01

Units and Measurements

Physics is a quantitative science, based on measurement of certain physical quantities. Measurement of any physical quantity involves comparison with some standard unit of that quantity. In this chapter, we shall study about the units and measurement.

IN THIS CHAPTER ....

Physics, Technology and Society

Physics is the study of matter and its motion, as well as space and time using concepts such as energy, force, mass and charge. It is an experimental science, creating theories that are tested against observation.

The connection between physics, technology and society can be seen in many examples like working of heat engines gave rise to thermodynamics. Wireless communication technology arose from basic laws of electricity and magnetism. Lately discovery of silicon chip triggered the computer revolution.

Physical Quantities

All the quantities which are used to describe the laws of physics are called physical quantities. To measure a physical quantity, some standard unit of that quantity is required, *e. g.* if length of some metal rod is measured to be 20 m, then m is the unit of length and 20 is the numerical value. So,

$$\text{Physical quantity} = \text{Numerical value} \times \text{Unit}$$

Note (i) If the numerical value of any physical quantity in different units u_1 and u_2 are n_1 and n_2 respectively, then $n_1u_1 = n_2u_2$.

(ii) As the unit will change, numerical value will also change, *e.g.* acceleration due to gravity, $g = 32\ \text{fts}^{-2} = 9.8\ \text{ms}^{-2}$.

Fundamental and Derived Units

Those physical quantities which are independent of other physical quantities and not defined in terms of other physical quantities, are called fundamental or *base quantities*. The quantities which can be expressed in terms of the fundamental quantities are called *derived quantities*.

Units of fundamental and derived quantities are respectively, known as the *fundamental units* and *derived units*. A complete set of these units, both fundamental and derived units, is known as the *system of units*.

System of Units

The common system of units are

(i) **FPS system** The units of length, mass and time are respectively foot, pound and second.

(ii) **CGS system** The units of length, mass and time are respectively centimetre, gram and second.

(iii) **MKS system** The units of length, mass and time are respectively metre, kilogram and second.

SI System (International System of Units)

The SI is based on the following seven fundamental units and two supplementary units.

Units and Symbol of Quantities

S. No.	Quantity	Unit	Symbol
Fundamental Units			
1.	Mass	kilogram	kg
2.	Length	metre	m
3.	Time	second	s
4.	Electric current	ampere	A
5.	Temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd
Supplementary Units			
1.	Plane angle	radian	rad
2.	Solid angle	steradian	Sr

Note (i) Angle and solid angle are considered supplementary base units because although these have units but they are both dimensionless.

(ii) 2π radians = 360°

Practical Units

A large number of units are used in general life for measurement of different quantities in comfortable manner. But they are neither fundamental units nor derived units.

Practical Units of Length, Mass and Time

Practical Units of Length	Practical Units of Mass	Practical Units of Time
1 light year $= 9.46 \times 10^{15}$ m	1 quintal = 10^2 kg	1 year = $365\frac{1}{4}$ solar days
1 astronomical unit or 1 AU $= 1.5 \times 10^{11}$ m	1 metric ton $= 10^3$ kg	1 lunar month = 27.3 solar days
1 parsec = 3.26 light year	1 atomic mass unit (amu) $= 1.66 \times 10^{-27}$ kg	1 solar day = 86400 s
1 seamile = 6020 ft	1 pound = 0.4537 kg	**Tropical year** It is that year in which solar eclipse occurs.
1 micron $= 1\ \mu m = 10^{-6}$ m	1 chandrasekhar limit = 1.4 times the mass of sun $= 2.8 \times 10^{30}$ kg	**Leap year** It is that year in which the month of February has 29 days.
1 angstrom $= 10^{-10}$ m	1 slug = 14.59 kg	1 shake = 10^{-8} s
1 fermi = 10^{-15} m		

Example 1. *What is the SI unit of surface tension?*

(a) Nm^{-1} (b) Nm^{-2} (c) Nm (d) N

Sol. (a) Surface tension $= \frac{\text{Force}}{\text{Length}} = \frac{\text{N}}{\text{m}} = \text{Nm}^{-1}$

Least Count and Percentage Uncertainty

The smallest value of a physical quantity which can be measured accurately with an instrument is called the Least Count (LC) of the measuring instrument.

$$\text{LC} = \frac{\text{Value of 1 main scale division}}{\text{Total number of vernier scale divisions}}$$

The instrument with the least uncertainty is taken to measure objects, as all measurements consider accuracy.

The percentage uncertainty is calculated with the following formula

$$= \frac{\text{Maximum possible error}}{\text{Measurement of object in question}} \times 100$$

The smaller the measurement, the larger the percentage uncertainty.

Least Count of Certain Measuring Instruments

- Vernier calliper,

  $$\text{Least count (LC)} = \frac{1\text{ mm}}{10\text{ divisions}} = 0.1\text{ mm}$$

- Screw gauge,

  $$\text{Least count} = \frac{1\text{ mm}}{100\text{ divisions}} = 0.01\text{ mm}$$

- Travelling microscope,

  $$\text{Least count} = \frac{0.5\text{ mm}}{50\text{ divisions}} = 0.01\text{ mm}$$

- Spectrometer,

  $$\text{Least count} = \frac{0.5\text{ degree}}{30\text{ divisions}} = \frac{30'}{30\text{ divisions}} = 1'$$

Accuracy and Precision of Measuring Instruments

In our real world, two terms, *i.e.* accuracy and precision are often used as interchangeably, but they have specific meaning.

Accuracy An instrument is said to be accurate, if the physical quantity measured by it resembles very closely to its true value.

Precision An instrument is said to have high degree of precision, if the measured value remains unchanged, how so ever, large number of times it may have been repeated.

Note (i) Resolution stands for least count or the minimum reading which an instrument can read.

(ii) The least count of an instrument is indirectly proportional to the precision of the instrument.

Example 2. *In an experiment, the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half a degree ($= 0.5°$), then the least count of the instrument is* **[AIEEE 2009]**

(a) one minute *(b) half minute*
(c) one degree *(d) half degree*

Sol. (*a*) Least count $= \dfrac{\text{Value of main scale division}}{\text{Number of divisions on vernier scale}}$

$$= \frac{1}{30}\text{ MSD} = \frac{1}{30} \times \frac{1°}{2} = \frac{1°}{60} = 1 \text{ min}$$

Example 3. *A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?* **[JEE Main 2014]**

(a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Sol. (*b*) If student measure 3.50 cm, it means that there is an uncertainty of order 0.01 cm.

For vernier scale with 1 MSD = 1mm and 9 MSD = 10 VSD

∴ LC of VC = 1 MSD − 1 VSD

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right) = \frac{1}{100} \text{ cm}$$

Errors in Measurement

The difference between the measured value and true value (mean value) of a quantity is called error of measurement. Different types of error are given below.

(i) **Absolute error** The difference between the true value and the measured value of a quantity is called an absolute error. Usually the mean value a_m is taken as the true value. So, if

$$a_{\text{mean}} = a_m = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

then absolute errors are

$$\Delta a_1 = a_m - a_1, \Delta a_2 = a_m - a_2,$$
$$\Delta a_3 = a_m - a_3, \ldots, \Delta a_n = a_m - a_n$$

The absolute error may be positive or negative.

(ii) **Mean absolute error** It is the arithmetic mean of the magnitudes of different values of absolute errors.

∴ Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|}{n}$$

The final result of measurement can be written as $a = a_m \pm \Delta\bar{a}$. This implies that value of a is likely to lie between $a_m + \Delta\bar{a}$ and $a_m - \Delta\bar{a}$.`

(iii) **Relative or fractional error**

∴ Relative error $= \dfrac{\text{Mean absolute error}}{\text{Mean value of measurement}}$

$$= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} = \frac{\Delta a_{\text{mean}}}{a_m}$$

(iv) **Percentage error**

∴ Percentage error $= \dfrac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

Example 4. *The average speed of a train is measured by 5 students. The results of measurements are given below*

Number of Students	*Speed (m/s)*
1	10.2
2	10.4
3	9.8
4	10.6
5	10.8

The percentage error in the measurement of average speed is

(*a*) 2.6% (*b*) 3.5% (*c*) 4.5% (*d*) 5.5%

Sol. (*a*) Mean value, $v_m = \dfrac{10.2 + 10.4 + 9.8 + 10.6 + 10.8}{5}$

$$= \frac{51.8}{5} = 10.4 \text{ ms}^{-1}$$

$$\Delta v_1 = v_m - v_1 = 10.4 - 10.2 = 0.2$$
$$\Delta v_2 = v_m - v_2 = 10.4 - 10.4 = 0.0$$
$$\Delta v_3 = v_m - v_3 = 10.4 - 9.8 = 0.6$$
$$\Delta v_4 = v_m - v_4 = 10.4 - 10.6 = -0.2$$
$$\Delta v_5 = v_m - v_5 = 10.4 - 10.8 = -0.4$$

Mean absolute error,

$$\overline{\Delta v} = \frac{|\Delta v_1| + |\Delta v_2| + |\Delta v_3| + |\Delta v_4| + |\Delta v_5|}{5}$$

$$= \frac{0.2 + 0.0 + 0.6 + 0.2 + 0.4}{5} = \frac{1.4}{5} = 0.28 \text{ ms}^{-1}$$

Relative error $= \pm \dfrac{\overline{\Delta v}}{v_m} = \pm \dfrac{0.28}{10.4}$

Percentage error $= \pm \dfrac{\overline{\Delta v}}{v_m} \times 100 = \pm \dfrac{0.28}{10.4} \times 100 = \pm 2.6\%$

Least Count Error

It is an instrumental or random error associated with the resolution of the instrument.

The least count error occurs with both systematic and random error. Instruments of higher precision and improving experimental techniques can reduce the least count error.

Combination of Errors

In Addition If $Z = A + B$, then $\Delta Z = \pm(\Delta A + \Delta B)$, maximum fractional error in this case $\frac{\Delta Z}{Z} = \frac{\Delta A + \Delta B}{A + B}$, *i.e.* when two physical quantities are added, then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

In Difference If $Z = A - B$, the maximum absolute error is $\Delta Z = \pm(\Delta A + \Delta B)$ and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A + \Delta B}{A - B}$$

Example 5. *The volumes of two bodies are measured to be $V_1 = (10.2 \pm 0.02)\,cm^3$ and $V_2 = (6.4 \pm 0.01)\,cm^3$. The sum and difference in volumes with error limits is*

(a) $(16.6 \pm 0.03)\ cm^3$ and $(3.8 \pm 0.03)\ cm^3$
(b) $(16.6 \pm 0.01)\ cm^3$ and $(3.8 \pm 0.01)\ cm^3$
(c) $(16.2 \pm 0.03)\ cm^3$ and $(3.6 \pm 0.03)\ cm^3$
(d) $(16.2 \pm 0.01)\ cm^3$ and $(3.6 \pm 0.01)\ cm^3$

Sol. (a) Given, $V_1 = (10.2 \pm 0.02)\ \text{cm}^3$

and $V_2 = (6.4 \pm 0.01)\ \text{cm}^3$

$$\Delta V = \pm(\Delta V_1 + \Delta V_2)$$
$$= \pm(0.02 + 0.01)\ \text{cm}^3 = \pm 0.03\ \text{cm}^3$$
$$V_1 + V_2 = (10.2 + 6.4)\ \text{cm}^3 = 16.6\ \text{cm}^3$$

and $V_1 - V_2 = (10.2 - 6.4)\ \text{cm}^3 = 3.8\ \text{cm}^3$

Hence, sum of volume $= (16.6 \pm 0.03)\ \text{cm}^3$

and difference of volume $= (3.8 \pm 0.03)\ \text{cm}^3$

In product If $Z = AB$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

In division If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

Note Maximum fractional error in product or division of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

Example 6. *The temperature of two bodies measured by a thermometer are $t_1 = 20°\,C + 0.5°\,C$ and $t_2 = 50°\,C \pm 0.5°\,C$. The maximum absolute error in temperature difference is*

(a) $\pm 1°C$ (b) $\pm 2°C$ (c) $\pm 3°C$ (d) $\pm 4°C$

Sol. (a) The temperature difference is given by

$$t' = t_2 - t_1 = (50°\ C \pm 0.5°\ C) - (20°\ C \pm 0.5°\ C)$$
$$t' = 30°\ C \pm 1°\ C$$

Example 7. *Two resistors of resistances $R_1 = 100 \pm 3\ \Omega$ and $R_2 = 200 \pm 4\ \Omega$ are connected in parallel, then the equivalent resistance in parallel is (in ohm)*

$\left(\text{Use } \frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} \text{ and } \frac{\Delta R'}{\Delta R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right)$

(a) 66.7 ± 1.8 (b) 300 ± 7
(c) 150.8 ± 2 (d) 92.3 ± 3

Sol. (a) The equivalent resistance of parallel combination is

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7\ \Omega$$

From, $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$, we get

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$
$$\Delta R' = (R'^2)\frac{\Delta R_1}{R_1^2} + (R'^2)\frac{\Delta R_2}{R_2^2}$$
$$= \left(\frac{R'}{R_1}\right)^2 \Delta R_1 + \left(\frac{R'}{R_2}\right)^2 \Delta R_2$$
$$= \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 = 1.8\ \Omega$$

Hence, $R' = (66.7 \pm 1.8)\ \Omega$

Example 8. *The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m, rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80\ m/s^2$ and the simplified relation $T = \frac{rhg}{2} \times 10^3 N/m$, the possible error in surface tension is closest to* **[JEE Main 2017]**

(a) 1.5% (b) 2.4%
(c) 10% (d) 0.15%

Sol. (a) By ascent formula, we have surface tension,

$$T = \frac{rhg}{2} \times 10^3 \frac{\text{N}}{\text{m}}$$
$$= \frac{dhg}{4} \times 10^3 \frac{\text{N}}{\text{m}} \qquad \left(\because r = \frac{d}{2}\right)$$
$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} \qquad \text{[given, } g \text{ is constant]}$$

So, percentage error $= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h}\right) \times 100$

$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}}\right) \times 100$$
$$= 1.5\%$$
$$\therefore \quad \frac{\Delta T}{T} \times 100 = 1.5\%$$

Example 9. *In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained 55.0 cm. The percentage error in the determination of g is close to*

[JEE Main 2019]

(a) 0.7% (b) 6.8%
(c) 3.5% (d) 0.2%

Sol. (b) Relation used for finding acceleration due to gravity by using a pendulum is

$$g = \frac{4\pi^2 l}{T^2}$$

So, fractional error in value of g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \quad \ldots(\text{i})$$

Given, $\Delta l = 0.1\,\text{cm}, l = 55\,\text{cm}, \Delta T = 1\,\text{s}$ and T for 20 oscillations = 30 s

Substituting above values in Eq. (i), we get

$$\frac{\Delta g}{g} = \frac{0.1}{55} + 2 \times \frac{1}{30}$$

Hence, percentage error in g is

$$= \frac{\Delta g}{g} \times 100$$

$$= \frac{10}{55} + \frac{20}{3} = 6.8\%$$

Significant Figures

"The significant figures are those number of digits in a quantity that are known reliably plus one digit that is uncertain." Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement.

