

Matrices

Question1

Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x)f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below

[27-Jan-2024 Shift 1]

Options:

A.

Statement I is false but Statement II is true

B.

Both Statement I and Statement II are false

C.

Statement I is true but Statement II is false

D.

Both Statement I and Statement II are true

Answer: D

Solution:

$$f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence statement- I is correct

Now, checking statement II

$$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(x) \cdot f(y) = f(x+y)$$

Hence statement-II is also correct.

Question2

Let A be a 2×2 real matrix and I be the identity matrix of order 2 . If the roots of the equation $|A - xI| = 0$ be -1 and 3 , then the sum of the diagonal elements of the matrix A^2 is.....

[27-Jan-2024 Shift 2]

Answer: 10

Solution:

$$|A - xI| = 0$$

Roots are -1 and 3

$$\text{Sum of roots} = \text{tr}(A) = 2$$

$$\text{Product of roots} = |A| = -3$$

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{We have } a + d = 2$$

$$ad - bc = -3$$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

$$\text{We need } a^2 + bc + bc + d^2$$

$$= a^2 + 2bc + d^2$$

$$= (a + d)^2 - 2ad + 2bc$$

$$= 4 - 2(ad - bc)$$

$$= 4 - 2(-3)$$

$$= 4 + 6$$

$$= 10$$

Question3

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$, Then a value of α is

[29-Jan-2024 Shift 1]

Options:

A.

3

B.

5

C.

17

D.

9

Answer: B

Solution:

$$|A| = \alpha^2 - \beta^2$$

$$|2A|^3 = 2^{21} \Rightarrow |A| = 2^4$$

$$\alpha^2 - \beta^2 = 16$$

$$(\alpha + \beta)(\alpha - \beta) = 16 \Rightarrow \alpha = 4 \text{ or } 5$$

Question4

Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A[(A + A^T)^2 + (A - A^T)^2]$ is equal to

[29-Jan-2024 Shift 1]

Options:

A.

$$A^2 + I$$

B.

$$A^3 + I$$

C.

$$A^2 + A^T$$

D.

$$A^3 + A^T$$

Answer: D

Solution:

$$AA^T = I = A^T A$$

On solving given expression, we get

$$\begin{aligned} & \frac{1}{2}A[A^2 + (A^T)^2 + 2AA^T + A^2 + (A^T)^2 - 2AA^T] \\ &= A[A^2 + (A^T)^2] = A^3 + A^T \end{aligned}$$

Question5

Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $|P^{-1}AP - 2I|$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

26

B.

27

C.

66

D.

23

Answer: A

Solution:

$$|P^{-1}AP - 2I| = |P^{-1}AP - 2P^{-1}P|$$

$$= |P^{-1}(A - 2I)P|$$

$$= |P^{-1}| |A - 2I| |P|$$

$$= |A - 2I|$$

$$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$$

So, Prime factor of 69 is 3 & 23

So, sum = 26

Question6

Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3×3 matrix, where

$$x \sin \theta = y \sin \left(\theta + \frac{2\pi}{3} \right) = z \sin \left(\theta + \frac{4\pi}{3} \right) \neq 0, \theta \in (0, 2\pi).$$

For a square matrix M , let trace (M) denote the sum of all the diagonal entries of M . Then, among the statements:

(I) Trace (R) = 0

(II) If trace(adj (adj(R))) = 0, then R has exactly one non-zero entry.

[30-Jan-2024 Shift 2]

Options:

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

Answer: C

Solution:

$$x \sin \theta = y \sin \left(\theta + \frac{2\pi}{3} \right) = z \sin \left(\theta + \frac{4\pi}{3} \right) \neq 0$$

$$\Rightarrow x, y, z \neq 0$$

Also,

$$\sin \theta + \sin \left(\theta + \frac{2\pi}{3} \right) + \sin \left(\theta + \frac{4\pi}{3} \right) = 0 \quad \forall \theta \in \mathbb{R}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

$$\Rightarrow xy + yz + zx = 0$$

$$(i) \quad \text{Trace}(R) = x + y + z$$

$$\text{If } x + y + z = 0 \text{ and } xy + yz + zx = 0$$

$$\Rightarrow x = y = z = 0$$

Statement (i) is False

$$(ii) \quad \text{Let } P : \text{trace}(\text{Adj}(\text{Adj}(R))) = 0$$

$Q : R$ has exactly one non zero entry

if P is false then $P \rightarrow Q$ is always true

Statement (ii) is True

Question7

Let A be a 3×3 matrix and $\det(A) = 2$. If

$$n = \det(\underbrace{\text{adj}(\text{adj}(\dots \text{adj}(A)))}_{2024 \text{ times}})$$

Then the remainder when n is divided by 9 is equal to _____

[31-Jan-2024 Shift 2]

Answer: 7

Solution:

$$|A| = 2$$

$$\underbrace{\text{adj}(\text{adj}(\text{adj} \dots (a)))}_{2024 - \text{times}} = |A|^{(n-1)^{2024}}$$

$$= |A|^{2024}$$

$$= 2^{2^{2024}}$$

$$2^{2024} = (2^2)2^{2022} = 4(8)^{674} = 4(9-1)^{674}$$

$$\Rightarrow 2^{2024} \equiv 4 \pmod{9}$$

$$\Rightarrow 2^{2024} \equiv 9m + 4, m \leftarrow \text{even}$$

$$2^{9m+4} \equiv 16 \cdot (2^3)^{3m} \equiv 16 \pmod{9}$$

$$\equiv 7$$

Question8

If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $C = ABA^T$ and $X = A^T C^2 A$, then $\det X$ is equal to :

[1-Feb-2024 Shift 1]

Options:

A.

243

B.

729

C.

27

D.

891

Answer: B

Solution:

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$$

$$\text{Now } C = ABA^T \Rightarrow \det(C) = (\det(A))^2 \times \det(B)$$

$$|C| = 9$$

$$\text{Now } |X| = |A^T C^2 A|$$

$$= |A^T| |C|^2 |A|$$

$$= |A|^2 |C|^2$$

$$= 9 \times 81$$

$$= 729$$

Question9

Let $A = I_2 - 2MM^T$, where M is real matrix of order 2×1 such that the relation $M^T M = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1 , then the sum of squares of all possible values of λ is equal to :

[1-Feb-2024 Shift 2]

Answer: 2

Solution:

$$A = I_2 - 2MM^T$$

$$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$$

$$= I_2 - 2MM^T - 2MM^T + 4MM^T MM^T$$

$$= I_2 - 4MM^T + 4MM^T$$

$$= I_2$$

$$AX = \lambda X$$

$$A^2X = \lambda AX$$

$$X = \lambda(\lambda X)$$

$$X = \lambda^2 X$$

$$X(\lambda^2 - 1) = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

Sum of square of all possible values = 2

Question 10

If **A** and **B** are two non-zero $n \times n$ matrices such that $A^2 + B = A^2B$, then
[24-Jan-2023 Shift 1]

Options:

A. $AB = I$

B. $A^2B = I$

C. $A^2 = I$ or $B = I$

D. $A^2B = BA^2$

Answer: D

Solution:

$$A^2 + B = A^2B$$

$$(A^2 - I)(B - I) = I$$

$$A^2 + B = A^2B$$

$$A^2(B - I) = B$$

$$A^2 = B(B - I)^{-1}$$

$$A^2 = B(A^2 - I)$$

$$A^2 = BA^2 - B$$

$$A^2 + B = BA^2$$

$$A^2B = BA^2$$

Question 11

The number of square matrices of order 5 with entries from the set $\{0, 1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is
[24-Jan-2023 Shift 2]

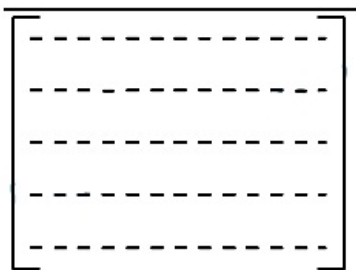
Options:

- A. 225
- B. 120
- C. 150
- D. 125

Answer: B

Solution:

Solution:



In each row and each column exactly one is to be placed -

$\therefore$ No. of such matrices = $5 \times 4 \times 3 \times 2 \times 1 = 120$

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightarrow 5 \text{ ways} \\ \rightarrow 4 \text{ ways} \\ \rightarrow 3 \text{ ways} \\ \rightarrow 2 \text{ ways} \\ \rightarrow 1 \text{ ways} \end{matrix}$$

Step-1 : Select any 1 place for 1 's in row 1. Automatically some column will get filled with 0 's.

Step-2 : From next now select 1 place for 1 's. Automatically some column will get filled with 0 's. $\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways = $5 \times 4 \times 3 \times 2 \times 1 = 120$

Question12

Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj} A))| = 12^4$. Then

$\left| A^{-1} \text{adj} A \right|$ is equal to

[24-Jan-2023 Shift 2]

Options:

- A. $2\sqrt{3}$
- B. $\sqrt{6}$

C. 12

D. 1

Answer: A

Solution:

Solution:

Given $|\text{adj}(\text{adj}(\text{adj} \cdot A))| = 12^4$

$$\Rightarrow |A|^{(n-1)^3} = 12^4$$

We are asked

$$\begin{aligned} & |A^{-1} \cdot \text{adj} A| \\ &= |A^{-1}| \cdot |\text{adj} A| \\ &= \frac{1}{|A|} \cdot |A|^{3-1} \\ &= |A| = 2\sqrt{3} \end{aligned}$$

Question13

Let $x, y, z > 1$ and

$$A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}.$$

Then $|\text{adj}(\text{adj} A^2)|$ is equal to
[25-Jan-2023 Shift 1]

Options:

A. 6^4

B. 2^8

C. 4^8

D. 2^4

Answer: B

Solution:

Solution:

$$|A| = \frac{1}{\log x \cdot \log y \cdot \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & 2 \log y & \log z \\ \log x & \log y & 3 \log z \end{vmatrix} = 2$$

$$\Rightarrow |\text{adj}(\text{adj} A^2)| = |A^2|^4 = 2^8$$

Question14

Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$. If $M = A^T B A$,

then the inverse of the matrix $A M^{2023} A^T$ is
[25-Jan-2023 Shift 2]

Options:

A. $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Answer: D

Solution:

Solution:

$$A A^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

⋮

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$$M = A^T B A$$

$$M^2 = M \cdot M = A^T B A A^T B A = A^T B^2 A$$

$$M^3 = M^2 \cdot M = A^T B^2 A A^T B A = A^T B^3 A$$

⋮

$$M^{2023} = \dots \dots \dots A^T B^{2023} A$$

$$A M^{2023} A^T = A A^T B^{2023} A A^T = B^{2023}$$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Question15

Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then.

[25-Jan-2023 Shift 2]

Options:

- A. Only S2 is true
- B. Only S1 is true
- C. Both S1 and S2 are false
- D. Both S1 and S2 are true

Answer: A

Solution:

Given, $A^T = A, B^T = -B, C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$$

$$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$$

$$= B^{26}A^{13} - A^{13}B^{26} = -M$$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

Question16

Let α and β be real numbers. Consider a 3×3 matrix A such that

$A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

[29-Jan-2023 Shift 1]

Options:

- A. $\alpha = 1$
- B. $\alpha = 4$

C. $\beta = 8$

D. $\beta = -8$

Answer: D

Solution:

Solution:

$$A^2 = 3A + \alpha I$$

$$A^3 = 3A^2 + \alpha A$$

$$A^3 = 3(3A + \alpha I) + \alpha A$$

$$A^3 = 9A + \alpha A + 3\alpha I$$

$$A^4 = (9 + \alpha)A^2 + 3\alpha A$$

$$= (9 + \alpha)(3A + \alpha I) + 3\alpha A$$

$$= A(27 + 6\alpha) + \alpha(9 + \alpha)$$

$$\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$$

$$\Rightarrow \beta = \alpha(9 + \alpha) = -8$$

Question 17

The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2 \cos t) & e^{-t}(-2 \sin t - \cos t) \\ e^t & e^{-t}(2 \sin t + \cos t) & e^{-t}(\sin t - 2 \cos t) \\ e^t & e^{-t} \cos t & e^{-t} \sin t \end{bmatrix}$$

is invertible, is

[29-Jan-2023 Shift 2]

Options:

A. $\left\{ (2k + 1) \frac{\pi}{2}, k \in \mathbb{Z} \right\}$

B. $\left\{ k\pi + \frac{\pi}{4}, k \in \mathbb{Z} \right\}$

C. $\{k\pi, k \in \mathbb{Z}\}$

D. $\mathbb{R}$

Answer: D

Solution:

Solution:

If its invertible, then determinant value $\neq 0$

So,

$$\begin{vmatrix} e^t & e^{-t}(\sin t - 2 \cos t) & e^{-t}(-2 \sin t - \cos t) \\ e^t & e^{-t}(2 \sin t + \cos t) & e^{-t}(\sin t - 2 \cos t) \\ e^t & e^{-t} \cos t & e^{-t} \sin t \end{vmatrix} \neq 0$$

$$\Rightarrow e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \sin t - 2 \cos t & -2 \sin t - \cos t \\ 1 & 2 \sin t + \cos t & \sin t - 2 \cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

Applying, $R_1 \rightarrow R_1 - R_2$ then $R_2 \rightarrow R_2 - R_3$

We get

$$e^{-t} \begin{vmatrix} 0 & -\sin t - \cos t & -3 \sin t + \cos t \\ 0 & 2 \sin t & -2 \cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$$e^{-t} \times 1(2 \sin t \cos t + 6 \cos^2 t + 6 \sin^2 t - 2 \sin t \cos t) \neq 0$$

$$\Rightarrow e^{-t} \times 6 \neq 0$$

for $\forall t \in$

Question18

Let A be a symmetric matrix such that $|A| = 2$ and

$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____.

