1

Sets

Numerical 2023

Question:1 The number of elements in the set

$\{n \in \mathbb{N} : 10 \leq n \leq 100$ and $3^n - 3$ is a multiple of $7\}$ is __________.

JEE Main 2023 (Online) 15th April Morning Shift

Question: 2

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by

$R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$. Then the number of elements in R is __________.

JEE Main 2023 (Online) 15th April Morning Shift

Question:3

Let $\mathrm{A} = \{-4, -3, -2, 0, 1, 3, 4\}$ and $\mathrm{R} = \{(a, b) \in \mathrm{A} \times \mathrm{A} : b = |a|$ or $b^2 = a + 1\}$ be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is _________

JEE Main 2023 (Online) 13th April Evening Shift

Question:4

The number of relations, on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is _________.

JEE Main 2023 (Online) 12th April Morning Shift

Question:5

The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is ________.

JEE Main 2023 (Online) 10th April Morning Shift

Question:6

Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ___________.

JEE Main 2023 (Online) 8th April Morning Shift

Question:7

Let $\mathrm{A} = \{1, 2, 3, 4, \ldots., 10\}$ and $\mathrm{B} = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \left\{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\right\}$ is _________.

JEE Main 2023 (Online) 6th April Morning Shift

Question:8 Let S = {1, 2, 3, 5, 7, 10, 11}. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is ___________.

JEE Main 2023 (Online) 25th January Morning Shift

Question:9 The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is _________.

JEE Main 2023 (Online) 24th January Evening Shift

Numerical Answer Key

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1. Ans. (15)
2. Ans. (6)
3. Ans. (7)
4. Ans. (4)
5. Ans. (6)
6. Ans. (19)
7. Ans. (18)
8. Ans. (43)
9. Ans. (13)

Numerical Explanation

Ans.1

To determine the number of elements in the given set, we need to find how many natural numbers n between 10 and 100 (inclusive) satisfy the condition that $3^n - 3$ is a multiple of 7.

Recall that for any integers a and b, a is a multiple of b if there exists an integer k such that $a = bk$. So in our case, we need to find how many n satisfy the equation $3^n - 3 = 7k$ for some integer k.

Notice that $3^n - 3 = 3(3^{n-1} - 1)$. We want this expression to be a multiple of 7. Let's explore a few powers of 3 modulo 7:

$3^1 \equiv 3 \pmod{7}$

$3^2 \equiv 9 \equiv 2 \pmod{7}$

$3^3 \equiv 27 \equiv 6 \pmod{7}$

$3^4 \equiv 81 \equiv 4 \pmod{7}$

$3^5 \equiv 243 \equiv 5 \pmod{7}$

$3^6 \equiv 729 \equiv 1 \pmod{7}$

We observe that $3^n \pmod{7}$ follows a cycle of length 6. So, $3^{n-1} \pmod{7}$ also follows the same cycle, but shifted:

$3^0 \equiv 1 \pmod{7}$

$3^1 \equiv 3 \pmod{7}$

$3^2 \equiv 2 \pmod{7}$

$3^3 \equiv 6 \pmod{7}$

$3^4 \equiv 4 \pmod{7}$

$3^5 \equiv 5 \pmod{7}$

We want $3(3^{n-1} - 1) \equiv 0 \pmod{7}$, which means that $3^{n-1} - 1 \equiv 0 \pmod{7}$. From the cycle above, we see that this is true when $n - 1$ is a multiple of 6, or equivalently, when n is one more than a multiple of 6.

Now let's find the multiples of 6 between 10 and 100:

$$12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96$$

Adding 1 to each of these values, we get the set of natural numbers n that satisfy the given condition:

$$13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, 91, 97$$

There are 15 elements in this set. Therefore, the number of elements in the given set is $\boxed{15}$.

Ans.2

$$2a + 3b = 4c + 5d$$

Given A = {1, 2, 3, 4}, the maximum value of $2a + 3b$ is 20, when (a, b) = (4, 4), anc the minimum value of $4c + 5d$ is 9, when (c, d) = (1, 1). Therefore, the possible values for $2a + 3b = 4c + 5d$ are 9, 13, 14, 17, 18, and 19.