Rules for Significant Figures

(i) All non-zero digits are significant figures.

Number	Significant figures
16	2
1683	4
16835	5

(ii) All zeros occurring between non-zero digits are significant figures.

Number	Significant figures
802	3
80004	5

(iii) All zeros to the right of the last non-zero digits are not significant.

Number	Significant number
40	1
410	2
40240	4

(iv) All zeros to the right of a decimal point and to the left of a non-zero digit are not significant.

Number	Significant number
0.08	1
0.008	1
0.0846	3

(v) All zeros to the right of a decimal point and to the right of a non-zero digit are significant.

Number	Significant number
0.40	2
0.430	3

(vi) The powers of ten are not counted as significant digits *e.g.*, 1.4×10^{-7} has only two significant figures 1 and 4.

Rounding off the Digits

Certain rules are applied in order to round off the measurements

(i) If the number lying to the right of digit to be rounded is less than 5, then the rounded digit is retained as such. However, if it is more than 5, then the digit to be rounded is increased by 1.

For example, $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 8.356$ is rounded off to 8.36 to three significant digits.

(ii) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is increased by 1.

For example, $x = 14.252$ is rounded off to $x = 14.3$ to three significant digits.

(iii) If the digit to be dropped is simply 5 or 5 followed by zeros, then the preceding digit is left unchanged if it is even and it is raised by one if it is odd.

For example, $x = 6.250$ or $x = 6.25$ becomes $x = 6.2$ after rounding off to two significant digits and $x = 6.350$ or $x = 6.35$ becomes $x = 6.4$ after rounding off to two significant digits.

Algebraic Operations with Significant Figures

The following rules for algebraic operations with significant figures, that make final result more consistent with the precision of the measured values.

Addition and Subtraction

The number of decimal places in the final result of any of these operations has to be equal to the smallest number of decimal places in any of the terms involved in calculation *e.g.*, sum of terms 2.29 and 62.7 is 64.99. After rounding off to one place of decimal, it will become 65.0. Subtraction of 62.7 from 82.27 gives 19.57. After rounding off to one place of decimal, it will become 19.6.

Multiplication and Division

In these operations, the number of significant figures in the result is same as the smallest number of significant figures in any of the factors.

e.g. $1.2 \times 1.3 = 1.56$. After rounding off to two significant figures, it becomes 1.6.

Similarly, if $\frac{1100}{10.2}$ gives 107.84. Then, the result when rounded off to three significant digits becomes 108.

Example 10. *The respective number of significant figures for the numbers* 23.023, 0.0003 *and* 2.1×10^{-3} *are* **[AIEEE 2010]**

(a) 5, 1, 2 (b) 5, 1, 5
(c) 5, 5, 2 (d) 4, 4, 2

Sol. (a) The reliable digit plus the first uncertain digit is known as significant figures.

For the number 23.023, all the non-zero digits are significant, hence 5.

For the number 0.0003, number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant, hence 1.

For the number 2.1×10^{-3}, significant figures are 2.

Example 11. *The area enclosed by a circle of diameter* 1.06 *m to correct number of significant figures is*

(a) $0.88\ m^2$ (b) $0.088\ m^2$
(c) $0.882\ m^2$ (d) $0.530\ m^2$

Sol. (c) Here, $r = \frac{1.06}{2} = 0.530$ m

Area enclosed $= \pi r^2 = 3.14 (0.53)^2$

$= 0.882026\ \text{m}^2 = 0.882\ \text{m}^2$

(rounded to three significant figures)

Example 12. *A body of mass m* = 3.513 *kg is moving along the X-axis with a speed of* 5.00 ms^{-1}. *The magnitude of its momentum is recorded as* **[AIEEE 2008]**

(a) 17.6 $kg\ ms^{-1}$ (b) 17.565 $kg\ ms^{-1}$
(c) 17.56 $kg\ ms^{-1}$ (d) 17.57 $kg\ ms^{-1}$

Sol. (a) So, momentum, $p = mv = 17.565\ \text{kg ms}^{-1}$

where $m = 3.513$ kg and $v = 5.00\ \text{ms}^{-1}$

As the number of significant digits in m is 4 and in v is 3, so, p must have 3 (minimum) significant digits.

Hence, $p = 17.6\ \text{kg ms}^{-1}$

Example 13. 5.74 *g of a substance occupies* 1.2 cm^3. *Keeping the significant figure in view, its density is given by*

(a) 4.8 $g\ cm^{-3}$ (b) 1.5 $g\ cm^{-3}$
(c) 2.1 $g\ cm^{-3}$ (d) 9.2 $g\ cm^{-3}$

Sol. (a) There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence, the density should be expressed to only 2 significant figures.

$\because$ Density $= \frac{\text{Mass}}{\text{Volume}}$

$\therefore$ Density $= \frac{5.74}{1.2}\ \text{g cm}^{-3} = 4.8\ \text{g cm}^{-3}$

Dimensions of Physical Quantities

Dimensions of a physical quantity are the powers to which the fundamental quantities are raised to represent that quantity. In mechanics, all physical quantities can be expressed in terms of mass [M], length [L] and time [T].

For example, Force = Mass × Acceleration

$$= \text{Mass} \times \frac{\text{Velocity}}{\text{Time}} = \frac{m \times s}{t \times t}$$

$$= \frac{m \times s}{t^2} = [\text{M}]\ [\text{L}]\ [\text{T}^{-2}]$$

So, the dimensions of force are 1 in mass, 1 in length and −2 in time.

Dimensional Formula and Dimensional Equations

The expression which shows how and which of the fundamental quantities represent the dimension of physical quantity is called the *dimensional formula* of the given physical quantity.

For example, as deduced above, $[\text{M}^1\text{L}^1\text{T}^{-2}]$ is the dimensional formula of force. It reveals that unit of force depends on [M], [L] and [T].

Further, if we represent force by [F], then $[\text{F}] = [\text{M}^1\text{L}^1\text{T}^{-2}]$ is called the *dimensional equation* of force.

Dimensionless Quantity

In the equation $[\text{M}^a\text{L}^b\ \text{T}^c]$, if $a = b = c = 0$, then the quantity is called dimensionless.

For example, strain, specific gravity and angle are dimensionless quantity because they are ratio of two similar quantities.

Some dimensionless quantities

Angle, solid angle, relative density, specific gravity, Poisson's ratio, Reynold's number, all trigonometric ratios, refractive index, relative permittivity, dielectric constant, magnetic susceptibility. A dimensionless quantity has same numeric value in all system of units.

Principle of Homogeneity

According to this principle, a correct dimensional equation must be homogeneous, *i.e.* dimensions of all the terms in a physical expression must be same.

LHS (dimension) = RHS (dimension)

Dimensions of Important Physical Quantities

Physical Quantity	SI Unit	Dimensional Formula
Power	Watt (W)	$[ML^2T^{-3}]$
Pressure, stress, coefficient of elasticity (ρ, σ, η)	Pascal (Pa) or Nm^{-2}	$[ML^{-1}T^{-2}]$
Frequency, angular frequency	Hz or s^{-1}	$[T^{-1}]$
Angular momentum	$kg\,m^2s^{-1}$	$[ML^2T^{-1}]$
Torque	Nm	$[ML^2T^{-2}]$
Gravitational constant (G)	$N\,m^2kg^{-2}$	$[M^{-1}L^3T^{-2}]$
Moment of inertia	kg m^2	$[ML^2]$
Acceleration, acceleration due to gravity	ms^{-2}	$[LT^{-2}]$
Force, thrust, tension, weight	Newton (N)	$[MLT^{-2}]$
Linear momentum, impulse	$kg\,ms^{-1}$ or Ns	$[MLT^{-1}]$
Work, energy, KE, PE, thermal energy, internal energy, etc.	Joule (J)	$[ML^2T^{-2}]$
Surface area, area of cross-section	m^2	$[L^2]$
Electric conductivity	Sm^{-1}	$[M^{-1}L^{-3}T^3A^2]$
Young's modulus, Bulk modulus	Pa	$[ML^{-1}T^{-2}]$
Compressibility	$m^2\ N^{-1}$	$[M^{-1}LT^2]$
Magnetic flux	Wb	$[ML^2T^{-2}A^{-1}]$
Magnetic flux density (σ)	Wb / m^2	$[MT^{-2}A^{-1}]$
Intensity of a wave	Wm^{-2}	$[MT^{-3}]$
Photon flux density	$m^{-2}\ s^{-1}$	$[L^{-2}T^{-1}]$
Luminous energy	Lm s	$[ML^2T^{-2}]$
Luminance	Lux	$[MT^{-3}]$
Specific heat capacity	$Jkg^{-1}K^{-1}$	$[L^2T^{-2}K^{-1}]$
Latent heat of vaporisation	Jkg^{-1}	$[L^2T^{-2}]$
Coefficient of thermal conductivity	$Wm^{-1}K^{-1}$	$[MLT^{-3}K^{-1}]$
Electric voltage	JC^{-1}	$[ML^2T^{-3}A^{-1}]$
Magnetisation	Am^{-1}	$[L^{-1}A]$
Magnetic induction	T	$[MT^{-2}A^{-1}]$
Planck's constant	J-s	$[ML^2T^{-1}]$
Radioactive decay constant	Bq	$[T^{-1}]$
Binding energy	MeV	$[ML^2T^{-2}]$

Dimensional Analysis and Its Applications

There are three applications of dimensional analysis

1. To check the correctness of a given physical equation

As per principle of homogeneity, if the dimensions of each term on both sides of a physical relation are same, then the relation is dimensionally correct otherwise wrong.

Example 14. *Is the given expression of velocity of sound given by* $v = \sqrt{\left(\frac{E}{\rho}\right)}$ *is dimensionally correct?*

Here, E = *coefficient of elasticity,*
ρ = *density of medium*

(a) *Yes* (b) *No*
(c) *Cannot be predicted* (d) *The correct expression is* $\frac{E}{\rho}$

Sol. (a) $[LHS] = [v] = [LT^{-1}]$

$$[RHS] = \left[\left(\frac{E}{\rho}\right)^{1/2}\right] = \left[\left(\frac{ML^{-1}T^{-2}}{ML^{-3}}\right)^{1/2}\right] = [LT^{-1}]$$

$[LHS] = [RHS]$

Hence, equation is dimensionally correct.

Example 15. *Given equation* $\frac{1}{2}mv^2 = mgh$, *where m is the mass of the body, v is velocity, g is the acceleration due to gravity and h is the height. Then the given equation is*

(a) *dimensionally incorrect*
(b) *dimensionally correct*
(c) *wrong*
(d) *None of the above*

Sol. (b) Given, $\frac{1}{2}mv^2 = mgh$

The dimensions of LHS are

$$[M][LT^{-1}]^2 = [M][L^2T^{-2}] = [ML^2T^{-2}]$$

The dimensions of RHS are

$$[M][LT^{-2}][L] = [M][L^2T^{-2}] = [ML^2T^{-2}]$$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

Example 16. *The SI unit of energy is* $J = kg\,m^2s^{-2}$, *that of speed v is* ms^{-1} *and of acceleration a is* ms^{-2}, *which one of the formula for kinetic energy given below is correct on the basis of dimensional arguments?*
(Given, m stands for the mass of body)

(a) $K = m^2v^2$ (b) $K = ma$
(c) $K = \frac{1}{2}mv^2 + ma$ (d) $K = \frac{1}{2}mv^2$

Sol. (a) Every correct formula or equation must have the same dimensions on both sides of the equation. Also, only quantities with the same physical dimensions can be added or subtracted. The dimensions of the quantity on the right side for (a) is

$$K = m^2v^2$$

Putting, $m = [M]$, $v = [LT^{-1}]$

$\therefore$ dimensions are $[M^2L^2T^{-2}]$

for (b), $K = ma$

putting $m = [M]$, $a = [LT^{-2}]$

$\therefore$ dimensions are $[MLT^{-2}]$

option (c) has no proper dimensions,

option (d), $K = \frac{1}{2}mv^2$, putting $m = [M]$

and $V = [LT^{-1}]$, we have

$$K = [M]\,[LT^{-1}]^2 = [ML^2T^{-2}]$$

in units it is written as $kg\ m^2\ s^{-2}$.

2. Derivation of formula

The method of dimensions is used to deduce the relation among the physical quantities. We should know the dependence of the physical quantity on the other quantities.

We explain the process in following examples.

Example 17. *The time period T of simple pendulum depends upon length l of the pendulum and gravitational acceleration. The formula for time period of simple pendulum is given by*

(a) $T = 2\pi\sqrt{\frac{l}{g}}$ (b) $T = 2\pi\sqrt{\frac{g}{l}}$

(c) $T = \frac{1}{2\pi}\sqrt{lg}$ (d) $T = \frac{2\pi l}{g}$

Sol. (a) Let $T \propto l^a$ and $T \propto g^b$

where a and b are dimensionless constants

$$T = kl^a g^b$$

where, k is dimensionless constant.

$$[LHS] = [T] = [M^0L^0T^1]$$

and $[RHS] = (l^a g^b) = [L]^a\,[LT^{-2}]^b$

$$= [L^{a+b}T^{-2b}] = [M^0L^{a+b}T^{-2b}]$$

According to homogeneity principle,

or $[M^0L^0T] = [M^0L^{a+b}T^{-2b}]$

For dimensional balance, dimensions on both sides should be same.

$\therefore$ $a + b = 0$ and $-2b = 1$

$\therefore$ $b = -\frac{1}{2}$ and $a = \frac{1}{2}$

$\therefore$ $T = 2\pi\sqrt{\frac{l}{g}}$

[since, numerical value of k in case of simple pendulum is 2π]

Example 18. *Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?* **[JEE Main 2019]**

(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$

(c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

Sol. (a) Let dimensions of resistance R, permittivity ε_0 and permeability μ_0 are [R], $[\varepsilon_0]$ and $[\mu_0]$, respectively.

