CHAPTER 9

SEQUENCES AND SERIES

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Let a_1, a_2, a_3, ...be the sequence, then the sum expressed as $a_1 + a_2 + a_3 + ... + a_n$ is called ....
(a) Sequence (b) Series
(c) Finite (d) Infinite

2. The third term of a geometric progression is 4. The product of the first five terms is :
(a) 4^3 (b) 4^5 (c) 4^4 (d) 4^7

3. In an AP. the pth term is q and the $(p+q)$th term is 0. Then the qth term is
(a) $-p$ (b) p (c) $p+q$ (d) $p-q$

4. If a, b, c, d, e, f are in A.P., then $e-c$ is equal to:
(a) $2(c-a)$ (b) $2(d-c)$ (c) $2(f-d)$ (d) $(d-c)$

5. The fourth, seventh and tenth terms of a G.P. are p, q, r respectively, then :
(a) $p^2 = q^2 + r^2$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $pqr + pq + 1 = 0$

6. If 1, a and P are in A. P. and 1, g and P are in G. P., then
(a) $1 + 2a + g^2 = 0$ (b) $1 + 2a - g^2 = 0$
(c) $1 - 2a - g^2 = 0$ (d) $1 - 2a + g^2 = 0$

7. For a, b, c to be in G.P. What should be the value of $\frac{a-b}{b-c}$?
(a) ab (b) bc
(c) $\frac{a}{b}$ or $\frac{b}{c}$ (d) None of these

8. What is the sum of terms equidistant from the beginning and end in an A.P. ?
(a) First term – Last term (b) First term × Last term
(c) First term + Last term (d) First term ÷ Last term

9. The first and eight terms of a G.P. are x^{-4} and x^{52} respectively. If the second term is x^t, then t is equal to:
(a) –13 (b) 4 (c) $\frac{5}{2}$ (d) 3

10. If the p^{th}, q^{th} and r^{th} terms of a G.P. are again in G.P., then which one of the following is correct?
(a) p, q, r are in A.P.
(b) p, q, r are in G.P.
(c) p, q, r are in H.P.
(d) p, q, r are neither in A.P. nor in G.P. nor in H.P.

11. If $5(3^{a-1}+1)$, $(6^{2a-3}+2)$ and $7(5^{a-2}+5)$ are in AP, then what is the value of a?
(a) 7 (b) 6
(c) 5 (d) None of these

12. If p^{th} term of an AP is q, and its q^{th} term is p, then what is the common difference ?
(a) – 1 (b) 0 (c) 2 (d) 1

13. If a, b, c are in geometric progression and a, 2b, 3c are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

14. If 1, x, y, z, 16 are in geometric progression, then what is the value of x + y + z ?
(a) 8 (b) 12 (c) 14 (d) 16

15. The product of first nine terms of a GP is, in general, equal to which one of the following?
(a) The 9th power of the 4th term
(b) The 4th power of the 9th term
(c) The 5th power of the 9th term
(d) The 9th power of the 5th term

16. In a G.P. if $(m+n)^{th}$ term is p and $(m-n)^{th}$ term is q, then m^{th} term is:
(a) $\frac{p}{q}$ (b) $\frac{q}{p}$ (c) pq (d) $\sqrt{pq}$

17. The following consecutive terms
$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ of a series are in:
(a) H.P. (b) G.P.
(c) A.P. (d) A.P., G.P.

18. The series $(\sqrt{2}+1), 1, (\sqrt{2}-1)$ ... is in :
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

19. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is:
(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $\sqrt{3}-2$ (d) $3+\sqrt{2}$

20. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61......., then n is equal to
(a) 10 (b) 12 (c) 11 (d) 13

21. There are four arithmetic means between 2 and –18. The means are
(a) –4, –7, –10, –13 (b) 1, –4, –7, –10
(c) –2, –5, –9, –13 (d) –2, –6, –10, –14

22. The arithmetic mean of three observations is x. If the values of two observations are y, z; then what is the value of the third observation ?
(a) x (b) $2x - y - z$
(c) $3x - y - z$ (d) $y + z - x$

23. What is the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...?$
(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

24. $\frac{1}{q+r}, \frac{1}{r+p}, \frac{1}{p+q}$ are in A.P. then,
(a) p, q, r are in A.P (b) p^2, q^2, r^2 are in A.P
(c) $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in A.P (d) $p + q + r$ are in A.P

25. If G be the geometric mean of x and y, then $\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} =$
(a) G^2 (b) $\frac{1}{G^2}$ (c) $\frac{2}{G^2}$ (d) $3G^2$

26. In a Geometric Progression with first term a and common ratio r, what is the Arithmetic Mean of the first five terms?
(a) $a + 2r$ (b) $a r^2$
(c) $a(r^5 - 1)/[5(r - 1)]$ (d) $a(r^4 - 1)/[5(r - 1)]$

27. If p, q, r are in A.P., a is G.M. between p & q and b is G.M. between q and r, then a^2, q^2, b^2 are in
(a) G.P. (b) A.P.
(c) H.P (d) None of these

28. Sum of n terms of series 1.3+3.5+5.7+.......... is
(a) $\frac{1}{3}n(n+1)(2n+1) - n$ (b) $\frac{3}{2}n(n+1)(2n+1) - n$
(c) $\frac{4}{5}n(n+1)(2n+1) - n$ (d) $\frac{2}{3}n(n+1)(2n+1) - n$

29. Let a_1, a_2, a_3............ be terms of an A.P. If $\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ...+ a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

30. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression equals
(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5} - 1)$
(c) $\frac{1}{2}(1 - \sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$.

31. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
(a) –4 (b) –12 (c) 12 (d) 4

32. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is :
(a) a (b) $\frac{a}{1 - a^2b^2}$
(c) $\frac{1}{1 - a^2b^2}$ (d) $\frac{a}{1 + a^2b^2}$

33. If arithmetic mean of a and b is $\frac{(a^{n+1} + b^{n+1})}{a^n + b^n}$, then the value of n is equal to
(a) –1 (b) 0 (c) 1 (d) 2

34. The H. M between roots of the equation $x^2 - 10x + 11 = 0$ is equal to :
(a) $\frac{1}{5}$ (b) $\frac{5}{21}$ (c) $\frac{21}{20}$ (d) $\frac{11}{5}$

35. If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7^{th} and $(m - 1)^{th}$ means 5 : 9, then the value of m is
(a) 10 (b) 11 (c) 12 (d) 14

36. Let S_n denote the sum of first n terms of an A.P. If $S_{2n} = 3\,S_n$, then the ratio $S_{3n}/\,S_n$ is equal to :
(a) 4 (b) 6 (c) 8 (d) 10

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

37. Consider the following statements
I. If $a_1, a_2, ..., a_n ...$ is a sequence, then the expression $a_1 + a_2 + ... + a_n + ...$ is called a series.
II. Those sequences whose terms follow certain patterns are called progressions.
Choose the correct option.
(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

38. Consider the following statements.
I. A sequence is called an arithmetic progression if the difference of a term and the previous term is always same.
II. Arithmetic Mean (A.M.) A of any two numbers a and b is given by $\frac{1}{2}(a + b)$ such that a, A, b are in A.P.
The arithmetic mean for any n positive numbers $a_1, a_2, a_3, ..., a_n$ is given by
$$\text{A.M.} = \frac{a_1 + a_2 + ... + a_n}{n}$$
Choose the correct option.
(a) Only I is true (b) Both are true
(c) Only II is true (d) Both are false

39. **Statement I:** Three numbers a, b, c are in A.P., then b is called the arithmetic mean of a and c.
Statement II: Three numbers a, b, c are in A.P. iff $2b = a + c$.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

40. **Statement I:** If 'a' is the first term and 'd' is the common difference of an A.P., then its n^{th} term is given by
$a_n = a - (n - 1)d$
Statement II: The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by $S_n = \frac{n}{2}\{2a + (n - 1)d\}$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

41. Consider the following statements.
I. The n^{th} term of a G.P. with first term 'a' and common ratio 'r' is given by $a_n = a.r^{n-1}$.
II. Geometric mean of a and b is given by $(ab)^{1/3}$
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

42. I. Three numbers a, b, c are in G.P. iff $b^2 = ac$
II. The reciprocals of the terms of a given G.P. form a G.P.
III. If $a_1, a_2, ..., a_n, ...$ is a G.P., then the expression $a_1 + a_2 + ... + a_n + ...$ is called a geometric series.
Choose the correct option.
(a) Only I and II are true
(b) Only II and III are true
(c) All are true
(d) Only I and III are true

43. I. If each term of a G.P. be raised to the same power, the resulting sequence also forms a G.P.
II. 25^{th} term of the sequence 4, 9, 14, 19, ... is 124.
Choose the correct option.
(a) Both are true (b) Both are false
(c) Only I is true (d) Only II is true

44. I. 18^{th} term of the sequence 72, 70, 68, 66, ... is 40.
II. 4^{th} term of the sequence $8 - 6i, 7 - 4i, 6 - 2i, ...$ is purely real.
Choose the correct option.
(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

45. I. 37 terms are there in the sequence 3, 6, 9, 12, ..., 111.
II. General term of the sequence 9, 12, 15, 18, ... is $3n + 8$.
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true (d) Both are false

46. I. 11^{th} terms of the G.P. 5, 10, 20, 40, ... is 5120
II. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtained quadratic equation is $x^2 - 16x + 25 = 0$
Choose the correct option.
(a) Only I is true (b) Only II is true.
(c) Both are true (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

47.

Column - I	Column - II
A. Sum of 20 terms of the A.P. 1, 4, 7, 10, ... is	1. 70336
B. Sum of the series 5+13+21+...+181 is	2. 156375
C. The sum of all three digit natural numbers, which are divisible by 7, is	3. 2139
D. The sum of all natural numbers between 250 and 1000 which are exactly divisible by 3, is	4. 590

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	3	1	4
(d)	2	1	3	4

48.

Column - I	Column - II
A. Sum of 7 terms of the G.P. 3, 6, 12, ... is	1. $\frac{10}{9}[10^n - 1] + n^2$
B. Sum of 10 terms of the G.P. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$ is	2. $\frac{1023}{512}$
C. Sum of the series $2 + 6 + 18 + ... + 4374$ is	3. 381
D. Sum to n terms of the series $11 + 103 + 1005 + ...$ is	4. 6560

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	1	4	2	3
(c)	3	4	2	1
(d)	3	2	4	1

49.

