is the most stable complex formed by Co^{3+} and strong field ligand CN^-.

$\therefore$ Octahedral low spin Co^{3+} complex, $[Co(CN)_6]^{3-}$ has the electronic configuration = $t_{2g}^6 \; e_g^1$

$\therefore$ $M_{S.O.}$ = $\sqrt{n(n+2)}$ = $\sqrt{1(1+2)}$ = $\sqrt{3}$ = 1.732 $\approx$ 2

56. 3.0

Explanation:

A, B, C are correct and D is incorrect because Fehling solution has Cu(II).

57. 10

Explanation:

Normality = $\frac{\text{No. of equivalents of solute}}{\text{Volume of solution (in L)}}$

0.1 = $\frac{1.43}{\frac{(106+18x)}{2} \times 0.1} \Rightarrow \frac{106+18x}{2}$ = 143

$\Rightarrow$ 18x = 286 - 106 = 180 $\Rightarrow$ x = 10

58. 55.95

Explanation:

Atomic mass of Fe $= \frac{\sum A_1 \cdot X_1}{\sum X_T}$

$= \frac{54 \times 5 + 56 \times 90 + 57 \times 5}{5 + 90 + 5} = 55.95$

59. 6

Explanation:

Total six planes contain HCH atoms in a plane.

60. 150

Explanation:

q = 0, $\triangle U = w$

$1 \times 20 \times [T_2 - 300] = -3000$

$T_2 - 300 = -150 \Rightarrow T_2 = 150$ K

MATHEMATICS (Section-A)

61.

(d) $^{50}P_{33}$

Explanation: Given that f is bijective function and

$f(3) \geq f(9) \geq f(15) \geq f(21) \geq ... \geq f(99)$

So, all elements 3, 9, 15... 99 i.e. 17 elements as 1 choice.

Remaining 50 - 17 = 33 elements has taken from 50 elements.

$\therefore$ Number of ways = $^{50}P_{33}$

62.

(d) 10

Explanation: Given equation is

$|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$

$\Rightarrow |\sqrt{x} - 2| + x - 4\sqrt{x} + 4 = 2$

$\Rightarrow |\sqrt{x} - 2| + (|\sqrt{x} - 2|)^2 = 0$

$\Rightarrow (|\sqrt{x} - 2|)^2 + |\sqrt{x} - 2| - 2 = 0$

Let $|\sqrt{x} - 2| = y$, then above equation reduced to

$y^2 + y - 2 = 0 \Rightarrow y^2 + 2y - y - 2 = 0$

$\Rightarrow y(y + 2) - 1(y + 2) = 0 \Rightarrow (y + 2)(y - 1) = 0$

$\Rightarrow y = 1, -2$

$\therefore$ y = 1 $[\because y = |\sqrt{x} - 2| \geq 0]$

$\Rightarrow \quad |\sqrt{x} - 2| = 1$

$\Rightarrow \quad \sqrt{x - 2} = \pm 1$

$\Rightarrow \quad \sqrt{x} = 3 \text{ or } 1$

$\Rightarrow$ x = 9 or 1

$\therefore$ Sum of roots = 9 + 1 = 10

63.

(b) 90

Explanation: Given A = {1, 2, 3, 4}

B = {1, 2, 3, 4, 5, 6}

Here f(3) can be 2, 3, 4, 5, 6

f(3) = 2, (f(1), f(2)) $\rightarrow$(1, 1) $\rightarrow$ Total 6 cases

f(3) = 3, (f(1), f(2)) $\rightarrow$ (1, 2), (2, 1)

$\rightarrow 2 \times 6$ = Total 12 cases

f(3) = 4,(f(1), f(2)) $\rightarrow$(1, 3), (3, 1), (2, 2)

$\rightarrow 3 \times 6$ = Total 18 cases

f(3) = 5, (f(1), f(2)) $\rightarrow$ (1, 4), (4, 1), (2, 3), (3, 2)

$\rightarrow 4 \times 6$ = Total 24 cases

f(3) = 6,(f(1), f(2)) $\rightarrow$ (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)

$\rightarrow 5 \times 6$ = Total 30 cases

Total number of cases = 6 + 12 + 18 + 24 + 30 = 90

64.

(d) 2^{58}

Explanation: Required sum = $^{59}C_{30}$ + $^{59}C_{31}$ + $^{59}C_{32}$ + ... + $^{59}C_{59}$

= $\frac{1}{2}$ [($^{59}C_{29}$ + $^{59}C_{30}$) + ($^{59}C_{28}$ + $^{59}C_{31}$) + ... + ($^{59}C_0$ + $^{59}C_{59}$)] ... [$\because$ ^nC_r = $^nC_{n-r}$]

= $\frac{1}{2}$ ($^{59}C_0$ + $^{59}C_1$ + $^{59}C_2$ + ... + $^{59}C_{59}$)

= $\frac{1}{2}$ (259) = 2^{58}

65.

