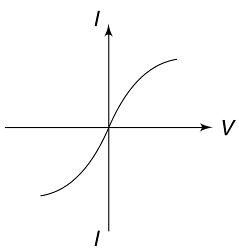
CHAPTER 8

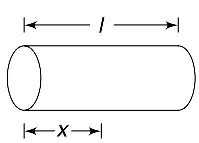
Current Electricity

LEVEL 1

Q. 1: The current – voltage graph for a resistor is as shown in the figure. Is it right to say that resistance decreases as the current through it increases?



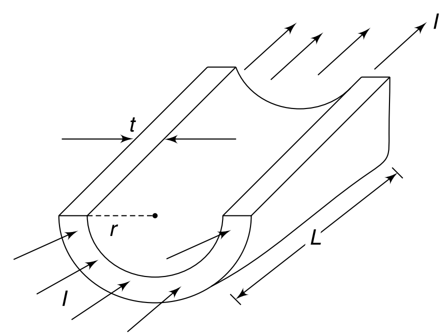
Q. 2: A cylindrical conductor has length l and area of cross section A. Its conductivity changes with distance (x) from one of its ends as $\sigma = \sigma_0 \frac{l}{x}$. [$\sigma_0$ is a constant]. Calculate electric field inside the conductor as a function of x, when a cell of emf V is connected across the ends.



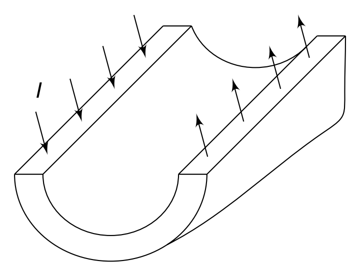
Q. 3: A conductor has density d and molar mass M. A wire made of this conductor has cross sectional radius r. Calculate the drift speed of electrons in this conductor when current through it is I. Assume that each atom contributes one free electron.

Q. 4: A conducting wire of length l and cross sectional area A is used to short the terminals of a cell having emf of ε and internal resistance r. The resistively, density and Molar mass of the material of the wire are r, d and M respectively. Calculate the average time needed for a free electron to travel from positive terminal of the cell to its negative terminal.

Q. 5: A conducting open pipe has shape of a half cylinder of length L. Its semicircular cross section has radius r and thickness of the conducting wall is $t(<<r)$. The resistance of the conductor when the current enters and leaves as shown in Figure (a) is R_1 and its resistance is R_2 when the current is as shown in Figure (b). Find $\frac{R_1}{R_2}$.



(a)



(b)

Q. 6: It was found that resistance of a cylindrical specimen of a wire does not change with small change in temperature. If its temperature coefficient of resistivity is α_R then find its thermal expansion coefficient (α).

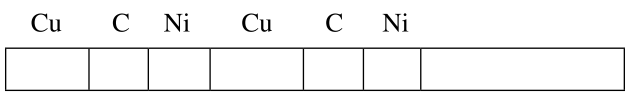
Q. 7: A cylindrical conductor is made so that its resistance is independent of temperature. It is done by stacking layers of copper, carbon and nichrome as shown in Figure. The length of each copper layer is 1 cm and sum of lengths of consecutive carbon and Nichrome Leyers is also 1 cm. Find the length of each Nichrome segment.

Given [ρ = resistivity, α = temperature coefficient of resistivity]

$\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega m^{-1}$; $\rho_C = 5 \times 10^{-5}\ \Omega m^{-1}$

$\rho_{Ni} = 1 \times 10^{-6}\ \Omega m^{-1}$; $\alpha_{Cu} = 3.9 \times 10^{-3}\,°C^{-1}$

$\alpha_C = -5 \times 10^{-4}\ °C^{-1}$; $\alpha_{Ni} = 4 \times 10^{-4}\,°C^{-1}$

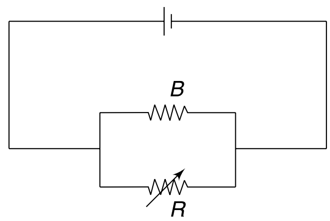


Assume no change in dimensions of the segment due to temperature change.

Q. 8: *A* bulb *B* is connected to a source having constant emf and some internal resistance. *A* variable resistance *R* is connected in parallel to the bulb. If the resistance *R* is increased, how will it affect the–

(a) Brightness of the bulb?

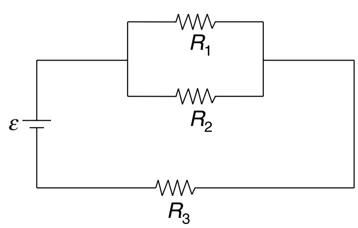
(b) Power spent by the source?



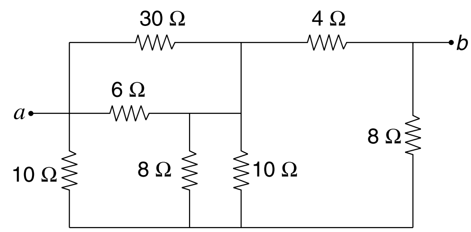
Q. 9: In the circuit shown in the Figure $R_1 = 3\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 5\ \Omega$. Emf of the cell is $\varepsilon = 6$ Volt.

(a) Infinitely many 1 Ω resistors are added in parallel to R_1 and R_2. Find current through R_2 and potential drop across it.

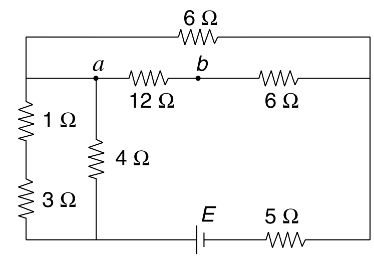
(b) Infinitely many 1 Ω resistors are added in series to R_3. Find the potential drop across R_3 and current through it.



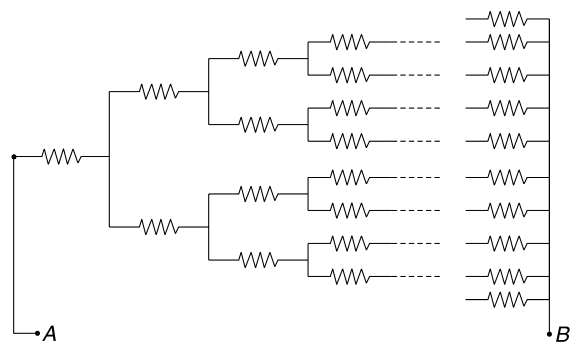
Q. 10: Find the equivalent resistance between point *a* and *b* in the network shown in figure.



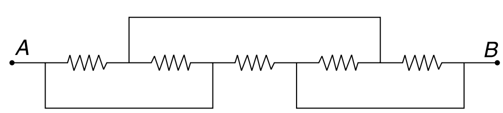
Q. 11: In the circuit shown in the figure, the potential difference between points *a* and *b* is $V_a - V_b = 4$ V. Find the emf, *E* of the battery.



Q. 12: An infinite network of resistances has been made as shown in the Figure. Each resistance is *R*. Find the equivalent resistance between *A* and *B*.



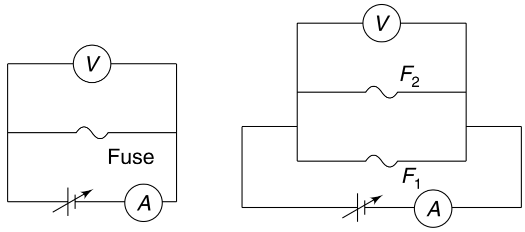
Q. 13: Find equivalent resistance between points *A* and *B* in the figure. Each resistance is *R*.



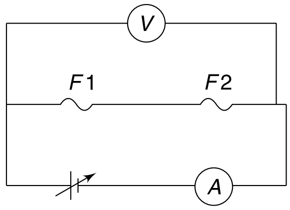
Q. 14: A fuse *F*1 is connected across a source of variable voltage and the voltage is increased gradually. The fuse blows out just when the reading of the voltmeter and ammeter reaches 1.0 V and 1.0 A respectively (see Figure (i)). The experiment is repeated with another fuse *F*2 and the reading of the voltmeter and ammeter when it blows out is 2.4 V and 1.2 A respectively.

(a) The two fuses are connected in parallel as shown in Figure (ii). Voltage is increased gradually. Find the reading of the ammeter when any one of the fuses blows out.

(b) The two fuses are connected in series as shown in Figure (iii). Find the reading of the voltmeter at the point one of the fuses blows out.

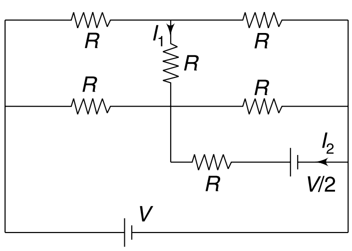


(i) (ii)

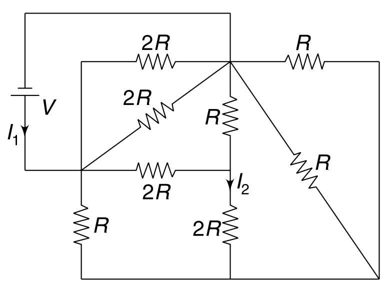


(iii)

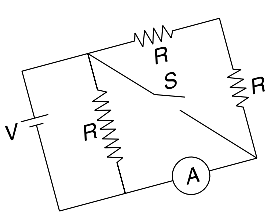
Q. 15: In the circuit shown in the Figure. Find I_1 and I_2.



Q. 16: In the network shown calculate current through the cell (I_1) and the current I_2 through the $2R$ resistance.

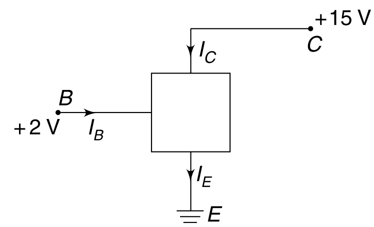


Q. 17: In the circuit shown, $R = 2\ \Omega$ and $V = 20$ volt. With switch S open the reading of ammeter is half its reading when 'S' is closed. Calculate the resistance of the ammeter.

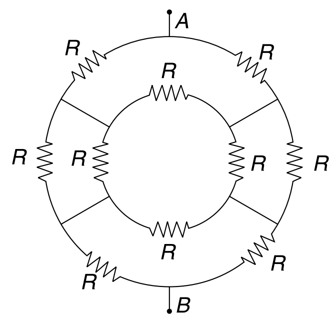


Q. 18: The box shown in the figure has a device which ensures that $I_C = 0.9\ I_E$.

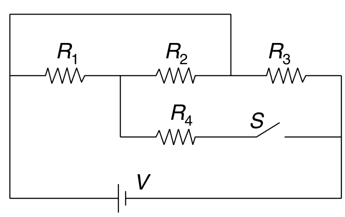
If a small change (ΔI_B) is made in I_B, calculate the corresponding change in I_C.



Q. 19: In the circuit shown in figure find the equivalent resistance across points A and B.



Q. 20: Find the percentage change in power supplied by the cell after the switch 'S' is closed in the circuit shown below. All resistances are identical

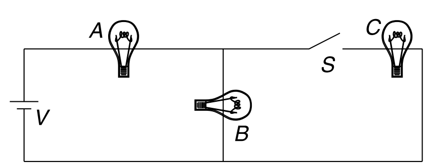


Q. 21: A and B are two identical bulbs of 40 W connected to a $V = 12$ volt cell. Switch S is closed to connect a third bulb C in the circuit. What happens to brightness of bulb A? Answer for two cases:

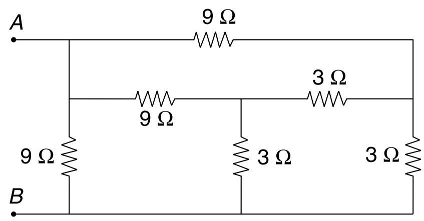
(i) Bulb C is a very high wattage bulb.

(ii) Bulb C is a very low wattage bulb.

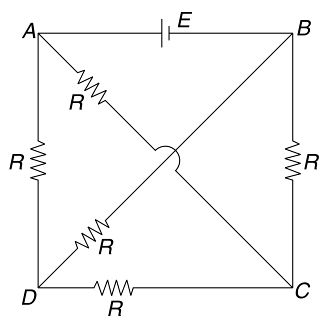
All the three bulbs have rated voltage of 12 volt.



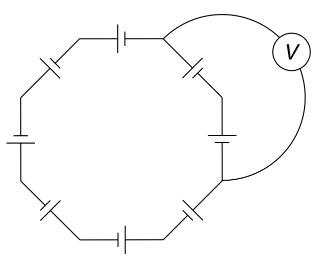
Q. 22: Find equivalent resistance between points A and B in the circuit shown.



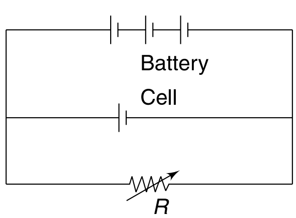
Q. 23: Find current through the cell and potential difference between A and D in the circuit shown in the Figure.



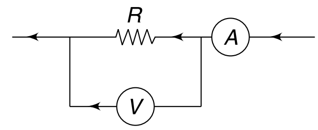
Q. 24: Eight identical 1 volt cells are connected to make a ring as shown in the Figure. An ideal voltmeter is connected as shown. What will be its reading?



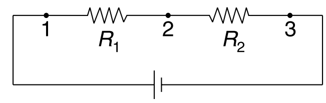
Q. 25: A battery of 120 V and internal resistance $r = 0.5\ \Omega$ is used to charge a 110 V cell in the circuit shown in the figure. Find the range of values of R for which the cell will never get charged.



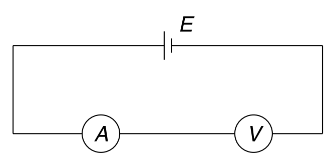
Q. 26: A voltmeter of resistance R_V and an ammeter of resistance R_A are connected as shown in an attempt to measure the resistance R. The measured value of the resistance is $R_M = \dfrac{V}{I_0}$ where V is reading of voltmeter and I_0 is reading of the ammeter. Find the true value of the resistance in terms of R_M, R_V and R_A.



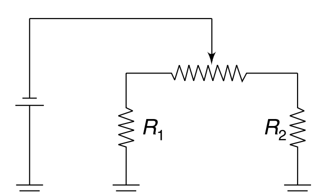
Q. 27: In the circuit shown in the Figure, cell is ideal and $R_2 = 100\ \Omega$. A voltmeter of internal resistance 200 Ω reads $V_{12} = 4$ V and $V_{23} = 6$ V between the pair of points 1-2 and 2-3 respectively. What will be the reading of the voltmeter between the points 1-3.



Q. 28: In the circuit shown, an ideal cell of emf E is connected in series to a non-ideal ammeter and voltmeter. Reading of the voltmeter is V_0. When a resistance is added in parallel to the voltmeter its reading becomes $\dfrac{V_0}{10}$ and the reading of the ammeter becomes 10 times the earlier value. Find V_0 in terms of E.



Q. 29: In the circuit shown, which way would you move the sliding contact, to the left or to the right, in order to increase current through resistance R_1? What will happen to current through R_2 as you move the contact?

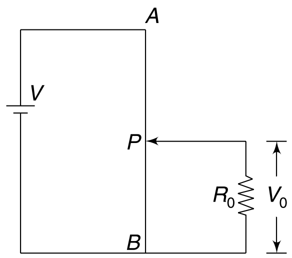


Q. 30: In the circuit shown in Figure. AB is a uniform wire of length L and resistance R. P is a sliding contact. Take the ratio of lengths PB to AB as α.

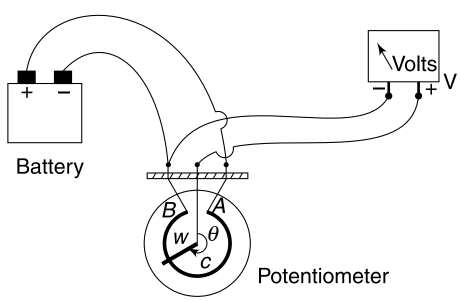
(a) Find the ratio $\dfrac{V_0}{V}$ in terms of R_0, R and α.

(b) Predict the value of $\dfrac{V_0}{V}$ when $R_0 \to \infty$. Use the result obtained in (a) to show that your prediction is correct.

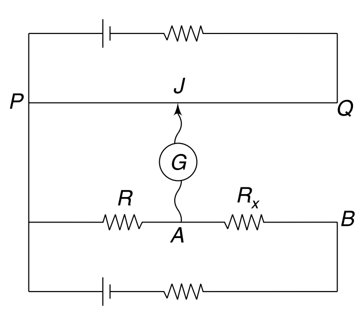
(c) Find the ratio $\frac{V_0}{V}$ when $R_0 = 2R$ and $\alpha = \frac{1}{2}$.



Q. 31: A rotary potentiometer has a circular resistance (C) and a conducting wiper (W) can rotate over it. An ideal battery and an ideal voltmeter are connected to it as shown in the figure. As the wiper is rotated on the circular resistance the reading of the voltmeter changes from zero to 20 V. What is reading of the voltmeter when wiper is at angular position θ = 120°? Assume that resistance C is almost a complete circle and resistance of all connecting wires and knobs in the circuit is zero.



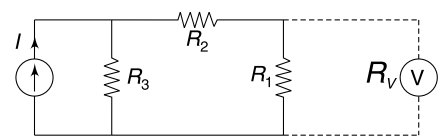
Q. 32: In the figure shown PQ is a potentiometer wire. When galvanometer is connected at A, it shows zero deflection when $PJ = x$. Now the galvanometer is connected to B and it shows zero deflection when $PJ = 3x$. Find the value of unknown resistance R_X in term of R.



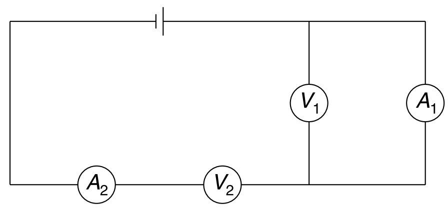
Q. 33: Three resistors R_1 = 3 kΩ, R_2 = 2 kΩ and R_3 = 5 kΩ have been connected to a constant current source as shown in figure. The current source supplies current I = 2 mA to the circuit. A voltmeter with R_V = 6 kΩ internal resistance is connected, as shown, to measure the potential difference across R_1.

(a) Find the percentage error in the measurement of potential difference (V_1) across. R_1 caused due to finite resistance of the voltmeter.

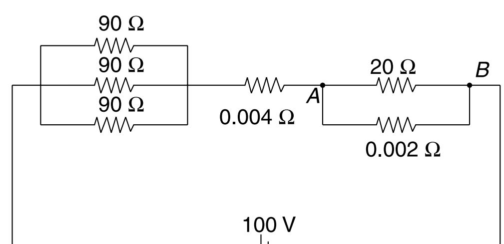
(b) If positions of R_2 and R_3 are interchanged will the percentage error in measurement of V_1 increase or decrease?

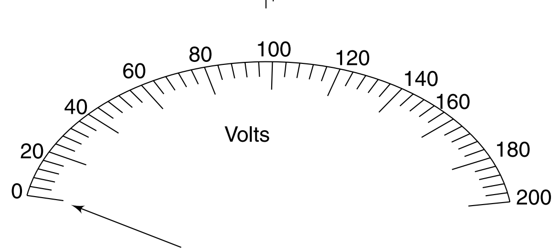


Q. 34: In the circuit shown in Figure the voltmeters V_1 & V_2 are identical. If readings of ammeter A_2, Voltmeter V_2 and V_1 are 400 μA, 100 V and 2 V respectively, find the resistance of ammeter A_1. Is the value realistic?



Q. 35: A lab voltmeter has scale as shown in Figure. A student uses this nearly ideal voltmeter to record potential difference between points A and B in the circuit shown. He could not see any deflection in the pointer and thinks that the voltmeter must be faulty. He replaces the voltmeter with another similar one. What reading will he record this time? Can you give some suggestion to record the potential difference across A and B?





Q. 36: A 10 wire potentiometer has first five wires of cross sectional radius r and next five wires of radius $2r$. The wires are uniform and made of same material. An ideal cell of emf 2V is connected across the potentiometer wire. What length of the potentiometer wire will balance the emf of–

(a) a Daniell cell (emf = 1.0 V)

(b) a Lechlanche cell (emf = 1.5 V)

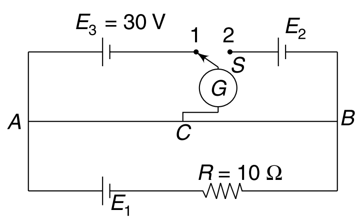
(c) a cell of emf 1.8 V?

Length of each wire is 100 cm.

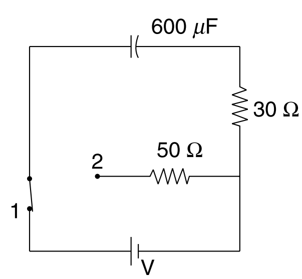
Q. 37: A water heater has a well insulated vertical cylindrical container of radius a in which water is filled to a height h. A resistor made of an allow is used to heat the water in the tank from a temperature θ_1 to θ_2 ($>\theta_1$) in a time interval Δt. The resistor wire has cross sectional radius b and its alloy material has resistivity ρ. Calculate the length of the resistor wire.

Density and specific heat capacity of water are d and s respectively. The power source connected to the resistor has emf ε.

Q. 38: In the circuit shown in Figure AB is a uniform wire of length L = 5m. It has a resistance of 2 Ω/m. When AC = 2.0 m, it was found that the galvanometer shows zero reading when switch s is placed in either of the two positions 1 or 2. Find the emf E_1.



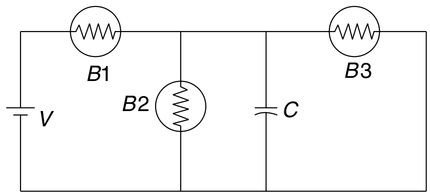
Q. 39: In the circuit shown, after the switch is shifted to position 2 the heat generated in 50 Ω resistance is 6 J. Find the emf (V) of the cell.



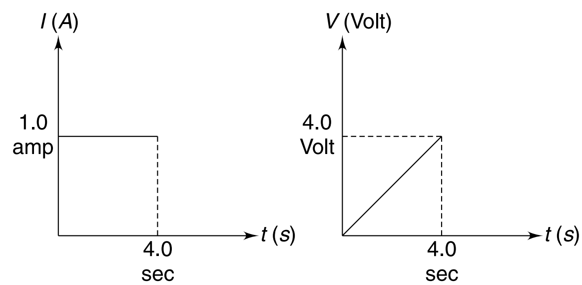
Q. 40: In the circuit shown in Figure $B1$, $B2$ and $B3$ are identical bulbs and C is a parallel plate capacitor.

(a) Which bulb is brightest?

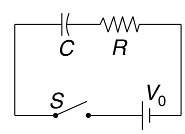
(b) If separation between the plates of the capacitor is increased very slowly to double its value, how does the brightness of each of the bulbs change?



Q. 41: When the applied potential difference across a circuit element is increased linearly with time as shown in the second figure, the current in the circuit remains constant with time as shown in the first Figure. At t = 4 s the lead wire connecting the element to the source breaks. What will be the voltage across the element after t = 4 s? Can you identify the element?



Q. 42: In the circuit shown the switch is closed at time t = 0. Plot the following graphs:



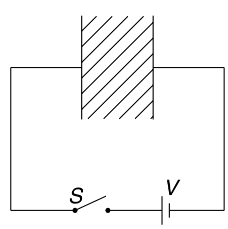
(i) current (I) in the circuit Vs time (t)

(ii) voltage across the capacitor (V_C) Vs 't'

(iii) power absorbed by the capacitor Vs 't'.

Q. 43: A capacitor having initial change q_0 and capacitance C is connected to a variable resistance R. The resistance R can have values ranging from O to R_0. How should we vary R with time (t) such that the discharging current remains constant with time?

Q. 44: A parallel plate capacitor has plate area A and separation between the plates equal to d. A material of dielectric constant K and resistivity ρ is filled between the plates. The switch is closed to connect the capacitor to a cell of emf V.



(a) Write the steady state current in the circuit and charge on the capacitor.

(b) When the circuit is in steady state, switch s is opened (at t = 0). Write charge on the capacitor as function of time (t) after this.

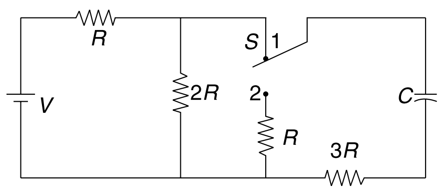
Q. 45: Consider the circuit shown in the figure. The switch has been in position 1 for a long time. Answer following questions:

(a) Find current through $2R$.