Column - I	Column - II
A. Sum to infinity of the G.P. $\frac{-5}{4}, \frac{5}{16}, \frac{-5}{64}, ...$ is	1. $\frac{2}{3}$
B. Value of $6^{1/2}.6^{1/4}.6^{1/8}$ .... ∞ is	2. $\frac{1}{3}$
C. If the first term of a G.P. is 2 and the sum to infinity is 6 then the common ratio is	3. – 1
D. If each term of an infinite G.P. is twice the sum of the terms following it, then the common ratio of the G.P. is	4. 6

Codes

	A	B	C	D
(a)	2	1	4	3
(b)	2	4	1	3
(c)	3	4	1	2
(d)	3	1	4	2

50. If the sequence is defined by $a_n = n(n + 2)$, then match the columns.

Column - I	Column - II
A. $a_1 =$	1. 35
B. $a_2 =$	2. 24
C. $a_3 =$	3. 8
D. $a_4 =$	4. 3
E. $a_5 =$	5. 15

Codes

	A	B	C	D	E
(a)	4	3	5	2	1
(b)	4	2	5	3	1
(c)	1	3	2	5	4
(d)	3	4	5	1	2

51. If the nth term of the sequence is defined as $a_n = \frac{2n-3}{6}$, then match the columns.

Column - I	Column - II
A. $a_1 =$	1. 1/6
B. $a_2 =$	2. 1/2
C. $a_3 =$	3. 5/6
D. $a_4 =$	4. –1/6
E. $a_5 =$	5. 7/6

Codes

	A	B	C	D	E
(a)	4	1	3	2	5
(b)	5	3	2	1	4
(c)	4	3	3	1	5
(d)	4	1	2	3	5

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

52. If the jth term and kth term of an A.P. are k and j respectively, the (k + j) th term is
(a) 0 (b) 1
(c) k + j + 1 (d) k + j –1

53. Third term of the sequence whose nth term is $a_n = 2^n$, is
(a) 2 (b) 4 (c) 8 (d) 3

54. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}$, n > 2. Then value of $\frac{a_{n+1}}{a_n}$ for n = 2, is
(a) 1 (b) 2 (c) 3 (d) 4

55. If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116. then the last term is
(a) 0 (b) 2 (c) 4 (d) 6

56. If the sum of first p terms of an A.P. is equal to the sum of the first q terms then the sum of the first $(p + q)$ terms, is
(a) 0 (b) 1 (c) 2 (d) 3

57. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then the value of n is
(a) 1 (b) 2 (c) 3 (d) 4

58. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°. The number of the sides of the polygon is
(a) 6 (b) 9 (c) 8 (d) 5

59. Which term of the following sequence $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$.......is $\frac{1}{19683}$?
(a) 3 (b) 9
(c) 6 (d) None of these

60. How many terms of G.P. 3, 3^2, 3^3, ... are needed to give the sum 120?
(a) 3 (b) 4 (c) 5 (d) 6

61. If f is a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in N$. such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, find the value of n.
(a) 2 (b) 4 (c) 6 (d) 8

62. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then the common ratio is
(a) 5 (b) 1 (c) 4 (d) 3

63. How many terms of the geometric series 1 + 4 + 16 + 64 + ... will make the sum 5461?
(a) 3 (b) 4 (c) 5 (d) 7

64. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n, T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals
(a) $\frac{1}{m} + \frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

65. **Assertion:** For x = ± 1, the numbers $\frac{-2}{7}$, x, $\frac{-7}{2}$ are in G.P.
Reason: Three numbers a, b, c are in G.P. if $b^2 = ac$.

66. **Assertion:** Sum to n terms of the geometric progression $x^3, x^5, x^7, ... (x \neq \pm 1)$ is $\frac{x^3(1-x^{2n})}{(1-x^2)}$.

Reason: If 'a' is the first term and r is common ratio of a G.P. then sum to n terms is given as

$$S_n = \frac{a(r^n-1)}{r-1} \text{ or } = \frac{a(1-r^n)}{1-r} \text{ if } r \neq 1.$$

67. **Assertion:** Value of a_{17}, whose n^{th} term is $a_n = 4n - 3$, is 65.
Reason: Value of a_9, whose n^{th} term is $a_n = (-1)^{n-1}.n^3$.

68. **Assertion:** If each term of a G.. P. is multiplied or divided by some fixed non-zero number, the resulting sequence is also a G P.
Reason: If $-1 < r < 1$, i.e. $|r| < 1$, then the sum of the infinite G P., $a + ar + ar^2 + ... = \frac{a}{1-r}$

i.e., $S_\infty = \frac{a}{1-r}$

69. **Assertion:** If the third term of a G.P. is 4, then the product of its first five terms is 4^5.
Reason: Product of first five terms of a G.P. is given as $a\,(ar)\,(ar^2)\,(ar^3)\,(ar^4)$

70. **Assertion:** If a, b, c are in A.P., then b + c, c + a, a + b are in A.P.
Reason: If a, b, c are in A.P., then $10^a, 10^b, 10^c$ are in G.P.

71. **Assertion:** If $\frac{2}{3}, k, \frac{5}{8}$ are in A.P, then the value of k is $\frac{31}{48}$.
Reason: Three numbers a, b, c are in A.P. iff 2b = a + c

72. **Assertion:** If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, then the value of m is 27.
Reason: 20^{th} term of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, ...$ is $\frac{5}{2^{20}}$

73. **Assertion:** The 20^{th} term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + ... +$ n terms is 1680.
Reason: If the sum of three numbers in A.P. is 24 and their product is 440. Then the numbers are 5, 8, 11 or 11, 8, 5.

74. **Assertion:** Sum of n terms of the A.P., whose k^{th} term is $5k + 1$, is $\frac{n(5n+7)}{2}$.
Reason: Sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 980.

75. **Assertion:** : The sum of n terms of two arithmetic progressions are in the ratio (7n + 1) : (4n + 17), then the ratio of their n^{th} terms is 7:4.
Reason: If $S_n = ax^2 + bx + c$, then $T_n = S_n - S_{n-1}$

76. Let sum of n terms of a series $S_n = 6n^2 + 3n + 1$.
Assertion: The series S_n is in A.P.
Reason: Sum of n terms of an A.P. is always of the form $an^2 + bn$.

77. **Assertion:** The arithmetic mean (A.M.) between two numbers is 34 and their geometric mean is 16.
The numbers are 4 and 64.
Reason: For two numbers a and b, A.M. = $A = \frac{a+b}{2}$
G.M = G = $\sqrt{ab}$.

78. **Assertion:** The ratio of sum of m terms to the sum of n terms of an A.P is $m^2 : n^2$. If T_k is the k^{th} term, then $T_5/T_2 = 3$.
Reason: For n^{th} term, $t_n = a + (n - 1)d$, where 'a' is first term and 'd' is common difference.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

79. Consider an infinite geometric series with first term *a* and common ratio *r*. If its sum is 4 and the second term is $\frac{3}{4}$, then :

(a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = \frac{3}{8}$
(c) $a = \frac{3}{2}, r = \frac{1}{2}$ (d) $a = 3, r = \frac{1}{4}$

80. If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then its common difference is
(a) ±1 (b) ±2 (c) ±3 (d) ±4

81. 4^{th} term from the end of the G.P. 3, 6, 12, 24., ..., 3072 is
(a) 348 (b) 843
(c) 438 (d) 384

82. If $a^x = b^y = c^z$, where a, b, c are in G.P. and $a, b, c, x, y, z \neq 0$; then $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in:
(a) A.P. (b) G.P. (c) H.P (d) None of these

83. The value of $3 - 1 + \frac{1}{3} - \frac{1}{9} + ...$ is equal to:
(a) $\frac{20}{9}$ (b) $\frac{9}{20}$ (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

84. $5^{1+x} + 5^{1-x}, \frac{a}{2}, 5^{2x} + 5^{-2x}$ are in A.P., then the value of a is:
(a) a < 12 (b) $a \leq 12$
(c) $a \geq 12$ (d) None of these

85. The product of *n* positive numbers is unity, then their sum is :
(a) a positive integer (b) divisible by *n*
(c) equal to $n + \frac{1}{n}$ (d) never less than *n*

86. An infinite G.P has first term *x* and sum 5, then
(a) $x < -10$ (b) $-10 < x < 0$
(c) $0 < x < 10$ (d) $x > 10$

87. Sum of the first *n* terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ is equal to :
(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $n + 2^{-n} - 1$ (d) $2^n + 1$

88. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the G.P. is

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$
(c) 4 (d) None of these

89. The sum of 11 terms of an A.P. whose middle term is 30,

(a) 320 (b) 330 (c) 340 (d) 350

90. The first term of an infinite G.P. is 1 and each term is twice the sum of the succeeding terms. then the sum of the series is

(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

91. There are four numbers of which the first three are in G.P. and the last three are in A.P., whose common difference is 6. If the first and the last numbers are equal then two other numbers are

(a) –2, 4 (b) –4, 2
(c) 2, 6 (d) None of these

92. If in a series $S_n = an^2 + bn + c$, where S_n denotes the sum of n terms, then

(a) The series is always arithmetic
(b) The series is arithmetic from the second term onwards
(c) The series may or may not be arithmetic
(d) The series cannot be arithmetic

93. If the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, then the ratio of the first term to the common difference is :

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

94. If the nth term of an arithmetic progression is 3n + 7, then what is the sum of its first 50 terms?

(a) 3925 (b) 4100
(c) 4175 (d) 8200

95. Let x be one A.M and g_1 and g_2 be two G.Ms between y and z. What is $g_1^3 + g_2^3$ equal to ?

(a) xyz (b) xy^2z
(c) xyz^2 (d) 2xyz

96. What is the sum of the first 50 terms of the series $(1 \times 3) + (3 \times 5) + (5 \times 7) + ...$?

(a) 1,71,650 (b) 26,600
(c) 26,650 (d) 26,900

97. The A.M. of the series 1, 2, 4, 8, 16, ..., 2^n is :

(a) $\frac{2^n - 1}{n}$ (b) $\frac{2^{n+1} - 1}{n+1}$

(c) $\frac{2^n + 1}{n}$ (d) $\frac{2^n - 1}{n+1}$

98. The 10 th common term between the series $3 + 7 + 11 + ...$ and $1 + 6 + 11 + ...$ is

(a) 191 (b) 193 (c) 211 (d) None of these

99. A man saves ` 135/- in the first year, ` 150/- in the second year and in this way he increases his savings by ` 15/- every year. In what time will his total savings be ` 5550/-?

(a) 20 years (b) 25 years
(c) 30 years (d) 35 years

100. Let a, b, c, be in A.P. with a common difference d. Then $e^{1/c}, e^{b/ac}, e^{1/a}$ are in :

(a) G.P. with common ratio e^d
(b) G.P with common ratio $e^{1/d}$
(c) G.P. with common ratio $e^{d/(b^2-d^2)}$
(d) A.P.