[29-Jan-2023 Shift 2]

Answer: 5

Solution:

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

Now $ac - b^2 = 2$ and $2a + b = 1$ and $2b + c = 2$

solving all these above equations we get

$$\frac{1-b}{2} \times \left(\frac{2-2b}{1} \right) - b^2 = 2$$

$$\Rightarrow (1-b)^2 - b^2 = 2$$

$$\Rightarrow 1 - 2b = 2$$

$$\Rightarrow b = -\frac{1}{2} \text{ and } a = \frac{3}{4} \text{ and } c = 3$$

$$\text{Hence } \alpha = 3a + \frac{3b}{2} = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$$

$$\text{and } \beta = 3b + \frac{3c}{2} = -\frac{3}{2} + \frac{9}{2} = 3$$

$$\text{also } s = a + c = \frac{15}{4}$$

$$\therefore \frac{\beta s}{\alpha^2} = \frac{3 \times \frac{15}{4}}{4 \times \frac{9}{4}} = 5$$

Question19

Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \neq 0$ $|A - d(\text{Adj } A)| = 0$

Then

[30-Jan-2023 Shift 1]

Options:

A. $(1 + d)^2 = (m + q)^2$

B. $1 + d^2 = (m + q)^2$

C. $(1 + d)^2 = m^2 + q^2$

D. $1 + d^2 = m^2 + q^2$

Answer: A

Solution:

Solution:

$$\begin{aligned} \text{Sol. } A &= \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \quad |A - d(\text{adj } A)| = 0 \\ \Rightarrow |A - d(\text{adj } A)| &= \left| \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d \begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right| \\ &= \left| \begin{bmatrix} m - qd & n(1 + d) \\ p(1 + d) & q - md \end{bmatrix} \right| = 0 \\ \Rightarrow (m - qd)(q - md) - np(1 + d)^2 &= 0 \\ \Rightarrow mq - m^2d - q^2d + mqd^2 - np(1 + d)^2 &= 0 \\ \Rightarrow (mq - np) + d^2(mq - np) - d(m^2 + q^2 + 2np) &= 0 \\ \Rightarrow d + d^3 - d((m + q)^2 - 2d) &= 0 \\ \Rightarrow 1 + d^2 &= (m + q)^2 - 2d \\ \Rightarrow (1 + d)^2 &= (m + q)^2 \end{aligned}$$

Question20

If P is a 3×3 real matrix such that $P^T = aP + (a - 1)I$, where $a > 1$, then
[30-Jan-2023 Shift 2]

Options:

A. P is a singular matrix

B. $|\text{Adj } P| > 1$

C. $|\text{Adj } P| = \frac{1}{2}$

D. $|\text{Adj } P| = 1$

Answer: D

Solution:

Solution:
 $P^T = aP + (a - 1)I$
 $\Rightarrow P = aP^T + (a - 1)I$
 $\Rightarrow P^T - P = a(P - P^T)$
 $\Rightarrow P = P^T, \text{ as } a \neq -1$
 Now, $P = aP + (a - 1)I$
 $\Rightarrow P = -I \Rightarrow |P| = 1$
 $\Rightarrow |\text{Adj } P| = 1$

Question21

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to:
 [31-Jan-2023 Shift 1]

- Options:**
- A. 6144
 - B. 4094
 - C. 4097
 - D. 2050

Answer: C

Solution:

Solution:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$$\Rightarrow A^3 = A^4 = \dots = A$$

$$(A + I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \dots + {}^{11}C_{10} A + {}^{11}C_{11} I$$

$$= ({}^{11}C_0 + {}^{11}C_1 + \dots + {}^{11}C_{10}) A + I$$

$$= (2^{11} - 1) A + I = 2047 A + I$$

$$\therefore \text{Sum of diagonal elements} = 2047(1 + 4 - 3) + 3$$

$$= 4094 + 3 = 4097$$

Question22

Let $A = [a_{ij}]$, $a_{ij} \in \mathbb{Z} \cap [0, 4]$, $1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is _____.
 [31-Jan-2023 Shift 2]

Answer: 204

Solution:

As given $a + b + c + d = 3$ or 5 or 7 or 11

if $\text{sum} = 3$

$$(1 + x + x^2 + \dots + x^4)^4 \rightarrow x^3$$

$$(1 - x^5)^4(1 - x)^{-4} \rightarrow x^3$$

$$\therefore {}^{4+3-1}C_3 = {}^6C_3 = 20$$

If $\text{sum} = 5$

$$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^5$$

$$\Rightarrow {}^{4+5-1}C_5 - 4x^{4+0-1}C_0 = {}^8C_5 - 4 = 52$$

If $\text{sum} = 7$

$$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^7$$

$$\Rightarrow {}^{4+5-1}C_4 - 4x^{4+0-1}C_0 = {}^8C_5 - 4 = 52$$

If $\text{sum} = 11$

$$(1 - 4x^5 + 6x^{10})(1 - x)^{-4} \rightarrow x^{11}$$

$$\Rightarrow {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-4}C_6 + 6 \cdot {}^{4+1-1}C_1$$

$$= {}^{14}C_{11} - 4 \cdot {}^9C_6 + 6 \cdot 4 = 364 - 336 + 24 = 52$$

$$\therefore \text{Total matrices} = 20 + 52 + 80 + 52 = 204$$

Question23

Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix $\text{Adj}(2 \cdot \text{Adj}(2A^{-1}))$ is 2^{84} , then n is equal to ____.

[31-Jan-2023 Shift 2]

Answer: 5

Solution:

$$\begin{aligned} & | \text{Adj}(2 \text{Adj}(2A^{-1})) | \\ &= | 2 \text{Adj}(\text{Adj}(2A^{-1})) |^{n-1} \\ &= 2^{n(n-1)} | \text{Adj}(2A^{-1}) |^{n-1} \\ &= 2^{n(n-1)} | (2A^{-1}) |^{(n-1)(n-1)} \\ &= 2^{n(n-1)} 2^{n(n-1)(n-1)} | A^{-1} |^{(n-1)(n-1)} \\ &= 2^{n(n-1) + n(n-1)(n-1)} \frac{1}{|A|^{(n-1)^2}} \\ &= \frac{2^{n(n-1) + n(n-1)(n-1)}}{2^{(n-1)^2}} \\ &= 2^{n(n-1) + n(n-1)^2 - (n-1)^2} \\ &= 2^{(n-1)(n^2 - n + 1)} \end{aligned}$$

$$\text{Now, } 2^{(n-1)(n^2 - n + 1)}$$

$$2^{(n-1)(n^2 - n + 1)} = 2^{84}$$

$$\text{So, } n = 5$$

Question24

If $A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

[1-Feb-2023 Shift 2]

Options:

A. $A^{30} - A^{25} = 2I$

B. $A^{30} + A^{25} + A = I$

C. $A^{30} + A^{25} - A = I$

D. $A^{30} = A^{25}$

Answer: C

Solution:

Solution:

$$A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

$$\text{If } A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \text{ Here } \alpha = \frac{\pi}{3}$$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$$

$$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{25} = A$$

$$A^{25} - A = 0$$

Question25

Let $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$. Then $3a^2 + 4b^2$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. 14

B. 4

C. 3

D. 7

Answer: B

Solution:

Solution:

$$A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 = b$$

$$\text{Let } A = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$$

$$A^2 = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = I$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta + \beta\delta \\ \alpha\gamma + \gamma\delta & \gamma\beta + \delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \alpha^2 + \beta\gamma = 1$$

$$(\alpha + \delta)\beta = 0 \Rightarrow \alpha + \delta = 0 = a$$

$$(\alpha + \delta)\gamma = 0$$

$$\beta\gamma + \delta^2 = 0$$

$$\text{Now } 3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$$

Question 26

Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to :
[6-Apr-2023 shift 2]

Options:

A. 40

B. 22

C. 24

D. 18

Answer: C

Solution:

Solution:

$$P^2 = I - P$$

$$P^\alpha + P^\beta = \gamma I - 29P$$

$$P^\alpha - P^\beta = \delta I - 13P$$

$$P^4 = (I - P)^2 = I + P^2 - 2P$$

$$P^4 = I + I - P - 2P = 2I - 3P$$

$$P^8 = (P^4)^2 = (2I - 3P)^2 = 4I + 9P^2 - 12P$$

$$= 4I + 9(I - P) - 12P$$

$$P^8 = 13I - 21P \dots (1)$$

$$P^6 = P^4 \cdot P^2 = (2I - 3P)(I - P)$$

$$= 2I - 5P + 3P^2$$

$$= 2I - 5P + 3(I - P)$$

$$= 5I - 8P \dots (2)$$

$$(1) + (2)$$

$$P^8 + P^6 = 18I - 29P$$

$$(1) - (2)$$

$$P^8 - P^6 = 8I - 13P$$

From (A) $\alpha = 8, \beta = 6$
 $\gamma = 18$
 $\delta = 8$
 $\alpha + \beta + \gamma - \delta = 32 - 8 = 24$

Question27

Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj } 2A))| = (16)^n$, then n is equal to

[8-Apr-2023 shift 1]

Options:

- A. 8
- B. 9
- C. 12
- D. 10

Answer: D

Solution:

Solution:

$$\begin{aligned} |A| &= 2[3] - 1[2] = 4 \\ \therefore |\text{adj}(\text{adj}(\text{adj } 2A))| \\ &= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n \\ &\Rightarrow (2^3 |A|)^8 = 16^n \\ &\Rightarrow (2^3 \times 2^2)^8 = 16^n \\ &= 2^{40} = 16^n \\ &= 16^{10} = 16^n \Rightarrow n = 10 \end{aligned}$$

Question28

Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. If

$P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to

[8-Apr-2023 shift 1]

Options:

- A. 2004

B. 2007

C. 2005

D. 2006

Answer: C

Solution:

Solution:

$$\begin{aligned} Q &= PAP^T \\ P^T \cdot Q^{2007} \cdot P &= P^T \cdot Q \cdot Q \dots Q \cdot P \\ &= P^T(PAP^T)(P \cdot AP^T) \dots (P^T)P. \\ &\Rightarrow (P^TP)A(P^TP)A \dots A(P^TP) \end{aligned}$$

$$P^T \cdot P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\therefore P^T \cdot Q^{2007} \cdot P = A^{2007}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = 1, b = 2007, c = 0, d = 1$$

$$2a + b - 3c - 4d = 2 + 2007 - 4 = 2005$$

Question29

If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to :
[8-Apr-2023 shift 2]

Options:

A. 14

B. 12

C. 19

D. 10

Answer: A

Solution:

Solution:

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0 \Rightarrow x^2 - 11x + 10 - 5\lambda = 0$$

$$\Rightarrow (10 - 5\lambda)A^{-1} = -A + 11I$$

$$\therefore \alpha = \frac{-1}{10 - 5\lambda} \quad \text{and} \quad \beta = \frac{+11}{10 - 5\lambda}$$

$$\alpha + \beta = -2 \Rightarrow \frac{10}{10 - 5\lambda} = -2 \Rightarrow 10 - 5\lambda = -5 \Rightarrow \lambda = 3$$

$$\therefore \alpha = \frac{1}{5} \text{ \& } \beta = \frac{-11}{5}$$

$$\therefore 4a^2 + \beta^2 + \lambda^2 = \frac{4}{25} + \frac{121}{25} + 3^2 = 14 \text{ Ans.}$$

Question30

If A is a 3×3 matrix and $|A| = 2$, then $|3 \operatorname{adj}(|3A|A^2)|$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. $3^{12} \cdot 6^{10}$

B. $3^{11} \cdot 6^{10}$

C. $3^{12} \cdot 6^{11}$

D. $3^{10} \cdot 6^{11}$

Answer: B

Solution:

Solution:

Given $|A| = 2$

Now, $|3 \operatorname{adj}(|3A|A^2)|$

$$|3A| = 3^3 \cdot |A|$$

$$= 3^3 \cdot (2)$$

$$\operatorname{Adj.} (|3A|A^2) = \operatorname{adj}\{(3^3 \cdot 2)A^2\}$$

$$= (2 \cdot 3^3)^2 (\operatorname{adj} A)^2$$

$$= 2^2 \cdot 3^6 \cdot (\operatorname{adj} A)^2$$

$$|3 \operatorname{adj}(|3A|A^2)| = |2^2 \cdot 3 \cdot 3^6 (\operatorname{adj} A)^2|$$

$$= (2^2 \cdot 3^7)^3 |\operatorname{adj} A|^2$$

$$= 2^6 \cdot 3^{21} (|A|^2)^2$$

$$= 2^6 \cdot 3^{21} (2^2)^2$$

$$= 2^{10} \cdot 3^{21}$$

$$= 2^{10} \cdot 3^{10} \cdot 3^{11}$$

$$|3 \operatorname{adj}(|3A|A^2)| = 6^{10} \cdot 3^{11}$$

Question31

If $A = \frac{1}{5!6!7!} \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\operatorname{adj}(\operatorname{adj}(2A))|$ is equal to