(b) Find potential difference across the capacitor.

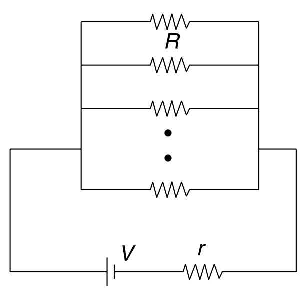
(c) At $t = 0$ the switch is moved to position 2. What current flows through the capacitor immediately after the switch is placed to position 2?

(d) Draw a graph of current *Vs* time for the current through the capacitor after the switch is moved to position 2. Indicate the time on the graph when the current becomes 37% of its value immediately after the switch is put to position 2.



Q. 46: An electric bulb has a solid cylindrical filament of length l and radius r and it consumes power P when connected to a power source. Another bulb having cylindrical filament of same material, operating at same voltage and emitting the same spectrum of light consumes 8 P power. Find length and radius of the new filament. Assume that the filaments do not radiate from the flat ends and radiation is the only source of heat loss from the filaments.

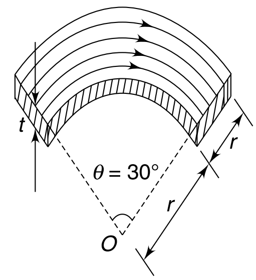
Q. 47: A battery of emf V and internal resistance r is connected to N identical bulbs, all in parallel. Resistance of each bulb is R. It is observed that maximum cumulative power is dissipated in the bulbs if $N = 10$. Can we say that $r = \dfrac{R}{10}$.



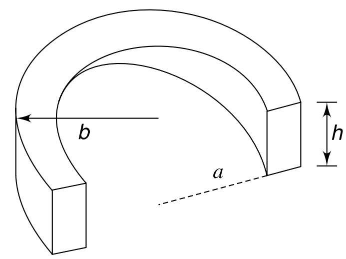
LEVEL 2

Q. 48: A copper wire of length 5 cm carries a current of density $j = 1\,\text{A}\,(\text{mm})^{-2}$. The density and molar mass of copper are 9000 Kg m^{-3} and 63 gmol^{-1}. Each copper atom contributes one free electron. The temperature of the wire is 27°C. Estimate the (average) distance travelled by a free electron during the time it moves from one end of the copper wire to the other end. Assume that thermal motion of electrons are similar to that of molecules of an ideal gas.

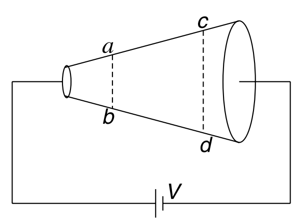
Q. 49: A conducting plate of thickness t is in the shape of an arc. Its inner and outer rim form a 30° arc of radius r and $2r$ respectively at point O (see Figure). Electric current flows through the strip along circular arcs as indicated in the diagram. Find resistance of the plate if its material has conductivity σ.



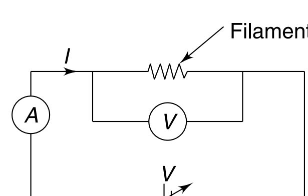
Q. 50: A conductor having resistivity ρ is bent in the shape of a half cylinder as shown in the figure. The inner and outer radii of the cylinder are a and b respectively and the height of the cylinder is h. A potential difference is applied across the two rectangular faces of the conductor. Calculate the resistance offered by the conductor.



Q. 51: A metallic wire has variable cross sectional area. Cross sectional area at cd is twice the area at ab. The wire is connected to a cell as shown. Find the ratio of heat dissipated per unit volume at section cd to that at section ab.

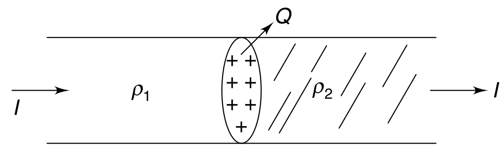


Q. 52: A tungsten filament bulb is connected to a variable voltage supply. The potential difference, V is varied and the current, I and steady temperature T of the filament is recorded. A graph is plotted for $\ln(VI)$ *Vs* $\ln(T)$. Find the slope of the graph. Assume that temperature of filament $T >> T_0$ = atmospheric temperature.



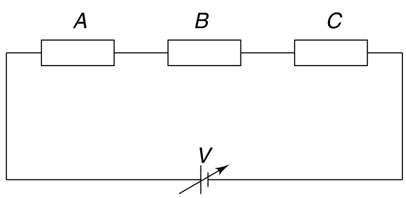
Q. 53: A tungsten filament, carrying current, has a constant diameter except a part of it which has half the diameter of the rest of the wire. If the temperature of thick part is 2000 K, then calculate the temperature of the thin part of the filament. Assume that the temperature is constant within each part and changes suddenly between the parts. Also assume that the temperature of the surrounding is very small compared to temperature of any part of the filament.

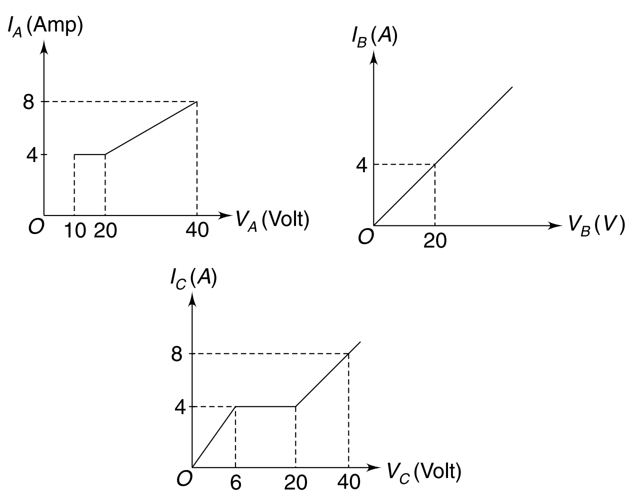
Q. 54: Two cylindrical rods, of different material, are joined as shown. The rods have same cross section (A) and their electrical resistivities are ρ_1 and ρ_2. When a current I is passed through the rods, a charge (Q) gets piled up at the junction boundary. Assuming the current density to be uniform throughout the cross section, calculate Q.



Under what condition the charge Q is negative?

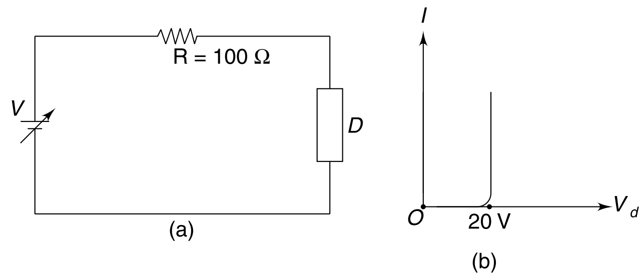
Q. 55: The current (I) – voltage (V) characteristic of three devices A, B and C connected in the circuit is as shown below.





Plot the variation of current through the cell when its emf is changed from 0 to 90 V.

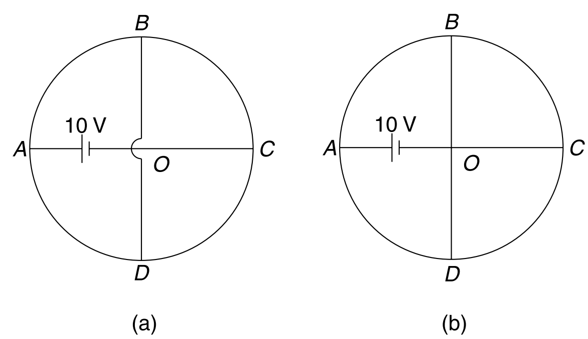
Q. 56: The current – voltage characteristic of an electric device is as shown in figure (b). The device gets damaged if power dissipated in it exceeds 1 Watt. This device is connected to a dc source of variable emf (V) and a resistance (R = 100 Ω) in series. What is possible range of V for which the device remains operational (*i.e.* consumes some power) and safe.



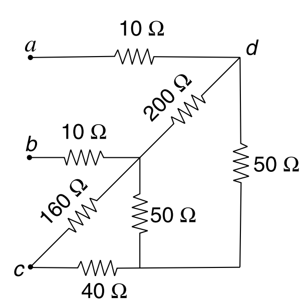
Q. 57: $ABCD$ is a uniform circular wire of resistance 16 Ω and AOC and BD are two uniform wires forming diameters at right angles, each of resistance 2 Ω. The two straight wires do not touch each other at O. A battery of emf 10 V is placed in AO as shown.

(a) Find the current through the battery (Figure a)

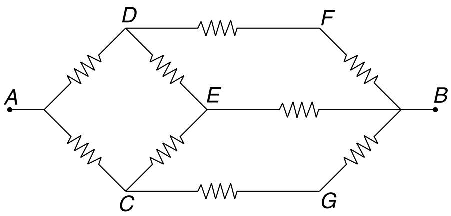
(b) If the straight wires are tied at O so as to form a junction, find the current through the battery (Figure b)



Q. 58: In the circuit shown in the fig the equivalent resistance between a and b is R_{ab} and the equivalent between a and c is $R_{ac.}$ Find the ratio $R_{ab}:R_{ac}$.



Q. 59: (a) In the circuit shown in figure, all resistances are identical when a 5V supply is connected across AB the current in branch CG is 3 mA. Find the effective resistance of the circuit between A and B.



(b) Twelve equal resistances, each equal to R, are placed along the sides of a cube. Find equivalent resistance across.

(i) Diagonally opposite points of the cube.

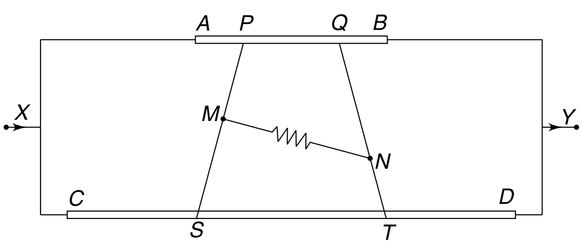
(ii) Diagonally opposite points on one face of the cube.

(iii) End points of a side of the cube.

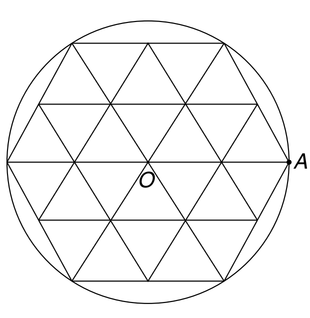
Q. 60: AB and CD are two resistance wires cut from a uniform long wire. Lengths of AB and CD are L and $2L$ respectively and resistance of AB is R. The two resistances are connected in parallel to a supply. P and Q are two points on the resistance AB such that $AP = QB = \frac{L}{3}$. Two conductors PS and QT are connected between two resistors such that no current flows through both the conductors. A resistance R is connected between points M and N as shown. Neglect resistance of PS and QT.

(a) Find the equivalent resistance of the circuit between X and Y.

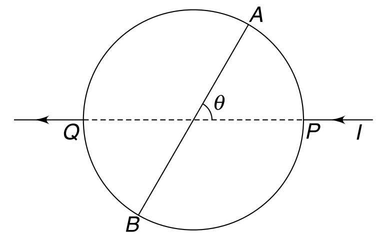
(b) Will there be any current in resistance connected across MN?



Q. 61: In the figure, each segment (side of small triangle) has resistance R and the wire used in the circumference of the circle has negligible resistance. Find equivalent resistance between point O and A.



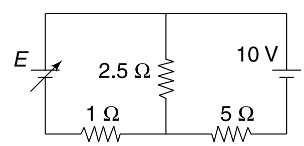
Q. 62: A uniform conducting wire is in the shape of a circle. The same wire has been used to make its diagonal AB. A current I enters at point P and leaves at the diagonally opposite point Q. AB makes an angle θ with the line PQ. Find current (i), through AB as a function of θ. Plot a graph showing variation of i with θ $(0° \leq \theta \leq 90°)$



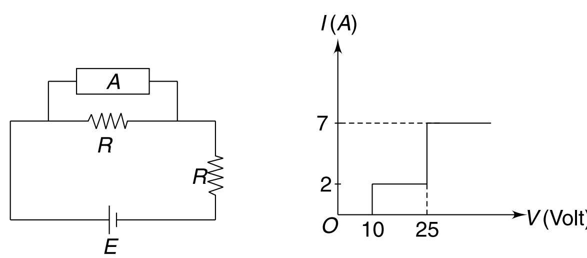
Q. 63: In the circuit shown in Figure the emf E of the battery is increased linearly from zero to 28 V in the interval $0 \leq t \leq 14$ s.

(a) Find the energy gained by the 10 V cell in the interval $0 \leq t \leq 14$ s.

(b) At what time the 10 V cell begin to charge?



Q. 64: In the circuit shown in figure (a), the emf of the ideal cell is $E = 100$ V and resistance R is 10 Ω. The current (I) – Voltage (V) characteristic of the circuit contained in box A is as shown in figure (b). Find the potential difference across the box A.



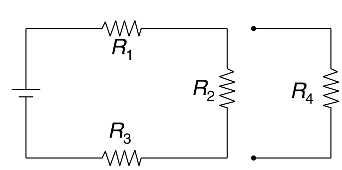
(a)

(b)

Q. 65: In the circuit shown in the Fig., $R_1 : R_2 : R_3 = 4 : 1 : 2$.

(a) Will the current through R_1 increase or decrease when a new resistance R_4 is added in parallel to R_2?

(b) Change in current through R_1 when R_4 is added is found to be $0.2A$. Calculate the current through R_4.

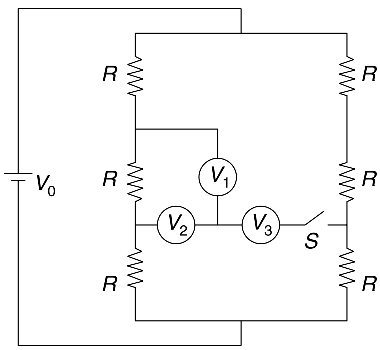


-

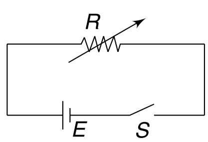
Q. 66: In the circuit shown in figure all resistances are identical (each equal to R) and the cell has an emf of V_0. The three voltmeters V_1, V_2 and V_3 are identical and are nearly ideal.

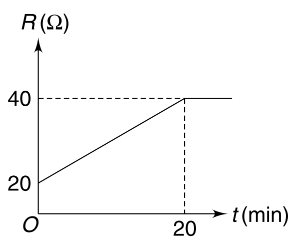
(a) Find the reading of the voltmeter V_1 when switch 'S' is open.

(b) Find the reading of the voltmeter V_1 after the switch is closed.

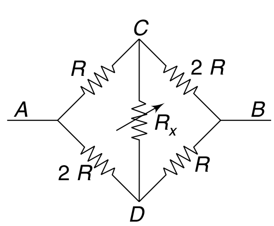


Q. 67: A chemical cell of emf E has negligible internal resistance. It is connected to a variable resistance (R) which changes linearly from 20 Ω to 40 Ω in 20 minute and thereafter becomes constant. It was found that the cell lost 10% of its total chemical energy in first 20 minute after the switch was closed. How long will the energy in the cell last?





Q. 68: In the circuit shown in the Figure R_X is a variable resistance. Find the equivalent resistance (R_{AB}) between A and B in terms of R and R_X. What are the possible range of values of R_{AB}?

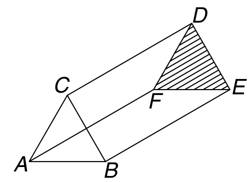


Q. 69: Six identical wires each of resistance R ohm are connected to form the edges of a tetrahedron. Find the equivalent resistance between any two vertices.

Q. 70: A prism shaped network of resistors has been shown in the figure. Each arm (like AB, AC, CD, DF...) has resistance R. Find the equivalent resistance of the network between

(a) A and B

(b) C and D.



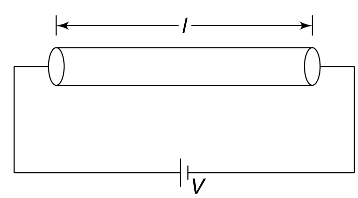
Q. 71: A cylindrical conductor has a resistance R. When the conductor is at a temperature (T) above its surrounding temperature (T_0), the ratio of thermal power dissipated by the conductor to its excess temperature ($\Delta T = T - T_0$) above surrounding is a known constant k. The conductor is connected to a cell of emf V. Initially, the conductor was at room temperature T_0. Mass and specific heat capacity of the conductor are m and s respectively.

(i) Find the time (t) dependence of the temperature (T) of the conductor after it is connected to the cell. Assume no change in resistance due to temperature.

(ii) Find the temperature of the conductor after a long time.

Q. 72: A conductor in the shape of a cylinder of length ℓ and cross sectional radius r is connected to a cell of emf V. The resistivity of the material of the conductor is ρ and does not change much with temperature. The emissivity of the curved surface of the conductor is e. [Take emissivity of the flat circular surfaces to be zero]. In steady state the temperature of the conductor is T when the environmental temperature is T_0. The difference between T and T_0 is much smaller than the environmental temperature. Stefan's Constant is σ.

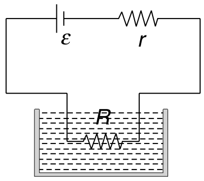
Find the steady state temperature T for the conductor.



Q. 73: In order to heat a liquid an electric heating coil is connected is to a cell of emf E = 12 V and internal resistance r = 1 Ω. There are three options for selecting the resistance (R) of the heating coil. R can be chosen as 1 Ω, 2 Ω or 4 Ω. The cell has a rating of 2000 mAh (milli Ampere hour) and it is to be used to heat the liquid till it expires. [The cell maintains constant emf till it lasts]

(a) Which value of R will you chose so that maximum heat (H_0) is transferred to the liquid before the cell expires? Calculate H_0.

(b) Which value of R will chose so that heat is transferred to the liquid at fastest possible rate? What percentage of H_0 (as obtained in (a)) is transferred to the liquid in this case by the time the cell expires?

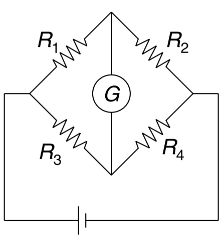


Q. 74: In a wheat stone bridge experiment to determine the unknown resistance R_4, the values of R_1 and R_2 were taken to be 10 Ω and 1 Ω respectively. It was found that the galvanometer will show exact zero deflection when value of R_3 is taken as 643 Ω < R_3 < 644 Ω.

Now R_1 is changed to 100 Ω (R_2 remains unchanged).

(a) If you have been asked to obtain a balanced bridge, which values of R_3 will you try with?

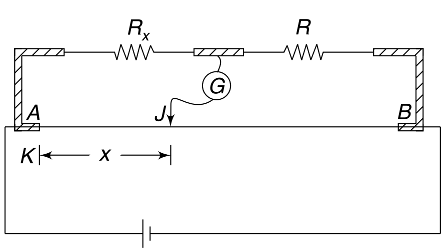
(b) If balance is obtained for 6432 Ω < R_3 < 6433 Ω write the measured value of unknown resistance R_4.



Q. 75: Figure shows an experimental set up to find the value of an unknown resistance (R_x) using a meter bridge. AB is the uniform meter bridge wire of length L = 100 cm. When the sliding jockey is placed at J ($AJ = x$), the galvanometer shows zero deflection. $AJ = x$ is known as balancing length and is measured using a scale having 1 mm as least count.

(a) In one experiment known resistance R was taken to be 20 Ω and balancing length was measured as x = (20.0 ± 0.1) cm. Find the value of R_x.

(b) Show that fractional error in calculated value of R_x is least when $x = \frac{L}{2}$. What shall we do to ensure that x is close to $L/2$?



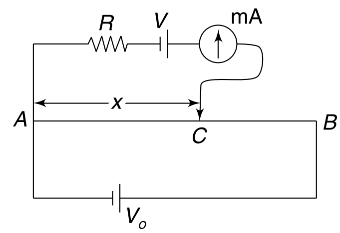
Q. 76: AB is a uniform wire of length L = 100 cm. A cell of emf V_0 = 12 volt is connected across AB. A resistance R, cell of emf V and a milliammeter (which can show deflection in both directions] is connected to the circuit as shown. Contact C can be slid on the wire AB. Distance $AC = x$. The current (I) through the milliammeter is taken positive when the cell of emf V is discharging. A graph of I Vs x has been shown.

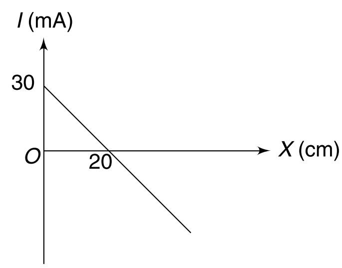
Neglect internal resistance of the cells.

(a) Find V

(b) Find R

(c) Find I when x = 100 cm

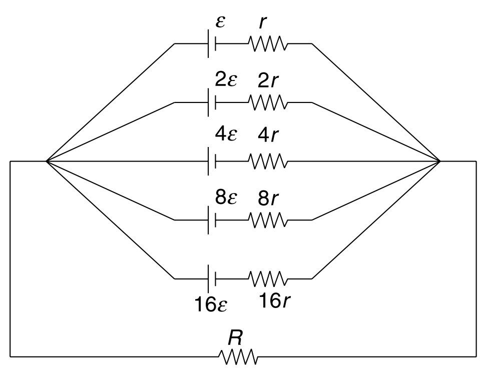




Q. 77: Five cells have been connected in parallel to form a battery. The emf and internal resistances of the cells have been shown in figure. A load resistance R is connected to the battery.

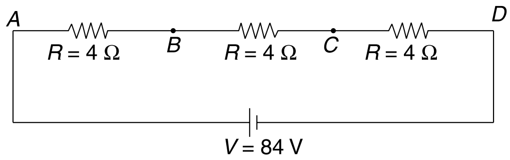
(a) Which of the 5 cells will have maximum current flowing through it?

(b) Find the current through load resistance R.

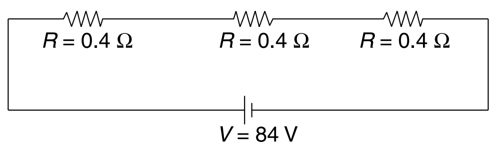


Q. 78: In the circuit shown, when a voltmeter is connected across any one of the three resistances, it shows a reading of 24 V.

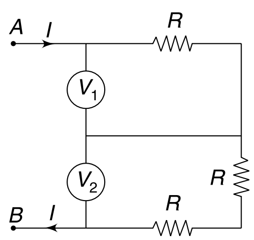
(a) Find the reading of the voltmeter when it is connected between points A and C.



(b) The same voltmeter is used to measure potential difference across resistances shown in figure below. Will the voltmeter be more accurate this time?



Q. 79: In the circuit shown in figure a current I = 600 μA enters through A and leaves through B. Reading of the identical voltmeters V_1 and V_2 are 20 V and 30 V respectively. Find R.

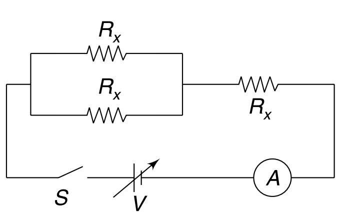


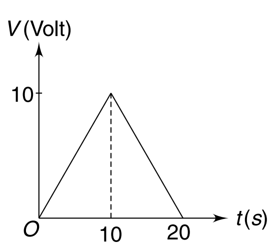
Q. 80: In the circuit shown, each resistor has a resistance R_X which depends on the voltage V_X across it.

For $V_X \leq 1$ V, $R_X = 1\ \Omega$

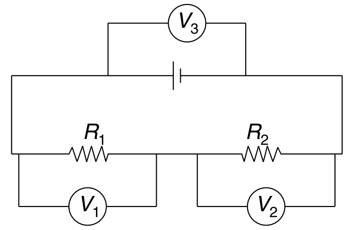
and for $V_X > 1$ V, $R_X = 2\ \Omega$.

The emf (V) of the source, changes with time (t) after the switch is closed at t = 0. The variation of V with time is depicted in the graph. Plot the variation of ammeter reading with time.

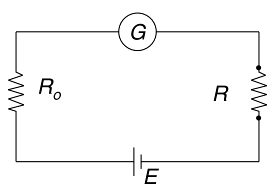


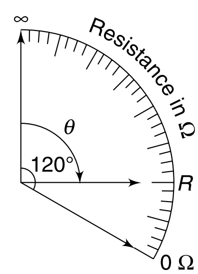


Q. 81: In the circuit shown in the figure, two resistors R_1 and R_2 have been connected in series to an ideal cell. When a voltmeter is connected across R_1 its reading is V_1 = 4.0 volt and when the same voltmeter is connected across R_2 its reading is V_2 = 6.0 volt. The reading of the voltmeter when it is connected across the cell is V_3 = 12.0 volt. Find the actual voltage across R_1 in the circuit.