So, $[R] = [\varepsilon_0]^\alpha\,[\mu_0]^\beta$ ...(i)

$$[R] = [M^1L^2T^{-3}A^{-2}],$$

$$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2],$$

$$[\mu_0] = [M^1L^1T^{-2}A^{-2}]$$

Now, from Eq. (i), we get

$$[M^1L^2T^{-3}A^{-2}] = [M^{-1}L^{-3}T^4A^2]^\alpha\,[M^1L^1T^{-2}A^{-2}]^\beta$$

$$[M^1L^2T^{-3}A^{-2}] = [M^{-\alpha+\beta}L^{-3\alpha+\beta}T^{4\alpha-2\beta}A^{2\alpha-2\beta}]$$

On comparing both sides, we get

$-\alpha + \beta = 1$ ...(ii)

$-3\alpha + \beta = 2$ ...(iii)

$4\alpha - 2\beta = -3$ ...(iv)

$2\alpha - 2\beta = -2$ ...(v)

Value of α and β can be found using any two Eqs. from (ii) to (v),

on subtracting Eq. (iii) from Eq. (ii), we get

$$(-\alpha + \beta) - (-3\alpha + \beta) = 1 - 2$$

$\Rightarrow$ $2\alpha = -1$

or $\alpha = \frac{-1}{2}$

Put the value of α in Eq. (ii), we get

$$\beta = +\frac{1}{2}$$

$\therefore$ $[R] = [\varepsilon_0]^{-1/2}\,[\mu_0]^{1/2} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

3. To convert a physical quantity from one system to the other

Let dimensional formula of a given physical quantity be $[M^aL^bT^c]$. If in a system having base units $[M_1L_1T_1]$ the numerical value of given quantity (Q) be n_1 and numerical value n_2 in another unit system having the base units $[M_2, L_2, T_2]$, then

$$Q = n_1u_1 = n_2u_2$$

$$n_1\,[M_1^aL_1^bT_1^c] = n_2\,[M_2^aL_2^bT_2^c]$$

$$\Rightarrow \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Example 19. *The density of a material in SI units is* 128 *kg* m^{-3}. *In certain units in which the unit of length is* 25 *cm and the unit of mass is* 50 *g, the numerical value of density of the material is* **[JEE Main 2019]**

(a) 40 (b) 16
(c) 640 (d) 410

Sol. (a) To convert a measured value from one system to another system, we use

$$n_1 u_1 = n_2 u_2$$

where, n is numeric value and u is unit.

We get $$128 \frac{\text{kg}}{\text{m}^3} = n_2 \frac{50\text{ g}}{(25\text{ cm})^3}$$

$$\Rightarrow \frac{128 \times 1000\text{ g}}{100 \times 100 \times 100\text{ cm}^3} = \frac{n_2 \times 50\text{ g}}{25 \times 25 \times 25\text{ cm}^3}$$

$$\left[\because \text{Density} = \frac{\text{Mass}}{\text{Volume}}\right]$$

$$\Rightarrow n_2 = \frac{128 \times 1000 \times 25 \times 25 \times 25}{50 \times 100 \times 100 \times 100} = 40$$

Limitations of Theory of Dimensions

Although dimensional analysis is very useful but it is not universal, it has some limitations as given below

(i) This method gives no information about dimensional constants. Such as universal constant of gravitation (G) or Planck's constant (h).

(ii) Numerical constant (k), having no dimensions such as 3/4, e, 2π, etc., cannot be deduced by the method of dimensions.

(iii) This technique is useful only for deducing and verifying power relations. Relationship involving exponential, trigonometric functions, etc., cannot be obtained or studied by this method.

(iv) We cannot use this method to obtain the required relation, if the quantity of interest depends upon more parameters than the number of fundamental quantities used.

(v) Even if a physical quantity depends on three physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions.

Practice Exercise

ROUND I Topically Divided Problems

Physical Quantities and Units

1. The SI unit of electrochemical equivalent is
(a) kg C (b) C kg^{-1} (c) kg C^{-1} (d) kg^2 C^{-1}

2. SI unit of intensity of wave is
(a) J $m^{-2}s^{-1}$ (b) J $m^{-1}s^{-2}$ (c) W m^{-2} (d) J m^{-2}

3. Which one of the following pairs of quantities and their unit is properly matched?
(a) Electric field - coulomb/m
(b) Magnetic flux - Weber/m^2
(c) Power - Farad
(d) Capacitance - Henry

4. Farad is not equivalent to
(q = coulomb, V = volt and J = joule)
(a) $\frac{q}{V}$ (b) qV^2 (c) $\frac{q^2}{J}$ (d) $\frac{J}{V^2}$

5. On which of the following factor universal time depends?
(a) Rotation of earth on its axis
(b) Oscillations of quartz crystal
(c) Vibrations of cesium atom
(d) Earth's orbital motion around the sun

6. A sextant is a double reflecting navigation instrument. Which of the following physical quantity, it can measure?
(a) Area of hill
(b) Angular distance between two visible objects
(c) Breadth of a tower
(d) Volume of the building

7. The 'rad' is the correct unit used to report the measurement of **[AIEEE 2006]**
(a) the ability of a beam of gamma ray photons to produce ions in a target
(b) the energy delivered by radiation to a target
(c) the biological effect of radiation
(d) the rate of decay of a radioactive source

8. A pressure of 10^6 dyne cm^{-2} is equivalent to
(a) 10^5 Nm^{-2} (b) 10^4 Nm^{-2} (c) 10^6 Nm^{-2} (d) 10^7 Nm^{-2}

9. How many wavelengths of Kr^{86} are there in one metre?
(a) 1553164.13 (b) 1650763.73
(c) 652189.63 (d) 2348123.73

10. Young modulus of steel is 1.9×10^{11} N/m^2. When expressed in CGS units of dyne/cm^2, it will be equal to (1 N = 10^5 dyne, 1 m^2 = $10^4 cm^2$) **[NCERT]**
(a) 1.9×10^{10} (b) 1.9×10^{11}
(c) 1.9×10^{12} (d) 1.9×10^{13}

11. 1 light year is defined as the distance travelled by light in one year. The speed of light 3×10^8 ms^{-1}. The same in metre is
(a) 3×10^{12}m (b) 9.461×10^{15} m
(c) 3×10^{15} m (d) None of these

12. 1 slug is equivalent to 14.6 kg. A force of 10 pound is applied on a body of 1 kg. The acceleration of the body is
(a) 43.8 ms^{-2} (b) 4.448 ms^{-2}
(c) 44.4 ms^{-2} (d) None of these

13. The time taken by an electron to go from ground state to excited state is one shake (1 shake = 10^{-8}s). This time in nanosecond will be
(a) 10 (b) 4 (c) 2 (d) 25

14. The concorde is the fastest airlines used for commercial service. It can cruise at 1450 mile per hour (about two times the speed of sound or in other words mach 2). What is it in m/s?
(a) 644.4 m/s (b) 80 m/s
(c) 40 m/s (d) None of these

15. The value of universal gas constant is $R = 8.3$ J/k-mol. The value of R in atmosphere litre per kelvin per mol
(a) 8.12 (b) 0.00812
(c) 81.2 (d) 0.0812

Least Count and Accuracy of Measuring Instruments

16. One main scale division of a vernier callipers is a cm and nth division of the vernier scale coincide with $(n-1)$th division of the main scale. The least count of the callipers (in mm) is **[JEE Main 2021]**
(a) $\frac{10na}{(n-1)}$ (b) $\frac{10a}{(n-1)}$
(c) $\left(\frac{n-1}{10n}\right)a$ (d) $\frac{10a}{n}$

17. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. The thickness of hair is **[NCERT]**

(a) 0.035 mm (b) 0.04 mm
(c) 0.35 mm (d) 0.40 mm

18. A vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier callipers, the least count is **[IIT JEE]**

(a) 0.02 mm (b) 0.05 mm
(c) 0.1 mm (d) 0.2 mm

19. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree.
Vernier scale reading : 9 division.

Given that, 1 division on main scale corresponds to 0.5 degree. Total division on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is **[AIEEE 2012]**

(a) 58.59° (b) 59.77°
(c) 58.65° (d) 59°

20. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm, Circular scale reading : 52 divisions

Given that, 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is

(a) 0.052 cm (b) 0.026 cm
(c) 0.005 cm (d) 0.52 cm

21. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(a) 3.32 mm
(b) 3.73 mm
(c) 3.67 mm
(d) 3.38 mm

22. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is **[JEE Main 2019]**

(a) 50 (b) 200
(c) 500 (d) 100

23. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91s, 92s and 95s. If the minimum division in the measuring clock is 1s, then the reported mean time should be **[JEE Main 2016]**

(a) (92 ± 2) s (b) (92 ± 5) s
(c) (92 ± 1.8) s (d) (92 ± 3) s

24. Which of the following is the most precise device for measuring length? **[NCERT]**

(a) A vernier callipers with 20 divisions on the sliding scale.
(b) A screw gauge of pitch 1 mm and 100 divisions on the circular scale.
(c) An optical instrument that can measure length to within a wavelength of light.
(d) All are equally precise device for measuring length.

Errors and Significant Figures

25. The density of a cube is measured by measuring its mass and length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the measurement of density is

(a) 7% (b) 5%
(c) 1% (d) 9%

26. A physical quantity is represented by $X = M^a L^b T^{-c}$. If percentage errors in the measurements of M, L and T are α%, β% and γ% respectively, then total percentage error is

(a) $(\alpha a + \beta b - \gamma c)$ % (b) $(\alpha a + \beta b + \gamma c)$ %
(c) $(\alpha a - \beta b - \gamma c)$ % (d) 0%

27. If there is a positive error of 50% in the measurement of speed of a body, then the error in the measurement of kinetic energy is

(a) 25% (b) 50%
(c) 100 % (d) 125%

28. The radius of the sphere is (4.3 ± 0.1) cm. The percentage error in its volume is

(a) $\frac{0.1}{4.3} \times 100$ (b) $3 \times \frac{0.1 \times 100}{4.3}$
(c) $\frac{1}{3} \times \frac{0.1 \times 100}{4.3}$ (d) $3 + \frac{0.1 \times 100}{4.3}$

29. The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1 mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 s using a clock of 1 s resolution. The percentage accuracy in the determination of g using this pendulum is x. The value of x to the nearest integer is **[JEE Main 2021]**

(a) 2% (b) 3% (c) 5% (d) 4%

30. The velocity of transverse wave in a string is $v = \sqrt{\frac{T}{M}}$ where T is the tension in the string and M is mass per unit length. If $T = 3.0$ kgf, mass of string is 2.5 g and length of string is 1.000 m, then the percentage error in the measurement of velocity is
(a) 0.5 (b) 0.7 (c) 2.3 (d) 3.6

31. The initial temperature of a liquid is $(80.0 \pm 0.1)^\circ$ C. After it has been cooled, its temperature is $(10.0 \pm 0.1)^\circ$ C. The fall in temperature in degree centigrade is
(a) 70.0 (b) 70.0 ± 0.3
(c) 70.0 ± 0.2 (d) 70.0 ± 0.1

32. A public park, in the form of a square, has an area of $(100 \pm 0.2)\,\text{m}^2$. The side of park is
(a) (10 ± 0.01)m (b) (10 ± 0.1)m
(c) (10.0 ± 0.1)m (d) (10.0 ± 0.2)m

33. Given, potential difference $V = (8 \pm 0.5)$ V and current $I = (2 \pm 0.2)$A. The value of resistance R (in Ω) is
(a) $4 \pm 16.25\%$ (b) $4 \pm 6.25\%$
(c) $4 \pm 10\%$ (d) $4 \pm 8\%$

34. The length, breadth and thickness of a block are measured to be 50 cm, 2.0 cm and 1.00 cm. The percentage error in the measurement of volume is
(a) 0.8% (b) 8%
(c) 10% (d) 12.5 %

35. One side of a cubical block is measured with the help of a vernier callipers of vernier constant 0.01 cm. This side comes out to be 1.23 cm. What is the percentage error in the measurement of area?
(a) $\frac{1.23}{0.01} \times 100$ (b) $\frac{0.01}{1.23} \times 100$
(c) $2 \times \frac{0.01}{1.23} \times 100$ (d) $3 \times \frac{0.01}{1.23} \times 100$

36. A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5%, respectively. The percentage of error in z is **[JEE Main 2020]**
(a) 13.5% (b) 16.5%
(c) 14.5% (d) 12.25%

37. The relative density of the material of a body is the ratio of its weight in air and the loss of its weight in water. By using a spring balance, the weight of the body in air is measured to be (5.00 ± 0.05) N. The weight of the body in water is measured to be (4.00 ± 0.05) N, then the maximum possible percentage error in relative density is
(a) 11% (b) 10% (c) 9% (d) 7%

38. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures? **[JEE Main 2019]**
(a) $4300 \pm 80\ \text{cm}^3$
(b) $4260 \pm 80\ \text{cm}^3$
(c) $4264.4 \pm 81.0\ \text{cm}^3$
(d) $4264 \pm 81\ \text{cm}^3$

39. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring $\pm$ 0.01V while measuring the current of 5 mA at 300K, what will be the error in the value of current (in mA)? **[JEE Main 2013]**
(a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

40. The result after adding 3.8×10^{-6} to 4.2×10^{-5} with due regard to significant figures is
(a) 4.58×10^{-5} (b) 0.458×10^{-4}
(c) 4.6×10^{-5} (d) 45.8×10^{-6}

41. The value of π^2 with due regard for significant figures is (Given $\pi = 3.14$)
(a) 9.86 (b) 9.859
(c) 9.8596 (d) 9.85960

42. You measure two quantities as $A = 1.0\ \text{m} \pm 0.2\ \text{m}, B = 2.0\ \text{m} \pm 0.2\ \text{m}$. We should report correct value for $\sqrt{AB}$ as **[NCERT Exemplar]**
(a) $1.4\ \text{m} \pm 0.4\ \text{m}$ (b) $1.41\ \text{m} \pm 0.15\ \text{m}$
(c) $1.4\ \text{m} \pm 0.3\ \text{m}$ (d) $1.4\ \text{m} \pm 0.2\ \text{m}$

43. The area of a square is $5.29\ \text{cm}^2$. The area of 7 such squares taking into account the significant figures is **[JEE Main 2019]**
(a) $37.030\ \text{cm}^2$ (b) $37.0\ \text{cm}^2$
(c) $37.03\ \text{cm}^2$ (d) $37\ \text{cm}^2$

44. For the four sets of three measured physical quantities as given below. Which of the following options is correct? **[JEE Main 2020]**
(i) $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$
(ii) $A_2 = 24.44, B_2 = 16.082, C_2 = 240.2$
(iii) $A_3 = 25.2, B_3 = 19.2812, C_3 = 236.183$
(iv) $A_4 = 25, B_4 = 236.191, C_4 = 19.5$
(a) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3 < A_2 + B_2 + C_2 < A_4 + B_4 + C_4$
(b) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3$
(c) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1 = A_3 + B_3 + C_3 < A_2 + B_2 + C_2$
(d) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2 = A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