101. If $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P. then $9^{ax+1}, 9^{bx+1}, 9^{cx+1}, x \neq 0$ are in :

(a) G.P. (b) G.P. only if $x < 0$
(c) G.P. only if $x > 0$ (d) None of these

102. The value of x + y + z is 15 if a, x, y, z, b are in A.P. while the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is $\frac{5}{3}$ if a, x, y, z, b are in H.P. Then the value of a and b are

(a) 2 and 8 (b) 1 and 9
(c) 3 and 7 (d) None of these

103. The A. M. between two positive numbers a and b is twice the G. M. between them. The ratio of the numbers is

(a) $(\sqrt{2} + 3) : (\sqrt{2} - 3)$
(b) $(2 + \sqrt{3}) : (2 - \sqrt{3})$
(c) $(\sqrt{3} + 1) : (\sqrt{3} - 1)$
(d) None of these

104. If S_n denotes the sum of n terms of a G.P. whose first term is a and the common ratio r, then value of $S_1 + S_3 + S_5 + ... + S_{2n-1}$ is

(a) $\frac{a}{1+r}\left[n + r.\frac{1-r^{2n}}{1-r^2}\right]$ (b) $\frac{2a}{1+r}\left[n + r.\frac{1-r^{2n}}{1+r^2}\right]$

(c) $\frac{a}{1+r}\left[n - r.\frac{1-r^{2n}}{1-r^2}\right]$ (d) $\frac{a}{1-r}\left[n - r.\frac{1-r^{2n}}{1-r^2}\right]$

105. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of an A.P., then value of

$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2)$ is

(a) $\frac{1}{2}$ (b) 0 (c) $-\frac{1}{2}$ (d) $\frac{3}{2}$

106. Find the sum up to 16 terms of the series

$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + ...$

(a) 448 (b) 445 (c) 446 (d) None of these

107. The sum of the first n terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

108. If sum of the infinite G.P. is $\frac{4}{3}$ and its first term is $\frac{3}{4}$ then its common ratio is :

(a) $\frac{7}{16}$ (b) $\frac{9}{16}$ (c) $\frac{1}{9}$ (d) $\frac{7}{9}$

109. If sixth term of a H. P. is $\frac{1}{61}$ and its tenth term is $\frac{1}{105}$, then the first term of that H.P. is

(a) $\frac{1}{28}$ (b) $\frac{1}{39}$ (c) $\frac{1}{6}$ (d) $\frac{1}{17}$

110. If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are the p^{th}, q^{th}, r^{th} terms respectively of an A.P. then the value of $ab(p-q)+bc(q-r)+ca(r-p)$ is

(a) -1 (b) 2 (c) 0 (d) -2

111. If the sum of an infinitely decreasing GP is 3, and the sum of the squares of its terms is 9/2, then sum of the cubes of the terms is

(a) $\frac{107}{12}$ (b) $\frac{105}{17}$ (c) $\frac{108}{13}$ (d) $\frac{97}{12}$

112. If x,y,z are in G.P. and $a^x = b^y = c^z$, then

(a) $\log_b a = \log_a c$ (b) $\log_c b = \log_a c$
(c) $\log_b a = \log_c b$ (d) None of these

113. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$ is

(a) 3 (b) 4 (c) 6 (d) 2

114. The fifth term of the H.P., $2, 2\frac{1}{2}, 3\frac{1}{3}$, ... will be

(a) $5\frac{1}{5}$ (b) $3\frac{1}{5}$
(c) $\frac{1}{10}$ (d) 10

115. If the 7^{th} term of a H.P. is $\frac{1}{10}$ and the 12^{th} term is $\frac{1}{25}$, then the 20^{th} term is

(a) $\frac{1}{37}$ (b) $\frac{1}{41}$
(c) $\frac{1}{45}$ (d) $\frac{1}{49}$

116. The harmonic mean of $\frac{a}{1-ab}$ and $\frac{a}{1+ab}$ is

(a) $\frac{a}{\sqrt{1-a^2b^2}}$ (b) $\frac{a}{1-a^2b^2}$
(c) a (d) $\frac{1}{1-a^2b^2}$

117. If the arithmetic, geometric and harmonic means between two distinct positive real numbers be A, G and H respectively, then the relation between them is

(a) $A>G>H$ (b) $A>G<H$
(c) $H>G>A$ (d) $G>A>H$

118. If the arithmetic, geometric and harmonic means between two positive real numbers be A, G and H, then

(a) $A^2=GH$ (b) $H^2=AG$
(c) $G=AH$ (d) $G^2=AH$

119. If b^2, a^2, c^2 are in A.P., then $\frac{1}{a+b},\frac{1}{b+c},\frac{1}{c+a}$ will be in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

120. If the arithmetic mean of two numbers be A and geometric mean be G, then the numbers will be

(a) $A\pm(A^2-G^2)$
(b) $\sqrt{A}\pm\sqrt{A^2-G^2}$
(c) $A\pm\sqrt{(A+G)(A-G)}$
(d) $\frac{A\pm\sqrt{(A+G)(A-G)}}{2}$

121. If $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$, then a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) In G.P. and H.P. both

122. If a, b, c are in A.P. and a, b, d in G.P., then a, a – b, d – c will be in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

123. If the ratio of H.M. and G.M. of two quantities is 12 : 13, then the ratio of the numbers is

(a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) None of these

124. If the ratio of H.M. and G.M. between two numbers a and b is 4 : 5, then the ratio of the two numbers will be

(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)**

2. **(b)** Here,
$t_3 = 4 \Rightarrow ar^2 = 4$
$\therefore$ Product of first five terms
$= a.\ ar.\ ar^2.\ ar^3.ar^4$
$= a^5 r^{10} = (ar^2)^5 = (4)^5$

3. **(b)** Let a, d be the first term and common difference respectively.
Therefore, $T_p = a + (p-1)d = q$ and ...(i)
$T_{p+q} = a + (p+q-1)d = 0$ ...(ii)
Subtracting (i), from (ii) we get $qd = -q$
Substituting in (i), we get
$a = q - (p-1)(-1) = q + p - 1$
Now $T_q = a + (q-1)d = q + p - 1 + (q-1)(-1)$
$= p + q - 1 - q + 1 = p$

4. **(b)** Let x be the common difference of the A.P.
$a, b, c, d, e, f.$
$\therefore e = a + (5-1)x \quad [\because a_n = a + (n-1)d]$
$\Rightarrow e = a + 4x$ ...(i)
and $c = a + 2x$ ...(ii)
$\therefore$ Using equations (i) and (ii), we get
$e - c = a + 4x - a - 2x$
$\Rightarrow e - c = 2x = 2(d - c).$

5. **(b)** Let a be the first term and r be common ratio.
Fourth term of G.P. : $p = T_4 = ar^3$ ...(i)
Seventh term of G.P. : $q = T_7 = ar^6$ ...(ii)
Tenth term of G.P. : $r = T_{10} = ar^9$ ...(iii)
Equ. (i) × Equ. (iii) :
$pr = ar^3 \times ar^9 \Rightarrow pr = a^2r^{12} \Rightarrow pr = (ar^6)^2 \Rightarrow pr = q^2$

6. **(d)** $2a = 1 + P$ and $g^2 = P$
$\Rightarrow g^2 = 2a - 1 \Rightarrow 1 - 2a + g^2 = 0$

7. **(c)** $\frac{a}{b}$ or $\frac{b}{c}$

8. **(c)** First term + last term

9. **(b)** Let a be the first term and r be the common ratio so, general term of G.P is $T_n = ar^{n-1}$
As given,
$T_1 = x^{-4} = a$ and, $T_8 = ar^7 = x^{52} \therefore ar^7 = x^{52}$
$\Rightarrow x^{-4} r^7 = x^{52} \Rightarrow r^7 = x^{56}$
$\Rightarrow r^7 = (x^8)^7 \Rightarrow r = x^8$
$\therefore T_2 = ar^1 = x^{-4}.x^8$
$T_2 = x^4$
But $T_2 = t\,x \Rightarrow x^t = x^4 \Rightarrow t = 4$

10. **(a)** Let R be the common ratio of this GP and a be the first term. pth term is aR^{p-1}, qth term is aR^{q-1} and rth term is aR^{r-1}.
Since p, q and r are in G.P. then
$(aR^{q-1})^2 = aR^{p-1}.\ aR^{r-1}$
$\Rightarrow a^2R^{2q-2} = a^2R^{p+r-2}$
$\Rightarrow R^{2q-2} = R^{p+r-2}$
$\Rightarrow 2q - 2 = p + r - 2$
$\Rightarrow 2q = p + r \Rightarrow$ p, q, r are in A.P.

11. **(d)** None of the options a, b or c satisfy the condition.

12. **(a)** Let first term and common difference of an AP are a and d respectively.
Its p^{th} term $= a + (p-1)d = q$ ...(i)
and q^{th} term $= a + (q-1)d = p$ ...(ii)
Solving eqs. (i) and (ii), we find
$a = p + q - 1,\ d = -1$

13. **(a)** Given that a, b, c, are in GP.
Let r be common ratio of GP.
So, $a = a$, $b = ar$ and $c = ar^2$
Also, given that a, 2b, 3c are in AP.
$\Rightarrow 2b = \frac{a + 3c}{2}$
$\Rightarrow 4b = a + 3c$ ... (i)
From eq. (i)
$4ar = a + 3ar^2$
$\Rightarrow 3r^2 - 4r + 1 = 0$
$\Rightarrow 3r^2 - 3r - r + 1 = 0$
$\Rightarrow 3r(r-1) - 1(r-1) = 0$
$\Rightarrow (r-1)(3r-1) = 0$
$\Rightarrow r = 1$ or $r = \frac{1}{3}$

14. **(c)** As given 1, x, y, z, 16 are in geometric progression.
Let common ratio be r,
$x = 1.\ r = r$
$y = 1.\ r^2 = r^2$
$z = 1.\ r^3 = r^3$
and $16 = 1.r^4 \Rightarrow 16 = r^4$
$\Rightarrow r = 2$
$\therefore x = 1.r = 2,\ y = 1.r^2 = 4,\ z = 1.r^3 = 8$
$\therefore x + y + z = 2 + 4 + 8 = 14$

15. **(d)** Let a be the first term and r, the common ratio
First nine terms of a GP are a, ar, ar^2, ... ar^8.
$\therefore P = a.ar.ar^2 .... ar^8$
$= a^9.r^{1+2+...+8}$
$= a^9.r^{\frac{8.9}{2}} = a^9r^{36}$
$= (ar^4)^9 = (T_5)^9$
= 9th power of the 5th term

16. (d) For a G. P, $a_{m+n} = p$ and $a_{m-n} = q$,
We know that $a_n = AR^{n-1}$ (in G.P.)
where A = first term and R = ratio
$\therefore\ a_{m+n} = p$
$\Rightarrow\ AR^{m+n-1} = p$ ...(i)
and $a_{m-n} = q$
$\Rightarrow\ AR^{m-n-1} = q$ ...(ii)
On multiply equations (i) and (ii), we have
$(AR^{m+n-1}).(AR^{m-n-1}) = pq$
$\Rightarrow\ A^2.R^{2(m-1)} = pq$
$\Rightarrow (AR^{m-1})^2 = pq$
$\Rightarrow AR^{m-1} = \sqrt{pq}$
$\Rightarrow\ a_m = \sqrt{pq}$

17. (c) The following consecutive terms
$\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ are in A.P because
$2\left(\frac{1}{1-x}\right) = \frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}} = \frac{2}{1-x}$
(i.e. $2b = a + c$)

18. (b) Consider series $(\sqrt{2}+1), 1, (\sqrt{2}-1)$, ...
$a = \sqrt{2}+1, r = \sqrt{2}-1$
Common ratios of this series are equal. Therefore series is in G.P.