Q. 82: An ohm-meter is a device that measures an unknown resistance. A simple ohm-meter can be constructed using a galvanometer as shown in the figure. The cell used in the circuit has emf E = 20 volt. The full scale deflection current and resistance of the galvanometer are 2 mA and 20 Ω respectively. R_0 is a fixed resistance and R is the unknown resistance whose value is directly given by the galvanometer scale. The galvanometer scale is shown in figure. When an unknown resistance R is placed in the circuit, the pointer deflects by $\theta = 90°$. Find R.

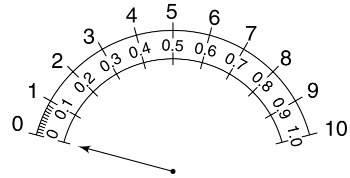


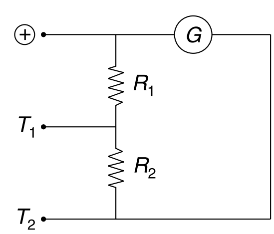


Q. 83: To enhance the sensitivity, an Ammeter is to be designed with two kinds of graduation on its scale — 0 to 10 A and 0 to 1 A. For that a galvanometer of resistance 50 Ω and full scale deflection current 1 mA was used along with two resistances R_1 and R_2 as shown. Either of T_1 or T_2 is to be used as negative terminal of the Ammeter.

(a) When measuring a current of the order of 0.1 A, which shall be used as negative terminal — T_1 or T_2?

(b) Find the values of R_1 and R_2.

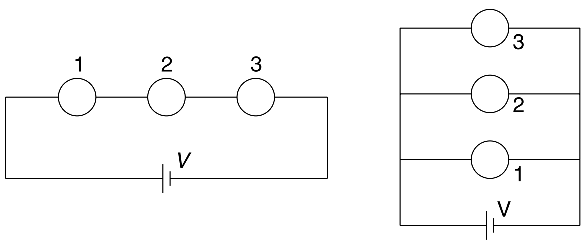




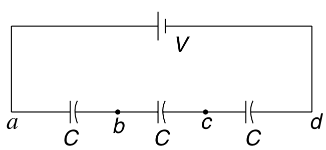
Q. 84: Three ammeters — 1, 2 and 3 have different internal resistances r_1, r_2 and r_3 respectively. Internal resistance r_1 is known but r_2 and r_3 are unknown. The angular deflection of pointer in each ammeter is proportional to the current. Initially, the three ammeters were connected in series to a voltage source (Fig. a) and deflections for the three ammeters were θ_1, θ_2 and θ_3 respectively. The three ammeters were then connected in parallel to the same voltage source (Fig. b). This time the deflections were observed to be θ_1', θ_2' and θ_3' respectively.

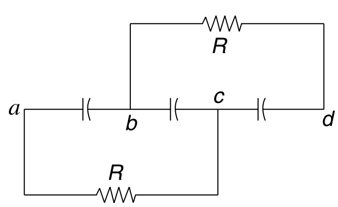
(a) Find r_2 and r_3.

(b) If $\theta_2 = \theta_3$ but $\theta_3' > \theta_2'$ then which one is larger r_2 or r_3?



Q. 85: Three identical capacitors, each of capacitance C are connected in series. The capacitors are charged by connecting a battery of emf V to the terminals (a and d) of the circuit. Now the battery is removed and two resistors of resistance R each are connected as shown. Find the heat dissipated in one of the resistors.





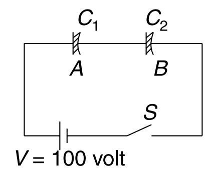
Q. 86: Assume that clouds are distributed around the entire earth at a height of 3000 m above the ground. The atmosphere can be modeled as a spherical capacitor with the earth as one plate and the cloud as other. When the electric field between the earth and the cloud becomes large, the air begins to conduct and the phenomena is called lightning. On a typical day 4×10^5 C of positive charge is spread over the surface of the earth and equal amount of negative charge is there on the cloud. Resistivity of the air is $\rho = 3 \times 10^{13}$ Ωm and radius of the earth = 6000 km.

(a) Find the resistance of the air gap between the earth's surface and cloud.

(b) Estimate the potential difference between the surface of the earth and the cloud.

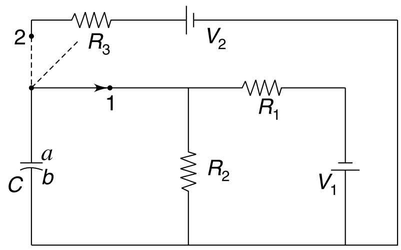
(c) In how much time the capacitor formed between the earth and the cloud will lose 63% of the charge?

Q. 87: The capacitor A shown in Fig. has a capacitance $C_1 = 3\ \mu$F. The dielectric filled in it has a breakdown voltage of 40 V and it has a resistance of 3 MΩ. The capacitor B has a capacitance of $C_2 = 2\ \mu$F and dielectric in it has a resistance of 2 MΩ. Breakdown voltage for B is 50 V. The switch is closed at $t = 0$. Will there be breakdown of any capacitor after the switch is closed? If yes, which will breakdown first and at what time?

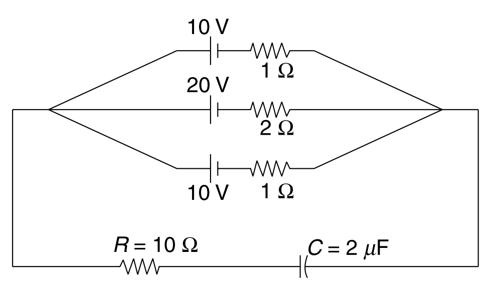


Q. 88: In the circuit shown in fig. the switch is kept closed in position 1 for a long time. At time $t = 0$ the switch is moved to position 2. Write the dependence of voltage (V_C) across the capacitor as a function of time (t). Take V_C to be positive when plate a is positive.

Given: $R_1 = 20\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 400$ kΩ, $V_1 = 40$ V, $V_2 = 90$ V and $C = 0.5\ \mu$F.

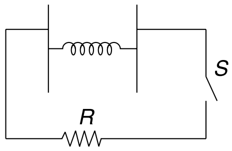


Q. 89: Find the charge on the capacitor in the circuit shown in Fig.



Q. 90: A parallel plate capacitor has its two plates connected to an ideal spring of force constant K. Relaxed length of spring is L and it is made of non conducting material. The area of each plate is A. The capacitor has a charge q_0 on it. To discharge the capacitor through the resistance R, switch S is closed.

If the time constant of the circuit is very large and discharge process is very slow, how much heat will be dissipated in the resistance? Assume that there is no friction and the plates always remains parallel to each other.

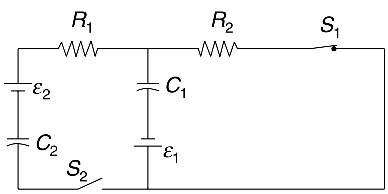


Q. 91: In the last problem calculate the amount of heat dissipated in the resistance assuming that the time constant of the circuit is very small and the discharge process is almost instantaneous.

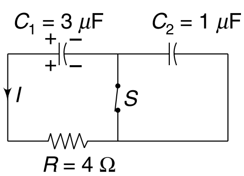
Q. 92: In the circuit shown in the figure $R_1 = R_2 = 5\ \Omega$, $C_1 = C_2 = 2\ \mu$F and $\varepsilon_1 = \varepsilon_2 = 5$ V. Switch S_1 is kept closed for a long time. Now switch S_2 is also closed. Immediately after S_1 is closed, find

(a) current through R_1

(b) current through R_2



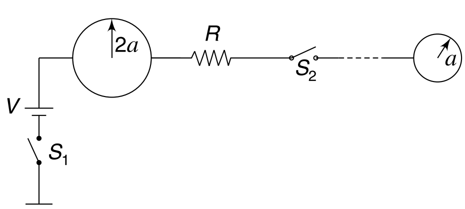
Q. 93: A charged capacitor ($C_1 = 3\ \mu$F) is getting discharged in the circuit shown. When the current I was observed to be 2.5A, switch 'S' was opened. Determine the amount of heat that will be liberated in the circuit after 'S' is opened.



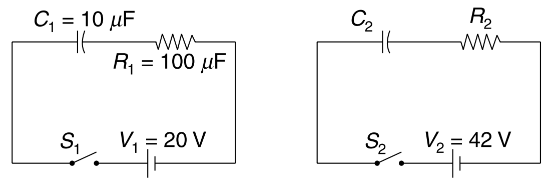
Q. 94: In the Fig. two neutral spherical conductors of radii $2a$ and a are separated by a large distance. Initially, switch S_1 is kept closed and S_2 is open. Now S_1 is opened and S_2 is closed at $t = 0$.

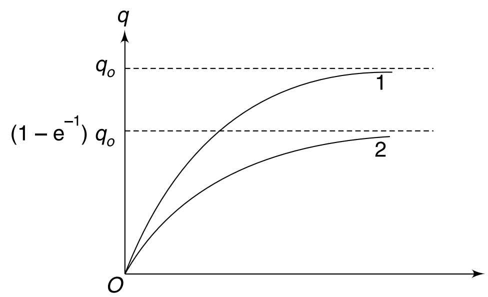
(a) Find the rate of fall in potential of the conductor of radius $2a$ as a function of time.

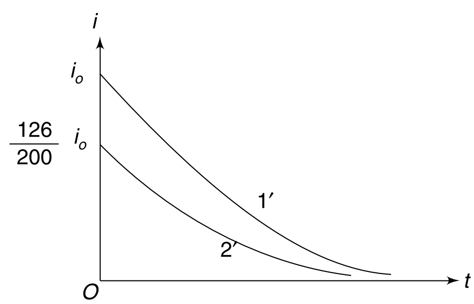
(b) Find the heat dissipated after S_2 is closed.



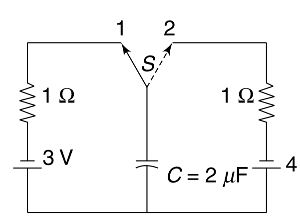
Q. 95: Consider two circuits given below. When switches S_1 and S_2 are closed at $t = 0$, the charge on capacitors C_1 and C_2 change with time as shown in graph 1 and 2 respectively. The current i_1 and i_2, in the two circuits change as shown in graph 1′ & 2′ respectively. Write the variation of current in the second circuit as function of time after the switch is closed at $t = 0$. $e^{-1} = 0.37$







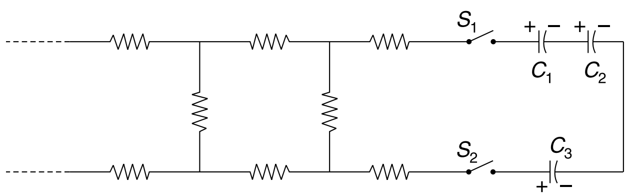
Q. 96: In the circuit shown in Fig. the capacitor is initially uncharged. Two way switch (s) is placed in position 1 for a very short interval of time (Δt) and then is moved to position 2. The switch is held in position 2 for equally short interval Δt and is then moved back to position 1. The process is repeated a large number of times until the charge on the capacitor stops changing. Find this final a value of charge on the capacitor. [Assume that in each contact the charge on the capacitor changes by a very small amount]



Q. 97: An infinite ladder network consisting of all equal resistances, $r = \dfrac{10}{2.732}\ \Omega$ is placed side by side to a capacitor system as shown in fig. Initially, all the switches are kept open and all the three capacitors are given equal charges of 30 μC each. The capacitances are $C_1 = 3\ \mu$F, $C_2 = 6\ \mu$F and $C_3 = 6\ \mu$F. Polarity of charges on the capacitor plates is shown in the Fig. Now all the three switches are closed simultaneously.

(a) Find the magnitude of rate of change of charge on the plates of the capacitors immediately after the switches are closed.

(b) Calculate heat generated in the circuit by the time steady sate condition is established.

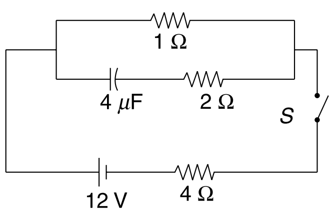


Q. 98: In the circuit shown in the figure, switch S is closed at time $t = 0$. Charge on positive plate of capacitor is q at time t.

(a) Derive a differential equation for q at time t.

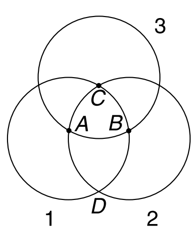
(b) Solve the equation to write q as a function of time.

(c) Put $t = 0$ and $t = \infty$ in your equation to get charge on the capacitor at these times.

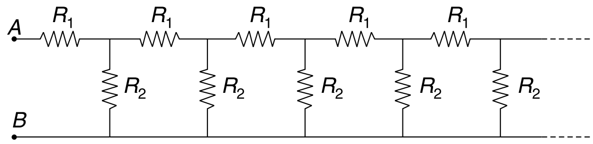


LEVEL 3

Q. 99: Three identical wire rings have been placed symmetrically as shown in the figure A, B and C are centres of the three rings. Resistance of each ring is $3R$. Find the equivalent resistance of this wire mesh across points C and D.

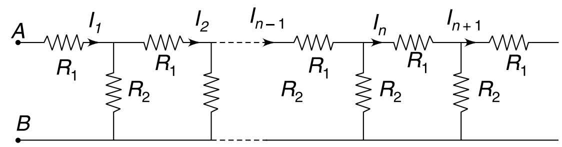


Q. 100: (a) The Fig shows a network consisting of an infinite number of pairs of resistors $R_1 = 2\ \Omega$ and $R_2 = 1\ \Omega$. Since the network is infinite, removing a pair of R_1 and R_2 from either end of the network will not make any difference. Using this calculate the equivalent (R) across points A and B.



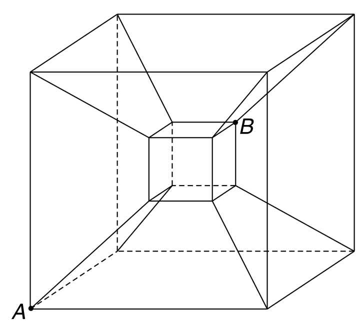
(b) Prove that $I_n = \dfrac{I_{n-1}R_2}{R_2 + R} = \dfrac{I_{n-1}}{\sqrt{3} + 2}$

Where I_n and I_{n-1} represent the current through R_1 in n^{th} and $(n - 1)^{\text{th}}$ segment respectively [see Fig]



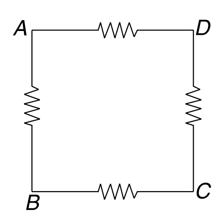
(c) If a 20 V battery is connected across A and B find I_{10}.

Q. 101: Consider the double cube resistor network shown in Fig. Each side of both cubes has resistance R and each of the wires joining the vertices of the two cubes also have same resistance R. Find the equivalent resistance between points A & B.



Q. 102: In the circuit shown in the figure, the power dissipated in the circuit is P_0 if an ideal cell is connected across A and B. Same power is dissipated in the circuit if the same cell is connected across C and D. When the cell was connected across A and D or across B and C, the power dissipated in the circuit is found to be $3P_0$.

Calculate the power dissipated in the circuit if the cell is connected across A and C.

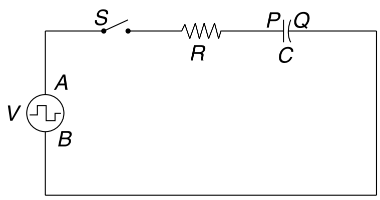


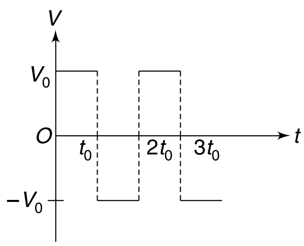
Q. 103: The voltage source shown in the Fig. is a square wave source. Its polarity changes after every $t_0 = 50\ \tau$ second [$\tau = RC$ is time constant of the $R - C$ circuit]. The magnitude of voltage across the source remains constant at V_0. When A is at higher potential compared to B the graph depicts the voltage as positive. Negative voltage means that terminal B is positive. Switch S is closed at $t = 0$.

(a) Taking charge on the capacitor to be positive when plate P is positive, plot the variation of charge on the capacitor as function of time (t).

(b) Write the magnitude of maximum current in the circuit.

(c) Plot the variation of current as function of time (t). Take clockwise current as positive.





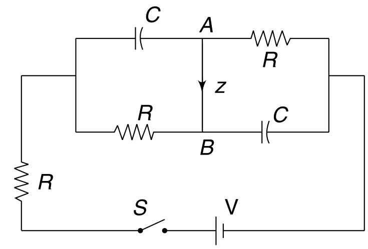
Q. 104: In the circuit shown in the fig, the switch 'S' is closed at time $t = 0$. The current in branch AB is represented by z and is taken to be positive when it is from A to B.

(a) Write the value of z immediately after the switch is closed.

(b) Write the value of z infinite time after the switch is closed.

(c) Write z as a function of time (t) and plot the variation of z with time.

(d) At what time t_0 the current z becomes zero?

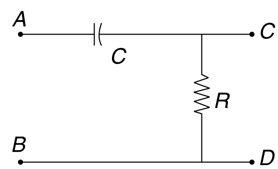


Q. 105: In the circuit shown in the figure, a voltage is applied between points A and B which changes with time as

$$V_0 = kt \text{ for } 0 \le t \le t_0$$

$$= kt_0 \text{ for } t > t_0$$

Plot the variation of potential difference (V) between C and D as a function of time.



ANSWERS

1. No.

2. $\dfrac{2Vx}{l^2}$

3. $\dfrac{IM}{\pi r^2 e \cdot d \cdot N_A}$

4. $t = \dfrac{ledN_A(\rho l + rA)}{\varepsilon M}$

5. $\dfrac{L^2}{\pi^2 r^2}$

6. $\alpha = \alpha_R$

7. 0.98 cm

8. (a) Increases (b) decreases

9. (a) zero, zero (b) zero, zero

10. 6 Ω

11. $E = \dfrac{46}{3}$ V

12. $2R$

13. $R/2$

14. (a) 1.5 A (b) 3.0 V

15. $I_1 = I_2 = 0$

16. $I_1 = \dfrac{2\text{ V}}{R}$; $I_2 = 0$

17. 4 Ω

18. $\Delta I_C = 9\Delta I_B$.

19. $\dfrac{5R}{4}$

20. Power supply increases by 67%

21. (i) A glows near its full brightness

(ii) A becomes slightly more brighter

22. 3.6 Ω

23. $\dfrac{E}{R}; \dfrac{E}{2}$

24. Zero

25. $R < 5.5\ \Omega$

26. $R = \dfrac{R_M R_V}{R_V - R_M}$

27. 12 V

28. $\dfrac{10E}{11}$

29. To left

30. (a) $\dfrac{\alpha}{(1-\alpha)\left(\dfrac{\alpha R}{R_0} + 1\right) + \alpha}$

(b) α

(c) $\dfrac{4}{9}$

31. 13.33 volt

32. $R_X = 2R$

33. (a) 25.9% (b) No change

34. 5.1 kΩ

35. Zero. He shall use a milli voltmeter.

36. (a) 3.125 m (b) 4.688 m

(c) 7.5 m

37. $\dfrac{\varepsilon^2 b^2}{\rho h d s a^2} \dfrac{\Delta t}{(\theta_2 - \theta_1)}$

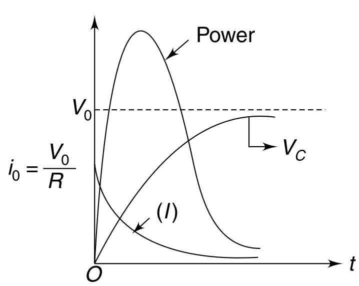
38. 150 V

39. 179 V

40. (a) $B1$ (b) No change

41. 4 V, Capacitor

42.



43. $R = R_0\left[1 - \dfrac{t}{R_0 C}\right]$

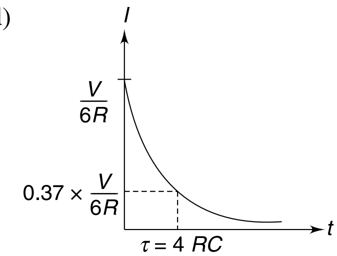
44. (a) $I = \dfrac{VA}{\rho d}; Q = \dfrac{K\in_0 AV}{d}$

(b) $q = \dfrac{K\in_0 AV}{d} e^{-\frac{t}{K\rho\in_0}}$

45. (a) $\dfrac{V}{3R}$ (b) $\dfrac{2V}{3}$

(c) $\dfrac{V}{6R}$

(d)



46. $2l$, $4r$

47. No

48. 8×10^4 km

49. $\dfrac{\pi}{6\ ln 2\ \sigma t}$

50. $R = \dfrac{\rho\pi}{h \ln(b/a)}$

51. $\dfrac{1}{4}$

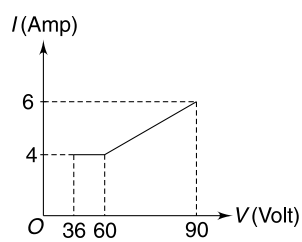
52. 4

53. $8^{1/4} \cdot 2000$ K

54. $Q = (\rho_2 - \rho_1) \in_0 I$

Q is negative if $\rho_1 > \rho_2$]

55.



56. 20 V < V < 25 V

57. (a) $\dfrac{5}{3} A$ (b) $\dfrac{35}{12} A$

58. $\dfrac{1190}{1189}$

59. (a) 500 Ω

(b) (i) $\dfrac{5R}{6}$ (ii) $\dfrac{3R}{4}$ (iii) $\dfrac{7R}{12}$

60. (a) $\dfrac{62R}{99}$ (b) Yes

61. $\dfrac{R}{4}$

62. $i = \frac{\pi - 2\theta}{\pi + 4}$

63. (a) zero (b) 7 s

64. 25 V

65. (a) Increase (b) $I_4 = 1.4A$

66. (a) $\frac{V_0}{6}$ (b) $\frac{2V_0}{9}$

67. $20 + 360\ ln2$

68. (a) $R_{AB} = \left(\frac{4R + 3R_x}{3R + 2R_x}\right)R$

(b) $\frac{4}{3}R < R_{AB} < \frac{3}{2}R$

69. $\frac{R}{2}$

70. (a) $\frac{8R}{15}$ (b) $\frac{3R}{5}$

71. (i) $T = T_0 + \frac{V^2}{kR}\left(1 - e^{-\frac{kt}{ms}}\right)$

(ii) $T = T_0 + \frac{V^2}{kR}$

72. $T = T_0 + \frac{V^2 r}{8e\sigma\rho\ell^2 T_0^3}$

73. (a) 4 Ω, 69.12 kJ (b) 1 Ω, 62.5%

74. (a) We should try with 6430 Ω < R_3 < 6440 Ω

(b) R_4 = 64.325 Ω.

75. (a) (5.00 ± 0.03) Ω

76. (a) 2.4 V (b) 80 Ω

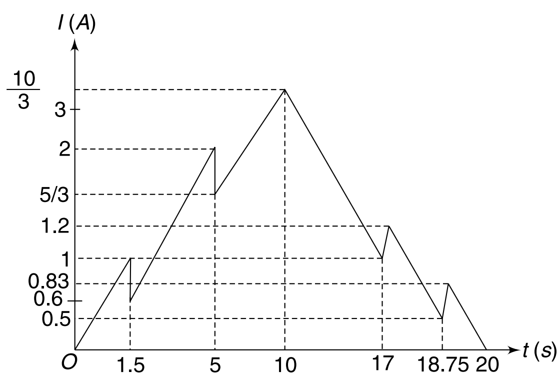
(c) – 120 mA

77. (a) cell with emf $16e$ (b) $I = \frac{80\varepsilon}{31R + 16r}$

78. (a) 48 V (b) Yes

79. 50 kΩ

80.