Dimensions

45. The damping force of an oscillating particle is observed to be proportional to velocity. The constant of proportionality can be measured in
(a) kg s^{-1} (b) kg s
(c) kg ms^{-1} (d) kg m^{-1} s^{-1}

46. The fundamental unit, which has the same power in the dimensional formulae of surface tension and viscosity is
(a) mass (b) length
(c) time (d) None of these

47. If the units of M and L are increased three times, then the unit of energy will be increased by
(a) 3 times (b) 6 times
(c) 27 times (d) 81 times

48. If L denotes the inductance of an inductor through which a current I is flowing, then the dimensional formula of LI^2 is
(a) $[MLT^{-2}]$
(b) $[ML^2T^{-2}]$
(c) $[M^2L^2T^{-2}]$
(d) not expressible in terms of M, L, T

49. The equation of alternating current is $I = I_0 e^{-t/CR}$ where t is time, C is capacitance and R is resistance of coil, then the dimensions of CR is
(a) $[MLT^{-1}]$ (b) $[M^0LT]$
(c) $[M^0L^0T]$ (d) None of these

50. Which of the following pairs has same dimensions?
(a) Current density and charge density
(b) Angular momentum and momentum
(c) Spring constant and surface energy
(d) Force and torque

51. Dimensions of $1/\mu_0\varepsilon_0$, where symbols have their usual meaning, are **[AIEEE 2003]**
(a) $[L^{-1}T]$ (b) $[L^2T^2]$ (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

52. The physical quantities not having same dimensions are **[AIEEE 2003]**
(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

53. The dimensions of magnetic field in M, L, T and C is given as **[AIEEE 2008]**
(a) $[MLT^{-1}C^{-1}]$
(b) $[MT^2C^{-2}]$
(c) $[MT^{-1}C^{-1}]$
(d) $[MT^2C^{-1}]$

54. The time dependence of a physical quantity P is given by $P = P_0 e^{-\alpha t^2}$, where α is a constant and t is time. Then constant α is
(a) dimensionless (b) dimension of t^{-2}
(c) dimensions of P (d) dimension of t^2

55. The dimensional formula of magnetic permeability is
(a) $[M^0L^{-1}T]$ (b) $[M^0L^2T^{-1}]$
(c) $[M^0L^2T^{-1}A^2]$ (d) $[MLT^{-2}A^{-2}]$

56. $[ML^{-2}T^{-2}]$ represents dimensional formula of which of the following physical quantities?
(a) Energy (b) Pressure
(c) Torque (d) Pressure gradient

57. Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Q denotes the electric charge? **[AIEEE 2006]**
(a) Wb/m^2 (b) henry (H)
(c) H/m^2 (d) weber (Wb)

58. Amount of solar energy received on the earth's surface per unit area per unit time is defined as solar constant. Dimensional formula of solar constant is **[JEE Main 2020]**
(a) $[MLT^{-2}]$ (b) $[ML^0T^{-3}]$
(c) $[M^2L^0T^{-1}]$ (d) $[ML^2T^{-2}]$

59. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is **[JEE Main 2020]**
(a) $[ML^{-1}T^{-2}]$ (b) $[MLT^{-2}]$
(c) $[ML^2T^{-1}]$ (d) $[ML^2T^{-2}]$

60. In SI units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is **[JEE Main 2019]**
(a) $[A^{-1}TML^3]$ (b) $[AT^2M^{-1}L^{-1}]$
(c) $[AT^{-3}ML^{3/2}]$ (d) $[A^2T^3M^{-1}L^{-2}]$

61. In the relation $y = r\sin(\omega t - kx)$, the dimensional formula of ω/k are
(a) $[M^0L^0T^0]$ (b) $[M^0L^1T^{-1}]$
(c) $[M^0L^0T^1]$ (d) $[M^0L^1T^0]$

62. Let $[\varepsilon_0]$ denotes the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then **[JEE Main 2013]**
(a) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$ (b) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$
(c) $[\varepsilon_0] = [M^{-2}L^2T^{-1}A^{-2}]$ (d) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^2]$

63. Let l, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{l}{rcv}$ in SI units will be **[JEE Main 2019]**
(a) $[LT^2]$ (b) $[LTA]$ (c) $[A^{-1}]$ (d) $[LA^{-2}]$

64. $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$. The value of n is
(a) 0 (b) –1
(c) 1 (d) None of these
(You may use dimensional analysis to solve the problem.)

65. The period of a body under SHM is represented by $T = p^a D^b S^c$, where p is pressure, D is density and S is surface tension. The value of a, b and c are

(a) $-\frac{3}{2}, \frac{1}{2}, 1$ (b) $-1, -2, 3$
(c) $\frac{1}{2}, \frac{-3}{2}, \frac{-1}{2}$ (d) $1, 2, \frac{1}{3}$

66. Taking frequency f, velocity v and density ρ to be the fundamental quantities, then the dimensional formula for momentum will be

(a) $[\rho v^4 f^{-3}]$ (b) $[\rho v^3 f^{-1}]$
(c) $[\rho v f^2]$ (d) $[\rho^2 v^2 f^2]$

67. If p represents radiation pressure, v represents speed of light and q represents radiation energy striking a unit area per second, then non-zero integers a, b and c are such that $p^a q^b v^c$ is dimensionless, then

(a) $a = 1, b = 1, c = -1$
(b) $a = 1, b = -1, c = 1$
(c) $a = -1, b = 1, c = 1$
(d) $a = 1, b = 1, c = 1$

68. The wavelength associated with a moving particle depends upon power p of its mass m, qth power of its velocity v and rth power of Planck's constant h. Then the correct set of values of p, q and r is

(a) $p = 1, q = -1, r = 1$
(b) $p = 1, q = 1, r = 1$
(c) $p = -1, q = -1, r = -1$
(d) $p = -1, q = -1, r = 1$

69. If speed V, area A and force F are chosen as fundamental units, then the dimensional formula of Young's modulus will be **[JEE Main 2020]**

(a) $[FA^2V^{-3}]$ (b) $[FA^{-1}V^0]$
(c) $[FA^2 V^{-2}]$ (d) $[FA^2V^{-1}]$

70. If momentum p, area A and time T are taken to be the fundamental quantities, then the dimensional formula for energy is **[JEE Main 2020]**

(a) $[p^2AT^{-2}]$ (b) $[pA^{-1}T^{-2}]$
(c) $[pA^{1/2}T^{-1}]$ (d) $[p^{1/2}AT^{-1}]$

71. If speed (v), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be **[JEE Main 2019]**

(a) $[v^{-4}A^{-2}F]$ (b) $[v^{-2}A^2 F^2]$
(c) $[v^{-2}A^2 F^{-2}]$ (d) $[v^{-4}A^2 F]$

72. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant h, speed of light c, gravitational constant G and ampere A is **[JEE Main 2020]**

(a) $h^{-2/3}c^{-1/3}G^{4/3}\ A^{-1}$ (b) $h^{1/3}G^{2/3}c^{1/3}\ A^{-1}$
(c) $h^0c^5G^{-1}\ A^{-1}$ (d) $h^{1/3}G^{2/3}c^{1/3}A^{-1}$

73. If surface tension (S), moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be **[JEE Main 2019]**

(a) $S^{1/2}I^{1/2}h^{-1}$ (b) $S^{3/2}I^{1/2}h^0$
(c) $S^{1/2}I^{1/2}h^0$ (d) $S^{1/2}I^{3/2}h^{-1}$

ROUND II Mixed Bag

Only One Correct Option

1. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that 0 on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is ……… cm. (Given, least count = 0.01 cm) **[JEE Main 2021]**

(a) 8.36 cm
(b) 8.54 cm
(c) 8.58 cm
(d) 8.56 cm

2. If 1 g cm s^{-1} = x N-s, then the number x is equal to

(a) 1×10^{-3} (b) 3.6×10^{-3}
(c) 1×10^{-5} (d) 6×10^{-4}

3. If muscle times speed equals power, what is the ratio of the SI units and the CGS unit of muscle?

(a) 10^5 (b) 10^3
(c) 10^7 (d) 10^{-5}

4. The pitch of a screw gauge 15 mm and there are 100 divisions on the circular scale. While measuring diameter of a thick wire. The pitch scale reads 1 mm and 63 rad division on the circular scale coincides with the reference. The length of the wire is 5.6 cm, then which one of the following option is correct?

(a) The least count of screw gauge is 0.002 cm
(b) The volume of the wire is 0.117 cm^3
(c) The diameter of the wire is 1.33 m
(d) The cross-section area of the wire is 0.0209 cm^3

5. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is **[AIEEE 2008]**

(a) 3.32 mm (b) 3.37 mm
(c) 3.67 mm (d) 3.38 mm

6. The dimensions of a rectangular block measured with callipers having least count of 0.01 cm are 5 mm × 10 mm × 5 mm. The maximum percentage error in the measurement of the volume of the block is
(a) 5% (b) 10 % (c) 15 % (d) 20%

7. A resistor of 10 kΩ having tolerance 10% is connected in series with another resistor of 20 kΩ having tolerance 20%. The tolerance of the combination will be approximately
(a) 10% (b) 13% (c) 17% (d) 20%

8. The following observations were taken for determining surface tension of water by capillary tube method. Diameter of capillary, $D = 1.25\times10^{-2}$ m and rise of water in capillary, $h = 1.46 \times 10^{-2}$ m. Taking $g = 9.80\,\text{ms}^{-2}$ and using the relation $T = (rgh/2)\times10^3\ \text{Nm}^{-1}$, what is the possible error in surface tension T?
(a) 2.4% (b) 15 % (c) 1.6% (d) 0.15%

9. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is **[AIEEE 2012]**
(a) 6% (b) zero (c) 1% (d) 3%

10. A wire has a mass (0.3 ± 0.003) g, radius (0.5 + 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of its density is **[IIT JEE 2004]**
(a) 1 (b) 2 (c) 3 (d) 4

11. In an experiment to measure the height of a bridge by dropping stone into water underneath, if the error in measurement of time is 0.1s at the end of 2 s, then the error in estimation of height of bridge will be **[Kerala CEE 2004]**
(a) 0.49 m (b) 0.98 m (c) 1.96 m (d) 2.12 m

12. A cube has a side of length 1.2×10^{-2} m. Calculate its volume. **[IIT JEE 2003]**
(a) $1.7 \times 10^{-6}\ \text{m}^3$ (b) $1.73 \times 10^{-6}\ \text{m}^3$
(c) $1.70 \times 10^{-6}\ \text{m}^3$ (d) $1.732 \times 10^{-6}\ \text{m}^3$

13. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is **[JEE Main 2021]**
(a) 3.9 (b) 8.4 (c) 7.5 (d) 3.0

14. A resistor of 4 kΩ with tolerance 10% is connected in parallel with a resistor of 6 kΩ with tolerance 100%. The tolerance of the parallel combination is nearly
(a) 10 % (b) 20 % (c) 30 % (d) 40 %

15. What is the unit of k in the relation where, $U = \dfrac{ky}{y^2 + a^2}$ where U represents the potential energy, y represents the displacement and a represents amplitude?
(a) m s^{-1} (b) m s (c) J m (d) J s^{-1}

16. In the equation $X = 3\,YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. In MKSQ system, the dimensional formula of Y is
(a) $[\text{M}^{-3}\text{L}^{-2}\text{T}^{-2}\text{Q}^{-4}]$ (b) $[\text{ML}^{-2}]$
(c) $[\text{M}^{-3}\text{L}^{-2}\text{Q}^{4}\text{T}^{8}]$ (d) $[\text{M}^{-3}\text{L}^{-2}\text{Q}^{4}\text{T}^{4}]$

17. The dimensional formula of $\dfrac{1}{\varepsilon_0}\dfrac{e^2}{hc}$ is
(a) $[\text{M}^0\text{L}^0\text{T}^0\text{A}^0]$ (b) $[\text{M}^{-1}\text{L}^3\text{T}^2\text{A}]$
(c) $[\text{ML}^3\text{T}^{-4}\text{A}^{-2}]$ (d) $[\text{M}^{-1}\text{L}^{-3}\text{T}^4]$

18. Given $X = (Gh/c^3)^{1/2}$, where G, h and c are gravitational constant, Planck's constant and the velocity of light respectively. Dimensions of X are the same as those of
(a) mass (b) time (c) length (d) acceleration

19. The thrust developed by a rocket-motor is given by $F = mv + A(p_1 - p_2)$, where m is the mass of the gas ejected per unit time, v is velocity of the gas, A is area of cross-section of the nozzle, p_1, p_2 are the pressures of the exhaust gas and surrounding atmosphere. The formula is dimensionally
(a) correct
(b) wrong
(c) sometimes wrong, sometimes correct
(d) data is not adequate

20. When a wave traverses a medium the displacement of a particle located at x at a time t is given by $y = a\sin(bt - cx)$, where a, b and c are constants of the wave. Which of the following is not dimensionless?
(a) $\dfrac{y}{a}$ (b) bt (c) cx (d) $\dfrac{b}{c}$

21. In a new system of units, unit of mass is 10 kg, unit of length is 1 km and unit of time is 1 min. The value of 1 J in this new hypothetical system is
(a) 3.6×10^{-4} new units (b) 6×10^{7} new units
(c) 10^{11} new units (d) 1.67×10^{4} new units

22. The number of particles given by $n = D\dfrac{n_2 - n_1}{x_2 - x_1}$ are crossing a unit area perpendicular to X-axis in unit time, where n_1 and n_2 are the number of particles per unit volume for the values x_1 and x_2 of x respectively. Then the dimensional formula of diffusion constant D is

(a) $[M^0LT^0]$ (b) $[M^0L^2T^{-4}]$
(c) $[M^0LT^{-3}]$ (d) $[M^0L^2T^{-1}]$

23. A calorie is a unit of heat and equals 4.2 J. Suppose we employ a system of units in which the unit of mass is α kg, the unit of length is β metre and the unit of time is γ s. In this new system, 1 calorie =

(a) $\alpha^{-1}\beta^{-2}\gamma^2$ (b) $4.2\,\alpha\beta^2\gamma^{-2}$
(c) $\alpha\beta^2\gamma^2$ (d) $4.2\,\alpha^{-1}\beta^{-2}\gamma^2$

24. An important milestone in the evolution of the universe just after the Big Bang is the Planck time t_p, the value of which depends on three fundamental constants speed c of light in vacuum, gravitational constant G and Planck's constant h. Then, $t_p \propto$

(a) Ghc^5 (b) $\dfrac{c^5}{Gh}$
(c) $\dfrac{Gh}{c^5}$ (d) $\left(\dfrac{Gh}{c^5}\right)^{1/2}$

25. Which of the following combinations have the dimensions of time? L-C-R represent inductance, capacitance and resistance, respectively.

