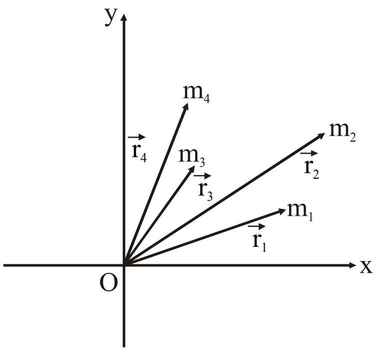
Chapter 7
System of Particles and Rotational Motion

CENTRE OF MASS

Centre of mass for a system of particles is defined as that point where the entire mass of the system is imagined to be concentrated.

If $\vec{r_1}, \vec{r_2}, \vec{r_3}$ .... be the position vectors of masses m_1, m_2, m_3 .... respectively from the origin O,



then the centre of mass of the system is

$$\overrightarrow{r_{cm}} = \frac{(m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + ...)}{(m_1 + m_2 + m_3 + ...)} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r_i}$$

where M is the total mass of the system of particles. The product of mass of the particles and its position vector w.r.t. the reference point is called **moment of mass**

i.e., *moment of mass* $= m \times \vec{r}$

MOTION OF CENTRE OF MASS

The **motion of the centre of mass** is governed by the equation

$$M\vec{A}_{cm} = \vec{F}_{ext} \text{ where } \vec{A}_{cm} = \frac{d^2(\vec{r}cm)}{dt^2}$$

Momentum conservation of a system of particles :

In the absence of external forces, the velocity of the centre of mass remains constant.

We have, $MA_{cm} = F_{ext}$

If $F_{ext} = 0$

$$M\frac{d}{dt}(v_{cm}) = 0$$

$\therefore$ v_{cm} = constant.

Hence, momentum (Mv_{cm} = constant) of the centre of mass system is conserved.

Rigid Bodies

If a body does not undergo any change in shape by the action of a force, it is said to be rigid.

If such body undergoes some displacement, every particle in it undergoes the same displacement. No real body can be perfectly rigid.

Rotatory Motion

A body rotating about a fixed axis then every particle of the body moves in a circle and the centres of all these circles lie on axis of rotation. The motion of the body is said to possess rotational motion.

Keep in Memory

1. The centre of mass of a system of two identical particles lies in between them on the line joining the particles.
2. If $m_1 = m_2$

$$\vec{r}_{cm} = \frac{(m_1\vec{r_1} + m_2\vec{r_2})}{(m_1 + m_2)} = \frac{(\vec{r_1} + \vec{r_2})}{2}$$

   so, for particles of equal masses the centre of mass is located at the **mean position vector** of the particles.
3. The position of centre of mass remains unchanged in pure rotatory motion. But it changes with time in translatory motion or rolling motion.
4. The position of centre of mass of a body is independent of the choice of co-ordinate system.
5. If we take the centre of mass at the origin, then the sum of the moments of the masses at of the system about the origin $\Sigma m_i \vec{r}_i$ is zero.
6. In pure rotatory motion, the axis of rotation passes through the centre of mass.
7. If external force is zero then the velocity of the centre of mass of a body remains constant.
8. The centre of mass and centre of gravity of a body coincide, if the value of g is same throughout the dimension of the body.
9. In kinematics and dynamics, whole of the mass of a body can be assumed to be concentrated at the centre of mass.
10. The location of the centre of mass depends on the shape and nature of distribution of mass of the body.
    (a) The position of centre of mass of continuous bodies can be found using integration as

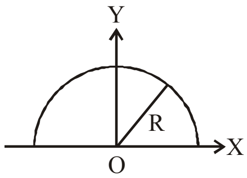
$$X_{cm} = \frac{1}{M}\int x\,dm,\ \ Y_{cm} = \frac{1}{M}\int y\,dm,\ \ Z_{cm} = \frac{1}{M}\int z\,dm$$

where, x, y and z are the co-ordinates of small mass dm and M is the total mass of the system.

(b) The C.M. of a uniform rod of length L is at its middle point.

(c) Centre of mass of a uniform semicircular wire is at $\left(0, \frac{2R}{\pi}\right)$, where R is the radius of the semicircular wire.

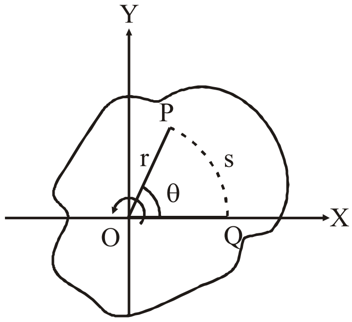
It does not depend on mass.



(d) For symmetrical bodies of uniform mass distribution, the C. M. lies at the geometrical centre

ANGULAR VELOCITY AND ANGULAR ACCELERATION

The angular velocity is defined as the angle covered by the radius vector per unit time. It is denoted by ω.



Average angular velocity $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$

The unit of angular velocity is **rad/sec**.

The instantaneous angular velocity ω (similar to instantaneous linear velocity) is defined as

$$\omega = \lim_{\Delta t \to o} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The angular acceleration is the rate of change of angular velocity. It is denoted by α.

The average angular acceleration $\alpha_{avg.}$ of a rotating body is

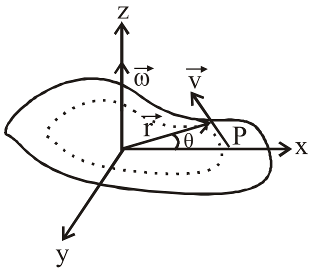
$$\alpha_{avg.} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

In analogy to linear acceleration $\vec{a}$, the instantaneous angular acceleration is defined as

$$\alpha = \lim_{\delta t \to o} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The unit of angular acceleration is **rad/sec²**.

Relationship between angular velocity and linear velocity:



$$\vec{v} = \vec{\omega} \times \vec{r}$$

$\Rightarrow$ v = rω sin θ

where θ is the angle between ω & r.

MOMENT OF INERTIA AND RADIUS OF GYRATION

A rigid body having constituent particles of masses m_1, m_2, ...m_n and r_1, r_2 ... r_n be their respective distances from the axis of rotation then **moment of inertia** is given by,

$$I = m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2 = \sum_{i=1}^{n} m_i r_i^2$$

The moment of inertia of continuous mass distribution is given by

$$I = \int r^2\,dm$$

where r is the perpendicular distance of the small mass dm from the axis of rotation.

Its **SI unit** is kgm^2. It is a **tensor**.

Radius of gyration :

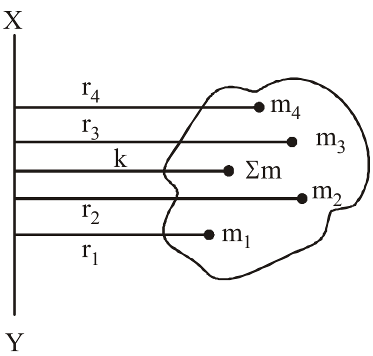
The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the entire mass of the body were concentrated, its moment of inertia about the given axis would be same as with its actual distribution of mass.

Radius of gyration k is given by,

$$I = MK^2$$

$$\text{or} \quad K = \left[\frac{I}{M}\right]^{\frac{1}{2}} = \left[\frac{\sum_i^n m_i r_i^2}{\Sigma m_i}\right]^{\frac{1}{2}}$$

where, $M = \Sigma m_i$



Also, $k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ...r_n^2}{n}}$

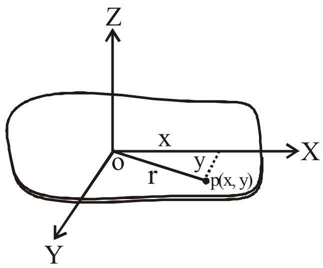
Therefore, radius of gyration (k) equals the root mean square of the distances of particles from the axis of rotation.

GENERAL THEOREMS ON MOMENT OF INERTIA

Theorem of perpendicular axis :

According to this theorem " the moment of inertia of a plane lamina (a plane lamina is a 2-dimensional body. Its third dimension is so small that it can be neglected.) *about an axis, perpendicular to the plane of lamina is equal to the sum of the moment of inertia of the lamina about two axes perpendicular to each other, in its own plane and intersecting each other at the point, where the perpendicular axes passes through it.*

If I_x and I_y be the moment of inertia of a plane lamina (or 2D rigid body) about the perpendicular axis OX and OY respectively, which lie in plane of lamina and intersect each other at O, then moment of inertia (I_z) about an axis passing through (OZ) and perpendicular to its plane is given by



$$I_x + I_y = I_z$$

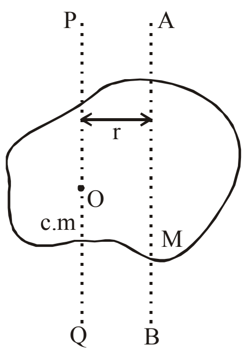
Let us consider a particle of mass m at point P distance r from origin O, where $r = \sqrt{x^2 + y^2}$

so $\quad I_x + I_y = \Sigma my^2 + \Sigma mx^2 = \Sigma mr^2$

i.e., $\quad I_z = I_x + I_y$

Theorem of parallel axes :

(Derived by Steiner) This theorem is true for both plane laminar body and thin 3D body. It states that *"the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the distance between two axes.*



Let AB be the axis in plane of paper about which, the moment of inertia (I) of plane lamina is to be determined and PQ an axis parallel to AB, passing through centre of mass O of lamina is at a distance 'r' from AB.

Consider a mass element m of lamina at point P distant x from PQ. Now the moment of inertia of the element about AB = $m(x + r)^2$

so moment of inertia of whole lamina about AB is

$$I = \Sigma m(x+r)^2 = \Sigma mx^2 + \Sigma mr^2 + 2\Sigma mxr$$

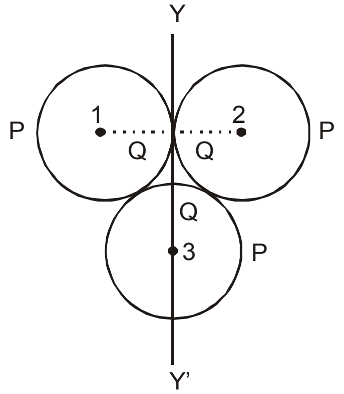
Where first term on R.H.S is $\Sigma mx^2 = I_{c.m.}$ moment of inertia of lamina about PQ through its centre of mass, second term on R.H.S. is $\Sigma mr^2 = r^2\Sigma m = Mr^2$, M is whole mass of lamina, third term on R.H.S is $(\Sigma mx)\, r = 0$, because Σmx is equal to moments of all particles of lamina about an axis PQ, passing through its centre of mass. Hence

$$I = I_{c.m.} + M.r^2$$

i.e., the moment of inertia of lamina about AB = its moment of inertia about a parallel axis PQ passing through its centre of mass + mass of lamina×(distance between two axes)^2

Example 1.

Three rings each of mass P and radius Q are arranged as shown in fig. What will be the moment of inertia of the arrangement about YY'?



Solution :

Moment of inertia of each ring about its diameter

$$= \frac{1}{2}PQ^2.$$

So total moment of inertia of all three rings about Y Y' is

$I_{total} = I_1 + I_2 + I_3$

Using theorem of parallel axes (for rings 1 and 2), we get

$$I_{total} = \left(\frac{1}{2}PQ^2 + PQ^2\right) + \left(\frac{1}{2}PQ^2 + PQ^2\right) + \frac{1}{2}PQ^2 = \frac{7}{2}PQ^2$$

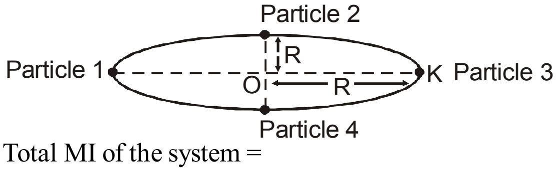
Example 2.

Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. Find MI of this system about an axis passing through one of the particles and perpendicular to plane of disc.

Solution :

According to the theorem of parallel axes, MI of disc about an axis passing through K and perpendicular to plane of disc, is

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$



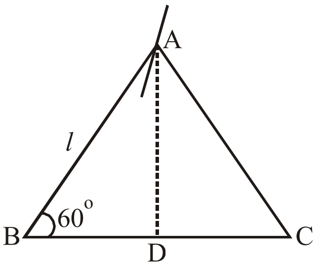
Total MI of the system =

$$\frac{3}{2}MR^2 + m(2R)^2 + m(\sqrt{2}R)^2 + m(\sqrt{2}R)^2 = 19\frac{MR^2}{2}$$

Example 3.

Three identical rods, each of length ℓ, are joined to from a rigid equilateral triangle. Find its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle.

Solution :



$$I = I_{AB} + I_{AC} + I_{BC} = \frac{ml^2}{3} + \frac{ml^2}{3} + (I_D + mh^2)$$

$$= \frac{2}{3}ml^2 + \frac{ml^2}{12} + m(l\sin 60)^2 = ml^2\left[\frac{2}{3} + \frac{1}{12} + \frac{3}{4}\right]$$

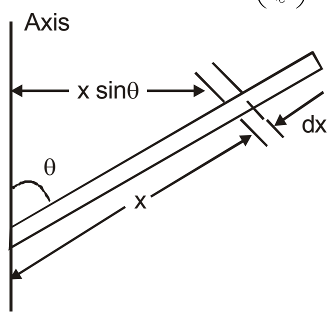
$$= \frac{3}{2}ml^2 = (3m)k^2 \text{ or } k = \frac{\ell}{\sqrt{2}}$$

Example 4.

A uniform rod of mass m and length ℓ makes a constant angle θ with the axis of rotation which passes through one end of the rod. Determine its moment of inertia about this axis.

Solution :

Mass of element of uniform rod = $\left(\frac{m}{\ell}\right)dx$



Moment of inertia of the element about the axis

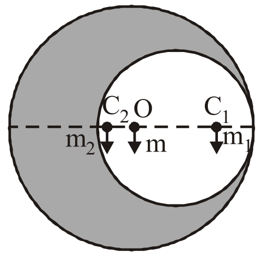
$$= \left(\frac{m}{\ell}dx\right)(x\sin\theta)^2.$$

$$I = \frac{m}{\ell}\sin^2\theta.\int_0^{\ell} x^2 dx = \frac{m\ell^2}{3}\sin^2\theta$$

Example 5.

A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown. Find the position of centre of mass of the remaining portion.

Solution :



Area of whole plate = $\pi(56/2)^2 = 784\,\pi$ sq. cm.
Area of cut portion = $\pi(42/2)^2 = 441\,\pi$ sq. cm. ;
Area of remaining portion = $784\pi - 441\pi = 343\,\pi\,cm^2$;
As mass $\propto$ area.

$$\therefore \frac{\text{Mass of cut portion}}{\text{Mass of remaining portion}} = \frac{m_1}{m_2} = \frac{441\pi}{343\pi} = \frac{9}{7}$$

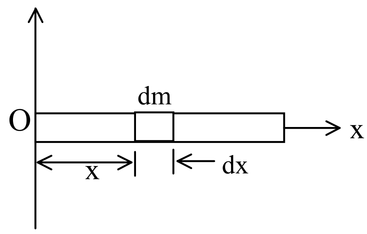
Let C_2 be centre of mass of remaining portion and C_1 be centre of mass of cut portion.
O is centre of mass of the whole disc.;
$OC_1 = r_1 = 28 - 21 = 7$ cm.
$OC_2 = r_2 = ?$;
Equating moments of masses about O,

we get $m_2 \times r_2 = m_1 \times r_1 \Rightarrow r_2 = \frac{m_1}{m_2}\times r_1 = \frac{9}{7}\times 7 = 9$

$\therefore$ Centre of mass of remaining portion is at 9 cm to the left of centre of disc.

Example 6.

Show that the centre of mass of a rod of mass M and length L lies midway between its ends, assuming the rod has a uniform mass per unit length.



Solution :

By symmetry, we see that $y_{CM} = z_{CM} = 0$ if the rod is placed along the x axis. Furthermore, if we call the mass per unit length λ (the linear mass density), then $\lambda = M/L$ for a uniform rod.