81. 4.8 V

82. 3333 Ω

83. (a) T_2 (b) $R_1 = \frac{5}{999}$ Ω

$R_2 = \frac{5}{111}$ Ω

84. (a) $r_2 = r_1\left(\frac{\theta_2}{\theta_1}\frac{\theta_1'}{\theta_2'}\right)$; $r_3 = r_1\left(\frac{\theta_3}{\theta_1}\cdot\frac{\theta_1'}{\theta_3'}\right)$

(b) $r_2 > r_3$

85. $\frac{2}{27}CV^2$

86. (a) 199 Ω (b) 3×10^5 V

(c) 265.3 s

87. Capacitor B, $6\ell n 6$ sec

88. $V_C = (90 - 120\ e^{-5t})$ volt

89. 24 μC

90. $\frac{q_0^2 L}{2\epsilon_0 A} - \frac{q_0^4}{8\epsilon_0^2 A^2 k}$

91. $\frac{q_0^2 L}{2\epsilon_0 A} - \frac{q_0^4}{4\epsilon_0^2 A^2 k}$

92. (a) 1A (b) zero

93. 121.87 μJ

94. (a) $\frac{V}{(8\pi\epsilon_0 a)R}\ e^{\frac{-3t}{8\pi\epsilon_0 aR}}$

(b) $\frac{4\pi\epsilon_0 aV^2}{3}$

95. $i = 0.126\ e^{-1000t}$ [t = in second]

96. 7 μF

97. (a) 1.0 C/s (b) 75 μJ

98. (a) $14\frac{dq}{dt} + \frac{5}{4}q = 12$ (b) $q = \frac{48}{5}\left[1 - e^{-\frac{5t}{56}}\right]$

(c) 0; $\frac{48}{5}$ μC.

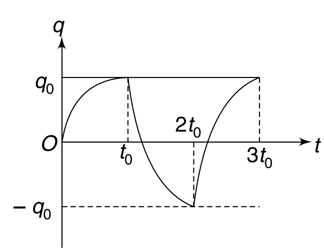
99. $\frac{63R}{164}$

100. (a) $R = (1 + \sqrt{3})\Omega$ (c) $\frac{20}{(\sqrt{3}+1)(\sqrt{3}+2)^9}$

101. $\frac{2R}{3}$

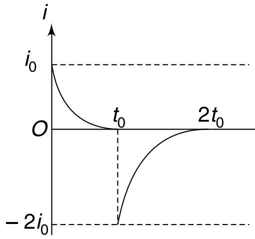
102. $\left(\frac{15 + 6\sqrt{7}}{16 + 6\sqrt{7}}\right)P_0$

103. (a)



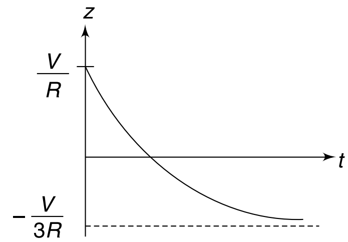
$q_0 = CV_0$

(b) $i_{max} = \dfrac{2V_0}{R}$

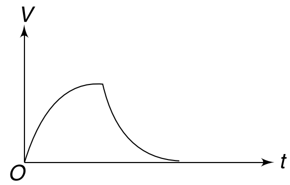
(c) 

104. (a) $\dfrac{V}{R}$ (b) $-\dfrac{V}{3R}$

(c) $Z = \dfrac{4V}{3R}\left[e^{-\frac{3t}{RC}} - \dfrac{1}{4}\right]$

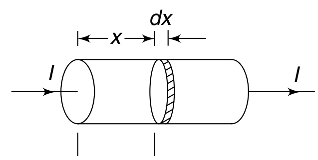


(d) $t_0 = \dfrac{2RC}{3}\,\ell n2$

105. 

SOLUTIONS

2. We will first calculate the resistance of the cylindrical conductor. For this let us consider the cylinder to be made of numerous thin discs. Resistance of one such disc (shown in Figure) will be



$$dR = \frac{dx}{\sigma A} = \frac{xdx}{A\sigma_0 l}$$

Resistances of all such discs are in series for current flowing along the axis of the cylinder.

$$\therefore \qquad R = \int dR = \frac{1}{Al\sigma_0}\int_0^l xdx = \frac{l}{2A\sigma_0}$$

Current through the Conductor is

$$I = \frac{V}{R} = \frac{2\ VA\sigma_0}{l}$$

Current density $\qquad j = \dfrac{I}{A} = \dfrac{2V\sigma_0}{l}$

Using ohm's law in microscopic form

$$E = \frac{j}{\sigma} = \frac{2V\sigma_0}{l}\frac{x}{\sigma_0 l}$$

$$E = \frac{2\ \text{V}}{l^2}x$$

3. Number of atoms in unit volume of the conductor is

$$= \frac{d}{M}N_A \qquad [N_A = \text{Avogadro's number}]$$

$\therefore$ No. of free electrons in unit volume

$$n = \frac{dN_A}{M}$$

Current density $\qquad j = \dfrac{I}{\pi r^2}$

$$\therefore \qquad nev_d = \frac{I}{\pi r^2}$$

$$v_d = \frac{I}{\pi r^2 e \cdot n} = \frac{IM}{\pi r^2 e \cdot d \cdot N_A}$$

4. Resistence of the wire $R = \frac{\rho \ell}{A}$

Current in the wire $I = \frac{\varepsilon}{R + r} = \frac{\varepsilon}{\frac{\rho \ell}{A} + r}$

Current density $j = \frac{I}{A} = \frac{\varepsilon}{\rho \ell + r \cdot A}$

$$neV_d = \frac{\varepsilon}{\rho \ell + r \cdot A}$$

$$V_d = \frac{\varepsilon}{ne(\rho \ell + r \cdot A)}$$

But n = no. of atoms in unit volume of the Conductor

$$= \frac{d}{M} N_A$$

$\therefore$ $V_d = \frac{\varepsilon M}{edN_A\ (\rho \ell + rA)}$

$\therefore$ Time to travel a distance ℓ for a free electron is

$$t = \frac{\ell}{V_d} = \frac{ledN_A(\rho \ell + r \cdot A)}{\varepsilon_M}$$

5. $R_1 = \frac{\rho L}{\pi r \cdot t}$

$$R_2 = \frac{\rho \pi r}{tL}$$

$\therefore$ $\frac{R_1}{R_2} = \frac{L^2}{\pi^2 r^2}$

6. A = Area of cross section of wire
L = Length of the wire
ρ = resistivity.

$$R = \frac{\rho L}{A}$$

If temperature changes by $\Delta\theta$, the resistance becomes.

$$R' = \frac{\rho(1 + \alpha_R \Delta\theta)\ L(1 + \alpha \Delta\theta)}{A(1 + 2\alpha \Delta\theta)}$$

$$R' = R\frac{(1 + \alpha_R \Delta\theta)(1 + \alpha \Delta\theta)}{(1 + 2\alpha \Delta\theta)}$$

$$R' = R\ \frac{1 + (\alpha_R + \alpha)\Delta\theta + \alpha_R \cdot \alpha \cdot \Delta\theta^2}{(1 + 2\alpha \Delta\theta)}$$

$$= R\ \frac{1 + (\alpha_R + \alpha)\Delta\theta}{1 + 2\alpha \Delta\theta}$$

If $\quad R' = R$ then

$\alpha = \alpha_R.$

7. Let R_0 represent the resistance at some initial temperature and R_T be resistance when temperature is changed by T.

$$R_T = R_{0\text{Cu}}(1 + \alpha_{\text{Cu}}T) + R_{0\text{C}}(1 + \alpha_{\text{C}}T) + R_{0\text{Ni}}(1 + \alpha_{\text{Ni}}T)$$

$$\Rightarrow \quad R_{0\text{Cu}} + R_{0\text{C}} + R_{0\text{Ni}} = R_{0\text{Cu}}(1 + \alpha_{\text{Cu}}T) + R_{0\text{C}}(1 + \alpha_{\text{C}}T) + R_{0\text{Ni}}(1 + \alpha_{\text{Ni}}T)$$

$$\Rightarrow \quad R_{0\text{Cu}}\alpha_{\text{Cu}} + R_{0\text{C}}\alpha_{\text{C}} + R_{0\text{Ni}}\alpha_{\text{Ni}} = 0$$

$$\rho_{\text{Cu}} l_{\text{Cu}} \alpha_{\text{Cu}} + \rho_{\text{C}} l_{\text{C}}\ \alpha_{\text{C}} + \rho_{\text{Ni}}\ l_{\text{Ni}}\ \alpha_{\text{Ni}} = 0$$

If $\quad l_{\text{Ni}} = x$ cm then $l_{\text{C}} = (1 - x)$ cm

$l_{\text{Cu}} = 1$ cm

$$\therefore \quad 1.7 \times 10^{-8} \times 1 \times 3.9 \times 10^{-3} + 5 \times 10^{-5} \times (1 - x)\,(-5 \times 10^{-4}) + 1 \times 10^{-6} \times x \times 4 \times 10^{-4} = 0$$

$$6.63 \times 10^{-11} - 2.5 \times 10^{-8} = \left(-4 \times 10^{-10} - 2.5 \times 10^{-8}\right)x$$

$$x = \frac{249.33}{254} = 0.98 \text{ cm}$$

8. Let resistance of the bulb be R_0.

The equivalent of two parallel resistances is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_0} + \frac{1}{R}$$

If R is increased, R_{eq} must also increase. This increases the overall resistance of the circuit and the current through the cell drops. It means power spent by the cell decrease. Potential drop across the internal resistance of the cell drops and the potential difference across the bulb increases. Hence, the bulb becomes brighter.

9. (a) With addition of each 1 Ω resistor, the equivalent resistance of the resistors in parallel goes on decreasing. If infinite resistors are added the equivalent will become Zero.

The equivalent resistance of entire circuit is $= R_3 = 5\ \Omega$.

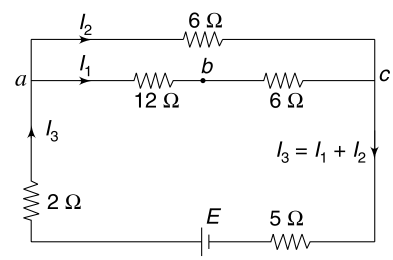
$$\therefore \quad I = \frac{6}{5} = 1.2A$$

Current through all the resistors in parallel is nearly zero since the current of $1.2A$ gets divided into infinitely many parts.

(b) In this case the resistance R_3 along with infinite many $1\,\Omega$ resistors has an equivalent $\to \infty$.

∴ Current in the circuit $\to 0$.

∴ Potential drop across each individual resistance is nearly zero.



11.

$$I_1 = \frac{4 \text{ volt}}{12\,\Omega} = \frac{1}{3}\,A$$

$$\therefore \quad V_b - V_c = 6 \times \frac{1}{3} = 2 \text{ V}$$

$$\therefore \quad V_a - V_c = 4 + 2 = 6 \text{ V}$$

$$I_2 = \frac{6\,\text{V}}{6\,\Omega} = 1A.$$

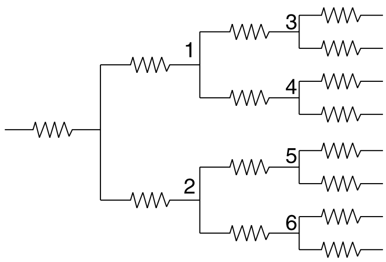
$$I_3 = I_1 + I_2 = \frac{1}{3} + 1 = \frac{4}{3}\,A$$

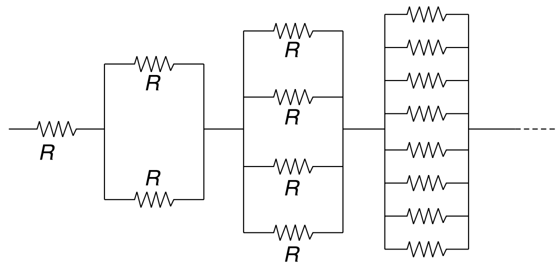
$$V_c - 5 \times \frac{4}{3} + E - 2 \times \frac{4}{3} = V_a$$

$$\Rightarrow \qquad -\frac{20}{3} + E - \frac{8}{3} = V_a - V_c$$

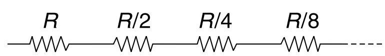
$$\Rightarrow \qquad E = 6 + \frac{20}{3} + \frac{8}{3} = \frac{46}{3} \text{ volt}$$

12. Points 1 and 2 are equipotential. They can be connected together. Points 3, 4, 5 and 6 are equipotential. Tie them together. And so on.



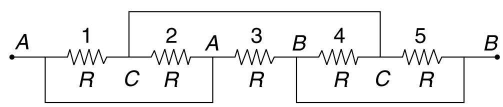


$\Rightarrow$

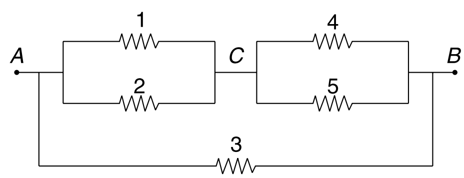


$\therefore$ Equivalent is
$$R_0 = R\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots \infty\right]$$

$$= R\left[\frac{1}{1 - \frac{1}{2}}\right] = 2R.$$



13. The circuit can be redrawn as shown in the figure.



1 and 2 are in parallel; 4 and 5 are in parallel. Equivalent of each pair is $\frac{R}{2}$. They add to become R which is in parallel to 3.

$$\therefore \qquad R_{eq} = \frac{R}{2}$$

14. Maximum current that the two fuses can tolerate are $I_{1\max} = 1.0\,\text{A}$ and $I_{2\max} = 1.2\,\text{A}$ respectively. Resistances of the two fuses are

$$R_1 = \frac{1.0\ \text{V}}{1.0\,\text{A}} = 1\ \Omega$$

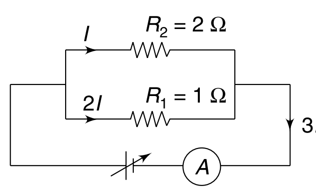
And
$$R_2 = \frac{2.4\ \text{V}}{1.2\,\text{A}} = 2\ \Omega.$$

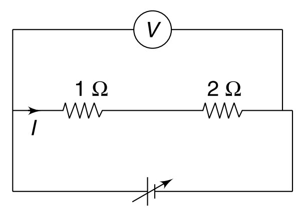
(a) Clearly, F1 blows out when $2I = 1.0\text{A}$

$\therefore$ reading of (A) is 1.5A

(b) F1 blows out when $I = 1.0\text{A}$

$$\therefore \qquad V = 3 \times 1 = 3.0\text{V}$$



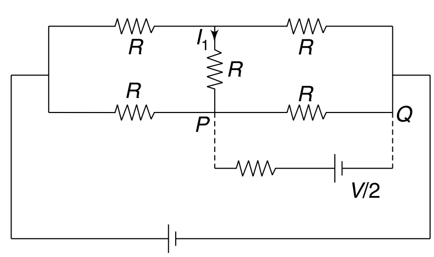


15. The circuit shown is a balanced wheastone bridge with $V_P - V_Q = \frac{V}{2}$.

When a cell having potential difference $\frac{V}{2}$ is added is parallel across P and Q, it will cause so difference to the circuit.

$$\therefore \qquad I_1 = 0$$

$$I_2 = 0.$$



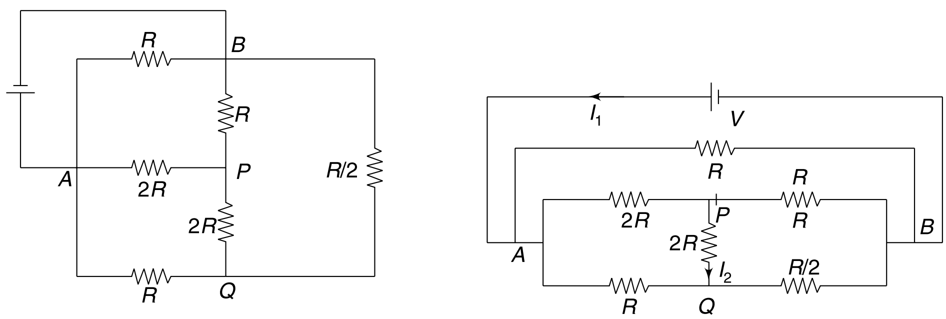
16. The circuit can be redrawn as shown.

Now you can see a wheatstone bridge (a balanced one) in parallel to R.

$\therefore \quad I_2 = 0$

Equivalent resistance is $\dfrac{R}{2}$

$\therefore \quad I_1 = \dfrac{2\ V}{R}$



17. When 'S' is open, Current through ammeter is

$$i_1 = \frac{20}{2+2+r} = \frac{20}{4+r}$$

When 'S' is closed,

$$i_2 = \frac{20}{r}$$

Given $\quad i_2 = 2i_1$

$$\frac{20}{r} = 2 \times \frac{20}{4+r}$$

$$4 + r = 2r$$

$\Rightarrow \quad r = 4\Omega$

18. From Kirchhoff's first law

$$I_E = I_C + I_B$$

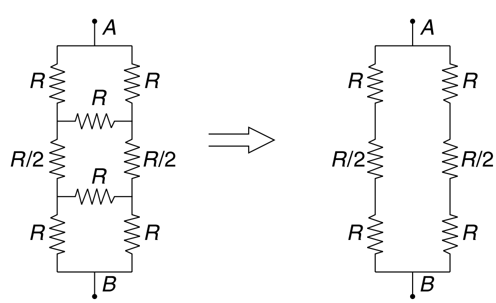
Put $\quad I_E = \dfrac{I_C}{0.9}$

$$\frac{I_C}{0.9} - I_C = I_B$$

$$I_C = 9 I_B$$

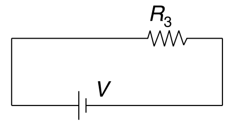
$\therefore \quad \Delta I_C = 9\,\Delta I_B$

19. The equivalent circuit is as shown in the figure. Equivalent resistance is $\dfrac{5R}{4}$.



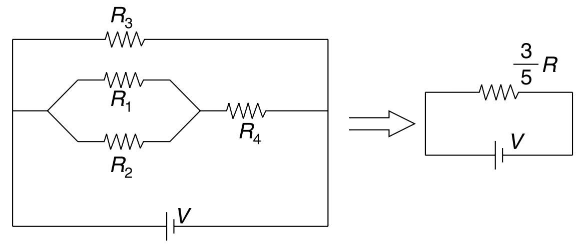
20. **Switch open**

$$P_{\text{cell}} = \frac{V^2}{R}$$



Switch closed

$$P'_{\text{cell}} = \frac{5V^2}{3R} = 1.67\ P_{\text{cell}}$$



21. B and C are in parallel.

Therefore equivalent resistance of B and C will be smaller than that of B.

Hence, the overall resistance decreases.

Current through A will increase and brightness of A will increase.

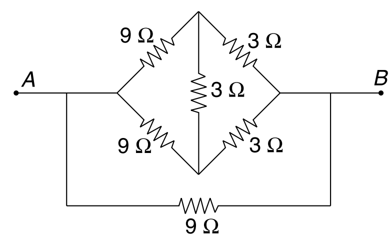
When C has high power rating, its resistance is very small. The equivalent of B and C is even smaller. Bulb A will nearly glow at its full brightness.

If C has low power rating, its resistance is too high.

Thus equivalent resistance of B and C will be only slightly less than that of B.

The brightness of A will increase slightly.

22. The given circuit is a balanced wheatstone bridge with a 9 Ω resistance in parallel.

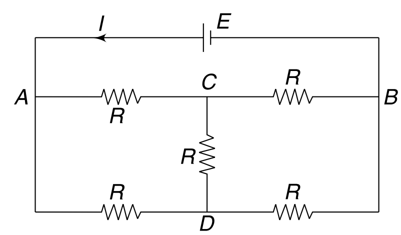


$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{12} + \frac{1}{9} = \frac{3+3+4}{36}$$

$\therefore$ $$R_{eq} = \frac{36}{10} = 3.6\ \Omega$$

23. Circuit is a balanced wheatstone bridge

Resistance between C and D can be removed. Equivalent resistance between A and B is R.



$\therefore$ $$I = \frac{E}{R}$$

$$V_{AD} = \frac{E}{2}$$

24. Voltmeter has infinite resistance. Current in the loop is $I = \dfrac{12 \text{ volt}}{12r}$ where r is internal resistance of each cell.

Potential difference across the terminals of each cell is = 1 V – Ir = 0

Hence voltmeter will show zero, irrespective of whether it is connected across one or more cells.

25. No current will flow through the cell if potential difference across the terminals of the battery is exactly 110 V.

$\Rightarrow$ $$120 - Ir = 110$$

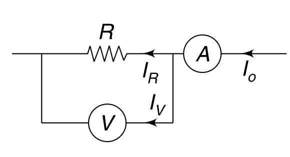
$\Rightarrow$ $$I = \frac{10}{0.5} = 20 \text{ A}$$

$\therefore$ $$R = \frac{110 \text{ V}}{20 \text{ A}} = 5.5\ \Omega$$

If $R > 5.5\ \Omega$ then the potential difference across the cell will be larger than 110 V and it will get charged.

If $R < 5.5\ \Omega$ than the potential difference across the cell will be smaller than 110 V and it will get discharged.

26. $$R_M = \frac{V}{I_0}$$



$\Rightarrow$ $$\frac{1}{R_M} = \frac{I_0}{V} = \frac{I_R + I_V}{V} = \frac{I_R}{V} + \frac{I_V}{V}$$

$$\Rightarrow \quad \frac{1}{R_M} = \frac{1}{R} + \frac{1}{R_V}$$

$$\Rightarrow \quad \frac{1}{R} = \frac{1}{R_M} - \frac{1}{R_V}$$

$$\Rightarrow \quad R = \frac{R_M \cdot R_V}{R_V - R_M}$$

27. Let emf of the cell be E.

Current through the voltmeter (when connected between 1-2) is $\frac{V_{12}}{R_V} = \frac{4}{200} = \frac{1}{50}$ A

Current through $R_1 = \frac{4}{R_1}$

$\therefore$ Current through $R_2 = \frac{4}{R_1} + \frac{1}{50}$

$\therefore$ Potential difference across $R_2 = \left(\frac{4}{R_1} + \frac{1}{50}\right) \times 100 = \frac{400}{R_1} + 2$

$$\therefore \quad E = 4 + \left(\frac{400}{R_1} + 2\right) = 6 + \frac{400}{R_1} \quad ...(1)$$

When the voltmeter is connected between 2-3

Current through voltmeter $= \frac{6}{200}$ A

Current through $R_2 = \frac{6}{100}$ A

$\therefore$ Current through $R_1 = \frac{6}{200} + \frac{6}{100} = \frac{18}{200} = \frac{9}{100}$ A

$$\therefore \quad E = \frac{9R_1}{100} + 6 \quad ...(2)$$

From (1) and (2)

$$\frac{9R_1}{100} + 6 = 6 + \frac{400}{R_1}$$

$$\Rightarrow \quad R_1 = \sqrt{\frac{40000}{9}} = \frac{200}{3}\ \Omega$$

Put this in (2)

$$E = \frac{9}{100} \times \frac{200}{3} + 6 = 12 \text{ V}$$

When connected across 1-3, the voltmeter will read $E = 12$ V.

28. Let current be I_0 in the original circuit. Potential difference across ammeter is $= E - V_0$

$\therefore$ Resistance of ammeter $\quad R_A = \frac{E - V_0}{I_0} \quad ...(1)$

After the resistance is added to the voltmeter, the potential difference across voltmeter becomes $\frac{V_0}{10}$.

$\therefore$ Potential difference across ammeter $= E - \frac{V_0}{10}$

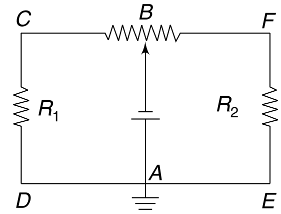
$$\therefore \quad R_A = \frac{E - V_0/10}{10 I_0} \quad ...(2) \text{ [the new current is } 10I_0]$$

From (1) and (2)

$$\frac{10E - V_0}{100I_0} = \frac{E - V_0}{I_0}$$

$\Rightarrow$ $$V_0 = \frac{90E}{99} = \frac{10E}{11}$$

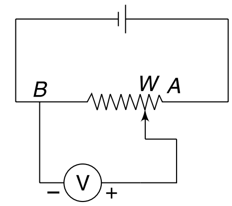
29. The effective circuit is as shown



If contact at B is moved to left, the resistance of loop $ABCD$ decreases and the current in the loop increases.

Resistance of the other loop increases and hence current through R_2 increases.

31. The equivalent circuit is as shown in figure.



When the contact is at A, reading of voltmeter = emf of cell = 20 V.

When the contact is at B, reading of voltmeter = zero.