(a) RC (b) LC (c) R/C (d) C/L

26. Photon is quantum of radiation with energy $E = h\nu$ where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of **[NCERT Exemplar]**

(a) linear impulse (b) angular impulse
(c) linear momentum (d) None of these

27. In order to determine the Young's modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1 m (measured using a scale of least count = 1 mm), a weight of mass 1 kg (measured using a scale of least count = 1 g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count = 0.001 cm). What will be the fractional error in the value of Young's modulus determined by this experiment? **[JEE Main 2021]**

(a) 0.14% (b) 0.9%
(c) 9% (d) 1.4%

28. What is dimensional formula of thermal conductivity?

(a) $[MLT^{-1}\theta^{-1}]$ (b) $[MLT^{-3}\theta^{-1}]$
(c) $[M^2LT^{-3}\theta^{-2}]$ (d) $[ML^2T^{-2}\theta]$

29. A quantity x is given by $x = \dfrac{IFv^2}{WL^4}$ where, I is moment of inertia, F is force, v is work and L is length. The dimensional formula for x is same as that of **[JEE Main 2020]**

(a) Planck's constant
(b) force constant
(c) coefficient of viscosity
(d) energy density

30. The quantities $x = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$, $y = \dfrac{E}{B}$ and $z = \dfrac{l}{CR}$ are defined, where C is capacitance, R is resistance, l is length, E is electric field, B is magnetic field, ε_0 is free space permittivity and μ_0 is permeability, respectively. Then, **[JEE Main 2020]**

(a) x, y and z have the same dimension
(b) Only x and z have the same dimension
(c) Only x and y have the same dimension
(d) Only y and z have the same dimension

31. Dimensional formula for thermal conductivity is (Here, K denotes the temperature) **[JEE Main 2020]**

(a) $[MLT^{-2}K]$ (b) $[MLT^{-2}K^{-2}]$
(c) $[MLT^{-3}K^{-1}]$ (d) $[MLT^{-3}K]$

32. A quantity f is given by $f = \sqrt{\dfrac{hc^5}{G}}$, where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of **[JEE Main 2020]**

(a) area (b) volume
(c) momentum (d) energy

33. The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\dfrac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is **[JEE Main 2019]**

(a) $[MLT^{-2}]$ (b) $[M^0L^2T^{-4}]$
(c) $[M^2LT^{-4}]$ (d) $[M^2L^2T^{-2}]$

34. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **[JEE Main 2019]**

(a) $[M^{-1}L^{-2}T^4A^{-2}]$ (b) $[M^{-2}L^0T^{-4}A^{-2}]$
(c) $[M^{-3}L^{-2}T^8A^4]$ (d) $[M^{-2}L^{-2}T^6A^3]$

Numerical Value Questions

35. If the unit of velocity is run, the unit of time is second and unit of force is strength in a hypothetical system of unit. In this system of unit, the unit of mass is $(\text{strength})^x\ (\text{second})^y\ (\text{run})^z$. The value of $\dfrac{y}{x}$ is …… .

36. The edge of a cube is measured using a vernier calliper (9 divisions of the main scale is equal to 10 divisions of vernier scale and 1 main scale division is 1 mm). The main scale division reading is 10 and 1 division of vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 g. The density (in gcm^{-3}) upto correct significant figures is .... **[IIT JEE 2005]**

37. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is .... **[JEE Main 2020]**

38. The focal length of a spherical mirror is given by $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$, where u is the position of object from pole of the mirror and v is position of image from pole of the mirror.

If $u=(30\pm 0.3)$ cm and $v=(60\pm 0.6)$ cm. The maximum percentage error in the measurement of focal length of the mirror is $n\%$, then the value of n is .... **[JEE Main 2021]**

39. If the speed of light (c), acceleration due to gravity (g) and pressure (p) are taken as base units. The dimension of g in the dimensional formula of universal gravitational constant is ....

Answers

Round I

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)
11. (b)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (a)	**18.** (d)	**19.** (c)	**20.** (a)
21. (d)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (b)	**30.** (d)
31. (c)	**32.** (a)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (a)	**40.** (c)
41. (a)	**42.** (d)	**43.** (c)	**44.** (*)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (b)	**49.** (c)	**50.** (c)
51. (c)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (d)	**56.** (d)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (d)
61. (b)	**62.** (b)	**63.** (c)	**64.** (a)	**65.** (a)	**66.** (a)	**67.** (b)	**68.** (d)	**69.** (b)	**70.** (c)
71. (d)	**72.** (c)	**73.** (c)							

Round II

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (d)
11. (c)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (d)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (d)
21. (a)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (a)
31. (c)	**32.** (d)	**33.** (c)	**34.** (c)	**35.** 1	**36.** 2.66	**37.** 1050	**38.** 1	**39.** 2	

Solutions

Round I

1. According to Faraday's first law of electrolysis, $m = Zq$ or $Z = \frac{m}{q}$. So, SI unit of Z is kg C^{-1}.

2. Intensity $(I) = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{J}}{\text{m}^2\ \text{s}}$ or $\text{Jm}^{-2}\text{s}^{-1}$

3. Magnetic flux has the unit as weber/ m^2.

4. Capacitance, $C = \frac{\text{Charge}}{\text{Potential}} = \frac{q}{V}$

Also, potential $= \frac{\text{work}}{\text{charge}}$ $\left(\because V = \frac{W}{q}\right)$

$\therefore \quad C = \frac{q^2}{J}$ as well as $C = \frac{J}{V^2}$

Thus, (a), (c) and (d) are equivalent to farad but (b) is not equivalent to Farad.

5. Time defined in terms of rotation of the earth is called Universal Time (UT).

6. The primary use of a sextant is to measure the angle between an astronomical object and the horizon.

7. 'rad' is used to measure biological effect of radiation.

8. 1 Newton = 10^5 dyne and 1 m = 100 cm

10^6 dyne $\text{cm}^{-2} = 10^6 \times 10^{-5}\ \text{N} \times (10^{-2}\ \text{m})^{-2} = 10^5\ \text{Nm}^{-2}$

9. According to definition, metre is the distance containing 1650763.73 wavelength in vacuum of radiation corresponding to orange red light emitted by an atom of Kr-86.

10. Young modulus, $Y = 1.9 \times 10^{11}\ \text{N/m}^2$

$\because \quad 1\ \text{N} = 10^5\ \text{dyne}, 1\ \text{m}^2 = 10^4\ \text{cm}^2$

So, $\quad Y \text{ (in CGS)} = \frac{1.9 \times 10^{11} \times 10^5}{10^4}\ \text{dyne/cm}^2$

$= 1.9 \times 10^{12}\ \text{dyne/cm}^2$

11. One light year $= 3 \times 10^8 \text{m/s} \times 1\ \text{yr}$

$= \frac{3 \times 10^8\ \text{m}}{\text{s}} \times 365 \times 24 \times 60 \times 60\,\text{s}$

$= 3 \times 10^8 \times 365 \times 24 \times 60 \times 60\ \text{m}$

$= 9.461 \times 10^{15}\ \text{m}$

12. Force, $F = ma$

$\therefore \quad a = \frac{F}{m} = \frac{10\ \text{pound}}{1\ \text{kg}}$

$= 10\,\frac{\text{pound}}{\text{kg}} = \frac{10\ \text{slug ft}}{\text{kg s}^2}$

$= 10 \times 14.6\ \text{kg}\,\frac{\text{ft}}{\text{kg s}^2} = 146\,\frac{\text{ft}}{\text{s}^2}$

$= 146 \times 0.30\ \text{ms}^{-2}$

$= 43.8\ \text{ms}^{-2}$

13. As, $\quad n_1 u_1 = n_2 u_2$

$\Rightarrow \quad n_2 = \frac{1\ \text{shake}}{1\ \text{ns}} = \frac{10^{-8}\ \text{s}}{10^{-9}\ \text{s}}$

$\therefore \quad n_2 = 10$

14. As, $\quad n_1 u_1 = n_2 u_2$

(n_1 and n_2 are numerical values and u_1 and u_2 are the unit in proper system)

$\therefore \quad n_2 = \frac{n_1 u_1}{u_2}$

$= \frac{1450\ \text{mile/h}}{\text{m/s}} = \frac{1450\ \text{s/mile}}{\text{mh}}$

$= \frac{1450\ \text{s} \times 1.6\ \text{km}}{10^{-3}\ \text{km} \times 60 \times 60\ \text{s}} = 644.4$

1450 mile/h = 644.4 m/s

15. As, $R = 8.3$ J/K-mol

Now, $\quad n_1 u_1 = n_2 u_2$

(u_1 and u_2 are units while n_1 and n_2 are the numerical values)

$\therefore \quad n_2 = \frac{n_1 u_1}{u_2} = \frac{8.3\ \text{J/K-mol}}{\text{atm L/K-mol}}$

$= \frac{8.3\ \text{J/K-mol}}{(1.013 \times 10^5\ \text{N/m}^2)\ (10^{-3}\text{m}^3)/\text{K-mol}}$

$= \frac{8.12}{10^2} = 0.0812$

$\therefore \quad$ 8.3 J/K-mol = 0.0812 atm L/K-mol

16. $na' = (n-1)a$

$\Rightarrow a' = \left(\frac{n-1}{n}\right)a$

$\therefore$ Least Count (LC) = 1MSD − 1VSD $= a - a'$

$= a - \left(\frac{n-1}{n}\right)a = \frac{a}{n}\ \text{cm} = \frac{10a}{n}\ \text{mm}$

17. Magnification of microscope = 100

Observed width of the hair = 3.5 mm

Magnification $= \frac{\text{Observed width}}{\text{Real width}}$

Real width $= \frac{\text{Observed width}}{\text{Magnification width}} = \frac{3.5}{100} = 0.035\ \text{mm}$

18. Least count of vernier callipers

LC = 1 MSD − 1 VSD

$= \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$

As, 20 divisions of vernier scale = 16 divisions of main scale

$\therefore \quad 1 \text{ VSD} = \frac{16}{20} \text{ mm} = 0.8 \text{ mm}$

$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD}$

$= 1 \text{ mm} - 0.8 \text{ mm} = 0.2 \text{ mm}$

19. Here, least count $= \frac{0.5}{30}$ degree

Now, total reading = main scale reading + (vernier scale reading) × (least count)

$$= 58.5 \text{ degrees} + (9)\left(\frac{0.5}{30}\right) \text{ degrees}$$

$$= \left(58.5 + \frac{1.5}{10}\right)$$

$$= 58.5 + 0.15$$

$$= 58.65 \text{ degree}$$

$$= 58.65°$$

20. Diameter of wire, $d = \text{MSR} + \text{CSR} \times \text{LC}$

$$= 0 + 52 \times \frac{1}{100}$$

$$= 0.52 \text{ mm} = 0.052 \text{ cm}$$

21. The diameter = main scale reading + circular scale reading × LC − zero error

$$= 3 + 35 \times \frac{1}{2 \times 50} - (-0.03)$$

$$= 3.38 \text{ mm}$$

22. In a screw gauge,

Least count

$$= \frac{\text{Measure of 1 main scale division (MSD)}}{\text{Number of division on circular scale}}$$

Here, minimum value to be measured/least count is

$$5\ \mu\text{m} = 5 \times 10^{-6} \text{ m}$$

$\therefore$ According to the given values,

$$5 \times 10^{-6} = \frac{1 \times 10^{-3}}{N}$$

or $\quad N = \frac{10^{-3}}{5 \times 10^{-6}} = \frac{1000}{5} = 200$ divisions

23. Arithmetic mean time of a oscillating simple pendulum

$$= \frac{\Sigma x_i}{N} = \frac{90 + 91 + 92 + 95}{4} = 92 \text{ s}$$

Mean deviation of a simple pendulum

$$= \frac{\Sigma |\bar{x} - x_i|}{N}$$

$$= \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Given, minimum division in the measuring clock, i.e. simple pendulum = 1 s. Thus, the reported mean time of a oscillating simple pendulum = (92 ± 2) s.

24. The instrument whose least count is minimum, is called the most precise device.

(a) Number of division (MSD) = 20

Main Scale Division (MSD) = 1 mm

As 20 divisions on vernier scale will be equal to the 19 divisions on main scale.

$\therefore$ Vernier Scale Division (VSD) $= \frac{19}{20}$ MSD

Least count of vernier callipers = 1 MSD − 1 VSD

$$= 1 \text{ MSD} - 1 \text{ VSD}$$

$$= 1 \text{ MSD} - \frac{19}{20} \text{ MSD}$$

$$= \frac{1}{20} \text{ MSD}$$

$$= \frac{1}{20} \text{ mm} = \frac{1}{200} \text{ cm}$$

$$= 0.005 \text{ cm}$$

(b) Pitch of screw guage = 1 mm

Number of divisions on circular scale = 100

Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$\frac{1}{100} \text{ mm} = \frac{1}{1000} \text{ cm} = 0.001 \text{ cm}$$

(c) Wavelength of light

$$(\lambda) \approx 10^{-7} \text{m} = 10^{-5} \text{ cm} = 0.00001 \text{ cm}$$

$\therefore$ As the given optical instrument can measure length to within a wavelength of light, therefore least count of the given optical instrument

= Wavelength of light

= 0.00001 cm

The least count is minimum for the given optical instrument, therefore the given optical instrument is the most precise.