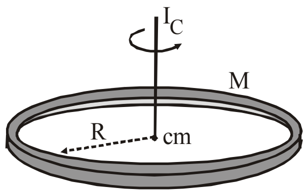
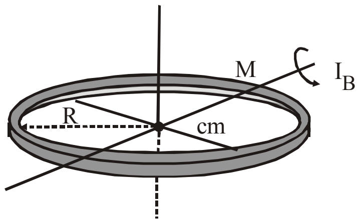
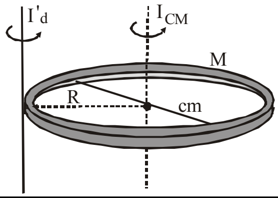
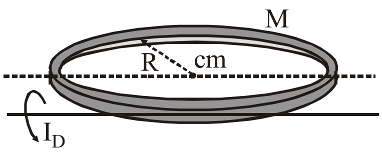
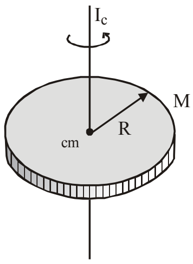
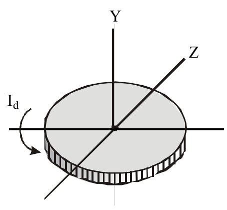
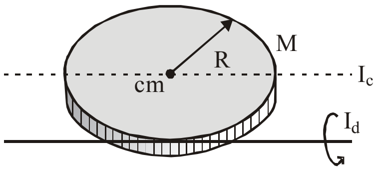
If we divide the rod into elements of length dx, then the mass of each element is $dm = \lambda\,dx$. Since an arbitrary element of each element is at a distance x from the origin, equation gives

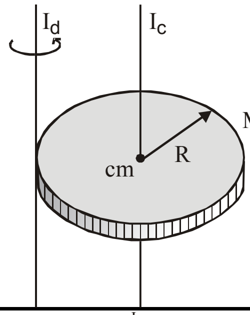
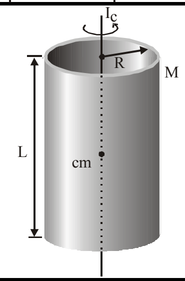
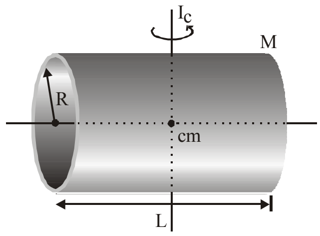
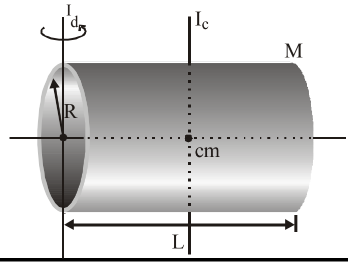
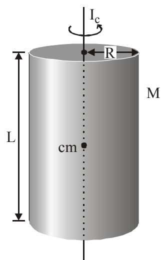
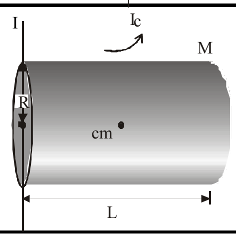
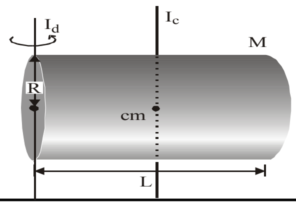
$$x_{CM} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x\lambda dx = \frac{\lambda L^2}{2M}$$

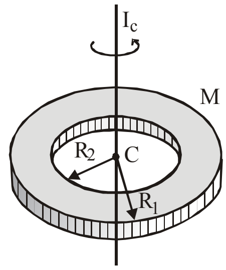
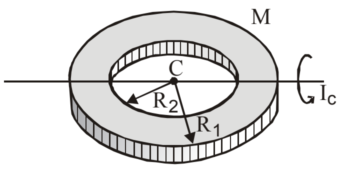
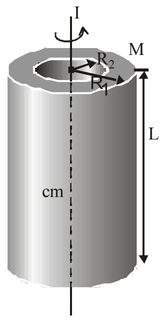
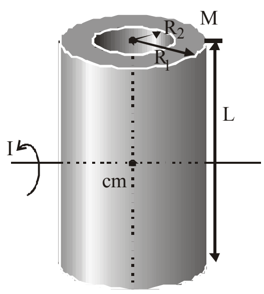
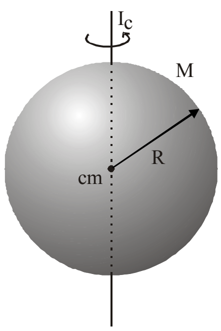
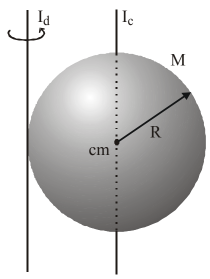
Because $\lambda = M/L$, this reduces to $x_{CM} = \frac{L^2}{2M}\left(\frac{M}{L}\right) = \frac{L}{2}$

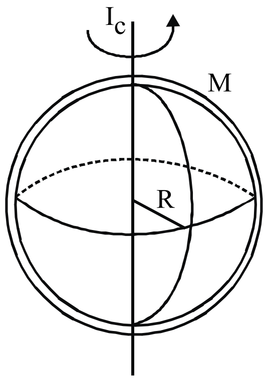
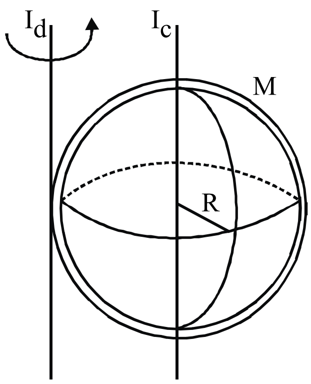
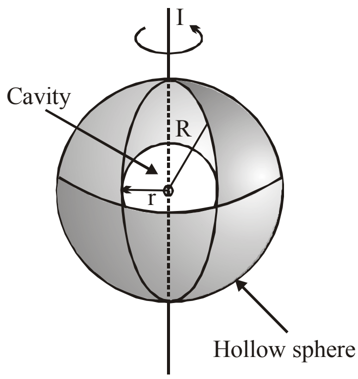
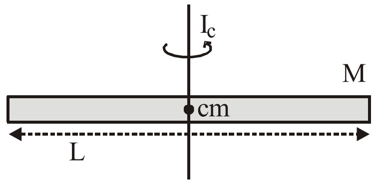
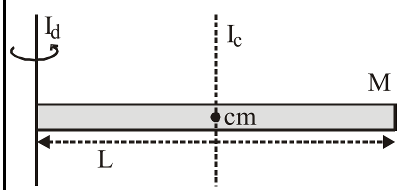
One can also argue that by symmetry, $x_{CM} = L/2$.

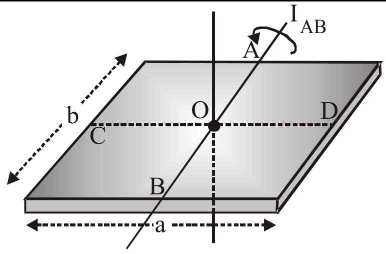
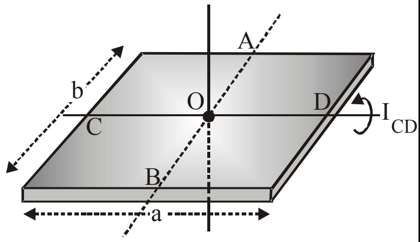
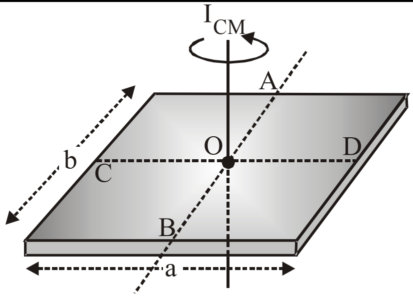
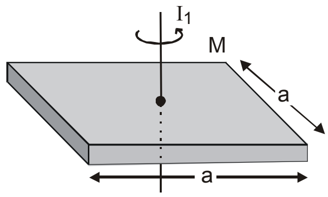
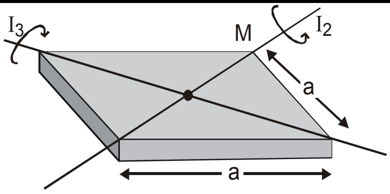
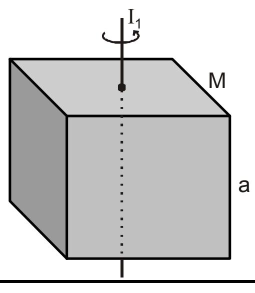
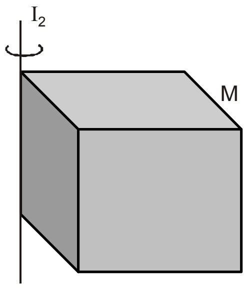
MOMENT OF INERTIA AND RADIUS OF GYRATION OF DIFFERENT OBJECTS

Shape of body	Rotational axis	Figure	Moment of inertia	Radius of gyration
(1) Ring M = mass R = radius	(a) Perpendicular to plane passing through centre of mass		MR^2	R
	(b) Diameter in the plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
	(c) Tangent perpendicular to plane		$2MR^2$	$\sqrt{2}\,R$
	(d) Tangent in the plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}\,R$
(2) Disc	(a) Perpendicular to plane passing through centre of mass		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
	(b) Diameter in the plane		$\frac{MR^2}{4}$	$\frac{R}{2}$
	(c) Tangent in the plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$

	(d) Tangent perpendicular to plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}\,R$
(3) Thin walled cylinder	(a) Geometrical axis		MR^2	R
	(b) Perpendicular to length passing through centre of mass		$M\left(\frac{R^2}{2}+\frac{L^2}{12}\right)$	$\sqrt{\frac{R^2}{2}+\frac{L^2}{12}}$
	(c) Perpendicular to length passing through one end		$M\left(\frac{R^2}{2}+\frac{L^2}{3}\right)$	$\sqrt{\frac{R^2}{2}+\frac{L^2}{3}}$
(4) Solid cylinder	(a) Geometrical axis		$\frac{MR^2}{2}$	$\frac{R}{\sqrt{2}}$
	(b) Perpendicular to length passing through centre of mass		$M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{12}}$
	(c) Perpendicular to length passing through one end		$M\left[\frac{R^2}{4}+\frac{L^2}{3}\right]$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{3}}$

(5) Annular disc	(a) Perpendicular to plane passing through centre of mass		$\frac{M}{2}\,[R_1^2+R_2^2]$	$\sqrt{\frac{R_1^2+R_2^2}{2}}$
	(b) Diameter in the plane		$\frac{M[R_1^2+R_2^2]}{4}$	$\sqrt{\frac{R_1^2+R_2^2}{4}}$
(6) Hollow cylinder	(a) Geometrical axis		$M\left[\frac{R_1^2+R_2^2}{2}\right]$	$\sqrt{\frac{R_1^2+R_2^2}{2}}$
	(b) Perpendicular to length passing through centre of mass		$M\left[\frac{L^2}{12}+\frac{(R_1^2+R_2^2)}{4}\right]$	$\sqrt{\frac{L^2}{12}+\frac{R_1^2+R_2^2}{4}}$
(7) Solid sphere	(a) Along the diameter		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}\,R$
	(b) Along the tangent		$\frac{7}{5}\,MR^2$	$\sqrt{\frac{7}{5}}\,R$

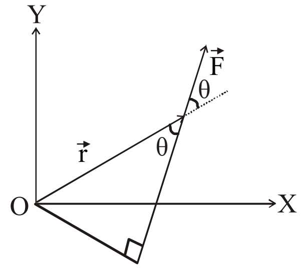
(8) Thin spherical shell	(a) Along the diameter		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}\,R$
	(b) Along the tangent		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}\,R$
(9) Hollow sphere	Along the diameter		$\frac{2}{5}M\left[\frac{R^5-r^5}{R^3-r^3}\right]$	$\sqrt{\frac{2}{5}\frac{(R^5-r^5)}{(R^3-r^3)}}$
(10) Thin rod	(a) Perpendicular to length passing through centre of mass		$\frac{ML^2}{12}$	$\frac{L}{2\sqrt{3}}$
	(b) Perpendicular to length passing through one end		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$

(11) Rectangular plate	(a) Perpendicular to length in the plane passing through centre of mass		$\frac{Ma^2}{12}$	$\frac{a}{2\sqrt{3}}$
	(b) Perpendicular to breadth in the plane passing through centre of mass		$\frac{Mb^2}{12}$	$\frac{b}{2\sqrt{3}}$
	(c) Perpendicular to plane passing through centre of mass		$\frac{M(a^2+b^2)}{12}$	$\frac{\sqrt{a^2+b^2}}{2\sqrt{3}}$
(12) Square Plate	(a) Perpendicular to plane passing through centre of mass		$I_1 = \frac{Ma^2}{6}$	$\frac{a}{\sqrt{6}}$
	(b) Diagonal passing through centre of mass		$I_2 = I_3 \frac{Ma^2}{12}$	$\frac{a}{2\sqrt{3}}$
(13) Cube	(a) Perpendicular to plane passing through centre of mass		$I_1 = \frac{Ma^2}{6}$	$\frac{a}{\sqrt{6}}$
	(b) Perpendicular to plane passing through one end		$I_2 = \frac{2Ma^2}{3}$	$\sqrt{\frac{2}{3}}\, a$

TORQUE, ANGULAR MOMENTUM AND ANGULAR IMPULSE

Torque :

The moment of force is called torque. *It is defined as the product of force and the perpendicular distance of the force from the axis of rotation.*



i.e., $\vec{\tau} = \vec{r} \times \vec{F}$

or, $\tau = rF \sin\theta$

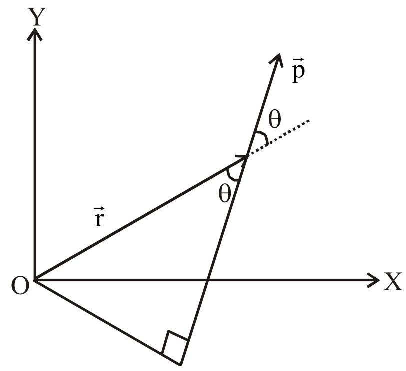
where θ is the angle between $\vec{r}$ and $\vec{F}$.

Its **S.I. unit** is (N-m). The **dimensions** of torque $[ML^2T^{-2}]$ are the same as that of energy but it is not energy.

Note : If the line of action of a force passes through axis of rotation then no torque will be formed.

Angular Momentum :

The angular momentum of a particle about an arbitrry point 'O' is the moment of linear momentum taken about that point.



It is given as $\vec{L} = \vec{r} \times \vec{p}$

or, $L = rp\sin\theta$

where θ is the angle between $\vec{r}$ and $\vec{F}$.

Angular Impulse :

$$J = \int_{t_1}^{t_2} \tau dt = L_2 - L_1 \quad (= \text{Change in angular momentum})$$

CONSERVATION OF ANGULAR MOMENTUM

From equation $\vec{\tau}_{ext} = \dfrac{d\vec{L}}{dt}$.

If $\quad \vec{\tau}_{ext} = 0 \Rightarrow \vec{L} = \text{constant} \qquad$ ...(i)

This is called law of **conservation of angular momentum.** According to this "*if resultant external torque $\vec{\tau}_{ext}$ acting on the system is zero then total angular momentum of the system is constant.*

The magnitude of angular momentum for a system is given by

$$|\vec{L}| = \sum_{i=1}^{n}(m_i r_i)v_i = \sum_{i=1}^{n}(m_i r_i^2)\omega \quad [\because v = r\omega]$$

$$\Rightarrow \quad |\vec{L}| = \sum_{i=1}^{n}(I_i \times \omega) = I\omega \qquad \text{...(ii)}$$

Where I_i is the moment of inertia of the i^{th} particle of that system and I is total moment of inertia of the system

$$I = \sum_{i=1}^{n} I_i = I_1 + I_2 + I_3 + ... + I_n$$

or $\quad I = m_1r_1^2 + m_2r_2^2 + ... + m_nr_n^2 \qquad$ ...(iii)

So if a system undergoes a redistribution of its mass, then its moment of inertia changes but since no external torque is applied on the system so total angular momentum is constant before and after the distribution of mass, even if moment of inertia of the system is changed.

$$\vec{L}_{\text{Initial}} = \vec{L}_{\text{Final}} \qquad \text{...(iv)}$$

or, $\quad I_1\omega_1 = I_2\omega_2 \qquad$ ...(v)

where L initial denote the state previous to the redistribution of mass and final denote the state after the redistribution of mass in that system.

A comparison of useful relations in rotational and translational or linear motion :

Rotational motion about a fixed axis	***Linear motion***
Angular velocity $\omega = \dfrac{d\theta}{dt}$	Linear velocity $v = \dfrac{dx}{dt}$
Angular acceleration $\alpha = \dfrac{d\omega}{dt}$	Linear acceleration $a = \dfrac{dv}{dt}$
Resultant torque $\tau = I\alpha$	Resultant force $F = ma$
Equations of rotational motion	Equations of linear motion
$\alpha = \text{constant then} \begin{cases} \omega = \omega_0 + \alpha t \\ \theta - \theta_0 = \omega_0 t + ½\alpha t^2 \\ \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \end{cases}$	$a = \text{constant then} \begin{cases} v = u + at \\ s = ut + \frac{1}{2}at^2 \\ v^2 - u^2 = 2as \end{cases}$ where $(s = x - x_0)$
Work $W = \int_{\theta_o}^{\theta} \vec{\tau} d\vec{\theta}$	work $W = \int_{x_o}^{x} \vec{F} d\vec{x}$
Kinetic energy $E_k = ½ I\omega^2$	Kinetic energy $E_k = ½ mv^2$
Power $P = \tau\omega$	Power $P = \vec{F}\vec{v}$
Angular momentum $L = \vec{I}\vec{\omega}$	Linear momentum $p = mv$
Torque $\tau = \dfrac{dL}{dt}$	Force $F = \dfrac{dp}{dt}$

Work Energy Theorem in Rotational Motion :

According to this theorem "*the work done by external forces in rotating a rigid body about a fixed axis is equal to the change in rotational kinetic energy of the body*."

Since, we can express the torque as

$$\tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}.\frac{d\theta}{dt}$$

$$\tau = I\omega\frac{d\omega}{d\theta} \quad \text{but} \quad \tau d\theta = dW$$

$\Rightarrow \quad \tau d\theta = dW = I\omega d\omega$

By integrating the above expression, we get total work done by all external force on the body, which is written as

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I\omega d\omega = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$$

where the angular velocity of the body changes from ω_1 to ω_2 as the angular displacement changes from θ_1 to θ_2 due to external force $\vec{F}_{ext.}$ on the body.

Rotational Kinetic Energy :

Let us consider a rigid body (collection of small particles) of high symmetry which is purely rotating about z-axis with an angular velocity ω. Each particle has some energy, determined by m_i and v_i. The kinetic energy of m_i particle is

$$(E_k)_i = ½m_i v_i^2 \quad \text{... (i)}$$

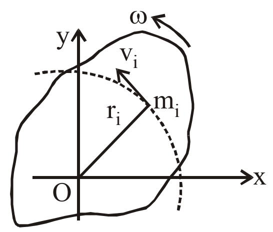


Fig. The total kinetic energy of the body is ½Iω².