When $\theta = 120°$; $$R_{AW} = \frac{R}{3}$$

Where R = resistance of C

$$R_{BW} = \frac{2R}{3}$$

$\therefore$ $$\text{Reading} = \frac{2}{3} \times 20 = \frac{40}{3} \text{ volt}$$

32. Let the potential gradient along the potentiometer wire PQ be K.

When galvanometer shows zero deflection the current in two loops are independent of each other. If current in loop having R and R_X is i, then

$$iR = Kx \quad ...(1)$$

and $$i(R + R_x) = K(3x) \quad ...(2)$$

(2) − (1)

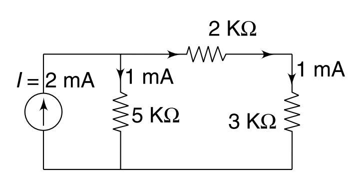
$$iR_X = 2Kx \quad ...(3)$$

(3) ÷ (1)

$$\frac{R_X}{R} = 2$$

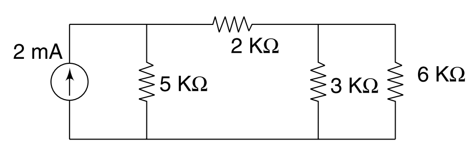
$\Rightarrow$ $$R_X = 2R.$$

33. **Without Voltmeter**



$$V = (3 \text{ k}\Omega)(1 \text{ mA}) = 3 \text{ Volt.}$$

With Voltmeter

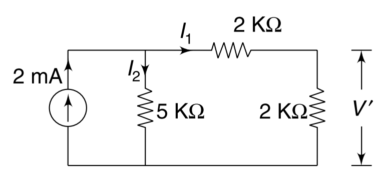


$$4i_1 = 5i_2$$

And $$i_1 + i_2 = 2 \text{ mA}$$

$\therefore$ $$i_1 = \frac{10}{9} \text{ mA}$$

$\therefore$ $$V' = 2 \times \frac{10}{9} = \frac{20}{9} \text{ Volt.}$$



$$\% \text{ error} = \left|\frac{V - V'}{V}\right| \times 100$$

$$= \frac{3 - \frac{20}{9}}{3} \times 100$$

$$= \frac{7}{27} \times 100 = 25.9\%.$$

34. Reading of ammeter A_2 = current through V_2

$$\therefore \quad R_{V2} = \frac{V_2}{I_2} = \frac{100}{400 \times 10^{-6}} = 0.25 \text{ M}\Omega$$

$p \cdot d$ across A_1 = reading of V_1 = 2 V

Current through V_1 is

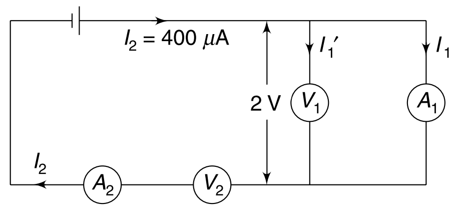
$$I_1' = \frac{V_1}{R_{V1}} \qquad [R_{V1} = R_{V2} = 0.25 \text{ M}\Omega]$$

$$= \frac{2}{0.25 \times 10^{+6}} = 8 \times 10^{-6}\, A = 8\, \mu\text{A}.$$

Current through A_1 is

$$I_1 = 400\, \mu\text{A} - 8\, \mu\text{A} = 392\, \mu\text{A}$$

$$\therefore \quad R_{A1} = \frac{2 \text{ volt}}{392\, \mu\text{A}} = \frac{2 \times 10^6}{392} = 5102\ \Omega \simeq 5.1 \text{ k}\Omega$$



36. If resistance of first 5 wires is R, then the resistance of next 5 wires must be $\frac{R}{4}$ (since area of cross section is 4 times).

Potential drop across first 5m length

$$= \frac{R}{R + \frac{R}{4}} \times 2 = \frac{8}{5} \text{ V} = 1.6 \text{ V}$$

Potential drop across next 5 m length = 0.4 Volt

Potential gradient along first 5 wires = $\frac{1.6 \text{ V}}{5 \text{ m}}$

Potential gradient along next 5 wires = $\frac{0.4 \text{ V}}{5 \text{ m}}$

(a) length needed to balance 1.0 V

$$l_1 = \frac{1.0 \text{ V}}{\frac{1.6}{5} \frac{V}{m}} = 3.125 \text{ m}$$

(b) $l_2 = \dfrac{1.5 \text{ V}}{\frac{1.6}{5} \frac{V}{m}} = 4.688 \text{ m}$

(c) $l_3 = 5 \text{ m} + \dfrac{0.2 \text{V}}{\frac{0.4 \text{V}}{5 \text{ m}}}$

$= 5 + 2.5 = 7.5 \text{ m}$

37. Let length of the wire be L.

Resistance of the wire $\quad R = \dfrac{\rho L}{\pi b^2}$

Power dissipated $\quad P = \dfrac{\varepsilon^2}{R} = \dfrac{\varepsilon^2 \pi b^2}{\rho L}$

Heat dissipated in time Δt

$$Q = P\,\Delta t = \frac{\varepsilon^2 \pi b^2}{\rho L}\,\Delta t \qquad ...(1)$$

Heat required to raise the temperature of water

$$Q = ms\,\Delta\theta = d\cdot(\pi a^2 h)\, s(\theta_2 - \theta_1) \qquad ...(2)$$

From (1) and (2)

$$d(\pi a^2 h)\, s(\theta_2 - \theta_1) = \frac{\varepsilon^2 \pi b^2}{\rho L}\,\Delta t$$

$$\therefore \quad L = \frac{\varepsilon^2 b^2}{\rho h d s a^2}\,\frac{\Delta t}{(\theta_1 - \theta_2)}$$

38.

$$V_{AC} = E_3 = 30 \text{ V}$$

$$V_{CB} = E_2$$

$$\therefore \quad \frac{E_2}{30} = \frac{V_{CB}}{V_{AC}}$$

$$\frac{E_2}{30} = \frac{3}{2} \qquad \left[\because \ \frac{V_{CB}}{V_{AC}} = \frac{l_{CB}}{l_{AC}} = \frac{3}{2}\right]$$

$$\therefore \quad E_2 = 45 \text{ V}$$

$$V_{AB} = 30 + 45 = 75 \text{ V}$$

$$R_{AB} = 2 \times 5 = 10\ \Omega$$

$\therefore$ Current through AB is $\dfrac{75}{10} = 7.5$ A

$$\therefore \quad E_1 = 7.5 \times (R_{AB} + R)$$

$$= 150 \text{ V.}$$

39. After the switch is moved to position 2, the energy stored in the capacitor gets dissipated in the two resistors. Current through two resistors will be same at any instant; therefore, heat dissipated in them will be in the ratio of their resistances.

$$\therefore \quad \frac{H_{30}}{H_{50}} = \frac{30}{50}$$

$$\Rightarrow \quad H_{30} = \frac{30}{50} \times 6 = 3.6 \text{ J}$$

Total energy stored in the capacitor was

$$U = 6 + 3.6 = 9.6 \text{ J}$$

$$\therefore \quad \frac{1}{2} CV^2 = 9.6$$

$$\frac{1}{2} \times (600 \times 10^{-6})\, V^2 = 9.6$$

$$\therefore \quad V^2 = 3.2 \times 10^4$$

$$\therefore \quad V = 179 \text{ V.}$$

40. (a) There is no current through the capacitor. $B2$ and $B3$ are in parallel. The current through $B1$ gets divided into two parts through $B2$ and $B3$. Hence $B1$ will be brightest.

(b) Still there is no current through the capacitor and there is no change in brightness of bulbs.

41. The element is a capacitor.

In case of capacitor

$$q = CV$$

$$\therefore \quad i = \frac{dq}{dt} = C\left(\frac{dV}{dt}\right)$$

$$\frac{dV}{dt} = \frac{4.0}{4.0} \text{ V/s} = 1.0 \text{ V/s}$$

Therefore, if $C = 1F$ then $I = 1 \times 1 = 1A$ (constant)

42.
$$I = i_0 e^{-t/\tau} \qquad \text{...(1)}$$

Where $\quad i_0 = \frac{V_0}{R}$ and $\tau = RC$

Charge on capacitor as function of time is

$$q = q_0[1 - e^{-t/\tau}]$$

Where $\quad q_0 = CV$

Energy stored in capacitor

$$U = \frac{q^2}{2C}$$

$$\frac{dU}{dt} = \frac{1}{C} q \frac{dq}{dt} = \frac{qi}{C}$$

$\therefore$ Power absorbed

$$P = \frac{dU}{dt} = \frac{q_0 i_0}{C} e^{-t/\tau}[1 - e^{-t/\tau}] \qquad \text{...(3)}$$

For drawing the graph of P versus t you need to acknowledge that P is zero both at $t = 0$ and $t = \infty$.

The graphs have been shown in answer.

43. As the charge on the capacitor decreases, the potential difference across it drops. To keep the current constant the resistance must be decreased progressively.

Initial value of resistance = R_0

Initial current $\quad i_0 = \frac{q_0/C}{R_0}$ = a constant. ...(1)

Charge on the capacitor after time t is

$$q = q_0 - i_0 t$$

$\therefore$ Potential difference across the capacitor

$$= \frac{q}{C} = \frac{q_0 - i_0 t}{C}$$

$$\therefore \quad \frac{q_0 - i_0 t}{C} = R i_0$$

$$\therefore \quad R = \frac{q_0}{C i_0} - \frac{t}{C}$$

$$\Rightarrow \quad R = R_0 - \frac{t}{C} \qquad \text{[Using (1)]}$$

$$= R_0\left[1 - \frac{t}{R_0 C}\right]$$

44. (a) $$C = \frac{K\epsilon_0 A}{d}$$

Resistance of dielectric material $R = \frac{\rho d}{A}$

$$I = \frac{V}{R} = \frac{VA}{\rho d}$$

$$Q = CV = \frac{K\epsilon_0 AV}{d}$$

(b) The charge will leak through the capacitor. Let the charge be q after time 't'.

$$\frac{q}{C} = Ri$$

$$\Rightarrow \quad -R\frac{dq}{dt} = \frac{q}{C}$$

$$\Rightarrow \quad \frac{dq}{q} = -\frac{1}{RC}dt$$

$$\Rightarrow \quad \int_Q^q \frac{dq}{q} = -\frac{1}{\tau}\int_0^t dt$$

$$\Rightarrow \quad ln\frac{q}{Q} = -\frac{t}{\tau}$$

$$\Rightarrow \quad q = Qe^{-t/\tau}$$

Where $Q = \frac{K\epsilon_0 AV}{d}$ and $t = RC = K\rho\epsilon_0$

46. Since the two filaments emit same spectrum, their temperature must be same. It means the power radiated is simply proportional to the surface area of the filament.

$P \; \alpha \; 2\pi rl$

$\Rightarrow \quad P \; \alpha \; rl \quad$ ...(1)

Power is also given by $P = \frac{V^2}{R} = \frac{V^2\pi r^2}{\rho l}$

$\therefore \quad P \; \alpha \; \frac{r^2}{l} \quad$ ...(2)

From (1) and (2) $rl \; \alpha \; \frac{r^2}{l} \Rightarrow l^2 \; \alpha \; r$

Putting this in (1) gives

$$l \; \alpha \; P^{1/3}$$

and $$r \; \alpha \; P^{2/3}$$

$\therefore$ length of new filament $8^{1/3} l = 2l$

Radius of new filament $8^{2/3} r = 4r$.

47. With 10 bulbs in parallel, their equivalent resistance is $\frac{R}{10}$.

Maximum power transfer theorem tells us that power dissipated in a resistance is maximum when it is equal to internal resistance of the battery. So we are tempted to assume that $r = \frac{R}{10}$. But we need to understand that in this situation, number of bulbs can be integer only and the resistance connected to the battery can be

$R, \frac{R}{2}, \frac{R}{3}, \frac{R}{4}$ ... Only.

We cannot have a resistance equal to $\frac{R}{10.5}$. So it may be that r is actually equal to $\frac{R}{10.5}$ but in no way we can make our resistance equal to that.

48. Number of free electrons per unit volume = Number of atoms per unit volume

$$\therefore \quad n = \frac{\text{Mass of unit volume}}{\text{Molar mass}} \times N_A$$

$$= \frac{9000}{63 \times 10^{-3}} \times 6.02 \times 10^{23} = 8.6 \times 10^{28}\ \text{m}^{-3}$$

Drift speed

$$v_d = \frac{j}{ne}$$

$$= \frac{1 \times 10^6 \cdot \text{A/m}^2}{8.6 \times 10^{28} \times 1.6 \times 10^{-19}} = 7.2 \times 10^{-5}\ \text{ms}^{-1}$$

Time needed to drift through 5 cm is

$$t = \frac{5 \times 10^{-2}}{7.2 \times 10^{-5}} = 695\ \text{s}$$

The thermal kinetic energy of electrons is of the order of $\frac{3}{2}$ kT where $K = 1.38 \times 10^{-23}$ JK^{-1} is Boltzmann constant.

$$\therefore \quad \frac{1}{2}\text{mv}^2 = \frac{3}{2}\text{kT}$$

$$v = \sqrt{\frac{3\text{kT}}{\text{m}}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}}$$

$$= 1.17 \times 10^5\ \text{ms}^{-1}$$

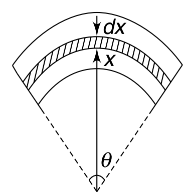
$\therefore$ Distance travelled by electron in $t = 695$ s is

$$S = vt = 1.17 \times 10^5 \times 695\ \text{m}$$

$$= 8.1 \times 10^4\ \text{km}$$

49. Consider a strip of width dx as shown in Figure. Resistance of the strip

$$dR = \frac{x\theta}{\sigma t dx} = \frac{\pi}{6\sigma t}\frac{x}{dx}$$



All such small resistances will be in parallel.

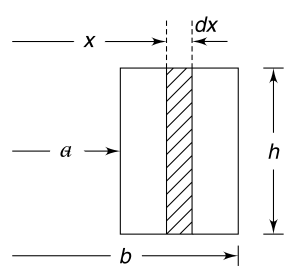
$$\therefore \quad \frac{1}{R} = \frac{1}{dR_1} + \frac{1}{dR_2} + \frac{1}{dR_3} + \dots$$

$$\Rightarrow \quad \frac{1}{R} = \int \frac{1}{dR} = \frac{6\sigma t}{\pi}\int_r^{2r}\frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{R} = \frac{6\sigma t}{\pi}[\ln x]_r^{2r} = \frac{6\ln 2}{\pi}\sigma t$$

$$\therefore \quad R = \frac{\pi}{6 \cdot \ln 2\sigma t}$$

50. Consider a strip of width dx shown on the rectangular face in the figure



Think of a half cylindrical conductor of radius x and infinitesimally small thickness dx.

Length of this conductor = πx

Cross sectional area of this conductor = $h\,dx$

$\therefore$ Resistance of this thin cylindrical conductor

$$dR = \frac{\rho \pi x}{h\,dx}$$

The given conductor is made of countless number of such thin conductors, all connected in parallel.

$$\therefore \qquad \frac{1}{R} = \int \frac{1}{dR} = \frac{h}{\rho\pi}\int_a^b \frac{dx}{x}$$

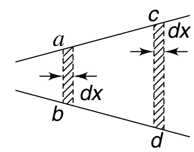
$$= \frac{h}{\rho\pi} \ln\left(\frac{b}{a}\right)$$

$$\therefore \qquad R = \frac{\rho\pi}{h\ln(b/a)}$$

51. let area at $ab = A$

And area at $cd = 2A$

Current in the wire $= I$. Current is same at all cross sections.



Current density at ab $\qquad J_1 = \frac{I}{A}$

Current density at cd $\qquad J_2 = \frac{I}{2A} = \frac{J_1}{2}$

Consider two discs of equal thickness dx at section ab and cd.

Resistance of disc at ab $dR_1 = \sigma \frac{dx}{A}$

Rate of Heat dissipation in disc at $ab = I^2(dR_1)$

Rate of heat dissipation per unit volume $= \frac{I^2(dR_1)}{A \cdot dx}$

$$= \frac{I^2\sigma}{A^2}$$

Similarly rate of heat dissipation per unit volume at section $cd = \frac{I^2\sigma}{(2A)^2}$.

$\therefore$ Required ratio $= \frac{1}{4}$.

52. Power radiated by the filament

$$P = \sigma e(T^4 - T_0^4)$$

$$\simeq \sigma e T^4 \qquad [\because T^4 >> T_0^4]$$

But $\qquad P = VI$

$$\therefore \qquad VI = \sigma e T^4$$

$$\therefore \qquad \ln(VI) = \ln(\sigma e) + 4\ln T$$

$\therefore$ graph has a slope $= 4$.

53. The power dissipated in a length L of a wire carrying current I is $P = I^2R$.

When the temperature of the wire is constant

$$I^2R = e\sigma\pi d \cdot L\left(T^4 - T_0^4\right)$$

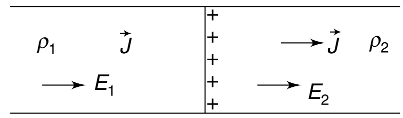
[σ is Stefan's constant and e is emissivity]

$$I^2R \simeq e\sigma\pi d \cdot LT^4$$

$$\Rightarrow \qquad \frac{I^2\rho L}{\pi\left(\frac{d^2}{4}\right)} = e\sigma\pi d \cdot LT^4 \quad \Rightarrow \quad T^4 \propto \frac{1}{d^3}$$

$$\therefore \qquad \left(\frac{T_{\text{thin}}}{2000\text{ K}}\right)^4 = \frac{2^3}{1} \qquad \therefore\ T_{\text{thin}} = 8^{1/4}\ 2000\text{ K}$$

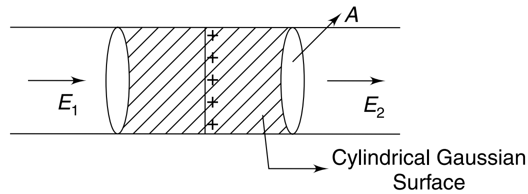
54. $$J = \frac{E_1}{\rho_1} = \frac{E_2}{\rho_2}$$



$\therefore$ $$E_2 = \frac{\rho_2}{\rho_1} E_1 \quad ...(1)$$

Consider a Gaussian surface as shown

$$\phi = E_2A - E_1A = \left(\frac{\rho_2}{\rho_1} - 1\right) E_1A$$



$\therefore$ $$\frac{Q}{\epsilon_0} = \left(\frac{\rho_2}{\rho_1} - 1\right) E_1A$$

$$Q = \left(\frac{\rho_2}{\rho_1} - 1\right) \epsilon_0 AE_1$$

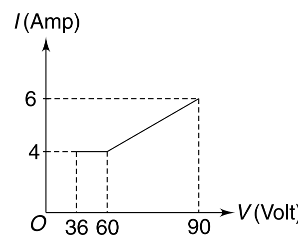
$\because$ $$I = \frac{E_1}{\rho_1} A$$

$\therefore$ $$Q = (\rho_2 - \rho_1) \epsilon_0 I$$

Q is positive if $\rho_2 > \rho_1$

Q is negative if $\rho_2 < \rho_1$

55. Minimum current that is possible through A is $4A$. Since three elements are in series, they must have same current.



Hence minimum current in the circuit is $4A$.

For $4A$, current

$$V_{\min} = V_A + V_B + V_C = 10 + 20 + 6 = 36 \text{ V}$$

And $$V_{\max} = V_A + V_B + V_C = 20 + 20 + 20 = 60 \text{ V}$$

$\therefore$ From 36 V to 60 V supply the current remains $4A$ and below 36 V there is no current.

For $V_A > 20$ V, $V_B > 20$ V, $V_C > 20$ V

All elements have same resistance (the slope of graph is same for all) $R = \frac{20}{4} = 5\ \Omega$.

$\therefore$ The effective resistance is 15 Ω for applied emf > 60 V.

At 90 V, the current is $\frac{90}{15} = 6A$.

56. The device conducts only when voltage across it is 20 V.

$\therefore$ $$V > 20 \text{ V}$$

$$I_{\max} = \frac{P_{\max}}{20 \text{ volt}} = \frac{1 \text{ Watt}}{20 \text{ V}} = \frac{1}{20} A.$$

From the circuit

$$V = 100\,I + 20$$

$\therefore$ $$I = \frac{V - 20}{100}$$

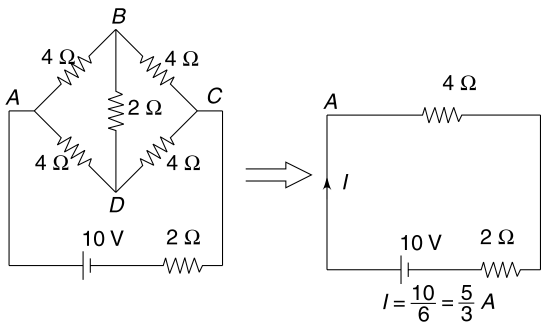
But $$I \le \frac{1}{20} A$$

$\therefore$ $$\frac{V - 20}{100} \le \frac{1}{20}$$

$$V - 20 \le 5$$

$$V \le 25 \text{ volt.}$$

57. (a) The circuit is a wheatstone bridge connected to a battery with a resistance in series. The bridge is balanced.



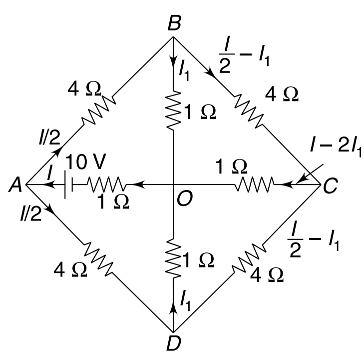
(b) The current distribution is as shown. Current in branch *AB* and *AD* are equal due to symmetry. Similarly current in *BO* and *OD* are same. Current in *BC* and *DC* are also same.

Applying Kirchhoff's second law to loop *OBCO*

$$1 \times I_1 - 4\left(\frac{I}{2} - I_1\right) - 1(I - 2I_1) = 0$$

$$I_1 = \frac{3I}{7} \qquad \text{...(1)}$$

Applying the same law to loop *ABOA*



$$4 \times \frac{I}{2} + 1 \times I_1 + 1 \times I = 10$$

$$3I + I_1 = 10$$

$$\Rightarrow \qquad 3I + \frac{3I}{7} = 10$$

$$\Rightarrow \qquad I = \frac{70}{24} = \frac{35}{12} A.$$

58. **Between *a* and *b*.**

40 Ω and 160 Ω in series makes 200 Ω.

200 Ω in parallel with 50 Ω makes $\frac{50 \times 200}{50 + 200} = 40$ Ω

40 Ω and 50 Ω is series makes 90 Ω.

90 Ω and 200 Ω in parallel make $\frac{90 \times 200}{90 + 200} = \frac{1800}{29}$.

Now 10 Ω, $\frac{1800}{29}$ and 10 Ω are in series

$\therefore$ $$R_{ab} = \frac{2380}{29} \ \Omega$$

Between *a* and *c*

The 10 Ω resistance connected to *b* is open circuit (not included in the circuit)

Between d and c we have a balanced wheatstone bridge $\left(\frac{50}{40} = \frac{200}{160}\right)$

$\therefore$ 50 Ω resistance can be removed

50 Ω and 40 Ω in series = 90 Ω

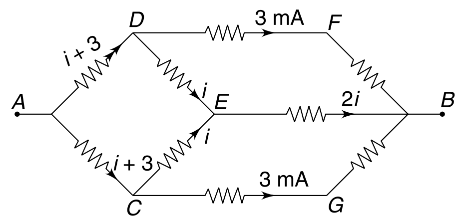
160 Ω and 200 Ω in series = 360 Ω

90 Ω and 360 Ω in parallel = 72 Ω

72 Ω and 10 Ω in series = 82 Ω.

$$\therefore \qquad \frac{R_{ab}}{R_{ac}} = \frac{2380}{19} \times \frac{1}{82} = \frac{1190}{1189}$$

59. (a) let each resistance be R. Due to symmetry current through DF is also 3 mA. If current in CE is i, then current through DE must also be i and current in EB is $2i$. Current in AC and AD is $(i + 3)$ mA



$$V_{CB} = (2R)(3) = iR + 2iR$$

$$\Rightarrow \qquad 3i = 6 \quad \Rightarrow \quad i = 2 \text{ mA}.$$

$\therefore$ Current drawn from the 5V supply is 10 mA

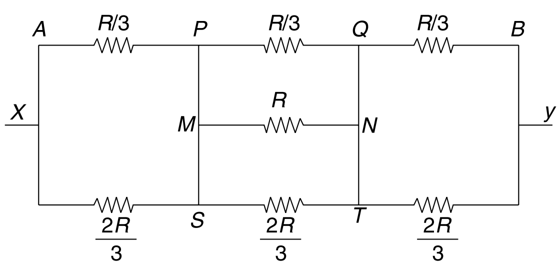
$$\therefore \qquad R_{\text{eff}} = \frac{5 \text{ V}}{10 \text{ mA}} = 500\ \Omega$$

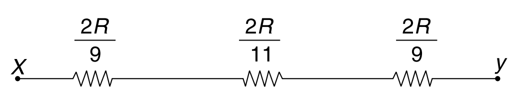
60. $\frac{R}{3}$ is in parallel to $\frac{2R}{3}$.