25. We know that, density, $\rho = \frac{M}{V} = \frac{[\text{M}]}{[\text{L}^3]}$

$$\therefore \quad \frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3\frac{\Delta L}{L} \times 100$$

$$= 3\% + 3\,(2\%) = 9\%$$

26. Given, $\quad X = [\text{M}^a \text{L}^b \text{T}^{-c}]$

$$\therefore \quad \frac{\Delta X}{X} = \pm\left[a\frac{\Delta M}{M} + b\frac{\Delta L}{L} + c\frac{\Delta T}{T}\right]$$

$$= \pm[a\alpha + \beta b + \gamma c]\%$$

27. Kinetic energy, $E = \frac{1}{2}mv^2$

$$\therefore \quad \frac{\Delta E}{E} \times 100 = \frac{\Delta v^2 - v^2}{v^2} \times 100$$

$$= [(1.5)^2 - 1] \times 100 = 125\%$$

28. Percentage error in radius is $\frac{0.1}{4.3} \times 100$

Again, $\quad V \propto R^3$

$\therefore$ Error in volume $= \frac{3 \times 0.1}{4.3} \times 100$

29. Given, $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{T^2}$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} = \frac{0.1}{10} + 2\left(\frac{1}{0.5 \times 200}\right)$$

$$\frac{\Delta g}{g} \times 100 = \frac{1}{100} \times 100 + \frac{1}{50} \times 100 = 3\%$$

30. $v = \sqrt{\frac{T}{m}} = \left[\frac{m'g}{M/l}\right]^{1/2} = \left[\frac{m'lg}{M}\right]^{1/2}$

It follows from here, $\frac{\Delta v}{v} = \frac{1}{2}\left[\frac{\Delta m'}{m} + \frac{\Delta l}{l} + \frac{\Delta M}{M}\right]$

$$= \frac{1}{2}\left[\frac{0.1}{3.0} + \frac{0.01}{1.000} + \frac{0.1}{2.5}\right]$$

$$= \frac{1}{2}[0.03 + 0.001 + 0.04]$$

$$= 0.036$$

Percentage error in the measurement = 3.6

31. When quantities are subtracted, their maximum absolute errors are added up.

$\therefore$ Result $= (80 - 10) \pm (0.1 + 0.1)$

$= 70 \pm 0.2$

32. Percentage error in side of public park

$$= \frac{1}{2}\left[\frac{0.2}{100} \times 100\right] = 0.1 \text{ m}$$

Absolute error in side of public park

$$= \frac{0.1}{100} \times 10 = 0.01 \text{ m}$$

Side of public park $= \sqrt{100} = 10\,\text{m}$

$\therefore$ Side $= (10 \pm 0.01)$ m

33. As, $V = (8 + 0.5)$ V

and $I = (2 + 0.2)$ A

$\therefore$ $R = \frac{8}{2} = 4\,\Omega$ (R = resistance)

$\Rightarrow$ $\frac{\Delta R}{R}\% = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)$

$$= \left(\frac{0.5}{8} + \frac{0.2}{2}\right) \times 100 = 16.25\%$$

$\therefore$ $R = (4 \pm 16.25\%)\,\Omega$

34. Percentage error in length $= \frac{1}{50} \times 100 = 2$

Percentage error in breadth $= \frac{0.1}{2.0} \times 100 = 5$

Percentage error in thickness $= \frac{0.1}{1.00} \times 100 = 1$

Percentage error in volume $= 2 + 5 + 1 = 8$

35. Percentage error in measurement of a side $= \frac{0.01}{1.23} \times 100$

$\therefore$ Percentage error in measurement of area

$$= 2 \times \frac{0.01}{1.23} \times 100$$

36. Given, $z = \frac{a^2 b^{2/3}}{\sqrt{c}\, d^3}$

According to question,

% error in z = (2)% error in $a + \left(\frac{2}{3}\right)$% error in $b + \left(\frac{1}{2}\right)$% error in c + (3)% error in d

$$\frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$$

$$= 2 \times 2\% + \frac{2}{3} \times 1.5\% + \frac{1}{2} \times 4\% + 3 \times 2.5\%$$

$$= 14.5\%$$

37. Weight in air = (5.00 ± 0.05) N

Weight in water = (4.00 ± 0.05) N

Loss of weight in water

= [(5.00 − 4.00) ± (0.05 + 0.05)] N

= (1.00 ± 0.1) N

Now, relative density $= \frac{\text{Weight in air}}{\text{Weight loss in water}}$

i.e. $\text{RD} = \frac{5.00 \pm 0.05}{1.00 \pm 0.1}$

Now, relative density with maximum permissible error

$$= \frac{5.00}{1.00} \pm \left(\frac{0.05}{5.00} + \frac{0.1}{1.00}\right) \times 100$$

$$= 5.0 \pm (1 + 10)\% = 5 \pm 11\%$$

So, maximum possible percentage error in relative density is ± 11%.

38. Volume of a cylinder of radius r and height h is given by

$$V = \pi r^2 h$$

or $V = \frac{1}{4}\pi D^2 h$, where D is the diameter of circular surface. Here, $D = 12.6$ cm and $h = 34.2$ cm

$\Rightarrow$ $V = \frac{\pi}{4} \times (12.6)^2 \times (34.2)$

$V = 4262.22 \text{ cm}^3$

$V = 4260$ (in three significant numbers)

Now, error calculation can be done as

$$\frac{\Delta V}{V} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta h}{h} = \frac{2 \times 0.1}{12.6} + \frac{0.1}{34.2}$$

$\Rightarrow$ $\frac{\Delta V}{V} = 0.0158 + 0.0029$

$\Rightarrow$ $\Delta V = (0.01879) \times (4262.22)$

$\Rightarrow$ $\Delta V = 79.7 \simeq 80 \text{ cm}^3$

$\therefore$ For proper significant numbers, volume reading will be $V = 4260 \pm 80 \text{ cm}^3$.

39. Given, $I = (e^{1000V/T} - 1)$ mA

$dV = \pm 0.01$ V, $T = 300$ K

$I = 5$ mA

$\Rightarrow \quad I = e^{1000V/T} - 1$

$I + 1 = e^{1000V/T}$

Taking log on both sides, we get

$$\log (I+1) = \frac{1000V}{T}$$

$$\Rightarrow \quad \frac{d(I+1)}{I+1} = \frac{1000}{T} dV$$

$$\frac{dI}{I+1} = \frac{1000}{T} dV$$

$$\Rightarrow \quad dI = \frac{1000}{T} \times (I+1)\, dV$$

$$dI = \frac{1000}{300} \times (5+1) \times 0.01 = 0.2 \text{ mA}$$

So, error in the value of current is 0.2 mA.

40. $3.8 \times 10^{-6} + 4.2 \times 10^{-5}$

$= (3.8 \times 10^{-1} + 4.2) \times 10^{-5}$

$= (0.38 + 4.2) \times 10^{-5}$

$= (4.58) \times 10^{-5}$

Rounding off to one place of decimal.

The sum $= 4.6 \times 10^{-5}$

41. As $\pi = 3.14$

$\therefore \quad \pi^2 = (3.14)^2 = 9.8596$

On rounding off $\pi^2 = 9.86$

42. Given $A = 1.0 \text{ m} \pm 0.2$ m

$B = 2.0 \text{ m} \pm 0.2$ m

$x = \sqrt{AB} = \sqrt{1.0 \times 2.0} = 1.414 \text{ m}$

Rounding-off to two significant digits,

$x = \sqrt{AB} = 1.4 \text{ m}$

Now, $\quad \frac{\Delta x}{x} = \frac{1}{2}\left[\frac{\Delta A}{A} + \frac{\Delta B}{B}\right]$

$= \frac{1}{2}\left[\frac{0.2}{1.0} + \frac{0.2}{0.2}\right]$

$= \frac{0.6}{2 \times 2.0}$

$\Delta x = \frac{0.6 \times x}{2 \times 2.0} = 0.15 \times 1.414 = 0.2121$

Rounding-off to one significant digits, $\Delta x = 0.2$ m

Thus, $\quad \sqrt{AB} = 1.4\text{m} \pm 0.2\text{m}$

43. Area of 1 square $= 5.29 \text{ cm}^2$

Area of seven such squares

= 7 times addition of area of 1 square

$= 5.29 + 5.29 + 5.29 + \ldots$ 7 times $= 37.03 \text{ cm}^2$

As we know that, if in the measured values to be added/subtracted the least number of significant digits after the decimal is *n*.

Then, in the sum or difference also, the number of significant digits after the decimal should be *n*.

Here, number of digits after decimal in 5.29 is 2, so our answer also contains only two digits after decimal point.

$\therefore$ Area required $= 37.03 \text{ cm}^2$

44. Given , $A_1 = 24.36$, $B_1 = 0.0724$, $C_1 = 256.2$

$\therefore \quad A_1 + B_1 + C_1 = 280.6324$

As, sum contains same number of digits after decimal as present in the number having the least number of decimal places.

So, $A_1 + B_1 + C_1 = 280.6$

Similarly, $\quad A_2 + B_2 + C_2 = 280.7$

$A_3 + B_3 + C_3 = 280.7$ (rounded off)

$A_4 + B_4 + C_4 = 280.7$ (rounded off)

So, $\quad A_1 + B_1 + C_1 < A_2 + B_2 + C_2$

$= A_3 + B_3 + C_3 = A_4 + B_4 + C_4$

None of the options is matching with result.

45. Force, $F = kv$, $[k] = \frac{[F]}{[v]} = \frac{[\text{MLT}^{-2}]}{[\text{LT}^{-1}]} = [\text{MT}^{-1}]$.

So, unit is kg s^{-1}.

46. [Surface tension] $= [\text{ML}^0\text{T}^{-2}]$, [viscosity] $= [\text{ML}^{-1}\text{T}^{-1}]$.

Clearly, mass has the same exponent in these physical quantities.

47. [Energy] $= [\text{ML}^2\text{T}^{-2}]$. Increasing *M* and *L* by a factor of 3, energy is increased 27 times.

48. LI^2 represents energy, *i.e.* $[\text{ML}^2\text{T}^{-2}]$.

49. *CR* is known as time constant

$$CR = [\text{T}]$$

50. Spring constant $= \frac{F}{L} = [\text{ML}^0\text{T}^{-2}]$

Surface energy $= \frac{\text{Energy}}{\text{Area}} = [\text{ML}^0\text{T}^{-2}]$

Spring constant and surface energy has same dimensions.

51. As we know that, formula of velocity is

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\Rightarrow \quad v^2 = \frac{1}{\mu_0 \varepsilon_0} = [\text{LT}^{-1}]^2$$

$$\therefore \quad \frac{1}{\mu_0 \varepsilon_0} = [\text{L}^2\text{T}^{-2}]$$

52. Planck's constant (in terms of unit)

$$h = \text{J-s} = [\text{ML}^2\text{T}^{-2}][\text{T}] = [\text{ML}^2\text{T}^{-1}]$$

Momentum $(p) = \text{kg-ms}^{-1}$

$= [\text{M}][\text{L}][\text{T}^{-1}] = [\text{MLT}^{-1}]$

53. From the relation $\quad F = qvB$

$\Rightarrow \quad [\text{MLT}^{-2}] = [\text{C}][\text{LT}^{-1}][B]$

$\Rightarrow \quad [B] = [\text{MC}^{-1}\text{T}^{-1}]$

54. Here αt^2 is a dimensionless, therefore $\alpha = \frac{1}{t^2}$ and has the dimension of $[t^{-2}]$.

55. As, $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

or $[\mu_0] = \frac{[F]}{[I_1 I_2]} = \frac{[MLT^{-2}]}{[A^2]} = [MLT^{-2}A^{-2}]$

56. As, $[ML^{-2}T^{-2}] = \frac{[MLT^{-2}]}{[L][L^2]} = \frac{\text{Force}}{\text{Distance} \times \text{Area}}$

$= \frac{\text{Pressure}}{\text{Distance}} = \text{Pressure gradient}$

57. Magnetic energy $= \frac{1}{2} LI^2 = \frac{LQ^2}{2t^2}$ $\left[\text{as, } I = \frac{Q}{t}\right]$

where, L = inductance and I = current.

Energy has the dimensions = $[ML^2T^{-2}]$

Equate the dimensions, we have

$$[ML^2T^{-2}] = [\text{henry}] \times \frac{[Q^2]}{[T^2]}$$

$\Rightarrow$ $[\text{henry}] = \frac{[ML^2]}{[Q^2]}$

58. Solar constant $= \frac{\text{Solar energy}}{\text{Area} \times \text{Time}} = \frac{[ML^2T^{-2}]}{[L^2][T]} = [ML^0T^{-3}]$

59. As, $\frac{B^2}{2\mu_0}$ = energy density of magnetic field

$= \frac{\text{Energy}}{\text{Volume}}$

So, $\left[\frac{B^2}{2\mu_0}\right] = [\text{Energy /Volume}] = \frac{[ML^2T^{-2}]}{[L^3]}$

$= [ML^{-1}T^{-2}]$

60. Dimensions of ε_0 (permittivity of free space) are

$$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$$

As, c = speed of light.

$\therefore$ Dimension of $[c] = [LT^{-1}]$

So, dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ are

$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \left[\sqrt{\frac{\varepsilon_0^2}{\varepsilon_0\mu_0}}\right] = [\varepsilon_0 c]$ $\left[\because c^2 = \frac{1}{\mu_0\varepsilon_0}\right]$

$= [M^{-1}L^{-3}T^4A^2][LT^{-1}] = [M^{-1}L^{-2}T^3A^2]$

61. Given, $y = r\sin(\omega t - kx)$

where, ωt = angle

$\therefore$ $\omega = \frac{1}{T} = [T^{-1}]$ ($\because$ angle is dimensionless)

Similarly, kx = angle

$\therefore$ $k = \frac{1}{x} = [L^{-1}]$

$\therefore$ $\frac{\omega}{k} = \frac{[T^{-1}]}{[L^{-1}]} = [LT^{-1}]$

62. Electrostatic force between two charges,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2} \Rightarrow \varepsilon_0 = \frac{q_1q_2}{4\pi FR^2}$$

Substituting the units.

Hence, $\varepsilon_0 = \frac{C^2}{N\text{-}m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$

63. Dimensions of given quantities are

l = inductance = $[M^1 L^2 T^{-2} A^{-2}]$

r = resistance = $[M^1 L^2 T^{-3} A^{-2}]$

c = capacitance = $[M^{-1} L^{-2} T^4 A^2]$

v = voltage = $[M^1 L^2 T^{-3} A^{-1}]$

So, dimensions of $\frac{l}{rcv}$ are

$$\left[\frac{l}{rcv}\right] = \frac{[M^1 L^2 T^{-2} A^{-2}]}{[M^1 L^2 T^{-2} A^{-1}]} = [A^{-1}]$$

64. Trigonometric function has no dimension, so

$\frac{x}{a}$ = dimensionless

Thus, a has the dimensions as x in equation.