[10-Apr-2023 shift 2]

Options:

A. 2^{16}

B. 2^8

C. 2^{12}

D. 2^{20}

Answer: A

Solution:

Solution:

$$|A| = \frac{1}{5!6!7!} \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_2 \rightarrow R_2 - R_1$$

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

$$\begin{aligned} |\text{adjadj}(2A)| &= |2A|^{(n-1)^2} \\ &= |2A|^4 \\ &= (2^3 |A|)^4 \\ &= 2^{12} |A|^4 \Rightarrow 2^{16} \end{aligned}$$

Question32

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where $a, c \in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval $(n - 1, n]$, where $n \in \mathbb{N}$, then n is equal to _____.
[11-Apr-2023 shift 1]

Answer: 2

Solution:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = A$$

$$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$$

$$\text{Given } A^3 = A$$

$$2ac+3 = 0 \dots (1) \text{ and } a+2+3c = 1$$

$$a+1+3c = 0$$

$$a+1 - \frac{9}{2a} = 0$$

$$2a^2 + 2a - 9 = 0$$

$$f(1) < 0, f(2) > 0$$

$$a \in (1, 2]$$

$$n = 2$$

Question33

Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 50

B. 75

C. 125

D. 100

Answer: D

Solution:

Solution:

$$\text{Let } C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B = CAD$$

$$B^n = (CAD)(CAD)(CAD) \dots (CAD)_{n\text{-times}}$$

$$\Rightarrow B^n = CA^nD \dots (1)$$

$$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

$$\text{Similarly } A^n = A^3 = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$$

$$B^n \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{n}{51} + 2 \\ -1 & -\frac{n}{51} - 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{n}{51} + 1 & \frac{n}{51} \\ -\frac{n}{51} & 1 - \frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25 - 50 & 25 \\ -25 & -25 - 50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

Sum of the elements = 100

Question34

The number of symmetric matrices of order 3 , with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is
[13-Apr-2023 shift 1]

Options:

A. 10^9

B. 10^6

C. 9^{10}

D. 6^{10}

Answer: B

Solution:

Solution:

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a, b, c, d, e, f \in \{0, 1, 2, \dots, 9\}, \text{ Number of matrices } = 10^6$$

Question35

Let $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$ be the adjoint of a matrix A and $|A| = 2$. then

$$\begin{bmatrix} \alpha & -2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$$
 is equal to

[13-Apr-2023 shift 1]

Options:

- A. 16
- B. 32
- C. 0
- D. -16

Answer: D

Solution:

Solution:

$$\text{Given, } B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$$

$$|B| = 4$$

$$1(8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) = 4$$

$$-\alpha^2 + 6\alpha - 8 = 0$$

$$\alpha = 2, 4$$

Given $\alpha > 2$

So, $\alpha = 2$ is rejected

$$\begin{bmatrix} 4 & -8 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix} = [-16]_{1 \times 1}$$

Question36

Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2 \operatorname{adj}(2 \operatorname{adj}(2A))| = 32^n$, then

$3n + \alpha$ is equal to

[13-Apr-2023 shift 2]

Options:

- A. 10
- B. 9
- C. 12
- D. 11

Answer: D

Solution:

$$\begin{aligned} |A| &= 2 \\ \text{adj}(kA) &= k^{m-1} \text{adj}A \quad \{m = \text{order of matrix}\} \\ \text{adj}(2A) &= 2^2 \text{adj}A = 4 \text{adj}(A) \\ \text{adj}(2 \text{adj}(2A)) &= \text{adj}(8 \text{adj}A) \\ &= 8^2 \text{adj} \text{adj}(A) \\ |2 \text{adj} 2 \text{adj}(2A)| &= |2^7 \text{adj} \text{adj}(A)| \\ &= (2^7)^3 |A|^{2^2} \\ &= 2^{21} |A|^4 \\ &= 2^{21} \cdot 2^4 \\ \Rightarrow 2^{25} &= (32)^n \\ \Rightarrow 2^{25} &= 2^{5n} \\ \Rightarrow n &= 5 \\ |A| &= 2 \\ (6-1) - 2(2\alpha-1) + 3(\alpha-3) &= 2 \\ \Rightarrow 5 - 4\alpha + 2 + 3\alpha - 9 &= 2 \\ \Rightarrow \alpha &= -4 \\ 3n + \alpha &= 11 \end{aligned}$$

Question37

Let the determinant of a square matrix A of order m be $m - n$, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If $\det(n \text{adj}(\text{adj}(mA))) = 3^a 5^b 6^c$, then $a + b + c$ is equal to [15-Apr-2023 shift 1]

Options:

- A. 101
- B. 84
- C. 109
- D. 96

Answer: D

Solution:

Solution:

$$\begin{aligned} |A| &= m - n \\ 4m + n &= 22 \\ 17m + 4n &= 93 \\ m = 5, n = 2 \\ |A| &= 3 \\ |2 \text{adj}(\text{adj}(5A))| &= 2^5 |5A|^{16} \\ &= 2^5 \cdot 5^{80} |A|^{16} \\ &= 2^5 \cdot 5^{80} \cdot 3^{16} \\ &= 3^{11} \cdot 5^{80} \cdot 6^5 \\ a + b + c &= 96 \end{aligned}$$

Question38

Let $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} ; a, b \in \{1, 2, 3, \dots, 100\} \right\}$ and let

$T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is
[24-Jun-2022-Shift-2]

Answer: 100

Solution:

Solution:

$$A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -a+ab \\ 0 & b^2 \end{bmatrix}$$

$$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$$

$\therefore b$ must be equal to 1

$\therefore$ In this case A^2 will become identity matrix and a can take any value from 1 to 100

$\therefore$ Total number of common element will be 100 .

Question39

Let A be a 3×3 real matrix such that

$$A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \text{ and } A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3 , then the system

$$(A - 2I)X = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

has :

[25-Jun-2022-Shift-1]

Options:

A. no solution

B. infinitely many solutions

C. unique solution

D. exactly two solutions

Answer: B

Solution:

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} a+b=1 \\ d+e=1 \\ g+h=0 \end{array}$$

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{array}{l} a+c=-1 \\ d+f=1 \\ g+i=0 \end{array}$$

$$A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{array}{l} rc=1 \\ f=1 \\ i=2 \end{array}$$

Solving will get

$$a=-2, b=3, c=1, d=-1, e=2, f=1, g=-1, h=1, i=2$$

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A = 2I = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$(A - 2I)x = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow -4x_1 + 3x_2 + x_3 = 4 \dots (i)$$

$$-x_1 + x_3 = 1$$

$$-x_1 + x_2 = 1 \dots (ii)$$

$$\text{So } 3(ii) + (ii) = (i)$$

$\therefore$ Infinite solution

Question40

Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is :
[25-Jun-2022-Shift-1]

Options:

- A. a non-identity symmetric matrix
- B. a skew-symmetric matrix
- C. neither symmetric nor skew-symmetric matrix
- D. an identity matrix

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$$

$$M = A^2 + A^4 + A^6 + \dots + A^{20}$$

$$= -4I + 16I - 64I + \dots \text{ upto 10 terms}$$

$$= -I[4 - 16 + 64 \dots + \text{ upto 10 terms }]$$

$$= -I \cdot 4 \left[\frac{(-4)^{10} - 1}{-4 - 1} \right] = \frac{4}{5}(2^{20} - 1)I$$

$$N = A^1 + A^3 + A^5 + \dots + A^{19}$$

$$= A - 4A + 16A + \dots \text{ upto 10 terms}$$

$$= A \left(\frac{(-4)^{10} - 1}{-4 - 1} \right) = - \left(\frac{2^{20} - 1}{5} \right) A$$

$$N^2 = \frac{(2^{20} - 1)^2}{2^5} A^2 = \frac{-4}{24}(2^{20} - 1)^2 I$$

$$MN^2 = \frac{-16}{125}(2^{20} - 1)^3 I = KI \quad (K \neq \pm 1)$$

$$(MN^2)^T = (KI)^T = KI$$

$\therefore A$ is correct

Question41

Let A be a 3×3 matrix having entries from the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all the entries equal to 5, is

[25-Jun-2022-Shift-1]

Answer: 414

Solution:

Solution:

Case-I:

$1 \rightarrow 7$ times

and $-1 \rightarrow 2$ times

$$\text{number of possible matrix} = \frac{9!}{7!2!} = 36$$

Case-II:

$1 \rightarrow 6$ times,

$-1 \rightarrow 1$ times

and $0 \rightarrow 2$ times

$$\text{number of possible matrix} = \frac{9!}{6!2!} = 252$$

Case-III:

$1 \rightarrow 5$ times

and $0 \rightarrow 4$ times

$$\text{number of possible matrix} = \frac{9!}{5!4!} = 126$$

Hence total number of all such matrix $A = 414$

Question42

Let $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$. Then the number of elements in the set $\{(n, m) : n, m \in \{1, 2, \dots, 10\} \text{ and } nA^n + mB^m = I\}$ is__
[25-Jun-2022-Shift-2]

Answer: 1

Solution:

$$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = A$$

$$\Rightarrow A^K = A, K \in \mathbb{I}$$

$$B^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = B$$

So, $B^K = B, K \in \mathbb{I}$

$$nA^n + mB^m = nA + mB$$

$$= \begin{bmatrix} 2n - 2n \\ n - n \end{bmatrix} + \begin{bmatrix} -m & 2m \\ -m & 2m \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $2n - m = 1, -n + m = 0, 2m - n = 1$

So, $(m, n) = (1, 1)$

Question43

Let A be a 3×3 invertible matrix. If $|\text{adj}(24A)| = |\text{adj}(3 \text{adj}(2A))|$, then $|A|^2$ is equal to :
[26-Jun-2022-Shift-1]

Options:

A. 6^6

B. 2^{12}

C. 2^6

D. 1

Answer: C

Solution:

We know, $|\text{adj } A| = |A|^{n-1}$

Now, $|\text{adj } 24A| = |\text{adj } 3(\text{adj } 2A)|$

$$\Rightarrow |24A|^{3-1} = |3\text{adj } 2A|^{3-1}$$

$$\Rightarrow |24A|^2 = |3\text{adj } 2A|^2$$

Also, we know, $|KA| = K^n |A|$

$$\Rightarrow ((24)^2)^2 |A|^2 = ((3)^3)^2 |\text{adj } 2A|^2$$

$$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot (|2A|^{3-1})^2$$

$$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot |2A|^4$$

$$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot (2^3)^4 \cdot |A|^4$$

$$\Rightarrow 3^6 \cdot 8^6 \cdot |A|^2 = 3^6 \cdot 8^4 \cdot |A|^4$$

$$\Rightarrow 8^2 = |A|^2$$

$$\Rightarrow |A|^2 = 64 = 2^6$$

Question44

Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $Y = \alpha I + \beta X + \gamma X^2$ and

$Z = \alpha^2 I - \alpha \beta X + (\beta^2 - \alpha \gamma) X^2$, $\alpha, \beta, \gamma \in \mathbb{R}$.

If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to__

[26-Jun-2022-Shift-2]

Answer: 100

Solution:

$$\therefore X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore Y = \alpha I + \beta X + \gamma X^2 \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$$\therefore Y \cdot Y^{-1} = I$$

$$\therefore \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{\alpha}{5} & \frac{\beta-2\alpha}{5} & \frac{\alpha-2\beta+\gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{\beta-2\alpha}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \alpha = 5, \beta = 10, \gamma = 15$$

$$\therefore (\alpha - \beta + \gamma)^2 = 100$$

Question45

The positive value of the determinant of the matrix A, whose

$$\text{Adj}(\text{Adj}(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}, \text{ is } \underline{\hspace{2cm}}$$

[27-Jun-2022-Shift-1]

Answer: 14

Solution:

Solution:

$$|\text{adj}(\text{adj}(A))| = |A|^{2^2} = |A|^4$$

$$\therefore |A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$$

$$= (14)^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= (14)^3 (3 - 2(-5) - 1(-1))$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

Question46

Let A and B be two 3×3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$. Then $|\text{adj}(B \text{ adj}(2A))|$ is equal to
[27-Jun-2022-Shift-2]

Options:

A. 16

B. 32

C. 64

D. 128

Answer: C

Solution:

Solution:

A and B are two matrices of order 3×3 .

and $AB = I$,

$$|A| = \frac{1}{8}$$

$$\text{Now, } |A| |B| = 1$$

$$|B| = 8$$

$$\therefore |\text{adj}(B \text{ adj}(2A))| = |B \text{ adj}(2A)|^2.$$

$$= |B|^2 |\text{adj}(2A)|^2$$

$$= 2^6 |2A|^{2 \times 2}$$

$$= 2^6 \cdot 2^{12} \cdot \frac{1}{2^{12}} = 64$$

Question47

Let A be a matrix of order 2×2 , whose entries are from the set $\{0, 1, 2, 3, 4, 5\}$. If the sum of all the entries of A is a prime number p, $2 < p < 8$, then the number of such matrices A is____
[27-Jun-2022-Shift-2]

Answer: 180

Solution:

Solution:

∵ Sum of all entries of matrix A must be prime p such that $2 < p < 8$ then sum of entries may be 3, 5 or 7.

