18. ELECTRIC CHARGES, FORCES AND FIELDS

1. INTRODUCTION

You must have felt the attraction of hair of your hand when you bring it near to your Television screen. Did you ever think of cause behind it? These all are the electric charges and their properties. Now we will extend our concept to electric charges and their effects.

1.1 Nature of Electricity

The atomic structure shows that matter is electrical in nature i.e. matter contains particles of electricity viz. protons and electrons. Whether a given body shows electricity (i.e. charge) or not depends upon the relative number of these particles in the body.

(a) If the number of protons is equal to the number of electrons in a body, the resultant charge is zero and the body will be electrically neutral. For example, the paper of this book is electrically neutral (i.e. exhibits no charge) because it has the same number of protons and electrons.

(b) If from a neutral body, some *electrons are removed, the protons outnumber the electrons. Consequently, the body attains a positive charge. Hence, a positively charged body has deficit of electrons from the normal due share.

2. TYPES OF CHARGES

Depending upon whether electrons are removed or added to a body, there are two types of charges viz

(i) Positive charge (ii) Negative charge

If a glass rod is rubbed with silk, some electrons pass from glass rod to silk. As a result, the glass rod becomes positively charged and silk attains an equal negative charge as shown in Fig. 18.1. It is because silk gains as many electrons as lost by the glass rod. It can be shown experimentally that like charges repel each other while unlike charges attract each other. In other words, if the two charges are of the same nature (i.e., both positive or both negative), the force between them is of repulsion. On the other hand, if one charge is positive and the other is negative, the force between them is of attraction. The following points may be noted:

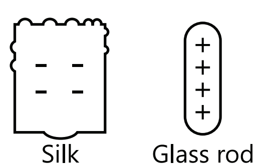

Figure 18.1

(a) The charges are not created by the rubbing action. There is merely transfer of electrons from one body to another.

(b) Electrons are transferred from glass rod to silk due to rubbing because we have done external work. Thus law of conservation of energy holds.

(c) The mass of negatively charged silk will increase and that of glass rod will decrease. It is because silk has gained electrons while glass rod has lost electrons.

3. PROPERTIES OF CHARGE

(a) Charge is a scalar quantity

(b) Charge is transferable

(c) Charge is conserved

(d) Charge is quantized

(e) Like point charges repel each other while unlike point charges attract each other.

(f) A charged body may attract a neutral body or an oppositely charged body but it always repels a similarly charged body

(g) Note: Repulsion is a sure test of electrification whereas attraction may not be.

(h) Charge is always associated with mass, i.e. charges cannot exist without mass though mass can exist without charge.

(i) Charge is relatively invariant: This means that charge is independent of frame of reference, i.e, charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.

(j) A charge at rest produces only electric field around itself; a charge having uniform motion produces electric as well as magnetic field around itself while a charge having acceleration emits electromagnetic radiation also in addition to producing electric and magnetic fields.

4. ELECTROSTATICS

The branch of physics which deals with charges at rest is called electrostatics. When a glass rod is rubbed with silk and then separated, the former becomes positively charged and the latter attains equal negative charge. It is because during rubbing, some electrons are transferred from glass to silk. Since glass rod and silk are separated by an insulating medium (i.e. air), they retain the charges. In other words, the charges on them are static or stationary. Note that the word 'electrostatic' means charges at rest.

5. CONDUCTORS AND INSULATORS

In general, the substances are divided into the following two classes on the basis of their ability to conduct electric charges:

(a) Conductors: Those substances through which electric charges can flow easily are called conductors e.g., silver, copper, aluminum, mercury, etc. In a metallic conductor, there are a large number of free electrons which act as charge carries. However, in a liquid conductor, both positive and negative ions are the charge carries. When a positively charged body is brought close to or touches a neutral conductor (metallic), the free electrons (charge carriers) in the conductor move quickly towards this positive charge. On the other hand, if a negatively charged body is brought close to or touches a neutral conductor, the free electrons in the conductor move away from the negative charge that is brought close.

(b) Insulators: Those substances through which electric charges cannot flow are called insulators e.g., glass, rubber, mica etc. When such materials are charged by rubbing, only the area that is rubbed becomes charged and there is no tendency of the charge to move into other regions of the substance. It is because there are practically no free electrons in an insulator.

6. CHARGING OF A BODY

A body can be charged by means of (a) friction, (b) conduction, (c) induction, (d) thermionic ionization, (e) photoelectric effect and (f) field emission.

(a) Charging by Friction: When a neutral body is rubbed with another neutral body (at least one of them should be insulator) then some electrons are transferred from one body to another. The body which gains electrons becomes negatively charged and the other becomes positively charged.

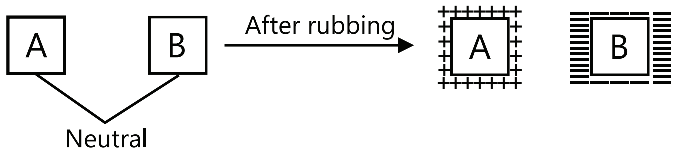

Figure 18.2

(b) Conduction (flow): There are two types of materials in nature.

(i) Conductor: Materials which have large number of free electrons.

(ii) Insulator or Dielectric or Nonconductors: Materials which do not have free electrons.

When a charged conductor is connected with a neutral conductor, then charge flows from one body to another body. In case of two charged conductors, charge flows from higher potential to lower potential. The charge stops flowing when the potential of the two bodies become same.

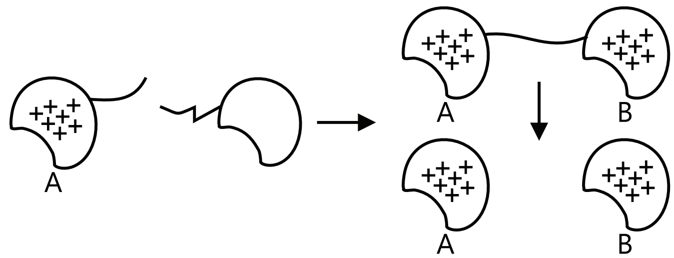

Figure 18.3

Note: If two identical shaped conductors kept at large distance are connected to each other, then they will have equal charges finally.

(c) Induction: When a charged particle is taken near to a neutral object, then the electrons move to one side and there is excess of electrons on that side making it negatively charged and deficiency on the other side making that side positively charged. Hence charges appear on two sides of the body (although total charge of the body is still zero). This phenomenon is called induction and the charge produced by it is called induced charge.

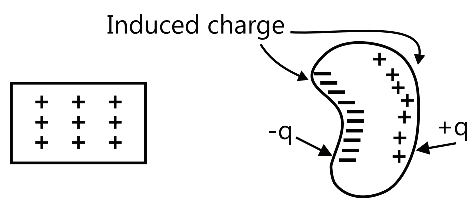

Figure 18.4

A body can be charged by induction in following two ways.

Method-I: The potential of conductor A becomes zero after earthing. To make potential zero some electrons flow from the Earth to the conductor A and now connection is removed making it negatively charged.

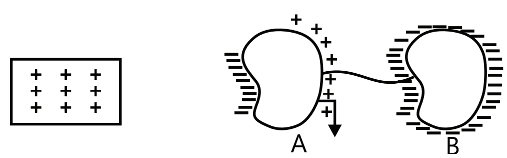

Figure 18.5

Method-II: The conductor which has included charge on it, is connected to a neutral conductor which makes the flow of charge such that their potentials become equal and now they are disconnected making the neutral conductor charged.

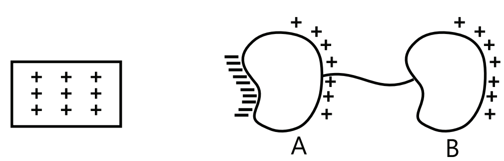

Figure 18.6

(d) Thermo-ionic emission: When the metal is heated at a high temperature then some electrons of metals are ejected and the metal gets ionized. It becomes positively charged.

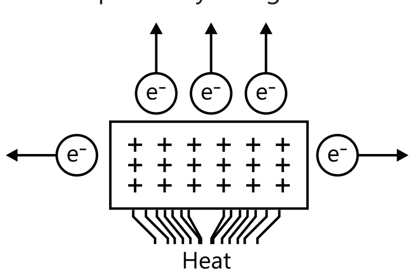

Figure 18.7

(e) Photoelectric effect: When light of sufficiently high frequency is incident on metal surface then some electrons come out and metal gets ionized.

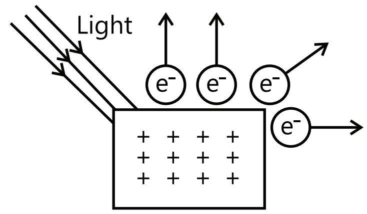

Figure 18.8

(f) Field emission: When electric field of large magnitude is applied near the metal surface then some electrons come out from the metal surface and hence the metal gets positively charged.

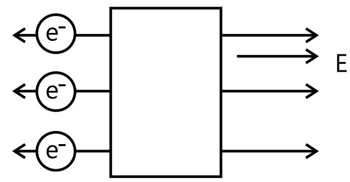

Figure 18.9

7. UNIT OF ELECTRIC CHARGE

We know that a positively charged body has deficit of electrons and a negatively charged body has excess of electrons from normal due share. Since the charge on an electron is very small, it is not convenient to select it as the unit of charge. In practice, coulomb is used as the unit of charge, i.e., SI unit of charge is coulomb abbreviated as C.

The charge on one electron in coulomb is $=-1.6\times10^{-19}$C

Note that charge on an electron has been found experimentally.

8. QUANTIZATION OF ELECTRIC CHARGE

The charge on an electron ($-e=1.6\times10^{-19}$C) or on a proton ($+e=+1.6\times10^{-19}$C) is minimum. We know that charge on a body is due to loss or gain of electrons by the body. Since a body cannot lose or gain a fraction of an electron, the charge on a body must be an integral multiple of electronic charge $\pm e$. In other words, charge on a body can only be $q=\pm ne$ where n=1, 2, 3, 4, and $e=1.6\times10^{-19}$C. This is called quantization of charge.

The fact that all free charges are integral multiple of electronic charge ($e=1.6\times10^{-19}$C) is known as quantization of electric charge.

$\therefore$ Charge on a body, $q=\pm ne$

Where n=1, 2, 3........ and $e=1.6\times10^{-19}$C

Suppose you measure the charge on a tiny body as $+4.5\times10^{-19}$C. This measurement is not correct because measured value is not an integral multiple of minimum charge (i.e., 1.6×10^{-19}C).

Note: **(i)** The quantization of charge shows that charge is discrete in nature and not of continuous nature.

(ii) Since the charge on an electron is so small ($e=1.6\times10^{-19}$C), we normally do not notice its discreteness in macroscopic charge (1μC charge requires about 10^{13} electrons) which thus seems continuous.

9. CONSERVATION OF ELECTRIC CHARGE

Just as total linear momentum of an isolated system always remains constant, similarly, the total electric charge of an isolated system always remains constant. This is called law of conservation of charge and may be stated as under: The total electric charge of an isolated system always remains constant.

In any physical process, the charges may get transferred from one part of the system to the other but total or net charge remains the same. In other words, charges can neither be created nor destroyed. No violation of this law has ever been found and it is as firmly established as the laws of conservation of linear momentum and energy.

Electrostatic Force-Coulomb's Law	
F = Electrostatic force q = Electric charge r= Distance between charge centers k= Coulomb constant $9.0\times10^9 N.m^2/C^2$	$F_s = k\frac{q_1q_2}{r^2}$

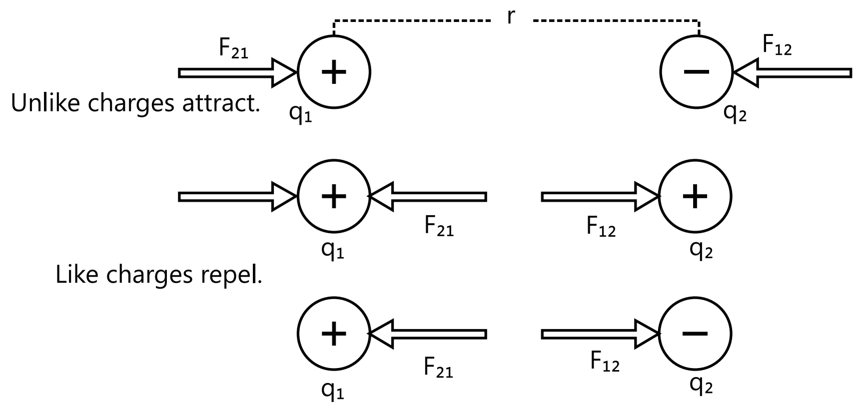

Figure 18.10

F_{21} is the force on charge 1 due to 2 and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$

PLANCESS CONCEPTS

In few problems of electrostatics, Lami's theorem is very useful.

According to this theorem, "if three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as shown in Fig. 18.11 are in equilibrium or if $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, then

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

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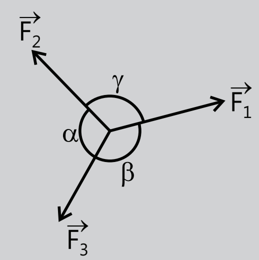


Figure 18.11

10. RELATIVE PERMITTIVITY OR DIELECTRIC CONSTANT

Permittivity is the property of a medium and affects the magnitude of force between two point charges. Air or vacuum has a minimum value of permittivity. The absolute (or actual) permittivity of air or vacuum is $8.854\times10^{-12} C^2N^{-1}m^{-2}$. The absolute permittivity ε of all other insulating materials is greater than ε_0. The ratio $\varepsilon/\varepsilon_0$ is called relative permittivity of the material and is denoted by K or (ε_r).

$K = \frac{\varepsilon}{\varepsilon_0} = \frac{\text{Absolute permittivity of medium}}{\text{Absolute permittivity of air (or vacuum)}}$ It may be noted that the relative permittivity is also called dielectric constant.

Another Definition. Force between two charges in air (or vacuum) is $F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ [See Fig. 18.12]

Force between the same two charges held same distance apart in a medium of absolute permittivity ε is

$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1q_2}{r^2}$ [see Fig. 18.12]

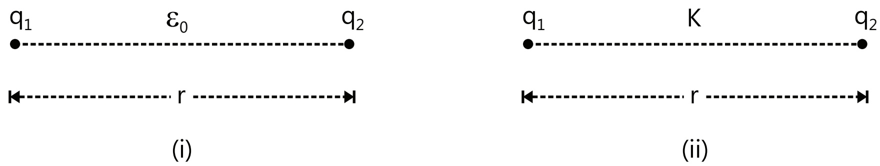


Figure 18.12

$\therefore \frac{F_{air}}{F_m} = \frac{\varepsilon}{\varepsilon_0} = K$ = Relative permittivity of the medium

Hence, relative permittivity (or dielectric constant) of a medium may be defined as the ratio of force between two charges separated by a certain distance in air (or vacuum) to the force between the same charges separated by the same distance in the medium.

Discussion. The following points may be noted:

(a) For air or vacuum, $K = \varepsilon_0 / \varepsilon_0 = 1$. For all other insulating materials, the value of K is more than 1.

(b) $F_m = F_{air} / K$. This implies that force between two charges is decreased when air is replaced by other insulating medium. For example, K for water is 80. It means that for the same charges (q_1, q_2) and same distance (r), the force between two charges in water is $1/80^{th}$ of that in air.

(c) K is number; being the ratio of two absolute permitivities. $\boxed{K = \frac{F_{air}}{F_{med}}}$ $K = \frac{\varepsilon}{\varepsilon_0}$

Comparison of Electrical Force with the Gravitational Force.

(a) Both electrical and gravitational forces follow the inverse square law.

(b) Both can act in vacuum also.

(c) Electrical forces may be attractive or repulsive but gravitational force is always attractive.

(d) Electrical forces are much stronger than gravitational forces.

(e) Both are central as well as conservative forces.

(f) Both the forces obey Newton's third law.

11. SUPERPOSITION OF ELECTROSTATIC FORCE

If in a region, more than 2 charges are present, then the net force acting on a particular charge will be the vector sum of the individual contribution of all other charges present in region, presence of any other charge in space cannot affect the force applied by a particular charge.

$\vec{F}_{1.net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + ... + \vec{F}_{1n}$,

Illustration 1: Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid. **(JEE ADVANCED)**

Sol: Inside the liquid, up thrust would act but simultaneously, electric force would also weaken due to dielectric of the liquid.

In vacuum each ball is in equilibrium under the following three forces:

(i) Tension, (ii) Electric force and (iii) Weight.

So, Lami's theorem can be applied.

In the liquid, $F_e^{'} = \frac{F_e}{K}$ Where, K=dielectric constant of liquid and W'=W-up thrust

Applying Lami's theorem in vacuum

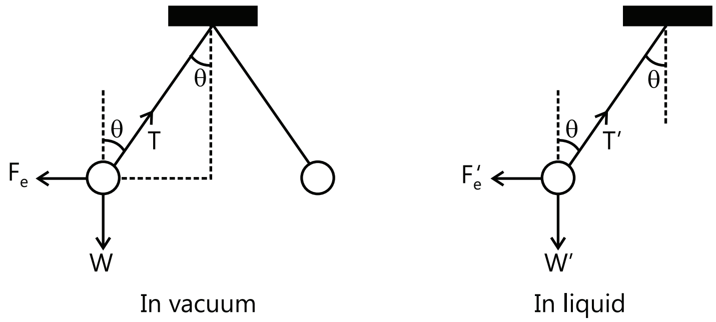


Figure 18.13

$$\frac{W}{\sin\left(90^o+\theta\right)} = \frac{F_e}{\sin\left(180^o-\theta\right)} \text{ or } \frac{W}{\cos\theta} = \frac{F_e}{\sin\theta} \quad \text{... (i)}$$

Similarly in liquid $\frac{W'}{\cos\theta} = \frac{F_e^{'}}{\sin\theta}$ ... (ii)

Dividing Eq.(i) by Eq.(ii), we get $\frac{W}{W'} = \frac{F_e}{F_e^{'}}$ or $K = \frac{W}{W-\text{upthrust}}$ $\left(\text{as } \frac{F_e}{F_e^{'}} = k\right)$

$\frac{V\rho g}{V\rho g - V\sigma g}$ (V=volume of ball) Or $K = \frac{\rho}{\rho-\sigma}$

Note: In the liquid F_e and W have been changed. Therefore, T will also change.

Illustration 2: A non-conducting rod of length L with a uniform positive charge density λ and a total charge Q is lying along the x-axis, as illustrated in Fig. 18.14. **(JEE ADVANCED)**

Calculate the force at a point P located along the axis of the rod and a distant x_0 from one end of the rod.

Sol: Consider rod as large number of small charges and apply principle of superposition of forces.

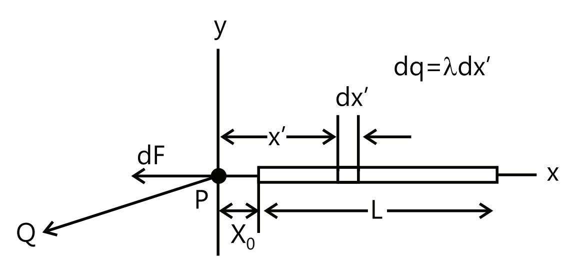


Figure 18.14

The linear charge density is uniform and is given by $\lambda = Q/L$. The amount of charge contained in a small segment of length dx' is $dq = \lambda dx'$.

Since the source carries a positive charge Q, the force at P points in the negative x direction, and the unit vector that points from the source to P is $\hat{r} = \hat{i}$. The contribution to the electric field due to dq is

$$d\vec{F} = \frac{Q}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r} = \frac{Q}{4\pi\varepsilon_o}\frac{\lambda dx'}{x'^2}\left(-\hat{i}\right) = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2 dx'}{Lx'^2}\hat{i}$$

Integrating over the entire length leads to

$$\vec{F} = \int d\vec{F} = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{L}\int_{x_0}^{x_0+L}\frac{dx'}{x'^2}\hat{i} = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{L}\left(\frac{1}{x_o} - \frac{1}{x_o+L}\right)\hat{i} = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{x_o\left(L+x_o\right)}\hat{i}$$

Notice that when P is very far away from the rod, $x_o >> L$ and the above expression becomes $\vec{F} \approx -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{x_o^2}\hat{i}$

The result is to be expected since at sufficiently far distance away, the distinction between a continuous charge distribution and a point charge diminishes.

12. ELECTRIC FIELD

A charged particle cannot directly interact with another particle kept at a distance. A charge produces something called an electric field in the space around it and this electric field exerts a force on any other charge (except the source charge itself) placed in it.

Thus, the region surrounding a charge or distribution of charge in which its electrical effects can be observed is called the electric field of the charge or distribution of charge. Electric field at a point can be defined in terms of either a vector function $\vec{E}$ called 'electric field strength' or a scalar function V called 'electric potential'. The electric field can also be visualized graphically in terms of 'lines of force'. The field propagates through space with the speed of light, c. Thus, if a charge is suddenly moved, the force it exerts on another charge a distance r away does not change until a time r/c later. In our forgoing discussion we will see that electric field strength $\vec{E}$ and electric potential V are interrelated. It is similar to a case where the acceleration, velocity and displacement of a particle are related to each other.

12.1 Electric Field Strength $(\vec{E})$

Like its gravitational counterpart, the electric field strength (often called electric field) at a point in an electric field is defined as the electrostatic force $\vec{F}_e$ per unit positive charge. Thus, if the electrostatic force experienced by a small test charge q_0 is $\vec{F}_e$, then field strength at that point is defined as, $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}_e}{q_0}$

($q_0 \to 0$ so that it doesn't interfere with the electrical field)

The electric field is a vector quantity and its direction is the same as the direction of the force $\vec{F}_e$ on a positive test charge. The SI unit of electric field is N/C. Here it should be noted that the test charge q_0 does not disturb other charges which produces $\vec{E}$. With the concept of electric field, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in this field.

An electric field leads to a force

Suppose there is an electric field strength $\vec{E}$ at some point in an electric field, then the electrostatic force acting on a charge $+q$ is qE in the direction of $\vec{E}$, while on the charge –q it is qE in the opposite direction of $\vec{E}$.

The electric field at a point is a vector quantity. Suppose $\vec{E}_1$ is the field at a point due to a charge q_1 and $\vec{E}_2$ is the field at the same point due to a charge q_2. The resultant field when both the charges are present is $\vec{E} = \vec{E}_1 + \vec{E}_2$

If the given charges distribution is continuous, we can use the technique of integration to find the resultant electric field at a point.

Illustration 3: A uniform electric field E is created between two parallel charged plates as shown in Fig. 18.15. An electron enters the field symmetrically between the plates with a speed v_o. The length of each plate is l. Find the angle of deviation of the path of the electron as it comes out of the field. **(JEE MAIN)**

Sol: Electron gains velocity in the vertical direction due to field between the plates.

The acceleration of the electron is $a = \frac{eE}{m}$ in the upward direction. The horizontal velocity remains v_o as there is no acceleration in this direction. Thus, the time taken in crossing the field is $t = \frac{l}{v_o}$. ... (i)

The upward component of the velocity of the electron as it emerges from the field region is

$$v_y = at = \frac{eEl}{mv_o}$$

The horizontal component of the velocity remains $v_x = v_o$.

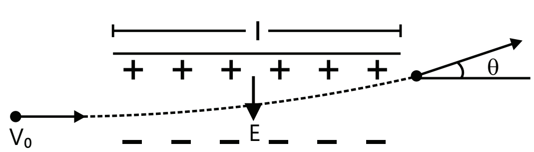


Figure 18.15

The angle θ made by the resultant velocity with the original direction is given by

$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv_o^2}$. Thus, the electron deviates by an angle $\theta = \tan^{-1}\frac{eEl}{mv_o^2}$.

PLANCESS CONCEPTS

Charge Densities

It is of three types:

(i) Linear charge density: It is defined as charge per unit length, i.e.

$\lambda = \frac{q}{l}$ its S.I. unit is coulomb/ metre and dimensional formula is $\left[ATL^{-1}\right]$

(ii) Surface charge density: It is defined as charge per unit area, i.e.

$\sigma = \frac{q}{A}$ its S.I. unit is $\text{coulomb} / \text{metre}^2$ and dimensional formula is $\left[ATL^{-2}\right]$

(iii) Volume charge density: It is defined as charge per unit volume i.e.

$\rho = \frac{q}{V}$ its S.I. unit is $\text{coulomb} / \text{metre}^3$ and dimensional formula is $\left[ATL^{-3}\right]$

Nitin Chandrol (JEE 2012 AIR 134)

12.2 Electric Fields Due to Continuous Charge Distributions

The electric field at a point P due to each charge element dq is given by Coulomb's law: $d\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r}$

Where r is the distance from dq to P and $\hat{r}$ is the corresponding unit vector. Using the superposition principle, the total electric field $\vec{E}$ is the vector sum (integral) of all these infinitesimal contributions: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\int_v \frac{dq}{r^2}\hat{r}$

This is an example of a vector integral which consists of three separate integrations, one for each component of the electric field.

12.3 Electric Field Due to a Point Charge

The electric field produced by a point charge q can be obtained in general terms from Coulomb's law. First note that the magnitude of the force exerted by the charge q on a test charge q_0 is,

$$F_e = \frac{1}{4\pi\varepsilon_o}.\frac{qq_0}{r^2}$$

Then divide this value by q_0 to obtain the magnitude of the field: $E = \frac{1}{4\pi\varepsilon_o}.\frac{q}{r^2}$

If q is positive, $\vec{E}$ is directed away from q. On the other hand, if q is negative, then $\vec{E}$ is directed towards q.

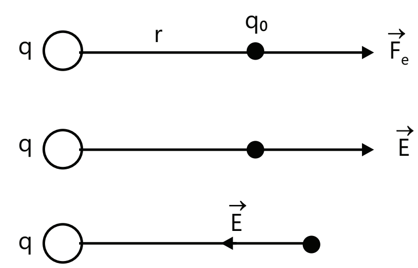


Figure 18.16

12.4 Electric Field Due to a Ring of Charge

A conducting ring of radius R has a total charge q uniformly distributed over its circumference. We are interested in finding the electric field at point P that lies on the axis of the ring at a distance x from its center. We divide the ring into infinitesimal segments of length dl. Each segment has a charge dq and acts as a point charge source of electric field.

Let $d\vec{E}$ be the electric field from one such segment; the net electric field at p is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. If we consider two ring segments at top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis) with no component perpendicular to that axis (i.e. no y or z component). So the field at P is described completely by its x component E_x.