19. (b) In G.P., let the three numbers be $\frac{a}{r}, a, ar$
If the middle number is double, then new numbers are in A.P.
i.e., $\frac{a}{r}, 2a, ar$, are in A.P.
$\therefore\quad 2a - \frac{a}{r} = ar - 2a$
$\Rightarrow\ a\left[2 - \frac{1}{r}\right] = a[r-2]$
$\Rightarrow\ 2 - \frac{1}{r} = r - 2$
$\Rightarrow\ r + \frac{1}{r} = 4$
$\Rightarrow\ r^2 - 4r + 1 = 0$
$\Rightarrow\ r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$
$\because\ r < 1$ not possible
$\therefore\ r = 2 + \sqrt{3}$

20. (c) Given, $\frac{2n}{2}\{2.2 + (2n-1)3\} = \frac{n}{2}\{2.57 + (n-1)2\}$
or $2(6n+1) = 112 + 2n$ or $10n = 110$
$\therefore\ n = 11$

21. (d) Let the means be X_1, X_2, X_3, X_4 and the common difference be b; then $2, X_1, X_2, X_3, X_4, -18$ are in A.P.;
$\Rightarrow -18 = 2 + 5b \Rightarrow 5b = -20 \Rightarrow b = -4$
Hence, $X_1 = 2 + b = 2 + (-4) = -2$;
$X_2 = 2 + 2b = 2 - 8 = -6$
$X_3 = 2 + 3b = 2 - 12 = -10$;
$X_4 = 2 + 4b = 2 - 16 = -14$
The required means are –2, –6, –10, –14.

22. (c) We take third observation as w
So, $x = \frac{y+z+w}{3}$
$\Rightarrow\ 3x = y + z + w$
$\Rightarrow\ w = 3x - y - z$

23. (d) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ can be written as
$1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \ldots$
[$\because$ This is a GP with first term = 1
and common ratio $= -\frac{1}{2}$]
So, sum of the series
$= \frac{1}{1 - \left(-\frac{1}{2}\right)} = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3}$

24. (b) $1/(q+r), 1/(r+p), 1/(p+q)$ are in A.P.
$\Rightarrow\ \frac{1}{r+p} - \frac{1}{q+r} = \frac{1}{p+q} - \frac{1}{r+p}$
$\Rightarrow\ q^2 - p^2 = r^2 - q^2$
$\Rightarrow\ p^2, q^2, r^2$ are in A.P.

25. (b) As given $G = \sqrt{xy}$
$\therefore \frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} = \frac{1}{xy - x^2} + \frac{1}{xy - y^2}$
$= \frac{1}{x-y}\left\{-\frac{1}{x} + \frac{1}{y}\right\} = \frac{1}{xy} = \frac{1}{G^2}$

26. (c) First five terms of given geometric progression are a, ar, ar^2, ar^3, ar^4
A.M. of these five terms
$= \frac{a + ar + ar^2 + ar^3 + ar^4}{5} = \frac{a(r^5 - 1)}{5(r-1)}$

27. (b) Since p, q, r are in A.P.
$\therefore\ q = \frac{p+r}{2}$ ...(i)
Since a is the G.M. between p, q
$\therefore\ a^2 = pq$ ...(ii)
Since b is the G.M. between q, r
$\therefore\ b^2 = qr$ ...(iii)
From (ii) and (iii)
$p = \frac{a^2}{q},\ r = \frac{b^2}{q}$
$\therefore$ (i) gives $2q = \frac{a^2}{q} + \frac{b^2}{q}$
$\Rightarrow\ 2q^2 = a^2 + b^2 \Rightarrow a^2, q^2, b^2$ are in A.P.

28. (d) $T_n = [n^{th}$ term of 1.3.5......] × [n^{th} term of 3.5.7....]

or $T_n = [1+(n-1)\times 2]\times[3+(n-1)\times 2]$

or $T_n = (2n-1)(2n+1) = (4n^2-1)$

$$S_n = \sum T_n = \sum (4n^2-1)$$

$$= 4.\sum n^2 - \sum 1$$

$$= \frac{4\times n(n+1)(2n+1)}{6} - n = \frac{2}{3}n\,(n+1)(2n+1) - n$$

29. (d) $$\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d} = \frac{p}{q}$$

$$\Rightarrow \frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p = 11, q = 41 \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

30. (b) Let the series a, ar, ar^2, ... are in geometric progression.

given, $a = ar + ar^2$

$\Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1\pm\sqrt{1-4\times -1}}{2} \Rightarrow r = \frac{-1\pm\sqrt{5}}{2}$$

$$\Rightarrow r = \frac{\sqrt{5}-1}{2}$$ [∵ terms of G.P. are positive ∴ r should be positive]

31. (b) As per question,

$a + ar = 12$ …(i)

$ar^2 + ar^3 = 48$ …(ii)

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$$

(∵ terms are +ve and –ve alternately)

$\Rightarrow a = -12$

32. (a) Let C be the required harmonic mean such that

$\frac{a}{1-ab}, C, \frac{a}{1+ab}$ are in H.P.

$\Rightarrow \frac{1-ab}{a}, \frac{1}{C}, \frac{1+ab}{a}$ are in A.P.

$$\Rightarrow \frac{2}{C} = \frac{1-ab}{a} + \frac{1+ab}{a} \Rightarrow \frac{2}{C} = \frac{2}{a} \Rightarrow C = a.$$

33. (b) Arithmetic mean between a and b is given by $\frac{a+b}{2}$

$$\therefore \frac{a+b}{2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

$\Rightarrow 2a^{n+1} + 2b^{n+1} = a^{n+1} + a^n b + b^n a + b^{n+1}$

$\Rightarrow (a^{n+1} - a^n b) + (b^{n+1} - ab^n) = 0$

$\Rightarrow a^n(a-b) + b^n(b-a) = 0$

$\Rightarrow (a^n - b^n)(a-b) = 0$

$\Rightarrow a^n - b^n = 0$ (∵ $a - b \neq 0$)

$$\Rightarrow \left(\frac{a}{b}\right)^n = 1 \Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0$$

$\Rightarrow n = 0$

34. (d) Let α and β be the root of equation

$x^2 - 10x + 11 = 0$

$\therefore \alpha+\beta = 10, \alpha\beta = 11$

$$\therefore HM = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2.11}{10} = \frac{+22}{10} = \frac{11}{5}$$

35. (d) Let the means be $x_1, x_2, ...x_m$ so that $1, x_1, x_2, ...x_m$, 31 is an A.P. of (m + 2) terms.

Now, $31 = T_{m+2} = a + (m+1)d = 1 + (m+1)d$

$$\therefore d = \frac{30}{m+1} \qquad \text{Given}: \frac{x_7}{x_{m-1}} = \frac{5}{9}$$

$$\therefore \frac{T_8}{T_m} = \frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$\Rightarrow 9a + 63d = 5a + (5m-5)d$

$$\Rightarrow 4.1 = (5m-68)\frac{30}{m+1}$$

$\Rightarrow 2m + 2 = 75m - 1020 \Rightarrow 73m = 1022$

$$\therefore m = \frac{1022}{73} = 14$$

36. (b) Since, S_n denote the sum of an A.P. series.

$\therefore S_n = \frac{n}{2}[2a+(n-1)d]$ where 'a' is the first term and 'd' is the common difference of an A.P.

Given, $S_{2n} = 3S_n$

Now, $S_{2n} = \frac{2n}{2}[2a+(2n-1)d]$

∴ From given equation, we have

$$\frac{2n}{2}[2a+(2n-1)d] = \frac{3n}{2}[2a+(n-1)d]$$

$\Rightarrow 2[2a+(2n-1)d] = 3[2a+(n-1)d]$

$\Rightarrow 4a + 2(2n-1)d = 6a + 3(n-1)d$

$\Rightarrow (4n-2)d = 2a + (3n-3)d$

$\Rightarrow 2a = (n+1)d$

Now, consider

$$\frac{S_{3n}}{S_n} = \frac{\frac{1}{2}(3n)[2a+(3n-1)d]}{\frac{1}{2}(n)[2a+(n-1)d]}$$

$$= \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{3[2a+3nd-d]}{[2a+nd-d]}$$

Put value of $2a = (n+1)d$, we get

$$\frac{S_{3n}}{S_n} = \frac{3[(n+1)d+3nd-d]}{(n+1)d+nd-d}$$

$$= \frac{3[nd+d+3nd-d]}{nd+d+nd-d} = \frac{3(4nd)}{2nd} = 6$$

STATEMENT TYPE QUESTIONS

37. **(d)** By definition, both the given statements are true.

38. **(b)**

39. **(c)** Both are statements are true.

40. **(b)** I. n^{th} term is $a_n = a + (n-1)d$

41. **(a)** II. Geometric mean of 'a' and 'b' = $\sqrt{ab}$

42. **(c)** All the given statements are true.

43. **(a)** Both the given statements are true

II. $a = 4, d = 5$

$a_n = 124 \Rightarrow a + (n-1)d = 124$

$\Rightarrow 4 + (n-1)5 = 124$

$\Rightarrow n = 25$

44. **(b)**

I. $a = 72, d = -2$

$a + (n-1)d = 40$

$\Rightarrow 72 + (n-1)(-2) = 40$

$\Rightarrow 2n = 34 \Rightarrow n = 17$

Hence, 17^{th} term is 40.

II. $a = 8 - 6i, d = -1 + 2i$

$a_n = (8 - 6i) + (n-1)(-1 + 2i)$

$= (9 - n) + i(2n - 8)$

a_n is purely real if $2n - 8 = 0 \Rightarrow n = 4$

Hence, 4^{th} term is purely real.

45. **(a)** I. $a = 3, d = 3$

$a + (n-1)d = 111 \Rightarrow 3 + (n-1)(3) = 111$

$\Rightarrow n = 37$

II. $a = 9, d = 3$

$a_n = a + (n-1)d = 9 + (n-1)3 = 3n + 6$

46. **(c)** I. $a.r^{n-1} = 5120 \Rightarrow 5(2^{n-1}) = 5120$

$\Rightarrow 2^{n-1} = 1024 \Rightarrow 2^{n-1} = 2^{10}$

$\Rightarrow n = 11$

II. Let α, β be the roots of the quadratic equation.

A.M. of $\alpha, \beta = \dfrac{\alpha + \beta}{2} = 8$;

G. M. of $\alpha, \beta = \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 5^2$

$\alpha + \beta = 16, \ \alpha\beta = 25$

Equation whose roots are α, β, is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 16x + 25 = 0$

MATCHING TYPE QUESTIONS

47. **(a)** (A) $S_{20} = \dfrac{20}{2}[2 \times 1 + (20-1)3]$

$= 10 \times 59 = 590$

(B) $a + (n-1)d = 181$

$\Rightarrow 5 + (n-1)8 = 181 \Rightarrow n = 23$

$\therefore$ Required sum $= \dfrac{n}{2}[a + l] = \dfrac{23}{2}[5 + 181]$

$= 2139$

(C) 105, 112, 119, ..., 994

$a_n = 994 \Rightarrow a + (n-1)d = 994$

$\Rightarrow 105 + (n-1)7 = 994$

$\Rightarrow n = 128$

$\therefore$ Required sum $= \dfrac{128}{2}[2 \times 105 + (128-1)7]$

$= 70336$

(D) 252, 255, 258, ..., 999

$a_n = 999 \Rightarrow 252 + (n-1)3 = 999$

$\Rightarrow n = 250$

$S_n = \dfrac{250}{2}[252 + 999] = 156375$

48. **(d)** (A) $S_7 = a\left(\dfrac{r^7 - 1}{r - 1}\right) = 3\left(\dfrac{2^7 - 1}{2 - 1}\right)$