If sum is 3 then possible entries are (0, 0, 0, 3), (0, 0, 1, 2) or (0, 1, 1, 1).

∴ Total number of matrices = $4 + 4 + 12 = 20$

If sum is 5 then possible entries are

(0, 0, 0, 5), (0, 0, 1, 4), (0, 0, 2, 3), (0, 1, 1, 3), (0, 1, 2, 2) and (1, 1, 1, 2).

∴ Total number of matrices = $4 + 12 + 12 + 12 + 12 + 4 = 56$

If sum is 7 then possible entries are

(0, 0, 2, 5), (0, 0, 3, 4), (0, 1, 1, 5), (0, 3, 3, 1), (0, 2, 2, 3), (1, 1, 1, 4), (1, 2, 2, 2), (1, 1, 2, 3) and (0, 1, 2, 4).

Total number of matrices with sum 7 = 104

∴ Total number of required matrices

= $20 + 56 + 104 = 180$

Question 48

Let A be a matrix of order 3×3 and $\det(A) = 2$. Then $\det(\det(A) \operatorname{adj}(5 \operatorname{adj}(A^3)))$ is equal to [28-Jun-2022-Shift-1]

Options:

A. 512×10^6

B. 256×10^6

C. 1024×10^6

D. 256×10^{11}

Answer: A

Solution:

Solution:

$|A| = 2$

$|A| = \operatorname{adj}(5 \operatorname{adj} A^3) \cdot$

$= |25| A | \operatorname{adj}(\operatorname{adj} A^3) \cdot$

$= 25^3 |A|^3 \cdot |\operatorname{adj} A^3|^2$

$= 25^3 \cdot 2^3 \cdot |A^3|^4$

$= 25^3 \cdot 2^3 \cdot 2^{12} = 10^6 \cdot 512$

Question 49

Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \dots, 100\} : A^n = A\}$ is ____
[28-Jun-2022-Shift-2]

Answer: 25

Solution:

Solution:

$$\therefore A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} = I$$

So $A^5 = A$, $A^9 = A$ and so on.

Clearly $n = 1, 5, 9, \dots, 97$

Number of values of $n = 25$

Question50

The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :
[29-Jun-2022-Shift-1]

Options:

A. $\frac{133}{10^4}$

B. $\frac{18}{10^3}$

C. $\frac{19}{10^3}$

D. $\frac{271}{10^4}$

Answer: C

Solution:

Solution:

Let matrix A is singular then $|A| = 0$

Number of singular matrix = All entries are same + only two prime number are used in matrix

$$= 10 + 10 \times 9 \times 2$$

$$= 190$$

$$\text{Required probability} = \frac{190}{10^4} = \frac{19}{10^3}$$

Question51

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all $i, j = 1, 2, 3$. Then, the matrix $A^2 + A^3 + \dots + A^{10}$ is equal to :
[29-Jun-2022-Shift-1]

Options:

A. $\left(\frac{3^{10}-3}{2} \right) A$

B. $\left(\frac{3^{10}-1}{2} \right) A$

C. $\left(\frac{3^{10}+1}{2} \right) A$

D. $\left(\frac{3^{10}+3}{2} \right) A$

Answer: A

Solution:

Solution:

Given, $a_{ij} = 2^{j-i}$

$$\text{Now, } A = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\ 2^{-1} & 2^0 & 2^1 \\ 2^{-2} & 2^{-1} & 2^0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & 2+2+2 & 4+4+4 \\ \frac{1}{2} + \frac{1}{2} + \frac{1}{2} & 1+1+1 & 2+2+2 \\ \frac{1}{4} + \frac{1}{4} + \frac{1}{4} & \frac{1}{2} + \frac{1}{2} + \frac{1}{2} & 1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 12 \\ \frac{3}{2} & 3 & 6 \\ \frac{3}{4} & \frac{3}{2} & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$$= 3A$$

Similarly, $A^3 = 3^2 A$

$$A^4 = 3^3 A$$

$$\therefore A^2 + A^3 + \dots + A^{10}$$

$$= 3A + 3^2 A + 3^3 A + \dots + 3^9 A$$

$$= A(3 + 3^2 + 3^3 + \dots + 3^9)$$

$$= A \left(\frac{3(3^9 - 1)}{3 - 1} \right) = \frac{3(3^9 - 1)}{2} A = \left(\frac{3^{10} - 3}{2} \right) A$$

Question52

Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1(\text{adj } A) + {}^5C_2(\text{adj } A)^2 - \dots - {}^5C_5(\text{adj } A)^5$,

then the sum of all elements of the matrix B is
[29-Jun-2022-Shift-2]

Options:

A. -5

B. -6

C. -7

D. -8

Answer: C

Solution:

Solution:

$$\text{Given } A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \text{ and}$$

$$B = I - {}^5C_1(\text{adj } A) + {}^5C_2(\text{adj } A)^2 - {}^5C_3(\text{adj } A)^3 + {}^5C_4(\text{adj } A)^4 - {}^5C_5(\text{adj } A)^5$$

$$= (I - (\text{adj } A))^5$$

$$\text{Cofactor of } A = \begin{bmatrix} (-1)^{1+1} \cdot 2 & (-1)^{1+2} \cdot 0 \\ (-1)^{2+1} \cdot (-1) & (-1)^{2+2} \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\text{Transpose of cofactor of } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

Now, $I - \text{adj } A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

Now let,

$$P = I - \text{adj}A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore P^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$$P^5 = P^4 \cdot P = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$$

$$\text{Now sum of elements} = -1 - 5 - 1 + 0 = -7$$

Question53

Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number and $N = \sum_{k=1}^{49} M^{2k}$. If

$(I - M^2)N = -2I$, then the positive integral value of α is ____
[29-Jun-2022-Shift-2]

Answer: 1

Solution:

Solution:

$$M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}; M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$$

$$N = M^2 + M^4 + \dots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \dots]I$$

$$= -\alpha^2 \frac{(1 - (-\alpha^2)^{49})}{1 + \alpha^2} \cdot I$$

$$I - M^2 = (1 + \alpha^2)I$$

$$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$$

$$\alpha = 1$$

Question54

Let $S = \{ \sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd} \}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.

If $\sum_{a \in S} \det(\text{adj } A) = 100\lambda$, then λ is equal to : ion:
[24-Jun-2022-Shift-1]

Options:

- A. 218
- B. 221
- C. 663
- D. 1717

Answer: B

Solution:

Solution:

$$\text{Given, } A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}_{3 \times 3} \quad S = \{ \sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd} \}$$

$$\therefore S = \{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, \dots, \sqrt{49}\}$$

We know,

$$|\text{adj } A| = |A|^{n-1}$$

Here, n = order of matrix.

Here, $n = 3$

$$\therefore |\text{adj } A| = |A|^{3-1} = |A|^2$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{vmatrix}$$

$$= 1(1-0) - 0 + a(0 - (-a))$$

$$= a^2 + 1$$

$$\therefore |\text{adj } A| = |A|^2 = (a^2 + 1)^2$$

$$\text{Now, } \sum_{a \in S} \det(\text{adj } A)$$

$$= \sum_{a \in S} (a^2 + 1)^2$$

$$= (1^2 + 1)^2 + ((\sqrt{3})^2 + 1)^2 + ((\sqrt{5})^2 + 1)^2 + \dots + ((\sqrt{49})^2 + 1)^2$$

$$= (1^2 + 1)^2 + (3 + 1)^2 + (5 + 1)^2 + \dots + (49 + 1)^2$$

$$= 2^2 + 4^2 + 6^2 + \dots + 50^2$$

$$= 2^2(1^2 + 2^2 + 3^2 + \dots + 25^2)$$

$$= 4 \cdot \frac{25 \cdot 26 \cdot 51}{6} = 100.221$$

$$\therefore 100K = 100.221$$

$$\Rightarrow K = 221$$

Question55

Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \dots, 100\} : A^n + (\omega B)^n = A + B\}$ is equal to
[25-Jul-2022-Shift-1]

Answer: 17

Solution:

Solution:

$$\text{Here } A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

We get $A^2 = A$ and similarly for

$$B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

We get $B^2 = -B \Rightarrow B^3 = B$

$\therefore A^n + (\omega B)^n = A + (\omega B)^n$ for $n \in \mathbb{N}$

For ω^n to be unity n shall be multiple of 3 and for B^n to be B , n shall be 3, 5, 7, ... 99

$\therefore n = \{3, 9, 15, \dots, 99\}$

Number of elements = 17

Question56

$$\text{Let } A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a, b \in \mathbb{C}. \text{ If for some } n \in \mathbb{N}, A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$$

then $n + a + b$ is equal to
[25-Jul-2022-Shift-2]

Answer: 24

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B^3 = 0$$

$$\therefore A^n = (I + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \dots$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and

$$na + \frac{n(n-1)}{2}ab = 2160$$

$$\Rightarrow a = 4, n = 12 \text{ and } b = 8$$

$$n + a + b = 24$$

Question57

Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(\text{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be :
[26-Jul-2022-Shift-1]

Options:

A. -1

B. 2

C. 1

D. $-\sqrt{2}$

Answer: B

Solution:

$$|(A+I)(adjA+I)| = 4$$

$$\Rightarrow |A \operatorname{adj} A + A + \operatorname{adj} A + I| = 4$$

$$\Rightarrow |(A)I + A + \operatorname{adj} A + I| = 4$$

$$|A| = -1 \Rightarrow |A + \operatorname{adj} A| = 4$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \operatorname{adj} A = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} (a+d) & 0 \\ 0 & (a+d) \end{vmatrix} = 4$$

$$\Rightarrow a+d = \pm 2$$

Question58

Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then the value of $A'BA$ is:

[26-Jul-2022-Shift-2]

Options:

A. 1224

B. 1042

C. 540

D. 539

Answer: D

Solution:

Solution:

$$A'BA = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} A$$

$$= \begin{bmatrix} 9^2 + 12^2 - 15^2 & -10^2 + 13^2 + 16^2 & 11^2 - 14^2 + 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} &= [9^2 + 12^2 - 15^2 - 10^2 + 13^2 + 16^2 + 11^2 - 14^2 + 17^2] \\ &= [(9^2 - 10^2) + (11^2 + 12^2) + (13^2 - 14^2) + (16^2 - 15^2) + 17^2] \\ &= [-19 + 265 + (-27) + 31 + 289] \\ &= [585 - 46] = [539] \end{aligned}$$

Question59

The number of matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where

$a, b, c, d \in \{-1, 0, 1, 2, 3, \dots, 10\}$, such that $A = A^{-1}$, is _____.
[26-Jul-2022-Shift-2]

Answer: 50

Solution:

Solution:

$$\because A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } A^2 = \begin{bmatrix} a^2 + bc & b(a + d) \\ c(a + d) & bc + d^2 \end{bmatrix}$$

For A^{-1} must exist $ad - bc \neq 0$ (i)

and $A = A^{-1} \Rightarrow A^2 = I$

$$\therefore a^2 + bc = d^2 + bc = 1$$

$$\text{and } b(a + d) = c(a + d) = 0$$

Case I : When $a = d = 0$, then possible values of (b, c) are $(1, 1)$, $(-1, 1)$ and $(1, -1)$ and $(-1, -1)$.

Total four matrices are possible.

Case II: When $a = -d$ then (a, d) be $(1, -1)$ or $(-1, 1)$.

Then total possible values of (b, c) are $(12 + 11) \times 2 = 46$.

$$\therefore \text{Total possible matrices} = 46 + 4 = 50.$$

Question60

Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. -10

B. -6

C. 6

D. 10

Answer: D

Solution:

Solution:

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

$$\alpha A^2 + \beta A = \begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix}$$

$$= \begin{bmatrix} -3\alpha + \beta & -8\alpha + 2\beta \\ 8\alpha - 2\beta & 21\alpha - 5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

On Comparing

$$8\alpha = 2\beta, -3\alpha + \beta = 2, 21\alpha - 5\beta = 2$$

$$\Rightarrow \alpha = 2, \beta = 8$$

$$\text{So, } \alpha + \beta = 10$$

Question61

Let S be the set containing all 3×3 matrices with entries from $\{-1, 0, 1\}$. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is _____.

[27-Jul-2022-Shift-1]

Answer: 5376

Solution:

Solution:

Sum of all diagonal elements is equal to sum of square of each element of the matrix.

$$\text{i.e., } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \text{ then } t_r(A \cdot A^T)$$

$$= a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$$

$$\because a_i, b_i, c_i \in \{-1, 0, 1\} \text{ for } i = 1, 2, 3$$

$\therefore$ Exactly three of them are zero and rest are 1 or -1 .

Total number of possible matrices ${}^9C_3 \times 2^6$

$$= \frac{9 \times 8 \times 7}{6} \times 64$$

$$= 5376$$

Question62

$$\text{Let } A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}.$$

If $A^2 + \gamma A + 18I = O$, then $\det(A)$ is equal to _____.

[27-Jul-2022-Shift-2]

Options:

A. -18

B. 18

C. -50

D. 50

Answer: B

Solution:

Solution:

Characteristic equation of A is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4 - \lambda & -2 \\ \alpha & \beta - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (4 + \beta)\lambda + (4\beta + 2\alpha) = 0$$

$$\text{So, } A^2 - (4 + \beta)A + (4\beta + 2\alpha)I = 0$$

$$|A| = 4\beta + 2\alpha = 18$$

Question63

Consider a matrix $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers. If $\frac{\det(\text{adj}(\text{adj}(\text{adj}(\text{adj} A))))}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3 - tuples (α, β, γ) is _____.