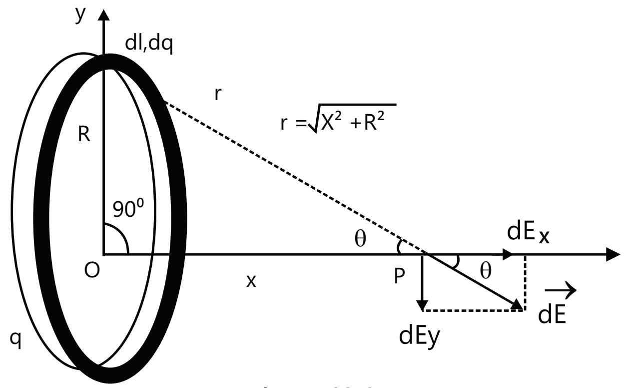


Figure 18.17

Calculation of E_x $dq = \left(\frac{q}{2\pi R}\right).dl$; $dE = \frac{1}{4\pi\varepsilon_o}.\frac{dq}{r^2}$

$$\therefore dE_x = dE\cos\theta = \left(\frac{1}{4\pi\varepsilon_o}\right)\left(\frac{dq}{x^2+R^2}\right)\left(\frac{x}{\sqrt{x^2+R^2}}\right) = \left(\frac{1}{4\pi\varepsilon_o}\right).\frac{(dq)x}{\left(x^2+R^2\right)^{3/2}}$$

$$\therefore E_x = \int dE_x = \frac{x}{4\pi\varepsilon_o\left(x^2+R^2\right)^{3/2}}\int dq \text{ ; or } E_x = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{qx}{\left(x^2+R^2\right)^{3/2}}$$

From the above expression, we can see that

(a) $E_x = 0$ at x=0, i.e., field is zero at the center of the ring. We should expect this, charges on opposite sides of the ring would push in opposite directions on a test charge at the center, and the forces would add to zero.

(b) $E_x = \frac{1}{4\pi\varepsilon_o}.\frac{q}{x^2}$ for x >> R i.e., when the point P is much farther from the ring, its field is the same as that of a point charge. To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

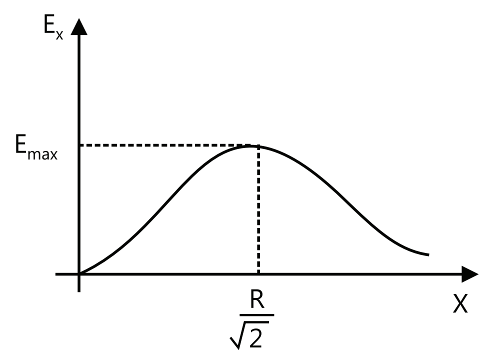


Figure 18.18

(c) E_x will be maximum where $\frac{dE_x}{dx} = 0$. Differentiating E_x w.r.t. x and putting it equal to zero we get $x = \frac{R}{\sqrt{2}}$ and E_{max} comes out to be, $\frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_o}.\frac{q}{R^2}\right)$.

12.5 Electric Field Due to a Line Charge

Positive charge q is distributed uniformly along a line with length 2a, lying along the y-axis between y=-a and y=+a. We are here interested in finding the electric field at point P on x-axis.

λ =charge per unit length= $\frac{q}{2a}$ $dq = \lambda dy = \frac{q}{2a}dy$; $dE = \frac{1}{4\pi\varepsilon_o}.\frac{dq}{r^2} = \frac{q}{4\pi\varepsilon_o}\frac{dy}{2a\left(x^2+y^2\right)}$

$$dE_x = dE\cos\theta = \frac{q}{4\pi\varepsilon_o}.\frac{xdy}{2a\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\sin\theta = \frac{q}{4\pi\varepsilon_o}\cdot\frac{ydy}{2a\left(x^2+y^2\right)^{3/2}}$$

$$\therefore E_x = \frac{1}{4\pi\varepsilon_o}\cdot\frac{qx}{2a}\int_{-a}^{a}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{q}{4\pi\varepsilon_o}\cdot\frac{1}{x\sqrt{x^2+a^2}}$$

$$\text{and } E_y = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{2a}\int_{-a}^{a}\frac{ydy}{\left(x^2+y^2\right)^{3/2}} = 0$$

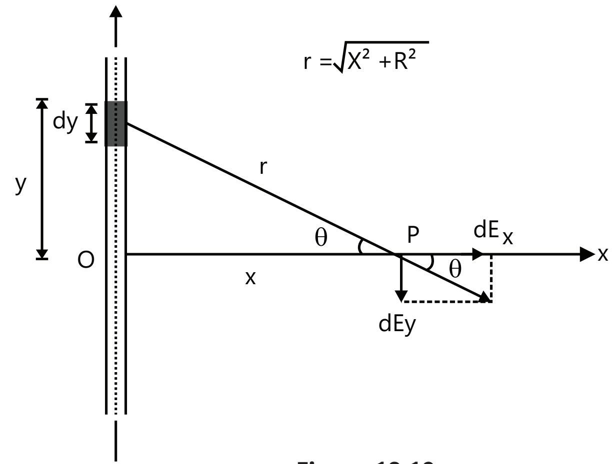


Figure 18.19

Thus, electric field is along x-axis only and which has a magnitude,

$$E_x = \frac{q}{4\pi\varepsilon_o x\sqrt{x^2+a^2}} \qquad \text{... (i)}$$

From the above expression, we can see that:

(a) If $x >> a$, $E_x = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{x^2}$, i.e., if point P is very far from the line charge, the field at P is the same as that of a point charge.

(b) Now assume that, we make the line of charge longer and longer, adding charge in proportion to the total length so that λ, the charge per unit length remains constant. In this case Eq(i) can be written as,

$$E_x = \frac{1}{2\pi\varepsilon_o}\cdot\left(\frac{q}{2a}\right)\cdot\frac{1}{x\sqrt{x^2/a^2+1}} = \frac{\lambda}{2\pi\varepsilon_o x\sqrt{x^2/a^2+1}}$$

Now, $= x^2/a^2 \to 0$ as $a >> x$, $E_x = \frac{\lambda}{2\pi\varepsilon_o x}$

13. ELECTRIC FIELD LINES

An electric line of force is an imaginary smooth curve in an electric field along which a free, isolated unit positive charge moves.

Properties

(a) Electric lines of force start at a positive and terminate at a negative charge.

(b) A tangent to a line of force at any point gives the direction of the force on positive charge and hence direction of electric field at that point.

(c) No two lines of force can intersect one another.

(d) The lines of force are crowded in the region of larger intensity and further apart in the region of weak field.

(e) Lines of force leave the surface of a conductor normally.

(f) Electric lines of force do not pass through a closed conductor.

Field of some special classes

We here highlight the following charge distributions.

(a) Single positive or negative charge (Fig. 18.20 (a) and (b))- The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward.

(b) Two equal positive charges (Fig18.20 (c))- the field lines around a system of two positive charges (q, q) give a vivid pictorial description of their mutual repulsion.

(c) Two equal and opposite charges (Fig18.20 (d))-The field around the configuration of two equal and opposite charges (q,-q), a dipole, show clearly the mutual attraction between the charges.

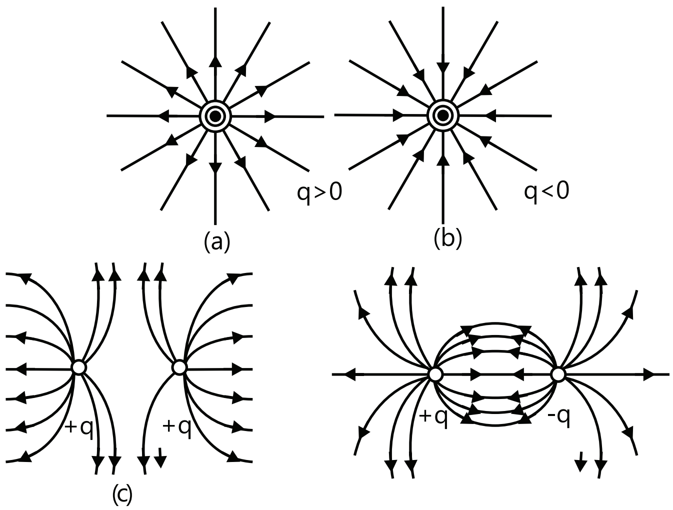


Figure 18.20

Properties:

(a) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge then lines start from positive charge and terminate at ∞. If there is only one negative charge then lines start from ∞ and terminate at negative charge.

(b) The electric intensity at a point is the number of lines of force streaming through per unit area normal to the direction of the intensity at that point. The intensity will be more where the density of lines is more.

(c) Number of lines originating (terminating) from (on) is directly proportional to the magnitude of the charge.

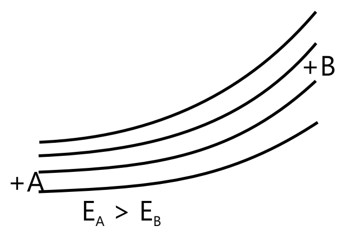


Figure 18.21

Note: A charge particle need not follow an Electric field lines.

(a) Electric field lines of resultant electric field can never intersect with each other.

(b) Electric field lines produced by static charges do not form close loop.

(c) Electric field lines end or start perpendicularly on the surface of a conductor.

(d) Electric field lines never enter in to conductors.

Illustration 4: Consider the situation shown in Fig. 18.22. What are the signs of q_1 and q_2? If the lines are drawn in proportion to the charge, what is the ratio q_1/q_2? **(JEE MAIN)**

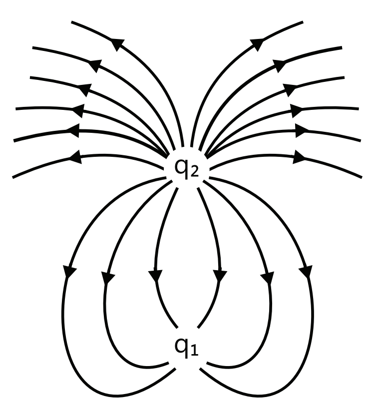


Figure 18.22

Sol: Use properties of field lines.

The basic concept of this question is that number density is directly proportional to electric field. If we take the entire area of the sphere around the charge, then area will be the same. Now, we just have to count the number of lines originating from the two charges.

In case of point charges, $E \propto q$

Thus, $E_1 / E_2 = q_1 / q_2 = n_1 / n_2 = 6/18 = 1/3$

However, this problem can also be seen by flux. Why don't you try it as an exercise?Plus, q_1 has to be negative, while q_2 would be positive.

14. ELECTRIC FLUX

The strength of an electric field is proportional to the number of field lines per unit area. The number of electric field lines that penetrates a given surface is called an "electric flux," which we denote as Φ_E. The electric field can therefore be thought of as the number of lines per unit area.

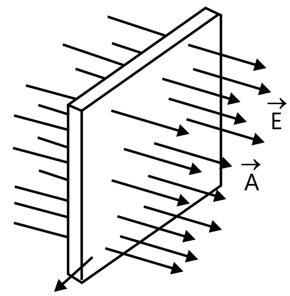


Figure 18.23

In Fig. 18.23 shows Electric field lines passing through a surface of area A.

Consider the surface shown in Fig. 18.24. Let $\vec{A} = A\hat{n}$ be defined as the area vector having a magnitude of the area of the surface, A, and pointing in the normal direction, $\hat{n}$. If the surface is placed in a uniform electric field $\vec{E}$ that points in the same direction as $\hat{n}$, i.e., perpendicular to the surface A, the flux through the surface is

$$\Phi_E = \vec{E}.\vec{A} = \vec{E}.\hat{n}A = EA$$

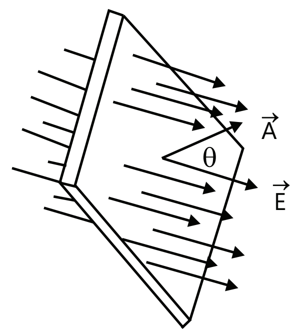


Figure 18.24

On the other hand, if the electric field $\vec{E}$ makes an angle θ with $\hat{n}$, the electric flux becomes $\Phi_E = \vec{E}.\vec{A} = \vec{E}.A\cos\theta = E_nA$

Where $E_n = \vec{E}.\hat{n}$ is the component of $\vec{E}$ perpendicular to the surface.

Note that with the definition for the normal vector $\hat{n}$, the electric flux Φ_E is positive if the electric field lines are leaving the surface, and negative if entering the surface.

In general, a surface S can be curved and the electric field $\vec{E}$ may vary over the surface. We shall be interested in the case where the surface is closed. A closed surface is a surface which completely encloses a volume. In order to compute the electric flux, we divide the surface into a large number of infinitesimal area elements $\Delta\vec{A}_i = \Delta A_i\hat{n}_i$, as shown in Fig. 18.25. Note that for a closed surface, the unit vector $\hat{n}_i$ is chosen to point in the outward normal direction.

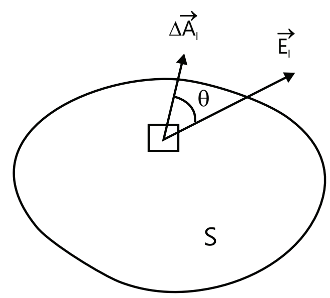


Figure 18.25

Electric field is passing through an area element $\Delta\vec{A}_i$, making an angle θ with the normal of the surface.

The electric flux through $\Delta\vec{A}_i$ is $\Delta\Phi_E = \vec{E}_i.\Delta\vec{A}_i = E_i\Delta A_i\cos\theta$

The total flux through the entire surface can be obtained by summing over all the area elements. Taking the limit $\Delta\vec{A}_i \to 0$ and the number of elements to infinity, we have $\Delta\Phi_E = \lim\limits_{\Delta A_i \to 0} \sum \vec{E}_i.d\vec{A}_i = \int \vec{E}.d\vec{A}$

In order to evaluate the above integral, we must first specify the surface and then sum over the dot product $\vec{E}.d\vec{A}$..

Let $\Delta\vec{A}_1 = \Delta A_1\hat{r}$ be

An area element on the surface of a sphere S_1 of radius r_1, as shown in Fig. 18.26.

The area element ΔA subtends a solid angle $\Delta\Omega$. The solid angle $\Delta\Omega$ subtended by $\Delta\vec{A}_1 = \Delta A_1\hat{r}$ at the center of the sphere is defined as

$$\Delta\Omega \equiv \frac{\Delta A_1}{r_1^2}$$

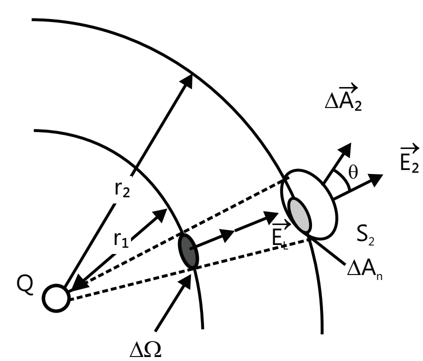


Figure 18.26

Solid angles are dimensionless quantities measured in steradians (sr). Since the surface area of the sphere S_1 is $4\pi r_1^2$, the total solid angle subtended by the sphere is

$$\Omega = \frac{4\pi r_1^2}{r_1^2} = 4\pi$$

In Fig. 18.26, the area element $\Delta\vec{A}_2$ makes an angle θ with the radial unit vector $\hat{r}$, then the solid angle subtended by ΔA_2 is

$$\Delta\Omega = \frac{\vec{\Delta A_2}.\hat{r}}{r_2^2} = \frac{\Delta A_2 \cos\theta}{r_2^2} = \frac{\Delta A_{2n}}{r_2^2}$$

Illustration 5: A non - uniform electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 18.28 (E is in newton per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? **(JEE ADVANCED)**

Sol: We can find the flux through the surface by integrating the scalar product $\vec{E}.d\vec{A}$ over each face.

Right face: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for the right face of the cube must point in the positive direction of the x axis. In unit-vector notation,

$d\vec{A} = dA\hat{i}$. The flux Φ, through the right face is then

$$\Phi = \int \vec{E}.d\vec{A} = \int\left(3.0x\hat{i} + 4.0\hat{j}\right).\left(dA\,\hat{i}\right) = \int\left[(3.0x)(dA)\hat{i}\cdot\hat{i} + (4.0)(dA)\hat{j}\cdot\hat{i}\right] = \int(3.0xdA + 0) = 3.0\int xdA.$$

We are about to integrate over the right face, but we note that x has the same value everywhere on that face-namely, x=3.0m. This means we can substitute that constant value for x. Then

$$\Phi_r = 3.0\int(3.0)dA = 9.0\int dA.$$

The integral $\int dA$ merely gives us the area A=4.0 m^2 of the right face; so

$$\Phi_r = (9.0N/C)\left(4.0m^2\right) = 36N.m^2/C.$$

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (i) The differential area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$. (ii) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, x=1.0m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_1 = -12N\cdot m^2/C.$$

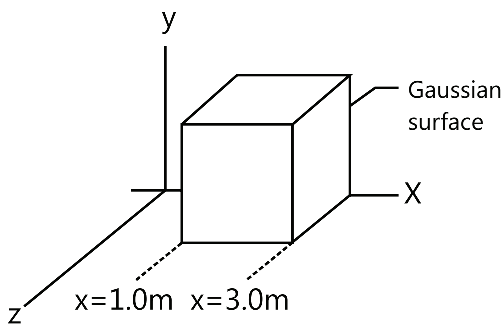


Figure 18.27

Top face: The differential area vector $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = +dA\hat{j}$. The flux Φ_l through the top face is then

$$\Phi_1 = \int\left(3.0x\hat{i} + 4.0\hat{j}\right)\cdot\left(dA\hat{j}\right) = \int\left[(3.0x)(dA)\hat{i}\cdot\hat{j} + (4.0)(dA)\hat{j}.\hat{j}\right] = \int(0 + 4.0dA) = 4.0\int dA = 16\ N\cdot m^2/C.$$

15. GAUSS' LAW

Consider a positive point charge Q located at the center of a sphere of radius r, as shown in Fig. 18.28. The electric field due to the charge Q is $\vec{E} = \left(Q/4\pi\varepsilon_0 r^2\right)\hat{r}$, which points in the radial direction. We enclose the charge by an imaginary sphere of radius r called the "Gaussian surface".

A spherical Gaussian surface enclosing a charge Q.

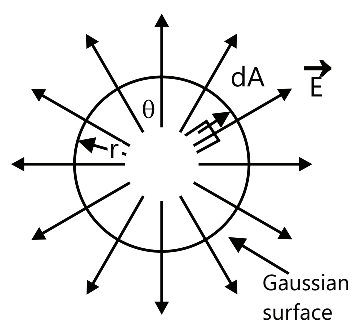


Figure 18.28

In spherical coordinates, a small surface area element on the sphere is given by $d\vec{A} = r^2 \sin\theta d\theta d\phi \hat{r}$

A small area element on the surface of a sphere of radius r.

Thus the net electric flux through the area element is

$$d\Phi_E = \vec{E} \cdot d\vec{A} = EdA = \left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)\left(r^2 \sin\theta d\theta d\phi\right) = \frac{Q}{4\pi\varepsilon_o}\sin\theta d\theta d\phi\sqrt{a^2+b^2}$$

The total flux through the entire surface is

$$\Phi_E = \oiint_s \vec{E} \cdot d\vec{A} = \frac{Q}{4\pi\varepsilon_o}\int_o^z \sin\theta d\theta \int_o^{2\pi} d\phi = \frac{Q}{\varepsilon_o}$$

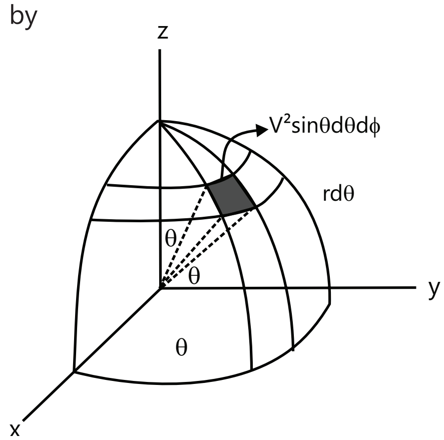


Figure 18.29

The same result can also be obtained by noting that a sphere of radius r has a surface area $A = 4\pi r^2$, and since the magnitude of the electric field at any point on the spherical surface is $E = Q / 4\pi\varepsilon_o r^2$, the electric flux through the surface is

$$\Phi_E = \oiint_s \vec{E} \cdot d\vec{A} = E\oiint_s dA = EA = \left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)4\pi r^2 = \frac{Q}{\varepsilon_o}$$

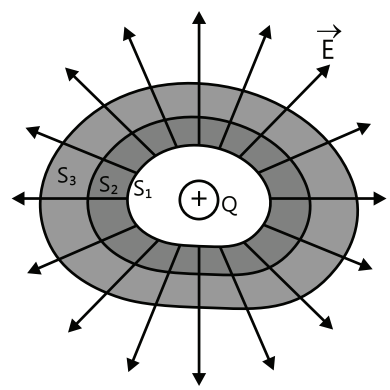


Figure 18.30

In the above, we have chosen a sphere to be the Gaussian surface. However, it turns out that the shape of the closed surface can be arbitrarily chosen. For the surfaces shown in Fig. 18.30, the same result $\left(\Phi_E = Q / \varepsilon_o\right)$ is obtained. Whether the choice is S_1, S_2 or S_3.

The statement that the net flux through any closed surface is proportional to the net charge enclosed is known as Gauss's law. Mathematically, Gauss's law is expressed as

$$\Phi_E = \oiint_s \vec{E} \cdot d\vec{A} = \frac{q_{ene}}{\varepsilon_o} \qquad \text{(Gauss's law)}$$

Where q_{ene} is the net charge inside the surface. One way to explain why Gauss's law holds is that the number of field lines that leave the charge is independent of the shape of the imaginary Gaussian surface we choose to enclose the charge.

Illustration 6: Fig. 18.31 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1\text{nC}$, $q_2 = q_5 = -5.9\text{nC}$, and $q_3 = -3.1\text{nC}$? Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the charged objects and the coin. **(JEE MAIN)**

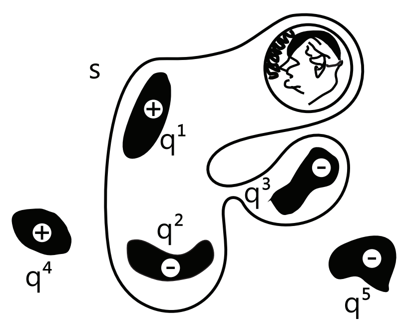


Figure 18.31

Sol: In Gauss's law, only enclosed charges used to calculate the flux.

The net flux Φ through the surface depends on the net charge q_{enc} enclosed by surface S.

The coin does not contribute to Φ because it is neutral and thus contains equal amounts of positive and negative charge. Charges q_4 and q_5 do not contribute because they are outside surface S. Thus, q_{enc} is $q_1 + q_2 + q_3$ and gives us

$$\Phi = \frac{q_{enc}}{\varepsilon_o} = \frac{q_1 + q_2 + q_3}{\varepsilon_o} = \frac{+3.1\times10^{-9}C - 5.9\times10^{-9}C - 3.1\times10^{-9}C}{8.85\times10^{-12}C^2 / N\cdot m^2} = -670 N.m^2 / C.$$

Conclude: The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

Illustration 7: Find the flux through the disk shown in Fig. 18.32. The line joining the charge to the center of the disk is perpendicular to the disk. **(JEE MAIN)**

Sol: The electric flux through the disk cannot be found by the equation

$\phi = \vec{E}\cdot\vec{A} = EA\cos\theta$ If we wish to use the basic formula, we can divide the disk into small rings as shown in Fig. 18.33 and find the electric field due to charge at all the rings:

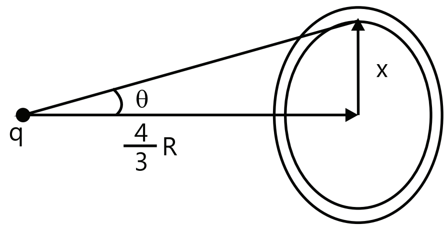


Figure 18.32

$\phi = \int \vec{E}.d\vec{s}$. Here we divide the entire disk into thin ring and find the flux due to the charge through the thin ring.

the electric field due to the point charge at the location of the ring shown is given by

$$E = \frac{kq}{(16/9)R^2 + x^2}.$$

As we discussed before, the area of the ring is $2\pi x dx$. But the electric field is not normal to the ring. The angle can be found as shown:

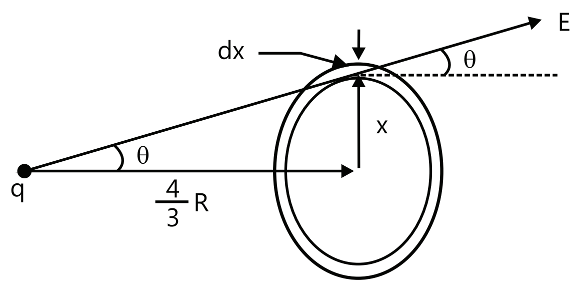


Figure 18.33

$$\cos\theta = \frac{4R/3}{\sqrt{(16R^2/9) + x^2}}, \qquad \phi = \int \vec{E}\cdot d\vec{s},$$

$$\phi = \int_0^{0.75R} \frac{kq \times (4R/3) \times 2\pi x dx}{\left[x^2 + (16R^2/9)\right] \times \sqrt{x^2 + (16R^2/9)}} = \frac{q}{10\varepsilon_o}.$$

Illustration 8: An infinitely long rod of negligible radius has a uniform charge density λ. Calculate the electric field at a distance r from the wire. **(JEE MAIN)**

Sol: We shall solve the problem by following the steps outlined above.

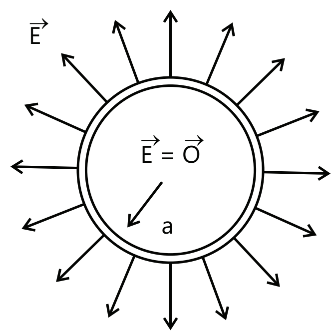


Figure 18.34

(a) An infinitely long rod possesses cylindrical symmetry.

(b) The charge density is uniformly distributed throughout the length, and the electric field $\vec{E}$ must point radially away from the symmetry axis of the rod (Fig. 18.34). The magnitude of the electric field is constant on cylindrical surface of radius r. Therefore, we choose a coaxial cylinder as our Gaussian surface.

(c) Field lines for an infinite uniformly charged rod (the symmetry axis of the rod and the Gaussian cylinder are perpendicular to plane of the page.)

(d) The amount of charge enclosed by the Gaussian surface, a cylinder of radius r and length ℓ (Fig. 18.35), is $q_{enc} = \lambda\ell$.

(e) As indicated in Fig. 18.36, the Gaussian surface consists of three parts: a two ends S_1 and S_2 plus the curved side wall S_3. The flux through the Gaussian surface is

$$\Phi_E = \oiint_S \vec{E}.d\vec{A} = \iint_{S_1} \vec{E}_1 \cdot d\vec{A}_1 + \iint_{S_2} \vec{E}_2 \cdot d\vec{A}_2 + \iint_{S_3} \vec{E}_3 \cdot d\vec{A}_3 = 0 + 0 + E_3A_3 = E(2\pi r\ell)$$

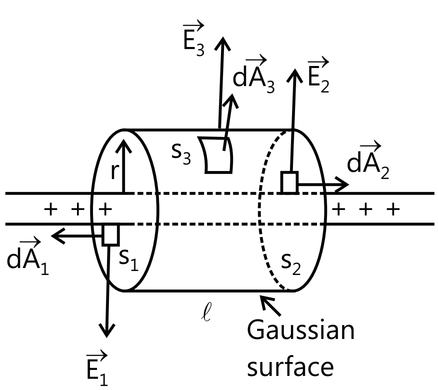


Figure 18.35

Where we have set $E_3 = E$. As can be seen from the Fig. 18.35, no flux passes through the ends since the area vectors $d\vec{A}_1$. and $d\vec{A}_2$ are perpendicular to the electric field which points in the radial direction.