Now we know that in rigid body every particle moves with same angular velocity, the individual linear velocities depends on the distance r_i from the axis of rotation according to the expression ($v_i = r_i\omega$). Hence the total kinetic energy of rotating rigid body is equal to the sum of kinetic energies of individual particles.

$$E_k = \Sigma(E_k)_i = \Sigma½m_i v_i^2 = (\Sigma m_i r_i^2)\frac{\omega^2}{2}$$

$$E_k = ½I\omega^2 \quad \text{... (ii)}$$

where $I = \Sigma m_i r_i^2$ is moment of inertia of the rigid body.

Now consider a rigid body which is rolling without slipping. In this case it possesses simultaneous translatory motion and rotatory motion and the total kinetic energy of the rigid body

$K.E_{Total}$ = rotation K.E. + translational K.E. of C.M.

$$E_k = (E_k)_{rotational} + (E_k)_{translational} \quad \text{... (iii)}$$

$$E_k = ½I\omega^2 + ½Mv_{cm}^2 \quad \text{... (iv)}$$

$$E_k = ½Mv_{cm}^2(1 + K^2/r^2) = ½I\omega^2(1 + r^2/K^2) \quad \text{... (v)}$$

where,

v_{cm} = linear velocity of the centre of mass of the rigid body

K = radius of gyration $(I = MK^2 = \Sigma M_i r_i^2)$

r = radius of moving rigid body

I = moment of inertia of the rigid body about centre of mass

½ $I\omega^2$ = rotational kinetic energy about the centre of mass.

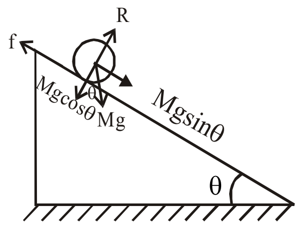
Hence it is clear from the expression that total kinetic energy of rolling body is equal to the sum of rotational kinetic energy about centre of mass (C.M.) and translational kinetic energy of the centre of mass of body.

Body Rolling without Slipping on an Inclined Plane :

When a body performed translatory as well as rotatory motion then we can say that the body is in rolling motion.

Acceleration for body rolling down an inclined plane without slipping.

Let M is the mass of the body, R is its radius and I is the moment of inertia about the centre of mass and K is the radius of gyration.



Force equation, $Mg\sin\theta - f = Ma$ ...(1)

Torque equation, $fR = I\alpha$ ...(2)

Also, $I = MK^2$ ...(3)

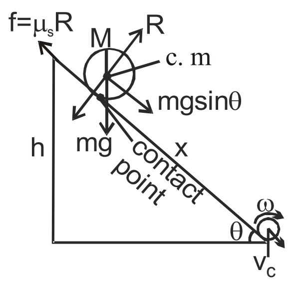
$\Rightarrow \quad f = \frac{MK^2\alpha}{R}$ ...(4)

Adding, eqn. (1) & (4)

$$Mg\sin\theta = M\left(\frac{K^2\alpha}{R} + a\right) \quad \alpha = \frac{a}{R} \quad (\because \text{ Motion is pure rolling})$$

$$Mg\sin\theta = M\left(\frac{K^2 a}{R^2} + a\right) \Rightarrow a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$$

Now assume in fig. that a body (which has high symmetry such as cylinder, sphere etc.) is rolling down an incline plane without slipping. This is possible only if friction is present between object and incline plane, because it provides net torque to the body for rotating about the centre of mass (since the line of action of the other forces such as mg and R pass through the centre of mass of rigid body, hence they do not produce torque in a body about the centre of mass).



A round object rolling down an incline, mechanical energy is conserved if no slipping occurs.

But mechanical energy of the body remains constant despite of friction because the contact point is at rest relative to the surface at any instant.

For pure rolling motion $v_c = r\omega$

so $\quad E_k = ½I(v_c / r)^2 + ½Mv_c^2$

$$E_k = \frac{1}{2}\left(\frac{I_c}{r^2} + M\right)v_c^2 \qquad \text{...(i)}$$

As body rolls down an incline, it loses potential energy Mgh (h is the height of incline). Since body starts from rest at top, hence its total kinetic energy at bottom given by eqn.(i) must be equal to Mgh at top i.e.,

$$\frac{1}{2}\left(\frac{I_c}{r^2} + M\right)V_c^2 = Mgh \Rightarrow v_c = \left(\frac{2gh}{1 + I_c / MR^2}\right)^{1/2} \qquad \text{...(ii)}$$

since h =x sin θ, where x is length of incline and $I_c = MK^2$ (K is radius of gyration), then

$$v_c = \left[\frac{2gx\sin\theta}{1 + K^2 / R^2}\right]^{½} \qquad \text{....(iii)}$$

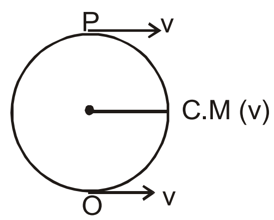
For rolling down an inclined plane without slipping, the essential condition is :

$\mu_s \geq \frac{I_{cm}}{MR^2 + I_{cm}}\tan\theta$; μ_s is the coefficient of static friction

where I_{cm} is moment of inertia of the body about its C.M.

DIFFERENT TYPES OF MOTION

Pure translational motion : In this case the velocities at all three points : (i) top most point P, (ii) C.M. and (iii) contact point O are same.

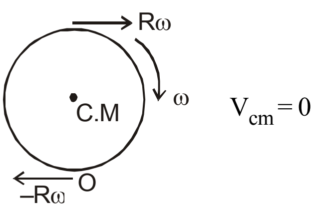
(A) 

Pure rotational motion : In this case the velocity of

(i) top most point P is Rω

(ii) C.M. is zero

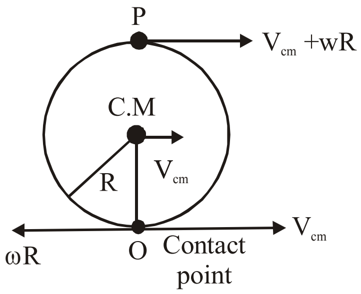
(iii) contact point O is – Rω

(B) 

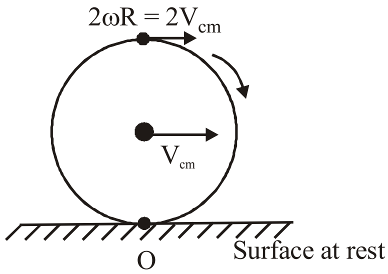
Rolling Motin Combination of translatory and rotatory motion :

(i) In pure rolling motion, the contact point O remains at rest.

(ii) In pure rolling, the velocity of top most point is, $V = V_{cm} + \omega R = 2V_{cm}$



(iii) In rolling without slipping $V_{cm} = \omega R$.



Keep in Memory

1. The axis of the rolling body is parallel to the plane on which the body rolls in case of sphere, disc, ring.
2. Let $(E_k)_r$ = rotational kinetic energy
   $(E_k)_t$ = translation kinetic energy
   (a) For solid sphere, $(E_k)_r = 40\%$ of $(E_k)_t$
   (b) For shell $(E_k)_r = 66\%$ of $(E_k)_t$
   (c) For disc, $(E_k)_r = 50\%$ of $(E_k)_t$
   (d) For ring, $(E_k)_r = (E_k)_t$
3. Translational kinetic energy is same for rolling bodies having same M, R and ω irrespective of their shape.
4. Total energy is minimum for solid sphere and maximum for ring having same mass and radius.
5. Rotational kinetic energy is maximum for ring and minimum for solid sphere of same mass and radius.
6. (i) The acceleration down the inclined plane for different shapes of bodies of same mass and radius are as follows :
   Sphere > disc > shell > ring
   (ii) The velocity down the plane is related as follows: Sphere > disc > shell > ring
   (iii) The time taken to reach the bottom of the inclined plane is related as follows :
   Ring > shell > disc > sphere.

This is because

(i) For ring $\frac{K^2}{R^2} = 1$

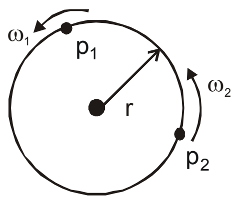
(ii) For disc $\frac{K^2}{R^2} = \frac{1}{2}$

(iii) For sphere $\frac{K^2}{R^2} = \frac{2}{5}$

(iv) For shell $\frac{K^2}{R^2} = \frac{2}{3}$

7. The angular speed of all particles of a rotating/revolving rigid body is same, although their linear velocities may be different.

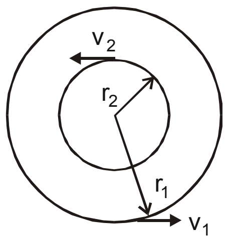
8. (i) Two particles moving with angular speed ω_1 and ω_2 on the same circular path and both in anti-clockwise direction then their relative angular speed will be $\omega_r = \omega_1 - \omega_2$.



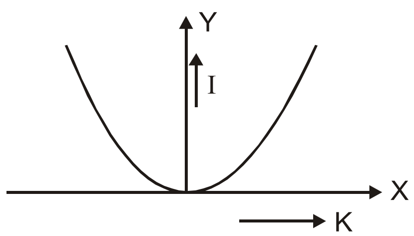
(ii) In the above case one particle will complete one revolution more or less as compared to the other in time

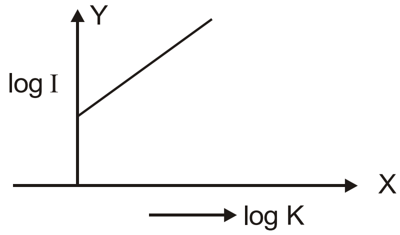
$$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{\omega_1 - \omega_2} = \frac{T_1 T_2}{T_1 - T_2} \text{ or } \frac{1}{T} = \frac{1}{T_2} - \frac{1}{T_1}$$

9. Let two particles move on concentric circles having radius r_1 and r_2 and their linear speeds v_1, v_2 both along anti-clockwise direction then their relative angular speed will be $\omega_r = \frac{|\vec{v}_1 - \vec{v}_2|}{|\vec{r}_1 - \vec{r}_2|}$



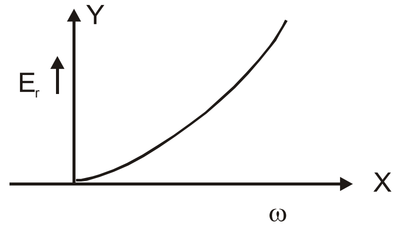
10. As $I \propto K^2$ and $I = MK^2$, hence graph between I and K will be a **parabola**. However graph between log I and log K will be straight line.



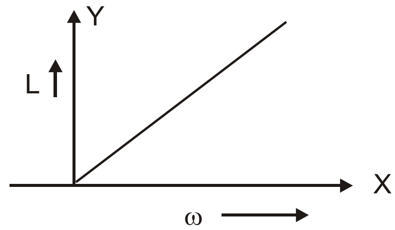


11. Rotational KE, $(E_k)_r = \frac{1}{2} I\omega^2 \quad \therefore E_r \propto \omega^2$

$\therefore$ graph between $(E_k)_r$ and ω will be as below



12. Angular momentum $L = I\omega$, hence $L \propto \omega$.
$\therefore$ Graph between L and ω is a straight line.



13. The moment of inertia is not a vector quantity because clockwise or anti-clockwise, direction is not associated with it. It is a tensor.

14. When a spherical/circular/cylindrical body is given a push, it only slips when the friction is absent, It may roll with slipping if friction is less than a particular value and it may roll without slipping if the friction is sufficient. (i.e. $\mu_s \geq \frac{I_{cm} \tan\theta}{(MR^2 + I_{cm})}$)

15. When a body rolls without slipping no work is done against friction.

Example 7.

A thin circular ring of mass M and radius R rotating about its axis with a constant angular speed ω. Two blocks, each of mass, m are attached gently to opposite ends of a diameter of the ring. Find the angular speed of the ring.

Solution :

As $L = I\omega$ = constant. Therefore,

$$I_2\omega_2 = I_1\omega_1 \text{ or } \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{MK^2\omega}{(M+2m)K^2} = \frac{M\omega}{M+2m}$$

Example 8

A solid sphere of mass 500 g and radius 10 cm rolls without slipping with a velocity of 20 cm/s. Find the total K.E. of the sphere.

Solution :

$$\text{Total K.E.} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\times\frac{2}{5}m\,r^2\omega^2 + \frac{1}{2}mv^2$$

$$\text{As, } I_{sphere} = \frac{2}{5}mr^2$$

$$= \frac{1}{5}mv^2 + \frac{1}{2}mv^2 = \frac{7}{10}mv^2$$

$$= \frac{7}{10}\times\frac{500}{1000}\times\left(\frac{1}{5}\right)^2 = 0.014\text{ J}.$$

Example 9.

A body of radius R and mass M is rolling horizontally without slipping with speed v, it then rolls up a hill to a maximum height h. If $h = 3v^2/4\,g$, (a) what is the moment of inertia of the body? (b) what might be the shape of the body?

Solution :

(a) $$K_{total} = K_{trans.} + K_{rot.} = \frac{1}{2}M\,v^2 + \frac{1}{2}I\,\omega^2$$

$$= \frac{1}{2}M\,v^2 + \frac{1}{2}I\left(\frac{v^2}{R^2}\right) = \frac{v^2}{2}\left[M + \frac{I}{R^2}\right]$$

When it rolls up a hill to height h, the entire kinetic energy is converted into potential energy M g h

$$\text{Thus } \frac{v^2}{2}\left[M + \frac{I}{R^2}\right] = M\,g\,h = M\,g\left[3\frac{v^2}{4g}\right]$$

$$\text{or } \left[M + \frac{I}{R^2}\right] = \frac{3}{2}M \quad \therefore \quad I = \frac{MR^2}{2}$$

(b) The body may be a circular disc or a solid cylinder.

Example 10.

The angular velocity of a body changes from ω_1 to ω_2 without applying torque but by changing moment of inertia. What will be the ratio of initial radius of gyration to the final radius of gyration?

Solution :

$$\text{Since } I_1\omega_1 = I_2\omega_2 \text{ or } M\,K_1^2\omega_1 = M\,K_2^2\omega_2$$

$$\Rightarrow \frac{K_1}{K_2} = \sqrt{\left(\frac{\omega_2}{\omega_1}\right)}$$

Example 11.

A rod of mass M and length ℓ is suspended freely from its end and it can oscillate in the vertical plane about the point of suspension. It is pulled to one side and then released. It passes through the equilibrium position with angular speed ω. What is the kinetic energy while passing through the mean position?

Solution :

$$\text{K.E,} = \frac{1}{2}I\omega^2 \text{ and } I = (M\,\ell^2/12) + M\,(\ell/2)^2 = \frac{M\ell^2}{3}$$

$$\therefore \quad K = \frac{1}{2}\left[\frac{M\,\ell^2}{3}\right]\omega^2 = \frac{1}{6}M\,\ell^2\omega^2$$

Example 12.

A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and moment of inertia about it is I. A weight mg is attached to the end of the cord and falls from rest. After falling through a distance h, what will be the angular velocity of the wheel?

Solution :

$$m\,g\,h = \frac{1}{2}I\omega^2 + \frac{1}{2}m\,v^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m\,r^2\,\omega^2$$

$$\text{or } 2\,m\,g\,h = (I + m\,r^2)\omega^2 \quad \therefore \quad \omega = \left[\frac{2mgh}{I + mr^2}\right]^{1/2}$$

Example 13.

Let g be the acceleration due to gravity at earth's surface and K be the rotational kinetic energy of the earth. Suppose the earth's radius decrease by 2%, keeping all other quantities same, then

(a) g decreases by 2% and K decreases by 4%
(b) g decreases by 4% and k increases by 2%
(c) g increases by 4% and K decreases by 4%
(d) g decreases by 4% and K increases by 4%

Solution : **(c)**

$$\text{We know that, } g = \frac{GM}{R^2}$$

$$\text{Differentiating, } \frac{dg}{g} = -\left(\frac{2dR}{R}\right) \qquad ....(1)$$

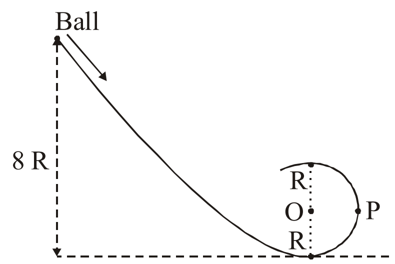
$$\text{Further, } K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{3}{5}M\,R^2\right]\omega^2$$

$$\text{or } \frac{dK}{K} = \frac{3}{10}M\,\omega^2\times\left(\frac{2dR}{R}\right) \qquad ....(2)$$

When radius decreases by 2%, then g increases by 4% and K decreases by 4%.

Example 14.

A small solid ball rolls without slipping along the track shown in fig. The radius of the circular part of track is R. If the ball starts from rest at a height 8 R above the bottom, what are the horizontal and vertical forces acting on it at P?



Solution :

Suppose m is the mass of the ball of radius r.

On reaching P, the net height through which the ball descends is

8 R – R = 7R, (from the fig.)

$\therefore$ decrease in P.E. of ball = mg × 7 R

This appears as total KE of ball at P.