[27-Jul-2022-Shift-2]

Answer: 42

Solution:

Solution:

$$\det(A) = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{vmatrix}$$

$$R_3 \rightarrow R_3 + R_1$$

$$\Rightarrow (\alpha + \beta + \gamma) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\therefore \det(A) = (\alpha + \beta + \gamma)(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$$

$$\text{Also, } \det(\text{adj}(\text{adj}(\text{adj}(\text{adj} A))))$$

$$= (\det(A))^{2^4} = (\det(A))^{16}.$$

$$\therefore \frac{(\alpha + \beta + \gamma)^{16}(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}}{(\alpha - \beta)^{16}(\beta - \gamma)^{16}(\gamma - \alpha)^{16}} = (4.13)^{16}$$

$$\Rightarrow \alpha + \beta + \gamma = 12$$

$$\Rightarrow (\alpha, \beta, \gamma) \text{ distinct natural triplets}$$

$$= {}^{11}C_2 - 1 - {}^3C_2(4) = 55 - 1 - 12 = 42$$

Question64

Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If

$B_n = \text{Adj}(B_{n-1})$ for all $n > 1$, then $\det(B_4)$ is equal to :

[28-Jul-2022-Shift-1]

Options:

A. 3^{28}

B. 3^{30}

C. 3^{32}

D. 3^{36}

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\text{Now } B_0 = A^{49} + 2A^{98} = (A^3)^{16} \cdot A + 2(A^3)^{32} \cdot A^2$$

$$B_0 = A + 2A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$|B_0| = 9$$

$$\text{Since, } B_n = \text{Adj } |B_{n-1}| \Rightarrow |B_n| = |B_{n-1}|^2$$

$$\text{Hence } |B_4| = |B_3|^2 = |B_2|^4 = |B_1|^8 = |B_0|^{16} = |3^2|^{16} = 3^{32}$$

Question65

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$, $\alpha, \beta \in \mathbb{R}$. Let α_1 be the value of α

which satisfies $(A + B)^2 - A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and α_2 be the value of α which

satisfies $(A + B)^2 - B^2$. Then $|\alpha_1 - \alpha_2|$ is equal to _____.

[28-Jul-2022-Shift-1]

Answer: 2

Solution:

Solution:

$$(A + B)^2 = A^2 + B^2 + AB + BA$$

$$= A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore B^2 + AB + BA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \dots\dots (1)$$

$$AB = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta - 1 & 1 \\ \alpha + 2\beta & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \beta + 2 & \alpha - \beta \\ 1 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta^2 + 1 & \beta \\ \beta & 1 \end{bmatrix}$$

By (1) we get

$$\begin{bmatrix} \beta^2 + 2\beta & \alpha + 1 \\ \alpha + 3\beta + 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \alpha = 1, \beta = 0 \Rightarrow \alpha_1 = 1$$

Similarly if $A^2 + AB + BA = 0$ then

$$\left(A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} -1 & -1 - \alpha \\ 2 + 2\alpha & \alpha^2 - 2 \end{bmatrix} \right)$$

$$\begin{bmatrix} 2\beta & \alpha - \beta + 1 - 1 - \alpha \\ \alpha + 2\beta + 1 + 2 + 2\alpha & \alpha^2 - 2 + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \beta = 0 \text{ and } \alpha = -1 \Rightarrow \alpha_2 = -1$$

$$\therefore |\alpha_1 - \alpha_2| = |2| = 2$$

Question66

Let A and B be any two 3×3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true?

[28-Jul-2022-Shift-2]

Options:

A. $A^4 - B^4$ is a symmetric matrix

B. $AB - BA$ is a symmetric matrix

C. $B^5 - A^5$ is a skew-symmetric matrix

D. $AB + BA$ is a skew-symmetric matrix

Answer: C

Solution:

Solution:

(A) $M = A^4 - B^4$

$$M^T = (A^4 - B^4)^T = (A^T)^4 - (B^T)^4 \\ = A^4 - (-B)^4 = A^4 - B^4 = M$$

(B) $M = AB - BA$

$$M^T = (AB - BA)^T = (AB)^T - (BA)^T \\ = B^T A^T - A^T B^T \\ = -BA - A(-B) \\ = AB - BA = M$$

(C) $M = B^5 - A^5$

$$M^T = (B^5 - A^5)^T = (B^T)^5 - (A^T)^5 = -(B^5 + A^5) \neq -M$$

(D) $M = AB + BA$

$$M^T = (AB + BA)^T = (AB)^T + (BA)^T \\ = B^T A^T + A^T B^T = -BA - AB = -M$$

Question67

Let A and B be two 3×3 non-zero real matrices such that AB is a zero matrix. Then
[29-Jul-2022-Shift-1]

Options:

- A. the system of linear equations $AX = 0$ has a unique solution
- B. the system of linear equations $AX = 0$ has infinitely many solutions
- C. B is an invertible matrix
- D. $\text{adj}(A)$ is an invertible matrix

Answer: B

Solution:

Solution:

AB is zero matrix

$$\Rightarrow |A| = |B| = 0$$

So neither A nor B is invertible

$$\text{If } |A| = 0$$

$$\Rightarrow |\text{adj } A| = 0 \text{ so } \text{adj } A$$

$AX = 0$ is homogeneous system and $|A| = 0$

So, it is having infinitely many solutions

Question68

The number of matrices of order 3×3 , whose entries are either 0 or 1 and the sum of all the entries is a prime number, is _____.
[29-Jul-2022-Shift-1]

Answer: 282

Solution:

Solution:

In a 3×3 order matrix there are 9 entries.

These nine entries are zero or one.

The sum of positive prime entries are 2, 3, 5 or 7 .

$$\text{Total possible matrices} = \frac{9!}{2!.7!} + \frac{9!}{3!.6!} + \frac{9!}{5!.4!} + \frac{9!}{7!.2!} = 36 + 84 + 126 + 36 = 282$$

Question69

Which of the following matrices can NOT be obtained from the matrix

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \text{ by a single elementary row operation?}$$

[29-Jul-2022-Shift-2]

Options:

A. $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

(1) $R_1 \rightarrow R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \rightarrow R_2 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

Question70

Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if $X^T A^k X = 33$, then k is equal to 10.
[29-Jul-2022-Shift-2]

Answer: 10

Solution:

Solution:

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$$

$$X^T A^k X = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\text{As } A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{for } K \rightarrow \text{Even } A^K = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X^T A^K X = 33 \text{ (This is not correct)}$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 3K+1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3K+3]$$

$$\therefore 3K+3 = 33 \therefore K = 10$$

But it should be dropped as 33 is not matrix

If K is odd

$$X^T A^K X = 33$$

$$X^T A A^{K-1} X = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3k-3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} -1 & 3 & 8 \end{bmatrix} \begin{bmatrix} 3k-2 \\ 1 \\ 1 \end{bmatrix} = [33]$$

$$[-3k + 13] = [33]$$

$$k = 20 / 3 \text{ (not possible)}$$

Question71

Let **M** be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is

24 Feb 2021 Shift 1

Answer: 540

Solution:

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$$

Case-I : Seven (1's) and two (0's)

$$\frac{9!}{7!2!} = 36$$

Case-II: One (2) and three (1's) and five (0's)

$$\frac{9!}{5!3!} = 504$$

$$\therefore \text{Total} = 540$$

Question72

If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

$13(a^2 + b^2)$ is

[25 Feb 2021 Shift 1]

Answer: 13

Solution:

Solution:

$$A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix} \text{ and } (I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow |(I_2 + A)(I_2 - A)^{-1}| = a^2 + b^2$$

$$\Rightarrow a^2 + b^2 = \frac{|I_2 + A|}{|I_2 - A|} \dots (i)$$

$$\text{Now, } I_2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

$$\text{Similarly, } I_2 - A = \begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

$$\text{Here, } |I_2 + A| = |I_2 - A| = \left(1 + \tan^2\left(\frac{\theta}{2}\right)\right)$$

$$\Rightarrow \frac{|I_2 + A|}{|I_2 - A|} = 1 \dots (ii)$$

From Eqs. (i) and (ii),

$$a^2 + b^2 = 1$$

$$\text{Now, } 13(a^2 + b^2) = 13 \times 1 = 13$$

Question73

If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$,

$AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is
[2021, 25 Feb. Shift-II]

Options:

A. 4

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

$$\text{Given, } A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}$$

Given, $AA^T = I_2$ i.e.

$$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 + \alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating these matrices,

$$1 + \alpha^2 = 1 \text{ gives, } \alpha = 0$$

$$\alpha(1 - \beta) = 0$$

$$\alpha^2 + \beta^2 = 1$$

Put $\alpha = 0$ in $\alpha^2 + \beta^2 = 1$, we get $0 + \beta^2 = 1$, gives $\beta = \pm 1$

where we take $\beta^4 = 1$

$$\therefore \alpha^4 + \beta^4 = 0^4 + 1 = 1$$

Question 74

Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

[2021, 26 Feb. Shift-I]

Options:

A. 4

B. 1

C. 6

D. 12

Answer: A

Solution:

Solution:

Let A be the matrix as follows, $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, since A is symmetric matrix.

$$\text{Now, } A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & b^2 + c^2 \end{bmatrix}$$

Given that, diagonal entries of A^2 is 1 . i.e. $a^2 + b^2 + b^2 + c^2 = 1$

$$\text{or } a^2 + 2b^2 + c^2 = 1$$

Case 1 $a = 0$

$$\text{Then, } 2b^2 + c^2 = 1$$

(a) $c = 0$ gives, $b^2 = \frac{1}{2}$ or $b = \pm \frac{1}{\sqrt{2}} \therefore a = 0, b = 1/\sqrt{2}, c = 0$ (2 matrices) $a = 0, b = -1/\sqrt{2}, c = 0$

(b) $b = 0$, gives $c^2 = 1$ or $c = \pm 1 \therefore a = 0, b = 0, c = 1$

and $a = 0, b = 0, c = -1$ (2 matrices)

Case $2b = 0$, then $a^2 + c^2 = 1$

(a) $a = 0$, then $c = \pm 1$ $a = 0, b = 0, c = 1$ and $a = 0, b = 0, c = -1$

This is repeated case.

(b) $c = 0$, then $a = \pm 1$ $a = 1, b = 0, c = 0$ and $a = -1, b = 0, c = 0$ Again 2 matrices.

Thus, only acceptable matrices are as follows

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Then possible number of such matrices are 4.

Question75

If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation

$$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for some real numbers } \alpha \text{ and } \beta, \text{ then } \beta - \alpha$$

is equal to

[2021, 26 Feb. Shift-II]

Answer: 4

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And so on,

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c So, " $A^{20} + \alpha A^{19} + \beta A$

$$= \begin{bmatrix} 1 + \alpha + \beta & 0 & 0 \\ 0 & 2^{20} + \alpha 2^{19} + 2\beta & 0 \\ 3\alpha + 2\beta & 0 & 1 - \alpha - \beta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing, we get

$$1 - \alpha - \beta = 1$$

$$\Rightarrow \alpha + \beta = 0$$

$$\text{and } 2^{20} + \alpha 2^{19} + 2\beta = 4$$

$$\Rightarrow 2^{20} + \alpha(2^{19} - 2) = 4 \text{ [use, } \alpha + \beta = 0 \text{]}$$

$$\Rightarrow \alpha = \frac{4 - 2^{20}}{2^{19} - 2} = -2$$

$$\Rightarrow \beta = 2$$

$$\therefore \beta - \alpha = 2 - (-2) = 2 + 2 = 4$$

Question76

Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$.