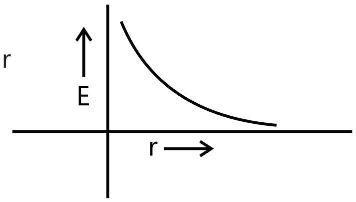


Figure 18.36

(f) Applying Gauss's Law gives $E(2\pi r\ell) = \lambda\ell / \varepsilon_o$, or $E = \dfrac{\lambda}{2\pi\varepsilon_o r}$

The result is in complete agreement with that obtained in equation using Coulomb's law. Notice that the result is independent of the length ℓ of the cylinder, and only depends on the inverse of the distance r from the symmetry axis. The qualitative behavior of E as a function of r is plotted in Fig. 18.36.

Illustration 9: Consider an infinitely large non-conduction plane in the xy-plane with uniform surface charge density σ. Determine the electric field everywhere in space. **(JEE MAIN)**

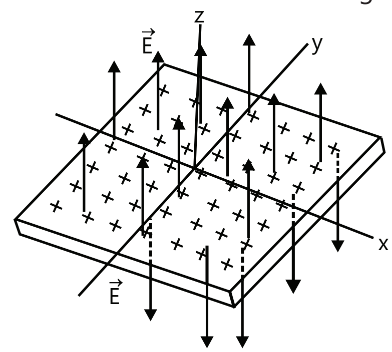


Figure 18.37

Sol: (i) An infinitely large plane possesses a planar symmetry.

(ii) Since the charge is uniformly distributed on the surface, the electric field $\vec{E}$ must point perpendicularly away from the plane, $\vec{E} = E\hat{k}$. The magnitude of the electric field is constant on planes parallel to the non-conducting plane.

We choose our Gaussian surface to be a cylinder, which is often referred to as a "pillbox"

The pillbox also consists of three parts: two end-caps S_1 and S_2, and a curved side S_3.

(ii) Since the surface charge distribution is uniform, the charge enclosed by the Gaussian "pillbox" is $q_{enc} = \sigma A$, where $A=A_1=A_2$ is the area of the end-caps.

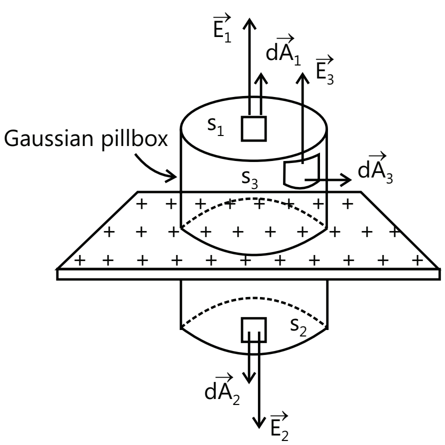


Figure 18.38

(iv) The total flux through the Gaussian pillbox flux is

$$\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = \oiint_{S_1} \vec{E}_1 \cdot d\vec{A}_1 + \oiint_{S_2} \vec{E}_2 \cdot d\vec{A}_2 + \oiint_{S_3} \vec{E}_3 \cdot d\vec{A}_3$$

$$= E_1A_1 + E_2A_2 + 0 = (E_1 + E_2)A$$

Since the two ends are at the same distance from the plane, by symmetry, the magnitude of the electric field must be the same: $E_1 = E_2 = E$. Hence, the total flux can be rewritten as

$$\Phi_E = 2EA$$

(v) By applying Gauss's law, we obtain $2EA = \dfrac{q_{enc}}{\varepsilon_o} = \dfrac{\sigma A}{\varepsilon_o}$ Which gives $E = \dfrac{\sigma}{2\varepsilon_o}$

In unit-vector notation, we have $\vec{E} = \begin{cases} \dfrac{\sigma}{2\varepsilon_o}\hat{k}, & z > 0 \\ -\dfrac{\sigma}{2\varepsilon_o}\hat{k}, & z < 0 \end{cases}$

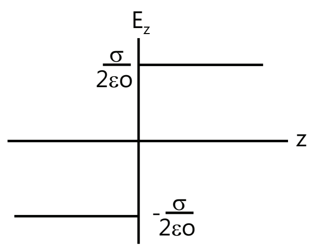


Figure 18.39

Thus, we see that the electric field due to an infinite large non-conducting plane is uniform in space. The result, plotted in Fig. 18.39, is the same as that obtained using Coulomb's law. Note again the discontinuity in electric field as we cross the plane:

$$\Delta E_z = E_{z+} - E_{z-} = \frac{\sigma}{2\varepsilon_o} - \left(-\frac{\sigma}{2\varepsilon_o}\right) = \frac{\sigma}{\varepsilon_o}$$

Illustration 10: A thin spherical shell of radius a has a charge +Q evenly distributed over its surface. Find the electric field both inside and outside the shell. **(JEE MAIN)**

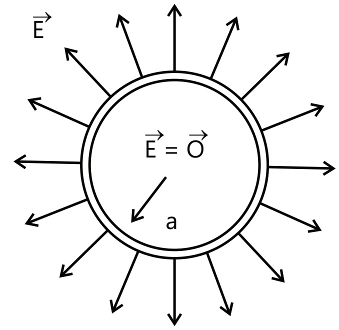


Figure 18.40

Sol: Apply Gauss's law, as the charge distribution is symmetric.

The charge distribution is spherically symmetric, with a surface charge density $\sigma = Q/A_s = Q/4\pi a^2$, where $A_s = 4\pi a^2$ is the surface area of the sphere. The electric field $\vec{E}$ must be radially symmetric and directed outward (Fig. 18.40). We treat the regions $r \le a$ and $r \ge a$ separately.

Electric field for uniform spherical shell of charge

Case 1: $r \le a$ We choose our Gaussian surface to be a sphere of radius $r \le a$, as shown in Fig. 18.41 (a).

The charge enclosed by the Gaussian surface is $q_{enc} = 0$ since all the charge is located on the surface of the shell. Thus, from Gauss's law, $\Phi_E = q_{enc}/\varepsilon_o$, we conclude E=0, r<a

Case 2: $r \ge a$ In this case, the Gaussian surface is a sphere of radius $r \ge a$, as shown in Fig. 18.42 (b). Since the radius of the "Gaussian sphere" is greater than the radius of the spherical shell, all the charge is enclosed: $q_{enc} = Q$

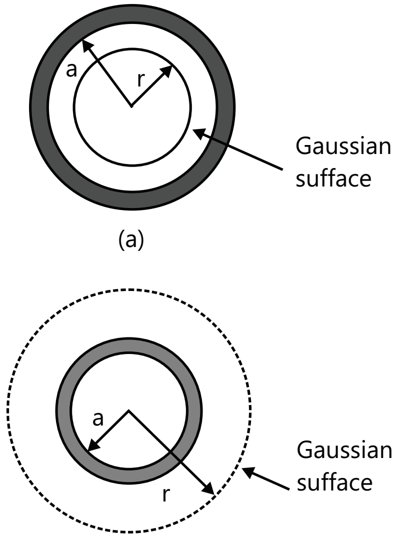


Figure 18.41

Since the flux through the Gaussian surface is $\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = EA = E\left(4\pi r^2\right)$

By applying Gauss's law, we obtain $E = \dfrac{Q}{4\pi\varepsilon_o r^2} = k_e\dfrac{Q}{r^2}$, $r \ge a$

Note that the field outside the sphere is the same as if all the charges were concentrated at the center of the sphere. The qualitative behavior of E as a function of r is plotted in Fig. 18.42 showing electric field as a function of r due to a uniformly charged spherical shell.

As in the case of a non-conducting charged plane, we again see a discontinuity in E as we cross the boundary at r=a. The change, from outer to the inner surface, is given by

$$\Delta E = E_+ - E_- = \frac{Q}{4\pi\varepsilon_o a^2} - 0 = \frac{\sigma}{\varepsilon_o}$$

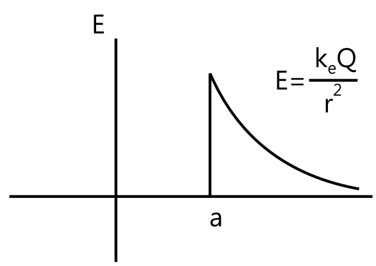


Figure 18.42

Illustration 11: Non-Conducting Solid Sphere

An electric charge +Q is uniformly distributed throughout a non-conducting solid sphere of radius a. Determine the electric field everywhere inside and outside the sphere. **(JEE MAIN)**

Sol: For non-conducting object. Charge distributed thoughout the mass.

The charge distribution is spherically symmetric with the charge density given by

$$\rho = \frac{Q}{V} = \frac{Q}{\left(4/3\right)\pi a^3}$$

Where V is the volume of the sphere. In this case, the electric field $\vec{E}$ is radially symmetric and directed outward. The magnitude of the electric field is constant on spherical surfaces of radius r. The regions $r \le a$ and $r \ge a$ shall be studied separately.

Case 1: $r \le a$

We choose our Gaussian surface to be a sphere of radius $r \le a$, as shown in Fig. 18.41 (a).

Fig. 18.41 (b) shows Gaussian surface for uniformly charged solid sphere, for (a) $r \le a$, and (b) $r > a$.

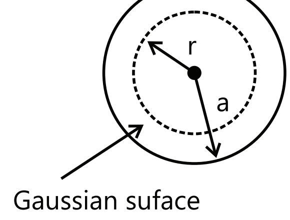


Figure 18.43

The flux through the Gaussian surface is $\Phi_E = \oint\!\!\oint_S \vec{E} \cdot d\vec{A} = EA = E\left(4\pi r^2\right)$ With uniform charge distribution, the charge enclosed is $q_{enc} = \int_V \rho dV = \rho V = \rho\left(\frac{4}{2}\pi r^3\right) = Q\left(\frac{r^3}{a^3}\right)$

Which is proportional to the volume enclosed by the Gaussian surface. Applying Gauss's law

$\Phi_E = q_{enc} / \varepsilon_o$, we obtain $E\left(4\pi r^2\right) = \frac{\rho}{\varepsilon_o}\left(\frac{4}{3}\pi r^3\right)$ or $\boxed{E = \frac{\rho r}{3\varepsilon_o} = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad r \le a}$

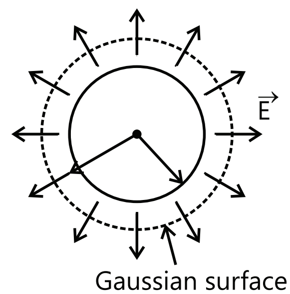


Figure 18.44

Case 2: $r \ge a$

In this case, our Gaussian surface is a sphere of radius $r \ge a$, as shown in Fig. 18.44 . Since the radius of the Gaussian surface is greater than the radius of the sphere all the charge is enclosed in our Gaussian surface: $q_{enc} = Q$. With the electric flux through the Gaussian surface given by $\Phi_E = E\left(4\pi r^2\right)$, upon applying Gauss's law, we obtain

$E\left(4\pi r^2\right) = Q / \varepsilon_o$, or $\boxed{E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}, \qquad r > a}$

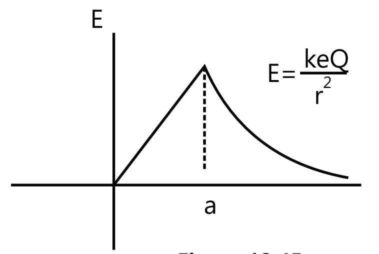


Figure 18.45

The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere. The qualitative behavior of E as a function of r is plotted in Fig. 18.45.

PROBLEM-SOLVING TACTICS

The following steps may be useful when applying Gauss's law:

(a) Identify the symmetry associated with the charge distribution.

(b) Determine the direction of the electric field, and a "Gaussian surface" on which the magnitude of the electric field is constant over portions of the surface.

(c) Divide the space into different regions associated with the charge distribution. For each region, calculate q_{enc}, the charge enclosed by the Gaussian surface.

(d) Calculate the electric flux Φ_E through the Gaussian surface for each region.

(e) Equate Φ_E with q_{enc} / ε_o, and deduce the magnitude of the electric field.

In this chapter, we have discussed how electric field can be calculated for both the discrete and continuous charge distributions. For the former, we apply the superposition principle: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$

For the latter, we must evaluate the vector integral $\vec{E} = \frac{1}{4\pi\varepsilon_o}\int \frac{dq}{r^2}\hat{r}$

Where r is the distance from dq to the field point P and $\hat{r}$ is the corresponding unit vector. To complete the integration, we shall follow the procedure outlined below:

(a) Start with $d\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r}$

(b) Rewrite the charge element dq as $dq = \begin{cases} \lambda d\ell & (\text{length}) \\ \sigma dA & (\text{area}) \\ \rho dV & (\text{volume}) \end{cases}$

Depending on whether the charge is distributed over a length, an area, or a volume.

(c) Substituting dq into the expression for $d\vec{E}$.

(d) Specify an appropriate coordinate system (Cartesian, cylindrical or spherical) and express the differential element $(d\ell, dA\, \text{or}\, dV)$ and r in terms of the coordinates (see table below for summary.)

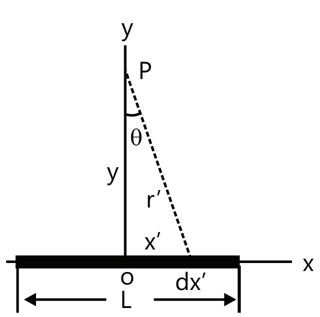
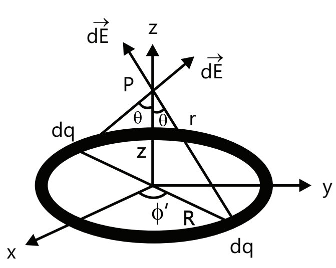
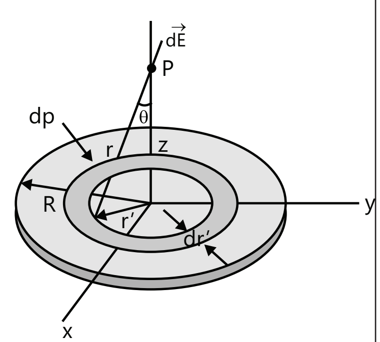
	Cartesian (x,y,z)	Cylindrical (ρ,ϕ,z)	Spherical (r,θ,ϕ)
Dl	dx, dy, dz	$d\rho,\ \rho d\phi,\ dz$	$dr, rd\theta, r\sin\theta d\phi$
dA	dxdy, dydz, dzdx	$d\rho dz,\ \rho d\phi dz,\ \rho d\phi d\rho$	$rdrd\theta, r\sin\theta drd\phi, r^2\sin\theta d\theta d\phi$
dV	dxdydz	$\rho d\,\rho d\phi dz$	$r^2\sin\theta dr\,d\theta\,d\phi$

Differential elements of length, area and volume in different coordinates

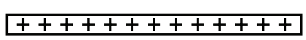
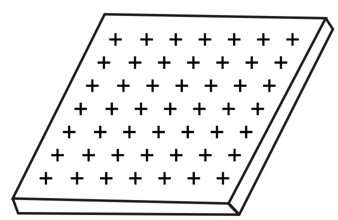
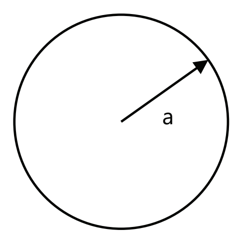
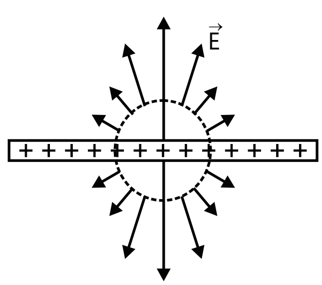
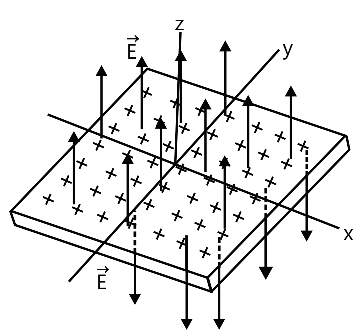
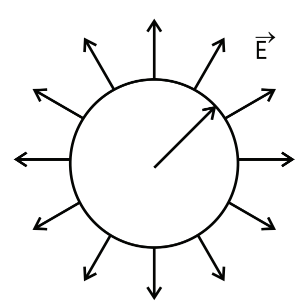
(a) Rewrite $d\vec{E}$ in terms of the integration variable(s), and apply symmetry argument to identify non-vanishing component(s) of the electric field.

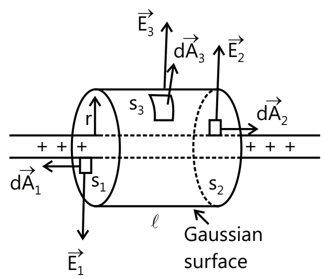
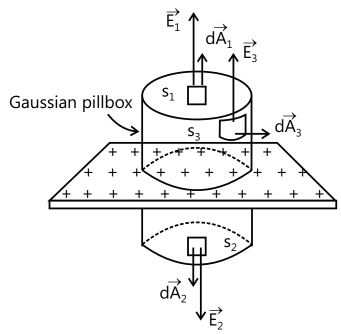
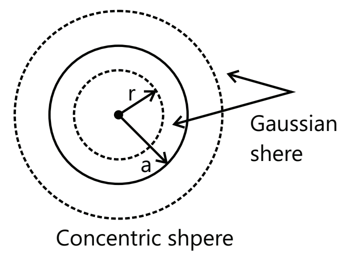
(b) Complete the integration to obtain $\vec{E}$.

In the Table below we illustrate how the above methodologies can be utilized to compute the electric field for an infinite line charge, a ring of charge and a uniformly charged disk.

	Line charge	**Ring of charge**	**Uniformly charged disk**
(1) Figure	 **Figure 18.46**	 **Figure 18.47**	 **Figure 18.48**
(2) Express dq in terms of charge density	$dq = \lambda dx'$	$dq = \lambda d\ell$	$dq = \sigma dA$
(3) write down dE	$dE = k_e\frac{\lambda dx'}{r'^2}$	$dE = k_e\frac{\lambda dl}{r^2}$	$dE = k_e\frac{\sigma dA}{r^2}$

	Line charge	Ring of charge	Uniformly charged disk
(4) Rewrite r and the differential element in terms of the appropriate coordinates	dx' $\cos\theta = \frac{y}{r'}$ $r' = \sqrt{x'^2 + y^2}$	$d\ell = Rd\phi'$ $\cos\theta = \frac{z}{r}$ $r = \sqrt{R^2 + z^2}$	$dA = 2\pi r'dr'$ $\cos\theta = \frac{z}{r}$ $r = \sqrt{r'^2 + z^2}$
(5) Apply symmetry argument to identify non-vanishing component(s) of dE	$dE_y = dE\cos\theta$ $= k_e \frac{\lambda y dx'}{\left(x'^2 + y^2\right)^{3/2}}$	$dE_y = dE\cos\theta$ $= k_e \frac{\lambda Rzd\phi'}{\left(R^2 + z^2\right)^{3/2}}$	$dE_y = dE\cos\theta$ $= k_e \frac{2\pi\sigma zr'dr'}{\left(r'^2 + z^2\right)^{3/2}}$
(6) Integrate to get E	$E_y = k_e\lambda y\int_{-\ell/2}^{+\ell/2} \frac{dx}{\left(x^2 + y^2\right)^{3/2}}$ $= \frac{2k_e\lambda}{y} \frac{\ell/2}{\sqrt{(\ell/2)^2 + y^2}}$	$E_z = k_e \frac{R\lambda z}{\left(R^2 + z^2\right)^{3/2}} \oint d\phi'$ $= k_e \frac{(2\pi R\lambda)z}{\left(R^2 + z^2\right)^{3/2}}$ $k_e \frac{Qz}{\left(R^2 + z^2\right)^{3/2}}$	$E_z = 2\pi\sigma k_e z\int_0^R \frac{r'dr'}{\left(r'^2 + z^2\right)^{3/2}}$ $= 2\pi\sigma k_e \left(\frac{z}{\lvert z\rvert} - \frac{z}{\sqrt{z^2 + R^2}}\right)$

System	Infinite line of charge	Infinite plane of charge	Uniformly Charged solid sphere
Figure	 **Figure 18.49**	 **Figure 18.50**	 **Figure 18.51**
Identify the symmetry	Cylindrical	Planar	Spherical
Determine the direction of $\vec{E}$	 **Figure 18.52**	 **Figure 18.53**	 **Figure 18.54**
Divide the space into different regions	r>0	Z >0 and z<0	$r \le a$ and $r \ge a$

Choose Gaussian surface	 Figure 18.55	 Figure 18.56	 Figure 18.57
Calculate electric flux	$\Phi_E = E(2\pi rl)$	$\Phi_E = EA + EA = 2EA$	$\Phi_E = E(4\pi r^2)$
Calculate enclosed charge q_{in}	$q_{enc} = \lambda l$	$q_{enc} = \sigma A$	$q_{enc} = \begin{cases} Q^{(r/a)^3} & r \le a \\ Q & r \ge a \end{cases}$
Apply Gauss's law $\Phi_E = q_{in} / \varepsilon_0$ to find E	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$	$E = \dfrac{\sigma}{2\varepsilon_0}$	$E = \begin{cases} \dfrac{Qr}{4\pi\varepsilon_0 a^3}, & r \le a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r \ge a \end{cases}$

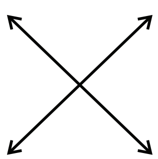
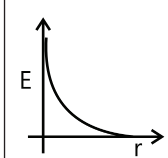
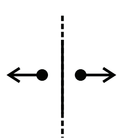
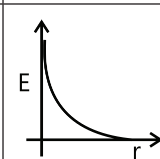
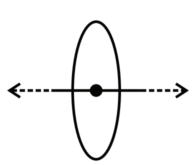
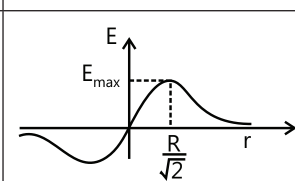
FORMULAE SHEET

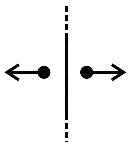
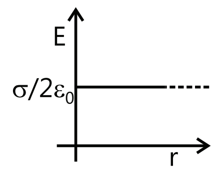
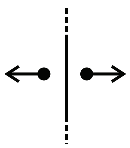
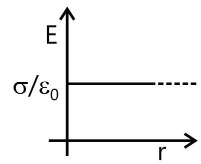
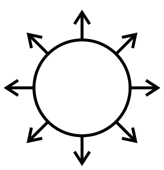
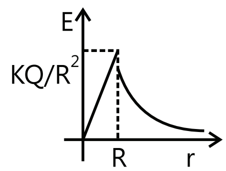
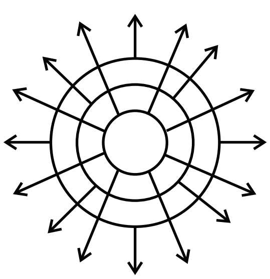
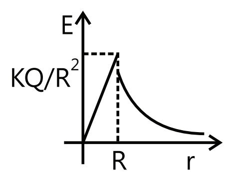
Electric Charges, Forces and Fields

S. No	Term	Description
1	Charge	Charges are of two types (a) Positive charge (b) Negative charge Like charges repel each other and unlike charges attract each other.
2	Properties of charge	1. Quantization:-q=ne where n=0, 1, 2...... and e is charge of an electron. 2. Additive: $-q_{net} = \sum q$ 3. Conservation: - total charge of an isolated system is constant
3	Coulomb's law	The mutual electrostatic force between the charges q_1 and q_2 separated by a distance r is given by Force on the charge q_1 $F_1 = Kq_1q_2r_{12} / r^2$ Where $\hat{r}_{12}$ is the unit vector in the direction from q_2 and q_1 . For more than two charges in the system, the force acting on any charge is vector sum of the coulomb force from each of the other charges. This is called principle of superposition for q_1 , q_2 , q_3 ..... q_n Charges are present in the system.

S. No	Term	Description
4	Electric Field	-The region around a particular charge in which its electrical effects can be observed is called the electric field of the charge -Electric field has its own existence and is present even if there is no charge to experience the electric force.
5	Electric field Intensity	$E=F/q_0$ Where F is the electric force experienced by the test charge q_0 at this point. It is a vector quantity. Some points to note on this 1. Electric field lines extend away from the positive charge and towards the negative charge. 2. Electric field produces the force so if a charge q is placed in the electric field E, the force experienced by the charge is F=qE 3. Principle of superposition also applies to electric field so $E = E_1 + E_2 + E_3 + E_4 + ...$ Electric field intensity due to point charge $\vec{E} = \frac{KQ\,\vec{r}}{r^2}$ Where r is the distance from the point charge and r is the unit vector along the direction from source to point.

Electric Field Intensities due to various Charge Distributions

Name/Type	Formula	Note	Graph
Point Charge 	$\frac{Kq}{\lvert\vec{r}\rvert^2}\hat{r} = \frac{Kq}{r^3}\vec{r}$	• q is source charge • $\vec{r}$ is vector drawn from source charge to the test point.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda}{r}\hat{r}$	• λ is linear charge density (assumed uniform) • r is perpendicular distance of point from line charge • $\hat{r}$ is radial unit vector drawn from the charge to test point	
Uniformly Charged Ring 	$E = \frac{KQx}{(R^2 + X^2)^{3/2}}$ $E_{centre} = 0$	• Q is total charge of the ring • x=distance of point on the axis from centre of the ring. • Electric field is always along the axis.	

Infinitely large non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	• σ is surface charge density (assumed uniform) • $\hat{n}$ is unit normal vector • Electric field intensity is independent of distance	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	• σ is surface charge density (assumed uniform) • $\hat{n}$ is unit normal vector • Electric field intensity is independent of distance	
Uniformly charged hollow conducting/non conducting sphere or solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \frac{KQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r<R$ $\vec{E} = 0$	• R is radius of the sphere • $\vec{r}$ is vector drawn from centre of the sphere to the test point. • Sphere acts like a point charge placed at the centre for point outside the sphere. • $\vec{E}$ is always along radial direction. • Q is total charge $(=\sigma 4\pi R^2)$. (σ = Surface charge density)	
Uniformly charged solid non conducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \frac{KQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{\rho}{3\varepsilon_0}\vec{r}$	• $\vec{r}$ is vector drawn from centre of the sphere to the test point. • Sphere acts like a point charge placed at the centre for points outside the sphere. • $\vec{E}$ is always along radial direction • Q is total charge $(=\rho\frac{4}{3}\pi R^3)$. ($\rho$ =volume charge density) • Inside the sphere $E \propto r$ • Outside the sphere $E \propto 1/r^2$	

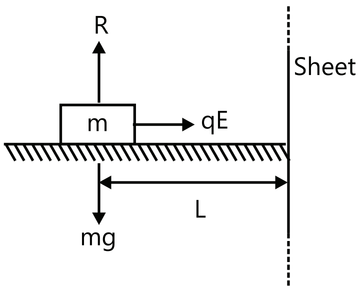
Note: (i) Net charge on a conductor remains only on the outer surface of a conductor.