(b) a, b, c, d are in G.P.

Explanation: Rearrange given equation, we get

(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) = 0

$\Rightarrow$ $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$\therefore$ ap - b = bp - c = cp - d = 0

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ $\therefore$ a, b, c, d are in GP.

66.

(d) 28

Explanation: 28

67.

(b) 6 : 1

Explanation: Since, given $l_1 + l_2 = 20 \Rightarrow \frac{\mathrm{d}l_2}{\mathrm{d}l_1} = -1$

Now, $A_1 = \left(\frac{l_1}{4}\right)^2$ and $A_2 = \pi\left(\frac{l_2}{2\pi}\right)^2$

Let S = $2A_1 + 3A_2 = \frac{l_1^2}{8} + \frac{3l_2^2}{4\pi}$

For max or min

$\frac{\mathrm{ds}}{\mathrm{d}l_1} = 0 \Rightarrow \frac{2l_1}{8} + \frac{6l_2}{4\pi} \cdot \frac{\mathrm{d}l_2}{\mathrm{d}l_1} = 0$

$\Rightarrow \frac{l_1}{4} = \frac{6l_2}{4\pi} \Rightarrow \frac{\pi l_1}{l_2} = 6$

68. **(a)** $\frac{x(x+2)}{2} + \frac{\log|x-1|}{2} - \frac{\log|x^2+1|}{4} - \frac{\tan^{-1}x}{2} + c$

Explanation: $\int \frac{x^4}{(x-1)(x^2+1)}dx$

= $\int \frac{x^4-1}{(x-1)(x^2+1)}dx$ + $\int \frac{1}{(x-1)(x^2+1)}dx$

= $\int \frac{(x+1)(x-1)(x^2+1)}{(x-1)(x^2+1)}dx$ + $\int \frac{dx}{(x-1)(x^2+1)}$

= $\int (x+1)dx$ + $\int \left[\frac{1}{2(x-1)} - \frac{x+1}{2(x^2+1)}\right] dx$

= $\int x dx + \int dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{4}\int \frac{2x}{x^2+1}dx - \frac{1}{2}\int \frac{1}{x^2+1}dx$

= $\frac{x^2}{2} + x + \frac{1}{2}\log|x-1| \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + c$

69.

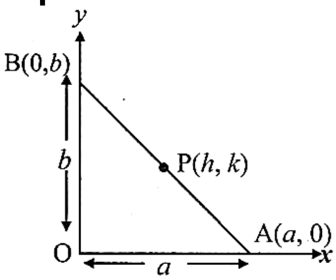
(c) 100

Explanation: 100

70.

(d) a circle

Explanation:



Let P(h, k) be the middle point of stick AB.

Then, P(h, k) $= \left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

$\Rightarrow h = \frac{a}{2}, k = \frac{b}{2}$

$\Rightarrow$ a = 2h, b = 2k

In ΔAOB, $OA^2 + OB^2 = AB^2$

$\Rightarrow a^2 + b^2 = l^2$

$\Rightarrow (2h)^2 + (2k)^2 = l^2$

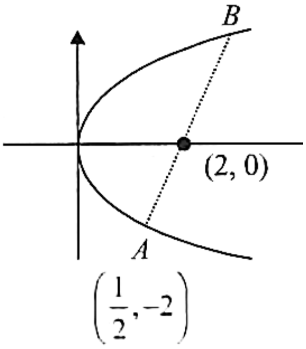
$\Rightarrow h^2 + k^2 = \frac{l^2}{4}$

$\therefore$ The locus of middle point P(h, k) is $x^2 + y^2 = \frac{l^2}{4}$ which is the equation of a circle.

71.

(b) x - 2y + 8 = 0

Explanation:



Let parabola $y^2 = 8x$ at point $\left(\frac{1}{2}, -2\right)$ is $(2t^2, 4t)$

$\Rightarrow t = \frac{-1}{2}$

Parameter of other end of focal chord is 2

So, coordinates of B is (8, 8)

$\Rightarrow$ Equation of tangent at B is 8y - 4(x + 8) = 0

$\Rightarrow$ 2y - x = 8

$\Rightarrow$ x - 2y + 8 = 0

72.