If $\text{Tr}(A)$ denotes the sum of all diagonal elements of the matrix A , then $\text{Tr}(A) - \text{Tr}(B)$ has value equal to
[2021, 18 March Shift-1]

Options:

A. 1

B. 2

C. 0

D. 3

Answer: B

Solution:

Solution:

$$\text{Given, } A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \dots (i)$$

$$\text{and } 2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \dots (ii)$$

Multiply with 2 in Eq. (ii), we get

$$4A - 2B = \begin{bmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{bmatrix} \dots\dots\dots (iii)$$

Adding Eqs. (i) and (iii),

$$5A = \begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore \text{Tr}(A) = 1 - 1 + 1 = 1$$

From Eq. (i),

$$B = \frac{1}{2} \left\{ \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\therefore \text{Tr}(B) = 0 - 1 + 0 = -1$$

$$\text{Hence, } \text{Tr}(A) - \text{Tr}(B) = 1 - (-1) = 2$$

Question77

Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in \mathbb{N}$ for which $P^n = 5I - 8P$ is equal to [2021, 18 March Shift-II]

Answer: 6

Solution:

Solution:

Method (1)

$$\text{Given, } P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$\text{Now, } P^n = 5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix}$$

$$\Rightarrow P^n = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} \dots\dots\dots (i)$$

$$\text{Now, } P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$\text{Again } P^3 = P \cdot P^2 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \text{ Now,}$$

$$P^6 = P^3 \cdot P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$$

$$\Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n \quad [\text{from Eq. (i)}]$$

$$\therefore n = 6$$

Method (2)

$$\text{Given } P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

Characteristics equation is $|P - \lambda I| = 0$

$$\begin{vmatrix} 2 - \lambda & -1 \\ 5 & -3 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (2 - \lambda)(-3 - \lambda) + 5 = 0$$

$$\Rightarrow \lambda^2 + \lambda - 1 = 0$$

By Cayley-Hamilton Theorem,

$$P^2 + P - I = 0$$

$$\Rightarrow P^2 = I - P$$

$$\Rightarrow P^3 + P^2 - P = 0$$

$$\Rightarrow P^3 = P - P^2 = P - (I - P) \quad [\text{from Eq. (i)}]$$

$$\Rightarrow P^3 = 2P - I$$

$$\Rightarrow P^3 = 2P - I$$

$$\text{Now, } P \cdot P^3 = P(2P - I)$$

$$\Rightarrow P^4 = 2P^2 - P = 2(I - P) - P \quad [\text{from Eq. (i)}]$$

$$\Rightarrow P^4 = -3P + I$$

$$\text{Again, } P \cdot (P^4) = P(-3P + I)$$

$$\Rightarrow P^5 = -3P^2 + P$$

$$= -3(I - P) + P \quad [\text{from Eq. (i)}]$$

$$= 5P - I$$

$$\text{and } P(P^5) = P(5P - I)$$

$$1I \Rightarrow P^6 = 5P^2 - P = 5(I - P) - P$$

$$\Rightarrow P^6 = 5I - 8P = P^n \quad (\text{given})$$

$$\therefore n = 6$$

Question78

Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations

$$A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \text{ has}$$

[2021, 16 March Shift-I]

Options:

- A. a unique solution.
- B. infinitely many solutions.
- C. no solution.
- D. exactly two solutions.

Answer: C

Solution:

Solution:

$$\text{Let } A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + i^2 & -i^2 - i^2 \\ -i^2 - i^2 & i^2 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2$$

$$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 4 & -4 - 4 \\ -4 - 4 & 4 + 4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$\text{Similarly, } A^8 = A^4 \cdot A^4$$

$$= \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 64 + 64 & -64 - 64 \\ -64 - 64 & 64 + 64 \end{bmatrix}$$

$$= \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \quad \text{Now, } A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$128x - 128y = 8$$

$$\Rightarrow x - y = \frac{1}{16} \quad \dots\dots\dots (i)$$

$$\text{and } -128x + 128y = 64$$

$$\Rightarrow x - y = -\frac{1}{2} \quad \dots\dots\dots (ii)$$

From Eqs. (i) and (ii), we get these two lines are parallel. So, there will be no solution.

Question79

The total number of 3×3 matrices A having entries from the set $(0, 1, 2, 3)$, such that the sum of all the diagonal entries of AA^T is 9, is equal to
[2021, 16 March Shift-I]

Answer: 766

Solution:

Solution:

Set $S : \{0, 1, 2, 3\}$

$$\text{Let } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{bmatrix}$$

$$\text{where } a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$c = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Now, as per question

$$ca \cdot a + b \cdot b + c \cdot c = 9$$

$$\Rightarrow (a_1^2 + a_2^2 + a_3^2) + (b_1^2 + b_2^2 + b_3^2)$$

$$+ (c_1^2 + c_2^2 + c_3^2) = 9$$

$$[a_i, b_i, c_i \in S]$$

$$9 = (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)$$

$$\text{or } 9 = (1 + 4 + 4 + 0 + 0 + 0 + 0 + 0 + 0 + 0)$$

$$\text{or } 9 = (9 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0)$$

$$\text{or } 9 = (4 + 1 + 1 + 1 + 1 + 1 + 1 + 0 + 0 + 0)$$

$$\text{Total permutations in case 1} = 1$$

$$\text{Total permutations in case 2} = \frac{9!}{6!2!} = 252$$

$$\text{In case 3} = \frac{9!}{8!} = 9$$

$$\text{In case 4} = \frac{9!}{5!3!} = 504$$

$$\text{Total permutations} = 1 + 252 + 9 + 504 = 766$$

Question80

Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $a = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries

such that $A = X B$, where $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ and $k \in \mathbb{R}$. If

$a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2 + 1)b_2^2 \neq -2b_1b_2$, then the value of k is

[2021, 16 March Shift-II]

Answer: 1

Solution:

Solution:

$$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$A = XB$$

$$X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$$

$$XB = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$XB = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$$

$$\text{As, } A = XB$$

$$\text{So, } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$$

$$\sqrt{3}a_1 = b_1 - b_2 \quad \dots\dots\dots (i)$$

$$\sqrt{3}a_2 = b_1 + kb_2 \quad \dots\dots\dots (ii)$$

$$\text{And as given, } a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

Adding, Eq. (i)² + Eq. (ii)²,

$$3a_1^2 + 3a_2^2 = (b_1 - b_2)^2 + (b_1 + kb_2)^2$$

$$\Rightarrow 2(b_1^2 + b_2^2)$$

$$= 2b_1^2 + b_2^2(k^2 + 1) + 2b_1b_2(k - 1)$$

$$\Rightarrow b_2^2(k^2 + 1 - 2) + 2b_1b_2(k - 1) = 0$$

$$\Rightarrow (k - 1)[b_2^2(k + 1) + 2b_1b_2] = 0$$

$$\text{So, } k = 1$$

Question81

$$\text{Let } P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix} \text{ and } A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$$

where, $\omega = \frac{-1 + i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3 . If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to.....

[16 Mar 2021 Shift 1]

Answer: 36

Solution:

Solution:

$$\text{Given, } P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$$

$$|(P^{-1}AP - I_3)|^2 = \alpha\omega^2$$

$$\Rightarrow |(P^{-1}AP - I_3)(P^{-1}AP - I_3)| = \alpha\omega^2$$

$$\Rightarrow P^{-1}APP^{-1}AP - P^{-1}API_3 - I_3P^{-1}AP + I_3 \cdot I_3 = \alpha\omega^2$$

$$\Rightarrow P^{-1}A^2P - P^{-1}AP - P^{-1}AP + I_3 = \alpha\omega^2$$

$$[\because PP^{-1} = I \text{ and } I A = A]$$

$$\Rightarrow |P^{-1}A^2P - 2P^{-1}AP + PP^{-1}| = \alpha\omega^2$$

$$\Rightarrow |P^{-1}(A^2 - 2A + I_3)P| = \alpha\omega\omega^2$$

$$\Rightarrow |P^{-1}| |A - I_3|^2 |P| = \alpha\omega^2$$

$$\Rightarrow |P^{-1}P| |A - I_3|^2 = \alpha\omega^2$$

$$\Rightarrow |A - I_3|^2 = \alpha\omega^2$$

Consider,

$$A - I_3 = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - I_3| = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$$

On applying $C_2 \rightarrow C_2 - C_3$, we get

$$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$$

On applying $C_2 \rightarrow C_2 - C_3$, we get

$$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$$

$$= -\omega[(-\omega - 2) - (-7 + \omega^2)]$$

$$= -\omega(-\omega - 2 + 7 - \omega^2)$$

$$= -\omega(1 - 2 + 7)$$

$$= -6\omega$$

$$|A - I_3| = -6\omega$$

$$|A - I_3|^2 = 36\omega^2 = \alpha\omega^2$$

$$\therefore \alpha = 36$$

Question 82

If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det[A^{10} - \text{Adj}(2A)^{10}]$ is equal to

[17 Mar 2021 Shift 1]

Answer: 16

Solution:

Solution:

$$\text{If } A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$$

$$\det(A^4) + \det[A^{10} - [\text{adj}(2A)]^{10}]$$

$$A \cdot A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^3 \cdot A$$

$$= \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 16 & 15 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 2^n & 2^n - (-1)^n \\ 0 & (-1)^n \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\text{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$$

$$\text{adj}(2A) = -2 \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \left[\begin{array}{l} \because x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ \text{adj}(x) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \end{array} \right]$$

$$[\text{adj}(2A)]^2 = 4 \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}$$

$$[\text{adj}(2A)]^3 = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2) \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8 \begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$$

$$[\text{adj}(2A)]^3 = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2) \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8 \begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$$

$$[\text{adj}(2A)]^n = (-2)^n \begin{bmatrix} 1 & (-1)^n [2^n - (-1)^n] \\ 0 & (-1)^n 2^n \end{bmatrix}$$

$$[\text{adj}(2A)]^{10} = 2^{10} \begin{pmatrix} 1 & -(2^{10} - 1) \\ 0 & 2^{10} \end{pmatrix}$$

$$\text{Now, } A^{10} - [\text{adj}(2A)]^{10}$$

$$= \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2^{10} & -(2^{20} - 2^{10}) \\ 0 & 2^{20} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \cdot 2^{10} - 2^{20} - 1 \\ 0 & 1 - 2^{20} \end{bmatrix}$$

$$\det | A^{10} - \text{adj}(2A)^{10} | = 0$$

$$\therefore \det(A^4) + \det[A^{10} - \text{adj}(2A)^{10}] = (16)^4 + 0 = 16$$

Question83

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \dots + A^{20}$, then the sum of all the elements of the matrix M is equal to
[2021, 27 July Shift-II]

Answer: 2020

Solution:

Solution:

$$\text{We have, } A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

.....
.....

$$A^n = \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, } M = A + A^2 + A^3 + \dots + A^{20}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} +$$

$$\dots + \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & \sum n & \sum(\sum r) \\ 0 & 20 & \sum n \\ 0 & 0 & 20 \end{bmatrix}$$

$$\text{Now, } \sum_{r=1}^{20} n = 1 + 2 + \dots + 20 = \frac{20 \times 21}{2} = 210$$

$$\sum_{r=1}^{20} (\sum r) = \sum_{r=1}^{20} \frac{r(r+1)}{2} = \frac{1}{2} \sum_{r=1}^{20} (r^2 + r)$$

$$= \frac{1}{2} \left[\frac{20 \times 21 \times 41}{6} + \frac{20 \times 21}{2} \right]$$

$$= \frac{1}{2} [2870 + 210] = 1540$$

$$\text{Hence, } M = \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$$

Sum of all elements = 2020.

Question84

$$S = \left\{ n \in \mathbb{N} \mid \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\}$$

$\forall a, b, c, d \in \mathbb{R}$

, where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is
[2021, 25 July Shift-1]

Answer: 11

Solution:

Solution:

$$\text{Let } X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and } A = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n$$

$$\Rightarrow AX = 1X$$

$$A = 1$$

$$A^2 = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$\Rightarrow A = i \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^8 = i^4 1 = 1$$

$$\therefore n = 8$$

$\Rightarrow n$ is a multiple of 8 .

16, 24,96

$$\text{Number of elements} = \frac{96 - 16}{8} + 1 = 11$$

Question85

$$\text{If } P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}, \text{ then } P^{50} \text{ is}$$

[2021, 25 July Shift-II]

Options:

A. $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Answer: A

Solution:

Solution:

Given, $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\vdots$

$$\Rightarrow P^n = \begin{bmatrix} 1 & 0 \\ \frac{n}{2} & 1 \end{bmatrix}$$

Hence, $P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

Question86

Let $A = [a_{ij}]$ be a real matrix of order 3×3 , such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to [2021, 22 July Shift-II]

Options:

A. 2

B. 1

C. 3

D. 9

Answer: C

Solution:

Solution:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$$

Sum of elements of each row is 1 .

Let X be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

$$\text{Then, } AX = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$AX = \begin{bmatrix} a_{11} + a_{12} + a_{13} \\ a_{21} + a_{22} + a_{23} \\ a_{31} + a_{32} + a_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$AX = X$$

Replace X with AX .

$$A \cdot AX = AX \Rightarrow A^2X = X$$

Again, replace X with AX ,

$$A^2(AX) = AX$$

$$A^3X = X$$

$$\text{Let } A^3 = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix}$$

Then,

$$A^3X = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^3X = \begin{bmatrix} a_{11}' + a_{12}' + a_{13}' \\ a_{21}' + a_{22}' + a_{23}' \\ a_{31}' + a_{32}' + a_{33}' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{So, } a_{11}' + a_{12}' + a_{13}' = 1$$

$$a_{21}' + a_{22}' + a_{23}' = 1$$

$$a_{31}' + a_{32}' + a_{33}' = 1$$

$$\therefore \text{ Sum } = 3$$

Question87

Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then, the number of 3×3 matrices B with entries from the set $\{1, 2, 3, 4, 5\}$ and satisfying $AB = BA$ is
[2021, 22 July Shift-II]

Answer: 3125

Solution:

Solution:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & a_1 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$$AB = BA$$

$$\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \begin{pmatrix} a_2 & a_1 & a_3 \\ b_2 & b_1 & b_3 \\ c_2 & c_1 & c_3 \end{pmatrix}$$

$$\begin{cases} b_1 = a_2 \\ b_2 = a_1 \\ b_3 = a_3 \end{cases} \quad \begin{cases} a_1 = b_2 \\ a_2 = b_1 \\ a_3 = b_3 \end{cases} \quad \begin{cases} C_1 = C_2 \\ C_2 = C_1 \\ C_3 = C_3 \end{cases}$$

$$B = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_2 & a_1 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

Number of distinct elements in B is 5 $\{a_1, a_2, a_3, c_1, c_3\}$ and according to question, $a_{ij} \in \{1, 2, 3, 4, 5\}$.