(ii) On the surface of spherical conductors charge is uniformly distributed.

Solved Examples

JEE Main/Boards

Example 1: A block having mass m and charge –q is resting on a frictionless plane at a distance L from fixed large non-conducting infinite sheet of uniform charge density s as shown in figure. Discuss the motion of the block assuming that collision of the block with the sheet is perfectly elastic. Is it SHM?



Sol: Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration.

Acceleration will be uniform because electric field E due to the sheet is uniform

$a=\frac{F}{m}=\frac{qE}{m}$, where $E=\sigma/2\varepsilon_0$

As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$L=\frac{1}{2}at^2$ i.e., $t=\sqrt{\frac{2L}{a}}=\sqrt{\frac{2mL}{aE}}=\sqrt{\frac{4mL\varepsilon_0}{a\sigma}}$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance L in same time t. After stopping it will be again accelerated towards the wall and so the block will execute oscillatory motion with 'span' L and time period.

$T=2t=2\sqrt{\frac{2mL}{aE}}=2\sqrt{\frac{4mL\varepsilon_0}{a\sigma}}$

However, as the restoring force F=qE is constant and not proportional to displacement x, the motion is not simple harmonic.

Example 2: How many electrons must be given to a neutral body so that it could acquire a charge of 4.0 pC?

Sol: Formula based.

On giving electrons, body acquires-ve charge and to acquire a net charge of 4pc

$q=4x10^{-12}\,C$

$e=1.6x10^{-19}\,C$

$\Rightarrow n=\frac{q}{e}=\frac{4x10^{-12}}{1.6x10^{-19}}=2.5x10^7$

$2.5x10^7$ electrons will have to be given.

Example 3: What is the value of charge on a body if it has an excess of 1.5 x 10^7 electrons?

Sol: Electrons are negatively charged

$n=1.5x10^7$ and the body has excess of electrons

$\Rightarrow$ it is- vely charged and charge on it is q=ne

$\Rightarrow q=1.5x10^7x1.6x10^{-19}\,C$

$\Rightarrow q=2.4\,pC$

Example 4: When 10^{22} electrons are removed from a neutral metal sphere, what is the charge on the sphere?

Sol: Loss of electrons make a body positively charged.

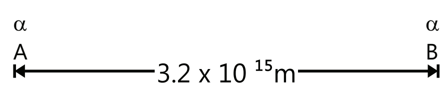
On removing electrons, body acquires +ve charge and its value is

$q=ne=10^{22}\,x1.6x10^{-19}$ =1600 coulomb.

Example 5: Calculate the coulomb force between two α-particles separated by a distance of $3.2x10^{-15}$ m.

Sol: Charge on α-particle

We have $q_\alpha=+2e=3.2x10^{-19}\,C$



$F=\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\alpha}{r^2}=9x10^9x\frac{3.2x10^{-19}x3.2x10^{-19}}{3.2x3.2x10^{-30}}$

=90 N (repulsive)

Example 6: Consider two identical spheres P and Q with charge q on each. A third sphere R of the same size but uncharged is successively brought in contact with the two spheres. What is the new force of repulsion between P and Q?

Sol: Charge on two spheres will be equally divided on two sphere each times on touching.

When R is kept in contact with R, charge q is equally distributed between P and R.

Charge on $P = \frac{q}{2}$

Charge on $R = \frac{q}{2}$

When R is kept in contact with Q, total charge will again be equally distributed.

Charge on $Q = \frac{q + (q/2)}{2} = \frac{3q}{4}$

Charge on $R = \frac{3q}{4}$

Initial force of repulsion between P and Q

$$F(say)\frac{1}{4\pi\varepsilon_0}\left(\frac{qxq}{r^2}\right)$$

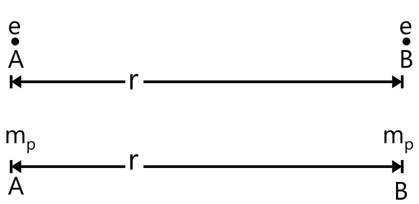
Final force of repulsion between P and Q

$$F^1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{\frac{q}{2}x\frac{3q}{4}}{r^2}\right) = \frac{3}{8}F$$

Example 7: Compare the electrostatic force and gravitational force taking two protons.

Sol: Simply apply the formula for Gravitational and Electrostatic force

$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}; F_g = G\frac{m_p^2}{r^2}$; Mass of proton = 1.67×10^{-27}kg



$$\Rightarrow \frac{F_e}{F_g} = 1.24 x 10^{36}$$

Example 8: A charge Q is to be divided on two objects. What should be the value of the charges on the two objects, so that the force between them can be maximum?

Sol: If a + b constant, than a x b is maximum when a = b.

Let the charges divided on the two objects be q and Q-q so that the force between them is $f = K\frac{q(Q-q)}{r^2}$

For maximum force, $\frac{dF}{dq} = 0$

$$\frac{d}{dq}\left[K\frac{q(Q-q)}{r^2}\right] = 0$$

$$\Rightarrow \frac{K}{r^2}\frac{d}{dq}[q(Q-q)] = 0$$

$$\Rightarrow \frac{d}{dq}[qQ - q^2] = 0$$

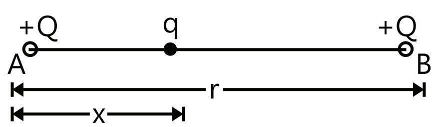
$$\Rightarrow Q - 2q = 0 \quad \Rightarrow q = \frac{Q}{2}$$

i.e, the charge must be equally divided.

Example 9: Two identical point charges of magnitude Q are kept at a distance r from each other. A third point charge q is placed on the line joining the above two charges, such that all the three charges are in equilibrium. What is the sign, magnitude and position of the third charge?

Sol: For equilibrium, net F on each charge = 0

Let identical charges Q be placed at A and B and another charge q is at a distance x from A so that it is in equilibrium.



$\therefore$ Force on q due to charge at A in the + X direction $= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2}$ and force on a due to charge at B in the-X direction $= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(r-x)^2}$

For equilibrium, these two forces must be equal i.e., $\frac{1}{x^2} = \frac{1}{(r-x)^2}$ or $x = \frac{r}{2}$ If q was a negative charge, the direction of force due to q at B would be in-X and at A in +X direction.

But, if all the three charges are of same nature, there would be repulsion between charges at A and B also. Hence to have equilibrium among three charges, Q must be opposite of q so that force of attraction between Q and q=force of repulsion between Q and q.

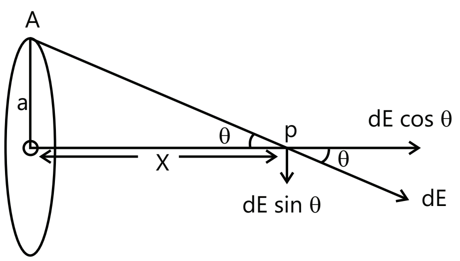
i.e.. $\dfrac{Q^2}{4\pi\varepsilon_0 r^2} = \dfrac{Oq}{4\pi\varepsilon_0 r^2} = \dfrac{Qq}{4\pi\varepsilon_0\left(\dfrac{r}{2}\right)^2}$

$\therefore q = \dfrac{Q}{4}$

Example 10: A charge Q is uniformly distributed on the circumference of a circular ring of radius a. Find the intensity of electric field at a point at a distance x from the center on the axis of ring.

Sol: Consider a small part of the ring. All points on the ring are symmetrical to any point on the axis of the ring. Given situation is depicted in the figure. Consider an infinitesimal element at point A on the circumference of the ring. Let charge on this element be dq. The magnitude of the intensity of electric field $d\vec{E}$ at a point P situated at a distance x from the center on its axis is,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{AP^2} = k\frac{dq}{(a^2+x^2)}.$$



Its direction is from A to P. Now consider two components of $d\vec{E}$ (i) dE $\sin\theta$, parallel to the axis of the ring and (ii) dE $\cos\theta$, parallel to the axis.

Here it is clear that in the vector sum of intensities due to all such elements taken all over the circumference, the dE $\sin\theta$ components of the diametrically opposite elements will cancel each other as they are mutually opposite. Hence only dE$\cos\theta$ components should be considered for integration.

$\therefore$ The total intensity of electric field at point P,

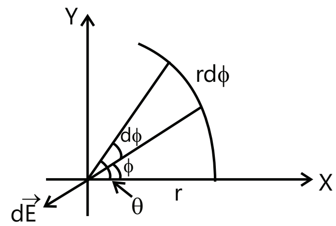
$$= \int dE\cos\theta = \int k\frac{dq}{(a^2+x^2)}\frac{OP}{AP} \quad E = k\int\frac{dq}{(a^2+x^2)}\frac{x}{(a^2+x^2)^{\frac{1}{2}}}$$

$$\therefore E = k\frac{x}{(a^2+x^2)^{\frac{3}{2}}}\int_{surface} dq = \frac{dxQ}{(a^2+x^2)^{\frac{3}{2}}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(a^2+x^2)^{\frac{3}{2}}}$$

JEE Advanced/Boards

Example 1: An arc of radius r subtends an angle θ at the center with x-axis in a Cartesian coordinate system. A charge is distributed over the arc such that the linear charge density is λ. Calculate the electric field at the origin.



Sol: Consider small element on the arc as point charge and then proceed by integrating for all such points. The electric charge distributed on the portion of the arc making an angle $d\phi$ is dQ= $\lambda r d\phi$. The electric field produced due to this portion at the origin will be, $dE = \dfrac{k\lambda r.d\phi}{r^2}$ The electric field vector $d\vec{E}$ of this portion of the arc is indicated in the diagram.

$d\vec{E}$ has two components

$$dE_x = -\frac{k\lambda r d\phi}{r^2}\cos\phi\ \hat{i} \text{ and } dE_y = -\frac{k\lambda r.d\phi}{r^2}\cos\phi\ \hat{j}$$

$$\therefore \overrightarrow{E_x} = \frac{k\lambda}{r}\int_0^0 \cos\phi d\phi\,\hat{i} = -\frac{k\lambda}{r}[\sin\phi]_0^0\ \hat{i}\theta$$

$$\therefore \overrightarrow{E_x} = -\frac{k\lambda}{r}\sin\phi\,\hat{i}\ (\theta \text{ not } \phi)$$

$$\text{Now, } \overrightarrow{E_y} = \frac{k\lambda}{r}\int\sin\phi d\phi\,\hat{j} = \frac{k\lambda}{r}[\cos\phi]_0^0\ \hat{j}\theta$$

$$\therefore \overrightarrow{E_y} = \frac{k\lambda}{r}[(\cos\theta - 1)\ \hat{j}]$$

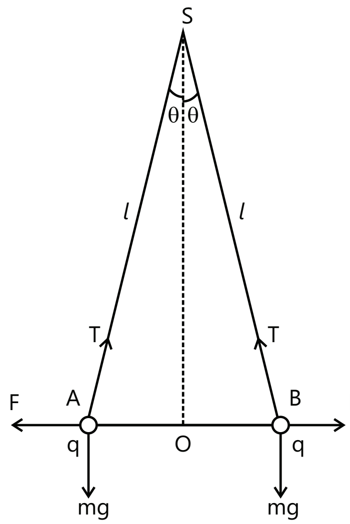
$$\therefore \overrightarrow{E_y} = \frac{k\lambda}{r}[(1-\sin\theta)\ \hat{i} + (\cos\theta - 1)\ \hat{j}]$$

$\left(\hat{i} \text{ component is just - } \sin\theta\right)$

Example 2: Two small spheres each having mass m kg and charge q coulomb are suspended from a point by insulating threads each 1 metre long but of negligible mass. If θ is the angle each string makes with the vertical when equilibrium has been attained, show that $q^2 = (4mgl^2\sin^2\theta\tan\theta)4\pi\varepsilon_{0.}$

Sol: Gravitational as well as electrostatic force act on each sphere.

Consider two small spheres A and B each of mass m kg and charge q coulomb. When the two spheres are suspended from point S by two threads each of length l, they repel each other and when equilibrium is attained, each string makes an angle θ with the vertical [See figure.].



Each of the two spheres is acted upon by the following three forces:

(i) The electrostatic force of repulsion f directed away from each other.

(ii) The weight mg of the sphere acting vertically downwards.

(iii) The tension T in the string directed towards point S.

Since the two spheres are in equilibrium, the three forces acting on a sphere can be represented by the three sides of the ΔAOS taken in order. For sphere A, we have at equilibrium by Lami's theorem

$$\frac{F}{OA}=\frac{mg}{SO}=\frac{T}{AS} \quad \text{... (i)}$$

Here, $OA = l\sin\theta$; $SO = l\cos\theta$ And $AB = 2AO = 2l\sin\theta$

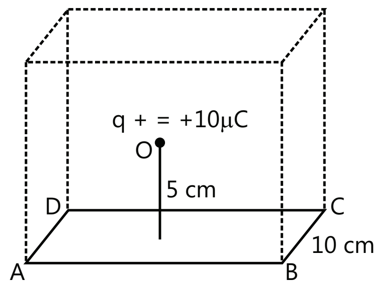
and $F=\frac{1}{4\pi\varepsilon_0}\cdot\frac{qxq}{AB^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{4l^2\sin^2\theta}$

From equation (i), we have $F = mgx\frac{OA}{SO}$

or $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{4l^2\sin^2\theta}=mgx\frac{l\sin\theta}{l\cos\theta}$

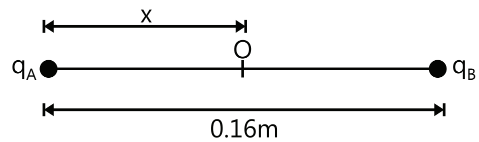
or $q^2 = (4mgl^2\sin^2\theta\tan\theta)4\pi\varepsilon_0$

Example 3: A point charge + 10μC is at a distance 5 cm directly above the center of a square of side 10 cm as shown in Fig. What is the magnitude of the electric flux through the square?

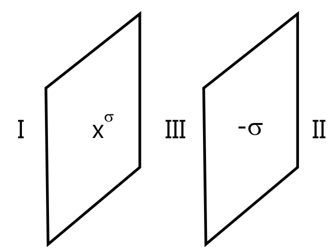


Sol: Charge is symmetric to all faces of the cube, hence by symmetry each face would have equal flux passing though it.

Here, $q = +10\mu C = 10^{-5}C$



Consider that the charge q is at a distance of 5cm from the square ABCD of each side 10 cm [figure]. The square ABCD can be considered as one of the six faces of a cube of each side 10cm. Then, according to Gauss's theorem, total electric flux through all the six faces of the cube, $\phi=\frac{q}{\varepsilon_0}$



Obviously, the flux through the square ABCD will be

$$\phi=\frac{1}{6}x\phi=\frac{1}{6}x\frac{q}{\varepsilon_0}$$

$$=\frac{1}{6}x\frac{10^{-5}}{8.854x10^{-12}}=1.88x10^5\ N\,m^2\,C^{-1}$$

Example 4: Two large thin metal plates are parallel and close to each other as shown in the figure. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0 $\times$ 10^{-22} Cm^{-2}. What is E (i) to the left of the plates, (ii) to the right of the plates and (iii) between the plates?

Sol: Apply formula for Electric field intensity due to charged plate.

Here $\sigma = 17.0x10^{-22}\,Cm^{-2}$

(i) To the left of plates: The region I is to the left of the plates. Therefore, the electric field to the left of plates is zero.

(ii) To the right of plates: The region II is to the right of the plates. Again, the electric field in the region II is zero.

(iii) Between the two plates, the electric field given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{17.0x10^{-22}}{8.854x10^{-12}} = 1.92x10^{-10}\ NC^{-1}$$

Example 5: A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 107 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionization.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Sol: Maximum field strength should be 10% of the dielectric strength of the material,.

10% of the given field i.e. $10^7\ Cm^{-1}$

Given $E = 0.1x10^7\ Cm^{-1}$

Using $E = -\frac{dV}{dr}$ i.e. $E = \frac{V}{r}$, we get

$$r = \frac{V}{E} = \frac{1000}{0.1x10^7} = 10^{-3}m$$

Using $C = \frac{\epsilon_0 \epsilon_r A}{d}$, we get

$$A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{Cr}{\epsilon_0 \epsilon_r} = \frac{(450x10^{-12})(10^{-3})}{8.854x10^{-12}x3} = 19\ cm^2.$$

Example 6: The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge-0.8μC in air is 0.2 N.

(i) What is the distance between the two spheres?

(ii) What is the force on the second sphere due to the first?

Sol: Consider each sphere as a point change and apply Coulomb's law.

(i) Force on charge 1 due to charge 2 is given by the relation

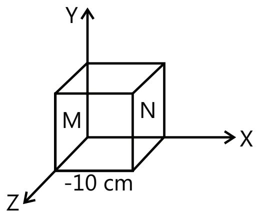
$$F_{12} = 9x10^9 \frac{q_1 q_2}{r^2}$$

$$\Rightarrow r^2 = \frac{F_{12}}{(9x10^9)q_1 q_2} = \frac{0.2}{(9x10^9)(0.8x10^{-6})(0.4x10^{-6})}$$

i.e. r=0.12m

(ii) $F_{12} = F_{12} = 0.2$ N, Attractive $F_{21}=F_{12}$.

Example 7: Electric field in the above figure is directed along + x direction and given by $E_x = 5Ax + 2B$, where E is in NC^{-1} and x is in meter. A and B are constants with dimensions.



Taking $A = 10NC^{-1}\ m^{-1}$ and $B = 5\ NC^{-1}$, Calculate

(i) The electric flux through the cube.

(ii) Net charge enclosed within the cube.

Sol: Vector rotation of area and Gauss's Law for net enclosed charged is applied.

(i) Given $E_x = 5Ax + 2B$. The electric field at face M where x=0 is $E_1 = 2B$ The electric field at face N where x = 10cm=0.010m is E_2 =5A x 0.10+ 2B= 0.5A+2B

The electric flux through face M is

$$\phi_1 = \vec{E_1}.S_1 = E_1 S_1\ \cos\pi = -E_1 S_1$$

$= -2Bxl^2$ where 1=10 cm= 0.01 m

The electric flux through face N

$$\phi_2 = \vec{E_2} \cdot S_2 = E_2 S_1\ \cos 0 = (0.5A + 2B)l^2$$

Net electric flux $\phi = \phi_1 + \phi_2$

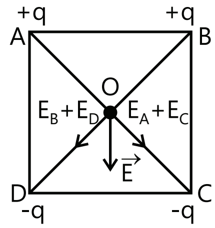
$$= -2Bl^2 + (0.5A + 2B)l^2 = 0.5Al^2$$

$$= 0.5x10x(0.10)^2 = 5x10^{-2}\ Vm$$

(ii) If θ is net charge enclosed within the cube, then by Gauss's theorem $\phi = \frac{1}{\varepsilon_0} q$

$$\phi = \varepsilon_0 \phi = 8.85x10^{-12}x5x10^{-2}C = 4.425x10^{-13}\ C$$

Example 8: Four electric charges, +q,+q, -q and –q are respectively placed on the vertices A, B, C and D of square. The length of the square is a. Calculate the intensity of the resultant electric field at the center.



Sol: Apply Superposition of electrostatic forces.

All the electric charges are equidistant from the center

O. If r is the distance of vertices from the center, we have, $E_A = E_B = E_C = E_D = \frac{kq}{r^2}$

The directions of these electric fields are as shown in figure.

If E′ is the resultant field of E_B and E_D

$$E' = E_B + E_D = 2\frac{kq}{r^2}$$

E is the resultant of E′ and E″. It is evident from the geometry of the figure that,

$E^2 = E'^2 + E''^2 = \frac{8k^2q^2}{r^4}$ Using

$r = \frac{a}{\sqrt{2}}, E = 4\sqrt{2}k\frac{q}{a^2}$

JEE Main/Boards

Exercise 1

Q.1 Electrostatic force between two charges is called central force. Why?

Q.2 In Coulomb's law, on what factors the value of electrostatic force constant k depends?

Q.3 Define dielectric constant of a medium.

Q.4 Dielectric constant of water is 80. What is its permittivity?

Q.5 State the principle of superposition of forces in electrostatics.

Q.6 How many electrons must be removed from a conductor, so that it acquires a charge of 3.5nC?

Q.7 A point charge of 10^{-7} coulomb is situated at the center of a cube of 1 m side. Calculate the electric flux through its surface.

Q.8 Find the electric flux through each face of a hollow cube of side 10 cm, if a charge of 8.854 μC is placed at its center.

Q.9 What is the force between two small charged spheres having charges of 2 x 10^{-7} C and 3 x 10^{-7} C placed 30 cm apart in air?

Q.10 A polythene piece rubbed with wool is found to have a negative charge of 3 x 10^{-7} C.

(i) Estimate the number of electrons transferred (from which to which?)

(ii) Is there a transfer of mass from wool to polythene?

Q.11 Give two properties of electric lines of force. Sketch them for an isolated positive charge.

Q.12 An infinite line charge produces a field of 9 x 10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

Q.13 Calculate the Coulomb's force between a proton and electron separated by 0.8 x 10^{-15} m.

Q.14 If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Q.15 Which is bigger, a coulomb or charge on an electron? How many electronic charge form one coulomb of charge?

Q.16 What is the amount of charge possessed by 1kg of electrons? Given that mass of an electron is 9.1 x 10^{-31} kg.

Q.17 Four charges +q, ..+q,-q,-q are placed respectively at the four corners of a square of side a. Find the magnitude and direction of the electric field at the center of the square.

Q.18 Four point charges $q_A = 2\mu C$, $q_B = -5\mu C$, $q_C = 2\mu C$ and $q_D = 5\mu C$ are located at corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the center of the square?

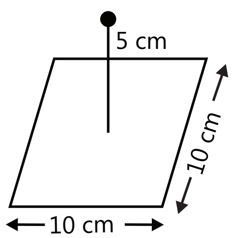
Q.19 Two point charges $q_A = 3\mu C$ and $q_B = 3\mu C$ are located 20 cm apart in vacuum.

(i) What is the electric field at the midpoint O of the line AB joining the two charges?

(ii) If a negative test charge of magnitude 1.5 x 10^{-9} C is placed at this point, what is the force experienced by the test charge?

Q.20 Consider a uniform electric field E=3 x 10^3 $\hat{i}$ N/C. (i) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (ii) What is the flux through the same square if the normal to its plane makes a 60^o angle with the x-axis?

Q.21 A point charge $+10\,\mu C$ is at a distance of 5 cm directly above the center of a square of side 10 cm, as shown in figure What is the magnitude of the electric flux through the square?



Q.22 Show that the electric field at the surface of a charged conductor is given by $\bar{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

Q.23 A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 g Let us now take two pieces of copper weighing 10 g. Let us transfer one electron from one piece. What will be the Coulomb force between the two pieces after the transfer of electrons, if they are 1 cm apart? Avogadro number = 6×1023 C mol^{-1}, charge on an electron = 1.6×10^{-19} C.

Q.24 Two fixed point charged 4Q and 2Q are separated by a distance x. Where a third point charge q should be placed for it to be in equilibrium?

Q.25 It is required to hold four equal point charges +q in equilibrium at the corners of a square. Find the point charge that will do this, if placed at the center of the square.

Q.26 Four point charges, each having a charge q are placed on the four corners A, B, C and D of a regular pentagon ABCEDE. The distance of each corner from the center is a. Find the electric field at the center of the pentagon.

Q.27 Define electric flux, Write its S.I. unit, A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface charge?

Q.28 A positive point charge (+q) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines originating from the point on to the surface of the plate.

Derive the expression of the electric field at the surface of a charged conductor.

Exercise 2

Single Correct Question

Q.1 A point charge $50\mu C$ is located in the XY plane at the point of position vector $\bar{r_0} = 2\hat{i} + 3\hat{j}$ what is the electric field at the point of position vector $\bar{r} = 8\hat{i} + 5\hat{j}$.

(A) 1200V/m (B) 0.04V/m

(C) 900V/m (D) 4500V/m

Q.2 A point charge q is placed at origin. Let E_A, E_B and E_C be the electric field at three points A (1, 2, 3), B (1, 1,-1) and C (2, 2, 2) due to charge q Then

[i] $E_A \perp E_B$ [ii] $E_B| = 4|E_C|$ Select the correct alternative

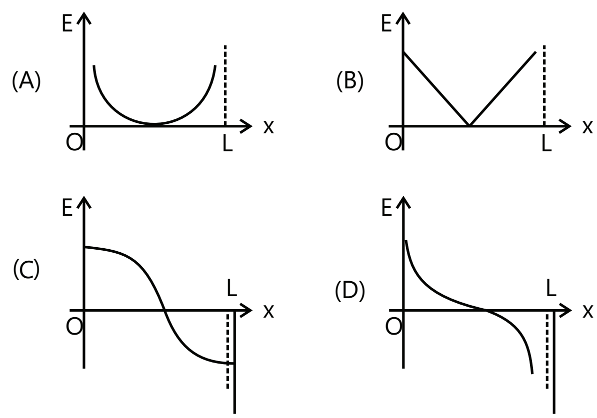
(A) Only [i] is correct

(B) (B) only [ii] is correct

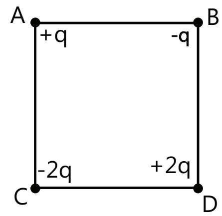
(C) Both [i] and [ii] are correct

(D) (D) both [i] and [ii] are wrong

Q.3 Two identical point charges are placed at a separation of l. P is a point on the line joining the charges, at a distance x from any one charge, The field at P is E. E is plotted against x for values of x from close to zero to slightly less than l. Which of the following best represents the resulting curve?



Q.4 Four charges are arranged at the corners of a square ABCD, as shown. The force on a +ve charge kept at the center of the square is



(A) Zero

(B) Along diagonal AC

(C) Along diagonal BD

(D) Perpendicular to the side AB

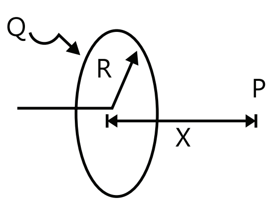
Q.5 Two free positive charges 4q and q are a distance l apart. What charge Q is needed to achieve equilibrium for the entire system and where should it be placed from charge q ?

(A) $Q = \frac{4}{9}q(\text{negative})\,\text{at}\,\frac{l}{3}$

(B) $Q = \frac{4}{9}q(\text{positive})\,\text{at}\,\frac{l}{3}$ (C) $Q = q(\text{positive})\,\text{at}\,\frac{l}{3}$

(D) $Q = q(\text{negative})\,\text{at}\,\frac{l}{3}$

Q.6 A small particle of mass m and charge –q is placed at point P on the axis of uniformly charged ring and released. If R >> x, the particle will undergo oscillation along the axis of symmetry with an angular frequency that is equal to



(A) $\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$ (B) $\frac{\sqrt{qQx}}{4\pi\varepsilon_0 mR^4}$

(C) $\frac{qQ}{4\pi\varepsilon_0 mR^3}$ (D) $\frac{qQx}{4\pi\varepsilon_0 mR^4}$

Q.7 Which of the following is a volt:

(A) Erg per cm

(B) Joule per coulomb

(C) Erg per ampere

(D) Newton/(Coulomb x m^2)

Q.8 A charged particle having some mass is resting in equilibrium at a height H above the center of a uniformly charged non-conducting horizontal ring of radius R. The force of gravity acts downwards. The equilibrium of the particle will be stable

(A) for all values of H (B) only if $H > \frac{R}{\sqrt{2}}$

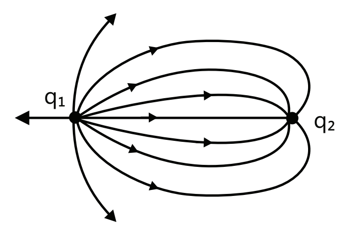
(C) only if $H < \frac{R}{\sqrt{2}}$ (D) only if $H = \frac{R}{\sqrt{2}}$

Q.9 Point P lies on the axis of a dipole. If the dipole is rotated by 90^o anti-clock wise, the electric field vector E at P will rotate by

(A) 90^o Clock wise (B) 180^o

(C) 90^o Anti clock wise (D) no ne

Q.10 The Fig. shows the electric field lines in the vicinity of two point charges. Which one of the following statements concerning this situation is true?