Thus mg × 7R = KE of translation + KE of rotational

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\left(\frac{2}{5}mr^2\right)\omega^2 \quad = \frac{7}{10}mv^2$$

$\therefore\ v^2 = 10\,g\,R$

(where v = rω and r is radius of solid ball)

The horizontal force acting on the ball,

F_h = centripetal force towards O

$$= \frac{mv^2}{R} = \frac{m(10gR)}{R} = 10\,mg$$

Vertical force on the ball, F_v = weight of the ball = mg.

Example 15.

A solid cylinder first rolls and then slides from rest down a smooth inclined plane. Compare the velocities in the two cases when the cylinder reaches the bottom of the incline.

Solution :

(i) We know that acc. of a body rolling down an inclined plane is $a_1 = \dfrac{g\sin\theta}{1+K^2/R^2}$

For a solid cylinder $K^2 = \dfrac{R^2}{2}$;

$$a_1 = \frac{g\sin\theta}{(1+1/2)} = \frac{2}{3}g\sin\theta\ ;\ \text{From } v^2 = u^2 + 2as$$

$$v_1^2 = 0 + 2\left(\frac{2}{3}g\sin\theta\right)\times\ell = \frac{4}{3}g\sin\theta\times\ell \qquad ...(i)$$

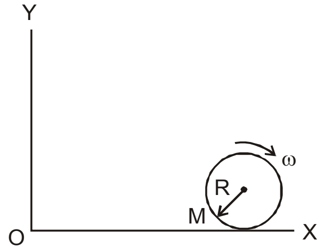
(ii) Acc. of the sliding cylinder, $a_2 = g\sin\theta$

From $v^2 = u^2 + 2as;\ v_2^2 = 0 + 2g\sin\theta\times\ell$ ...(ii)

$$\therefore\ \frac{v_1^2}{v_2^2} = \frac{4g\sin\theta\times\ell}{3\times2g\sin\theta\times\ell} = \frac{2}{3}\ ;\ \frac{v_1}{v_2} = \sqrt{2/3}\ .$$

Example 16.

A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane (fig.). Determine the magnitude of angular momentum of the disc about the origin O.



Solution :

The angular momentum L is given by

$$L = I_{cm}\omega + MvR = \left(\frac{1}{2}MR^2\right)\omega + M(\omega R)R \quad = \frac{3}{2}MR^2\omega$$

CONCEPT MAP

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Centre of mass
The point where whole mass of the system is supposed to be concentrated

Position of centre of mass of the system
$$r_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum m_i}$$

Position of centre of mass of an object changes in translatory motion but remains unchanged in rotatory motion

Velocity of centre of mass of the system
$$v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum m_i}$$

Acceleration of centre of mass of the system
$$a_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{\sum m_i}$$

Position of centre of mass depends upon, shape, size distribution of mass of the body

Centre of mass for rigid bodies is independent of the state i.e, rest or motion of the body

Rigid body
A body with a perfectly definite and unchanging shape

Moment of inertia
Inertia of rotational motion

M.I., $I = \sum_{i=1}^{n} m_i r_i^2$

Radius of gyration
$$K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$
$$K = \sqrt{\frac{I}{M}}$$

Theorems of moment of inertia

Theorem of perpendicular axes $I_z = I_x + I_y$
I_x, I_y and I_z moments of inertia about perpendicular axes x, y and z respectively.

Theorem of parallel axes
$I = I_{cm} + mr^2$
I_{cm} = M.I. about the parallel axis through the centre of mass

Moment of inertia of some regular shaped body
(1) Rod $I_{cm} = Ml^2/12$
$I_{one\ edge} = Ml^2/3$
(2) Ring $I_{cm} = MR^2$
$I_{diameter} = MR^2/2$
(3) Disc $I_{cm} = MR^2/2$
$I_{diameter} = MR^2/4$
(4) Solid sphere $I_{diameter} = 2/5\ MR^2$

Equations of rotational motion
(i) $\omega_2 = \omega_1 + \alpha t$
(ii) $\theta = \omega_0 t + ½ \alpha t^2$
(iii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

***Torque or couple* or moment of force**
$\tau = r \times F = rF \sin\theta\ \hat{n}$
$\tau_{rotational} = I\alpha$

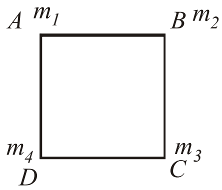
Angular momentum
$L = I\omega = mvr = r \times mv$
$L = r \times P = rP \sin\theta$

Kinetic energy
$K.E._{rotational} = ½ I\omega^2$
$K.E._{rolling} = ½ mv^2 + ½ I\omega^2$

EXERCISE - 1
Conceptual Questions

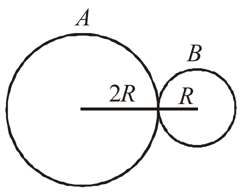
1. Centre of mass of the earth and the moon system lies
(a) closer to the earth
(b) closer to the moon
(c) at the mid-point of line joining the earth and the moon
(d) cannot be predicted

2. Four particles of masses m_1,m_2,m_3 and m_4 are placed at the vertices A,B,C and D as respectively of a square shown. The COM of the system will lie at diagonal AC if



(a) $m_1 = m_3$
(b) $m_2 = m_4$
(c) $m_1 = m_2$
(d) $m_3 = m_4$

3. Two spheres A and B of masses m and $2m$ and radii $2R$ and R respectively are placed in contact as shown. The COM of the system lies



(a) inside A (b) inside B
(c) at the point of contact (d) None of these

4. Moment of inertia does not depend upon
(a) distribution of mass
(b) axis of rotation
(c) point of application of force
(d) None of these

5. A disc is given a linear velocity on a rough horizontal surface then its angular momentum is
(a) conserved about COM only
(b) conserved about the point of contact only
(c) conserved about all the points
(d) not conserved about any point.

6. A body cannot roll without slipping on a
(a) rough horizontal surface
(b) smooth horizontal surface
(c) rough inclined surface
(d) smooth inclined surface

7. A body is projected from ground with some angle to the horizontal. The angular momentum about the initial position will
(a) decrease
(b) increase
(c) remains same
(d) first increase then decrease

8. A ball tied to a string is swung in a vertical circle. Which of the following remains constant?
(a) tension in the string
(b) speed of the ball
(c) centripetal force
(d) earth's pull on the ball

9. Angular momentum of a system of a particles changes, when
(a) force acts on a body
(b) torque acts on a body
(c) direction of velocity changes
(d) None of these

10. Angular momentum is
(a) a polar vector (b) an axial vector
(c) a scalar (d) None of these

11. If a running boy jumps on a rotating table, which of the following is conserved?
(a) Linear momentum
(b) K.E
(c) Angular momentum
(d) None of these

12. A gymnast takes turns with her arms & legs stretched. When she pulls her arms & legs in
(a) the angular velocity decreases
(b) the moment of inertia decreases
(c) the angular velocity stays constant
(d) the angular momentum increases

13. Moment of inertia of a circular wire of mass M and radius R about its diameter is
(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$.

14. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ?
(a) Angular velocity
(b) Angular momentum
(c) Moment of inertia
(d) Rotational kinetic energy

15. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B Such that
(a) $I_A < I_B$ (b) $I_A > I_B$
(c) $I_A = I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where d_A and d_B are their densities.

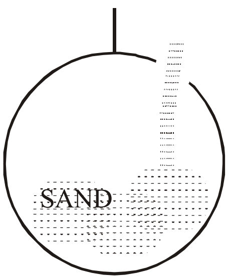
16. Angular momentum is
(a) moment of momentum
(b) product of mass and angular velocity
(c) product of M.I. and velocity
(d) moment of angular motion

17. The angular momentum of a system of particle is conserved
(a) when no external force acts upon the system
(b) when no external torque acts upon the system
(c) when no external impulse acts upon the system
(d) when axis of rotation remains same

18. Analogue of mass in rotational motion is
(a) moment of inertia (b) angular momentum
(c) gyration (d) None of these

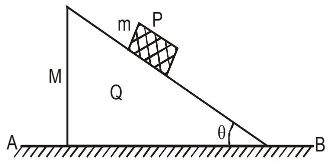
19. Moment of inertia does not depend upon
(a) angular velocity of body
(b) shape and size
(c) mass
(d) position of axis of rotation

20. A hollow sphere is held suspended. Sand is now poured into it in stages.



The centre of gravity of the sphere with the sand
(a) rises continuously
(b) remains unchanged in the process
(c) First rises and then falls to the original position
(d) First falls and then rises to the original position

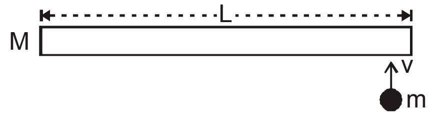
21. A block Q of mass M is placed on a horizontal frictionless surface AB and a body P of mass m is released on its frictionless slope. As P slides by a length L on this slope of inclination θ, the block Q would slide by a distance



(a) (m/M) L cos θ (b) m L/(M + m)
(c) (M + m)/(m L cos θ) (d) (m L cos θ) / (m + M)

22. A solid sphere and a hollow sphere of the same material and of same size can be distinguished without weighing
(a) by determining their moments of inertia about their coaxial axes
(b) by rolling them simultaneously on an inclined plane
(c) by rotating them about a common axis of rotation
(d) by applying equal torque on them

23. A stick of length L and mass M lies on a frictionless horizontal surface on which it is free to move in any way. A ball of mass m moving with speed v collides elastically with the stick as shown in fig.



If after the collision ball comes to rest, then what should be the mass of the ball?
(a) m = 2 M (b) m = M
(c) m = M/2 (d) m = M/4

24. A flywheel rotates about an axis. Due to friction at the axis, it experiences an angular retardation proportional to its angular velocity. If its angular velocity falls to half while it makes n rotations, how many more rotations will it make before coming to rest?
(a) 2n (b) n
(c) n/2 (d) n/3

25. A raw egg and a hard boiled egg are made to spin on a table with the same angular momentum about the same axis. The ratio of the time taken by the two to stop is
(a) = 1 (b) < 1
(c) > 1 (d) None of these

EXERCISE - 2
Applied Questions

1. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad/s. The radius of the cylinder is 0.25 m. The K.E. associated with the rotation of the cylinder is
(a) 3025 J (b) 3225 J
(c) 3250 J (d) 3125 J

2. What is the moment of inertia of a solid sphere of density ρ and radius R about its diameter?
(a) $\frac{105}{176} R^5 \rho$ (b) $\frac{105}{176} R^2 \rho$
(c) $\frac{176}{105} R^5 \rho$ (d) $\frac{176}{105} R^2 \rho$

3. Two particles A and B, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is 2 v, the speed of centre of mass is
(a) zero (b) v
(c) 1.5 v (d) 3 v

4. Point masses 1, 2, 3 and 4 kg are lying at the points (0, 0, 0), (2, 0, 0), (0, 3, 0) and (–2, –2, 0) respectively. The moment of inertia of this system about X-axis will be
(a) 43 kg –m^2 (b) 34 kg–m^2
(c) 27 kg – m^2 (d) 72 kg – m^2

5. A body having moment of inertia about its axis of rotation equal to 3 kg-m^2 is rotating with angular velocity equal to 3 rad/s. Kinetic energy of this rotating body is the same as that of a body of mass 27 kg moving with a speed of

(a) 1.0 m/s (b) 0.5 m/s
(c) 1.5 m/s (d) 2.0 m/s

6. A particle moves in a circle of radius 0.25 m at two revolutions per second. The acceleration of the particle in metre per second^2 is

(a) π^2 (b) $8\pi^2$
(c) $4\pi^2$ (d) $2\pi^2$

7. The radius of gyration of a body about an axis at a distance 6 cm from its centre of mass is 10 cm. Then, its radius of gyration about a parallel axis through its centre of mass will be

(a) 80 cm (b) 8 cm
(c) 0.8 cm (d) 80 m

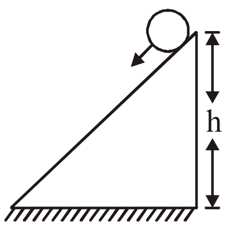
8. A particle of mass m is moving in a plane along a circular path of radius r. Its angular momentum about the axis of rotation is L. The centripetal force acting on the particle is

(a) L^2/mr (b) L^2m/r
(c) L^2/mr^3 (d) L^2/mr^2

9. A small disc of radius 2 cm is cut from a disc of radius 6 cm. If the distance between their centres is 3.2 cm, what is the shift in the centre of mass of the disc?

(a) 0.4 cm (b) 2.4 cm
(c) 1.8 cm (d) 1.2 cm

10. A solid cylinder of mass m & radius R rolls down inclined plane without slipping. The speed of its C.M. when it reaches the bottom is



(a) $\sqrt{2gh}$
(b) $\sqrt{4gh/3}$
(c) $\sqrt{3/4gh}$
(d) $\sqrt{4gh}$

11. The ratio of moment of inertia of circular ring & circular disc having the same mass & radii about on axis passing the c.m & perpendicular to plane is

(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1

12. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane. Let T_1 and T_2 be the tensions at the points L/4 and 3L/4 away from the pivoted end. Then

(a) $T_1 > T_2$
(b) $T_2 > T_1$
(c) $T_1 - T_2$
(d) the relation between T_1 and T_2 depends on whether the rod rotates clockwise and anticlockwise.

13. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a uniform speed v. The centrifugal force on it is

(a) $\frac{mv^2}{r}$ towards the centre
(b) $\frac{mv^2}{r}$ away from the centre
(c) $\frac{mv^2}{r}$ along the tangent through the particle
(d) zero

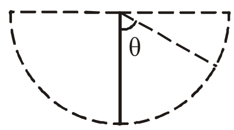
14. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 32 I_X$ (b) $I_Y = 16 I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64 I_X$

15. A particles performing uniform circular motion. Its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

(a) $\frac{L}{4}$ (b) $2L$
(c) $4L$ (d) $\frac{L}{2}$

16. A simple pendulum is vibrating with angular amplitude of 90° as shown in figure.



For what value of θ is the acceleration directed
(i) vertically upwards
(ii) horizontally
(iii) vertically downwards

(a) $0°, \cos^{-1}\frac{1}{\sqrt{3}}, 90°$ (b) $\cos^{-1}\frac{1}{\sqrt{3}}, 0°, 90°$
(c) $90°, \cos^{-1}\frac{1}{\sqrt{3}}, 0°$ (d) $\cos^{-1}\frac{1}{\sqrt{3}}, 90°, 0°$

17. A mass is tied to a string and rotated in a vertical circle, the minimum velocity of the body at the top is

(a) $\sqrt{gr}$ (b) g/r
(c) $\left(\frac{g}{r}\right)^{3/2}$ (d) gr

18. A couple is acting on a two particle systems. The resultant motion will be

(a) purely rotational motion
(b) purely linear motion
(c) both a and b
(d) None of these

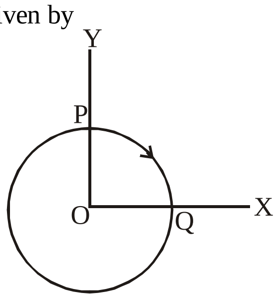
19. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

(a) is zero (b) remains constant
(c) goes on increasing (d) goes on decreasing.

20. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$
(c) $\omega_A = \omega$ (d) $\omega_B = \omega$

21. A particle moves in a circle of radius 4 cm clockwise at constant speed 2 cm s^{-1}. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X and Y respectively (in cm s^{-2}), the acceleration of the particle at the instant half way between P and Q is given by



(a) $-4(\hat{x}+\hat{y})$ (b) $4(\hat{x}+\hat{y})$
(c) $-(\hat{x}+\hat{y})/\sqrt{2}$ (d) $(\hat{x}-\hat{y})/4$

22. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct?

(a) $\vec{L}$ (angular momentum) is conserved about the centre
(b) only direction of angular momentum $\vec{L}$ is conserved
(c) It spirals towards the centre
(d) its acceleration is towards the centre.

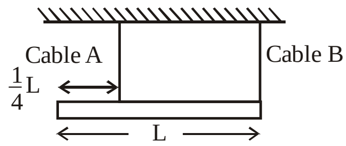
23. Two rings of radius R and nR made up of same material have the ratio of moment of inertia about an axis passing through centre as 1 : 8. The value of n is

(a) 2 (b) $2\sqrt{2}$
(c) 4 (d) $\frac{1}{2}$

24. A cylinder rolls down an inclined plane of inclination 30°, the acceleration of cylinder is

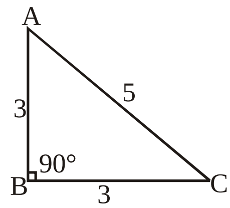
(a) $\frac{g}{3}$ (b) g
(c) $\frac{g}{2}$ (d) $\frac{2g}{3}$

25. A uniform bar of mass M and length L is horizontally suspended from the ceiling by two vertical light cables as shown. Cable A is connected 1/4th distance from the left end of the bar. Cable B is attached at the far right end of the bar. What is the tension in cable A?



(a) 1/4 Mg (b) 1/3 Mg
(c) 2/3 Mg (d) 3/4 Mg

26. ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC and CA as axes respectively. Which one of the following relations is correct?



(a) $I_{AB} > I_{BC}$
(b) $I_{BC} > I_{AB}$
(c) $I_{AB} + I_{BC} = I_{CA}$
(d) I_{CA} is maximum

27. In carbon monoxide molecule, the carbon and the oxygen atoms are separated by a distance 1.12×10^{-10} m. The distance of the centre of mass, from the carbon atom is

(a) 0.64×10^{-10} m (b) 0.56×10^{-10} m
(c) 0.51×10^{-10} m (d) 0.48×10^{-10} m

28. A weightless ladder 20 ft long rests against a frictionless wall at an angle of 60º from the horizontal. A 150 pound man is 4 ft from the top of the ladder. A horizontal force is needed to keep it from slipping. Choose the correct magnitude from the following.