(d) $e^{x^2}\left(y^2 - 1\right) + e^{y^2} = C$

Explanation: Let $y^2 = v \Rightarrow 2y\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \left(e^{x^2} + e^{v^2}\right)\frac{1}{2}\left(\frac{dy}{dx}\right) + e^{x^2}$ (xv - x) = 0

$\Rightarrow e^{x^2} + e^{v} + 2xe^{x^2}(v - 1)\frac{dx}{dv} = 0$ ...(i)

Let $e^{x^2} = t \Rightarrow 2xe^{x^2}\frac{dx}{dv} = \frac{dt}{dv}$

(i) $\Rightarrow t + e^{v} + (v - 1)\frac{dt}{dv} = 0$

$\Rightarrow \frac{dt}{dv} + \frac{t}{v-1} = \frac{-e^v}{v-1}$ (Linear)

I.F. $= e^{\int \frac{1}{v-1}dv} = e^{\log(v-1)} = v - 1$

General solution of the given equation is

t(v - 1) = $-\int e^v dv$ + c

$\Rightarrow$ t (v - 1) = $-e^v$ + c

$\Rightarrow e^{x^2}$ (y^2 - 1) = $-e^{y^2}$ + c

$\Rightarrow e^{x^2}$ (y^2 - 1) = $-e^{y^2}$ + c

73. **(a)** $\cos^{-1}\left(\frac{31}{49}\right)$

Explanation: Direction ratios of the line joining (1, -2, 4) to (-1, 1,-2) are proportional to -2, 3, - 6

cos θ = $\frac{|(2)(-2)+(3)(3)+(6)(-6)|}{\sqrt{2^2+3^2+6^2}\sqrt{(-2)^2+3^2+(-6)^2}}$

= $\frac{31}{40}$

$\Rightarrow \theta$ = $\cos^{-1}\left(\frac{31}{49}\right)$

74.

(b) 3

Explanation: Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$,

A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Since, area of $\triangle$PQR $= \frac{1}{2}|\overrightarrow{q} \times \overrightarrow{r}|$

Area of $\triangle$ABC $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

As $\overrightarrow{AB} = \frac{\overrightarrow{r}-2\overrightarrow{q}}{3}, \overrightarrow{AC} = \frac{-\overrightarrow{r}-\overrightarrow{q}}{3}$

So, Area of $\triangle$ABC $= \frac{1}{6}|\overrightarrow{q} \times \overrightarrow{r}|$

Now, $\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)}$ = 3

75.

(c) a$\bar{x}$ + b

Explanation: Required mean = $\frac{(ax_1+b)+(ax_2+b)+\ldots+(ax_n+b)}{n}$

$= \frac{a(x_1+x_2+\ldots+x_n)+nb}{n}$ = a$\bar{x}$ + b,

$\left(\because \frac{x_1+x_2+\ldots+x_n}{n} = \bar{x}\right)$

76.

(d) $\frac{99}{200}$

Explanation: If A draws 1, then B has options 2 to 100.

If A draws 2, then B has options 3 to 100 and so on.

Required probability = $\frac{1}{100} \times \frac{99}{100} + \frac{1}{100} \times \frac{98}{100} + \ldots + \frac{1}{100} \cdot \frac{1}{100}$

= $\frac{1}{100} \times \frac{99+98+\ldots+1}{100} = \frac{99}{200}$

77.

(d) $\frac{1}{2^{n/2}}$

Explanation: $\frac{1}{2^{n/2}}$

78.

(b) y = x + 1

Explanation: We have $\frac{x^2}{3} - \frac{y^2}{2} = 1$

Since, tangent is equally inclined to the axes i.e., tanθ = 1 = m.

Equation of tangent in slope form is

y = mx + $\sqrt{a^2m^2 - b^2}$

Here, a^2 = 3, b^2 = 2

$\Rightarrow$ y = 1.. x + $\sqrt{3 \times (1)^2 - 2}$

$\Rightarrow$ y = x + 1

79.

(d) B $\cap$ C'

Explanation: (A $\cup$ B $\cup$ C) $\cap$ (A $\cap$ B' $\cap$ C')' $\cap$ C'

= (A $\cup$ B $\cup$ C) $\cap$ (A' $\cup$ B $\cup$ C) $\cap$ C'

= [(A $\cap$ A') $\cup$ (B $\cup$ C)] $\cap$ C'

= (ϕ $\cup$ B $\cup$ Q $\cap$ C'

= (B $\cap$ C') $\cup$ (C $\cap$ C')

= (B $\cap$ C') $\cup$ ϕ

= B $\cap$ C'

80. **(a)** - 2 - $\sqrt{3}$

Explanation: Since the given determinant is equal to zero

$\Rightarrow$ 0 (0 - cosx sinx) - cos x (0 - $\cos^2$x) - sin x($\sin^2$x - 0) = 0