(A) q_1 is negative and q_2 is positive

(B) The magnitude of the ratio (q_2 / q_1) is less than one

(C) Both q_1 and q_2 have the same sign of charge

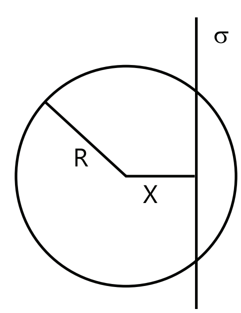
(D) The electric field is strongest midway between the charges.

Q.11 Electric flux through a surface of area 100 m^2 lying in the xy plane is (in V-m) if $E = \hat{i} + \sqrt{2}\hat{j} + \sqrt{3}\hat{k}$

(A) 100 (B) 141.4

(C) 173.2 (D) 200

Q.12 An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the Fig. 18.80. The electric flux Φ through the Gaussian surface is



(A) $\frac{\pi R^2 \sigma}{\varepsilon_0}$ (B) $\frac{2\pi(R^2 - x^2)}{\sigma\ \varepsilon_0}$

(C) $\frac{\pi(R - x)^2}{\sigma\varepsilon_0}$ (D) $\frac{\pi(R^2 - x^2)^2\sigma}{\varepsilon_0}$

Q.13 Two identical small conducting spheres, having charges of opposite sign, attract each other with a force of 0.108 N when separated by 0.5 m. The spheres are connected by a conducting wire, which is then removed, and thereafter, they repel each other with a force of 0.036 N. The initial charges on the spheres are

(A) $\pm 5 \times 10^{-6}$ C and $\mp 15 \times 10^{-6}$ C

(B) $\pm 1.0 \times 10^{-6}$ C and $\mp 3.0 \times 10^{-6}$ C

(C) $\pm 2.0 \times 10^{-6}$ C and $\mp 6.0 \times 10^{-6}$ C

(D) $\pm 0.5 \times 10^{-6}$ C and $\mp 1.5 \times 10^{-6}$ C

Previous Years' Questions

Q.1 An alpha particle of energy 5 MeV is scattered through 180^o by a fixed uranium nucleus. The distance of closest approach is of the order of ***(1981)***

(A) l Å (B) 10^{-10} cm

(C) 10^{-12} cm (D) 10^{-15} cm

Q.2 Two equal negative charges – q are fixed at points (0,-a) and (0,a) on y-axis. A positive charge Q is released from rest at the point (2a, 0) on the x-axis. The charge Q will ***(1984)***

(A) Execute simple harmonic motion about the origin

(B) Move to the origin and remain at rest

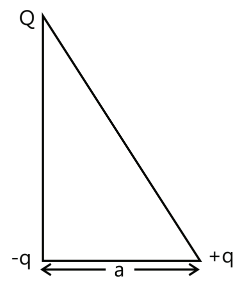
(C) Move to infinity

(D) Execute oscillatory but not simple harmonic motion

Q.3 A charge q is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to ***(1987)***

(A) $-\frac{Q}{2}$ (B) $-\frac{Q}{4}$ (C) $+\frac{Q}{4}$ (D) $+\frac{Q}{2}$

Q.4 The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor ***(1996)***



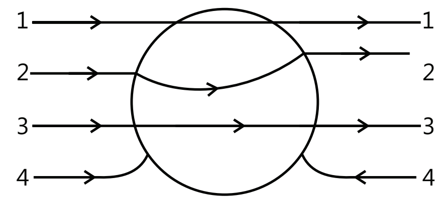
(A) Is same throughout

(B) Is higher near the outer cylinder than near the inner cylinder

(C) Varies as 1/r where r is the distance from the axis

(D) Varies as $1/r^2$ where r is the distance from the axis

Q.5 A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as ***(1996)***



(A) 1 (B) 2 (C)3 (D) 4

Q.6 An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to. ***(1997)***

(A) 1 (B) $(m_p/m_e)^{1/2}$

(C) $(m_e/m_p)^{1/2}$ (D) 1836

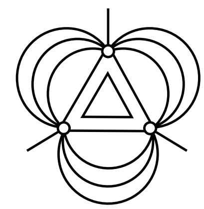
Q.7 A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{t=\infty}^{t=0} -\vec{E}\cdot\vec{dl}$ (l=0 being center of the ring) in volt is ***(1997)***

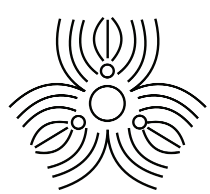
(A) +2 (B) -1 (C) -2 (D) zero

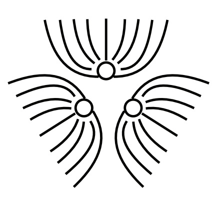
Q.8 Three charges Q, +q and +q are placed at the vertices of a right angle triangle (isosceles triangle) as shown. The net electrostatic energy of the configuration is zero, if Q is equal to ***(2000)***

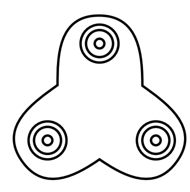
(A) $\frac{-q}{1+\sqrt{2}}$ (B) $\frac{-2q}{2+\sqrt{2}}$ (C)-2q (D) +q

Q.9 Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in ***(2001)***

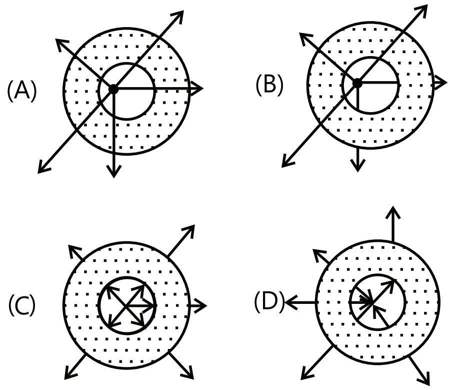
(A)


(B)


(C)


(D)


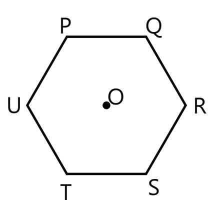
Q.10 A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represent the electric lines of force? ***(2003)***



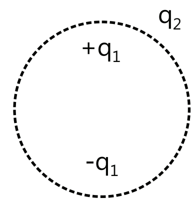
Q.11 Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charge is possible for, P, Q, R, S, T and U respectively? ***(2004)***

(A) +, -, +, -, -,+ (B) +, -, +, -, +, -

(C) +, +, -, +, -, - (D) -, +, +, -, +, -



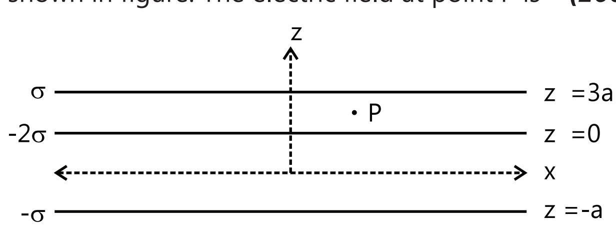
Q.12 Consider the charge configuration and a spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to ***(2004)***



(A) q_2 (B) Only the positive charges

(C) All the charges (D) $+q_1$ and $-q_1$

Q.13 Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is ***(2005)***



(A) $\frac{2\sigma}{\varepsilon_0}\hat{k}$ (B) $-\frac{2\sigma}{\varepsilon_0}\hat{k}$ (C) $\frac{4\sigma}{\varepsilon_0}\hat{k}$ (D) $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

Q.14 Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then ***(2007)***

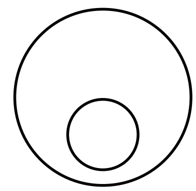
(A) Negative and distributed uniformly over the surface of the sphere

(B) Negative and appears only at the point on the sphere closest to the point charge

(C) Negative and distributed non-uniformly over the entire surface of the sphere

(D) zero

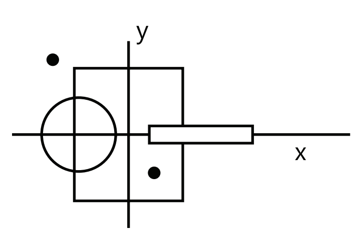
Q.15 A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is ***(2007)***



(A) Zero everywhere (B) Non-zero and uniform

(C) Non-uniform (D) Zero only at its center

Q.16 A disk of radius $\frac{a}{4}$ having a uniformly distributed charge 6C and 6C is placed in the x-y plane with its center at $\left(\frac{-a}{2},0,0\right)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from $x=\frac{a}{4}$ to $x=\frac{5a}{4}$. Two point charges -7C and 3C are placed at $\left(\frac{a}{4},\frac{-a}{4},0\right)$ and $\left(\frac{-3a}{4},\frac{3a}{4},0\right)$. Respectively. Consider a cubical surface formed by six surfaces $x=\pm\frac{a}{2}$, $y=\pm\frac{a}{2}$, $z=\pm\frac{a}{2}$. The electric flux through this cubical surface is ***(2009)***



(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$ (C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

Q.17 Three concentric metallic spherical shells of radii R, 2R and 3R are given charges Q_1, Q_2 and Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is ***(2009)***

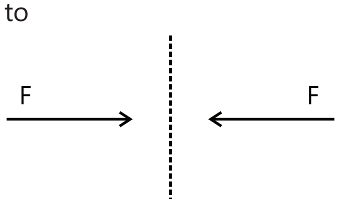
(A) 1:2:3 (B) 1:3:5 (C) 1:4:9 (D) 1:8:18

Q.18 A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7} \times 10^5\ Vm^{-1}$, When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}\ ms^{-1}$. Given $g = 9.8 ms^{-2}$, viscosity of the air $= 1.8 \times 10^{-5}\ Ns\,m^{-2}$ and the density of oil = 900 kg m^{-3}, the magnitude of q is ***(2010)***

(A) 1.6×10^{-19}C (B) 3.2×10^{-19}C

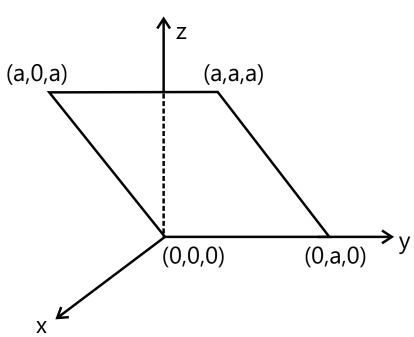
(C) 4.8×10^{-19}C (D) 8.0×10^{-19}C

Q.19 A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to ***(2010)***



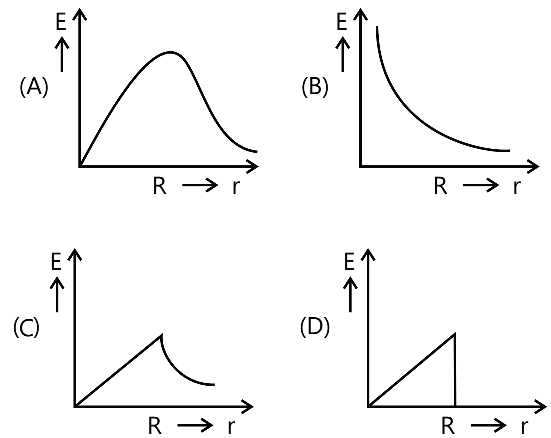
(A) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2 R$ (C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

Q.20 Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is ***(2011)***



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$ (C) E_0a^2 (D) $\frac{E_0a^2}{\sqrt{2}}$

Q.21 In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be ***(2012)***

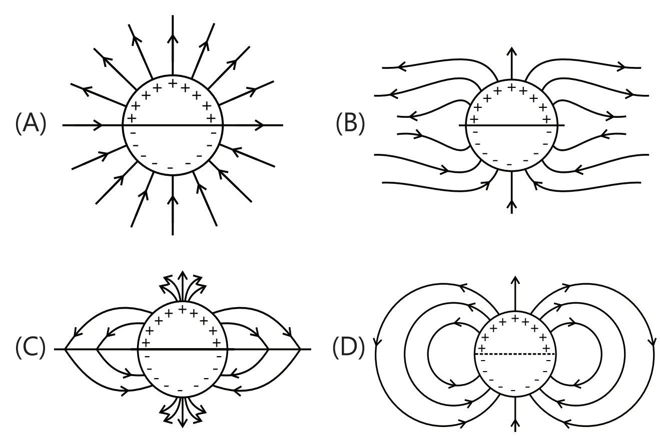


Q.22 Two charges, each equal to q, are kept at x = −a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y << a) along the y-axis, the net force acting on the particle is proportional to: ***(2013)***

(A) – y (B) $\frac{1}{y}$

(C) $-\frac{1}{y}$ (D) y

Q.23 A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale) ***(2015)***

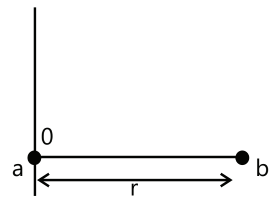


JEE Advanced/Boards

Exercise 1

Q.1 A negative point charge 2q and a positive charge q are fixed at a distance l apart. Where should a positive test charge Q be placed on the line connecting the charge for it to be in equilibrium? What is the nature of the equilibrium with respect to longitudinal motion?

Q.2 Draw E-r graph for 0<r<b, if two point charges a & b are located r distance apart, when



(i) Both are + ve

(ii) Bothe are – ve

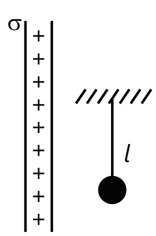
(iii) a is + ve and b is – ve

(iv) a is – ve and b is + ve

Q.3 A clock face has negative charges –q, -2q, -3q, ..., , -12q fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction as electric field at the center of the dial.

Q.4 A charge + 10^{-9} C is located at the origin in free space & another charge Q at (2, 0, 0). If the X-component of the electric field at (3, 1, 1) is zero, calculate the value of Q. Is the Y-component zero at (3, 1, 1)?

Q.5 A simple pendulum of length l and bob mass m is hanging in front of a large non - conducting sheet having surface charge density σ. If suddenly a charge +q is given to the bob & it is released from the position shown in figure. Find the maximum angle through which the string is deflected from vertical.

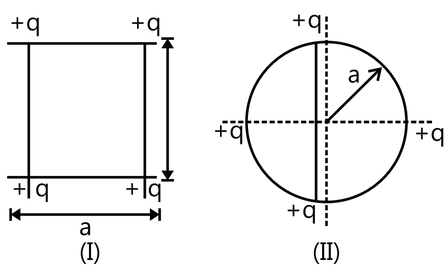


Q.6 A particle of mass m and charge –q moves along a diameter of a uniformly charged sphere of radius R and carrying a total charge + Q. Find the frequency of S.H.M. of the particle if the amplitude does not exceed R.

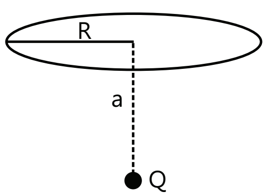
Q.7 A charge +Q is uniformly distributed over a thin ring with radius R. A negative point charge –Q and mass m starts from rest at a point far away from the center of the ring and moves towards the center. Find the velocity of this particle at the moment it passes through the center of the ring.

Q.8 A point charge +q & mass 100 gm experiences a force of 100 N at a point at a distance 20 cm from a long infinite uniformly charged wire. If it is released find its speed when it is at a distance 40 cm from wire

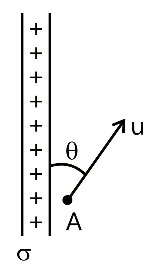
Q.9 consider the configuration of a system of four charges each of value +q. Find the work done by external agent in changing the configuration of the system from figure (i) and figure (ii).



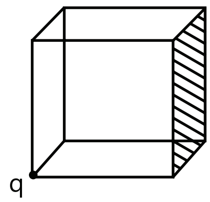
Q.10 Two identical particles of mass m carry charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first from a large distance with an initial speed V. find the closest distance of approach.



Q.11 A particle of mass m and negative charge q is thrown in a gravity free space with speed u from the point A on the large non conducting charged sheet with surface charge density σ, as shown in figure. Find the maximum distance from A on sheet where the particle can strike.

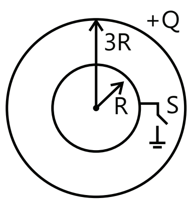


Q.12 The length of each side of a cubical closed surface is l. If charge q is situated on one of the vertices of the cube, then find the flux passing through shaded face of the cube.



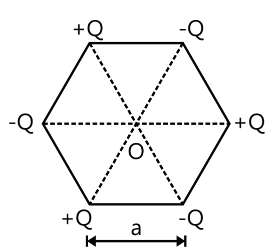
Q.13 A point charge Q is located on the axis of a disc of radius R at a distance a from the plane of the disc. If one fourth (1/4th) of the flux from the charge passes through the disc, then find the relation between a & R.

Q.14 Two thin conducting shells of radii R and 3R are shown in figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of switch S. find the charge attained by the inner shell.



Q.15 Consider three identical metal spheres A, B and C. spheres A carries charge + 6q and sphere B carries charge -3q. Sphere C carries no charge. Spheres A and B are touched together and then separated. Sphere C is then touched to sphere A and separated from it. Finally the sphere C is touched to sphere B and separated from it. Find the final charge on the sphere C.

Q.16 Six charges are placed at the vertices of a regular hexagon as shown in the figure. Find the electric field on the line passing through O and perpendicular to the plane of the figure as a function of distance x from point O.



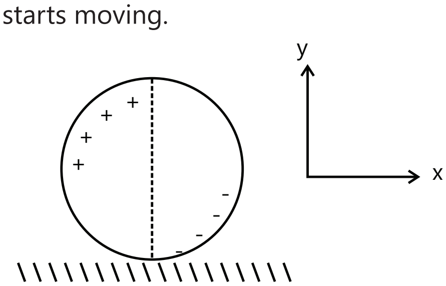
Q.17 A circular ring of radius R with uniform positive charge density λ per unit length is fixed in the Y-Z plane with its center at the origin O. A particle of mass m and positive charge q is projected from the point P $(\sqrt{3}R, 0, 0)$ on the positive X-axis directly towards O, with initial velocity v. Find the smallest value of the speed v such that the particle does not return to P.

Q.18 2 small balls having the same mass & charge & located on the same vertical at heights h_1 & h_2 are thrown in the same direction along the horizontal at the same velocity v. The 1st ball touches the ground at a distance l from the initial vertical. At what height will the 2nd ball be at this instant? The air drag & the charges induced should be neglected.

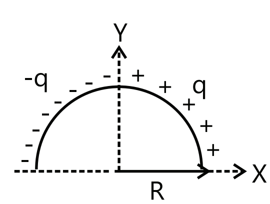
Q.19 Two identical balls of charges q_1 & q_2 initially have equal velocity of the same magnitude and direction. After a uniform electric field is applied for some time, the direction of the velocity of the first ball changes by 60^o and the magnitude is reduced by half. The direction of the velocity of the second ball changes by 90^o. In what proportion will the velocity of the second ball changes?

Q.20 Small identical balls with equal charges are fixed at vertices of regular 2008- g on with side a. At a certain instant, one of the balls is released & a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by K at a sufficiently long distance from the polygon. Determine the charge q of each part.

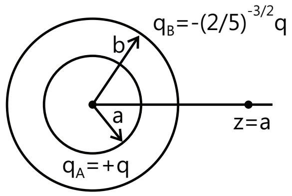
Q.21 A non - conducting ring of mass m and radius R is charged as shown. The charged density i.e. charge per unit length is λ. It is then placed on a rough non - conducting horizontal surface plane. At time t=0, a uniform electric field $E = E_0 i$ is switched on and the ring start rolling without sliding. Determine the frictional force (magnitude and direction) acting on the ring, when it starts moving.



Q.22 Find the electric field at the center of semicircular ring shown in figure.

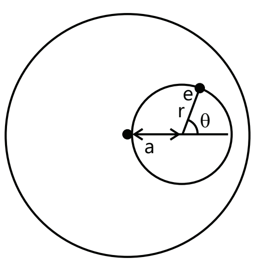


Q.23 Two concentric rings, one of radius 'a' and the other of radius 'b' have the charges +q and $-(2/5)^{-3/2}$ q respectively as shown in the figure. Find the ratio b/a if a charge particle placed on the axis at z=a is in equilibrium.



Q.24 A positive charge Q is uniformly distributed throughout the volume of a non - conducting sphere of radius R. A point mass having charge +q and mass m is fired towards the center of the sphere with velocity v from a point at distance r(r > R) from the center of the sphere. Find the minimum velocity v so that it can penetrate R/2 distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remain constant throughout the motion.

Q.25 A cavity of radius r is present inside a solid dielectric sphere of radius R, having a volume charge density of ρ. The distance between the centers of the sphere and the cavity is a. An electron e is kept inside the cavity at an angle $\theta = 45^o$ as shown. How long will it take to touch the sphere again?



Exercise 2

Single Correct Choice Type

Q.1 Mid way between the two equal and similar charges, we placed the third equal and similar charge. Which of the following statements is correct, concerning to the equilibrium along the line joining the charges

(A) The third charge experienced a net force inclined to the line joining the charges.

(B) The third charge is in stable equilibrium.

(C) The third charge is in unstable equilibrium.

(D) The third charge experiences a net force perpendicular to the line joining the charges.

Q.2 Select the correct statement: (Only force on a particle is due to electric field)

(A) A charged particle always moves along the electric lines of force.

(B) A charged particle may move along the line of force.

(C) A charged particle never moves along the line of force.

(D) A charged particle moves along the line of force only if released from rest.

Q.3 A conducting sphere of radius r has a charge. Then

(A) The charge is uniformly distributed over its surface, if there is an external electric field.

(B) Distribution of charge over its surface will be non-uniform if no external electric field exists in space.

(C) Electric field strength inside the sphere will be equal to zero only when no external electric field exists.

(D) Potential at every point of the sphere must be same.

Multiple Correct Choice Type

Q.4 Two fixed charges 4Q (positive) and Q (negative) are located at A and B, the distance AB being 3 m.

(A) The point P where the resultant field due to both is zero is on AB outside AB.

(B) The point P where the resultant field due to both is zero is on AB inside AB.

(C) If a positive charge is placed at P and displaced slightly along AB it will execute oscillation

(D) If a negative charge is placed at P and displaced slightly along AB it will execute oscillations.

Q.5 Three point charges Q, 4Q and 16Q are placed on a straight line 9 cm long. Charges are placed in such a way that the system has minimum potential energy. Then

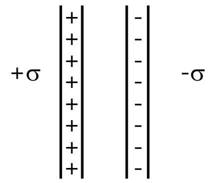
(A) 4Q and 16Q must be at the ends and Q at a distance of 3 cm from the 16Q.

(B) 4Q and 16Q must be at the ends and Q at a distance of 6 cm from the 16Q.

(C) Electric field at the position of Q is zero.

(D) Electric field at the position of Q is $\frac{Q}{4\pi\varepsilon_0}$.

Q.6 Two infinite sheets of uniform charge density $+\sigma$ and $-\sigma$ are parallel to each other as shown in the Fig. 18.103, Electric field at the



(A) Points to the left or to the right of the sheets is zero.

(B) Midpoint between the sheets is zero.

(C) Midpoint of the sheets is σ / ε_0 and is directed towards right.

(D) Midpoint of the sheet is $2\sigma / \varepsilon_0$ and is directed towards right.

Q.7 A particle of mass m and charge q is thrown in a region where uniform gravitational field and electric field are present. The path of particle

(A) May be a straight line (B) May be a circle

(C) May be a parabola (D) May be a hyperbola

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.8: Statement-I: A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)

Statement-II: Electric lines of force represents path of charged particle which is released from rest in it.

Q.9: Statement-I: For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.

Statement-II: For a non-uniformly charged thin circular ring with net charges zero, the electric field at any point on axis of the ring is zero.

Q.10: Statement-I: If a concentric spherical Gaussian surface is drawn inside this spherical shell of charge, electric field (E) at each point of surface must be zero.

tatement-II: In accordance with Gauss's law

$$\phi_E = \int E.dA = \frac{Q_{net\ enclosed}}{\varepsilon_0}$$

$$Q_{net\,enclosed} = 0 \text{ implies } \phi_E = 0$$

Q.11: Statement-I: In a given situation of arrangement of charges, an extra charge is placed outside the Gaussian surface. In the Gauss Theorem $\int \bar{E}.d\bar{s} = \frac{Q_{in}}{\varepsilon_0} Q_{in}$ remains unchanged whereas electric field E at the site of the element is changed.

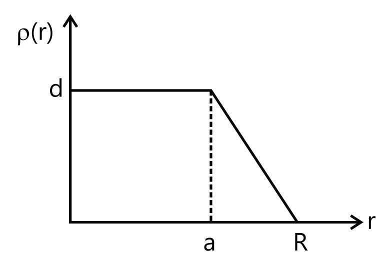
Statement-II: Electric field E at any point on the Gaussian surface is due to inside charge only.

Q.12: Statement-I: The flux crossing through a closed surface is independent of the location of enclosed charge.

Statement-II: Upon the displacement of charges within a closed surface, the E at any point on surface does not charge.

Previous Years' Questions

Paragraph: (Q.1-Q.4) The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density p(r) (change per unit volume) is dependent only on the radial distance r from the center of the nucleus as shown in figure, the electric field is only along the radial direction.



Q.1 The electric field r=R is ***(2008)***

(A) Independent of a

(B) Directly proportional to a

(C) Directly proportional to a^2

(D) Inversely proportional to a

Q.2 For a=0, the value of d (maximum value of p as shown in the figure) is ***(2008)***

(A) $\frac{3Ze}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$ (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

Q.3 The electric field within the nucleus is generally observed to be linearly dependent on r. This implies ***(2008)***

(A) $a = 0$ (B) $a = \frac{R}{2}$ (C) $a = R$ (D) $a = \frac{2R}{3}$

Q.4 Under the influence of the coulomb field of charge +Q, a charge –q is moving around it in an elliptical orbit. Find out the correct statement(s). ***(2008)***

(A) The angular momentum of the charge –q is constant

(B) The linear momentum of the charge –q is constant

(C) The angular velocity of the charge –q is constant

(D) The linear speed of the charge –q is constant.

Q.5 A positively charged thin metal ring of radius R is fixed in the x-y plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is ***(1998)***

(A) Periodic for all values of z_0 satisfying $0 < z_0 < \infty$.