$\frac{R}{3}$, R and $\frac{2R}{3}$ are in parallel.

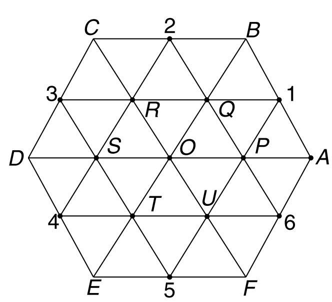
$\frac{R}{3}$ and $\frac{2R}{3}$ are in parallel.

$\therefore \quad R_{eq} = \frac{62R}{99}$

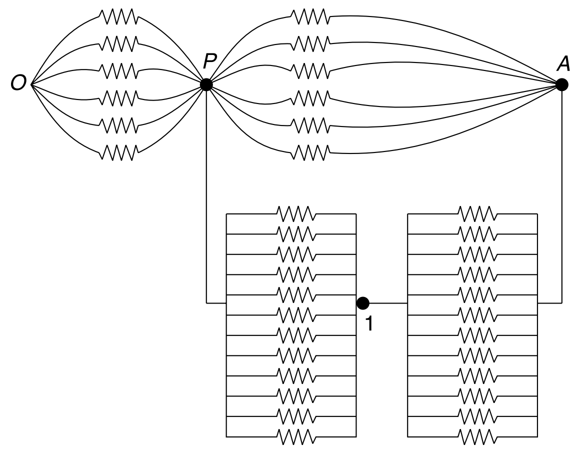




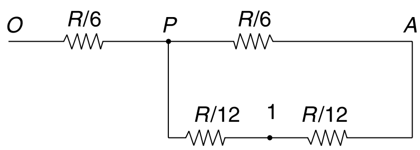
61. Points A, B, C, D, E & F are equipotentials.



From symmetry, points P, Q, R, S, T and U are equipotential and 1, 2, 3, 4, 5 and 6 are also equipotential. The circuit can be redrawn as



$\Rightarrow \quad \therefore \quad R_{eq} = \dfrac{R}{4}$



62. Resistance of PA = Resistance of QB is

$R_1 = K(a\theta)$ $\quad$ K = a constant

$R_1 = C\theta$ $\quad$ a = radius

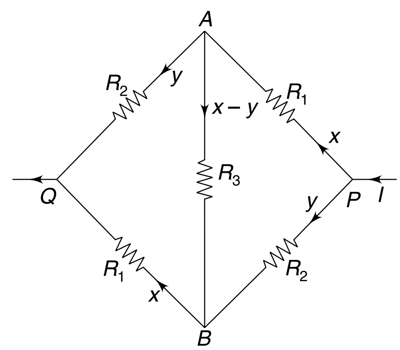
[$C = k \cdot a$ = another constant]

Resistance of PB = Resistance of QA is

$$R_2 = C(\pi - \theta)$$

Resistance of AB

$$R_3 = K(2a) = 2C.$$



The circuit can be redrawn as shown.

Taking into account the symmetry, the current distribution can be assumed as shown in the Figure.

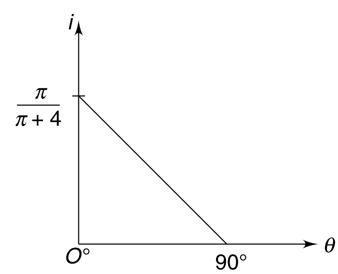
Using Kirchhoff's law in loop ABP we get

$$R_1 x + R_3(x - y) = R_2 y$$

$$C\theta x + 2C(x - y) = C(\pi - \theta) y$$

$$\Rightarrow \quad (2 + \theta)x = (\pi + 2 - \theta)y \qquad \text{...(1)}$$

And
$$x + y = I \qquad \text{...(2)}$$



Solving (1) and (2)

$$x = \frac{(\pi + 2 - \theta)I}{\pi + 4} \quad \text{and} \quad y = \frac{(2 + \theta)I}{\pi + 4}$$

$$\therefore \quad x - y = \frac{\pi - 2\theta}{\pi + 4} \quad \Rightarrow \quad i = \frac{\pi - 2\theta}{\pi + 4}$$

63. (a) Using Kirchhoff's voltage law in the bigger loop we get

$$E - 10 = 5i_1 + I \qquad \text{...(1)}$$

In the left loop we get

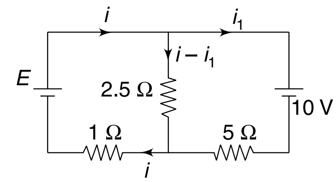
$$E = (I - i_1)\, 2.5 + I$$

$$\Rightarrow \quad E = 3.5\, I - 2.5 i_1 \qquad \text{...(2)}$$

Multiplying (1) by 3.5 and subtracting (2) from it

$$2.5\,E - 3.5 = 20 i_1$$

$$\Rightarrow \quad i_1 = \frac{2.5\,E - 35}{20} = \frac{E - 14}{8} \qquad \text{...(3)}$$



Since $\quad E = 2t$

$$\therefore \quad i_1 = \frac{2t - 14}{8} = \frac{t}{4} - \frac{7}{4}$$

Energy gained by 10 V cell is

$$U = \int_0^{14} i_1 \times 10\, dt = 2.5 \int_0^{14} (t - 7)\, dt$$

$$= 2.5\left[\frac{14^2}{2} - 7 \times 14\right] = 0$$

(b) **From (3)** i_1 is positive when $E > 14$ volt

$\therefore$ Cell starts charging after $t = 7$ s.

Before this it was discharging.

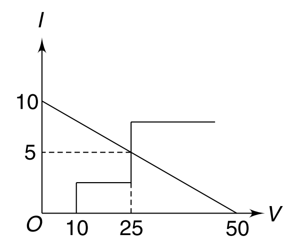
64. Let current in the circuit be I.

Potential difference across A is $V = E - IR$

or, $\quad V = 100 - 10\,I \qquad$ ...(1)

Current through R connected in parallel to A is

$$I' = \frac{V}{R} = \frac{100}{10} - \frac{10I}{10} = 10 - I$$



$\therefore$ Current through the box is

$$i = I - I'$$

$$i = I - (10 - I) = 2I - 10$$

Using (1) $\quad I = 10 - \frac{V}{10}$

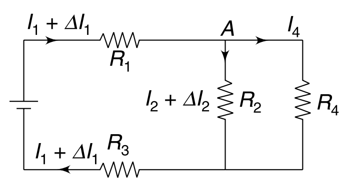
$$\therefore \quad I = 2\left(10 - \frac{V}{10}\right) - 10$$

$$I = 10 - \frac{V}{5} \qquad \text{...(2)}$$

Plot the equation (2) on the given graph

The intersection point gives the correct value of V. Hence answer is 25 V.

65. (a) When R_4 is added, the equivalent resistance of the circuit decreases. Hence, current through R_1 increases.



(b) let $R_1 = 4\,R$, $R_2 = R$, $R_3 = 2\,R$.

Note: Kirchoff's laws are not valid only for currents and voltages, but also for voltage increments ΔV_i and current increments ΔI_i

$$\Delta I_1 R_1 + \Delta I_2 R_2 + \Delta I_3 R_3 = 0$$

$$[\Delta I_1 = \Delta I_3 = 0.2A]$$

$$\therefore \quad 0.2 \times 4R + \Delta I_2 \times R + 0.2 \times 2R = 0$$

$$\Delta I_2 = -1.2A$$

Negative sign means I_2 decreases.

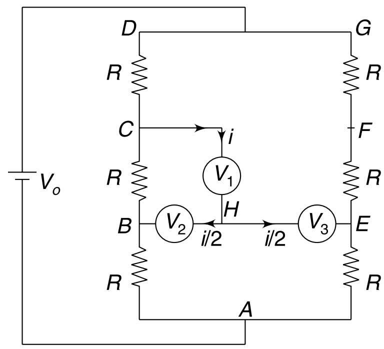
At junction A

$$\Delta I_1 = \Delta I_2 + I_4$$

$$0.2 = -1.2 + I_4$$

$$\therefore \quad I_4 = 1.4\,A.$$

66. (a) When the switch is open, potential drop across each R is $= \frac{V_0}{3}$.



Voltmeters are nearly ideal. A very small current will flow through V_1 and V_2 and both of them will show equal reading of $\frac{V_0}{6}$.

(b) **After 'S' is closed.**

p.d. across each 'R' is still $\frac{V_0}{3}$.

Assuming
$$V_A = 0$$
$$V_B = V_E = \frac{V_0}{3}$$
$$V_C = V_F = \frac{2V_0}{3}$$
$$V_D = V_G = V_0$$

If current through V_1 is i, then current through V_2 and V_3 will be $\frac{i}{2}$ as shown. (Remember i is very small] If resistance of each voltmeter is R_V

$$iR_V = V_C - V_H$$
$$iR_V = \frac{2V_0}{3} - V_H \quad ...(1)$$

And
$$\frac{i}{2} R_V = V_H - V_B$$
$$\frac{i}{2} R_V = V_H - \frac{V_0}{3} \quad ...(2)$$

(1) + (2)
$$\frac{3}{2} iR_V = \frac{V_0}{3}$$

$\therefore$
$$iR_V = \frac{2V_0}{9}$$

67. Total charge flow in first 20 min can be calculated as–

$$Q_0 = \int_0^t idt = \int_0^{t=20} \frac{E}{R} dt = \int_0^{t=20} \frac{E}{20+t} dt$$

$\Rightarrow$
$$Q_0 = E\,\ell n\left(\frac{20+20}{20}\right) = E\,\ell n(2)$$

After 20 min the resistance remains constant at 40 Ω and current remains constant at $I = \frac{E}{40}$

$\therefore$ Charge flow in next Δt time is

$$\Delta Q = I\Delta t = \frac{E\Delta t}{40}$$

Energy lost by a cell is proportional to the charge flow through it. Hence

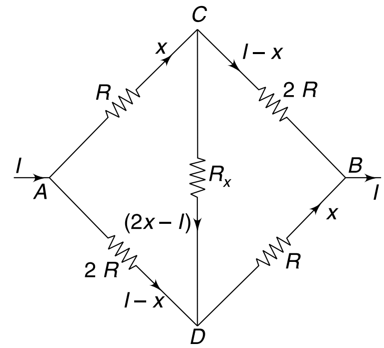
$$\frac{\Delta Q}{Q_0} = \frac{9}{1}$$

$\Rightarrow$
$$\frac{\frac{E\Delta t}{40}}{E\ell n2} = 9 \Rightarrow \Delta t = 330\ \ell n2$$

$\therefore$ Total time for which the cell lasts is

$$T = (20 + 360\ \ell n 2) \text{ minute.}$$

68. Taking into account the symmetry, current distribution is as shown in the Figure.



Using Kirchhoff's voltage law in loop $ACDA$–

$$Rx + R_x(2x - I) = 2R(I - x)$$

$$\Rightarrow \qquad x = \left(\frac{2R + R_x}{3R + 2R_x}\right) I$$

Now
$$V_A - V_B = Rx + 2R(I - x)$$
$$= 2RI - R \cdot x$$
$$= R\left[2I - \frac{2R + R_x}{3R + 2R_x} \cdot I\right]$$
$$= R\left[\frac{4R + 3\,R_x}{3R + 2R_x}\right] I$$

Equivalent resistance
$$R_{AB} = \frac{V_A - V_B}{I}$$

$$\therefore \qquad R_{AB} = \left[\frac{4R + 3R_x}{3R + 2R_x}\right] R$$
$$= \left[\frac{4 + 3\dfrac{R_x}{R}}{3 + 2\dfrac{R_x}{R}}\right] R$$

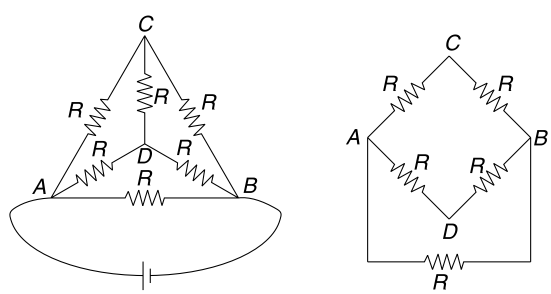
When $\dfrac{R_x}{R} \to 0$

$$R_{AB} = \frac{4}{3}R$$

When $\dfrac{R_x}{R} \to \infty$

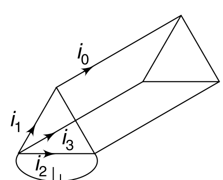
$$R_{AB} = \frac{3}{2}R.$$

69. Equivalent circuit is as shown. If the resistance connected directly between A and B is removed, the remaining circuit is a balanced wheatstone bridge with $V_C = V_D$. Resistance between C and D can be removed.

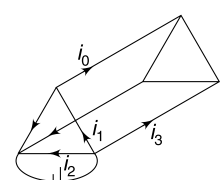


$$R_{eq} = \frac{R}{2}$$

70. **(a)** If a cell is connected across A and B with A positive, then let's assume that current through CD is i_0 – from C to D. If polarity of cell is reversed (*i.e.*, B is made positive) the direction of current in each branch must reverse without change in magnitude. But symmetry tells us that direction of current in CD will remain same even after the polarity is reversed. (see Fig. (a) & (b))



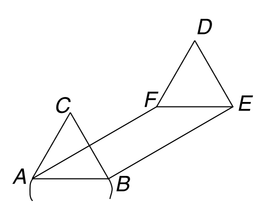
(a)



(b)

This can be true only if $i_0 = 0$.

Remove CD. Now the circuit is simple series parallel combination of resistors (Fig. (c))



(c)

$$R_{FE} = \frac{R \cdot 2R}{R + 2R} = \frac{2R}{3}$$

.

$$R_{AFEB} = R + \frac{2R}{3} + R = \frac{8R}{3}$$

$$R_{AB} = \frac{2R}{3}$$

$\therefore$
$$R_{eq} = \frac{\frac{8R}{3}\cdot\frac{2R}{3}}{\frac{8R}{3}+\frac{2R}{3}} = \frac{16R}{30} = \frac{8R}{15}$$

(b) In this case A and B are equipotential points. Similarly, E and F are equipotential. Remove the arm AB and EF and join A and B together. Also join E and F together. Now, it is a series parallel circuit. Prove yourself that $R_{eq} = \frac{3R}{5}$.

71. (i) Power dissipated in resistance = Rate of heat dissipation to surrounding + Rate of heat absorption by conductor

$$\frac{V^2}{R} = k(T - T_0) + ms\,\frac{dT}{dt}$$

$$ms\,\frac{dT}{dt} = \frac{V^2}{R} - k(T - T_0)$$

$$\int_{T_0}^{T} \frac{dT}{\frac{V^2}{R} - k(T - T_0)} = \frac{1}{ms}\int_0^t dt$$

$$\left\{\ell n\left[\frac{V^2}{R} - k(T - T_0)\right]\right\}_{T_0}^{T} = -\frac{kt}{ms}$$

$$\ell n\left[\frac{V^2}{R} - k(T - T_0)\right] - \ell n\left[\frac{V^2}{R}\right] = -\frac{kt}{ms}$$

$$1 - \frac{Rk(T - T_0)}{V^2} = e^{-\frac{kt}{ms}}$$

$\therefore$
$$T - T_0 = \frac{V^2}{kR}\left(1 - e^{-\frac{kt}{ms}}\right)$$

$$T = T_0 + \frac{V^2}{kR}\left(1 - e^{-\frac{kt}{ms}}\right)$$

(ii) when $t \to \infty$; $e^{-\frac{kt}{ms}} \to 0$

$\therefore$
$$T = T_0 + \frac{V^2}{kR}$$

72. Electric power dissipated

$$= \frac{V^2}{R} = \frac{V^2}{\rho\frac{\ell}{\pi r^2}} = \frac{\pi r^2 V^2}{\rho\ell}$$

Radiant power loss

$$= e\cdot\sigma\, 2\pi r\ell\left[T^4 - T_0^4\right]$$

$$= 2\pi e\,\sigma r\ell\left[(T_0 + \Delta T)^4 - T_0^4\right]$$

$$= 2\pi e\sigma r\ell\, T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$= 2\pi e\sigma r\ell T_0^4\left[1 + \frac{4\Delta T}{T_0} \ldots - 1\right]$$

$$= 2\pi e\sigma r\ell T_0^4\left[\frac{4\Delta T}{T_0}\right]$$

[Because $\Delta T << T_0$]

$$= 8\pi e\sigma r\ell\ T_0^3\ \Delta T$$

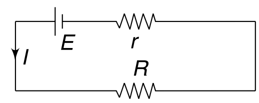
For steady state

$$8\pi e\sigma r\ell T_0^3\ \Delta T = \frac{\pi r^2 V^2}{\rho\ell}$$

$$\Rightarrow \qquad \Delta T = \frac{V^2 r}{8e\sigma\rho\ell^2 T_0^3}$$

$$\Rightarrow \qquad T = T_0 + \frac{V^2 r}{8e\sigma\rho\ell^2 T_0^3}$$

73. (a) Power dissipated in R; $P_R = I^2 R$.



Power spent by the cell; $P_{\text{cell}} = I^2(R + r)$

Fraction of energy spent by the cell that is dissipated in R is

$$\eta = \frac{P_R}{P_{\text{cell}}} = \frac{R}{R + r}$$

$$\eta = \frac{1}{1 + \frac{r}{R}}$$

η will be maximum if R is maximum (*i.e.* = 4Ω)

Total energy dissipated in R will be

$$H_0 = \eta\cdot(EQ) \text{ [}Q = \text{total charge flow through the cell]}$$

$$= \frac{1}{1 + \frac{1}{4}}.\ 12 \times [2000 \times 10^{-3} \times 3600]$$

$$= \frac{4}{5} \times 12 \times 7200 = 69.12 \text{ KJ.}$$

(b) Power transfer to R is maximum when $R = r = 1\Omega$.

In this case, half the energy spent by the cell will get dissipated in R

$$\therefore \qquad H = \frac{EQ}{2}$$

$\therefore$ Required answer is

$$\frac{H}{H_0} \times 100$$

$$= \frac{\frac{EQ}{2}}{\eta EQ} \times 100 = \frac{1}{2 \times \frac{4}{5}} \times 100 = 62.5\%$$

74. (a) In first experiment

$$\frac{R_1}{R_2} = 10$$

For balance

$$\frac{R_3}{R_4} = 10$$

$$\therefore \qquad R_4 = \frac{R_3}{10}$$

$$\therefore \qquad 64.3 < R_4 < 64.4\Omega.$$

In second experiment

$$\frac{R_1}{R_2} = 100$$

$\therefore$ For balance $\quad \dfrac{R_3}{R_4} = 100$

$\therefore \quad R_4 = \dfrac{R_3}{100}$

$\therefore$ We should try with $\quad 6430 < R_3 < 6440\ \Omega$

$\therefore \quad 64.32 < R_4 < 64.33\ \Omega$

$R_4 \simeq 64.325\ \Omega$

75. (a) $\quad \dfrac{R_x}{R} = \dfrac{x}{L-x}$

$$R_x = R\left(\frac{x}{L-x}\right) \qquad \text{...(i)}$$

For $\quad R = 20\,\Omega,\ x = 20$ cm

$$R_x = 20\left[\frac{20}{100-20}\right] = 5\,\Omega$$

From (i) $\ell n R_x = \ell n R + \ell n x - \ell n (L-x)$

$$\frac{dR_x}{R_x} = \frac{dx}{x} + \frac{dx}{L-x}$$

$$\frac{\Delta R_x}{R_x} = \frac{L\Delta x}{x(L-x)} \quad [\Delta x = \text{maximum error in } x] \qquad \text{...(2)}$$

For $R_x = 5\,\Omega$; $|\Delta x| = 0.1$ cm; $L = 100$ cm $x = 20$ cm

$\therefore \quad |\Delta R_x| = \dfrac{5 \times 100 \times 0.1}{20 \times 80} = 0.03\ \Omega$

$\therefore \quad R_x = (5.00 \pm 0.03)\ \Omega$

(b) From equation (2), the fractional error $\dfrac{\Delta R_x}{R_x}$ is minimum for a given error in x (*i.e.* for given Δx)

When $x(L-x)$ is maximum.

This happens when $x = \dfrac{L}{2}$

To minimize the error, we need to change the known resistance R so as to bring it close to R_x. In that case $x = \dfrac{L}{2}$.

You can show that if R were equal to 5 Ω in the first part of the question, $R_x = (5.00 \pm 0.02)\ \Omega$.

76. (a) when $I = 0$

$x = 20$ cm

It means $\quad V_{AC} = \dfrac{V_0 x}{L} = \dfrac{12 \times 20}{100} = 2.4$ volt

Since there is no current through the milliammeter

Hence $\quad V_{AC} = V = 2.4$ volt.

(b) When $x = 0$; $I = 30$ mA

$\therefore \quad 2.4 = I\,R$

$2.4 = 30 \times 10^{-3} \cdot R$

$R = 80\ \Omega$

(c) When $x = 100$

$$V_{AB} = V_0 = 12 \text{ volt}$$

$$\therefore \quad 80I = 12 - 2.4$$

$$I = 0.12\ A = 120 \text{ mA}$$

This current is in negative direction

77. The internal resistance of equivalent cell is given by

$$\frac{1}{r_0} = \frac{1}{r} + \frac{1}{2r} + \frac{1}{4r} + \ldots + \frac{1}{16r}.$$

$$\frac{1}{r_0} = \frac{1}{r}\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^4}\right]$$

$$= \frac{1}{r}\cdot\frac{1\left[1 - \left(\frac{1}{2}\right)^5\right]}{\left[1 - \frac{1}{2}\right]}$$

$$= \frac{1}{r}\cdot\frac{31}{16}$$

$$\therefore \quad r_0 = \frac{16r}{31}$$

Equivalent emf

$$\varepsilon_0 = \frac{\frac{\varepsilon}{r} + \frac{2\varepsilon}{2r} + \frac{4\varepsilon}{4r} + \frac{8\varepsilon}{8r} + \frac{16\varepsilon}{16r}}{\frac{1}{r_0}}$$

$$= \frac{5\varepsilon}{r}\cdot\frac{16r}{31} = \frac{80\varepsilon}{31}$$

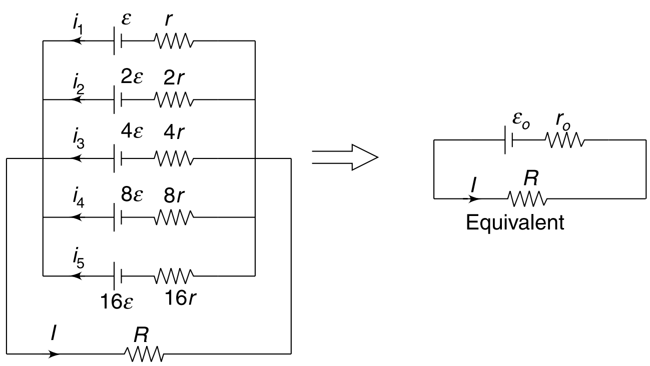
$$\therefore \quad I = \frac{\varepsilon_0}{R + r_0}$$

$$I = \frac{\frac{80\varepsilon}{31}}{R + \frac{16r}{31}} = \frac{80\varepsilon}{31R + 16r}$$

If we write the voltage equation for a loop containing any cell (n^{th} cell) and load resistance R, we get

$$i_n r_n = \varepsilon_n - IR$$

$$i_n = \frac{\varepsilon_n}{r_n} - \frac{IR}{r_n}$$



value of $\frac{\varepsilon_n}{r_n} = \frac{\varepsilon}{r}$ for all cells

$\therefore$ $$i_n = \frac{\varepsilon}{r} - \frac{IR}{r_n}$$

$\therefore$ i_n is maximum for the cell which has largest value of r_n (*i.e.*, cell having internal resistance $16r$)

78. (a) Let the resistance of voltmeter be r

$$V_{AB} = 24\text{V}; \quad V_{BC} = V_{CD} = 30\text{V}$$

$\therefore$ $$\frac{R_{AB}}{R} = \frac{24}{30} \Rightarrow R_{AB} = \frac{16}{5}$$

$\Rightarrow$ $$\frac{4r}{4+r} = \frac{16}{5}$$

$\Rightarrow$ $$r = 16\ \Omega.$$

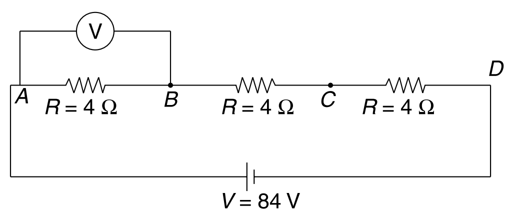
When connected across A and C

$$R_{AC} = \frac{(2R)(r)}{2R+r} = \frac{2 \times 4 \times 16}{2 \times 4 + 16} = \frac{16}{3}\ \Omega$$

$\therefore$ $$\frac{V_{AC}}{V_{CD}} = \frac{R_{AC}}{R}$$

$\therefore$ $$V_{AC} = \left(\frac{R_{AC}}{R_{AC}+R}\right) \times 84$$

$$= \frac{16/3}{16/3+4} \times 84 = 48\ \text{V}$$



(b) Yes, the voltmeter will now be more close to an ideal voltmeter as its resistance will far exceed the resistance in the circuit.