$= 3(128 - 1) = 381$

(B) $S_{10} = 1\left[\dfrac{\left(\dfrac{1}{2}\right)^{10} - 1}{\left(\dfrac{1}{2}\right) - 1}\right] = 2\left(1 - \dfrac{1}{210}\right)$

$= \dfrac{1024 - 1}{512} = \dfrac{1023}{512}$

(C) $a = 2, r = 3, l = 4374$

Required sum $= \dfrac{lr - a}{r - 1} = \dfrac{(4374 \times 3) - 2}{3 - 1}$

$= 6520$

(D) $S_n = 11 + 103 + 1005 + \ldots$ to n terms

$= (10+1) + (10^2 + 3) + (10^3 + 5) + \ldots + \{10^n + (2n-1)\}$

$= (10 + 10^2 + 10^3 + \ldots + 10^n) + \{1 + 3 + 5 + \ldots + (2n-1)\}$

$= \dfrac{10(10^n - 1)}{(10 - 1)} + \dfrac{n}{2}(1 + 2n - 1)$

$= \dfrac{10}{9}(10^n - 1) + n^2$

49. **(c)** (A) $S = \dfrac{a}{1 - r} = \dfrac{\frac{-5}{4}}{1 - \left(-\frac{1}{4}\right)} = -1$

(B) $6^{\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \infty\right)} = 6^{\left(\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right)} = 6^1 = 6$

(C) $S_\infty = 6 \Rightarrow \dfrac{2}{1 - r} = 6 \Rightarrow r = \dfrac{2}{3}$

(D) $a_n = 2[a_{n+1} + a_{n+2} + \ldots \infty] \ \forall \ n \in N$

$\Rightarrow a.r^{n-1} = 2[a.r^n + a.r^{n+1} + \ldots \infty]$

$= \dfrac{2.a.r^n}{1 - r}$

$\Rightarrow 1 = \dfrac{2r}{1 - r} \Rightarrow r = \dfrac{1}{3}$

50. **(a)** $a_n = n(n+2)$

For $n=1$, $a_1 = 1(1+2) = 3$

For $n=2$, $a_2 = 2(2+2) = 8$

For $n=3$, $a_3 = 3(3+2) = 15$

For $n=4$, $a_4 = 4(4+2) = 24$

For $n=5$, $a_5 = 5(5+2) = 35$

Thus first five terms are 3, 8, 15, 24, 35.

51. **(d)** Here $a_n = \dfrac{2n-3}{6}$

Putting $n = 1, 2, 3, 4, 5$, we get

$a_1 = \dfrac{2\times1-3}{6} = \dfrac{2-3}{6} = \dfrac{-1}{6}$;

$a_2 = \dfrac{2\times2-3}{6} = \dfrac{4-3}{6} = \dfrac{1}{6}$;

$a_3 = \dfrac{2\times3-3}{6} = \dfrac{6-3}{6} = \dfrac{3}{6} = \dfrac{1}{2}$;

$a_4 = \dfrac{2\times4-3}{6} = \dfrac{8-3}{6} = \dfrac{5}{6}$;

and $a_5 = \dfrac{2\times5-3}{6} = \dfrac{10-3}{6} = \dfrac{7}{6}$

$\therefore$ The first five terms are $-\dfrac{1}{6}, \dfrac{1}{6}, \dfrac{1}{2}, \dfrac{5}{6}$ and $\dfrac{7}{6}$

INTEGER TYPE QUESTIONS

52. **(a)** Let a, be the first term and d, the common difference.

General term (n_{th} term) of the AP is

$T_n = a + (n-1)d$

As given, $T_j = a + (j-1)d = k$ ....(i)

$T_k = a + (k-1)d = j$ ....(ii)

Subtracting (ii) from (i), we get

$(j-k)d = k - j \Rightarrow d = -1$

On putting $d = -1$ in (i), we get

$a + (j-1)(-1) = k$

$\Rightarrow a = k + j - 1$

Now, $T_{k+j} = a + (k+j-1)d = k + j - 1 + [(k+j)-1](-1)$

$= (k+j-1) - (k+j-1) = 0$

53. **(c)** $a_n = 2^n \Rightarrow a_3 = 2^3 = 8$

54. **(b)** For n = 1, $\dfrac{a_{n+1}}{a_n} = \dfrac{a_2}{a_1} = \dfrac{1}{1} = 1$ $(\because a_1 = a_2 = 1)$

and $a_n = a_{n-1} + a_{n-2}, n > 2$ …(A)

n = 3 in equation (A) $a_3 = a_2 + a_1 = 1 + 1 = 2$

for n = 2, $\dfrac{a_{n+1}}{a_n} = \dfrac{a_3}{a_2} = \dfrac{2}{1} = 2$;

55. **(c)** $a = 25, d = 22 - 25 = -3$. Let n be the no. of terms

Sum = 116 ; Sum $= \dfrac{n}{2}[2a + (n-1)d]$

$116 = \dfrac{n}{2}[50 + (n-1)(-3)]$

or $232 = n[50 - 3n + 3] = n[53 - 3n]$

$= -3n^2 + 53n$

$\Rightarrow 3n^2 - 53 + 232 = 0$

$\Rightarrow (n-8)(3n-29) = 0$

$\Rightarrow n = 8$ or $n = \dfrac{29}{3}$, $n \neq \dfrac{29}{3}$ $\therefore n = 8$

$\therefore$ Now, $T_8 = a + (8-1)d = 25 + 7\times(-3)$

$= 25 - 21$

$\therefore$ Last term = 4

56. **(a)** Let a be the first term and d be the common difference of A.P.

Sum of first p terms $= \dfrac{p}{2}[2a + (p-1)d]$ … (i)

Sum of first q terms $= \dfrac{q}{2}[2a + (q-1)d]$ … (ii)

Equating (i) & (ii)

$\dfrac{p}{2}[2a + (p-1)d] = \dfrac{q}{2}[2a + (q-1)d]$

Transposing the term of R.H.S to L.H.S

or $2a(p-q) + p(p-1) - q(q-1)d = 0$

$\Rightarrow 2a(p-q) + [(p^2 - q^2 - (p-q)d] = 0$

or $2a(p-q) + (p-q)[(p+q) - d] = 0$

$\Rightarrow (p-q)[2a + (p+q-1)d] = 0$

$\Rightarrow 2a + (p+q-1)d = 0$ …(iii)

$(\because p \neq q)$

Sum of first $(p+q)$ term $= \dfrac{p+q}{2}[2a + (p+q-1)d]$

$= \dfrac{p+q}{2} \times 0 = 0$

$\therefore$ $2a + (p+q-1)d = 0$ [from (iii)]

57. **(a)** A. M. between a and b $= \dfrac{a+b}{2}$

$\therefore \dfrac{a^n + b^n}{a^{n-1} + b^{n-1}} = \dfrac{a+b}{2}$

$2a^n + 2b^n = a^n + ab^{n-1} + a^{n-1}b + b^n$

$\Rightarrow a^n a^{n-1}b - ab^{n-1} + b^n = 0$

$\Rightarrow a^{n-1}(a-b) - b^{n-1}(a-b) = 0$

$\Rightarrow (a-b)(a^n - b^{n-1}) + b^n = 0$ $[\because a \neq 0]$

$\Rightarrow a^{n-1} - b^{n-1} = 0 \Rightarrow a^{n-1} = b^{n-1}$

$\left(\dfrac{a}{b}\right)^{n-1} = 1 = \left(\dfrac{a}{b}\right)^0 \Rightarrow n - 1 = 0 \Rightarrow n = 1$

58. (b) The angles of a polygon of n sides form an A.P. whose first term is 120° and common difference is 5°.
The sum of interior angles

$$= \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2\times 120+(n-1)5]$$

$$= \frac{n}{2}[240+5n-5] = \frac{n}{2}(235+5n)$$

Also the sum of interior angles = 180 × n – 360

$$\therefore \frac{n}{2}(235+5n) = 180n-360$$

Multiplying by $\frac{2}{5}$, $n(47+n) = 2(36n-72)$

$n(47+n) = 72n - 144$
$\Rightarrow n^2 + (47-72)n + 144 = 0$
$\Rightarrow n^2 - 25n + 144 = 0$
$\Rightarrow (n-16)(n-9) = 0$
$\Rightarrow n \neq 16 \ \therefore n = 9$

59. (b) $a = \frac{1}{3}$, $r = \frac{1/9}{1/3} = \frac{1}{9}\times\frac{3}{1} = \frac{1}{3}$

Let $T_n = \frac{1}{19683} \Rightarrow ar^{n-1} = \frac{1}{19683}$

$$\Rightarrow \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)^{n} = \left(\frac{1}{3}\right)^{9} \Rightarrow n = 9$$

60. (b) Let n be the number of terms of the G.P. 3, 3^2, 3^3, ... makes the sum = 120
we have $a = 3$, $r = 3$

$$S = \frac{a(r^n-1)}{r-1},\ r>1;\ \text{Sum} = \frac{3(3^n-1)}{3-1} = 120$$

or $\frac{3}{2}(3^n-1) = 120$

Multiplying both sides by $\frac{3}{2}$

$\therefore 3^n - 1 = 80$
$\therefore 3^n = 80 + 1 = 81 = 3^4 \Rightarrow n = 4$
$\therefore$ Required number of terms of given G. P. is 4

61. (b) $f(1) = 3, f(x+y) = f(x)f(y)$
$f(2) = f(1+1) = f(1)f(1) = 3.3 = 9$
$f(3) = f(1+2) = f(1)f(2) = 3.9 = 27$
$f(4) = f(1+3) = f(1)f(3) = 3.27 = 81$
Thus we have

$$\sum_1^n f(x) = f(1)+f(2)+f(3)+........+f(n) = 120$$

$\Rightarrow 3 + 9 + 27 + ...$ to n term = 120

or $\frac{3(3^n-1)}{3-1} = 120 \qquad [a=3, r=3]$

$$\therefore \frac{3(3^n-1)}{2} = 120 \Rightarrow 3^n - 1 = 120\times\frac{2}{3} = 80$$

$3^n = 80 + 1 = 81 = 3^4 \Rightarrow n = 4$

62. (c) Let the G.P. be a, ar, ar^2, ...
$S = a + ar + ar^2 + ...+$ to $2n$ term

$$= \frac{a(r^{2n}-1)}{r-1}$$

The series formed by taking term occupying odd places is $S_1 = a + ar^2 + ar^4 + ...$ to n terms

$$S_1 = \frac{a\left[(r^2)^n-1\right]}{r^2-1} \Rightarrow S_1 = \frac{a(r^{2n}-1)}{r^2-1}$$

Now, $S = 5S_1$

or $\frac{a(r^{2n}-1)}{r-1} = 5\frac{a(r^{2n}-1)}{r^2-1}$

$$\Rightarrow 1 = \frac{5}{r+1}$$

$\Rightarrow r + 1 = 5 \therefore r = 4$

63. (d) $a\left(\frac{r^n-1}{r-1}\right) = 5461 \Rightarrow \frac{4^n-1}{4-1} = 5461$

$\Rightarrow 4^n = 4^7$
$\Rightarrow n = 7$

64. (d) $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)

$T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)

(i) - (ii) $\Rightarrow (m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

ASSERTION - REASON TYPE QUESTIONS

65. (a) The numbers $\frac{-2}{7}, x, \frac{-7}{2}$ will be in G.P.