(a) 175 lb (b) 100 lb
(c) 120 lb (d) 17.3 lb

29. The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter, is

(a) $\frac{3}{2}MR^2$ (b) $\frac{2}{3}MR^2$
(c) $\frac{5}{4}MR^2$ (d) $\frac{4}{5}MR^2$

30. A tube one metre long is filled with liquid of mass 1 kg. The tube is closed at both the ends and is revolved about one end in a horizontal plane at 2 rev/s. The force experienced by the lid at the other end is

(a) $4\pi^2$N (b) $8\pi^2$N
(c) $16\pi^2$N (d) 9.8 N

31. If the linear density (mass per unit length) of a rod of length 3m is proportional to x, where x is the distance from one end of the rod, the distance of the centre of gravity of the rod from this end is

(a) 2.5m (b) 1m
(c) 1.5m (d) 2m

32. A composite disc is to be made using equal masses of aluminium and iron so that it has as high a moment of inertia as possible. This is possible when

(a) the surfaces of the disc are made of iron with aluminium inside

(b) the whole of aluminium is kept in the core and the iron at the outer rim of the disc

(c) the whole of the iron is kept in the core and the aluminium at the outer rim of the disc

(d) the whole disc is made with thin alternate sheets of iron and aluminium

33. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(a) $\frac{R^2}{K^2+R^2}$ (b) $\frac{K^2+R^2}{R^2}$

(c) $\frac{K^2}{R^2}$ (d) $\frac{K^2}{K^2+R^2}$

34. When a bucket containing water is rotated fast in a vertical circle of radius R, the water in the bucket doesn't spill provided

(a) the bucket is whirled with a maximum speed of $\sqrt{2gR}$

(b) the bucket is whirled around with a minimum speed of $\sqrt{[(1/2)gR]}$

(c) the bucket is having a rpm of $\sqrt{900g/(\pi^2 R)}$

(d) the bucket is having a rpm of $\sqrt{3600g/(\pi^2 R)}$

35. A coin placed on a gramophone record rotating at 33 rpm flies off the record, if it is placed at a distance of more than 16 cm from the axis of rotation. If the record is revolving at 66 rpm, the coin will fly off if it is placed at a distance not less than

(a) 1 cm (b) 2 cm

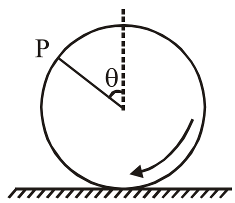
(c) 3 cm (d) 4 cm

36. Two fly wheels A and B are mounted side by side with frictionless bearings on a common shaft. Their moments of inertia about the shaft are 5.0 kg m^2 and 20.0 kg m^2 respectively. Wheel A is made to rotate at 10 revolution per second. Wheel B, initially stationary, is now coupled to A with the help of a clutch. The rotation speed of the wheels will become

(a) $2\sqrt{5}$ rps (b) 0.5 rps

(c) 2 rps (d) None of these

37. A wheel is rolling straight on ground without slipping. If the axis of the wheel has speed v, the instantenous velocity of a point P on the rim, defined by angle θ, relative to the ground will be



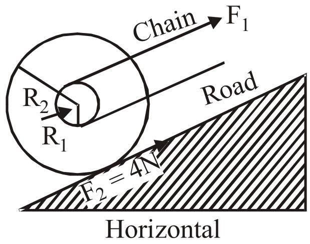
(a) $v\cos\left(\frac{1}{2}\theta\right)$

(b) $2v\cos\left(\frac{1}{2}\theta\right)$

(c) $v(1+\sin\theta)$

(d) $v(1+\cos\theta)$

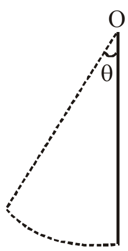
38. Acertain bicycle can go up a gentle incline with constant speed when the frictional force of ground pushing the rear wheel is $F_2 = 4$ N. With what force F_1 must the chain pull on the sprocket wheel if R_1=5 cm and R_2 = 30 cm?



(a) 4 N (b) 24 N

(c) 140 N (d) $\frac{35}{4}$ N

39. Auniform rod of length l is free to rotate in a vertical plane about a fixed horizontal axis through O. The rod begins rotating from rest from its unstable equilibrium position. When it has turned through an angle θ, its angular velocity ω is given as



(a) $\sqrt{\frac{6g}{l}}\sin\theta$ (b) $\sqrt{\frac{6g}{l}}\sin\frac{\theta}{2}$

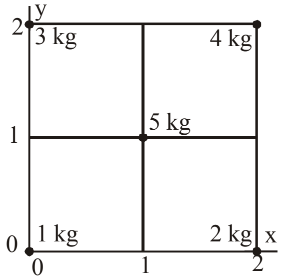
(c) $\sqrt{\frac{6g}{l}}\cos\frac{\theta}{2}$ (d) $\sqrt{\frac{6g}{l}}\cos\theta$

40. A sphere of mass 2000 g and radius 5 cm is rotating at the rate of 300 rpm .Then the torque required to stop it in 2π revolutions, is

(a) 1.6×10^2 dyne cm (b) 1.6×10^3 dyne cm

(c) 2.5×10^4 dyne cm (d) 2.5×10^5 dyne cm

41. Five masses are placed in a plane as shown in figure. The coordinates of the centre of mass are nearest to



(a) 1.2, 1.4 (b) 1.3, 1.1

(c) 1.1, 1.3 (d) 1.0, 1.0

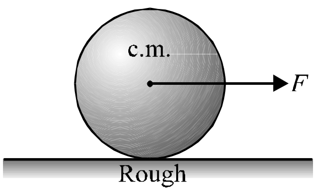
42. A solid sphere of mass 1 kg rolls on a table with linear speed 1 ms^{-1}. Its total kinetic energy is

(a) 1 J (b) 0.5 J
(c) 0.7 J (d) 1.4 J

43. A boy and a man carry a uniform rod of length L, horizontally in such a way that the boy gets $1/4^{th}$ load. If the boy is at one end of the rod, the distance of the man from the other end is

(a) L/3 (b) L/4
(c) 2 L/3 (d) 3 L/4

44. A solid sphere of mass M and radius R is pulled horizontally on a sufficiently rough surface as shown in the figure.



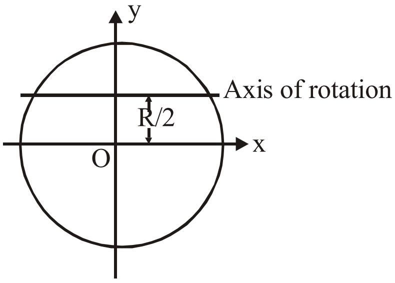
Choose the correct alternative.

(a) The acceleration of the centre of mass is F/M
(b) The acceleration of the centre of mass is $\frac{2}{3}\frac{F}{M}$
(c) The friction force on the sphere acts forward
(d) The magnitude of the friction force is F/3

45. Three identical particles each of mass 1 kg are placed touching one another with their centres on a straight line. Their centres are marked A, B and C respectively. The distance of centre of mass of the system from A is

(a) $\frac{AB+AC+BC}{3}$ (b) $\frac{AB+AC}{3}$
(c) $\frac{AB+BC}{3}$ (d) $\frac{AC+BC}{3}$

46. M.I of a circular loop of radius R about the axis in figure is



(a) MR^2 (b) $(3/4)\,MR^2$
(c) $MR^2/2$ (d) $2MR^2$

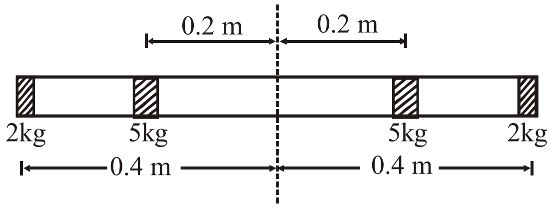
47. If the earth is treated as a sphere of radius R and mass M, its angular momentum about the axis of its diurnal rotation with period T is

(a) $\frac{4\pi MR^2}{5T}$ (b) $\frac{2\pi MR^2}{5T}$
(c) $\frac{MR^2T}{T}$ (d) $\frac{\pi MR^3}{T}$

48. The wheel of a car is rotating at the rate of 1200 revolutions per minute. On pressing the accelerator for 10 seconds it starts rotating at 4500 revolutions per minute. The angular acceleration of the wheel is

(a) 30 radian / $second^2$ (b) 1880 degrees/ $second^2$
(c) 40 radian / $second^2$ (d) 1980 degree/$second^2$

49. Four masses are fixed on a massless rod as shown in the adjoining figure. The moment of inertia about the dotted axis is about

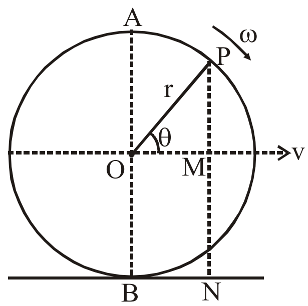


(a) 2 kg × m^2 (b) 1 kg × m^2
(c) 0.5 kg × m^2 (d) 0.3 kg × m^2

50. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest . In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of 25 rad s^{-2} must be applied about that axis for a duration of

(a) 4 s (b) 2 s
(c) 8 s (d) 10 s

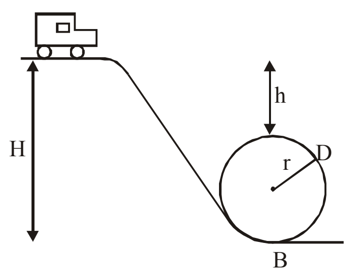
51. Fig. shows a disc rolling on a horizontal plane with linear velocity v. Its linear velocity is v and angular velocity is ω. Which of the following gives the velocity of the particle P on the rim of the disc



(a) v (1 + cos θ) (b) v (1 – cos θ)
(c) v (1 + sin θ) (d) v (1 – sin θ)

52. A toy car rolls down the inclined plane as shown in the fig. It loops at the bottom. What is the relation between H and h?

(a) $\frac{H}{h}=2$
(b) $\frac{H}{h}=3$
(c) $\frac{H}{h}=4$
(d) $\frac{H}{h}=5$



53. A sphere rolls down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom?

(a) $\frac{5}{7} g \sin\theta$ (b) $\frac{3}{5} g \sin\theta$

(c) $\frac{2}{7} g \sin\theta$ (d) $\frac{2}{5} g \sin\theta$

54. In a bicycle, the radius of rear wheel is twice the radius of front wheel. If r_F and r_F are the radii, v_r and v_r are the speed of top most points of wheel. Then

(a) $v_r = 2v_F$ (b) $v_F = 2v_r$

(c) $v_F = v_r$ (d) $v_F > v_r$

55. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $\frac{F_1}{F_2}$ is

(a) $\left(\frac{R_1}{R_2}\right)^2$ (b) $\frac{R_2}{R_1}$

(c) $\frac{R_1}{R_2}$ (d) 1

56. A wheel having moment of inertia 2 kg-m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis, The torque which can stop the wheel's rotation in one minute would be

(a) $\frac{\pi}{18}$Nm (b) $\frac{2\pi}{15}$Nm

(c) $\frac{\pi}{12}$Nm (d) $\frac{\pi}{15}$Nm

57. Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass particles through a distance d, by what distance would the particle of mass m_2 move so as to keep the mass centre of particles at the original position?

(a) $\frac{m_2}{m_1} d$ (b) $\frac{m_1}{m_1 + m_2} d$

(c) $\frac{m_1}{m_2} d$ (d) d

58. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

(a) $1 : \sqrt{2}$ (b) $1 : 3$

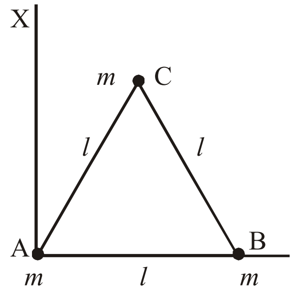
(c) $2 : 1$ (d) $\sqrt{5} : \sqrt{6}$

59. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

(a) $\frac{(I_1 + I_2)\omega}{I_1}$ (b) $\frac{I_2 \omega}{I_1 + I_2}$

(c) ω (d) $\frac{I_1 \omega}{I_1 + I_2}$

60. Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side/cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram-cm^2 units will be



(a) $\frac{3}{2} m\ell^2$

(b) $\frac{3}{4} m\ell^2$

(c) $2 m\ell^2$

(d) $\frac{5}{4} m\ell^2$

61. The moment of inertia of a uniform circular disc of radius 'R' and mass 'M' about an axis passing from the edge of the disc and normal to the disc is

(a) MR^2 (b) $\frac{1}{2} MR^2$

(c) $\frac{3}{2} MR^2$ (d) $\frac{7}{2} MR^2$

62. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

(a) $2 : 1$ (b) $1 : 2$

(c) $\sqrt{2} : 1$ (d) $1 : \sqrt{2}$

63. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force

(a) dissipates energy as heat.

(b) decreases the rotational motion.

(c) decreases the rotational and translational motion.

(d) converts translational energy to rotational energy

64. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with uniform angular speed ω. What is the force exerted by the liquid at the other end?

(a) $\frac{ML\omega^2}{2}$ (b) $ML\omega^2$

(c) $\frac{ML\omega^2}{4}$ (d) $\frac{ML\omega^2}{8}$

65. A circular disk of moment of inertia I_t is rotating in a horizontal plane, its symmetry axis, with a constant angular speed ω_i. Another disk of moment of inertia I_b is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed ω_f. The energy lost by the initially rotating disk to friction is

(a) $\frac{1}{2}\frac{I_b^2}{(I_t+I_b)}\omega_i^2$ (b) $\frac{I_t^2}{(I_t+I_b)}\omega_i^2$

(c) $\frac{I_b-I_t}{(I_t+I_b)}\omega_i^2$ (d) $\frac{1}{2}\frac{I_bI_t}{(I_t+I_b)}\omega_i^2$

66. The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is

(a) $I_0 + ML^2/2$ (b) $I_0 + ML^2/4$

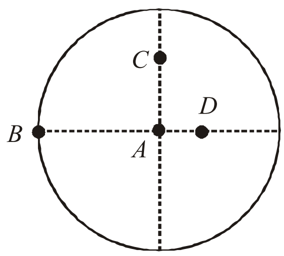
(c) $I_0 + 2ML^2$ (d) $I_0 + ML^2$

67. The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at

(a) t = 1s (b) t = 0.5 s

(c) t = 0.25 s (d) t = 2s

68. The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through



(a) *B* (b) *C*

(c) *D* (d) *A*

69. Three masses are placed on the *x*-axis : 300 g at origin, 500 g at $x = 40$ cm and 400 g at $x = 70$ cm. The distance of the centre of mass from the origin is

(a) 40 cm (b) 45 cm

(c) 50 cm (d) 30 cm

70. If the angular velocity of a body rotating about an axis is doubled and its moment of inertia halved, the rotational kinetic energy will change by a factor of :

(a) 4 (b) 2

(c) 1 (d) $\frac{1}{2}$

71. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is

(a) $\frac{1}{2}mR^2$ (b) $\frac{1}{4}mR^2$

(c) $\frac{1}{8}mR^2$ (d) $\sqrt{2}mR^2$

Directions for Qs. (72 to 75) : Each question contains STATEMENT-1 and STATEMENT-2. Choose the correct answer (ONLY ONE option is correct) from the following-

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

72. **Statement 1 :** When you lean behind over the hind legs of the chair, the chair falls back after a certain angle.

Statement 2 : Centre of mass lying outside the system makes the system unstable.

73. **Statement 1:** A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.

Statement 2 : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

74. **Statement 1 :** If no external force acts on a system of particles, then the centre of mass will not move in any direction.

Statement 2 : If net external force is zero, then the linear momentum of the system remains constant.

75. **Statement 1 :** A wheel moving down a frictionless inclined plane will slip and not roll on the plane.

Statement 2 : It is the frictional force which provides a torque necessary for a body to roll on a surface.