(B) Simple harmonic for all values of z_0 satisfying $0 < z_0 \le R$.

(C) Approximately simple harmonic provided $z_0 << R$.

(D) Such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$.

Q.6 A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its center. ***(1998)***

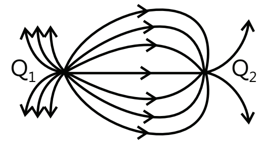
(A) Increases as r increases for r<R

(B) Decreases as r increases for $0 < r < \infty$

(C) Decreases as r increases for $R < r < \infty$

(D) Is discontinuous at r=R

Q.7 A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(2010)

(A) $|Q_1| > |Q_2|$

(B) $|Q_1| < |Q_2|$

(C) At a finite distances to the left Q_1 the electric field is zero.

(D) At a finite distance to the right of Q_2 the electric field is zero.

Q.8 A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B (< R_A)$ are kept for apart and each is given charge +Q. Now they are connected by a thin mental wire. Then ***(2011)***

(A) $E_A^{inside} = 0$ (B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (D) $E_A^{on surface} < E_B^{on surface}$

Q.9 A cubical region of side a has its centre at the origin. It encloses three fixed point charges , -q at (0, -a/4, 0), +3q at (0, 0, 0) and -q at (0, +a/4, 0). Choose the correct option(s) ***(2012)***

(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = −a/2

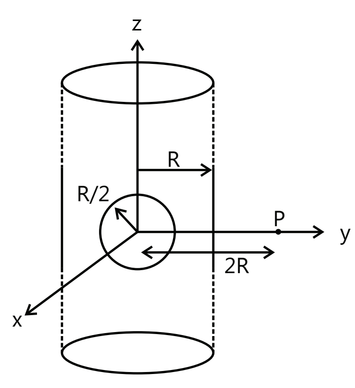
(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = −a/2.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane

z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

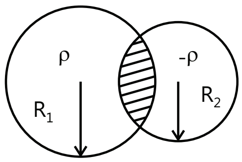
Q.10 An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is ***(2012)***



Q.11 Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centre of the spheres is zero. The ratio ρ_1 / ρ_2 can be ***(2013)***

(A) – 4 (B) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D) 4

Q. 12 Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region, ***(2013)***



(A) The electrostatic field is zero

(B) The electrostatic potential is constant

(C) The electrostatic field is constant in magnitude

(D) The electrostatic field has same direction

Q.13 Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then ***(2014)***

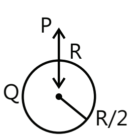
(A) $Q = 4\sigma\pi r_0^2$ (B) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$ (D) $E_2(r_0/2) = 4E_3(r_0/2)$

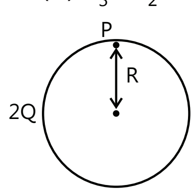
Q.14 Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_3 respectively, then ***(2014)***

(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$

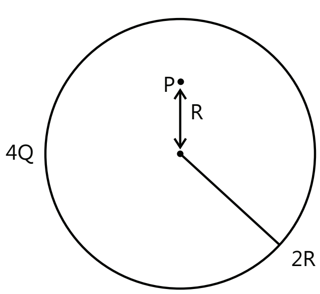
(C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$



Sphere 1



Sphere 2



Sphere 3

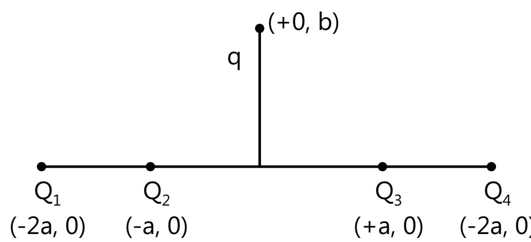
Q.15 Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = -2a, - a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists. ***(2014)***

	List I		**List II**
P.	Q_1, Q_2, Q_3, Q_4 all positive	1.	+ x
Q.	Q_1, Q_2 positive , Q_3, Q_4 negative	2.	- x
R.	Q_1, Q_4 positive , Q_2, Q_3 negative	3.	+ y
S.	Q_1, Q_3 positive , Q_2, Q_4 negative	4.	- y

Codes:

(A) P-3, Q-1, R-4, S-2 (B) P-4, Q-2, R-3, S-1

(C) P-3, Q-1, R-2, S-4 (D) P-4, Q-2, R-1, S-3



Q.16 The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and -q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statement(s) is (are) ***(2015)***

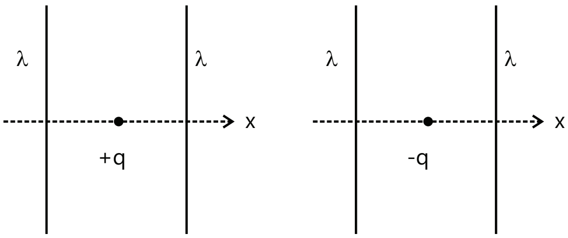
(A) Both charges execute simple harmonic motion.

(B) Both charges will continue moving in the direction of their displacement.

(C) Charge +q executes simple harmonic motion while charge - q continues moving in the direction of its displacement.

(D) Charge -q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

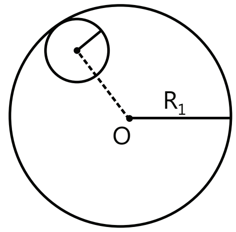
Q.17 Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are) ***(2015)***



(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$ (B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$



PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 17 Q.18 Q.19
Q.23

Exercise 2

Q. 1 Q.3

Previous Years' Questions

Q.7 Q.8 Q.11

JEE Advanced/Boards

Exercise 1

Q.4 Q.20 Q.23
Q.24 Q.25

Exercise 2

Q.6

Previous Years' Questions

Q.1 Q.2 Q.3
Q.4 Q.5 Q.8

Answer Key

JEE Main/Boards

Exercise 1

Q.2 System of Units and nature of medium

Q.6 2.1875 $\times 10^{10}$

Q.7 $1.13\times10^{4}Nm^{2}C^{-1}$

Q.8 $1.67\times10^{5}Nm^{2}C^{-1}$

Q.9 $6\times10^{-3}N$(repulsive)

Q.10 (i) 2×10^{12}, from wool to polythene,

(ii) Yes, but of a negligible amount ($=2\times10^{18}$ kg in the example).

Q.12 0.1 μC/m

Q.13 -360 N

Q.14 No change

Q.15 One coulomb, 6.25×10^{18}

Q.16 $1.76\times10^{11}C$

Q.17 $4\sqrt{2}kq/a^2$

Q.18 Zero N

Q.19 (i) $5.4\times10^{6}Nm^{-1}$ along OB

(ii) $8.1\times10^{-3}N$ along OA

Q.20 (i) $30Nm^2/C$, (ii) $15Nm^2/C$

Q.21 $22\times10^{5}Nm^2/C$

Q.23 2.06×10^{18} N (attractive)

Q.24 At a distance 2a/3 from the charge +4 q; Q=4q/9 (negative)

Q.25 $\frac{1+2\sqrt{2}}{4}q$ (negative)

Q.26 kq/a^2 along OE

Q.27 No change

Q.28 (i) dV=4E, (ii) $V_c > V_A$

Exercise 2

Q.1 D	**Q.2** C	**Q.3** D	**Q.4** D	**Q.5** A	**Q.6** A
Q.7 B	**Q.8** B	**Q.9** A	**Q.10** B	**Q.11** C	**Q.12** D
Q.13 B					

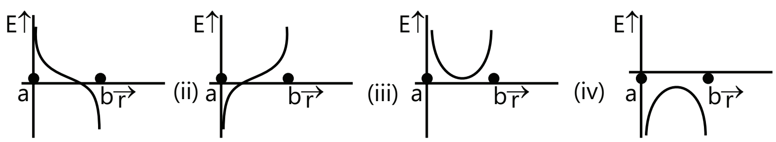
Previous Years' Questions

Q.1 C	**Q.2** D	**Q.3** B	**Q.4** C	**Q.5** B	**Q.6** B
Q.7 A	**Q.8** B	**Q.9** C	**Q.10** C	**Q.11** D	**Q.12** C
Q.13 B	**Q.14** D	**Q.15** B	**Q.16** A	**Q.17** B	**Q.18** D
Q.19 A	**Q.20** C	**Q.21** C	**Q.22** D	**Q.23** D	

JEE Advanced/Boards

Exercise 1

Q.1 $a=\ell(1+\sqrt{2})$, the equilibrium will be stable

Q.2 (i) 

Q.3 9:30

Q.4 $-3\left(\sqrt{\frac{3}{11}}\right)^3\times10^{-9}C$, No field along y-axis

Q.5 $2\tan^{-1}\left(\frac{\sigma q_0}{2\varepsilon_0 mg}\right)$

Q.6 $\frac{1}{2\pi}\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$

Q.7 $\sqrt{\frac{2kQ^2}{mR}}$

Q.8 $20\sqrt{\ln 2}$

Q.9 $-\frac{kq^2}{a}\left(3-\sqrt{2}\right)$

Q.10 $r=\frac{4KQ^2}{mV^2}$

Q.11 $\frac{2\varepsilon_0 u^2 m}{q\sigma}$

Q.12 $\frac{q}{24 \in_0}$

Q.13 $a = \frac{R}{3}$

Q.14 -Q/3

Q.15 1.125 q

Q.16 0

Q.17 $\sqrt{\frac{\lambda q}{2\varepsilon_0 m}}$

Q.18 $H_2 = h_1 + h_2 - g\left(\frac{\ell}{V}\right)^2$

Q.19 $\frac{v}{\sqrt{3}}$

Q.20 $\sqrt{4\pi\varepsilon_0 Ka}$

Q.21 $\lambda R E_0 \hat{i}$

Q.22 $-\frac{4kq}{\pi R^2}\hat{i}$

Q.23 2

Q.24 $\sqrt{\frac{2kQq}{m}\left[\frac{-1}{r} + \frac{11}{8R}\right]}$

Q.25 $\frac{\sqrt{6\sqrt{2}mr\ \varepsilon_0}}{\sqrt{epa}}$

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** B **Q.3** D

Multiple Correct Choice Type

Q.4 A, D **Q.5** B, C **Q.6** A, C **Q.7** A, C

Assertion Reasoning Type

Q.8 C **Q.9** C **Q.10** D **Q.11** C **Q.12** C

Previous Years' Questions

Q.1 A **Q.2** B **Q.3** C **Q.4** A, C **Q.5** A, C **Q.6** A

Q.7 A, D **Q.8** A, B, C, D **Q.9** A, C, D **Q.10** 6 **Q.11** B, D **Q.12** C, D

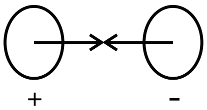
Q.13 C **Q.14** C **Q.15** A **Q.16** C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Because, the forces act towards or away from centre of the charge



Sol 2: The value of a quantity depends on the units it's been given. Electrostatic force constant k = $\frac{1}{4\pi\varepsilon}$ where ε = permittivity of medium

∴ k is dependent on nature of medium

Sol 3: Dielectric constant of a medium is the ratio of permittivity of medium to permittivity of vacuum,

$k = \frac{\varepsilon}{\varepsilon_0}$

Sol 4: Given, dielectric constant = 80

$\Rightarrow \varepsilon = 80 \times \varepsilon_0 = 80 \times 8.854 \times 10^{-12}$

$= 0.708 \times 10^{-9}\ C^2/N\text{-}m^2$

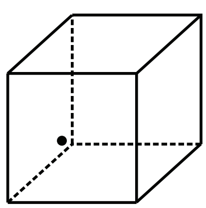
Sol 5: If a system contains many number of particles then the force on the system is the sum of forces on the particles.

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ \vec{F}_n$.

Sol 6: q = ne

$\Rightarrow n = \frac{q}{e} = \frac{3.5\times10^{-9}}{1.6\times10^{-19}} = 2.1875 \times 10^{10}$ electrons

Sol 7:

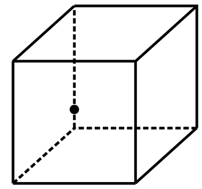


Electric flux through the surfaces of cube

$= \frac{\text{charge enclosed}}{\epsilon_o}$

$= \frac{10^{-7}}{8.854\times10^{-12}} = 1.13 \times 10^4\ Nm^2\ C^{-1}$

Sol 8:



Electric flux through all surfaces

cube $= \frac{\text{charge enclosed}}{\varepsilon_o} = \frac{8.854\times10^{-6}}{8.854\times10^{-12}} = 10^6\ Nm^2\ C^{-1}$

flux through one surface

$= \frac{1}{6}(10^6)\ Nm^2\ C^{-1} = 1.67 \times 10^5\ Nm^2\ C^{-1}$

(By symmetry)

Sol 9:

$F = \frac{k.q_1q_2}{r^2} = \frac{9\times10^9\times6\times10^{-14}}{9\times10^{-2}} = 6 \times 10^{-3}N$ (repulsive)

Sol 10: (i) q = ne

$\Rightarrow n = \frac{q}{e} = \frac{3\times10^{-7}}{1.6\times10^{-19}} \cong 2 \times 10^{12}$

electrons should be present in polythene.

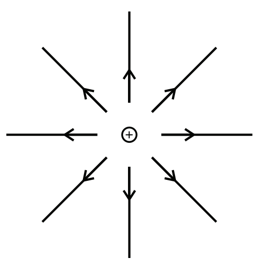
∴ Direction of flow of electrons is from wool to polythene.

(ii) Yes, since electrons have mass, there is mass transfer.

Sol 11: Properties:

→ Electric lines of forces start at a positive charge and terminate at a negative charge.

→ No two lines of forces can intersect one another.



Sol 12:

Electric flux through the imaginary cylinder

$= \frac{\text{charge enclosed}}{\varepsilon_o}$

$\Rightarrow E(2\pi rl) = \frac{\lambda \ell}{\varepsilon_o}$

$\Rightarrow E = \frac{1}{2\pi\,\varepsilon_o} \cdot \frac{\lambda}{r}$

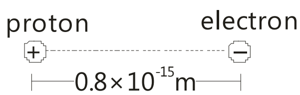
$\lambda = 2\pi\,\varepsilon_o\,E\,r$

$\Rightarrow \lambda = 2\pi \times (8.854 \times 10^{-12}) \times 9 \times 10^{4} \times 2 \times 10^{-2}$

$\Rightarrow \lambda = 10^{-7}$ C/m

$\Rightarrow \lambda = 0.1\ \mu$C/m

Sol 13:



$F = \frac{k.q_1q_2}{r^2}$

$= \frac{9\times10^{9}\times(1.6\times10^{-19})\times(-1.6\times10^{-19})}{(0.8\times10^{-15})^2}$

= -360 N (attractive)

Sol 14: $F_1 = \frac{kq_1q_2}{r^2}$

$F_2 = \frac{k(2q_1)(2q_2)}{(2r)^2} = \frac{kq_1q_2}{r^2} = F_1$

∴ No change is observed.

Sol 15: Electron charge 1.6×10^{-19} C << 1 C

∴ Coulomb is bigger

$q = ne \Rightarrow n = \frac{q}{e} = \frac{1}{1.6\times10^{-19}}$

$= 6.25 \times 10^{18}$ electrons are required

Sol 16: Given,

1 kg of electrons

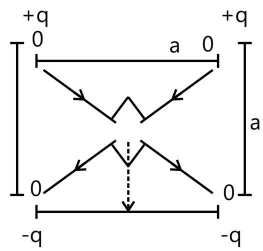
mass of electrons = 9.1×10^{-31} kg

No. of electron $= \frac{1}{9.1\times10^{-31}}$

Charge of 1 kg of electrons = n.e

$= \frac{1}{9.1\times10^{-31}} \times 1.6 \times 10^{-19}\ \text{C} = 1.76 \times 10^{11}$ C

Sol 17:



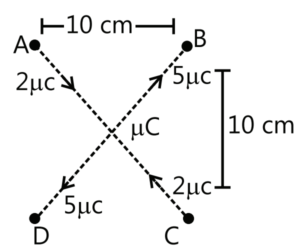
Electric field due +q at center

$= \frac{kq}{r^2} = \frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{2kq}{a^2}$

Addition of the four vectors gives field $2\sqrt{2}$ (E_q) downward

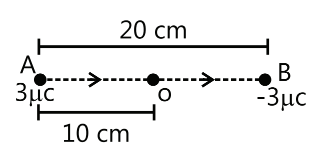
∴ Electric field $= 4\sqrt{2} \cdot \frac{kq}{a^2}$

Sol 18:



We can see that the forces acting on 1μC are pairs of forces with equal magnitude and opposite direction

∴ Net force $= \vec{0}$



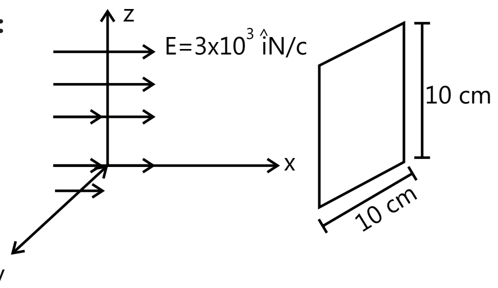
Sol 19: (i) Electric field at O

$= \frac{k(3mC)}{(10cm)^2}\,\hat{i} + \left(\frac{k(-3\imath\ C)}{(10cm)^2}(-\hat{i})\right) = \frac{2k(3\imath\ C)}{10^{-2}}\,\hat{i}$

$= 5.4 \times 10^{6}$ Nm^{-1} along OB

(ii) If a charge of -1.5×10^{-9} C is placed at O, then the force it experiences=E×q= -8.1×10^{-3} N along OA

Sol 20:

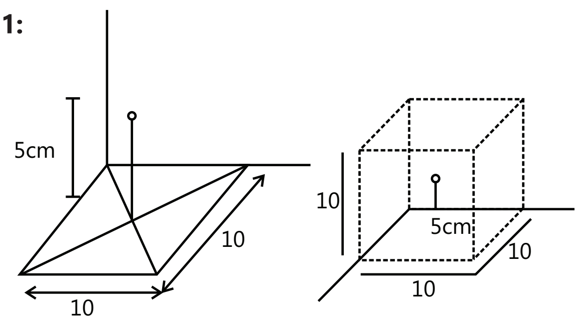


(i) Flux= $\vec{E}.\vec{A} = 3\times10^3\ \hat{i} \times (10^{-2})\,\hat{i} = 30\ Nm^2/C$

(ii) Flux= $\vec{E}.\vec{A} = 3\times10^3\ \hat{i}\ (10^{-2})\left(\frac{\hat{i}}{2}+\frac{\sqrt{3}\,\hat{j}}{2}\right)$

$= 15\ Nm^2/C$

Sol 21:



Construct a Gaussian surface as shown

The electric flux through the surfaces of cube = $\frac{\text{charge enclosed}}{\varepsilon_o}$

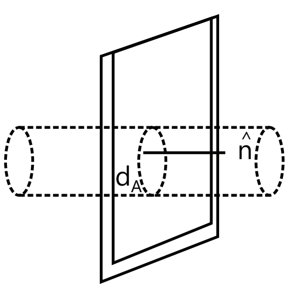
$= \frac{10\mu C}{\varepsilon_o} = 4\pi \times 9 \times 10^9 \times 10 \times 10^{-6} = 36\pi \times 10^4\ Nm^2\ C^{-1}$

Flux through one plate (bottom plate)

$= \frac{1}{6} \times$ (total/flux) ($\because$ Symmetry)

$= \frac{1}{6} \times \frac{10\mu C}{\varepsilon_o} = 6\pi \times 10^4\ Nm^2\ C^{-1} = 2 \times 10^5\ Nm^2\ C^{-1}$

Sol 22:



flux = E. $\pi r^2 = \frac{q_{enclosed}}{\epsilon_0}$ (gauss law)

$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$ ($\because \sigma = \frac{q}{\pi r^2}$)

electric field as in direction of $\hat{n}$

$\Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0}\cdot\hat{n}$

Sol 23: No. of copper molecules

$= \frac{109}{63.59} \times 6.023 \times 10^{23} = 0.95 \times 10^{23}$ atoms

No. of electrons transferred

$= \frac{0.95\times10^{23}}{100} = 0.95\times10^{21}$ electrons

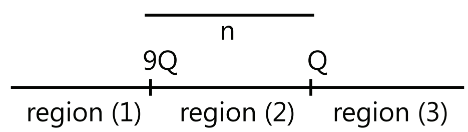
Charge of the pieces = n.e.

$= 1.52 \times 10^2 = 152$ C

Force between the two pieces

$= \frac{kq_1q_2}{r^2} = \frac{9\times10^9\times(152)\times(-152)}{10^{-4}}$

Sol 24:



Charge q should be negative to achieve equilibrium

Also if charge is placed in region (1) or (3) the charge will attract the charge in the middle while the other positive charge pushes the middle charge towards q. So only region (2) is appropriate

Let distance between 4Q and q be 'd' then for equilibrium

$\frac{k(4Q)(Q)}{x^2} = \frac{(k)(4Q)(q)}{d^2}$

$\frac{q}{Q} = \frac{d^2}{x^2}$

Also $\frac{k(4Q)(Q)}{x^2} = \frac{(k)(Q)(q)}{(x-d)^2}$

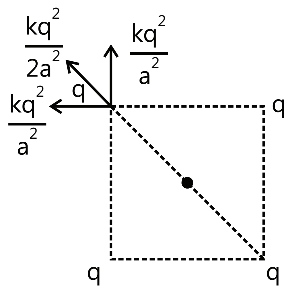
$\Rightarrow \frac{q}{Q} = \frac{4(x-d)^2}{x^2}$

$\Rightarrow d^2 = 4(x-d)^2 \Rightarrow x = d \pm \frac{d}{2}$

$\Rightarrow d = \frac{2x}{3}$ or d = 2x and $\frac{q}{Q} = \frac{4}{9} \Rightarrow q = \frac{4Q}{9}$

$\therefore q = \frac{-4Q}{9}$ (negative charge)

Sol 25:



The force on one charge due to others is

$$= \sqrt{\left(\frac{kq^2}{a^2}\right)^2 + \left(\frac{kq^2}{a^2}\right)^2} + \frac{kq^2}{2a^2} = \left(\sqrt{2} + \frac{1}{2}\right)\frac{kq^2}{q^2}$$

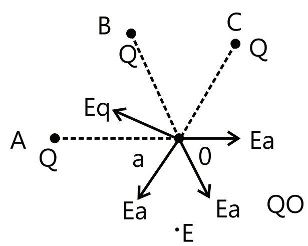
The charge to be placed at the center should be negative and let value be Q

$$\frac{kQ(q)}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{kq^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$\Rightarrow Q = \left(\frac{1 + 2\sqrt{2}}{4}\right) q$$

$$\therefore Q = -\frac{(1 + 2\sqrt{2})}{4} q$$

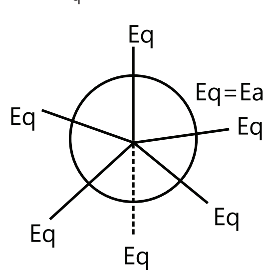
Sol 26:



$$E = \frac{kq}{a^2}$$

The system will be stable if a force E_q is placed at O along EO (∵ symmetric and equal forces are acting)

∴ By adding a force E_q along EO and OE we get



∴ final electric field is E along OE

$$E = \frac{kq}{a^2}$$

Sol 27: Electric flux is the rate of flow of the electric field through a given area

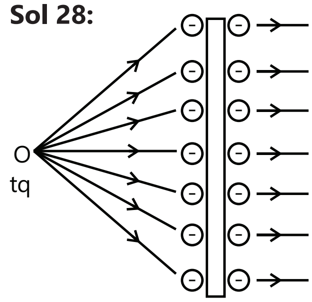
$\phi = \vec{E}.\vec{A}$

SI units of flux is Volt-meter

Electric flux is independent of the radius of spherical surface since flux $= \frac{q_{enclosed}}{\varepsilon_0}$ (Gauss law)

∴ No change will be observed.

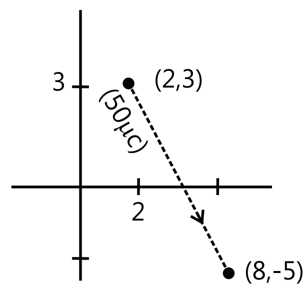
Sol 28:



For derivation of the expression, please refer the theory.

Exercise 2

Sol 1: (D)

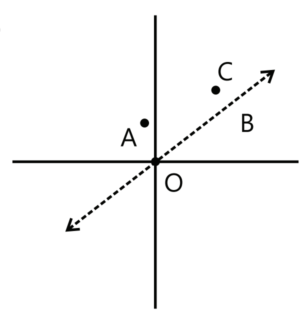


Direction of field $= \left(6\hat{i} - 8\hat{j}\right)$ m

distance = 10 m

Magnitude of field $= \frac{kq}{r^2} = \frac{9 \times 10^9 \times (90\mu c)}{10^2} = 4500$ V/m

Sol 2: (C)



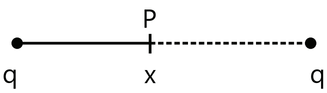
$\vec{E}_A$ is parallel to $\overrightarrow{OA}$ and similarly are others.

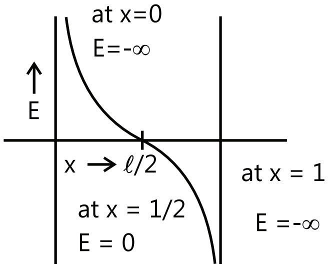
$\overrightarrow{OA} \cdot \overrightarrow{OB} = 1 + 2 - 3 = 0$

$\Rightarrow \overrightarrow{OA} \perp \overrightarrow{OB} \Rightarrow \vec{E}_A \perp \vec{E}_B$

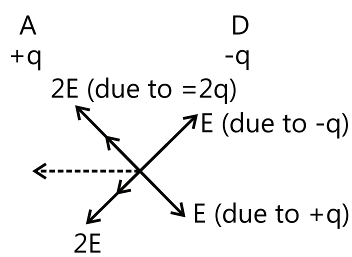
$\left[\overrightarrow{OB}\right] = \sqrt{3}\ \left[\overrightarrow{OC}\right] = 2\sqrt{3}$

$\Rightarrow E_B \propto \dfrac{1}{\left|\overrightarrow{OB}\right|^2}$, $E_C \propto \dfrac{1}{\left|\overrightarrow{OC}\right|^2} \Rightarrow \dfrac{E_C}{E_B} = \dfrac{1}{4}$

Sol 3: (D) 

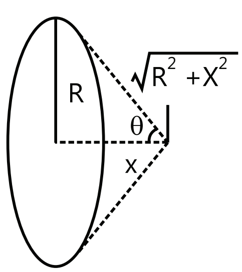


$E_p = \dfrac{kq}{x^2} - \dfrac{kq}{(\ell - x)^2}$

Sol 4: (D) 

The vector sum gives field in direction perpendicular to AB

Sol 5: (A) Refer question 24 of exercise I JEE Main

Sol 6: (A) 

Electric field due to ring at x is

$= E = \int dE = \int dE\cos\theta\,\hat{i} + \int dE\sin\theta\,\hat{j}$

$\Rightarrow E = \int_0^Q \dfrac{K\cos\theta\, dq}{(x^2 + R^2)}\hat{i} + \int_0^Q \dfrac{K\sin\theta\, dq}{(x^2+R^2)}\hat{j}$

$\Rightarrow E = \int_0^Q \dfrac{x}{(x^2+R^2)^{3/2}} dq + 0$

($\because \hat{j}$ components get cancelled while integration)

$\Rightarrow E = \dfrac{kQx}{(R^2 + x^2)^{3/2}}$

$\Rightarrow E = \dfrac{kQx}{R^3}$ if $R \gg> x$

$\Rightarrow F = m_o a = - Eq$

$\Rightarrow a = -\dfrac{kQq}{m_0 R^3}.x$

$\omega^2 = \dfrac{kQq}{m_0 R^3}$

$\Rightarrow \omega = \sqrt{\dfrac{Qq}{4\pi\,\varepsilon_0 m_0 R^3}}$

Sol 7: (B) Volt = joule/coulomb

(Since volt is S.I. unit of electric potential =W/q)

Sol 8: (B) $F = \dfrac{kQx(+q)}{(R^2 + x^2)^{3/2}}$ -mg

if $\dfrac{dF}{dx} < 0$ then the particle is in stable equilibrium

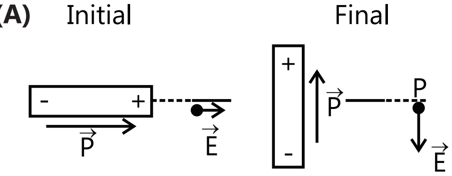
$\Rightarrow \dfrac{(R^2+x^2)^{3/2} - \frac{3}{2}(R^2+x^2)^{1/2}(2x^2)}{(R^2+x^2)^3} < 0$

$\Rightarrow R^2 + x^2 - 3x^2 < 0$

$\Rightarrow x > \dfrac{R}{\sqrt{2}}$ or $x < -\dfrac{R}{\sqrt{2}}$

$\therefore$ Only if $x > \dfrac{R}{\sqrt{2}}$, the equilibrium will be stable.