If $\frac{x}{-\frac{2}{7}} = \frac{-\frac{7}{2}}{x} \Rightarrow x^2 = -\frac{7}{2}\times-\frac{2}{7} = 1 \Rightarrow x = \pm 1$

66. (a) Here $a = x^3$, $r = \frac{x^5}{x^3} = x^2, x \neq \pm 1$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{x^3(1-x^{2n})}{(1-x^2)}$$

67. (b) **Assertion:** $a_n = 4n - 3$

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Reason: $a_n = (-1)^{n-1}.n^3$

$a_9 = (-1)^{9-1}.(9)^3 = (-1)^8(729) = 729$

68. (b) Both are true but Reason is not the correct explanation for the Assertion.

69. (a) **Assertion:** $a_3 = 4 \Rightarrow ar^2 = 4$

$\therefore$ Product of first five terms = a (ar) $(ar^2)(ar^3)(ar^4)$

$= a^5.r^{10} = (ar^2)^5 = 4^5$

70. (b) **Assertion:** b + c, c + a, a + b will be in A.P.

if (c + a) – (b + c) = (a + b) – (c + a)

i.e. if 2b = a + c

i.e. if a, b, c are in A.P.

Reason: $10^a, 10^b, 10^c$ are in G.P. if $\frac{10^b}{10^a} = \frac{10^c}{10^b}$

i.e. if $10^{b-a} = 10^{c-b}$

i.e. if b – a = c – b $\Rightarrow$ 2b = a + c which is true.

71. (a) **Assertion:** $2k = \frac{2}{3} + \frac{5}{8} = \frac{16+15}{24}$

$2k = \frac{31}{24}$

$k = \frac{31}{24 \times 2} = \frac{31}{48}$

72. (b) **Assertion:** Let the sum of n term is denoted by S_n

$\therefore\ S_n = 3n^2 + 5n$

Put $n = 1, 2.\ \ T_1 = S_1 = 3.1^2 + 5.1 = 3 + 5 = 8;$

$S_2 = T_1 + T_2 = 3.\,2^2 + 5\,.\,2 = 12 + 10 = 22$

$\therefore\ T_2 = S_2 - S_1 = 22 - 8 = 14$

$\therefore$ Common difference $d = T_2 - T_1 = 14 - 8 = 6$

$a = 8,\ d = 6$

m^{th} term $= a + (m-1)d = 164 \Rightarrow 8 + (m-1)\,.\,6 = 164$

$6m + 2 = 164 \Rightarrow 6m = 164 - 2 = 162$

$\therefore\ m = \frac{162}{6} = 27$

Reason: $T_n = ar^{n-1}$

$T_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2}.\frac{1}{2^{19}} = \frac{5}{2^{20}}$

73. (b) **Assertion:** First factor of the terms are 2, 4, 6, ...

$\therefore$ First factor of n^{th} term $= 2n$ ...(i)

Second factor of the term are 4, 6, 8 ....

$\therefore$ Second factor of n^{th} term

$= 4 + (n-1)2 = 2(n+1)$ ...(ii)

$\therefore\ n^{th}$ term of the given series

$= 2n \times 2(n+1) = 4n(n+1)$

$\therefore$ putting $n = 20$

20^{th} term of the given series $= 4 \times 20 \times 21$

$= 80 \times 21 = 1680$

Reason: Let three number in A.P be $a - d, a, a + d$

Their sum $= a - d + a + a + d = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Their product $= (a-d)(a)(a+d) = 440$

$a(a^2 - d^2) = 440 \Rightarrow 8(64 - d^2) = 440$

$\Rightarrow 64 - d^2 = 55 \Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow d = \pm 3$

Hence, the numbers are 8 – 3, 8, 8 + 3 or 8 + 3, 8, 8 – 3 i.e., 5, 8, 11, or 11, 8, 5

74. (c) **Assertion:** $T_k = 5k + 1$ Putting $k = 1, 2$

$T_1 = 5 \times 1 + 1 = 5 + 1 = 6$;

$T_2 = 5 \times 2 + 1 = 10 + 1 = 11$

$\therefore\ d = T_2 - T_1 = 11 - 6 = 5$

$a = 6,\ d = 5$

Sum of n term $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 6 + (n-1)5]$

$= \frac{n}{2}[12 + 5n - 5] = \frac{n(5n+7)}{2}$

Reason: We have to find the sum

105 + 110 + 115 + ...+ 995

Let 995 = n^{th} term

$\therefore\ a + [n-1]d = 995$ or $105 + [n-1]5 = 995$

Dividing by 5,

$21 + (n-1) = 199$ or $n = 199 - 20 = 179$

$\therefore$ 105 + 110 + 115 + ...+ 995

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{179}{2}[2 \times 105 + (179-1)5]$

$= \frac{179}{2}[2 \times 105 + 5 \times 178] = 98450$

75. (a) $\because \frac{S_n}{S'_n} = \frac{(7n+1)}{(4n+17)} = \frac{n(7n+1)}{n(4n+17)}$

$\therefore S_n = (7n^2 + n)\lambda,\ S'_n = (4n^2 + 17n)\lambda$

Then, $\frac{T_n}{T'_n} = \frac{S_n - S_{n-1}}{S'_n - S'_{n-1}} = \frac{7(2n-1)+1}{4(2n-1)+17} = \frac{14n-6}{8n+13}$

$\Rightarrow T_n : T'_n = (14n - 6) : (8n + 13)$

76. (d) We have, $S_n = 6n^2 + 3n + 1$

$\therefore S_1 = 6 + 3 + 1 = 10$

$S_2 = 24 + 6 + 1 = 31$

$S_3 = 54 + 9 + 1 = 64$ and so on.

So, $T_1 = 10$

$T_2 = S_2 - S_1 = 31 - 10 = 21$

$T_3 = S_3 - S_2 = 64 - 31 = 33$

So, the sequence is 10, 21, 33,...

Now, 21 – 10 = 11 and 33 – 21 = 12 $\neq$ 11

$\therefore$ The given series is not in A.P.

So, Assertion is false and Reason is true.

77. **(a)** Let the numbers be a and b.

Then, $A.M. = \frac{a+b}{2} = 34 \Rightarrow a + b = 68$ ...(i)

Also, $G.M. = \sqrt{ab} = 16 \Rightarrow ab = 256$ ...(ii)

Now, $a - b = \pm\sqrt{(a+b)^2 - 4ab}$

$= \pm\sqrt{(68)^2 - 4\times 256} = \pm\sqrt{4624 - 1024} = \pm\sqrt{3600}$

$\Rightarrow a - b = \pm 60$

$\therefore$ a – b = 60 or a – b = – 60 ...(iii)

when a – b = 60, then solving (i) and (iii), we get a = 64 and b = 4.

Then, numbers are 64 and 4.

When a – b = –60, then solving (i) and (iii), we get a = 4, b = 64

$\therefore$ Numbers are 4 and 64.

78. **(b)** $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ (given)

Also

$$\frac{T_m}{T_n} = \frac{S_m - S_{m-1}}{S_n - S_{n-1}} = \frac{m^2 - (m-1)^2}{n^2 - (n-1)^2} = \frac{2m-1}{2n-1}$$

Substituting m = 5 and n = 2, we get

$$\frac{T_5}{T_2} = \frac{2(5)-1}{2(2)-1} = \frac{9}{3} = 3$$

CRITICALTHINKING TYPE QUESTIONS

79. **(d)** Since, sum = 4

and second term = $\frac{3}{4}$

$\Rightarrow \frac{a}{1-r} = 4$, and $ar = \frac{3}{4}$

$\Rightarrow \frac{a}{1 - \frac{3}{4a}} = 4$

$\Rightarrow (a-1)(a-3) = 0$

$\Rightarrow a = 1$ or $a = 3$

80. **(c)** Let roots be α, β, γ and $a = a - d, b = a, c = a + d$.

Then $\alpha + \beta + \gamma = 3a = -(-12) \Rightarrow a = 4$

$\alpha\beta\gamma = a(a^2 - d^2) = -(-28) \Rightarrow d = \pm 3$

81. **(d)** Clearly, the given progression is a G.P. with common ratio $r = 2$.

$\therefore$ 4th term from the end $= \ell\left(\frac{1}{r}\right)^{4-1}$

$= (3072)\left(\frac{1}{2}\right)^{4-1} = 384$

82. **(a)** As given : $a^x = b^y = c^z$

Let, $a^x = b^y = c^z = k$ (say)

$\Rightarrow a = k^{1/x}, b = k^{1/y}, c = k^{1/z}$

As given : a, b, c are in G.P.

$\Rightarrow b^2 = ac$

i.e., $k^{2/y} = k^{1/x} k^{1/z} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$

$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

83. **(c)** The given sereis $3 - 1 + \frac{1}{3} - \frac{1}{9}.......\infty$ is in G.P.

Its common ratio $r = -\frac{1}{3}$ and first term a = 3

$$S_\infty = \frac{a}{1-r} = \frac{3}{1+\frac{1}{3}} = \frac{3\times 3}{4} = \frac{9}{4}$$

84. **(d)** Given : $5^{1+x} + 5^{1-x}, \frac{a}{2}, 5^{2x} + 5^{-2x}$ are in A.P.

We know that if a, b, c are in A.P. then 2b = a + c

$\therefore\ 2.\frac{a}{2} = 5^{1+x} + 5^{1-x} + 5^{2x} + 5^{-2x}$

$\Rightarrow a = 5.5^x + 5(5^x)^{-1} + (5^x)^2 + (5^x)^{-2}$

Let $5^x = t$

$\therefore\ a = 5t + \frac{5}{t} + t^2 + \frac{1}{t^2}$

$\Rightarrow a = t^2 + \frac{1}{t^2} + 5\left(t + \frac{1}{t}\right)$

$\Rightarrow a = \left(t + \frac{1}{t}\right)^2 - 2 + 5\left(t + \frac{1}{t}\right)$

Put $t + \frac{1}{t} = A$

$\therefore\ a = A^2 + 5A - 2$ [add & subtract $\left(\frac{b}{2a}\right)^2$]

$\Rightarrow a = \left[A^2 + 5A - \left(\frac{5}{2}\right)^2\right] + \left(\frac{5}{2}\right)^2 - 2$

$\Rightarrow a = \left(A - \frac{5}{2}\right)^2 + \frac{17}{4}$

$\Rightarrow a \ge \frac{17}{4}$.

85. **(d)** Since, product of n positive number is unity.