79. Let resistance of each voltmeter be R_0.

$$Ri = 20 = R_0(I - i) \quad ...(1)$$

And $$2Ri' = 30 \quad ...(2)$$

$$(2) \div (1)\ \frac{2i'}{i} = \frac{3}{2}$$

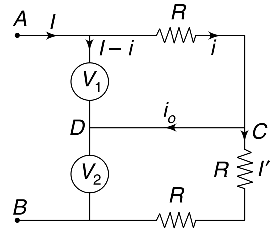
$$i' = \frac{3}{4}i$$

$\therefore$ $$i_0 = i - \frac{3i}{4} = \frac{i}{4}$$

$\therefore$ current through V_2 is $I - i + \frac{i}{4} = I - \frac{3i}{4}$

$$R_0\left(I - \frac{3i}{4}\right) = 2Ri' = 30 \quad ...(3)$$

From (1) and (3)



$$\frac{I-i}{I-\frac{3i}{4}} = \frac{2}{3}$$

$$\Rightarrow \quad 3I - 3i = 2I - \frac{3i}{2}$$

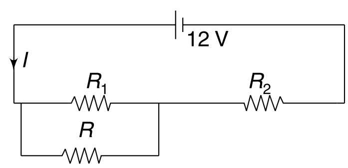
$$\Rightarrow \quad I = \frac{3i}{2} \quad \Rightarrow \quad i = \frac{2}{3} \times 600 = 400\ \mu A$$

$$\because \quad Ri = 20$$

$$R \times 400 \times 10^{-6} = 20$$

$$R = 50\ k\Omega.$$

81. The reading of voltmeter is absolutely correct when it is connected across the battery.



It means the emf of the cell is = 120 volt.

Let the resistance of the voltmeter be R.

The current I is

$$I = \frac{12}{\frac{R_1 R}{R_1 + R} + R_2}$$

$\therefore$ Voltmeter reading = voltage drop across R_1(or R)

$$\Rightarrow \quad 4 = \frac{12}{\frac{R_1 R}{R_1 + R} + R_2} \cdot \frac{R_1 R}{R_1 + R}$$

$$\therefore \quad 1 + \frac{R_2(R_1 + R)}{R_1 R} = 3$$

$$\Rightarrow \quad \frac{R_2(R_1 + R)}{R_1 R} = 2$$

$$\Rightarrow \quad \frac{R_2}{R} + \frac{R_2}{R_1} = 2 \qquad ...(1)$$

Similarly

$$V_2 = \frac{12}{\frac{R_2 R}{R_2 + R} + R_1} \cdot \frac{R_2 R}{R_2 + R}$$

$$\Rightarrow \quad 6 = \frac{12}{1 + \frac{R_1(R_2 + R)}{R_2 R}}$$

$$\Rightarrow \quad \frac{R_1 R_2 + R_1 R}{R_2 R} = 1$$

$$\Rightarrow \quad \frac{R_1}{R} + \frac{R_1}{R_2} = 1 \qquad ...(2)$$

Eliminating R between (1) and (2)

$$\frac{R_1}{R_2} = \frac{2}{3}$$

The actual voltage across R_1 is

$$V_1 = \frac{12 \cdot R_1}{R_1 + R_2} = \frac{12}{1 + \dfrac{R_2}{R_1}} = \frac{12}{1 + \dfrac{3}{2}}$$

$$= \frac{24}{5} = 4.8 \text{ Volt.}$$

82. The ohm meter reads $R = 0\,\Omega$ when full scale deflection current passes through it.

$\therefore$ $$2 \text{ mA} = \frac{20 \text{ volt}}{R_0 + 20\ \Omega}$$

$\therefore$ $$R_0 = 9980\ \Omega$$

When $R = \infty$, current in the galvanometer is zero and there is no deflection.

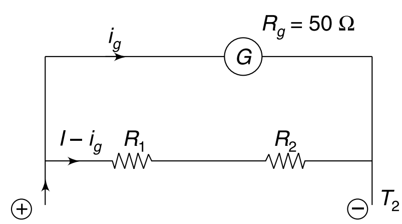
For $\theta = 90°$, current through the galvanometer is

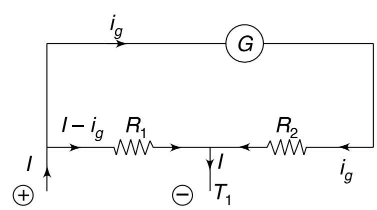
$$i = \frac{2 \text{ mA}}{120°} \times 90° = 1.5 \text{ mA}$$

$\therefore$ $$\frac{20 \text{ volt}}{9980 + 20 + R} = 1.5 \times 10^{-13}$$

$\Rightarrow$ $$13333 = 10{,}000 + R$$

$\Rightarrow$ $$R = 3333\ \Omega$$





83. (a) A smaller shunt is required to measure higher current. Hence, T_1 shall be (–) terminal if current is higher.

For current of the order of 0.1 A, T_2 shall be the (–) terminal

(b) For the range 0 – 1 A, R_1 and R_2 are in series.

$$i_g R_g = (I - i_g)(R_1 + R_2)$$

$\Rightarrow$ $$R_1 + R_2 = \frac{1 \times 10^{-3} \times 50}{1 - 10^{-3}} = \frac{50}{999} \qquad \text{...(1)}$$

For range 0 – 10 A, T_1 is negative terminal. Hence R_g and R_2 are in series and R_1 is in parallel to them.

$$i_g(R_g + R_2) = (I - i_g)R_1$$

$$10^{-3}(50 + R_2) = (10 - 10^{-3})R_1$$

$\Rightarrow$ $$10^{-3}(R_1 + R_2) + 0.050 = 10R_1$$

Using (1)

$$\frac{0.050}{999} + 0.050 = 10R_1$$

$\Rightarrow$ $$R_1 = \frac{5}{999}\Omega$$

From (1) $$R_2 = \frac{45}{999} = \frac{5}{111}\Omega.$$

84. (a) **In series**

current $$i = k_1\theta_1 = k_2\theta_2 = k_2\,\theta_3 \qquad \text{...(1)}$$

In parallel

$$V_1 = V_2 = V_3$$

$$k_1\ \theta_1' \cdot r_1 = k_2\theta_2' r_2 = k_3\ \theta_3' r_3 \qquad \text{...(2)}$$

From (2) $$r_2 = r_1\left(\frac{k_1\ \theta_1'}{k_2\ \theta_2'}\right)$$

$\Rightarrow$ $$r_2 = r_1\left(\frac{\theta_2\ \theta_1'}{\theta_1\ \theta_2'}\right)$$ [using (1)]

Similarly, $$r_3 = r_1\left(\frac{\theta_3}{\theta_1}\cdot\frac{\theta_1'}{\theta_3'}\right)$$

(b) From (1), if $\theta_2 = \theta_3$

Then $k_2 = k_3$.

From (2) $k_2\theta_2' r_2 = k_3\theta_3' r_3$

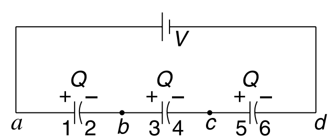
$\Rightarrow$ $\theta_2' r_2 = \theta_3' r_3$

If $\theta_3' > \theta_2'$

It means $r_2 > r_3$

85. Charge on each plate (polarity shown) when capacitors are charged

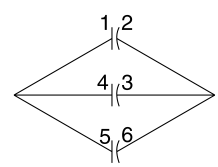
$$Q = \frac{CV}{3}.$$



The cell is removed and the capacitors are connected in parallel using resistors. Plate 1, 4 and 5 are kept together and 2, 3 and 6 are held together.

Total charge available for distribution amongst the capacitors = $Q + Q - Q = Q$.

When charge flow stops, charge on each capacitor = $\frac{Q}{3} = \frac{CV}{9}$.



Heat dissipated = loss in energy stored in capacitor system

$$= \frac{1}{2}\cdot\frac{C}{3}V^2 - \frac{1}{2C}\left(\frac{CV}{9}\right)^2 \times 3$$

$$= \left(\frac{1}{6} - \frac{1}{54}\right)CV^2 = \frac{4}{27}CV^2$$

Heat dissipated in each resistor

$$= \frac{2}{27}CV^2$$

86. (a) Resistance of the air gap

$$R = \rho\frac{L}{A} = 3 \times 10^{13}\cdot\frac{3000}{4\pi(6 \times 10^6)^2} = 199\,\Omega$$

(b) The potential difference between two concentric spheres is

$$V = kq\left(\frac{1}{R} - \frac{1}{R + L}\right)$$

$$= \frac{kq}{R}\left[1 - \left(1 + \frac{L}{R}\right)^{-1}\right]$$

$$= \frac{kq}{R}\left[1 - \left(1 - \frac{L}{R}\right)\right] = \frac{kqL}{R^2}$$

$$= \frac{9 \times 10^9 \times 4 \times 10^5 \times 3000}{(6 \times 10^6)^2} = 3 \times 10^5 \text{ volt.}$$

(c) Capacitance; $$C = \frac{Q}{V} = \frac{4 \times 10^5}{3 \times 10^5} = \frac{4}{3}\text{ F.}$$

..

Time constant $\tau = RC = 199 \times \frac{4}{3} = 265.3$ s.

Time needed for 63% discharging $= \tau = 265.3$ s.

87. The final voltage across A will be

$$V_1 = \frac{C_2}{C_1 + C_2} V = \frac{2}{3+2} \times 100 = 40 \text{ V}$$

and across B it will be

$$V_2 = \frac{C_1}{C_1 + C_2} \cdot V = \frac{3}{3+2} \times 100 = 60 \text{ V}$$

Obviously, capacitor B will breakdown first as soon as voltage across it reaches 50 V (at that time voltage across A will be less than 40 V]

The effective capacitance of the circuit is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 2}{3 + 2} = \frac{6}{5} \mu\text{F}$$

The effective resistance is $R = 3 + 2 = 5$ MΩ.

Time constant of the circuit $\tau = RC = 6$ sec

Charge on capacitors as a function of time

$$q = q_0[1 - e^{-t/\tau}]$$

$$q_0 = CV = \frac{6}{5} \mu\text{F} \times 100 \text{ V} = 120 \ \mu\text{C}$$

$$q = (120 \ \mu C)(1 - e^{-t/\tau})$$

Breakdown of B will take place when

$$Q = 2\mu\text{F} \times 50 \text{ V} = 100 \ \mu\text{C}$$

$$\therefore \quad 100 \ \mu\text{C} = (120 \ \mu\text{C}) \ (1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{5}{6} = 1 - e^{-t/\tau}$$

$$\Rightarrow \quad e^{-t/\tau} = \frac{1}{6}$$

$$\frac{t}{\tau} = \ell n 6$$

$$t = \tau \ell n 6 = 6 \ell n 6 \text{ sec.}$$

88. When switch is in position 1, potential difference across R_2 is

$$V = \frac{R_2}{R_1 + R_2} V_1 = \frac{60}{20 + 60} \times 40 = 30 \text{ volt.}$$

This is also the potential difference across the capacitor with plate b positive.

Charge on capacitor is

$$q_0 = CV = (0.5 \ \mu\text{F}) \ (30 \text{ V}) = 15 \ \mu\text{C}.$$

As per sign convention given in the problem, initial charge (q_0) on the capacitor is negative

Let charge on it be q at time t. Then

$$R_3 \frac{dq}{dt} + \frac{q}{C} = V_2$$

$$R_3 \frac{dq}{dt} = V_2 - \frac{q}{C}$$

$$\Rightarrow \quad \int_{q_0}^{q} \frac{dq}{V_2 - \frac{q}{C}} = \frac{1}{R_3} \int_0^t dt$$

$$\Rightarrow \quad \left[\ell n\left(V_2 - \frac{q}{C}\right)\right]_{q_0}^{q} = -\frac{t}{R_3C}$$

$$\Rightarrow \quad \ell n\left(V_2 - \frac{q}{C}\right) - \ell n\left(V_2 - \frac{q_0}{C}\right) = -\frac{t}{R_3C}$$

$$\Rightarrow \quad \frac{V_2 - q/C}{V_2 - q_0/C} = e^{-t/R_3C}$$

$$\Rightarrow \quad V_2 - \frac{q}{C} = \left(V_2 - \frac{q_0}{C}\right)e^{-\frac{t}{R_3C}}$$

$$\Rightarrow \quad q = C\left[V_2 - \left(V_2 - \frac{q_0}{C}\right)e^{-t/R_3C}\right]$$

$$\therefore \quad q = CV_2 - (CV_2 - q_0)\, e^{-t/R_3C}$$

Now $\quad CV_2 = 0.5\ \mu\text{F} \times 90\ \text{V} = 45\ \mu\text{C}$

$$q_0 = -15\ \mu\text{C}$$

$$R_3C = (400\,\text{k}\Omega)\ (0.5\ \mu\text{F}) = 0.2\ \text{s}$$

$$\therefore \quad \frac{1}{R_3C} = \frac{1}{0.2} = 5$$

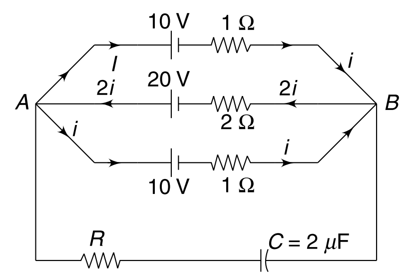
$$\therefore \quad q = [45 - 60\,e^{-5t}]\ \mu\text{C}.$$

$\therefore$ Potential difference across C is

$$V = \frac{q}{C} = (90 - 120\ e^{-5t})\ \text{volt}.$$

89. There will be no current through the branch having the capacitor.

Using Kirchhoff's voltage law in a loop containing 20 V cell and one 10 V cell, we get



$$2i(2) + i(1) = 20 - 10$$

$$i = 2A$$

$$\therefore \quad V_A - V_B = 12\ \text{volt}.$$

[OR, you can find the equivalent emf of the three cells in parallel]

$$\therefore \quad q = (V_A - V_B)\ C = 24\ \mu\text{C}.$$

90. Force between capacitor plates is

$$F = \frac{q_0^2}{2\epsilon_0 A}$$

The spring must be compressed by x such that $kx = F$

$$\Rightarrow \quad x = \frac{q_0^2}{2A\epsilon_0 k}$$

$\therefore$ Separation between plates $d = L - x = L - \dfrac{q_0^2}{2A\,\epsilon_0 k}$

Capacitance has initial value of $C_0 = \dfrac{\epsilon_0 A}{d}$

Energy stored in the capacitor is

$$U_0 = \frac{q_0^2}{2C_0} = \frac{q_0^2 d}{2\epsilon_0 A}$$

$$= \frac{q_0^2}{2\epsilon_0 A}\left(L - \frac{q_0^2}{2A\epsilon_0 k}\right) = \frac{q_0^2 L}{2\epsilon_0 A} - \frac{q_0^4}{4\epsilon_0^2 A^2 k}$$

Energy stored in the spring is

$$U_s = \frac{1}{2}kx^2 = \frac{q_0^4}{8\epsilon_0^2 A^2 k}$$

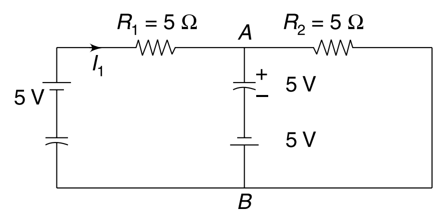
The entire energy stored in the capacitor and the spring gets dissipated as heat in the resistor.

$$\therefore \quad \text{Heat} = U_0 + U_s = \frac{q_0^2 L}{2\epsilon_0 A} - \frac{q_0^4}{8\epsilon_0^2 A^2 k}$$

91. The spring will get no time to expand during the discharge and the heat dissipated will be = U_0

$$= \frac{q_0^2 L}{2\epsilon_0 A} - \frac{q_0^4}{4\epsilon_0^2 A^2 k}$$

92. With $S1$ closed (and $S2$ open) for long time the charge on capacitance C_1 will become C_1E_1 and potential difference across it will be E_1. Potential difference between A and B is zero. After $S2$ is closed, the potential difference across A and B cannot change instantaneously (since charge on the capacitor cannot change instantaneously)



Hence current through R_2 is zero immediately after S_2 is closed.

For $\quad V_A - V_B = 0$

$$I_1R_1 = 5,\ I = 1\ A$$

93. Just before the switch is opened

$$V_R = IR = 10 \text{ Volt.}$$

$\therefore$ p.d. across C_1 at this instant

$$V_0 = V_R = 10 \text{ Volt.}$$

Energy stored in C_1 at this instant is

$$U_1 = \frac{1}{2}C_1V_0^2 = \frac{1}{2} \times 3 \times 10^{-6} \times 10^2 = 150\ \mu\text{J.}$$

Charge on C_1 at this instant $\quad Q_0 = 3 \times 10^{-6} \times 10$

$$= 30\ \mu\text{C}$$

Now this charge gets shared between C_1 and C_2 so that p.d across both of them becomes equal.

Final common p.d

$$V = \frac{Q_0}{C_1 + C_2} = \frac{30\ \mu\text{C}}{4\ \mu\text{F}} = 7.5 \text{ Volt.}$$

$\therefore$ Final energy stored in the capacitor system

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (7.5)^2$$

$$= 28.13\ \mu\text{J}$$

$\therefore$ Heat liberated = Energy lost

$$= 150 - 28.13 = 121.87\ \mu\text{J}$$

94. Conductor of radius $2a$ gets charged to a potential V when S_1 is closed.

Capicitance $\quad C_1 = 4\pi\epsilon_0(2a) = 8\pi\epsilon_0 a.$

Charge on the sphere $\quad Q_0 = C_1V = 8\pi\epsilon_0 aV$

After S_2 is closed, both the spheres will acquire same potential. Let final charge on the two spheres be q_1 and q_2.

$$q_1 + q_2 = Q_0 \qquad \text{...(1)}$$

$$\frac{q_1}{8\pi\in_0 a} = \frac{q}{4\pi\in_0 a} \qquad \left[\because C_2 = \text{capacitance of small sphere} = 4\pi\varepsilon_0 a = \frac{C_1}{2}\right]$$

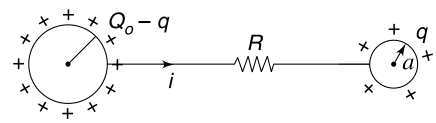
$$q_1 = 2q_2 \qquad ...(2)$$

Solving (1) and (2)

$$q_1 = \frac{2Q_0}{3} = \frac{16\pi\in_0 aV}{3}$$

$$q_2 = \frac{Q_0}{3} = \frac{8\pi\in_0 aV}{3}$$

(a) let the charge on bigger sphere be $(Q_0 - q)$ and that on smaller sphere be q, at time t' after S_2 is closed.



$$\frac{dq}{dt} = i = \text{current through } R$$

Potential difference between two spheres = iR

$$\frac{Q_0 - q}{C_1} - \frac{q}{C_2} = iR$$

$$\frac{Q_0}{C_1} - q\left(\frac{C_1 + C_2}{C_1 C_2}\right) = iR$$

differentiating, w.r.t. time

$$-\frac{C_1 + C_2}{C_1 C_2}\frac{dq}{dt} = R\frac{di}{dt}$$

$$\Rightarrow \qquad -\frac{3}{8\pi\in_0 a} i = R\frac{di}{dt}$$

$$\Rightarrow \qquad \int_{i_0}^{i}\frac{di}{i} = -\frac{3}{8\pi\in_0 aR}\int_0^t dt$$

$$\Rightarrow \qquad \ell n\frac{i}{i_0} = -\frac{3t}{8\pi\in_0 aR}$$

$$\Rightarrow \qquad i = i_0\, e^{-\frac{3t}{(8\pi\in_0 a)R}} = \frac{V}{R}\, e^{-\frac{3t}{(8\pi\in_0 a)R}}$$

$$i_0 = \text{current at } t = 0^+ = \frac{V}{R}$$

Rate of change of potential

$$V = \frac{Q - q}{C}$$

$$\frac{dV}{dt} = -\frac{1}{C}\frac{dq}{dt} = -\frac{i}{C}$$

$$\left|\frac{dV}{dt}\right| = \frac{i}{C} = \frac{V}{(8\pi\in_0 a)R}\, e^{-\frac{3t}{8\pi\in_0 aR}}$$

(b) Heat dissipated = loss in electrostatic potential energy

$$= \frac{Q_0^2}{2C_1} - \left(\frac{q_1^2}{2C_1} + \frac{q_2^2}{\frac{2\cdot C_1}{2}}\right)$$

$$= \frac{Q_0^2}{2C_1} - \left[\frac{2Q_0^2}{9C_1} + \frac{Q_0^2}{9C_1}\right]$$

$$= \frac{Q_0^2}{2C_1} - \frac{Q_0^2}{3C_1} = \frac{Q_0^2}{6C_1} = \frac{(C_1V)^2}{6C_1}$$

$$= \frac{C_1V^2}{6} = \frac{4}{3}\pi\epsilon_0 aV^2.$$

95. $$q_0 = C_1V_1 = 200\ \mu C$$

Maximum charge on C_2 is $(1 - e^{-1})\, q_0 = 0.63 \times 200 = 126\ \mu C$

$\therefore$ $$C_2V_2 = 126\ \mu C \quad \Rightarrow \quad C_2 = 3\ \mu F$$

$$i_0 = \frac{V_1}{R_1} = 0.2A$$

Maximum current in the second circuit is

$$\frac{126}{200} \times 0.2 = \frac{V_2}{R_2}$$

$\Rightarrow$ $$0.126A = \frac{42}{R_2}$$

$$R_2 = \frac{1000}{3}\ \Omega$$

Time constant of the second circuit is $\tau_2 = R_1C_2 = \frac{1000}{3} \times 3\ \mu s$

$$= 100\ \mu s$$

$$= 10^{-3} s$$

$\therefore$ $$i = 0.126\ e^{-t/\tau_2}$$

$$i = 126\ e^{-1000t}$$ [t is in second]

96. Initially when 'S' is in position 1 or 2 the capacitor gets charged. This continues till p.d across capacitor becomes 3.5 volt.

Now, when in position 1, the current is

$$i = \frac{0.5\,\text{V}}{1\,\Omega} = 0.5\,\text{A}.$$

This current discharges the capacitor. Its charge gets reduced by $\Delta q = i\Delta t = 0.5\,\Delta t$

When in position 2, the current is once again $i = \frac{0.5\ \text{V}}{1\Omega} = 0.5$ A but this time it charges the capacitor.

Charge gained in time Δt is

$$\Delta q = 0.5\ \Delta t$$

Therefore, once the capacitor gets charged to 3.5 V, charge on it will become almost constant.

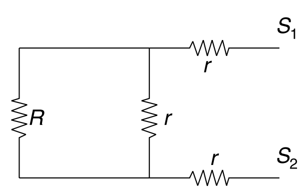
97. (a) Let's first find the equivalent resistance of the infinite ladder network between S_1 and S_2.

Since, there are infinite number of sets of resistance, removing one set will not affect the overall resistance.

Let the equivalent resistance be R.