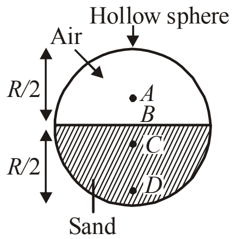
EXERCISE - 3
Exemplar & Past Years NEET/AIPMT Questions

Exemplar Questions

1. For which of the following does the centre of mass lie outside the body?

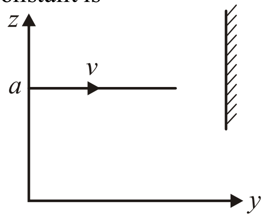
(a) A pencil (b) A shotput
(c) A dice (d) A bangle

2. Which of the following points is the likely position of the centre of mass of the system shown in figure?



(a) A (b) B
(c) C (d) D

3. A particle of mass m is moving in yz-plane with a uniform velocity v with its trajectory running parallel to +ve y-axis and intersecting z-axis at $z = a$ in figure. The change in its angular momentum about the origin as it bounces elastically from a wall at y = constant is

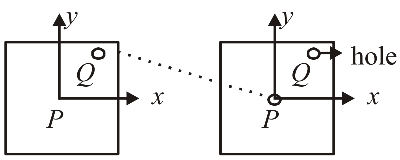


(a) $mva\hat{\boldsymbol{e}}_x$
(b) $2mva\hat{\boldsymbol{e}}_x$
(c) $ymv\hat{\boldsymbol{e}}_x$
(d) $2ymv\hat{\boldsymbol{e}}_x$

4. When a disc rotates with uniform angular velocity, which of the following is not true?

(a) The sense of rotation remains same
(b) The orientation of the axis of rotation remains same
(c) The speed of rotation is non-zero and remains same
(d) The angular acceleration is non-zero and remains same

5. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind in figure. The moment of inertia about the z-axis is then,



(a) increased
(b) decreased
(c) the same
(d) changed in unpredicted manner

6. In problem-5, the CM of the plate is now in the following quadrant of x-y plane.

(a) I (b) II
(c) III (d) IV

7. The density of a non-uniform rod of length 1 m is given by $\rho(x) = a(1 + bx^2)$ where, a and b are constants and $0 \le x \le 1$. The centre of mass of the rod will be at

(a) $\frac{3(2+b)}{4(3+b)}$ (b) $\frac{4(2+b)}{3(3+b)}$

(c) $\frac{3(3+b)}{4(2+b)}$ (d) $\frac{4(3+b)}{3(2+b)}$

8. A merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round of afterwards is

(a) 2ω (b) ω

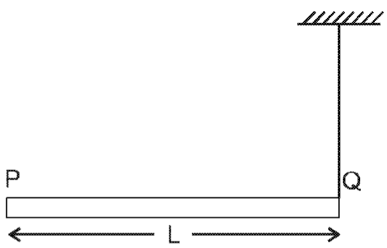
(c) $\frac{\omega}{2}$ (d) 0

NEET/AIPMT (2013-2017) Questions

9. A small object of uniform density rolls up a curved surface with an initial velocity 'v'. It reaches upto a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is a

(a) solid sphere (b) hollow sphere **[2013]**
(c) disc (d) ring

10. A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is **[2013]**



(a) g /L (b) 2g/L

(c) $\frac{2g}{3L}$ (d) $\frac{3g}{2L}$

11. Two discs are rotating about their axes, normal to the discs and passing through the centres of the discs. Disc D_1 has 2 kg mass and 0.2 m radius and initial angular velocity of 50 rad s^{-1}. Disc D_2 has 4kg mass, 0.1 m radius and initial angular velocity of 200 rad s^{-1}. The two discs are brought in contact face to face, with their axes of rotation coincident. The final angular velocity (in rad s^{-1}) of the system is

(a) 40 (b) 60 ***[NEET Kar. 2013]***
(c) 100 (d) 120

12. The ratio of radii of gyration of a circular ring and a circular disc, of the same mass and radius, about an axis passing through their centres and perpendicular to their planes are

(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$ ***[NEET Kar. 2013]***
(c) $3:2$ (d) $2:1$

13. The ratio of the accelerations for a solid sphere (mass 'm' and radius 'R') rolling down an incline of angle 'θ' without slipping and slipping down the incline without rolling is :

(a) $5:7$ (b) $2:3$ ***[2014]***
(c) $2:5$ (d) $7:5$

14. A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions s^{-2} is :

(a) 25 N (b) 50 N ***[2014]***
(c) 78.5 N (d) 157 N

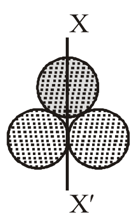
15. Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is ***[2015]***

(a) $3mr^2$

(b) $\frac{16}{5}mr^2$

(c) $4mr^2$

(d) $\frac{11}{5}mr^2$

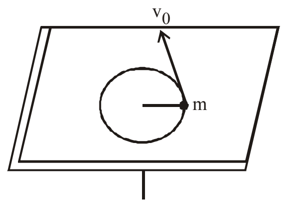


16. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is ***[2015]***

(a) $\frac{Wd}{x}$ (b) $\frac{W(d-x)}{x}$

(c) $\frac{W(d-x)}{d}$ (d) $\frac{Wx}{d}$

17. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to string which passes through a smooth hole in the plane as shown.



The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is ***[2015]***

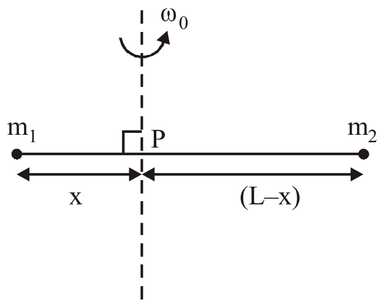
(a) $\frac{1}{4}mv_0^2$ (b) $2mv_0^2$

(c) $\frac{1}{2}mv_0^2$ (d) mv_0^2

18. A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of α for which angular momentum about origin is conserved is : ***[2015 RS]***

(a) 2 (b) zero
(c) 1 (d) –1

19. Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by : ***[2015 RS]***



(a) $x = \frac{m_1}{m_2}L$ (b) $x = \frac{m_2}{m_1}L$

(c) $x = \frac{m_2 L}{m_1 + m_2}$ (d) $x = \frac{m_1 L}{m_1 + m_2}$

20. An automobile moves on a road with a speed of 54 km h^{-1}. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kg m^2. If the vehicle is brought to rest in 15s, the magnitude of average torque transmitted by its brakes to the wheel is : ***[2015 RS]***

(a) 8.58 kg m^2 s^{-2} (b) 10.86 kg m^2 s^{-2}
(c) 2.86 kg m^2 s^{-2} (d) 6.66 kg m^2 s^{-2}

21. A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first ?

(a) Disk **[2016]**

(b) Sphere

(c) Both reach at the same time

(d) Depends on their masses

22. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2.0s is approximately :

(a) 8.0 (b) 7.0 **[2016]**

(c) 6.0 (d) 3.0

23. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre ? **[2016]**

(a) $15\ MR^2/32$ (b) $13\ MR^2/32$

(c) $11\ MR^2/32$ (d) $9\ MR^2/32$

24. Which of the following statements are **correct** ? **[2017]**

(A) Centre of mass of a body always coincides with the centre of gravity of the body

(B) Centre of mass of a body is the point at which the total gravitational torque on the body is zero

(C) A couple on a body produce both translational and rotation motion in a body

(D) Mechanical advantage greater than one means that small effort can be used to lift a large load

(a) (A) and (B) (b) (B) and (C)

(c) (C) and (D) (d) (B) and (D)

25. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2 . They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is:- **[2017]**

(a) $\frac{1}{4}I(\omega_1-\omega_2)^2$ (b) $I(\omega_1-\omega_2)^2$

(c) $\frac{1}{8}(\omega_1-\omega_2)^2$ (d) $\frac{1}{2}I(\omega_1+\omega_2)^2$

26. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? **[2017]**

(a) $0.25\ rad/s^2$ (b) $25\ rad/s^2$

(c) $5\ m/s^2$ (d) $25\ m/s^2$

Hints & Solutions

EXERCISE - 1

1.	(a)	2.	(b)	3.	(c)	4.	(c)
5.	(b)	6.	(d)	7.	(b)		

8. (d) The pull of earth changes only when the body moves so that g changes. It is an exceptional case and should not be considered unless otherwise mentioned.

9. (b) If we apply a torque on a body, then angular momentum of the body changes according to the relation

$$\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow \text{if } \vec{\tau} = 0 \text{ then, } \vec{L} = \text{constant}$$

10. (b) Angular momentum $\vec{L}$ is defined as $\vec{L} = \vec{r} \times m(\vec{v})$

so $\vec{L}$ is, an axial vector.

11. (c) The boy does not exert a torque to rotating table by jumping, so angular momentum is conserved i.e.,

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$$

12. (b) Since no external torque act on gymnast, so angular momentum (L=I ω) is conserved. After pulling her arms & legs, the angular velocity increases but moment of inertia of gymnast, decreases in, such a way that angular momentum remains constant.

13. (a)

14. (b) Angular momentum will remain the same since external torque is zero.

15. (a) $I_B = MR^2 \quad I_A = \frac{1}{2}MR^2$

$\therefore I_A < I_B$

16. (a) Angular momentum $= \vec{r} \times (\text{linear momentum})$

17. (b) We know that $\tau_{ext} = \frac{dL}{dt}$

if angular momentum is conserved, it means change in angular momentum $= 0$

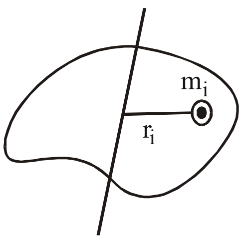
$$\text{or, } dL = 0, \quad \frac{dL}{dt} = 0 \Rightarrow \tau_{ext} = 0$$

Thus total external torque $= 0$.

18. (a) Analogue of mass in rotational motion is moment of inertia. It plays the same role as mass plays in translational motion.

19. (a) Basic equation of moment of inertia is given

by $I = \sum_{i=1}^{n} m_i r_i^2$



where m_i is the mass of i^{th} particle at a distance of r_i from axis of rotation.

Thus it does not depend on angular velocity.

20. (d) Initially centre of gravity is at the centre. When sand is poured it will fall and again after a limit, centre of gravity will rise.

21. (d) Here, the centre of mass of the system remains unchanged. in the horizontal direction. When the mass m moved forward by a distance L cos θ, let the mass (m + M) moves by a distance x in the backward direction. hence

$(M + m)\,x - m\,L \cos\theta = 0$

$\therefore \quad x = (m\,L\cos\theta)/(m + M)$

22. (b) Acceleration of solid sphere is more than that of hollow sphere, it rolls faster, and reaches the bottom of the inclined plane earlier.

Hence, solid sphere and hollow sphere can be distinguished by rolling them simultaneously on an inclined plane.

23. (d) Applying the law of conservation of momentum

$m\,v = M\,V$ ...(1)

By conservation of angular momentum

$$m\,v\,(L/2) = \left(\frac{M L^2}{12}\right)\omega \qquad ...(2)$$

As the collision is elastic, we have

$$\frac{1}{2}m\,v^2 = \frac{1}{2}M\,V^2 + \frac{1}{2}I\,\omega^2 \qquad ...(3)$$

Substituting the values, we get m = M/4

24. (b) α is propotional to ω

Let $\alpha = k\omega$ ($\because$ k is a constant)

$$\frac{d\omega}{dt} = k\omega \qquad \left[\text{also } \frac{d\theta}{dt} = \omega \Rightarrow dt = \frac{d\theta}{\omega}\right]$$

$$\therefore \frac{\omega\, d\omega}{d\theta} = k\omega \Rightarrow d\omega = k\,d\theta$$

$$\text{Now} \int_{\omega}^{\omega/2} d\omega = k\int d\theta$$

$$\int_{\omega/2}^{0} d\omega = k\int_{0}^{\theta} d\theta \Rightarrow -\frac{\omega}{2} = k\theta \Rightarrow -\frac{\omega}{2} = k\theta_1$$

$$(\because \theta_1 = 2\pi n)$$

$\therefore \theta = \theta_1$ or $2\pi n_1 = 2\pi n$

$n_1 = n$

25. (b) So raw egg is like a spherical shell & hard bioled egg is like solid sphere. Let I_1, I_2 be M. I. of raw egg and boiled egg respectively.

Given that angular momentum L, is same

$\therefore I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 > \omega_1 \because I_1 > I_2$

Now from first equation of angular motion ($\omega_f = \omega_i + \alpha t$) here α is retarding decceleration for both cases & $\omega_f = 0$ for both case.

$$\text{So } \frac{t_1(\text{raw egg})}{t_2(\text{hard egg})} = \frac{\omega_1/\alpha}{\omega_2/\alpha} \Rightarrow \frac{t_1}{t_2} < 1$$

EXERCISE - 2

1. (d) K.E. of rotation $= \frac{1}{2} I\omega^2$

$$= \frac{1}{2} \times \left(\frac{1}{2} mr^2\right) \omega^2$$

$$= \frac{1}{4} \times 20 \times (0.25)^2 \times 100 \times 100 = 3125 \text{ J}$$

2. (c) For solid sphere

$$I = \frac{2}{5} MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3 \rho\right) R^2$$

$$= \frac{8}{15} \times \frac{22}{7} R^5 \rho = \frac{176}{105} R^5 \rho$$

3. (a) Force F_A on particle A is given by

$$F_A = m_A a_A = \frac{m_A v}{t} \quad ...(1)$$

Similarly $F_B = m_B a_B = \frac{m_B \times 2v}{t}$ ...(2)

Now $\frac{m_A v}{t} = \frac{m_B \times 2v}{t}$ $(\because F_A = F_B)$

So $m_A = 2m_B$

For the centre of mass of the system

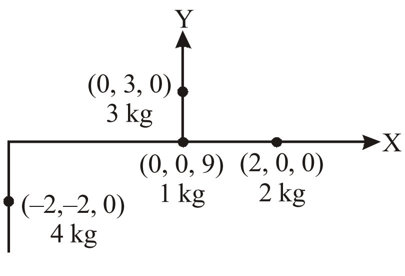
$$v = \frac{m_A v_A + m_B v_B}{m_A + m_B}$$

or $v = \frac{2 m_B v - m_B \times 2v}{2m_B + m_B} = 0$

Negative sign is used because the particles are travelling in opposite directions.

Alternatively, if we consider the two masses in a system then no external force is acting on the system. Mutual forces are internal forces. Since the centre of mass is initially at rest, it well remain at rest

4. (a) Moment of inertia of the whole system about the axis of rotation will be equal to the sum of the moments of inertia of all the particles.



$\therefore$ $I = I_1 + I_2 + I_3 + I_4$
$= 0 + 0 + 27 + 16 = 43 \text{ kg m}^2$

5. (a) $E_r = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 3 \times (3)^3 = 13.5 \text{ J}$

$$\text{K.E.} = \frac{1}{2} mv^2 = \frac{1}{2} \times 27 \times v^2 = 13.5$$

$$v = 1 \text{ m/s}$$

6. (c) Centripetal acceleration

$$a_c = 4\pi^2 v^2 r = 4\pi^2 \times 2 \times 2 \times 0.25 = 4\pi^2 \text{ms}^{-2}$$

7. (b) From the theorem of parallel axes, the moment of inertia $I = I_{CM} + Ma^2$

where I_{CM} is moment of inertia about centre of mass and a is the distance of axis from centre.

$mk^2 = m(k^1)^2 + m(6)^2$

$(\because I = mk^2$ where k is radius of gyration)

or, $k^2 = (k^1)^2 + 36$

or, $100 = (k^1)^2 + 36$

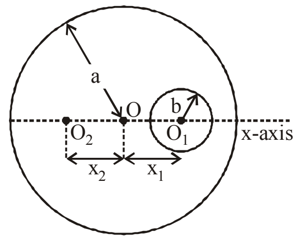
$\Rightarrow$ $(k^1)^2 = 64$ cm

$\therefore$ $k^1 = 8$ cm

8. (c) $L = mvr$ or $v = L/mr$

Centripetal force $\frac{mv^2}{r} = \frac{m(L/mr)^2}{r} = \frac{L^2}{mr^3}$

9. (a) The situation can be shown as : Let radius of complete disc is a and that of small disc is b. Also let centre of mass now shifts to O_2 at a distance x_2 from original centre.



The position of new centre of mass is given by

$$X_{CM} = \frac{-\sigma . \pi b^2 . x_1}{\sigma . \pi a^2 - \sigma . \pi b^2}$$

Here, a = 6 cm, b = 2 cm, $x_1 = 3.2$ cm

Hence, $X_{CM} = \frac{-\sigma \times \pi (2)^2 \times 3.2}{\sigma \times \pi \times (6)^2 - \sigma \times \pi \times (2)^2}$

$$= \frac{12.8\pi}{32\pi} = -0.4 \text{ cm.}$$

10. (b) By energy conservation

$$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$$

$$(K.E)_i = 0, (P.E)_i = mgh, (P.E)_f = 0$$

$$(K.E)_f = ½I\omega^2 + ½mv_{cm}^2$$

Where I (moment of inertia) = ½mR²

(for solid cylinder)

so $mgh = ½(½mR^2)\left(\frac{v_{cm}^2}{R^2}\right) + ½mv_{cm}^2$

$$\Rightarrow v_{cm} = \sqrt{4gh/3}$$

11. (b) Moment of inertia (M.I) of circular ring $I_1 = mr^2$
moment of inertia (M.I) of circular disc $I_2 = ½mr^2$

$$\Rightarrow \frac{I_1}{I_2} = \frac{2}{1}$$

12. (a) Tension provides the necessary centripetal force for the rest of rod.

13. (d) Centrifugal force is observed when the observer is in a frame undergoing circular motion.

14. (d) $I = \frac{1}{2} m R^2$

$M \propto t \propto R^2$

For disc X, $I_X = \frac{1}{2}(m)(R)^2 = \frac{1}{2}(\pi r^2 t).(R)^2$

for disc Y, $I_Y = \frac{1}{2}[\pi(4R)^2.t/4][4R]^2$

$$\Rightarrow \frac{I_X}{I_Y} = \frac{1}{(4)^3} \Rightarrow I_Y = 64\, I_X$$

15. (a) Angular momentum $\propto \frac{1}{\text{Angular frequency}}$

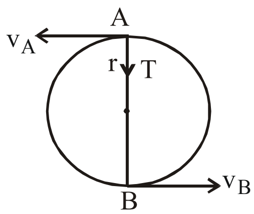
$\propto$ Kinetic energy

$$\Rightarrow \vec{L} = \frac{K.E.}{\omega}$$

$$\frac{L_1}{L_2} = \left(\frac{K.E_1}{\omega_1}\right) \times \frac{\omega_2}{K.E_2} = 4 \Rightarrow L_2 = \frac{L}{4}$$

16. (a) When $\theta = 0°$, the net force is directed vertically upwards.

17. (a) Let velocity at $A = v_A$ and velocity at $B = v_B$



Applying conservation of energy at A & B

$$\frac{1}{2}mv_A^2 + 2gmr = \frac{1}{2}mv_B^2 \quad [\because (P.E)_A = mg\,(2r)]$$

$$v_B^2 = v_A^2 + 4gr \text{.........(i)}$$

Now as it is moving in circular path it has centripetal force.