$\Rightarrow x_1 x_2 x_3 ....x_n = 1$ ...(i)

Using A.M. $\ge$ GM

$\Rightarrow \frac{x_1 + x_2 + ... + x_n}{n} \ge (x_1 x_2 .....x_n)^{\frac{1}{n}}$

$\Rightarrow x_1 + x_n + ... + x_n \ge n\ (1)^{\frac{1}{n}}$ [From eq^n (i)]

$\Rightarrow$ Sum of n positive number is never less than n.

86. **(c)** We know that, the sum of infinite terms of GP is

$$S_\infty = \begin{cases} \dfrac{a}{1-r}, & |r|<1 \\ \infty, & |r| \ge 1 \end{cases}$$

$\therefore S_\infty = \dfrac{x}{1-r} = 5 \quad (\because |r|<1)$

or, $1 - r = \dfrac{x}{5}$

$\Rightarrow r = \dfrac{5-x}{5}$ exists only when $|r| < 1$

i.e., $-1 < \dfrac{5-x}{5} < 1$

or, $-10 < -x < 0$

or, $0 < x < 10$

87. **(c)** Sum of the n terms of the series

$\dfrac{1}{2}+\dfrac{3}{4}+\dfrac{7}{8}+\dfrac{15}{16}+.......$ upto n terms, can be written as

$\left(1-\dfrac{1}{2}\right)+\left(1-\dfrac{1}{4}\right)+\left(1-\dfrac{1}{8}\right)+\left(1-\dfrac{1}{16}\right)....$ upto n terms

$= n - \left(\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+.....+n \text{ terms}\right)$

$= n - \dfrac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}$

$= n + 2^{-n} - 1$

88. **(c)** Let us consider a G.P. $a, ar, ar^2, ...$with $2n$ terms.

We have $\dfrac{a(r^{2n}-1)}{r-1} = \dfrac{5a\left[(r^2)^n - 1)\right]}{(r^2-1)}$

(Since common ratio of odd terms will be r^2 and number of terms will be n)

$\Rightarrow \dfrac{a(r^{2n}-1)}{r-1} = 5\dfrac{a(r^{2n}-1)}{(r^2-1)}$

$\Rightarrow a(r+1) = 5a$, i.e., $r = 4$

89. **(b)** Middle term $= 6^{th}$ term $= 30$

$\Rightarrow a + 5d = 30$

$S_{11} = \dfrac{11}{2}[2a+10d] = \dfrac{11}{2}\times 2[a+5d] = 11 \times 30 = 330$

90. **(c)** Let the G.P. be $1, r, r^2, ... \infty$

Given $x_n = 2(x_{n+1} + x_{n+2} + ...\text{ to } \infty)$

$\therefore x_n = 2\dfrac{x_{n+1}}{1-r}$ [common ratio is r]

$\therefore \dfrac{x_{n+1}}{x_n} = \dfrac{1-r}{2} \Rightarrow r = \dfrac{1-r}{2} \quad \therefore r = \dfrac{1}{3}$

The sum of required series is

$1+\dfrac{1}{3}+\left(\dfrac{1}{3}\right)^2+.....\infty = \dfrac{1}{1-\frac{1}{3}} = \dfrac{3}{2}$

91. **(b)** Let the last three numbers in A.P. be $a, a+6, a+12$, then the first term is also $a + 12$.

But $a+12, a, a+6$ are in G.P.

$\therefore a^2 = (a+12)(a+6) \Rightarrow a^2 = a^2 + 18a + 72$

$\therefore a = -4.$

$\therefore$ The numbers are 8, –4, 2, 8.

92. **(b)** $S_n = an^2 + bn + c$

$\therefore S_{n-1} = a(n-1)^2 + b(n-1) + c$ for $n \ge 2$

$\therefore t_n = S_n - S_{n-1}$

$= a\{n^2 - (n-1)^2\} + b\{n-(n-1)\}$

$= a(2n-1) + b$

$\therefore t_n = 2an + b - a,\ n \ge 2$

$\therefore t_{n-1} = 2a(n-1) + b - a$ for $n \ge 3$

$\therefore t_n - t_{n-1} = 2a(n-n+1) = 2a$ for $n \ge 3$

$\therefore t_3 - t_2 = t_4 - t_3 = ...2a$

Now $t_2 - t_1 = (S_2 - S_1) - S_1 = S_2 - 2S_1$

$= (a.2^2 + b.2 + c) - \{a.1^2 + b.1 + c\}$

$= 2a - c \ne 2a$

$\therefore$ Series is arithmetic from the second term onwards.

93. **(a)** Sum of n terms of A.P with first term = a and common difference, = d is given by

$S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = 5[2a + 9d]$

$S_5 = \dfrac{5}{2}[2a + 4d]$

According to the given condition,

$S_{10} = S_5 \Rightarrow 5[2a+9d] = 4 \times \dfrac{5}{2}[2a+4d]$

$\Rightarrow \quad 2a + 9d = 2[2a + 4d]$

$\Rightarrow \quad 2a + 9d = 4a + 8d \Rightarrow d = 2a$

$\Rightarrow \quad \dfrac{a}{d} = \dfrac{1}{2} \Rightarrow a : d = 1 : 2$

94. **(c)** As given, n^{th} term is

$T_n = 3n + 7$

Sum of n term, $S_n = \sum T_n$

$= \sum(3n+7) = 3\sum n + 7\sum 1$

$= \dfrac{3n(n+1)}{2} + 7n = n\left[\dfrac{3n+3+14}{2}\right]$

$= n\left[\dfrac{3n+17}{2}\right]$

Sum of 50 terms $= S_{50} = 50\left[\dfrac{3\times 50+17}{2}\right]$

$= 50\left[\dfrac{167}{2}\right] = 25 \times 167 = 4175$

95. (d) Since x is A.M

$\Rightarrow \quad x = \frac{y+z}{2}$,

$\Rightarrow \quad 2x = y + z$ ...(i)

and y, g_1, g_2, z ....are in G.P.

$\Rightarrow \quad \frac{g_1}{y} = \frac{g_2}{g_1} = \frac{z}{g_2}$

$\Rightarrow \quad g_1^2 = g_2 y$

$\Rightarrow \quad g_1^3 = g_1 g_2 y$ ...(ii)

Also, $g_2^2 = g_1 z$

$g_2^3 = g_1 g_2 z$ ...(iii)

$\Rightarrow \quad g_1^2 g_2^2 = g_1 g_2 yz$

$\Rightarrow \quad yz = g_1 g_2$ ...(iv)

Adding equations (ii) and (iii)

$g_1^3 + g_2^3 = y g_1 g_2 + z g_1 g_2 = g_1 g_2 (y + z)$

$= yz . 2x = 2xyz$

96. (a) The given series is

$(1 \times 3) + (3 \times 5) + (5 \times 7) + ...$

Its general term is given by

$T_n = (2n - 1)(2n + 1) = 4n^2 - 1$

Sum of series $= 4\Sigma n^2 - \Sigma 1$

$S_n = \frac{4n(n+1)(2n+1)}{6} - n$

$S_n = n\left[\frac{2(2n^2 + 3n + 1)}{3} - 1\right]$

$S_n = n\left[\frac{4n^2 + 6n + 2 - 3}{3}\right]$

$S_n = \left[\frac{n(4n^2 + 6n - 1)}{3}\right]$

For sum of first 50 terms of the series,

n = 50,

$S_{50} = \frac{50[4(50)^2 + 6(50) - 1]}{3}$

$= \frac{50 \times (10000 + 300 - 1)}{3}$

$= \frac{50 \times 10299}{3} = 171650$

97. (b) We know that A.M. $= \frac{S_n}{n+1}$

Given sequence 1, 2, 4, 8, 16,....., 2^n.

$\Rightarrow \quad S_n = 1 + 2 + 2^2 + 2^3 + 2^4 + ... + 2^n$

$= \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1 \left[\because S_n = \frac{a(r^n - 1)}{(r - 1)}\right]$

$\therefore \quad$ A.M. $= \frac{2^{n+1} - 1}{n+1}$.

98. (a) The first common term is 11.

Now the next common term is obtained by adding L.C.M. of the common difference 4 and 5, i.e., 20.

Therefore, 10^{th} common term = T_{10} of the AP whose $a = 11$ and $d = 20$

$T_{10} = a + 9d = 11 + 9(20) = 191$

99. (a) Given statement makes an AP series where, a = ` 135, d = ` 15 and S_n = ` 5550

Let total savings be ` 5550 in n years

So, $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$5550 = \frac{n}{2}[2 \times 135 + (n-1)15]$

$\Rightarrow \quad 11100 = n[270 + 15n - 15]$

$\Rightarrow \quad 15n^2 + 255n - 11100 = 0$

$\Rightarrow \quad n^2 + 17n - 740 = 0$

$\Rightarrow \quad n^2 + 37n - 20n - 740 = 0$

$\Rightarrow \quad (n + 37)(n - 20) = 0$

$n = 20 \; (\because n \neq -37)$

100. (c) a, b, c are in A.P. $\Rightarrow \quad 2b = a + c$

Now,

$e^{1/c} \times e^{1/a} = e^{(a+c)/ac} = e^{2b/ac} = (e^{b/ac})^2$

$\therefore e^{1/c}, \; e^{b/ac}, e^{1/a}$ in G.P. with common ratio

$= \frac{e^{b/ac}}{e^{1/c}} = e^{(b-a)/ac} = e^{d/(b-d)(b+d)}$

$= e^{d/(b^2 - d^2)}$

[$\because$ a, b, c are in A.P. with common difference d

$\therefore$ b – a = c – b = d]

101. (a) $\frac{2}{\sqrt{c} + \sqrt{a}} = \frac{1}{\sqrt{b} + \sqrt{c}} + \frac{1}{\sqrt{a} + \sqrt{b}}$

$= \frac{2\sqrt{b} + \sqrt{a} + \sqrt{c}}{(\sqrt{b} + \sqrt{c})(\sqrt{a} + \sqrt{b})}$

$\Rightarrow 2\sqrt{ab} + 2b + 2\sqrt{ac} + 2\sqrt{bc}$

$= 2\sqrt{bc} + 2\sqrt{ac} + c + 2\sqrt{ab} + a$

$\Rightarrow 2b = a + c$

$\therefore$ a, b, c, are in A.P.

$\Rightarrow$ ax, bx, cx, are in A.P.

$\Rightarrow$ ax + 1, bx + 1, cx + 1, are in A.P.

$\Rightarrow 9^{ax+1}, 9^{bx+1}, 9^{cx+1}$ are in G.P.