So, number of matrices = $5 \times 5 \times 5 \times 5 \times 5 = 3125$

Question88

Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity

matrix of order 3×3 . If $B = [b_{ij}]$, then b_{13} is equal to ...

[2021, 20 July Shift-1]

Answer: 910

Solution:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = 7A^{20} - 20A^7 + 21$$

$$\therefore A^2 = A \cdot A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^2 = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^4 = A^3 A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^4 = \begin{pmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^n = \begin{pmatrix} 1 & -n & \frac{n^2 - n}{2} \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore b_{13} = 7 \left(\frac{20 \times 19}{2} \right) - 20 \left(\frac{7 \times 6}{2} \right) + 2(0)$$

$$\Rightarrow b_{13} = 1330 - 420 = 910$$

Question 89

If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and $Q = A^T B A$, then the

inverse of the matrix $AQ^{2021}A^T$ is equal to
[26 Aug 2021 Shift 1]

Options:

A. $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

B. $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Answer: B

Solution:

Solution:

$$AA^T = \begin{vmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix}$$

$$AA^T \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\text{Now, } Q^2 = A^T B A A^T B A$$

$$\Rightarrow Q^2 = A^T B^2 A$$

$$\text{Again, } Q^3 = (A^T B A)(A^T B^2 A) = A^T B^3 A$$

Similarly,

$$Q^{2021} = A^T B^{2021} A$$

$$A Q^{2021} A^T = A (A^T B^{2021} A) A^T$$

$$= (A A^T) B^{2021} (A A^T) = B^{2021}$$

$$B = \begin{vmatrix} 1 & 0 \\ i & 1 \end{vmatrix}$$

$$B^2 = \begin{vmatrix} 1 & 0 \\ 2i & 1 \end{vmatrix}, \text{ similarly } B^{2021} = \begin{vmatrix} 1 & 0 \\ 2021i & 1 \end{vmatrix}$$

$$(B^{2021})^{-1} = \frac{\text{adj}(B^{2021})}{|B^{2021}|} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$$

Question90

Let A be a 3×3 real matrix. If $\det(2 \text{ Adj}(2 \text{ Adj}(\text{Adj}(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equal
[26 Aug 2021 Shift 2]

Answer: 4

Solution:

Solution:

We have, A is a square matrix of 3×3 . Now,
 $2 \text{ Adj}(2 \text{ Adj}(\text{Adj}(2A)))$

$$\begin{aligned}
&= 2 \operatorname{Adj}(2 \operatorname{Adj}(2^{3-1} \operatorname{adj} A)) [\because \operatorname{adj}(KA) = K^{n-1} \operatorname{adj} A] \\
&= 2 \operatorname{Adj}(2 \operatorname{Adj}(4 \operatorname{Adj} A)) \\
&= 2 \operatorname{Adj}(2 \times 4^{3-1} \operatorname{Adj} \operatorname{Adj} A) \\
&= 2 \operatorname{Adj}(32 \operatorname{Adj} \operatorname{Adj} A) \\
&= 2 \times 32^{3-1} \operatorname{Adj} \operatorname{Adj} \operatorname{Adj} A \\
&= 2^{11} \operatorname{Adj} \operatorname{Adj} \operatorname{Adj} A \\
&= 2^{11} \operatorname{Adj}(|A|^{3-2} A) \\
&= 2^{11} \operatorname{Adj}(|A| A) \\
&= 2^{11} \times |A|^{3-1} \operatorname{Adj} A \\
&= 2^{11} \times |A|^2 \operatorname{Adj} A
\end{aligned}$$

$$\text{Now, } |2 \operatorname{Adj}(2 \operatorname{Adj}(\operatorname{Adj}(2A)))| = 2^{41}$$

$$\Rightarrow |2^{11} \times |A|^2 \operatorname{Adj} A| = 2^{41}$$

$$\Rightarrow (2^{11})^3 (|A|^2)^3 |\operatorname{Adj} A| = 2^{41}$$

$$\Rightarrow 2^{33} |A|^6 |A|^{3-1} = 2^{41}$$

$$\Rightarrow |A|^6 \times |A|^2 = 2^8$$

$$\Rightarrow |A|^8 = 2^8$$

$$\Rightarrow |A| = \pm 2$$

$$\text{Now, } |A^2| = |A|^2 = (\pm 2)^2 = 4$$

Question91

Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbb{R}$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to :
[27 Jul 2021 Shift 1]

Options:

- A. 5
- B. $\frac{8}{3}$
- C. 2
- D. 4

Answer: D

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}, |A| = 6$$

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{6} \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\left. \begin{aligned} \alpha + \beta &= \frac{2}{3} \\ \beta &= -\frac{1}{6} \end{aligned} \right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$4(\alpha - \beta) = 4(1) = 4$$

Question92

Let A and B be two 3×3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to :
[27 Jul 2021 Shift 2]

Options:

- A. 2
- B. 4
- C. 1
- D. 0

Answer: D

Solution:

Solution:

$$C = A^2 - B^2; |C| \neq 0$$

$$A^5 = B^5 \text{ and } A^3B^2 = A^2B^3$$

$$\text{Now, } A^5 - A^3B^2 = B^5 - A^2B^3$$

$$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$$

$$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$$

$$\text{Post multiplying inverse of } A^2 - B^2 :$$

$$A^3 + B^3 = 0$$

Question93

Let $A = \{a_{ij}\}$ be a 3×3 matrix, where $a_{ij} = \begin{cases} (-1)^{j-1} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$ then

$\det(3\text{Adj}(2A^{-1}))$ is equal to _____.
[20 Jul 2021 Shift 2]

Answer: 108

Solution:

Solution:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$|A| = 4$$

$$\begin{aligned}
 |3\text{adj}(2A^{-1})| &= |3 \cdot 2^2 \text{adj}(A^{-1})| \\
 &= 12^3 |\text{adj}(A^{-1})| = 12^3 |A^{-1}|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108
 \end{aligned}$$

Question94

If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is
[27 Aug 2021 Shift 1]

Options:

- A. $\frac{1}{2}$
- B. $-\frac{1}{2}$
- C. -1
- D. 1

Answer: A

Solution:

Solution:

Given matrix,

$$A = \begin{bmatrix} 0 & 2 \\ K & -1 \end{bmatrix}$$

Characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 2 \\ K & -1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(1 + \lambda) - 2K = 0$$

$$\Rightarrow \lambda^2 + \lambda - 2K = 0$$

$\therefore$ Every square matrix satisfied its own characteristic equation.

$$\therefore A^2 + A - 2KI = 0$$

$$\Rightarrow A^2 = 2KI - A$$

$$\Rightarrow A^4 = 4K^2I + A^2 - 2(2KI)(A)$$

$$\Rightarrow A^4 = 4K^2I + 2KI - A - 4KA$$

$$\Rightarrow A^4 = (4K^2 + 2K)I - (1 + 4K)A \dots (i)$$

$$\text{Now, } A(A^3 + 3I) = 2I$$

$$\Rightarrow A^4 = 2I - 3A \dots (ii)$$

Comparing Eqs. (i) and (ii), we get

$$1 + 4K = 3$$

$$\Rightarrow K = \frac{1}{2}$$

Question95

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

[2021, 26 Aug. Shift-II]

Options:

A. $A^6 - A$

B. A^5

C. $A^5 - A$

D. A^6

Answer: A

Solution:

Solution:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Now,

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

.....

.....

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n-2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^{2025} - A^{2020} = \begin{pmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$A^6 - A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A^{2025} - A^{2020} = A^6 - A$$

Question96

Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}, \text{ then the matrix } A^{31} \text{ is equal to:}$$

[Jan. 7, 2020 (I)]

Options:

A. A

B. I_3

C. A^2

D. A^3

Answer: D

Solution:

Solution:

Solution of $x^2 + x + 1 = 0$ is ω, ω^2

So, $\alpha = \omega$ and

$\omega^4 = \omega^3 \cdot \omega = 1 \cdot \omega = \omega$

$$A^2 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow A^4 = I$

$\Rightarrow A^{30} = A^{28} \times A^3 = A^3$

Question97

The number of all 3×3 matrices A, with enteries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is_____.

[NA Jan. 8, 2020 (I)]

Answer: 672

Solution:

Let $A = [a_{ij}]_{3 \times 3}$

It is given that sum of diagonal elements of AA^T is 3 i.e., $\text{tr}(AA^T) = 3$

$$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \dots + a_{33}^2 = 3$$

Possible cases are

$$\left. \begin{array}{ll} 0,0,0,0,0,1,1,1 & \rightarrow 1 \\ 0,0,0,0,0,0,-1,-1,-1, & \rightarrow 1 \\ 0,0,0,0,0,0,1,1,-1 & \rightarrow 3 \\ 0,0,0,0,0,0,-1,1,-1 & \rightarrow 3 \end{array} \right\} {}^9C_6 \times 8 = 84 \times 8 = 672$$

Question98

If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to:

[Jan. 8, 2020 (II)]

Options:

A. $A - 4I$

B. $6I - A$

C. $A - 6I$

D. $4I - A$

Answer: C

Solution:

Solution:

Characteristics equation of matrix 'A' is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2 - \lambda & 2 \\ 9 & 4 - \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 6\lambda - 10 = 0$$

$$\therefore A^2 - 6A - 10I = 0$$

$$\Rightarrow A^{-1}(A^2) - 6A^{-1} - 10IA^{-1} = 0$$

$$\Rightarrow 10A^{-1} = A - 6I$$

Question99

If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24} \right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then

which one of the following is not true?

[Sep. 04, 2020 (I)]

Options:

A. $0 \leq a^2 + b^2 \leq 1$

B. $a^2 - d^2 = 0$

C. $a^2 - c^2 = 1$

D. $a^2 - b^2 = \frac{1}{2}$

Answer: D

Solution:

Solution:

$$\therefore A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbb{N}$$

$$\therefore A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\therefore A^5 = \begin{bmatrix} \cos 5\theta & i \sin 5\theta \\ i \sin 5\theta & \cos 5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\therefore a = \cos 5\theta, b = i \sin 5\theta = c, d = \cos 5\theta$$

$$\therefore a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$$

$$a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$$

$$a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$$

$$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos \frac{10\pi}{24}$$

$$\text{and } 0 < \cos \frac{5\pi}{12} < 1 \Rightarrow 0 \leq a^2 + b^2 \leq 1$$

$$\therefore a^2 - b^2 = \frac{1}{2} \text{ is wrong.}$$

Question 100

Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to

 .
[NA Sep. 03, 2020 (I)]

Answer: 10

Solution:

Solution:

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix}$$

$$x^4 = \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x^2 + 1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2 + 1)^2 + x^2 & x(x^2 + 1) + x \\ x(x^2 + 1) + x & x^2 + 1 \end{bmatrix}$$

$$\text{Given that } (x^2 + 1)^2 + x^2 = 109$$

$$x^4 + 3x^2 - 108 = 0$$

$$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$$

$$\therefore x^2 = 9$$

$$a_{22} = x^2 + 1 = 9 + 1 = 10$$

Question101

Let $a, b, c \in \mathbb{R}$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} \text{ satisfies } A^T A = I, \text{ then a value of } abc \text{ can be:}$$

[Sep. 02, 2020 (II)]

Options:

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. 3

D. $\frac{2}{3}$

Answer: B

Solution:

Solution:

Given : $A^T A = I$

$$\Rightarrow = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \sum a^2 & \sum ab & \sum ab \\ \sum ab & \sum a^2 & \sum ab \\ \sum ab & \sum ab & \sum a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $\sum a^2 = 1$ and $\sum ab = 0$

Now, $a^3 + b^3 + c^3 - 3abc$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= (a + b + c)(1 - 0)$$

$$= \sqrt{(a + b + c)^2} = \sqrt{\sum a^2 + 2\sum ab} = \pm 1$$

$$\Rightarrow 2 - 3abc = 1 \Rightarrow abc = \frac{1}{3}$$

$$\text{or } 2 - 3abc = -1 \Rightarrow abc = 1.$$

Question102

Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is :

[Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{6}}$

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$$

$$\therefore A \cdot A^T = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \times \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix}$$

$$= \begin{bmatrix} 4q^2 + r^2 & 2q^2 - r^2 & -2q^2 + r^2 \\ 2q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -2q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix}$$

Given, $AA^T = I$

$$\therefore 4q^2 + r^2 = p^2 + q^2 + r^2 = 1$$

$$\Rightarrow p^2 - 3q^2 = 0 \text{ and } r^2 = 1 - 4q^2$$

$$\text{and } 2q^2 - r^2 = 0 \Rightarrow r^2 = 2q^2$$

$$\therefore p^2 = \frac{1}{2}, q^2 = \frac{1}{6} \text{ and } r^2 = \frac{1}{3}$$

$$\therefore |p| = \frac{1}{\sqrt{2}}$$

Question103

Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that

$Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to:

[Jan. 12, 2019 (I)]

Options:

- A. 10
- B. 135
- C. 15
- D. 9

Answer: A

Solution:

Solution:

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}$$

$$\Rightarrow P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$$\therefore Q - P^5 = I_3$$

$$\therefore Q = I_3 + P^5 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

Question104

Let $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, ($\alpha \in \mathbb{R}$) such that $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Then a value of