LHS of given equation is dimensionless and hence, a^n is dimensionless

$\Rightarrow$ $n = 0$

65. By substituting the dimensions of each quantity, we get

$$T = [ML^{-1}T^{-2}]^a [L^3M]^b [MT^{-2}]^c$$

By solving, we get $a = -\frac{3}{2}$, $b = \frac{1}{2}$ and $c = 1$

66. Momentum, $p \propto f^a v^b \rho^c$

$[MLT^{-1}] = [T^{-1}]^a [LT^{-1}]^b [ML^{-3}]^c$

$[MLT^{-1}] = [M^c L^{b-3c} T^{-a-b}]$

$\Rightarrow$ $c = 1$

$b - 3c = 1$

$\Rightarrow$ $b = 4$

$-a - b = -1$

$a + b = 1, a = -3$

$[p] = [f^{-3}v^4\rho]$

67. Here, $[M^0L^0T^0] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [LT^{-1}]^c$

or $[M^0L^0T^0] = [M^{a+b}L^{-a+c}T^{-2a-3b-c}]$

Comparing powers of M, L and T, we get

$$a + b = 0, -a + c = 0, -2a - 3b = 0$$

Solving, $a = 1, b = -1, c = 1$

68. $\lambda = m^p v^q h^r$

$[M^0LT^0] = [M^p][LT^{-1}]^q [ML^2T^{-2}]^r$

$[M^0LT^0] = [M^{p+r}L^{q+2r}T^{-q-r}]$

$\therefore$ $p + r = 0, q + 2r = 1, -q - r = 0$

After solving, we get

$p = -1, q = -1, r = 1$

69. Let Young's modulus is related to speed, area and force, as $Y = F^x A^y V^z$

Substituting dimensions, we have

$$[ML^{-1}T^{-2}] = [MLT^{-2}]^x [L^2]^y [LT^{-1}]^z$$

Comparing power of similar quantities, we have

$$x = 1, x + 2y + z = -1 \text{ and } -2x - z = -2$$

Solving these, we get

$$x = 1, y = -1, z = 0$$

So, $[Y] = [FA^{-1}V^0]$

70. Let dimensions of energy E in terms of momentum p, area A and time T are

$$[E] = [p]^x [A]^y [T]^z$$

Substituting dimensions of fundamental quantities for E, p, A and T, we have

$$[ML^2T^{-2}] = [MLT^{-1}]^x [L^2]^y [T]^z$$

$$[ML^2T^{-2}] = [M^x L^{x+2y} T^{-x+z}]$$

Equating powers of same physical quantities on both sides, we have

$$x = 1, x + 2y = 2 \text{ and } -x + z = -2$$

So, $x = 1, y = \frac{1}{2}, z = -1$

$\therefore$ Dimensional formula of $[E] = [pA^{1/2}T^{-1}]$

71. Dimensions of speed are $[v] = [LT^{-1}]$

Dimensions of acceleration are $[A] = [LT^{-2}]$

Dimensions of force are $[F] = [MLT^{-2}]$

Dimension of Young modulus is, $[Y] = [ML^{-1}T^{-2}]$

Let dimensions of Young's modulus is expressed in terms of speed, acceleration and force as

$$[Y] = [v]^\alpha [A]^\beta [F]^\gamma \quad \ldots(i)$$

Then substituting dimensions in terms of M, L and T, we get

$$[ML^{-1}T^{-2}] = [LT^{-1}]^\alpha [LT^{-2}]^\beta [MLT^{-2}]^\gamma$$

$$= [M^\gamma L^{\alpha+\beta+\gamma} T^{-\alpha-2\beta-2\gamma}]$$

Now comparing powers of basic quantities on both sides, we get $\gamma = 1$

$$\alpha + \beta + \gamma = -1$$

and $-\alpha - 2\beta - 2\gamma = -2$

Solving these, we get

$$\alpha = -4, \beta = 2, \gamma = 1$$

Substituting α, β, and γ in Eq. (i), we get

$$[Y] = [v^{-4}A^2F^1]$$

72. Let $V_0 = (h)^a \cdot (c)^b \cdot (G)^c \cdot (A)^d \quad \ldots(i)$

Then, $[V_0] = [\text{potential}] = \left[\frac{\text{potential energy}}{\text{charge}}\right]$

$$= \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$$

$$[h] = \left[\frac{\text{Energy}}{\text{Frequency}}\right] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

$$[c] = [\text{Speed}] = [LT^{-1}]$$

$$[G] = \left[\frac{\text{Force} \times (\text{Distance})^2}{(\text{Mass})^2}\right]$$

$$= \frac{[MLT^{-2}][L^2]}{[M^2]}$$

$$= [M^{-1}L^3T^{-2}]$$

Substituting the dimensions of V_0, h, C, G and A in Eq. (i) and equating dimension on both sides, we get

$$[ML^2T^{-3}A^{-1}] = [ML^2T^{-1}]^a \times [LT^{-1}]^b \times [M^{-1}L^3T^{-2}]^c \times [A]^d$$

$\Rightarrow$

$$a - c = 1 \quad \ldots(ii)$$

$$2a + b + 3c = 2 \quad \ldots(iii)$$

$$-a - b - 2c = -3 \quad \ldots(iv)$$

$$d = -1 \quad \ldots(v)$$

On solving above equations, we get

$$a = 0, b = 5, c = -1, d = -1$$

Substituting these values in Eq. (i), we get

$$V_0 = h^0 \cdot c^5 \cdot G^{-1} \cdot A^{-1}$$

73. Suppose, linear momentum (p) depends upon the Planck's constant (h) raised to the power (a), surface tension(S) raised to the power (b) and moment of inertia (I) raised to the power (c).

Then, $p \propto (h)^a (S)^b (I)^c$ or $p = kh^a S^b I^c$

where, k is a dimensionless proportionality constant.

Thus, $[p] = [h]^a [S]^b [I]^c \quad \ldots(i)$

Then, the respective dimensions of the given physical quantities, *i.e.*

$$[p] = [\text{mass} \times \text{velocity}] = [MLT^{-1}]$$

$$[I] = [\text{mass} \times (\text{distance})^2] = [ML^2T^0]$$

$$[S] = [\text{force} \times \text{length}] = [ML^0T^{-2}]$$

$$[h] = [ML^2T^{-1}]$$

Then, substituting these dimensions in Eq. (i), we get

$$[MLT^{-1}] = [ML^2T^{-1}]^a [MT^{-2}]^b [ML^2]^c$$

For dimensional balance, the dimensions on both sides should be same.

Thus, equating dimensions, we have

$$a + b + c = 1$$

$$2(a + c) = 1 \quad \text{or} \quad a + c = \frac{1}{2}$$

$$-a - 2b = -1 \quad \text{or} \quad a + 2b = 1$$

Solving these three equations, we get

$$a = 0, b = \frac{1}{2}, c = \frac{1}{2}$$

$\therefore$ $p = h^0 S^{\frac{1}{2}} I^{\frac{1}{2}}$

or $p = S^{\frac{1}{2}} I^{\frac{1}{2}} h^0$

Round II

1. Vernier scale reading (VSR) $= 6 \times 0.01 = 0.06$ cm

Correct value $= 8.5 + 0.06 - 0.02 = 8.54$ cm

2. As, $x = \frac{1 \text{ g cm s}^{-1}}{\text{N-s}} = \frac{1 \text{ g cm s}^{-1}}{1 \text{ kg} \times 1 \text{ ms}^{-1} \times 1 \text{ s}}$

$$= \frac{1 \text{ g cm s}^{-1}}{10^3 \text{ g} \times 10^2 \text{ cm s}^2 \times 1 \text{ s}} = 10^{-5}$$

3. Muscle $\times$ Speed = Power

$\therefore$ Muscle $= \frac{\text{Power}}{\text{Speed}} = \frac{\text{Work}}{\text{Time} \times \text{Speed}}$

$$= \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}]\ [\text{LT}^{-1}]} = [\text{MLT}^{-2}]$$

= Mass $\times$ acceleration = Force

Hence, $\frac{\text{SI unit of force}}{\text{CGS unit of force}} = \frac{\text{kg} \times \text{m} \times \text{s}^{-2}}{\text{g} \times \text{cm} \times \text{s}^{-2}}$

$$= 10^3 \times 10^2 = 10^5$$

4. Least count $1 = \frac{1}{100}$ mm $= 0.01 = 0.001$ cm

Diameter of wire $D = 1$ mm + 63×0.01 mm

= 1.63 mm or 0.163 cm

Volume of wire $= \frac{\pi D^2 l}{4} = \frac{3.14 \times (0.163)^2 \times 5.6}{4}$

$= 0.117 \text{ cm}^3$

5. As, pitch $= \frac{1}{2}$ mm $= 0.5$ mm

Least count $= \frac{0.5}{50}$ mm $= 0.01$ mm

Zero error = -0.03 mm

Zero correction = + 0.03 mm

Observed diameter of wire $= 3 + 35 \times 0.01$

= 3.35 mm

Corrected diameter of wire = 3.35 mm + 0.03 mm

= 3.38 mm

6. Given, $l = 10$ mm $= 1$ cm

$b = 5$ mm $= 0.5$ cm and

$h = 5$ mm $= 0.5$ cm

Error in measurements,

$\Delta l = \Delta b = \Delta h = 0.01$ cm

$\therefore$ Maximum percentage error in volume,

$$\frac{\Delta V}{V} \times 100 = \frac{\Delta l}{l} \times 100 + \frac{\Delta b}{b} \times 100 + \frac{\Delta h}{h} \times 100$$

$$= \frac{0.01}{1} \times 100 + \frac{0.01}{0.5} \times 100 + \frac{0.01}{0.5} \times 100$$

$= 1\% + 2\% + 2\% = 5\%$

7. As, $\Delta R_s = \Delta R_1 + \Delta R_2 = \left[\frac{10}{100} \times 10 + \frac{20}{100} \times 20\right] \text{k}\,\Omega = 5 \text{ k}\Omega$

$$\frac{\Delta R_s}{R_s} \times 100 = \frac{5}{30} \times 100 = \frac{50}{3} = 17\%$$

8. $T = \frac{rgh}{2} = \frac{dgh}{4}$ $\quad \left[\because r = \frac{d}{2}\right]$

$$\therefore \quad \frac{\Delta T}{T} \times 100 = \frac{\Delta d}{d} \times 100 + \frac{\Delta g}{g} \times 100 + \frac{\Delta h}{h} \times 100$$

$$= \frac{0.01}{1.26} \times 100 + \frac{0.01}{9.80} \times 100 + \frac{0.01}{1.45} \times 100$$

$= 0.8 + 0.1 + 0.7 = 1.6\%$

9. $R = \frac{V}{i}$

$\therefore \quad \log R = \log V - \log i$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta i}{i} = 3\% + 3\% = 6\%$$

10. Here, $\frac{\Delta m}{m} = \frac{0.003}{0.3}$,

$\frac{\Delta r}{r} = \frac{0.005}{0.5} \cdot \frac{\Delta L}{L} = \frac{0.06}{6}$

As $\quad \rho = \frac{m}{(\pi r^2) L}$

$$\therefore \left(\frac{\Delta \rho}{\rho}\right) \times 100 = \left(\frac{\Delta m}{m} + \frac{2\,\Delta r}{r} + \frac{\Delta L}{L}\right) \times 100$$

$$= \left(\frac{0.003}{0.3} + \frac{2 \times 0.005}{0.5} + \frac{0.06}{6}\right) \times 100$$

$= 1 + 2 + 1 = 4\%$

11. From, $s = ut + \frac{1}{2} at^2$

$$h = 0 + \frac{1}{2} \times 9.8\ (2)^2 = 19.6 \text{ m}$$

$$\frac{\Delta h}{h} = \pm \left(\frac{\Delta t}{t}\right) \qquad (\because a = g = \text{constant})$$

$$= \pm 2 \left(\frac{0.1}{2}\right) = \pm \frac{1}{10}$$

$$\Delta h = \frac{h}{10} = \frac{19.6}{10} = 1.96 \text{ m}$$

12. Here, $L = 1.2 \times 10^{-2}$ m, $V = ?$

$$V = L^3 = (1.2 \times 10^{-2})^3 = 1.728 \times 10^{-6} \text{ m}^3$$

As the result can have only two significant digits, therefore rounding off, we get

$$V = 1.7 \times 10^{-6} \text{ m}^3$$

13. Resistance, $R = \frac{\rho l}{A}$

Also, $\quad R = \frac{V}{I}$

$$\Rightarrow \quad \rho = \frac{AV}{Il} = \frac{\pi D^2 V}{4Il} \qquad \left(\therefore A = \frac{\pi D^2}{4}\right)$$

$$\therefore \quad \frac{\Delta \rho}{\rho} = 2\frac{\Delta D}{D} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta l}{l}$$

$$= 2\left(\frac{0.01}{5.00}\right) + \left(\frac{0.1}{5.0}\right) + \left(\frac{0.01}{2.00}\right) + \left(\frac{0.1}{10.0}\right)$$

$$\frac{\Delta \rho}{\rho} \times 100 = 0.4 + 2 + 0.5 + 1 = 3.9\%$$

14. Equivalent, $R_s = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\Delta R_s}{R_s}\times 100 = \frac{\Delta R_1}{R_1}\times 100 + \frac{\Delta R_2}{R_2}\times 100 + \frac{\Delta (R_1 + R_2)}{R_1 + R_2}\times 100$$

Now, $\Delta R_1 = \dfrac{10}{100}\times 4\ \text{k}\Omega = 0.4\ \text{k}\Omega$

$\Delta R_2 = \dfrac{10}{100}\times 6\ \text{k}\Omega = 0.6\ \text{k}\Omega$

Again, $\dfrac{\Delta R_s}{R_s}\times 100 = \dfrac{0.4}{4}\times 100 + \dfrac{0.6}{6}\times 100 + \dfrac{0.4+0.6}{10}\times 100$

$= 10 + 10 + 10 = 30\%$

15. The right hand side of the given relation is basically $\dfrac{k}{\text{metre}}$. But, since the left hand side is joule, therefore k should be Jm.

16. Capacitance $X = [\text{M}^{-1}\text{L}^{-2}\text{T}^2\text{Q}^2]$

Magnetic induction $Z = [\text{MT}^{-1}\text{Q}^{-1}]$

$[Z^2] = [\text{M}^2\text{T}^{-2}\text{Q}^{-2}]$

Given, $X = 3YZ^2$

or $Y = \dfrac{X}{3Z^2}$ or $[Y] = \dfrac{[X]}{[Z]^2}$

$\therefore$ $[Y] = \dfrac{[\text{M}^{-1}\text{L}^{-2}\text{T}^2\text{Q}^2]}{[\text{M}^2\text{T}^{-2}\text{Q}^{-2}]} = [\text{M}^{-3}\text{L}^{-2}\text{T}^4\text{Q}^4]$

17. The formula for fine structure constant $= \dfrac{e^2}{4\pi\varepsilon_0\left(\dfrac{h}{2\pi}\right)c}$

It is dimensionless.

18. $[X] = \left[\dfrac{\text{M}^{-1}\text{L}^3\text{T}^{-2}\times \text{ML}^2\text{T}^{-1}}{\text{L}^3\text{T}^{-3}}\right]^{1/2} = [\text{L}]$

19. Each of the three terms in the given equation has the dimensional formula of force.

20. Given equation, $y = a\sin(bt - cx)$

Comparing the given equation with general wave equation

$$y = a\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

we get, $b = \dfrac{2\pi}{T}, c = \dfrac{2\pi}{\lambda}$

Dimension of $\dfrac{b}{c} = \dfrac{2\pi/T}{2\pi/\lambda} = [\text{LT}^{-1}]$ and other three quantities are dimensionless.