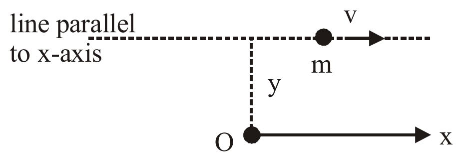
At point $A \Rightarrow T + mg = \frac{mv_A^2}{r}$

for minimum velocity $T \geq 0$

or $\frac{mv_A^2}{r} \geq mg \Rightarrow v_A^2 \geq gr \Rightarrow v_A \geq \sqrt{gr}$

18. (a) A couple consists of two equal and opposite forces whose lines of action are parallel and laterally separated by same distance. Therefore, net force (or resultant) of a couple is null vector, hence no translatory motion will be produced and only rotational motion will be produced.

19. (b) Angular momentum of mass m moving with a constant velocity about origin is



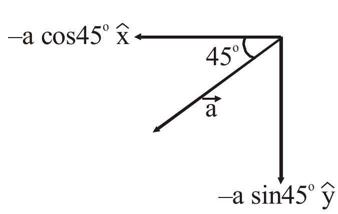
L = momentum × perpendicular distance of line of action of momentum from origin

$L = mv \times y$

In the given condition mvy is a constant. Therefore angular momentum is constant.

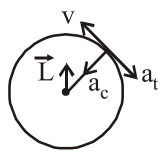
20. (c) Since the spheres are smooth, there will be no transfer of angular momentum from the sphere A to sphere B. The sphere A only transfers its linear velocity v to the sphere B and will continue to rotate with the same angular speed ω.

21. (c) $\hat{n} = \frac{-(\hat{x} + \hat{y})}{\sqrt{2}}$



$$\vec{a} = \frac{v^2}{r}\hat{n} = \frac{4}{4} \times \frac{-(\hat{x} + \hat{y})}{\sqrt{2}}$$

22. (b) Since v is changing (decreasing), L is not conserved in magnitude. Since it is given that a particle is confined to rotate in a circular path, it can not have spiral path. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.



The direction of $\vec{L}$ remains same even when the speed decreases.

23. (a) The moment of inertia (I) of circular ring whose axis of rotation is passing through its center, $I_1 = m_1 R^2$

Also, $I_2 = m_2 (nR)^2$

Since both rings have same density,

$$\Rightarrow \frac{m_2}{2\pi\,(nR) \times A_2} = \frac{m_1}{2\pi R \times A_1}$$

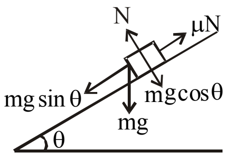
Where A is cross-section of ring,

$A_1 = A_2$ (Given) $\quad \therefore m_2 = nm_1$

Given $\frac{I_1}{I_2} = \frac{1}{8} = \frac{m_1 R^2}{m_2 (nR)^2} = \frac{m_1 R^2}{nm_1 (nR)^2}$

$\Rightarrow \frac{1}{8} = \frac{1}{n^3}$ or $n = 2$

24. (a) Remember that acceleration of a cylinder down a smooth inclined plane is

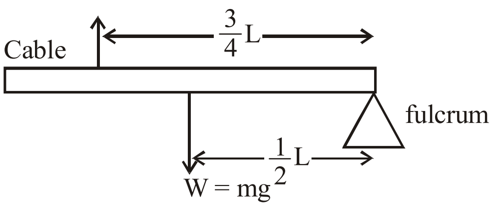


$a = \frac{g\sin\theta}{\left(1 + \frac{I}{mR^2}\right)}$ where $I = \frac{mR^2}{2}$ is the moment of

Inertia for cylinder $a = \frac{g \sin 30°}{\left(1 + \frac{mR^2}{2} \times \frac{1}{mR^2}\right)}$

$$= \frac{g \times \frac{1}{2}}{1 + \frac{1}{2}} = \frac{g}{3}$$

25. (c) This is a torque problem. While the fulcrum can be placed anywhere, placing it at the far right end of the bar eliminated cable B from the calculation. There are now only two forces acting on the bar ; the weight that produces a counterclockwise rotation and the tension in cable A that produces a clockwise rotation. Since the bar is in equilibrium, these two torques must sum to zero.



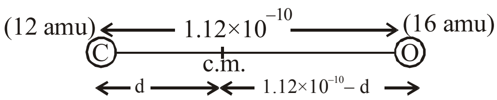
$\Sigma\tau = T_A (3/4L) - Mg(1/2L) = 0$

Therefore

$T_A = (MgL/2)/(3L/4) = (MgL/2)(4/3L) = 2Mg/3$

26. (b) The intersection of medians is the centre of mass of the triangle. Since distances of centre of mass from the sides are related as : $x_{BC} < x_{AB} < x_{AC}$ therefore $I_{BC} > I_{AB} > I_{AC}$ or $I_{BC} > I_{AB}$.

27. (a)



From definition of centre of mass.

$d = \frac{16 \times 1.12 \times 10^{-10} + 12 \times 0}{16 + 12} = \frac{16 \times 1.12 \times 10^{-10}}{28}$

$= 0.64 \times 10^{-10}$ m.

28. (d) AB is the ladder, let F be the horizontal force and W is the weigth of man. Let N_1 and N_2 be normal reactions of ground and wall, respectively. Then for vertical equilibrium

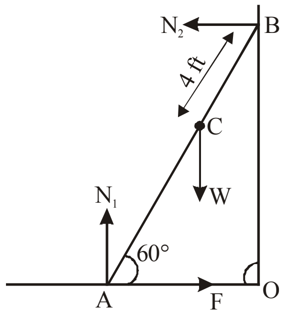
$W = N_1$ .....(1)

For horizontal equilibrium $N_2 = F$ .....(2)

Taking moments about A

$N_2(AB \sin 60°) - W(B \cos 60°) = 0$ ......(3)

Using (2) and AB = 20 ft, BC = 4 ft, we get



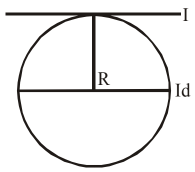
$F\left(20 \times \frac{\sqrt{3}}{2}\right) - w\left(4 \times \frac{1}{2}\right) = 0$

$\Rightarrow F = \frac{2w \times 2}{20\sqrt{3}} = \frac{w}{5\sqrt{3}} = \frac{150}{5\sqrt{3}}$ pound (lb)

$= 10\sqrt{3} = 10 \times 1.73 = 17.3$ pound

29. (c) Moment of inertia of disc about its diameter is

$I_d = \frac{1}{4} MR^2$



MI of disc about a tangent passing through rim and in the plane of disc is

$I = I_G + MR^2 = \frac{1}{4} MR^2 + MR^2 = \frac{5}{4} MR^2$

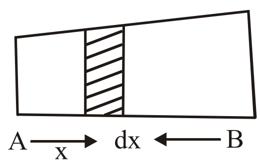
30. (b) $F = mr\omega^2 = 1 \times \frac{1}{2} \times 4\pi^2 \times 2 \times 2 = 8\pi^2 N$

31. (d) Consider an element of length dx at a distance x from end A.

Here : mass per unit length λ of rod

$(\lambda \propto x \Rightarrow \lambda = kx)$

$\therefore \; dm = \lambda dx = kx\, dx$



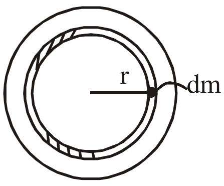
Position of centre of gravity of rod from end A

$$x_{CG} = \frac{\int_0^L x\,dm}{\int_0^L dm}$$

$$\therefore \quad x_{CG} = \frac{\int_0^3 x(kx\,dx)}{\int_0^3 kx\,dx} = \frac{\left[\frac{x^3}{3}\right]_0^3}{\left[\frac{x^2}{2}\right]_0^3} = \frac{\frac{(3)^3}{3}}{\frac{(3)^2}{2}} = 2\text{m}$$

32. (b) Density of iron > density of aluminium

moment of inertia $= \int r^2 dm$.



$\therefore$ Since $\rho_{iron} > \rho_{aluminium}$
so whole of aluminium is kept in the core and the iron at the outer rim of the disc.

33. (d) Rotational energy $= \frac{1}{2}I(\omega)^2 = \frac{1}{2}(mK^2)\omega^2$

Linear kinetic energy $= \frac{1}{2}m\omega^2R^2$

$\therefore$ Required fraction

$$= \frac{\frac{1}{2}(mK^2)\omega^2}{\frac{1}{2}(mK^2)\omega^2 + \frac{1}{2}m\omega^2R^2} = \frac{K^2}{K^2+R^2}$$

34. (c) At the highest point

$$mg = mR4\pi^2\nu^2 \Rightarrow \nu = \sqrt{\frac{g}{4\pi^2R}}$$

Revolution per minute $= 60\nu = \sqrt{\frac{900\,g}{\pi^2R}}$

35. (d) $\mu\,mg = mr4\pi^2\nu^2$

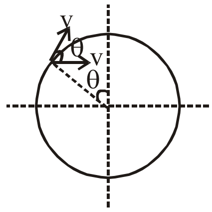
$$\Rightarrow r_2\nu_2^2 = r_1\,\nu_1^2 \Rightarrow r_2 = \frac{r_1\nu_1^2}{\nu_2^2} = \frac{r_1\nu_1^2}{4\nu_1^2}$$

$$r_2 = \frac{r_1}{4} = 4\ \text{cm}$$

36. (c) By conservation of angular momentum,

$$5\times10 = 5\omega + 20\omega \Rightarrow \omega = \frac{50}{25} = 2\ \text{rps}$$

37. (b)



$$v_R = \sqrt{v^2+v^2+2v^2\cos\theta} = \sqrt{2v^2(1+\cos\theta)}$$

$$= 2v\cos\frac{\theta}{2}$$

38. (b) For no angular acceleration $\tau_{net} = 0$
$\Rightarrow F_1\times5 = F_2\times30$ (given $F_2 = 4$N) $\Rightarrow F_1 = 24$N

39. (c) $\frac{1}{2}I\omega^2$ = Loss of gravitational potental energy

$$\frac{1}{2}\times\frac{ml^2}{3}\omega^2 = \frac{mgl}{2}(1+\cos\theta)$$

$$\Rightarrow \omega^2 = \frac{3g}{l}\left(2\cos^2\frac{\theta}{2}\right) \Rightarrow \omega = \sqrt{\frac{6g}{l}}\cos\frac{\theta}{2}$$

40. (d) Use $\tau = I\alpha$

and $\omega^2 - \omega_0^2 = 2\alpha\theta$ Here $I = \frac{2}{5}MR^2$

Given M = 2000g = 2kg
R = 5cm = 5×10^{-2}m
$\theta = 2\pi\times2\pi$ radians
n = 300 rpm = 5 rps

$$\therefore \quad \tau = I\alpha = \left(\frac{2}{5}MR^2\right)\left(\frac{\omega^2-\omega_0^2}{2\theta}\right)$$

$$= \frac{2}{5}\times2\times\left(5\times10^{-2}\right)^2\frac{4\pi^2\left(5^2-0^2\right)}{2\times4\pi^2}$$

= 0.025 N – m.
= 2.5×10^5 dyne cm.

41. (c) $X_{C.M.} = \frac{1\times0+2\times2+3\times0+4\times2+5\times1}{1+2+3+4+5}$

$$= \frac{4+8+5}{15} = \frac{17}{15} = 1.1$$

$$Y_{C.M} = \frac{1\times0+2\times0+3\times2+4\times2+5\times1}{1+2+3+4+5}$$

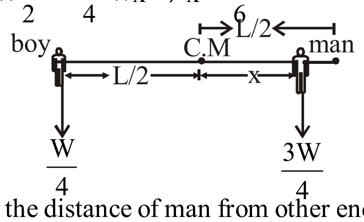
$$= \frac{6+8+5}{15} = 1.3$$

42. (c) $E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\times\frac{2}{5}MR^2\times\frac{v^2}{R^2} = \frac{7}{10}Mv^2 = 0.7\ \text{J}$$

43. (a) So couple about the C.M must be zero for rotational equilibrium, It means that

$$\frac{W}{4}\times\frac{L}{2}=\frac{3W}{4}\times x \Rightarrow x=\frac{L}{6}$$

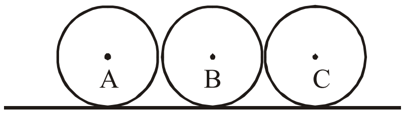


and the distance of man from other end is

$$\frac{L}{2}-x=\frac{L}{2}-\frac{L}{6}=\frac{L}{3}$$

44. (b,c)

45. (b) Position of C.M w.r. to A



$$=\frac{1\times 0+1\times AB+1\times AC}{1+1+1}=\frac{AB+AC}{3}$$

46. (b) Use theorem of parallel axes.

47. (a) Angular momentum $=I\omega=\frac{2}{5}MR^2.\frac{2\pi}{T}=\frac{4\pi MR^2}{5T}$

48. (d) $\alpha=\frac{\omega_2-\omega_1}{t_2-t_1}$

$\omega_1=\frac{2\pi\times 1200}{60}=40\pi$; $\quad \omega_2=\frac{2\pi\times 4500}{60}=150\pi$

$\alpha=\frac{110\pi}{10}\text{rad/sec}^2$

Now, π radian $=180°$

$\therefore$ 1 rad $=\frac{180}{\pi}$ degree

$\therefore\ \alpha=\frac{11\pi\times 180}{\pi}\text{degree/sec}^2=1980\text{ degree/sec}^2$

49. (b) $I=2\times 5\times(0.2)^2+2\times 2\times(0.4)^2=1\text{kg}\times\text{m}^2$

50. (b) $E=1500=1/2\times 1.2\,\omega^2$

$\omega^2=\frac{3000}{1.2}=2500$

$\omega=50\text{rad/sec}$

$t=\frac{\omega}{\alpha}=\frac{50}{25}=2\sec$

51. (c) Velocity of $P=(NP)\omega=(NM+MP)\omega$

$=r(r+\sin\theta)\omega=v(1+\sin\theta)$

52. (d) Velocity at the bottom and top of the circle is $\sqrt{5gr}$ and $\sqrt{gr}$. Therefore $(1/2)M(5gr)=MgH$ and $(1/2)M(gr)=Mgh$.

53. (a) Acceleration of a body rolling down an inclined plane is given by, $a=\frac{g\sin\theta}{1+\frac{K^2}{R^2}}$

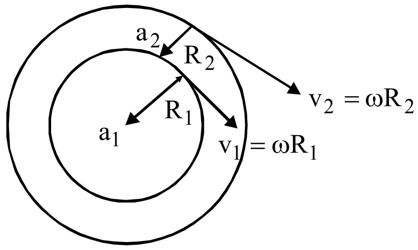
In case of a solid sphere, we have

$$\frac{K^2}{R^2}=\frac{[(I/M)]}{R^2}=\frac{I}{MR^2}=\frac{(2/5)MR^2}{MR^2}=\frac{2}{5}$$

Substituting $\frac{K^2}{R^2}=\frac{2}{5}$ in equation (1) we get

$a=\frac{5}{7}g\sin\theta$

54. (c) The velocity of the top point of the wheel is twice that of centre of mass. And the speed of centre of mass is same for both the wheels.



55. (c) $a_1=\frac{v_1^2}{R_1}=\frac{\omega^2R_1^2}{R_1}=\omega^2R_1$

$a_2=\frac{v_2^2}{R_2}=\omega^2R_2$

Taking particle masses equal

$\frac{F_1}{F_2}=\frac{ma_1}{ma_2}=\frac{a_1}{a_2}=\frac{R_1}{R_2}$

Alternative method : The force experienced by any particle is only along radial direction or the centripetal force.

Force experienced by the particle, $F=m\omega^2R$

$\therefore\ \frac{F_1}{F_2}=\frac{R_1}{R_2}$

56. (d) $\tau\times\Delta t=L_0$ $\qquad \{\because \text{ since } L_f=0\}$

$\Rightarrow\tau\times\Delta t=I\omega$

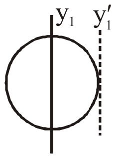
or $\tau\times 60=2\times 2\times 60\pi/60$

$\left(\because\ f=60\text{rpm}\ \therefore\ \omega=2\pi f=2\pi\times\frac{60}{60}\right)$

$\tau=\frac{\pi}{15}\text{N}-\text{m}$

57. (c) $m_1d=m_2\,d_2\Rightarrow d_2=\frac{m_1d}{m_2}$

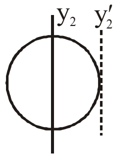
58. (d)



Circular disc (1)

$I_{y_1}=\frac{MR^2}{4}$

$$\therefore I'_{y_1} = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$



$$I_{y_2} = \frac{MR^2}{2}$$

$$\therefore I'_{y_2} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

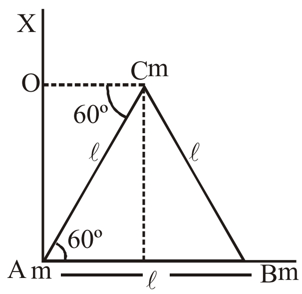
$$I'_{y_1} = MK_1^2,\ I'_{y_2} = MK_2^2$$

$$\therefore \frac{K_1^2}{K_2^2} = \frac{I'_{y_1}}{I'_{y_2}} \Rightarrow K_1 : K_2 = \sqrt{5} : \sqrt{6}$$

59. (d) Angular momentum will be conserved

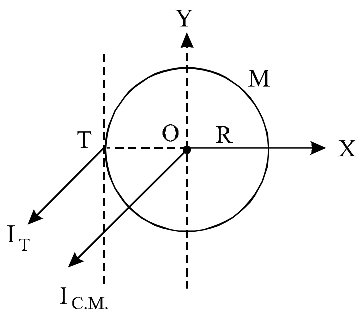
$$I_1\omega = I_1\omega' + I_2\omega' \Rightarrow \omega' = \frac{I_1\omega}{I_1 + I_2}$$

60. (d) $I_{AX} = m(AB)^2 + m(OC)^2 = m\ell^2 + m(\ell\cos 60^\circ)^2$
$= m\ell^2 + m\ell^2/4 = 5/4\, m\ell^2$



61. (c) M.I. of a uniform circular disc of radius 'R' and mass 'M' about an axis passing through C.M. and normal to the disc is

$$I_{C.M.} = \frac{1}{2}MR^2$$



From parallel axis theorem

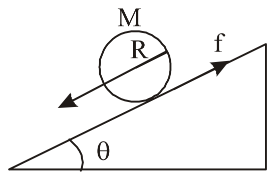
$$I_T = I_{C.M.} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

62. (d) $K = \frac{L^2}{2I} \Rightarrow L^2 = 2KI \Rightarrow L = \sqrt{2KI}$

$$\frac{L_1}{L_2} = \sqrt{\frac{K_1}{K_2}\cdot\frac{I_1}{I_2}} = \sqrt{\frac{K}{K}\cdot\frac{I}{2I}} = \frac{1}{\sqrt{2}}$$

$L_1 : L_2 = 1 : \sqrt{2}$

63. (d) Net work done by frictional force when drum rolls down without slipping is zero.