Sol 9: (A) Initial Final



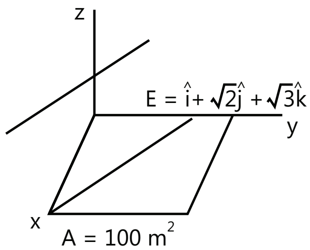
The field vector is rotated by 90° clockwise

Sol 10: (B) q_1 is positive, (emission of field lines), q_2 is negative, (termination of field lines).

$\dfrac{q_2}{q_1} = \dfrac{7}{10} = \dfrac{\text{number of lines absorbed}}{\text{number of lines emitted}} < 1$

Electric field is strongest at some point closer to q_2.

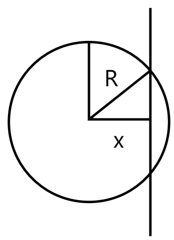
Sol 11: (C)



Only z-component of field is responsible for flux through plate

$\Rightarrow$ Flux= $\vec{E}.\vec{A}$ = $(\hat{i}+\sqrt{2}\hat{j}+\sqrt{3}\hat{k})$. (100 $\hat{k}$) = 173.2 V-m

Sol 12: (D)

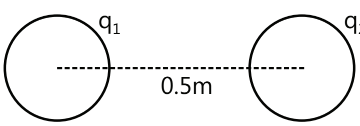


$q_{enclosed} = \sigma.\ A_{enclosed}$

$A_{enclosed} = \pi r^2 = \pi(R^2 - x^2)$

$\therefore$ Flux through sphere $= \dfrac{q_{enclosed}}{\epsilon_o} = \dfrac{\pi(R^2 - x^2)\sigma}{\epsilon_0}$

Sol 13: (B)



$$\frac{kq_1q_2}{(0.5)^2} = 0.108 \text{ N}$$

When connected with a wire, the charges on them will be distributed equally giving

$q = \dfrac{-q_1 + q_2}{2}$ on each sphere

(Since one of them is negative)

$$\frac{kq^2}{(0.5)^2} = 0.036 \text{ N}$$

$$\Rightarrow \frac{q^2}{q_1q_2} = \frac{0.036}{0.108} = \frac{1}{3}$$

$$\Rightarrow (q_2 - q_1)^2 = \frac{4q_1q_2}{3}$$

$$\Rightarrow q_2^2 + q_1^2 - \frac{10q_1q_2}{3} = 0$$

$$\Rightarrow q_2 = \frac{q_1}{3} \text{ or } q_1 = \frac{q_2}{3}$$

substituting $q_2 = \dfrac{q_1}{3}$ in $\dfrac{kq_1q_2}{r^2}$ = 0.108N gives

$\Rightarrow q_1 = \pm 3 \times 10^{-6}$ C and $q_2 = \pm 1 \times 10^{-6}$ C

Previous Years' Questions

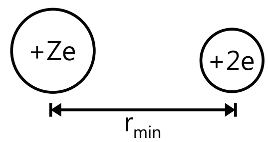
Sol 1: (C) From conservation of mechanical energy

Decrease in kinetic energy = increase in potential energy

or $\dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(2e)}{r_{min}} = 5\text{MeV} = 5 \times 1.6 \times 10^{-13}$ J

$$\therefore r_{min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5 \times 1.6 \times 10^{-13}}$$

$$= \frac{(9 \times 10^9)(2)(92)(1.6 \times 10^{-19})^2}{5 \times 1.6 \times 10^{-13}} \qquad (\ Z = 92)$$



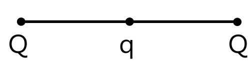
$= 5.3 \times 10^{-14}$ m

$= 5.3 \times 10^{-12}$ cm

i.e., r_{min} is of the order of 10^{-12} cm

Sol 2: (D) Motion is simple harmonic only if Q is released from a point not very far from the origin on x-axis. Otherwise motion is periodic but not simple harmonic.

Sol 3: (B) Since, q is at the centre of two charges Q and Q, net force on it is zero, whatever the magnitude and sign of charge on it.



For the equilibrium of Q, q should be negative because other charge Q will repel it, so q should attract it. Simultaneously these attractions and repulsions should be equal.

$$\frac{1}{4\pi\varepsilon_0}\frac{QQ}{r^2}\ \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(r/2)^2}$$

or $q = \dfrac{Q}{4}$

or with sign $q = -\dfrac{Q}{4}$

Sol 4: (C) The magnitude of electric field at a distance r from the axis is given as:

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

i.e., $E\propto\frac{1}{r}$

Here, λ is the charge per unit length of the capacitor.

Sol 5: (B) Electric Field lines never enter a metallic conductor (E = 0, inside a conductor) and they fall normally on the surface of a metallic conductor (because whole surface is at same potential and lines are perpendicular to equipotential surface)

Sol 6: (B) Electrostatic force, $F_e = eE$ (for both the particles)

But acceleration of electron, $a_e = F_e/m_e$ and acceleration of proton, $a_p = F_e/m_p$

$$S = \frac{1}{2}a_e t_1^2 = \frac{1}{2}a_p t_2^2$$

$$\therefore \frac{t_2}{t_1} = \sqrt{\frac{a_e}{a_p}} = \sqrt{\frac{m_p}{m_e}}$$

Sol 7: (A) $-\int_{\ell=\infty}^{\ell=0}\vec{E}.d\vec{\ell} = \int_{\ell=\infty}^{\ell=0} dV = V\ (\text{centre}) - V\ (\text{infinity})$

But V(infinity) = 0

$\therefore -\int_{\ell=\infty}^{\ell=0}\vec{E}.d\vec{\ell}$ corresponds to potential at centre of ring.

And V(centre) $= \frac{1}{4\pi\varepsilon_0}.\frac{q}{R} = \frac{(9\times10^9)(1.11\times10^{-10})}{0.5} = 2$ volt

Sol 8: (B) Net electrostatic energy of the configuration will be

$$U = K\left[\frac{q.q}{a}+\frac{Q.q}{\sqrt{2}a}+\frac{Q.q}{a}\right] \text{ Here, } K=\frac{1}{4\pi\varepsilon_0}$$

Putting U = 0 we get, $Q = \frac{-2q}{2+\sqrt{2}}$

Sol 9: (C) Electric lines of force never form closed loops.

Sol 10: (C) Electric field is zero everywhere inside a metal (conductor) i.e., field lines do not enter a metal. Simultaneously these are perpendicular to a metal surface (equipotential surface).

Sol 11: (D) According to option (d) the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up.

Sol. 12: (C) At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1$, $-q_1$ and q_2. Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

Sol 13: (B) All the three plates will produce electric field at P along negative z-axis, Hence,

$$\vec{E}_P = \left[\frac{\sigma}{2\varepsilon_0}+\frac{2\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

$\therefore$ Correct answer is (b)

Sol 14: (D) Charge will be induced in the conducting sphere, but net charge on it will be zero.

$\therefore$ Option (d) is correct.

Sol 15: (B) Inside the cavity, field at any point is uniform and non-zero.

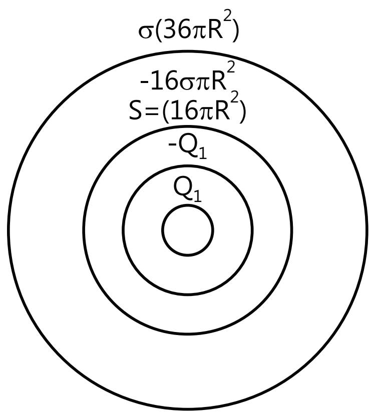
Therefore, correct option is (b).

Sol 16: (A) Total enclosed charge as already shown is

$$q_{net} = \frac{6C}{2} + \frac{8C}{4} - 7C = -2C$$

From Gauss theorem, net flux, $\phi_{net} = \frac{q_{net}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$

Sol 17: (B)



$Q_1 = \sigma(4\pi R^2) = 4\pi\sigma R^2$

$Q_2 = 16\pi\sigma R^2 - Q_1 = 12\pi\sigma R^2$

$Q_3 = 36\pi\sigma R^2 - 16\pi\sigma R^2 = 20\pi\sigma R^2$

$Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

Sol 18: (D) $qE = mg$ ... (i)

$6\pi\eta rv = mg$

$\frac{4}{3}\pi r^3 \rho g = mg$ ... (ii)

$\therefore r = \left(\frac{3mg}{4\pi\rho g}\right)^{1/3}$ ... (iii)

Substituting the value of r in Eq. (i) we get,

$6\pi\eta v\left(\frac{3mg}{4\pi\rho g}\right)^{1/3} = mg$

or $(6\pi\eta v)^3\left(\frac{3mg}{4\pi\rho g}\right) = (mg)^3$

Again substituting mg = qE we get.

$(qE)^2 = \left(\frac{3}{4\pi\rho g}\right)(6\pi\eta v)^3$

or $qE = \left(\frac{3}{4\pi\rho g}\right)^{1/2}(6\pi\eta v)^{3/2}$

$\therefore q = \frac{1}{E}\left(\frac{3}{4\pi\rho g}\right)^{1/2}(6\pi\eta v)^{3/2}$

Substituting the values we get

$q = \frac{7}{81\pi\times 10^5}\sqrt{\frac{3}{4\pi\times 900\times 9.8}\times 216\pi^3}$

$\times\sqrt{(1.8\times 10^{-5}\times 2\times 10^{-3})^3}$

$= 8.0 \times 10^{-19}$ C

Sol 19: (A) Electrical force per unit area = $\frac{1}{2}\varepsilon_0 E^2$

$= \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{\sigma^2}{2\varepsilon_0}$

Projected area = πR^2

$\therefore$ Net electrical force = $\left(\frac{\sigma^2}{2\varepsilon_0}\right)(\pi R^2)$

In equilibrium, this force should be equal to the applied force.

$\therefore F = \frac{\pi\sigma^2 R^2}{2\varepsilon_0}$ or $F \propto \frac{\sigma^2 R^2}{\varepsilon_0}$

Sol 20: (C) Electric flux, $\phi = \vec{E}\,.\,\vec{S}$

or $\phi = ES\cos\theta$

Here, θ is the angle between $\vec{E}$ and $\vec{S}$

In this question $\theta = 45°$, because $\vec{S}$ is perpendicular to surface.

$E = E_0$

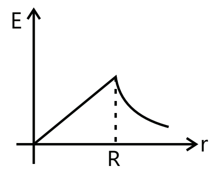
$S = (\sqrt{2}\,a)(a) = \sqrt{2}\,a^2$

$\therefore \phi = (E_0)(\sqrt{2}\,a^2)\cos 45° = E_0 a^2$

Sol 21: (C) $\bar{E}_{inside} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\right)\bar{r}$

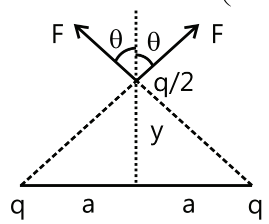
$\bar{E}_{outside} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\right)\bar{r}$

$\therefore$



Sol 22: (D)

$F_{net} = 2F\cos\theta = 2\frac{k.q.q/2}{\left(\sqrt{a^2+y^2}\right)}.\frac{y}{\sqrt{a^2+y^2}} = \frac{kq^2y}{a^3}\,(y << a)$

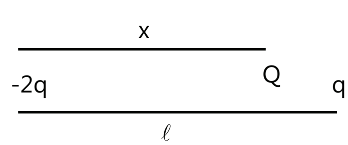


Sol 23: (D) It originates from +Ve charge and terminates at - Ve charge. It can not form close loop.

JEE Advanced/Boards

Exercise 1

Sol 1:



For equilibrium $x > \ell$ and Q should be positive balancing force equations,

$\frac{k(2q)(Q)}{(x)^2} = \frac{kq(Q)}{(x-\ell)^2}$

$\Rightarrow \left(1-\frac{\ell}{x}\right)^2 = \frac{1}{2} \Rightarrow 1 - \frac{\ell}{x} = \pm\frac{1}{\sqrt{2}}$

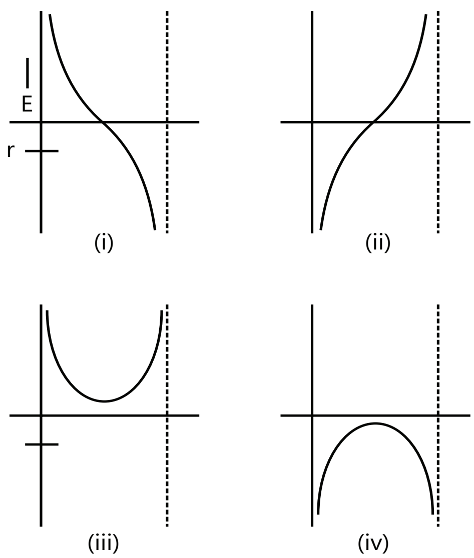
$$\Rightarrow \frac{\ell}{x} = \frac{\sqrt{2} \pm 1}{\sqrt{2}} \Rightarrow x = 2 \pm \sqrt{2}\ \ell$$

$x > \ell \Rightarrow x = (2 + \sqrt{2})\ \ell$

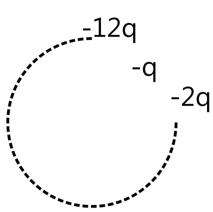
$(1 + \sqrt{2})\ell$ from q

It is in stable equilibrium w.r.t. longitudinal motion

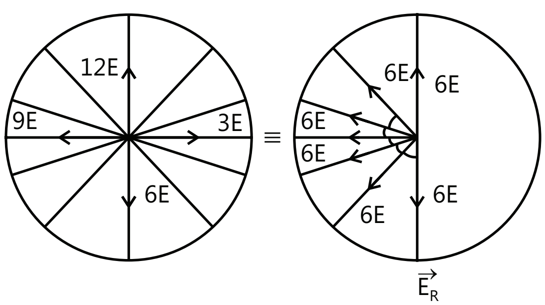
Sol 2:

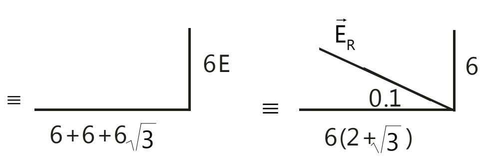


Sol 3:



Direction of electric field at center is





$$\tan\theta = \frac{1}{2+\sqrt{3}}$$

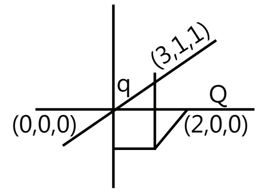
$$\tan\theta = 2 - \sqrt{3}$$

$\Rightarrow \theta = 15°$

The hour hand should be midway of between 9 and 10

$\therefore$ Time = 9 : 30

Sol 4:



$$\vec{E}_q = \frac{kQ}{\sqrt{(3^2+1^2+1^2)}}\left(\frac{3\hat{i}+\hat{j}+\hat{k}}{\sqrt{11}}\right)$$

$$\vec{E}_Q = \frac{kQ}{\left(\sqrt{1^2+1^2+1^2}\right)}\left(\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}\right)$$

At P x-Component of field is zero

$$\Rightarrow (\vec{E}_P + \vec{E}_Q)_x = 0 \Rightarrow \frac{3kq}{\left(\sqrt{11}\right)^3} = \frac{-QK}{\left(\sqrt{3}\right)^3}$$

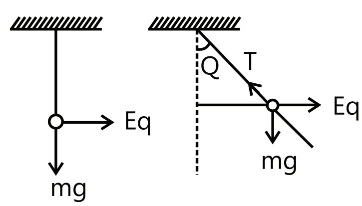
$$\Rightarrow Q = -3\left(\sqrt{\frac{3}{11}}\right)^3 \times 10^{-9}\ C$$

y-component has zero field.

Sol 5: Electric field due to plate = $\frac{\sigma}{2\epsilon_0}$

(Non-conducting plate)

The force that is being applied on bob = Eq



Change in gravitational potential = $mg\ell\ (1 - \cos\theta)$

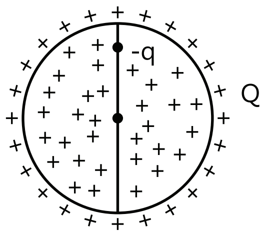
Change in electrical potential = $Eq\ \ell\ \sin\theta$

$mg\ \ell\ (2\sin^2\theta/2) = Eq\ell\ (2\sin\theta/2\cos\theta/2)$

$$\Rightarrow \tan\theta/2 = \frac{Eq}{mg}$$

$$\Rightarrow \theta = 2\tan^{-1}\left(\frac{\sigma q}{2\ \varepsilon_0 mg}\right)$$

Sol 6:



At any point x from center, the acceleration of the charge is

$$a = -\frac{Eq}{m}$$

But, electric field at the point is

$$E.(4\pi r^2) = \frac{q_{encloses}}{\varepsilon_0} \text{ (Gauss's law)}$$

$$\Rightarrow E.4\pi r^2 = \frac{1}{\varepsilon_0} \cdot \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3$$

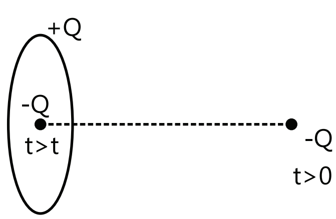
$$\Rightarrow E = \frac{Qr}{4\pi\ \varepsilon_0 R^3}$$

$$\Rightarrow a = \frac{-Qq}{4x\ \varepsilon_0 R^2}.x$$

$$\Rightarrow \therefore \omega = \sqrt{\frac{Qq}{4\pi\ \varepsilon_0 R^3}}$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{Qq}{4\pi\ \varepsilon_0 R^3}}$$

Sol 7:



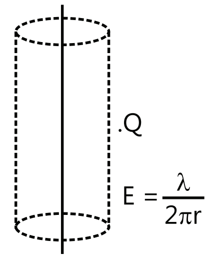
Electric potential at center of ring = $\frac{kQ}{R}$

By energy conservation,

$$\frac{1}{2}mv^2 = \frac{kQ(Q)}{R}$$

$$\Rightarrow v = \sqrt{\frac{2kQ^2}{mR}}$$

Sol 8:



$$V = -\int Edr = \left[\frac{-\lambda}{2\pi}\ell nr\right]_{r_1}^{r_2}$$

$$\Delta V = \frac{\lambda}{2\pi}\ \lambda n\ \frac{r_1}{r_2}$$

By energy conservation

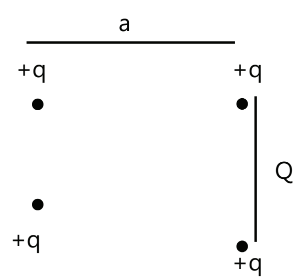
$$\frac{1}{2}mv^2 = \frac{\lambda}{2\pi}\ \ln\left(\frac{r_2}{r_1}\right)$$

$$v = \sqrt{\frac{\lambda}{\pi m}\ln\left(\frac{r_2}{r_1}\right)} \Rightarrow v = \sqrt{\frac{2Er}{m}.\ln 2}$$

$$v = \sqrt{\frac{2\times 100\times 0.2}{0.1}.\ln 2}$$

$$\Rightarrow v = 20\sqrt{\ln 2}$$

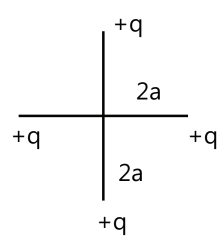
Sol 9:



Initial configuration

Initial potential energy = $\frac{kq^2}{a} \times 4 + \frac{kq^2}{\sqrt{2}a} \times 2$

$= (4 + \sqrt{2})\ kq^2/a$



Final configuration

Final potential energy

$= \frac{kq^2}{\sqrt{2}a} \times 4 + \frac{kq^2}{2a} \times 2 = (2\sqrt{2} + 1)\ kq^2/a$

work done = $U_f - U_i$

$$= \left(\left(2\sqrt{2}+1\right)-\left(4+\sqrt{2}\right)\right)\ \frac{kq^2}{a}\ = -\left(3-\sqrt{2}\right)\ \frac{kq^2}{a}$$

Sol 10: Q Q

$\leftarrow$ V Initial

Q Q

$\leftarrow$ $\leftarrow$

V′ V′

r

At closest distance of approach

By momentum conservation V′ = V/2

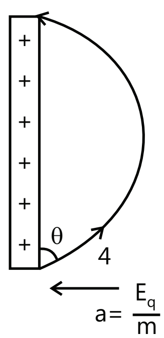
By energy conservation,

$$\frac{1}{2}\ mV^2 = \frac{1}{2}\ m\left(\frac{V'}{2}\right)^2 \times 2 + \frac{kQ^2}{r}$$

$$\Rightarrow \frac{1}{r}\ kQ^2 = \frac{1}{4}\ mV^2$$

$$\Rightarrow r = \frac{4KQ^2}{mV^2}$$

Sol 11:



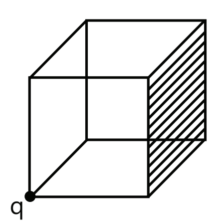
$$\Rightarrow \text{Maximum horizontal distance} = \frac{u^2 \sin 2\theta}{a}$$

$$\Rightarrow H_{max} = \frac{u^2}{a}$$

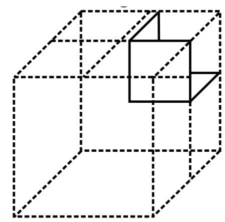
$$\Rightarrow H_{max} = \frac{mu^2}{E_q} = \frac{2\ \varepsilon_0 mu^2}{\sigma q}$$

($\because$ $E = \frac{\sigma}{2\ \varepsilon_0}$ for non-conducting plate)

Sol 12:



Construct Gaussian surface as below



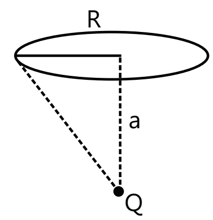
such that the original is $\frac{1}{8}$ th of it

$$\text{Flux} = \frac{q_{enclosed}}{\varepsilon_0}$$

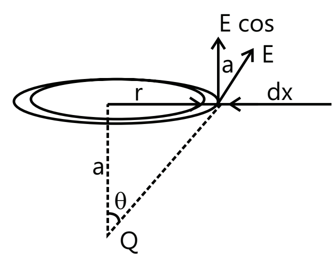
flux through one forth of one surface

$$= \frac{1}{4} \cdot \frac{1}{6}\ \frac{q}{\varepsilon_0} = \frac{q}{24\ \varepsilon_0}\ \text{(By symmetry)}$$

Sol 13:



Take an elemental part with thickness dr as below



The electric field at the elemental part is

$$E = \frac{kQ}{a^2 + r^2}$$

flux through the element $d\phi = E.\ dA \cos\theta$

$$\Rightarrow \phi = \int_0^R \frac{Ea\left(2\pi r\right)}{\left(a^2+r^2\right)^{1/2}} dr$$

$$\Rightarrow \phi = 2\pi kQa \int_0^R \frac{r}{(a^2+r^2)^{3/2}} dr$$

$$\Rightarrow \phi = a\pi kQ \left(\frac{(a^2+r^2)^{-1/2}}{-\frac{1}{2}}\right)_0^R$$

$$\Rightarrow \phi = a\pi kQ\ [-2] \left[\frac{1}{(a^2+R^2)^{1/2}} - \frac{1}{a}\right]$$

$$\Rightarrow \phi = 2\pi kQ\left[1-\frac{a}{(a^2+R^2)^{1/2}}\right]$$

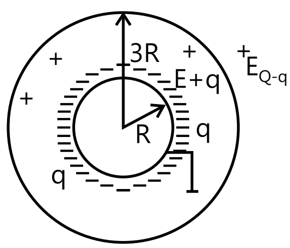
Given

$$\phi = \frac{Q}{4\,\varepsilon_0}$$

$$\Rightarrow \frac{Q}{2\,\varepsilon_0}\left(1-\frac{a}{(a^2+R^2)^{1/2}}\right) = \frac{Q}{4\,\varepsilon_0}$$

$$\Rightarrow \frac{a}{(a^2+R^2)^{1/2}} = \frac{1}{2} \Rightarrow 3a^2 = R^2 \Rightarrow a = \frac{R}{\sqrt{3}}$$

Sol 14:

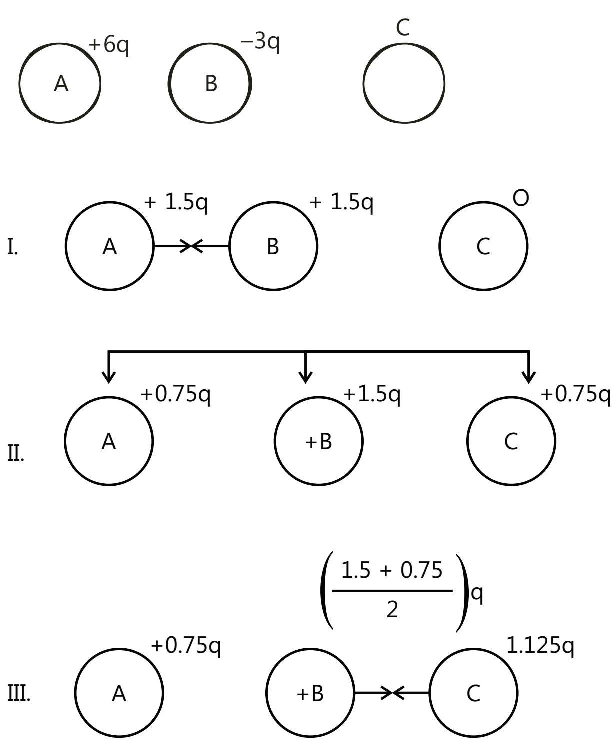


The potential at r = R is zero

$$\Rightarrow \frac{k(Q-q)}{3R} + \frac{kq}{3R} - \frac{kq}{R} = 0$$

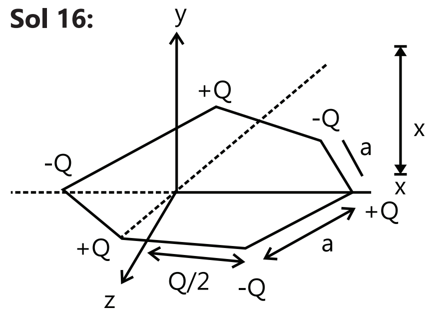
$$\Rightarrow q = \frac{Q}{3} \text{ (negative)}$$