$\Rightarrow$ $\cos^3$x - $\sin^3$x = 0

$\Rightarrow$ $\tan^3$ = 1 $\Rightarrow$ tan x = 1

$\therefore \sum\limits_{\mathrm{x} \in \mathrm{s}} \tan\left(\frac{\pi}{3} + \mathrm{x}\right) = \sum\limits_{x \in S} \frac{\tan \pi/3 + \tan x}{1 - \tan \pi/3 \cdot \tan x}$

$\sum\limits_{x \in s} \frac{\sqrt{3}+1}{1-\sqrt{3}}$

$\sum\limits_{x \in s} \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \sum\limits_{x \in s} \frac{1+3+2\sqrt{3}}{-2}$

= - 2 - $\sqrt{3}$

MATHEMATICS (Section-B)

81. 9

Explanation:

Given f^3(x) = $= \int\limits_0^x$ t f^2 (t) dt ...(i)

Differentiating both the sides of equation (i) by using Leibnitz rule, we get

3f^2(x) f'(x) = x f^2(x)

As f(x) is strictly increasing, so f(x) $\neq$ 0

$\therefore$ 3 f'(x) = x $\Rightarrow$ f'(x) = $\frac{x}{3}$

$\therefore$ f(x) = $\frac{x^2}{6}$ + C (On integrating both sides with respect to x.)

But f(0) = 0 $\Rightarrow$ C = 0

$\therefore$ f(x) = $\frac{x^2}{6}$, x $\geq$ 0

Slope of the tangent at P(x_1, y_1) to the curve f(x) = $\frac{x_1}{3}$

$\therefore$ Slope of normal = $\frac{-3}{x_1}$

i.e., $\frac{-1}{2} = \frac{-3}{x_1}$ (Given)

$\Rightarrow$ x_1 = 6 and y_1 = 6

So, Equation of normal is (y - 6) = $\frac{-1}{2}$ (x - 6)

$\therefore$ For y-intercept, put x = 0, we get y = 9

82. 16

Explanation:

$g^3(x) - (x^3 + 2)g^2(x) + (2x^3 + 1) g(x) - x^3 = 0$

$\Rightarrow (g(x) - 1)^2 g(x) - x^3) = 0$

So, invertible function is

$g(x) = x^3$

$g^{-1}(x) = x^{1/3}$

$g'(8) \cdot \left(g^{-1}\right)'(8) = 16$

83. 285.0

Explanation:

$[\vec{b} \cdot (\vec{a} \times \vec{c})]^2 = [\vec{a} \cdot (\vec{b} \times \vec{c})]^2$

$= \begin{vmatrix} a^2 & \vec{a}.\vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b}.\vec{a} & b^2 & \vec{b}.\vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & c^2 \end{vmatrix}$

As $|\vec{a}| = 14, |\vec{b}| = \sqrt{3}, \vec{a} \cdot \vec{b} = 0$

$\vec{a}.\vec{c} = 11, \vec{b}.\bar{c} = -\sqrt{3}|b| = -\sqrt{3} \times \sqrt{3} = -3$

& $\vec{b}.(\bar{a} \times \vec{c}) = 27$

$\Rightarrow (27)^2 = \begin{vmatrix} 14 & 0 & 11 \\ 0 & 3 & -3 \\ 11 & -3 & c^2 \end{vmatrix}$

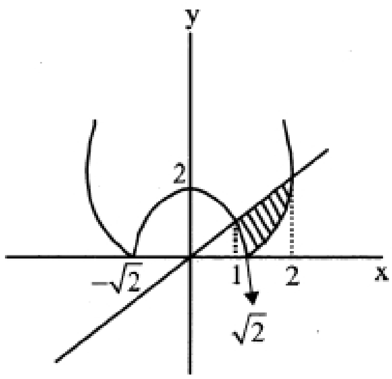
$\Rightarrow 14\left(3c^2 - 9\right) + 11(-33) = 729 \Rightarrow c^2 = 29$

So, $|\vec{a} \times \vec{c}|^2 = a^2c^2 - (\vec{a} \cdot \vec{c})^2 = 14 \times 29 - 11^2 = 285$

84. 27.0

Explanation:

$\left|x^2 - 2\right| \leq y \leq x$



$A = \int_1^{\sqrt{2}} \left(x - \left(2 - x^2\right)\right) dx + \int_{\sqrt{2}}^{2} \left(x - \left(x^2 - 2\right)\right) dx$