102. (b) As x, y, z, are A.M. of a and b

$\therefore x + y + z = 3\left(\frac{a+b}{2}\right)$

$\therefore 15 = \frac{3}{2}(a + b) \Rightarrow a + b = 10$ ...(i)

Again $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are A.M. of $\frac{1}{a}$ and $\frac{1}{b}$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\therefore \frac{5}{3}=\frac{3}{2}.\frac{a+b}{ab}$

$\Rightarrow \frac{10}{9}=\frac{10}{ab} \Rightarrow ab=9$ ...(ii)

Solving (i) and (ii), we get
a = 9, 1, b = 1, 9

103. (b) Given $2\sqrt{ab}=\frac{a+b}{2}$

$\Rightarrow \sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}=4$

$\Rightarrow t^2-4t+5=0$, where $\sqrt{\frac{a}{b}}=t$

$\therefore \quad t=2\pm\sqrt{3} \Rightarrow \sqrt{\frac{a}{b}}=2\pm\sqrt{3}$

$\therefore \quad \frac{a}{b}=\frac{(2\pm\sqrt{3})^2}{4-3}=\frac{(2\pm\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$

$\therefore$ a : b = $2+\sqrt{3}:2-\sqrt{3}$

or $2-\sqrt{3}:2+\sqrt{3}$

104. (d) We have $S_n=\frac{a(1-r^n)}{1-r}$

$\therefore \quad S_{2n-1}=\frac{a}{1-r}[1-r^{2n-1}]$

Putting 1, 2, 3,........, n for n in it and summing up we have

$S_1+S_3+S_5+..........+S_{2n-1}$

$=\frac{a}{1-r}$ [(1 +1 + ...n term) – (r +r³ + r⁵ + ...n term)]

$=\frac{a}{1-r}\left[n-\frac{r\{1-(r^2)^n\}}{1-r^2}\right]=\frac{a}{1-r}\left[n-r.\frac{1-r^{2n}}{1-r^2}\right]$

105. (b) We have ,

$S_1=\frac{n_1}{2}[2a+(n_1-1)d] \Rightarrow \frac{2S_1}{n_1}=2a+(n_1-1)d$

$S_2=\frac{n_2}{2}[2a+(n_2-1)d] \Rightarrow \frac{2S_2}{n_2}=2a+(n_2-1)d$

$S_3=\frac{n_3}{2}[2a+(n_3-1)d] \Rightarrow \frac{2S_3}{n_3}=2a+(n_3-1)d$

$\therefore \frac{2S_1}{n_1}(n_2-n_3)+\frac{2S_2}{n_2}(n_3-n_1)+\frac{2S_3}{n_3}(n_1-n_2)$

$=[2a+(n_1-1)d](n_2-n_3)+[2a+(n_2-1)d](n_3-n_1)$
$+[2a+(n_3-1)d](n_1-n_2)=0$

106. (c) We have $t_n=\frac{1^3+2^3+3^3+.......+n^3}{1+3+5+.....\text{upto n terms}}$

$=\frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n}{2}\{2+2(n-1)\}}=\frac{\frac{n^2(n+1)^2}{4}}{n^2}$

$=\frac{(n+1)^2}{4}=\frac{n^2}{4}+\frac{n}{2}+\frac{1}{4}$

$\therefore \quad S_n=\Sigma t_n=\frac{1}{4}\Sigma n^2+\frac{1}{2}\Sigma n+\frac{1}{4}\Sigma 1$

$=\frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{1}{2}\frac{n(n+1)}{2}+\frac{1}{4}n$

$S_{16}=\frac{16.17.33}{24}+\frac{16.17}{4}+\frac{16}{4}=446$

107. (b) If n is odd, the required sum is

$1^2+2.2^2+3^2+2.4^2+......+2.(n-1)^2+n^2$

$=\frac{(n-1)(n-1+1)^2}{2}+n^2$

[$\because$ (n–1) is even
$\therefore$ using given formula for the sum of (n–1) terms.]

$=\left(\frac{n-1}{2}+1\right)n^2=\frac{n^2(n+1)}{2}$

108. (a) $S_\infty=\frac{a}{1-r}$ where 'a' be the first term and r be the common ratio of G.P.

$\therefore \quad \frac{4}{3}=\frac{3/4}{1-r}$

$\Rightarrow 1-r=\frac{3/4}{4/3} \Rightarrow 1-\frac{9}{16}=r \Rightarrow r=\frac{7}{16}$

109. (c) Let six term of H.P. = $\frac{1}{61}$

$\Rightarrow$ six term of AP = 61
Similarly tenth term of A.P. = 105
Let first term of AP is a and common diff. = d
$\therefore$ a + 5d = 61
and a + 9d = 105
solving these equation, we get
a = 6, d = 11

Hence, first term of H.P. = $\frac{1}{6}$

110. (c) Let x be the first term and y be the c.d. of corresponding A.P., then

$\frac{1}{a}=x+(p-1)y$ ...... (i)

$\frac{1}{b}=x+(q-1)y$ ...... (ii)

$\frac{1}{c}=x+(r-1)y$ ...... (iii)

Multiplying (i), (ii) and (iii) respectively by abc (q – r), abc (r – p), abc (p – q) and then adding, we get, bc (q – r) + ca (r – p)+ ab (p – q) = 0

111. (c) Let the GP be a, ar, ar^2,........, where $0 < r < 1$.
Then, $a + ar + ar^2 + ... = 3$
and $a^2 + a^2r^2 + a^2r^4 + ... = 9/2$.

$\Rightarrow \frac{a}{1-r} = 3$ and $\frac{a^2}{1-r^2} = \frac{9}{2}$

$\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$

Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get a = 2

Now, the required sum of the cubes is

$a^3 + a^3r^3 + a^3r^6 + ... = \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$

112. (c) x,y,z are in G.P. $\Rightarrow y^2 = xz$ ......(i)

We have, $a^x = b^y = c^z = \lambda$ (say)

$\Rightarrow x \log a = y \log b = z \log c = \log \lambda$

$\Rightarrow x = \frac{\log\lambda}{\log a}, y = \frac{\log\lambda}{\log b}, z = \frac{\log\lambda}{\log c}$

Putting x,y,z in (i), we get

$$\left(\frac{\log\lambda}{\log b}\right)^2 = \frac{\log\lambda}{\log a} \cdot \frac{\log\lambda}{\log c}$$

$(\log b)^2 = \log a . \log c$

or $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

113. (a) We have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...\infty \quad ....(i)$$

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...\infty \quad ....(ii)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$$

114. (d) Series $2, 2\frac{1}{2}, 3\frac{1}{3}$, ... are in H.P.

$\Rightarrow \frac{1}{2}, \frac{2}{5}, \frac{3}{10}$, ... will be in A.P.

Now, first term a = $\frac{1}{2}$

and common difference d = $-\frac{1}{10}$

So, 5^{th} term of the A.P. = $\frac{1}{2} + (5-1)\left(-\frac{1}{10}\right) = \frac{1}{10}$.

Hence, 5^{th} term in H.P. is 10.

115. (d) Considering corresponding A.P.

$a + 6d = 10$ and $a + 11d = 25$

$\Rightarrow d = 3, a = -8$

$\Rightarrow T_{20} = a + 19d = -8 + 57 = 49$

Hence, 20^{th} term of the corresponding H.P. = $\frac{1}{49}$.

116. (c) $$\text{H.M.} = \frac{2\left(\frac{a}{1-ab}\right)\left(\frac{a}{1+ab}\right)}{\frac{a}{1-ab} + \frac{a}{1+ab}}$$

$$= \frac{2\left(\frac{a^2}{1-a^2b^2}\right)}{\frac{a}{1-ab} + \frac{a}{1+ab}} = \frac{2a^2}{2a} = a.$$

117. (a) It is a fundamental concept.

118. (d) Let $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.

Then, $G^2 = ab$ ...(i)

and $AH = \left(\frac{a+b}{2}\right) \cdot \frac{2ab}{a+b} = ab$ ...(ii)

From (i) and (ii), we have $G^2 = AH$

119. (a) Given that b^2, a^2, c^2 are in A.P.

$\therefore a^2 - b^2 = c^2 - a^2$

$\Rightarrow (a - b)(a + b) = (c - a)(c + a)$

$\Rightarrow \frac{1}{b+c} - \frac{1}{a+b} = \frac{1}{c+a} - \frac{1}{b+c}$

$\Rightarrow \frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+a}$ are in A.P.

120. (c) A.M. = $\frac{a+b}{2}$ = A and G.M. = $\sqrt{ab}$ = G

On solving a and b are given by the values

$A \pm \sqrt{(A+G)(A-G)}$.

Trick: Let the numbers be 1, 9. Then, A = 5 and G = 3. Now, put these values in options.

Here, (c) $\Rightarrow 5 \pm \sqrt{8 \times 2}$, i.e. 9 and 1.

121. (c) Since the reciprocals of a and c occur on R.H.S., let us first assume that a, b, c are in H.P.

So, that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b} = d$, say

$\Rightarrow \frac{a-b}{ab} = d = \frac{b-c}{bc} \Rightarrow a - b = abd$ and $b - c = bcd$

Now, L.H.S. $= -\frac{1}{a-b}+\frac{1}{b-c} = -\frac{1}{abd}+\frac{1}{bcd}$

$= \frac{1}{bd}\left(\frac{1}{c}-\frac{1}{a}\right) = \frac{1}{bd}(2d) \Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$ = R.H.S.

$\therefore$ a, b, c are in H.P. is verified.

Aliter: $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$

$\frac{1}{b-a}-\frac{1}{c}=\frac{1}{a}-\frac{1}{b-c}$

$\Rightarrow \frac{c-b+a}{c(b-a)}=\frac{b-c-a}{a(b-c)} \Rightarrow -\frac{1}{c(b-a)}=\frac{1}{a(b-c)}$

$\Rightarrow ac - bc = ab - ac \Rightarrow b = \frac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P.

122. (b) Given that a, b, c are in A.P.

$\Rightarrow b = \frac{a+c}{2}$ ... (i)

and $b^2 = ad$ ... (ii)

Hence, a, a – b, d – c are in G.P. because

$(a-b)^2 = a^2 - 2ab + b^2 = a(a-2b) + ad$

$\Rightarrow a(-c) + ad = ad - ac$.

123. (d) Given that $\frac{\text{H.M.}}{\text{G.M.}}=\frac{12}{13}$

$\Rightarrow \frac{\frac{2ab}{a+b}}{\sqrt{ab}}=\frac{12}{13}$ or $\frac{a+b}{2\sqrt{ab}}=\frac{13}{12}$

$\Rightarrow \frac{(a+b)+2\sqrt{ab}}{(a+b)-2\sqrt{ab}}=\frac{13+12}{13-12}=\frac{25}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}=\frac{5^2}{1} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{5}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})}=\frac{5+1}{5-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}}=\frac{6}{4} \Rightarrow \left(\frac{a}{b}\right)^{\frac{1}{2}}=\frac{6}{4} \Rightarrow a : b = 9 : 4$

124. (c) We have H.M. $=\frac{2ab}{a+b}$ and G.M. $=\sqrt{ab}$

So, $\frac{\text{H.M.}}{\text{G.M.}}=\frac{4}{5} \Rightarrow \frac{\frac{2ab}{(a+b)}}{\sqrt{ab}}=\frac{4}{5}$

$\Rightarrow \frac{2\sqrt{ab}}{(a+b)}=\frac{4}{5} \Rightarrow \frac{a+b}{2\sqrt{ab}}=\frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{5+4}{5-4} \Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}=\frac{9}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{3}{1}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})}=\frac{3+1}{3-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}}=\frac{4}{2} \Rightarrow \left(\frac{a}{b}\right) = 2^2 = 4$

$\Rightarrow a : b = 4 : 1$