Then the equivalent of resistance network can be drawn as in figure.

Equivalent resistance between S_1 and S_2 is also $\simeq R$

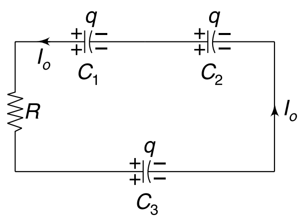


$\therefore$ $$R = \frac{Rr}{R + r} + 2r$$

or, $$R^2 - 2rR - 2r^2 = 0$$

or, $$R = \frac{2r \pm \sqrt{4r^2 + 8r^2}}{2}$$

$$= r(1 + \sqrt{3})$$



[Negative sign is impossible as R cannot be negative]

$$= \frac{10}{2.732} \times 2.732$$

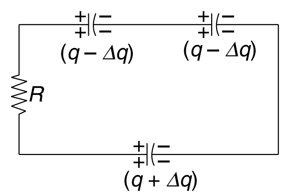
$$= 10\ \Omega.$$

Now at initial moment the circuit arrangement looks like the one shown in second figure.

where I_0 = current just after closing of switches

From Kirchhoff's voltage law, we get

$$\frac{q}{C_1} + \frac{q}{C_2} - \frac{q}{C_3} - I_0 R = 0$$



or, $$\frac{30}{3} + \frac{30}{6} - \frac{30}{6} - I_0 \times 10 = 0$$

or, $$I_0 = 1.0 \text{ Amp. Answers}$$

(b) Initially, the energy stored in the capacitors is

$$U_i = \frac{q^2}{2C_1} + \frac{q^2}{2C_2} + \frac{q^2}{2C_3}$$

$$= \frac{900}{2 \times 3} + \frac{900}{2 \times 6} + \frac{900}{2 \times 6}$$

$$= 300\ \mu\text{J}$$

Let Δq charge flow through the circuit till a steady state is reached. Then charges on C_1, C_2 and C_3 will be as shown in third figure.

For steady state

$$\frac{q - \Delta q}{C_1} + \frac{q - \Delta q}{C_2} - \frac{q + \Delta q}{C_3} = 0$$

or, $$\Delta q = 15\ \mu\text{C}$$

Thus, final energy stored in capacitors is

$$U_f = \frac{(q - \Delta q)^2}{2C_1} + \frac{(q - \Delta q)^2}{2C_2} + \frac{(q + \Delta q)^2}{2C_3}$$

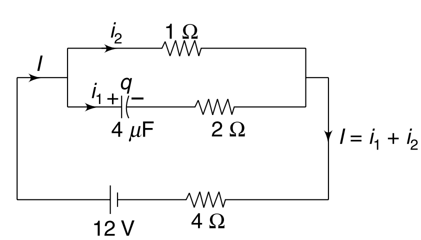
$$= \frac{15^2}{2 \times 3} + \frac{15^2}{2 \times 6} + \frac{45^2}{2 \times 6} = 225\ \mu\text{J}$$

$\therefore$ Heat generated = loss in stored energy

$$= U_i - U_f = 75\ \mu\text{J}$$

98. (a) Situation at time t is as shown in the figure.

Applying Kirchhoff's loop law in outer loop we get



$$1(i_2) + 4(i_1 + i_2) = 12$$

$$\Rightarrow \quad 4i_1 + 5i_2 = 12 \quad \Rightarrow \quad i_2 = \frac{12 - 4i_1}{5} \qquad ...(1)$$

Applying Kirchhoff's law in the loop having the capacitor and 1Ω resistor.

$$\frac{q}{C} + 2i_1 = 1(i_2)$$

$$\Rightarrow \qquad \frac{q}{4} + 2i_1 = \frac{12 - 4i_1}{5} \qquad [q \text{ is in } \mu\text{C}]$$

$$\Rightarrow \qquad 14 i_1 = 12 - \frac{5}{4} q$$

$$\Rightarrow \qquad 14 \frac{dq}{dt} = 12 - \frac{5}{4} q$$

(b)
$$\int_0^q \frac{dq}{12 - \frac{5}{4} q} = \frac{1}{14} \int_0^t dt$$

$$\Rightarrow \qquad \left[\ell n\left(12 - \frac{5}{4}q\right)_0^q \right] = -\frac{5t}{56}$$

$$\Rightarrow \qquad \ell n\left(\frac{12 - \frac{5}{4}q}{12}\right) = -\frac{5t}{56}$$

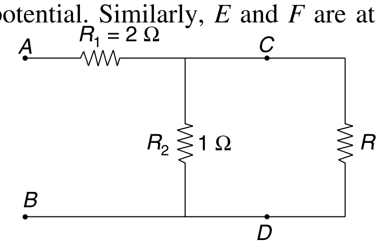
$$\Rightarrow \qquad 1 - \frac{5}{48} q = e^{-\frac{5t}{56}}$$

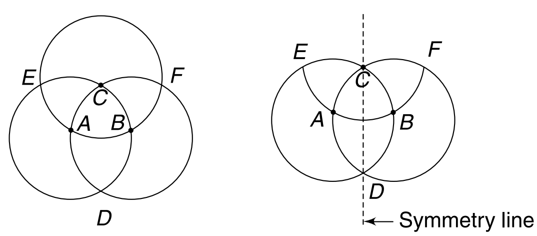
$$\Rightarrow \qquad q = \frac{48}{5}\left[1 - e^{-\frac{5t}{56}}\right]$$

(c) At $t = 0$, $q = 0$

At $t = \infty$, $q = \frac{48}{5}$ μC.

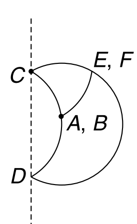
99. If a battery is connected across points C and D, points A and B will be at same potential. Similarly, E and F are at same potential. There will be no current in the resitance directly connecting A to B and E to F. The circuit can be redrawn as

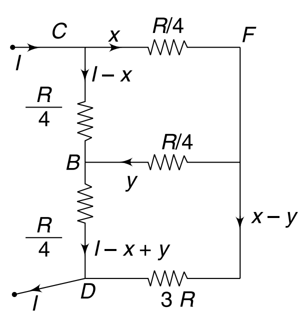




$$R_{AC} = R_{CB} = R_{EC} = R_{FC} = R_{AD} = R_{BD} = \frac{3R}{6} = \frac{R}{2}$$

The circuit can be folded about the line of symmetry shown in the figure so that E overlaps F and A falls on B.





The circuit has been redrawn as shown in figure. We assume a current I flowing into the circuit at C. The current has been distributed using x and y as unknowns.

Using Kirchhoff's laws we can write following equations–

$$2x + y = I \qquad ...(1)$$

$$13x - 14y = I \qquad ...(2)$$

Solving $$x = \frac{15I}{41} \quad \text{and} \quad y = \frac{11I}{41}$$

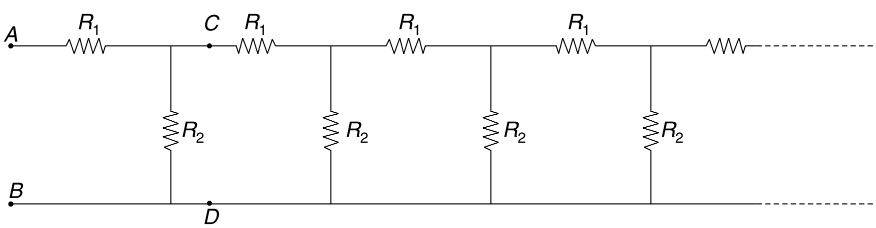
Now $$V_C - V_D = \frac{R}{4}(I - x) + \frac{R}{4}(I - x + y)$$

$$= \frac{R}{4}[2I - 2x + y] = \frac{R}{4}\cdot\left(\frac{63}{41}I\right)$$

$\therefore$ $$\frac{V_C - V_D}{I} = \frac{63}{164}R$$

$\therefore$ Equivalent resistance is $\frac{63R}{164}$.

100. (a) Equivalent across A and $B = R$



If first pair of R_1 and R_2 is removed than equivalent across C and D will remain R. Hence, the circuit can be drawn as shown. Equivalent of this network across A and B is R.

$\therefore$ $$\frac{RR_2}{R + R_2} + R_1 = R$$

Solving $$R = \frac{R_1 \pm \sqrt{R_1^2 + 4R_1R_2}}{2}$$

Since R cannot be negative

$$R = \frac{R_1 + \sqrt{R_1^2 + 4R_1R_2}}{2} = \frac{2 + \sqrt{4 + 4 \times 2 \times 1}}{2}$$

$$= (1 + \sqrt{3})\ \Omega$$

(b) The equivalent across M and N after removing all the previous sections is still R (the network is infinite!)

$$\left(I_{n-1} - I_n\right)R_2 = I_nR$$

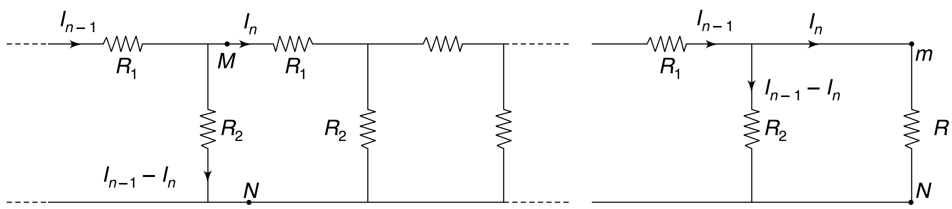
$\Rightarrow$ $$I_n = \frac{I_{n-1}R_2}{R_2 + R} = \frac{I_{n-1}}{\sqrt{3} + 2} \qquad ...(1)$$

(c) $$I_1 = \frac{20}{R} = \frac{20}{\sqrt{3} + 1}$$

Using (1) $$I_2 = \frac{I_1}{\sqrt{3} + 2} = \frac{20}{(\sqrt{3} + 1)(\sqrt{3} + 2)}$$

$$I_3 = \frac{I_2}{\sqrt{3}+2} = \frac{20}{(\sqrt{3}+1)(\sqrt{3}+2)^2}$$

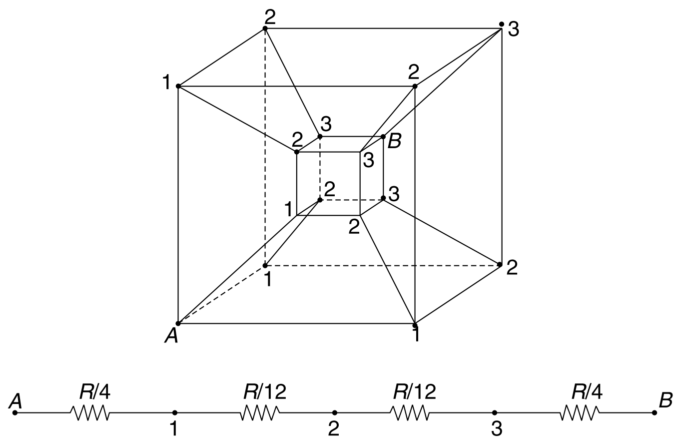
Similarly, we get $$I_{10} = \frac{20}{(\sqrt{3}+1)(\sqrt{3}+2)^9}$$



101. Due to Symmetry

Points marked 1 are equipotential points.

Similarly all 2s are at same potential & all 3s are also at same potential.



102. Equivalent resistance across $A - B$ is given by

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2 + R_3 + R_4} \quad ...(1)$$

Resistance across $C - D$ is

$$\frac{1}{R_{CD}} = \frac{1}{R_3} + \frac{1}{R_2 + R_1 + R_4} \quad ...(2)$$

Same power will be dissipated in two cases if

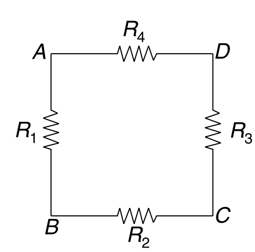
$$R_{AB} = R_{CD} \Rightarrow R_1 = R_3$$

[Have a careful look at (1) and (2). Don't try any mathematics]

Similarly, one can prove that $R_2 = R_4$.

When cell is connected across $B - C$(or $A - D$) the equivalent resistance is

$$\frac{1}{R_{BC}} = \frac{1}{R_2} + \frac{1}{R_1 + R_2 + R_4}$$



Given $$\frac{\varepsilon^2}{R_{AB}} = P_0 \text{ and } \frac{\varepsilon^2}{R_{BC}} = 3P_0$$

$\therefore$ $$\frac{1}{R_{BC}} = \frac{3}{R_{AB}}$$

$$\Rightarrow \quad \frac{1}{R_2} + \frac{1}{2R_1 + R_2} = 3\left[\frac{1}{R_1} + \frac{1}{R_1 + 2R_2}\right]$$

$$\Rightarrow \quad \frac{2R_1 + 2R_2}{2R_1R_2 + R_2^2} = 3\left(\frac{2R_1 + 2R_2}{R_1^2 + 2R_1R_2}\right)$$

$$\Rightarrow \quad R_1^2 + 2R_1R_2 = 3R_2^2 + 6R_1R_2$$

$$\Rightarrow \quad R_1^2 - 4R_2\cdot R_1 - 3R_2^2 = 0$$

$$\Rightarrow \quad R_1 = \frac{4R_2 \pm \sqrt{16R_2^2 + 12R_2^2}}{2}$$

$$R_1 = 2R_2 + \sqrt{7}R_2 = (2 + \sqrt{7})R_2$$

$$\therefore \quad P_0 = \frac{\varepsilon^2}{R_{AB}} = \varepsilon^2\left[\frac{1}{(2 + \sqrt{7})R_2} + \frac{1}{(4 + \sqrt{7})R_2}\right]$$

$$= \frac{\varepsilon^2}{R_2}\left[\frac{6 + 2\sqrt{7}}{15 + 6\sqrt{7}}\right] = \frac{2\varepsilon^2}{R_2}\ \frac{(3 + \sqrt{7})}{(15 + 6\sqrt{7})}$$

Resistance across $A - C$ is

$$\frac{1}{R_{AC}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} = \frac{1}{R_1 + R_2} + \frac{1}{R_1 + R_2}$$

$$= \frac{2}{R_1 + R_2} = \frac{2}{(3 + \sqrt{7})R_2}$$

$$\therefore \quad P_{AC} = \frac{\varepsilon^2}{R_{AC}} = \frac{2\varepsilon^2}{(3 + \sqrt{7})R_2}$$

$$= P_0\,\frac{(15 + 6\sqrt{7})}{(3 + \sqrt{7})(3 + \sqrt{7})}$$

$$= \left(\frac{15 + 6\sqrt{7}}{16 + 6\sqrt{7}}\right)P_0$$

103. After the switch is closed, the capacitor gets (almost) fully charged in an interval of $t_0 = 50\ \tau$. In fact, few time constants are good enough to charge it completely.

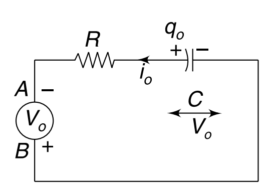
During charging the charge on the capacitor varies as

$$q = q_0[1 - e^{-t/\tau}] \qquad [q_0 = CV_0]$$

After the polarity of source gets reversed, the capacitor will get discharged and then it will get recharged with polarity of plates changed. There is enough time to assume that the capacitor is fully charged with polarity changed. The situation at time t_0^+ is shown below. Clearly, the potential difference across R is $2V_0$. Current is $i_0 = \dfrac{2V_0}{R}$.

This is maximum current in the circuit.

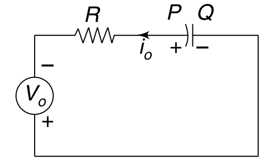
For simplicity in calculation, let's begin counting time at the instant end B becomes positive for the first time. At time 't' the situation is as shown.



Charge on plate Q is taken positive.

$$\frac{q}{C} + V_0 = Ri$$

But $\quad i = -\dfrac{dq}{dt} \qquad [\therefore\ \ q \text{ is decreasing}]$



$$\therefore \qquad -R\frac{dq}{dt} = \frac{q}{C} + V_0$$

$$\Rightarrow \qquad \int_{q_0}^{q} \frac{dq}{\dfrac{q}{RC} + \dfrac{V_0}{R}} = -\int_0^t dt$$

$$\Rightarrow \qquad \left[\ell n\left[\frac{q}{RC} + \frac{V_0}{R}\right]\right]_{q_0 = CV_0}^{q} = \frac{-t}{RC}$$

$$\Rightarrow \qquad \ell n\left(\frac{q}{RC} + \frac{V_0}{R}\right) - \ell n\left(\frac{CV_0}{RC} + \frac{V_0}{R}\right) = \frac{-t}{RC}$$

$$\Rightarrow \qquad \ell n\left(\frac{\dfrac{q}{RC} + \dfrac{V_0}{R}}{2\,\dfrac{V_0}{R}}\right) = \frac{-t}{\tau} \qquad [RC = \tau]$$

$$\Rightarrow \qquad \frac{q}{2CV_0} + \frac{1}{2} = e^{-t/\tau}$$

$$\Rightarrow \qquad q = 2CV_0\left[e^{-t/\tau} - \frac{1}{2}\right]$$

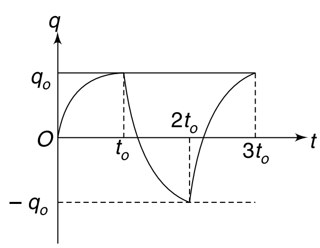
q will became zero when

$$e^{-t/\tau} = \frac{1}{2}$$

$$\therefore \qquad \frac{t}{\tau} = \ell n 2 \quad \Rightarrow \quad t = (\ell n 2)\,\tau$$

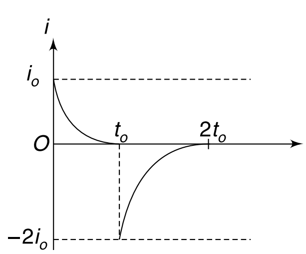
Here we have taken q to be positive when plate Q is positive. But question asks us to take P as positive. This needs to be taken care while plotting the graph.

(a)

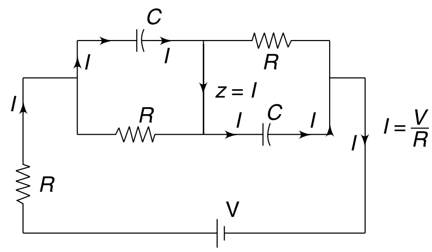


(b) $i_{\max} = \dfrac{2V_0}{R}$

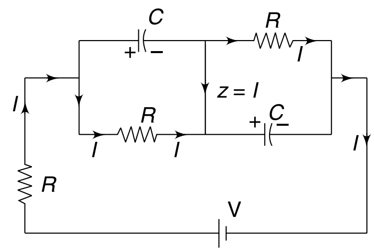
(c) current can be obtained as $\dfrac{dq}{dt}$.



104. (a) at $t = 0^+$ the current flow will be as shown below

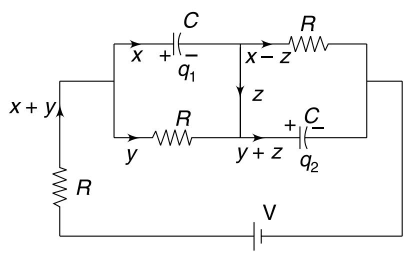


(b) At $t = \infty$, the current flow is as shown below



$$I = \frac{V}{3R} \qquad \text{(in direction shown)}$$

(c) At time t let the charge on capacitors and current in different branches be as shown



The relevant equations are

$$R(x + y) + Ry + R(x - z) = V$$

$$\Rightarrow \qquad 2x + 2y - z = \frac{V}{R} \qquad ...(1)$$

$$\frac{q_1}{C} = Ry \qquad ...(2)$$

$$\frac{q_2}{C} = R(x - z) \qquad ...(3)$$

$$\frac{dq_1}{dt} = x \qquad ...(4)$$

$$\frac{dq_2}{dt} = y + z \qquad ...(5)$$

From (2) $$\frac{dq_1}{dt} = RC\frac{dy}{dt}$$

$$\Rightarrow \qquad x = RC\frac{dy}{dt} \qquad ...(6)$$

From (3) $$\frac{dq_2}{dt} = RC\left(\frac{dx}{dt} - \frac{dz}{dt}\right)$$

$$\Rightarrow \quad y + z = RC\left(\frac{dx}{dt} - \frac{dz}{dt}\right) \quad ...(7)$$

(6) + (7) $$\frac{x + y + z}{RC} = \frac{dx}{dt} + \frac{dy}{dt} - \frac{dz}{dt} \quad ...(8)$$

From equation (1) $$x + y = \frac{V}{2R} + \frac{z}{2}$$

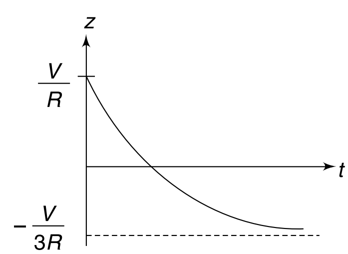
Also $$\frac{dx}{dt} + \frac{dy}{dt} = \frac{1}{2}\frac{dz}{dt}$$

Put both the above results into equation (8)

$$\frac{V}{2R} + \frac{z}{2} + z = \left[\frac{1}{2}\frac{dz}{dt} - \frac{dz}{dt}\right] RC$$

$$\frac{V}{R} + 3z = -RC\frac{dz}{dt}$$

$$\int_{z=\frac{V}{R}}^{z} \frac{dz}{\frac{V}{R} + 3z} = -\frac{1}{RC}\int_{t=0}^{t} dt$$



$$\left[\ell n\left(\frac{V}{R} + 3z\right)\right]_{\frac{V}{R}}^{z} = -\frac{3t}{RC}$$

$$\ell n\left(\frac{V}{R} + 3z\right) - \ell n\left(\frac{4V}{R}\right) = -\frac{3t}{RC}$$

$$\ell n\left(\frac{1}{4} + \frac{3Rz}{4V}\right) = -\frac{3t}{RC}$$

$$z = \frac{4V}{3R}\left[e^{-\frac{3t}{RC}} - \frac{1}{4}\right]$$

(d) $$z = 0 \quad \text{when} \quad e^{-\frac{3t}{RC}} = \frac{1}{4}$$

$$\frac{3t}{RC} = \ell n 4$$

$$t = \frac{2RC}{2}\,\ell n 2$$

105. For $0 \le t \le t_0$

$$V_0 = \frac{q}{C} + Ri$$

Where q and i are instantaneous values of charge on the capacitor and the current.

$$kt = \frac{q}{C} + Ri$$

Differentiate wrt time (t)

$$k = \frac{1}{C}\frac{dq}{dt} + R\frac{di}{dt}$$

$$\Rightarrow \quad k - \frac{i}{C} = R\frac{di}{dt}$$

$$\Rightarrow \qquad \int_0^i \frac{di}{k - \frac{i}{C}} = \frac{1}{R}\int_0^t dt$$

$$\Rightarrow \qquad \left[\ell n\left(k - \frac{i}{C}\right)\right]_0^i = -\frac{t}{RC}$$

$$\Rightarrow \qquad \ell n\left(k - \frac{i}{C}\right) - \ell n k = -\frac{t}{RC}$$

$$\Rightarrow \qquad \ell n\left(1 - \frac{i}{kC}\right) = -\frac{t}{RC}$$

$$\Rightarrow \qquad i = kC\left(1 - e^{-t/RC}\right)$$

$$\therefore \qquad V = Ri = kCR\left(1 - e^{-t/RC}\right)$$

Hence voltage across C and D increases as per above equation till $t = t_0$.

Let the current (i) and voltage V at $t = t_0$ be

$$i_1 = kC\left(1 - e^{-t_0/RC}\right)$$

$$V_1 = kCR\left(1 - e^{-t_0/RC}\right)$$

For $t > t_0$, the applied voltage remains constant at kt_0.

For simplicity, let's begin our time count from this instant itself.

$$kt_0 = \frac{q}{C} + iR$$

Differentiate wrt 't'

$$\frac{1}{C}\frac{dq}{dt} + R\frac{di}{dt} = 0$$

$$\Rightarrow \qquad \frac{1}{C}i + R\frac{di}{dt} = 0$$

$$\Rightarrow \qquad \int_{i_1}^{i} \frac{di}{i} = -\frac{1}{RC}\int_0^t dt$$

$$\Rightarrow \qquad \ell n\left(\frac{i}{i_1}\right) = -\frac{t}{RC}$$

$$\therefore \qquad i = i_1 e^{-t/RC}$$

$$\therefore \qquad V = iR = i_1 R\, e^{-t/RC}$$

Hence V decresses exponentially after $t = t_0$.

The graph will be as shown in the figure