$W_{net} = 0,\ W_{trans.} + W_{rot.} = 0$
$\Delta K_{trans.} + \Delta K_{rot.} = 0$
$\Delta K_{trans} = -\Delta K_{rot}.$
i.e., converts translation energy to rotational energy.

64. (a) Tube may be treated as a particle of mass M at distance L/2 from one end.

$$\text{Centripetal force} = Mr\omega^2 = \frac{ML}{2}\omega^2$$

65. (d) By conservation of angular momentum, $I_t\,\omega_i = (I_t + I_b)\,\omega_f$

where ω_f is the final angular velocity of disks

$$\therefore \qquad \omega_f = \left(\frac{I_t}{I_t + I_b}\right)\omega_i$$

Loss in K.E., ΔK = Initial K.E. – Final K.E.

$$= \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}(I_t + I_b)\,\omega_f^2$$

$$\Rightarrow \Delta K = \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}(I_t + I_b)\frac{I_t^2}{(I_t + I_b)^2}\omega_i^2$$

$$= \frac{1}{2}\omega_i^2\frac{I_t}{I_t + I_b}(I_t + I_b - I_t) = \frac{1}{2}\omega_i^2\frac{I_t I_b}{I_t + I_b}$$

66. (b) By theorem of parallel axes,
$I = I_{cm} + Md^2$
$I = I_0 + M(L/2)^2 = I_0 + ML^2/4$

67. (a) When angular acceleration (α) is zero then torque on the wheel becomes zero.
$\theta(t) = 2t^3 - 6t^2$

$$\Rightarrow \frac{d\theta}{dt} = 6t^2 - 12t$$

$$\Rightarrow \alpha = \frac{d^2\theta}{dt^2} = 12t - 12 = 0$$

$\therefore$ t = 1 sec.

68. (a) According to parallel axis theorem of the moment of Inertia

$$I = I_{cm} + md^2$$

d is maximum for point B so I_{max} about B.

69. (a) $X_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$X_{cm} = \frac{300 \times (0) + 500(40) + 400 \times 70}{300 + 500 + 400}$$

$$X_{cm} = \frac{500 \times 40 + 400 \times 70}{1200}$$

$$X_{cm} = \frac{50 + 70}{3} = \frac{120}{3} = 40 \text{ cm}$$

70. (b) $K = \dfrac{1}{2} I\omega^2$

$$K' = \frac{1}{2}\left(\frac{1}{2}\right)(2\omega)^2 = 2K$$

71. (a) For complete disc with mass '4M', M.I. about given axis $= (4M)(R^2/2) = 2\,MR^2$

Hence, by symmetry, for the given quarter of the disc

$$\text{M.I.} = 2\,MR^2/4 = \frac{1}{2}MR^2$$

72. (d)

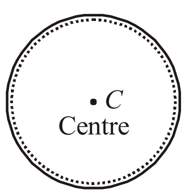
73. (a) For a disc rolling without slipping on a horizontal rough surface with uniform angular velocity, the acceleration of lowest point of disc is directed vertically upwards and is not zero (Due to translation part of rolling, acceleration of lowest point is zero. Due to rotational part of rolling, the tangential acceleration of lowest point is zero and centripetal acceleration is non-zero and upwards). Hence statement 1 is false.

74. (a) 75. (b)

EXERCISE - 3

Exemplar Questions

1. (d) A bangle is in the form of a ring as shown in figure. The centre of mass of the ring lies at its centre, which is outside the ring or bangle.



2. (c) Centre of mass of a system lies towards the part of the system, having bigger mass. In the above diagram, lower part is heavier, hence CM of the system lies below the horizontal diameter at C point.

3. (b) The initial velocity is $v_i = v\hat{e}_y$ and after reflection from the wall, the final velocity is $v_f = -v\hat{e}_y$. The trajectory is at constant distance a on z axis and as particle moves along y axis, its y component changes .
So position vector (moving along y-axis),

$$\vec{r} = y\hat{e}_y + a\hat{e}_z$$

Hence, the change in angular momentum is

$$\vec{r} \times m(v_f - v_j) = 2mva\hat{e}_x .$$

4. (d) Angular acceleration $\alpha = \dfrac{d\omega}{dt}$

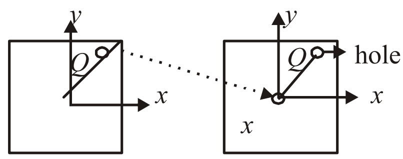
where ω is angular velocity of the disc and is uniform or constant.

$$\alpha = \frac{d\omega}{dt} = 0$$

Hence, angular acceleration is zero.

5. (b) According to the question, when the small piece Q removed it is stick at P through axis of rotation passes, but axis of rotation does not passes through Q and It is glued to the centre of the plate, the mass comes closer to the z-axis, hence, moment of inertia decreases.

6. (c) Let us consider the diagram of problem 5, there is a line shown in the figure drawn along the diagonal. First, centre of mass of the system was on the dotted line and was shifted towards Q from the centre (1st quadrant).



When the mass Q is decrease, it will be on the same line but shifted towards other side of Q on the line joining QPor away from the centre so, new CM will lies in IIIrd quadrant in x-y plane.

7. (a) As given that density :

$$\rho(x) = a(1 + bx^2)$$

where a and b are constants and $0 \le x \le 1$

At $b = 0$, $\rho(x) = a =$ constant

In that case, centre of mass will be at

$x = 0.5$ m (i.e., mid-point of the rod)

Putting, $b = 0$ in options.

(a) $\dfrac{3}{4} \times \dfrac{2}{3} = \dfrac{1}{2} = 0.5$ m

(b) $\frac{4}{3}\times\frac{2}{3}\neq 0.5$ m

(c) $\frac{3}{4}\times\frac{3}{2}\neq 0.5$ m

(d) $\frac{4}{3}\times\frac{3}{2}\neq 0.5$ m

So, only (a) option gives 0.5.

8. (a) As there is external torque acting on the system, angular momentum should be conserved.

Hence $I\omega$ = constant ...(i)

where, I is moment of inertia of the system and ω is angular velocity of the system.

From Eq. (i),

$I_1\omega_1 = I_2\omega_2$

where ω_1 and ω_2 are angular velocities before and after jumping.

$(\because I = mr^2)$ So, given that,

$m_1 = 2M, m_2 = M, \omega_1 = \omega, \omega_2 = ?$

$r_1 = r_2 = R \quad m_1r_1^2\omega_1 = m_2r_2^2\omega_2$

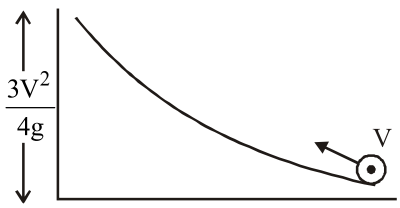
$2MR^2\omega = MR^2\omega_2$

as mass reduced to half, hence, moment of inertia also reduced to half.

$\omega_2 = 2\omega$

NEET/AIPMT (2013-2017) Questions

9. (c)



From law of conservation of mechanical energy

$$\frac{1}{2}I\omega^2 + 0 + \frac{1}{2}mv^2 = mg\times\frac{3v^2}{4g}$$

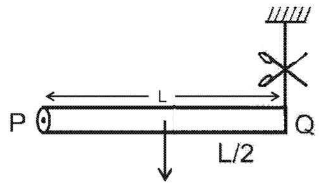
$$\Rightarrow \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 - \frac{1}{2}mv^2$$

$$= \frac{mv^2}{2}\left(\frac{3}{2}-1\right)$$

or, $\frac{1}{2}I\frac{V^2}{R^2} = \frac{mv^2}{4}$ or, $I = \frac{1}{2}mR^2$

Hence, object is a disc.

10. (d)



Weight of the rod will produce the torque

$$\tau = mg\frac{L}{2} = I\alpha = \frac{mL^2}{3}\alpha \quad \left[\because I_{rod} = \frac{ML^2}{3}\right]$$

Hence, angular acceleration $\alpha = \frac{3g}{2L}$

11. (c) Given: $m_1 = 2$ kg $\quad m_2 = 4$ kg

$r_1 = 0.2$ m $\quad r_2 = 0.1$ m

$w_1 = 50$ rad s^{-1} $\quad w_2 = 200$ rad s^{-1}

As, $I_1W_1 = I_2W_2$ = Constant

$$\therefore W_f = \frac{I_1W_1 + I_2W_2}{I_1 + I_2} = \frac{\frac{1}{2}m_1r_1^2w_1 + \frac{1}{2}m_2r_2^2w_2}{\frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2}$$

By putting the value of m_1, m_2, r_1, r_2 and solving we get = 100 rad s^{-1}

12. (a) $\because I = MK^2 \therefore K = \sqrt{\frac{I}{M}}$

$I_{ring} = MR^2$ and $I_{disc} = \frac{1}{2}MR^2$

$$\frac{K_1}{K_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{MR^2}{\left(\frac{MR^2}{2}\right)}} = \sqrt{2}:1$$

13. (a) For solid sphere rolling without slipping on inclined plane, acceleration

$$a_1 = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

For solid sphere slipping on inclined plane without rolling, acceleration

$a_2 = g\sin\theta$

Therefore required ratio $= \frac{a_1}{a_2}$

$$= \frac{1}{1+\frac{K^2}{R^2}} = \frac{1}{1+\frac{2}{5}} = \frac{5}{7}$$

14. (d) Here $\alpha = 2$ revolutions/s$^2 = 4\pi$ rad/s^2 (given)

$$I_{cylinder} = \frac{1}{2}MR^2 = \frac{1}{2}(50)(0.5)^2$$

$$= \frac{25}{4}\ \text{Kg-m}^2$$

As $\tau = I\alpha$ so $TR = I\alpha$

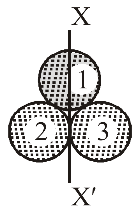
$$\Rightarrow T = \frac{I\alpha}{R} = \frac{\left(\frac{25}{4}\right)(4\pi)}{(0.5)} N = 50\pi N = 157\,N$$

15. (c) Moment of inertia of shell 1 along diameter

$$I_{diameter} = \frac{2}{3}MR^2$$

Moment of inertia of shell 2 = m. i of shell 3

$$= I_{tangential} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$



So, I of the system along x x^1

$$= I_{diameter} + (I_{tangential}) \times 2$$

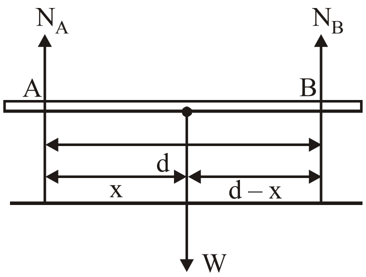
or, $$I_{total} = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right) \times 2$$

$$= \frac{12}{3}MR^2 = 4MR^2$$

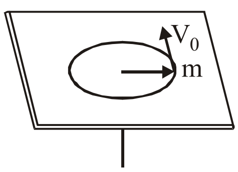
16. (c) By torque balancing about B

$N_A\,(d) = W\,(d - x)$

$$N_A = \frac{W(d-x)}{d}$$



17. (b) Applying angular momentum conservation



$$mV_0R_0 = (m)\,(V^1)\left(\frac{R_0}{2}\right)$$

$\therefore \quad v^1 = 2V_0$

Therefore, new KE $= \frac{1}{2}m\,(2V_0)^2 = 2mv_0^2$

18. (d) From Newton's second law for rotational motion,

$$\vec{\tau} = \frac{d\vec{L}}{dt}, \text{ if } \vec{L} = \text{constant then } \vec{\tau} = 0$$

So, $\vec{\tau} = \vec{r} \times \vec{F} = 0$

$$\left(2\hat{i} - 6\hat{j} - 12\hat{k}\right) \times (\alpha\hat{i} + 3\hat{j} + 6\hat{k}) = 0$$

Solving we get $\alpha = -1$

19. (c) Work required to set the rod rotating with angular velocity ω_0

$$K.E. = \frac{1}{2}I\omega^2$$

Work is minimum when I is minimum.

I is minimum about the centre of mass

So, $(m_1)\,(x) = (m_2)\,(L - x)$

or, $m_1x = m_2L - m_2x$

$$\therefore x = \frac{m_2L}{m_1 + m_2}$$

20. (d) **Given :** Speed V = 54 kmh^{-1} = 15 ms^{-1}

Moment of inertia, I = 3 kgm^2

Time t = 15s

$$\omega_i = \frac{V}{r} = \frac{15}{0.45} = \frac{100}{3} \quad \omega_f = 0$$

$$\omega_f = \omega_i + \alpha t$$

$$0 = \frac{100}{3} + (-\alpha)(15) \Rightarrow \alpha = \frac{100}{45}$$

Average torque transmitted by brakes to the wheel

$$\tau = (I)\,(\alpha) = 3 \times \frac{100}{45} = 6.66\ \text{kgm}^2\text{s}^{-2}$$

21. (b) Time of descent $\propto \frac{K^2}{R^2}$

Order of value of $\frac{K^2}{R^2}$

for disc; $\frac{K^2}{R^2} = \frac{1}{2} = 0.5$

for sphere; $\frac{K^2}{R^2} = \frac{2}{5} = 0.4$

(sphere) < (disc)

$\therefore$ Sphere reaches first

22. (a) **Given:** Radius of disc, R = 50 cm

angular acceleration $\alpha = 2.0$ rads^{-2}; time t = 2s

Particle at periphery (assume) will have both radial (one) and tangential acceleration

$a_t = R\alpha = 0.5 \times 2 = 1$ m/s^2

From equation,

$\omega = \omega_0 + \alpha t$

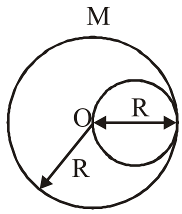
$\omega = 0 + 2 \times 2 = 4$ rad/sec

$a_c = \omega^2 R = (4)^2 \times 0.5 = 16 \times 0.5 = 8 m/s^2$

Net acceleration,

$a_{total} = \sqrt{a_t^2 + a_c^2} = \sqrt{1^2 + 8^2} \approx 8 \ m/s^2$

23. (b) Moment of inertia of complete disc about point 'O'.



$I_{Total\ disc} = \frac{MR^2}{2}$

Mass of removed disc

$M_{Removed} = \frac{M}{4}$ (Mass $\propto$ area)

Moment of inertia of removed disc about point 'O'.

$I_{Removed}$ (about same perpendicular axis)

$= I_{cm} + mx^2$

$= \frac{M}{4}\frac{(R/2)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$

Therefore the moment of inertia of the remaining part of the disc about a perpendicular axis passing through the centre,

$I_{Remaing\ disc} = I_{Total} - I_{Removed}$

$= \frac{MR^2}{2} - \frac{3}{32}MR^2 = \frac{13}{32}MR^2$

24. (d) Centre of mass may or may not coincide with centre of gravity. Net torque of gravitational pull is zero about centre of mass.

$\tau_g = \Sigma\tau_i = \Sigma r_i \times m_{ig} = 0$

Mechanical advantage, M. A.= $\frac{\text{Load}}{\text{Effort}}$

If M.A. > 1 $\Rightarrow$ Load > Effort

25. (a) Here, $I\omega_1 + I\omega_2 = 2I\omega$

$\Rightarrow \quad \omega = \frac{\omega_1 + \omega_2}{2}$

$(K.E.)_i = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2$

$(K.E.)_f = \frac{1}{2} \times 2I\omega^2 = I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$

Loss in K.E. $= (K.E)_f - (K.E)_i = \frac{1}{4}I(\omega_1 - \omega_2)^2$

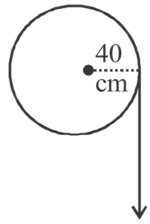
26. (b) Given, mass of cylinder m = 3kg

R = 40 cm = 0.4 m

F = 30 N ; α = ?

As we know, torque $\tau = I\alpha$

$F \times R = MR^2\alpha$



$\alpha = \frac{F \times R}{MR^2}$

$\alpha = \frac{30 \times (0.4)}{3 \times (0.4)^2}$ or, $\alpha = 25 rad/s^2$