21. We know that, the dimensional formula of energy is $[\text{ML}^2\text{T}^{-2}]$.

$$n_2 = 1\left[\frac{1\ \text{kg}}{10\ \text{kg}}\right]^1\left[\frac{1\ \text{m}}{1\ \text{km}}\right]^2\left[\frac{1\ \text{s}}{1\ \text{min}}\right]^2$$

$$= \frac{1}{10}\times\frac{1}{10^6}\times\frac{1}{(60)^{-2}}$$

$$= \frac{3600}{10^7} = 3.6\times 10^{-4}$$

22. From the given relation, $D = \dfrac{n(x_2 - x_1)}{n_2 - n_1}$

Here, $[n] = \left[\dfrac{1}{\text{area}\times\text{time}}\right] = \dfrac{1}{[\text{L}^2\text{T}]} = [\text{L}^{-2}\text{T}^{-1}]$

$x_2 - x_1 = [\text{L}]$

and $n_2 - n_1 = \left[\dfrac{1}{\text{volume}}\right] = \left[\dfrac{1}{\text{L}^3}\right] = [\text{L}^{-3}]$

So, $[D] = \dfrac{[\text{L}^{-2}\text{T}^{-1}\text{L}]}{[\text{L}^{-3}]} = [\text{L}^2\text{T}^{-1}]$

23. [Calorie] $= [\text{ML}^2\text{T}^{-2}]$

Comparing with general dimensional formula $[\text{M}^a\text{L}^b\text{T}^c]$, we get

$a = 1, b = 2, c = 2$

$$n_2 = 4.2\left[\frac{1\ \text{kg}}{\alpha\ \text{kg}}\right]^1\left[\frac{1\ \text{m}}{\beta\text{m}}\right]^2\left[\frac{1s}{\gamma\text{s}}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

24. Note carefully that every alternative has Gh and c^5.

$[Gh] = [\text{M}^{-1}\text{L}^3\text{T}^{-2}]\ [\text{ML}^2\text{T}^{-1}] = [\text{M}^0\text{L}^5\text{T}^{-3}]$

$[c] = [\text{LT}^{-1}]$

$\therefore$ $\left(\dfrac{Gh}{c^5}\right)^{1/2} = [\text{T}]$

25. We know that,

$R = [\text{M}^1\text{L}^2\text{T}^{-3}\text{A}^{-2}]$

$C = [\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2]$

$L = [\text{ML}^2\text{T}^{-2}\text{A}^{-2}]$

$\therefore$ $RC = [\text{T}]$ and $\sqrt{LC} = [\text{T}]$

26. From $E = h\nu$

$$h = \frac{E}{\nu} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}^{-1}]} = [\text{ML}^2\text{T}^{-1}]$$

Angular impulse $= \tau\times t = [\text{ML}^2\text{T}^{-2}\times \text{T}] = [\text{ML}^2\text{T}^{-1}]$

27. $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{mgL}{\pi R^2 l}$

$\therefore$ $\dfrac{\Delta Y}{Y} = \dfrac{\Delta m}{m} + \dfrac{\Delta L}{L} + 2\dfrac{\Delta R}{R} + \dfrac{\Delta l}{l}$

$$\frac{\Delta Y}{Y}\times 100 = \left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]\times 100$$

$= 0.1 + 0.1 + 1 + 0.2 = 1.4\%$

28. Heat ΔQ transferred through a rod of length L and area A in time Δt is $\Delta Q = KA\left(\dfrac{T_1 - T_2}{L}\right)\Delta t$

where, K = coefficient of thermal conductivity,

$T_1 - T_2$ = temperature difference.

$\therefore$ $K = \dfrac{\Delta Q\times L}{A(T_1 - T_2)\Delta t}$ ...(i)

Substituting dimensions for corresponding quantities in Eq. (i), we have

$$[K] = \frac{[\text{ML}^2\text{T}^{-2}]\ [\text{L}]}{[\text{L}^2]\ [\theta]\ [\text{T}]} = [\text{MLT}^{-3}\theta^{-1}]$$

29. Given that, $x = \frac{IFv^2}{WL^4}$

Dimensionally,

$$[x] = \frac{[I]\,[F]\,[v]^2}{[W]\,[L]^4} = \frac{[M^1L^2]\,[M^1L^1T^{-2}]\,[L^1T^{-1}]^2}{[M^1L^2T^{-2}]\,[L^1]^4}$$

$$= [M^1L^2]\frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[M^1L^2T^{-2}][L^4]}$$

$$= [M^1L^{-1}T^{-2}] \quad \ldots(i)$$

On checking the alternatives

(a) Planck's constant $\Rightarrow [h] = [M^1L^2T^{-1}]$ doesn't match with dimensional formula of x.

(b) Force constant $\Rightarrow [K] = [M^1T^{-2}]$ doesn't match with dimensional formula of x.

(c) Coefficient of viscosity $\Rightarrow [\eta] = [M^1L^{-1}T^{-1}]$ doesn't match with dimensional formula of x.

(d) Energy density $\Rightarrow [E_d] = [M^1L^{-1}T^{-2}]$ matches with dimensional formula of x.

30. $x = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = speed of light in vacuum

$\therefore$ Dimension of x, $[x] = [M^0L^1T^{-1}]$

$$y = \frac{E}{B} = \text{speed of EM wave}$$

$\therefore$ Dimension of y, $[y] = [M^0L^1T^{-1}]$

$$z = \frac{l}{RC} = \frac{l}{\tau} = \frac{\text{length}}{\text{time}}$$

$\therefore$ Dimension of z, $[z] = [M^0L^1T^{-1}]$

Thus, all quantities have same dimensions, *i.e.* of velocity.

31. For conduction of heat,

$$\frac{dQ}{dt} = KA\frac{dT}{dx}$$

$$K = \frac{\left(\frac{dQ}{dt}\right)}{A\left(\frac{dT}{dx}\right)} = \frac{dQ \times dx}{A \times dt \times dT}$$

$$= \frac{\text{joule} \times \text{metre}}{(\text{metre})^2 \times \text{second} \times \text{kelvin}}$$

$$= \frac{\frac{\text{kilogram} \times (\text{metre})^2}{(\text{second})^2} \times \text{metre}}{(\text{metre})^2 \times \text{second} \times \text{kelvin}}$$

$$= \frac{\text{kg} \cdot \text{m}^2 \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2 \cdot \text{s} \cdot \text{K}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^3 \cdot \text{K}}$$

$$[K] = \frac{[M^1][L^1]}{[T^3][K^1]}$$

$$\Rightarrow \quad [K] = [M^1L^1T^{-3}K^{-1}]$$

32. Dimensions of quantity f are

$$[f] = \frac{[h]^{\frac{1}{2}}\,[c]^{\frac{5}{2}}}{[G]^{\frac{1}{2}}} \quad \ldots(i)$$

As, $h = \frac{E}{\nu}$; $[h] = [ML^2T^{-2}]\,[T] = [ML^2T^{-1}]$

$$c = [LT^{-1}] \text{ and } G = \frac{F \cdot r^2}{m^2}$$

$$\Rightarrow \quad [G] = \frac{[MLT^{-2}]\,[L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$$

So, dimensions of f using Eq. (i),

$$[f] = \frac{[ML^2T^{-1}]^{\frac{1}{2}}\,[LT^{-1}]^{\frac{5}{2}}}{[M^{-1}L^3T^{-2}]^{\frac{1}{2}}}$$

$$= \left[M^{\frac{1}{2}+\frac{1}{2}}, L^{\frac{5}{2}-\frac{3}{2}+1}, T^{-\frac{1}{2}-\frac{5}{2}+\frac{2}{2}}\right] = [ML^2T^{-2}]$$

Thus, it is the dimensions of energy.

33. Force of interaction between two atoms is given as

$$F = \alpha\beta \exp(-x^2/\alpha kT)$$

As we know, exponential terms are always dimensionless, so

dimensions of $\left(\frac{-x^2}{\alpha kT}\right) = [M^0L^0T^0]$

$\Rightarrow$ Dimensions of α = Dimension of (x^2/kT)

Now, substituting the dimensions of individual term in the given equation, we get

$$= \frac{[M^0L^2T^0]}{[M^1L^2T^{-2}]}$$

{$\because$ Dimensions of kT equivalent to the dimensions of energy $= [M^1L^2T^{-2}]$}

$$= [M^{-1}L^0T^2] \quad \ldots(i)$$

Now from given equation, we have dimensions of

F = dimensions of $\alpha \times$ dimensions of β

$\Rightarrow$ Dimensions of β = Dimensions of $\left(\frac{F}{\alpha}\right)$

$$= \frac{[M^1L^1T^{-2}]}{[M^{-1}L^0T^2]} \quad [\because \text{using Eq. (i)}]$$

$$= [M^2L^1T^{-4}]$$

34. To find dimensions of capacitance in the given relation, we can use formula for energy.

Capacitor energy, $U = \frac{1}{2}CV^2$

So, dimensionally,

$$\Rightarrow \quad [C] = \left[\frac{U}{V^2}\right]$$

As, V = potential = $\frac{\text{potential energy}}{\text{charge}}$

We have, $[C] = \frac{[U]}{\left[\frac{U^2}{q^2}\right]} = \frac{[q^2]}{[U]} = \left[\frac{A^2T^2}{ML^2T^{-2}}\right]$

$$\therefore \quad X = [M^{-1}L^{-2}A^2T^4]$$

To get dimensions of magnetic field, we use force on a current carrying conductor in magnetic field,

$$F = BIl \Rightarrow [B] = \frac{[F]}{[I][l]} = \left[\frac{MLT^{-2}}{AL}\right]$$

$\therefore$ $Z = [M^1L^0T^{-2}A^{-1}]$

Now, using given relation,

$X = 5YZ^2$

$$[Y] = \frac{[X]}{[Z^2]} = \left[\frac{M^{-1}L^{-2}A^2T^4}{(M^1L^0T^{-2}A^{-1})^2}\right] = \frac{M^{-1}L^{-2}A^2T^4}{M^2T^{-4}A^{-2}}$$

$\therefore$ $[Y] = [M^{-3}L^{-2}T^8A^4]$

35. $\because$ $F = ma$

$$\Rightarrow \quad m = \frac{F}{a} = (\text{Force}) \div \frac{\text{Change in velocity}}{\text{Time}}$$

$$= \frac{\text{Force} \times \text{Time}}{\text{Change in velocity}}$$

$= (\text{strength})\ (\text{second})\ (\text{run})^{-1}$

Thus, $x = 1$, $y = 1$ and $z = -1$

$\therefore$ $\frac{y}{x} = 1$

36. 1 MSD = 1 mm

9 MSD = 10 VSD

$\therefore$ Least count, LC = 1 MSD – VSD

$$= 1\text{ mm} - \frac{9}{10}\text{ mm}$$

$$= \frac{1}{10}\text{ mm}$$

Measure reading of edge = MSR + VSR (LC)

$$= 10 + 1 \times \frac{1}{10}$$

$= 10.1$ mm

Volume of cube, $V = (1.01)^3\ \text{cm}^3 = 1.03\ \text{cm}^3$

[after rounding off upto 3 significant digits, as edge length is measured upto 3 significant digits]

$$\therefore \quad \text{Density of cube} = \frac{2.736}{1.03} = 2.6563\ \text{gcm}^{-3}$$

$= 2.66\ \text{gcm}^{-3}$

(after rounding off to 3 significant digits)

37. Given, relative error in mass, $\frac{\Delta m}{m} \times 100 = 6\%$

Relative error in diameter, $\frac{\Delta d}{d} \times 100 = 1.5\%$

$$\text{Density of sphere, } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

$$\rho = \frac{6}{\pi} Md^{-3} \text{ or } \rho \propto Md^{-3}$$

For maximum error in density,

$$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3 \times \frac{\Delta d}{d} = 6\% + 3 \times 1.5\%$$

$$\frac{\Delta\rho}{\rho} \times 100 = 10.5\%$$

$$\Rightarrow \quad \frac{1050}{100}\% = \frac{x}{100}\% \quad \text{(given)}$$

$\therefore$ $x = 1050$

38. $\because$ $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

or $\frac{1}{f} = \frac{1}{30} + \frac{1}{60}$

or $\frac{1}{f} = \frac{2+1}{60} = \frac{3}{60}$

$\therefore$ $f = 20$ cm

$\because$ $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

or $\frac{-1}{f^2}\frac{df}{dx} = -\frac{1}{v^2}\frac{dv}{dx} - \frac{1}{u^2}\frac{du}{dx}$

$\Rightarrow$ $\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$

$\Rightarrow$ $\frac{\Delta f}{400} = \frac{0.6}{3600} + \frac{0.3}{900}$

$\Rightarrow$ $\frac{\Delta f}{20} = \frac{0.6}{180} + \frac{0.3}{45} = \frac{1}{300} + \frac{1}{150}$

$\Rightarrow$ $\Delta f = \left(\frac{2+1}{300}\right)20 = \frac{60}{300} = 0.2$

Percentage error in focal length

$$= \frac{\Delta f}{f} \times 100 = \frac{0.2}{20} \times 100 = 1\%$$

$\therefore$ $n = 1$

39. Dimensions of $[c] = [LT^{-1}]$

$\Rightarrow$ $[L] = [cT]$

The dimensions of $[g] = [LT^{-2}] = [cT\ T^{-2}]$

$\therefore$ $[T] = [cg^{-1}]$

The dimensions of $[p] = [ML^{-1}T^{-2}]$

$= [M\ (cT)^{-1}\ (cg^{-1})^{-2}]$

$= [Mc^{-3}\ g^2\ T^{-1}]$

$= [M\ c^{-3}\ g^2(cg^{-1})^{-1}]$

$= [M\ c^{-4}\ g^3]$

$\therefore$ $[M] = [p\ c^4\ g^{-3}]$

The dimensions of universal gravitational constant,

$[G] = [M^{-1}L^3T^{-2}]$

$= [(p\ c^4g^{-3}0]^{-1}\ (cT)^3\ T^{-2}]$

$= [p^{-1}\ c^{-4}g^3\ c^3T]$

$= [p^{-1}\ c^{-1}\ g^3\ (c^{-1}g^{-1})]$

$= [p^{-1}\ c^0\ g^2]$

Thus,the dimensions of g in the dimensional formula of universal gravitational constant is 2.