Sol 15:



$\therefore$ Charge on C = 1.125 q

Sol 16:



Consider electric field due to +Q charges,

We will get,

$$\vec{E}_{+Q} = \frac{kQ^2}{a^2+x^2}\;\frac{x}{\sqrt{a^2+x^2}}\;(+\hat{j})$$

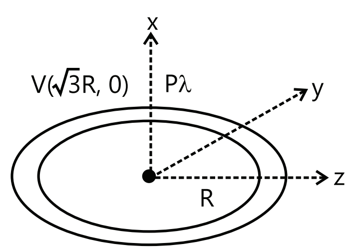
while due to negative charges,

$$\vec{E}_{-Q} = \frac{kQ^2}{a^2+x^2}\;\frac{x}{\sqrt{a^2+x^2}}\;(-\hat{j})$$

$\therefore$ Electric field at point on the y-axis

$= \vec{E}_{+Q} + \vec{E}_{-Q} = 0$

Sol 17:



Initial electric potential energy = $q.V_P$

$$= q.\frac{kQ}{\sqrt{R^2+(\sqrt{3}R)^2}} = \frac{kQq}{2R} = \frac{k\lambda(2\pi R)q}{2R} = \frac{\lambda q}{4\,\varepsilon_0}$$

Final potential energy = $\frac{kQ}{R}\,q = \frac{\lambda q}{2\,\varepsilon_0}$

for minimum velocity, final kinetic energy = 0

By conservation of Energy,

$K.E._i + P.E._i = K.E._f + P.E._f$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{\lambda q}{4\,\varepsilon_0} = 0 + \frac{\lambda q}{2\,\varepsilon_0}$$

$$\Rightarrow v = \sqrt{\frac{\lambda q}{2\,\varepsilon_0 m}}$$

Sol 18: Consider the two balls of system, the only external force is gravitational force. Initial position of COM is at $\frac{h_1+h_2}{2}$. The vertical distance moved by

COM during time $t = \left(\frac{\ell}{v}\right)$ is $h = \frac{1}{2} g \left(\frac{\ell}{v}\right)^2$

Final height of COM is

$$H = \frac{h_1 + h_2 - g\left(\frac{\ell}{V}\right)^2}{2}$$

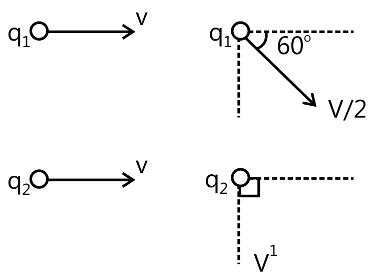
If the COM is at H, and one particle is on ground then the height of the other will be at height $H_2 = \frac{2(H_{COM}) - H_1}{2-1}$

$\Rightarrow H_2 = h_1 + h_2 - g\left(\frac{\ell}{v}\right)^2$

$\therefore$ Height at which the body is located is

$H = h_1 + h_2 - g\left(\frac{\ell}{v}\right)^2$

Sol 19: I. t = 0 $\rightarrow$ II. t = t



Acceleration of first ball in x-direction due to

field = $(a_1)_x = \frac{3V}{4t}\ (\hat{i})$

Similarly $(a_1)_y = \frac{\sqrt{3}V}{4t}\ (-\hat{j})$

$(a_2)_x = \frac{-V}{t}\ (\hat{i})$

$(a_2)_y = \frac{-V'}{t}\ (\hat{j})$

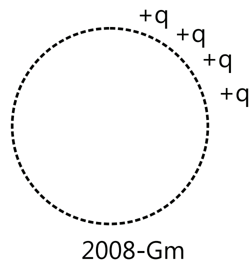
$\vec{a}_1 = \frac{\vec{E}.q_1}{m} = -\frac{3v}{4t}\hat{i} - \frac{\sqrt{3}v}{4t}\hat{j}$

$\Rightarrow \vec{E} = \frac{-3mv}{4q_1 t}\hat{i} - \frac{\sqrt{3}mv}{4q_1 t}\hat{j}$

$\vec{a}_2 = \frac{\vec{E}q_2}{m} = \frac{-3vq_2\hat{i}}{4q_1 t} - \frac{\sqrt{3}vq_2}{4q_1 t}\hat{j} = \frac{-v}{t}\hat{i} - \frac{v^1}{t}\hat{j}$

$\Rightarrow \frac{q_2}{q_1} = \frac{4}{3}$ and $V^1 = \frac{\sqrt{3}V}{4}\left(\frac{4}{3}\right) = \frac{v}{\sqrt{3}}$

Sol 20:



The potential energy of the system be $U = U_{12} + U_{13} + \ldots\ldots U_{20062007}$

The K.E. of the first ball after being released for a long time

By energy conservation

$K.E_f + P.E_f = K.E_i + P.E_i$

$\Rightarrow K_1 = 0 + (P.E_i - P.E_f)$

$\Rightarrow K_1 = U_{12} + U_{13} + U_{14} + \ldots\ldots U_{2007}$

The K.E. of second ball after being released for a long time

By energy conservation

$K_2 = (P.E'_i - P.E'_f)$

$= U_{23} + U_{24} + U_{25} + \ldots\ldots + U_{2\ 2007}$

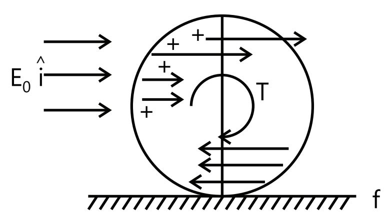
$= U_{12} + U_{13} + U_{14} + \ldots. + U_{1\ 2006}$

($\because U_{n\ n+1} = U_{n-1\ n}$ by symmetry)

$\Rightarrow K_1 - K_2 = U_{1\ 2007} = \frac{kq^2}{a^2} = K(\because K_1 - K_2 = K$ Given)

$\Rightarrow q = \sqrt{4\pi\,\varepsilon_0 aK}$

Sol 21:



Friction acts in forward direction decreases angular acceleration and increasing linear acceleration

$\Rightarrow f = ma$

and $T_e - f.R = I\alpha = mR^2\alpha$

also $a = R\alpha$

$\Rightarrow f = \frac{T_e}{2R}$

but $T_e = \int dT_e = \int_0^Q E_0 dq.R\cos\theta$

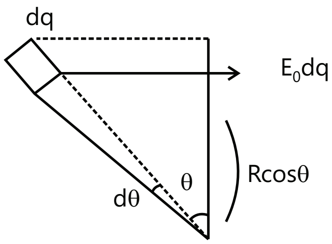
$$\Rightarrow T_e = 2 \times \int_0^{\pi/2} E_0 \lambda . R^2 \cos\theta \, d\theta$$

(2 is multiplied considering –ve changes also)

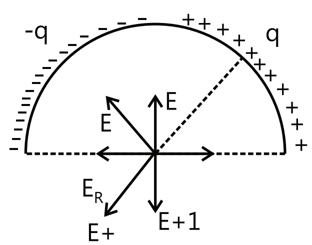
$$= 2 \times E_0 \lambda R^2 \left[\sin\theta\right]_0^{\pi/2}$$

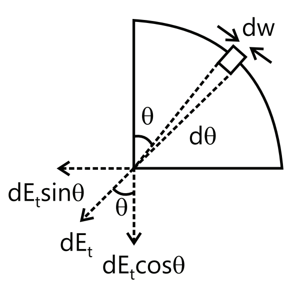
$$\Rightarrow T_e = 2E_0 \lambda R^2$$

$$\Rightarrow f = E_0 \lambda R \ \hat{i}$$



Sol 22:





The x-component of field $= \int dE_+ \sin\theta$

$$= \int \frac{k.dq}{R^2} \sin\theta = \frac{k}{R} \lambda \int_0^{\pi/2} \sin\theta \, d\theta$$

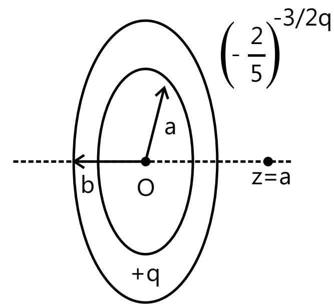
$$= \frac{k\lambda}{R}\left[-\cos\theta\right]_0^{\pi/2} = \frac{k\lambda}{R}(1) = \frac{kq}{R\left(\frac{\pi R}{2}\right)} = \frac{2kq}{\pi R^2}$$

The y-component of positive charges' field cancels the y-component field of negative charges' field.

$\therefore$ The total electric field will be

$$\vec{E}_{total} = \vec{E}_+ + \vec{E}_- = \frac{4kQ}{\pi R^2} \ (-\hat{i})$$

Sol 23:



$$\vec{E}_A = \frac{kQ(a)}{(a^2 + a^2)^{3/2}} \ (\hat{i})$$

$$\vec{E}_B = \frac{kQ'(a)}{(b^2 + a^2)^{3/2}} \ (-\hat{i})$$

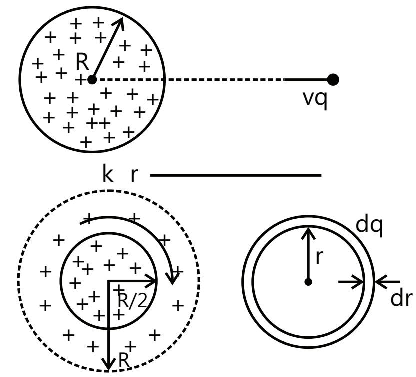
$$\vec{E} = \vec{E}_A + \vec{E}_B = 0 \text{ (given)}$$

$$\Rightarrow \frac{Q}{(2a^2)^{3/2}} = \frac{-Q'}{(b^2 + a^2)^{3/2}}$$

$$\Rightarrow b^2 + a^2 = 2a^2\left(\frac{5}{2}\right)$$

$$\Rightarrow b = 2a \Rightarrow \frac{b}{a} = 2$$

Sol 24:



$\therefore$ potential at R/2 is

$$V = \int dV = \int_0^{Q_1} \frac{k.dq}{\left(\frac{R}{2}\right)} + \int_{Q'}^{Q} \frac{kdq}{r}$$

(element part is a hollow sphere of rad radius r)

$$\Rightarrow V = \int_0^{R/2} \frac{k.\rho.4\pi r^2 dr}{\left(\frac{R}{2}\right)} + \int_{R/2}^{R} \frac{k\rho 4\pi r^2 dr}{r} \qquad \left(\because \rho = \frac{Q}{(4/3)\pi R^3}\right)$$

$$\Rightarrow V = \frac{2k\rho}{R} \times 4\pi \left[\frac{r^3}{3}\right]_0^{\frac{R}{2}} + 4\pi\rho k \left[\frac{r^2}{2}\right]_{\frac{R}{2}}^{R}$$

$$\Rightarrow V = \frac{2k}{R}\,\frac{Q}{8} + \frac{9}{8}\,\frac{kQ}{R} = \frac{11}{8}\,\frac{kQ}{R}$$

By energy conservation we get,

$K.E._i + P.E._i = K.E._f + P.E._f$

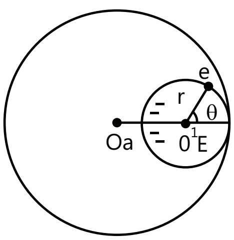
$$\Rightarrow \frac{1}{2}mv^2 + \frac{kQq}{r} = 0 + \frac{11kQq}{8R}$$

$$\Rightarrow V = \sqrt{\frac{2kQq}{m}\left[\frac{-1}{r} + \frac{11}{8R}\right]}$$

Sol 25:

The electric field inside the cavity will be

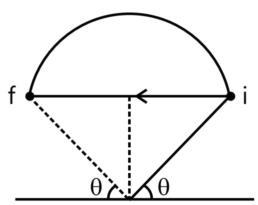
$E = \frac{kQ}{a^2}$ along OO′ (proof next page)



The distance the electron has to travel is

$2a\cos\theta = \sqrt{2}r$

$s = 1/2\ at^2$

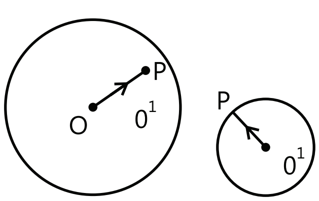


$$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\sqrt{2}r}{\frac{Eq}{m}}}\ \left[\because a = \frac{Eq}{m}\right]$$

$$\Rightarrow t = \sqrt{\frac{2m\sqrt{2}r}{e.\frac{1}{4\pi\varepsilon_0}\frac{\rho\times(4/3)\pi a^3}{a^2}}} = \sqrt{\frac{6\sqrt{2}mr\,\varepsilon_0}{e\rho a}}$$

$\vec{E}_{required} = \vec{E}_{whole} - \vec{E}_{cut}$

=



$= \overrightarrow{OP} - \overrightarrow{O'P} = \overrightarrow{OO'}$

Field at center of cavity $= \frac{kq}{a^2}$ where $q = \frac{4}{3}\rho\pi a^3$

(Included charge)

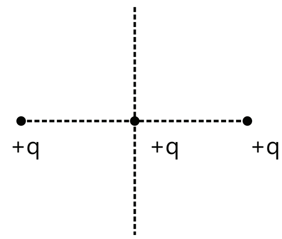
$$\Rightarrow \vec{E}_{required} = \frac{\rho a}{3\varepsilon_0} \text{ along } \overrightarrow{OO'}$$

It is true for any point inside cavity.

Exercise 2

Single Correct Choice Type

Sol 1: (B)



The charge in the middle experiences force along the line.

The equilibrium is stable along the line connecting charges while

The equilibrium is unstable along the line perpendicular to the line of charges

∴ Only option B is correct (given consider equilibrium only along line joining charges)

Sol 2: (B) It is not necessary for particle to move along lines of force. Lines of forces only denote the direction of force that exists on particle.

∴ Option B is correct (It may move in a uniform electric field)

To contradict option D, take negative charge.

Sol 3: (D) Charge won't be uniformly distributed if there is an external field, even in an external electric field, the field strength inside sphere is zero (by Gauss law)

Potential must be same at every point of sphere

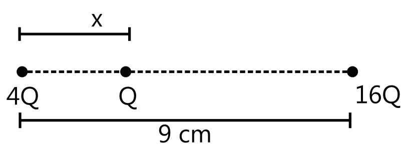
Multiple Correct Choice Type

Sol 4: (A, D)

The resultant electric field will be zero at point closer to B and outside AB (by analysing directions of field and magnitudes)

If a positive charge is placed at P and distributed, the positive charge either goes towards, -Q or moves away from –Q but won't oscillate ($\because \frac{d^2v_P}{dx^2} > 0$) (unstable equilibrium) while negative charge oscillate ($\because \frac{d^2U_n}{d^2x} = -\frac{d^2U_P}{d^2x} < 0$) (Stable equilibrium)

Sol 5: (B, C) For the system to be at minimum potential energy, the higher charged particles should be far apart.



and now potential energy

$$U = \frac{(k)(64Q^2)}{d^2} + \frac{k(16Q^2)}{d-x} + \frac{k4Q^2}{x}$$

$$\frac{dU}{dx} = 0 \Rightarrow \frac{+16}{(d-x)^2} = \frac{4}{x^2}$$

$\Rightarrow \pm 2x = d - x$

$\Rightarrow x \pm 2x = d$

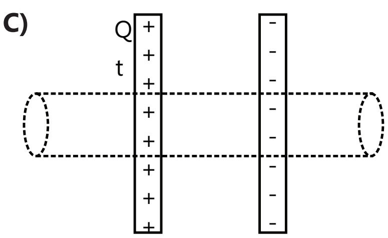
$\Rightarrow x = d/3$ or $x = -d$

$$\Rightarrow x = \frac{9}{3} = 3\text{cm}.$$

Field at Q is

$$= \frac{k(4Q)}{(3cm)^2} - \frac{k(16Q)}{(6cm)^2} = 0$$

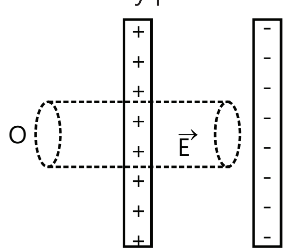
Sol 6: (A, C)



$$\text{flux} = E\,(2\pi r^2) = \frac{q_{enclosed}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

$\therefore E = 0$

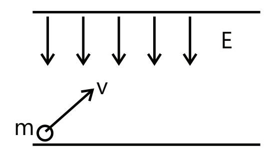
Also for any point between plates



$$\Rightarrow \text{flux} = E.(\pi r^2) = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\sigma \pi r^2}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0} = \text{towards right}$$

Sol 7: (A, C)



$$a = \frac{Eq}{m} + g \quad (\because F = ma = Eq + mg)$$

if $\frac{Eq}{m} + g = 0 \Rightarrow$ (linear motion)

if $\frac{Eq}{m} + g = k$ (constant) $\Rightarrow$ parabolic motion

Assertion Reasoning Type

Sol 8: (C) Electric lines of force represent the force acting on particle at that point

Sol 9: (C) Refer to question 30 of exercise - III

Sol 10: (D) Drawing Gaussian surface won't change electric field.

Sol 11: (C) Statement-I is true, since $Q_{enclosed}$ is same but E at that site changes depending on external charge. But Gauss law is still valid since the flux by the external change is zero.

Sol 12: (C) Statement-I is true by Gauss law.

Statement-II is false since distance between the point charge and the site decreases which changes electric field.

Previous Years' Questions

Sol 1: (A) At r = R. From Gauss's law

$$E\,(4\pi R^2) = \frac{q_{net}}{\varepsilon_0} = \frac{Ze}{\varepsilon_0}$$

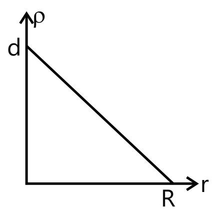
$$\text{or } E = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{R^2}$$

E is independent of a.

Sol 2: (B) For a = 0

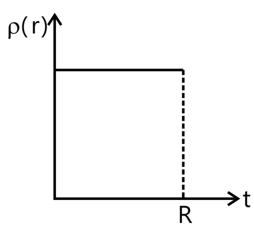
$\rho(r) = \left(-\frac{d}{R}.r + d\right)$

Now $\int_a^R (4\pi r^2)\left(d - \frac{d}{R}r\right)dr$ = net charge = Ze.



Solving this equation, we get $d = \frac{3Ze}{\pi R^3}$

Sol 3: (C) In case of solid sphere of charge of uniform volume density



$E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R^3}.r$

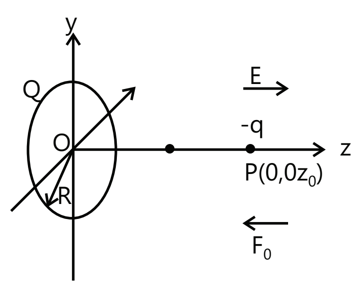
or $E \propto r$

Thus, for E to be linearly dependent on r, volume charge density should be constant.

or a = R

Sol 4: (A, C) Net torque on (–q) about a point (say P) lying over +Q is zero. Therefore, angular momentum of (–q) about point P should remain constant.

Sol 5: (A, C) Let Q be the charge on the ring, the negative charge –q is released from point P (0, 0, z_0). The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be



$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz_0}{(R^2 + z_0^2)^{3/2}}$

E = 0 at centre of the ring because $z_0 = 0$

Force on charge at P will be towards centre as shown, and its magnitude is

$F_e = qE = \frac{1}{4\pi\varepsilon_0}.\frac{Qq}{(R^2 + Z_0^2)^{3/2}}.z_0$ ....(i)

Similarly, when it crosses the origin, the force is again towards centre O.

Thus, the motion of the particle is periodic for all values of z_0 lying between 0 and ∞.

Secondly, if $z_0 << R$, $(R^2 + z_0^2)^{3/2} = R^3$

$F_e = \frac{1}{4\pi\varepsilon_0}.\frac{Qq}{R^3}.z_0$ [From Eq. (i)]

i.e., the restoring force $F_e \propto -z_0$. Hence, the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position.)

Sol 6: (A) Inside the sphere $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$

$\Rightarrow E \propto r$ for $r \le R$

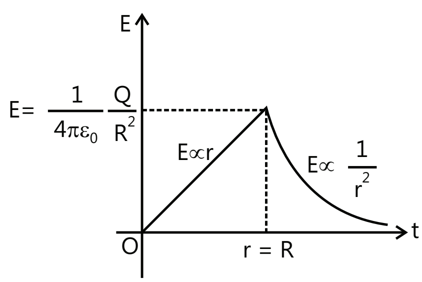
i.e., E at centre = 0 as r = 0

and E at surface = $\frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2}$ as r = R

Outside the sphere

$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2}$ for $r \ge R$ or $E \propto \frac{1}{r^2}$

Thus, variation of electric field (E) with distance (r) from the centre will be as shown



Sol 7: (A, D) From the behaviour of electric lines, we can say that Q_1 is positive and Q_2 is negative. Further, $|Q_1| > |Q_2|$

At some finite distance to the right of Q_2, electric field will be zero. Because electric field due to Q_1 is towards right (away from Q_1) and due to Q_2 is towards left (towards Q_2). But since magnitude of Q_1 is more, the two fields may cancel each other because distance of that point form Q_1 will also be more

Sol 8: (A, B, C, D) Inside a conducting shell electric field is always zero. Therefore, option (a) is correct. When the two are connected, their potentials become the same.

$\therefore V_A = V_B$

or $\frac{Q_A}{R_A} = \frac{Q_B}{R_B}\left(V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\right)$

Since, $R_A > R_B \therefore Q_A > Q_B$

Potential is also equal to, $V = \frac{\sigma R}{\varepsilon_0}$, $V_A = V_B$

$\therefore \sigma_A R_A = \sigma_B R_B$ or $\frac{\sigma_A}{\sigma_B} = \frac{R_A}{R_B}$ or $\sigma_A < \sigma_B$

Electric field on surface, $E = \frac{\sigma}{\varepsilon_0}$ or $E \propto \sigma$

Since $\sigma_A < \sigma_B \therefore E_A < E_B$

Sol 9: (A, C, D) $\phi_{out} = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

By symmetry

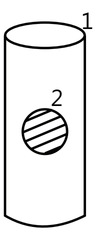
Sol 10: (1) + (2) = Complete cylinder

$E_1 + E_2 = E$

$E = \frac{\rho \times \pi R^2}{2\pi\varepsilon_0(2R)} = \frac{\rho R}{4\varepsilon_0}$

$E_2 = \rho \times \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 \times \frac{1}{4\pi\varepsilon_0\left(4R^2\right)} = \frac{\rho R}{24 \times 4\varepsilon_0}$

$E_1 = E - E_2 \Rightarrow \frac{\rho R}{4\varepsilon_0}\left[1 - \frac{1}{24}\right] = \frac{\rho R}{4\varepsilon_0}\frac{23}{4\times 6} = \frac{23\rho R}{16\varepsilon_0 \times 6}$



Sol 11: (B, D)

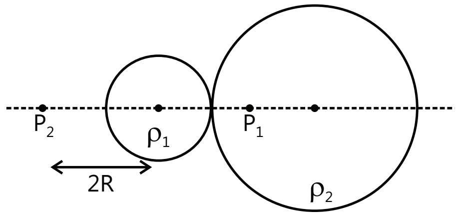
At point P_1, $\frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\pi R^3}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$

$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3}$

$\frac{\rho_1}{\rho_2} = 4$

At point P_2, $\frac{\rho_1(4/3)\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\pi 8R^3}{(5R)^2} = 0$

$\therefore \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}$



Sol 12: (C, D) In triangle PC_1C_2

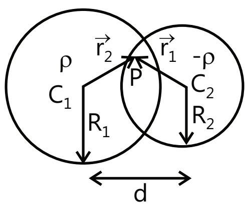
$\vec{r}_2 = \vec{d} + \vec{r}_1$

The electrostatic field at point P is

$\bar{E} = \frac{K\left(\rho\frac{4}{3}\pi R_1^3\right)\vec{r}_2}{R_1^3} + \frac{K\left(\rho\frac{4}{3}\pi R_2^3\right)(-\vec{r}_1)}{R_2^3}$

$\bar{E} = K\rho\frac{4}{3}\pi\left(\vec{r}_2 - \vec{r}_1\right)$

$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{d}$



Sol 13: (C) $\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$

$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi\varepsilon_0 r_0^2}$, $E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{\pi\varepsilon_0 r_0}$, $E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\varepsilon_0}$

$\therefore \quad E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$

Sol 14: (C)

For point outside dielectric sphere $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

For point inside dielectric sphere $E = E_s\frac{r}{R}$

Exact Ratio $E_1 : E_2 : E_3 = 2 : 4 : 1$

Sol 15: (A)

P: By Q_1 and Q_4, Q_3 and Q_2 F is in +y

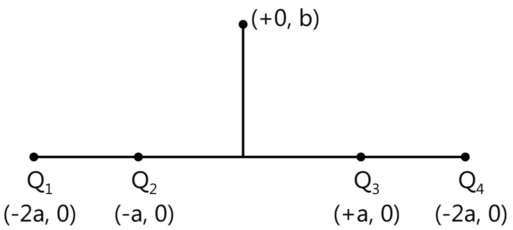
Q: By Q_1 and Q_4, Q_2 and Q_3 F is in + ve x.

R: By Q_1 and Q_4, F is in + ve y

By Q_2 and Q_3, F is in –ve y

But later has more magnitude, since its closer to (0, b). Therefore net force is in –y

S: By Q_1 and Q_4, F is in +ve x and by Q_2 and Q_3, F is in –x, but later is more in magnitude, since its closer to (0, b). Therefore net force is in –ve x.



Sol 16: (C) In Case I:

$$\bar{F} = \frac{\lambda q}{2\pi\varepsilon_0(r+x)}\hat{i} + \frac{\lambda q}{2\pi\varepsilon_0(r-x)}(-\hat{i}) = \frac{\lambda q}{\pi\varepsilon_0 r^2}x(-\hat{i})$$

Hence +q, charge will performs SHM with time period

$$T = 2\pi\sqrt{\frac{\pi r^2 \varepsilon_0 m}{\lambda q}}$$

In case II: Resultant force will act along the direction of displacement.

Sol 17: (D) $\vec{E} = \frac{\rho}{3\varepsilon_0}\overline{C_1C_2}$

$C_1 \Rightarrow$ Centre of sphere and $C_2 \Rightarrow$ centre of cavity.