α is :

[April 8, 2019 (I)]

Options:

- A. $\frac{\pi}{32}$
- B. 0
- C. $\frac{\pi}{64}$
- D. $\frac{\pi}{16}$

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$\text{Similarly, } A^4 = A^2 \cdot A^2 = \begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix}$$

$$\text{and so on } A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Then $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$$\Rightarrow 32\alpha = n\pi + (-1)^n \frac{\pi}{2} \text{ and } 32\alpha = 2n\pi + \frac{\pi}{2}$$

$$\Rightarrow \alpha = \frac{n\pi}{32} + (-1)^n \frac{\pi}{64} \text{ and } \alpha = \frac{n\pi}{16} + \frac{\pi}{64} \text{ where } n \in \mathbb{Z}$$

$$\text{Put } n = 0, \alpha = \frac{\pi}{64}$$

Hence, required value of α is $\frac{\pi}{64}$

Question 105

The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, ($x, y \in \mathbb{R}$, $x \neq y$) for

which $A^T A = 3I_3$ is:

[April 09, 2019 (II)]

Options:

A. 2

B. 3

C. 6

D. 4

Answer: D

Solution:

Solution:

Given, $A^T A = 3I_3$

$$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = 3I$$

$$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow 8x^2 = 3 \text{ and } 6y^2 = 3 \Rightarrow x = \pm \sqrt{\frac{3}{8}} \text{ and } y = \pm \sqrt{\frac{1}{2}}$$

$$\text{Number of combinations of } (x, y) = 2 \times 2 = 4$$

Question 106

If **A** is a symmetric matrix and **B** is a skew-symmetric matrix such that

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}, \text{ then } \mathbf{AB} \text{ is equal to:}$$

[April 12, 2019 (I)]

Options:

A. $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

D. $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

Answer: B

Solution:

Solution:

$$\text{Let } \mathbf{A} = \begin{bmatrix} a & c \\ c & b \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 & d \\ -d & 0 \end{bmatrix}$$

$$\text{Then, } \mathbf{A} + \mathbf{B} = \begin{bmatrix} a & c+d \\ c-d & b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

On comparing each term,

$$a = 2, b = -1, c - d = 5, c + d = 3$$

$$\Rightarrow a = 2, b = -1, c = 4, d = -1$$

$$\text{Now, } \mathbf{AB} = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

Question 107

Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $\mathbf{B} = \mathbf{A}^{20}$. Then the sum of the elements of the first column of **B** is?

[Online April 16, 2018]

Options:

A. 211

B. 210

C. 231

D. 251

Answer: C

Solution:

Solution:

$$\text{Here } A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\text{also } A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$$

$$\text{and, } A^4 = A^3 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$$

On observing the pattern, we come to a conclusion that,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ \frac{n(n+1)}{2} & n & 1 \end{bmatrix}$$

$$\therefore A^{20} = [1, 0, 0; 20, 1, 0; 210, 20, 1]$$

Therefore, sum of first column of $A^{20} = [1 + 20 + 210] = 231$

Question108

For two 3×3 matrices A and B, let $A + B = 2B^T$ and $3A + 2B = I_3$, where B^T is the transpose of B and I_3 is 3×3 identity matrix. Then : quad

[Online April 9, 2017]

Options:

A. $5A + 10B = 2I_3$

B. $10A + 5B = 3I_3$

C. $B + 2A = I_3$

D. $3A + 6B = 2I_3$

Answer: B

Solution:

Solution:

$$A^T + B^T = 2B$$

$$\because [(A + B)^T = (2B^T)^T]$$

$$\Rightarrow B = \frac{A^T + B^T}{2} = A + \left(\frac{B^T + A^T}{2} \right) = 2B^T$$

$$\Rightarrow 2A + A^T = 3B^T \Rightarrow A = \frac{3B^T - A^T}{2}$$

$$\text{Also, } 3A + 2B = I_3 \dots\dots(i)$$

$$\Rightarrow 3(3B^T - A^T) + 2 \left(\frac{A^T + B^T}{2} \right) = I_3$$

$$\Rightarrow 11B^T - A^T = 2I_3 \dots\dots(ii)$$

Add (i) and (ii)

$$35B = 7I_3$$

$$\Rightarrow B = \frac{I_3}{5} \Rightarrow 11 \frac{I_3}{5} - A = 2I_3$$

$$\Rightarrow 11 \frac{I_3}{5} - 2I_3 = A \Rightarrow A = \frac{I_3}{5}$$

$$\because 5A = 5B = I_3$$

$$\Rightarrow 10A + 5B = 3I_3$$

Question109

If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is ;

[Online April 9, 2016]

Options:

A. $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

Answer: C

Solution:

Solution:

$$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\begin{aligned} PP^T &= P^T P = I \\ Q^{2015} &= (PAP^T)(PAP^T) \dots (PAP^T) \text{ (2015 terms)} \\ &= PA^{2015}P^T \\ P^T Q^{2015} P &= A^{2015} \end{aligned}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2015} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$$

Question 110

If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct?

[Online April 10, 2015]

Options:

A. $A^2 + I = A(A^2 - I)$

B. $A^4 - I = A^2 + I$

C. $A^3 + I = A(A^3 - I)$

D. $A^3 - I = A(A - I)$

Answer: A

Solution:

Solution:

Given that

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow A^2 = -I$$

$$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^2 + I = A^3 - A$$

$$-I + I = A^3 - A$$

$$A^3 \neq A$$

Question111

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where

I is 3×3 identity matrix, then the ordered pair (a, b) is equal to:
[2015]

Options:

A. (2,1)

B. (-2,-1)

C. (2,-1)

D. (-2,1)

Answer: B

Solution:

Solution:

Given that $AA^T = 9I$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -1 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2-4 & a+4+2b \\ 2+2-4 & 4+1+4 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow a+4+2b=0 \Rightarrow a+2b=-4 \dots\dots(i)$$

$$2a+2-2b=0 \Rightarrow 2a-2b=-2 \dots\dots(ii)$$

$$\Rightarrow a-b=-1$$

Subtract (ii) from (i)

$$a+2b=-4$$

$$a-b=-1$$

$$\begin{array}{rrr} - & + & + \\ \hline \end{array}$$

$$3b=-3$$

$$b=-1$$

$$\text{and } a=-2$$

$$(a, b) = (-2, -1)$$

Question112

If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, then:

[Online April 12, 2014]

Options:

A. $y = 2x$

B. $y = -2x$

C. $y = x$

D. $y = -x$

Answer: A

Solution:

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y + 2x + x \\ 3y - x + 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y + 3x \\ 3y - x + 2 \end{bmatrix}$$

$$\Rightarrow y + 3x = 6 \text{ and } 3y - x = 6$$

On solving, we get

$$x = \frac{6}{5} \text{ and } y = \frac{12}{5}$$

$$\Rightarrow y = 2x$$

Question113

Let A and B be any two 3×3 matrices. If A is symmetric and B is skewsymmetric, then the matrix $AB - BA$ is:

[Online April 19, 2014]

Options:

A. skewsymmetric

B. symmetric

C. neither symmetric nor skewsymmetric

D. I or $-I$, where I is an identity matrix

Answer: B

Solution:

Solution:

Let A be symmetric matrix and B be skew symmetric matrix.

$\therefore A^T = A$ and $B^T = -B$

Consider $(AB - BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T = (-B)(A) - (A)(-B)$

$= -BA + AB = AB - BA$

This shows $AB - BA$ is symmetric matrix.

Question114

If p, q, r are 3 real numbers satisfying the matrix equation,

$$\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix} \text{ then } 2p + q - r \text{ equals :}$$

[Online April 22, 2013]

Options:

A. -3

B. -1

C. 4

D. 2

Answer: A

Solution:

Solution:

Given

$$\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3p + 3q + 2r & 4p + 2q & p + 3q + 2r \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 3p + 3q + 2r = 3 \text{(i)}$$

$$4p + 2q = 0 \Rightarrow q = -2p \text{(ii)}$$

$$p + 3q + 2r = 1 \text{(iii)}$$

On solving (i), (ii) and (iii), we get

$$p = 1, q = -2, r = 3$$

$$\therefore 2p + q - r = 2(1) + (-2) - (3) = -3.$$

Question115

The matrix $A^2 + 4A - 5I$, where I is identity matrix and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, equals
[Online April 9, 2013]

Options:

A. $4 \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

B. $4 \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

C. $32 \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

D. $32 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Answer: A

Solution:

Solution:

$$\begin{aligned} A^2 + 4A - 5I &= A \times A + 4A - 5I \\ &= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4 \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 9+4-5 & -4+8-0 \\ -8+16-0 & 17-12-5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix} = 4 \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix} \end{aligned}$$

Question116

If $A = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix}$, $B = \begin{pmatrix} \alpha + 1 \\ 0 \\ 0 \end{pmatrix}$ be two matrices, then AB^T is a non-zero matrix for $|\alpha|$ not equal to
[Online May 7, 2012]

Options:

A. 2

B. 0

C. 1

D. 3

Answer: C

Solution:

Solution:

$$\text{Let } A = \begin{pmatrix} \alpha - 1 & & \\ 0 & & \\ & 0 & \end{pmatrix}, B = \begin{pmatrix} \alpha + 1 & & \\ 0 & & \\ & 0 & \end{pmatrix}$$

be two matrices.

$$AB^T = \begin{pmatrix} \alpha - 1 & & \\ 0 & & \\ & 0 & \end{pmatrix} (\alpha + 1 \ 0 \ 0) = \begin{pmatrix} \alpha^2 - 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus, AB^T is non-zero matrix for $|\alpha| \neq 1$

Question117

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$ then AB equals

[Online May 26, 2012]

Options:

- A. I
- B. A
- C. B
- D. 0

Answer: A

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Question118

If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then

H^{70} is equal to
[2011RS]

Options:

- A. 0
- B. $-H$
- C. H^2
- D. H

Answer: D

Solution:

Solution:

$$H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$\text{We observed that } H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega^k \end{bmatrix}$$

$$\therefore H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega^{69}\omega & 0 \\ 0 & \omega^{69}\omega \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H \quad [\because \omega^{3n} = 1]$$

Question119

Let A and B be two symmetric matrices of order 3.

Statement-1: $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

[2011]

Options:

- A. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
- B. Statement-1 is true, Statement-2 is false.
- C. Statement-1 is false, Statement-2 is true.
- D. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:

Given that A and B are symmetric matrix

$$A' = A$$

$$B' = B$$

Now $(A(BA))' = (BA)'A' = (A'B')A' = (AB)A = A(BA)$ ($\because$ product of matrices are associative)

Similarly, $((AB)A)' = A'(B'A') = A(BA) = (AB)A$

So, $A(BA)$ and $(AB)A$ are symmetric matrices.

Again $(AB)' = B'A' = BA$

Now if $BA = AB$, then AB is symmetric matrix

Question120

The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is [2010]

Options:

- A. 5
- B. 6
- C. at least 7
- D. less than 4

Answer: C

Solution:**Solution:**

$\begin{bmatrix} 1 & \dots & \dots \\ \dots & 1 & \dots \\ \dots & \dots & 1 \end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

$\begin{bmatrix} 1 & \dots & \dots \\ \dots & 1 & \dots \\ \dots & \dots & 1 \end{bmatrix}$ are 6 non-singular matrices.

Total = $6 + 6 = 12$

So, required cases are more than 7, non-singular 3×3 matrices.

Question121

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in \mathbb{N}$. Then

[2006]

Options:

- A. there cannot exist any B such that $AB = BA$
- B. there exist more than one but finite number of B's such that $AB = BA$
- C. there exists exactly one B such that $AB = BA$

D. there exist infinitely many B's such that $AB = BA$

Answer: D

Solution:

Solution:

$$\text{Given that } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

Hence, $AB = BA$ only when $a = b$

$\therefore$ There can be infinitely many B's for which $AB = BA$

Question122

If A and B are square matrices of size $n \times n$ such that

$A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?
[2006]

Options:

A. $A = B$

B. $AB = BA$

C. either of A or B is a zero matrix

D. either of A or B is identity matrix

Answer: B

Solution:

Solution:

$$\text{Given that } A^2 - B^2 = (A - B)(A + B)$$

$$A^2 - B^2 = A^2 + AB - BA - B^2$$

$$\Rightarrow AB = BA$$

Question123

If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds
for all $n \geq 1$, by the principle of mathematical induction
[2005]

Options:

A. $A^n = nA - (n - 1)I$

B. $A^n = 2^{n-1}A - (n - 1)I$

C. $A^n = nA + (n - 1)I$

D. $A^n = 2^{n-1}A + (n - 1)I$

Answer: A

Solution:

Solution:

Given that $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Therefore we observed that $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

$$\text{Now } nA - (n - 1)I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$$

$$\therefore nA - (n - 1)I = A^n$$

Question124

If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

[2003]

Options:

A. $\alpha = 2ab, \beta = a^2 + b^2$

B. $\alpha = a^2 + b^2, \beta = ab$

C. $\alpha = a^2 + b^2, \beta = 2ab$

D. $\alpha = a^2 + b^2, \beta = a^2 - b^2$.

Answer: C

Solution:

Solution:

$$A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \Rightarrow A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$

$\alpha = a^2 + b^2; \beta = 2ab$