$= \left[\frac{x^2}{2} - 2x + \frac{x^3}{3}\right]_1^{\sqrt{2}} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{\sqrt{2}}^{2}$

$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$

$= -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} = \frac{-8\sqrt{2}}{3} + \frac{9}{2}$

Then 6A = $-16\sqrt{2} + 27 \therefore 6A + 16\sqrt{2} = 27$

85. 14.0

Explanation:

Shortest distance between the lines

Since $\vec{a}_1$ = (2, -1, 6), $\vec{a}_2$ = (6, 1, - 8) and

$\vec{b}_1$ = (3, 2, 2), $\vec{b}_2$ = (3, -2, 0)
Now ($\vec{a}_2$ - $\vec{a}_1$) = (4, 2, -14)
$\vec{b}_1 \times \vec{b}_2$ = (4, 6, -12)
So, shortest distance = $\left|\frac{(\vec{a}_2-\vec{a}_1)\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$
$= \left|\frac{16+12+168}{\sqrt{16+36+144}}\right| = \left|\frac{196}{14}\right| = 14$

86. 17
Explanation:
n(S) = 36
Let E be the event of getting the sum of digits on the dice equal to 7, then
n(E) = 6
P(E) = $\frac{6}{36} = \frac{1}{6}$ = P
then P(E') = q = $\frac{5}{6}$
probability of not throwing the sum 7 in first m trails = q^m
Therefore P(at least one 7 in m throw) = 1 - q^m = 1 - $\left(\frac{5}{6}\right)^m$
According to the question
$1 - \left(\frac{5}{6}\right)^m > 0.95 \Rightarrow \left(\frac{5}{6}\right)^m > 0.05$
$\Rightarrow m(\log_{10}5 - \log_{10}6) < \log_{10}1 - \log_{10}20$
$\therefore$ m > 16.44
Hence, the least number of trails = 17

87. 5
Explanation:
5

88. 16
Explanation:
$\tan A \cdot \tan B = \frac{1}{2}$
$\Rightarrow \tan A = \frac{1}{2\tan B}$
$\Rightarrow \tan^2 A = \frac{1}{4\tan^2 B}$
Now, (5 − 3 cos2A)(5 − 3 cos2B)
$= \left(5 - 3\left(\frac{1-\tan^2 A}{1+\tan^2 A}\right)\right)\left(5 - 3\left(\frac{1-\tan^2 B}{1+\tan^2 B}\right)\right)$
$= \left(\frac{5+5\tan^2 A-3+3\tan^2 A}{1+\tan^2 A}\right)\left(\frac{5+5\tan^2 B-3+3\tan^2 B}{1+\tan^2 B}\right)$
$= \left(\frac{2+8\tan^2 A}{1+\tan^2 A}\right)\left(\frac{2+8\tan^2 B}{1+\tan^2 B}\right)$
$= 4 \times \left(\frac{1+4\tan^2 A}{1+\tan^2 A}\right)\left(\frac{1+4\tan^2 B}{1+\tan^2 B}\right)$
$= 4 \times \left(\frac{1+4\left(\frac{1}{4\tan^2 B}\right)}{1+\left(\frac{1}{4\tan^2 B}\right)}\right)\left(\frac{1+4\tan^2 B}{1+\tan^2 B}\right)$
$= 4 \times \left(\frac{4\left(1+\tan^2 B\right)}{1+4\tan^2 B}\right)\left(\frac{1+4\tan^2 B}{1+\tan^2 B}\right)$
= 4 × 4 = 16

89. 8.0
Explanation:

Given that

$(I - A)^2 = I^3 - A^3 - 3A(I - A) = I - A^3$

$\Rightarrow 3A(I - A) = 0 \Rightarrow A^2 = A$

$\Rightarrow \begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$

$\Rightarrow a^2 = a, b(a + d - 1) = 0, d^2 = d$

Case I: b = 0 $\Rightarrow$ (a, d) = (0, 1), (0, 0), (1, 1), (1, 0) $\rightarrow$ 4ways

Case II: a + d = 1 $\Rightarrow$ (1, 0), (0, 1) and b = $\pm 1 \rightarrow$ 4 ways

$\Rightarrow$ Total 8 matrices

90. 3

Explanation:

For x-intercept y = 0

$\therefore ||x-2|-a| = 3$

$\Rightarrow |x-2|$ - a = 3 or -3

$\Rightarrow |x-2|$ = a + 3 or a - 3

For 3x-intercepts

a + 3 > 0 and a - 3 = 0 ...(i)

or a + 3 = 0 and a - 3 > 0 ...(ii)

From Eq. (i) a = 3 and Eq. (ii) is rejected.

Hence, sum = 3