Now, let's find the combinations of (a, b), (c, d) that satisfy the given equation:

1. $2a + 3b = 9 \Rightarrow (a, b) = (3, 1) \Rightarrow (c, d) = (1, 1)$

2. $2a + 3b = 13 \Rightarrow (a, b) = (2, 3) \Rightarrow (c, d) = (2, 1)$

3. $2a + 3b = 14 \Rightarrow (a, b) = (4, 2) \Rightarrow (c, d) = (1, 2)$

4. $2a + 3b = 14 \Rightarrow (a, b) = (1, 4) \Rightarrow (c, d) = (1, 2)$

5. $2a + 3b = 17 \Rightarrow (a, b) = (4, 3) \Rightarrow (c, d) = (3, 1)$
6. $2a + 3b = 18 \Rightarrow (a, b) = (3, 4) \Rightarrow (c, d) = (2, 2)$

There are a total of 6 elements in the relation R for the given equation with the specified values of a, b, c, and d.

Ans.3

$$A=\{-4,-3,-2,0,1,3,4\}$$

$$R=\{(-4,4),(-3,3),(0,0),(1,1)$$
$$(3,3),(4,4),(0,1),(3,-2)\}$$

Relation to be reflexive $(a,a) \in R \forall a \in A$

$\Rightarrow (-4,-4),(-3,-3),(-2,-2)$ also should be added in R.

Relation to be symmetric if $(a,b) \in R$, then $(b,a) \in R \forall a,b \in A$

$\Rightarrow (4,-4),(3,-3),(1,0),(-2,3)$ also should be added in R

$\Rightarrow$ Minimum number of elements to be added to $R = 3+4=7$

Ans.4

$(1,1),(2,2),(3,3) \in R$

Since $(1,2),(2,3) \in R$, $(1,3)$ must $\in R$

Possible cases :

Case-1 : All of $(2,1),(3,2),(3,1) \notin R \rightarrow 1$ relation.

Case-2 : Only one of $(2,1),(3,2),(3,1) \in R \rightarrow 3$ relations.

Note that exactly two of $(2,1),(3,2),(3,1) \in R$ is not possible because if two of these $\in R$, third must $\in R$ to make relation transitive.

1. R_1 = {(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)}

Here, none of (2,1), (3,2), (3,1) are in R_1.

2. R_2 = {(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (2,1)}

Here, only (2,1) is in R_2, and neither (3,2) nor (3,1) are in R_2.

3. R_3 = {(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (3,2)}

Here, only (3,2) is in R_3, and neither (2,1) nor (3,1) are in R_3.

4. R_4 = {(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (3,1)}

Here, only (3,1) is in R_4, and neither (2,1) nor (3,2) are in R_4.

Ans.8

Elements of the type $3\mathrm{k} = 3$

Elements of the type $3\mathrm{k} + 1 = 1, 7, 9$

Elements of the type $3\mathrm{k} + 2 = 2, 5, 11$

Subsets containing one element $S_1 = 1$

Subsets containing two elements

$$S_2 = {}^3C_1 \times {}^3C_1 = 9$$

Subsets containing three elements

$$\mathrm{S}_3 = {}^3\mathrm{C}_1 \times {}^3\mathrm{C}_1 + 1 + 1 = 11$$

Subsets containing four elements

$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$

Subsets containing five elements

$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$

Subsets containing six elements $S_6 = 1$

Subsets containing seven elements $S_7 = 1$

$\Rightarrow$ sum $= 43$

Ans.9

$R = \{(a, b)(b, c)(b, d)\}$

$S : \{a, b, c, d\}$

Adding $(a, a), (b, b), (c, c), (d, d)$ make reflexive.

Adding $(b, a), (c, b), (d, b)$ make Symmetric

And adding $(a, d), (a, c)$ to make transitive

Further $(d, a) \& (c, a)$ to be added to make Symmetricity.

Further $(c, d) \& (d, c)$ also be added.

So total 13 elements to be added to make equivalence.

MCQ 2023 (Single Correct Answer)

Question:1

Let $\mathrm{A}=\{1,3,4,6,9\}$ and $\mathrm{B}=\{2,4,5,8,10\}$. Let R be a relation defined on $\mathrm{A} \times \mathrm{B}$ such that $\mathrm{R}=\{((a_1, b_1),(a_2, b_2)): a_1 \leq b_2$ and $b_1 \leq a_2\}$. Then the number of elements in the set R is :

JEE Main 2023 (Online) 11th April Evening Shift

A 180

B 26

C 52

D 160

Question:2

An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

JEE Main 2023 (Online) 11th April Morning Shift

A 10

B 15

C 21

D 9

Question:3

Let $\mathrm{A}=\{2,3,4\}$ and $\mathrm{B}=\{8,9,12\}$. Then the number of elements in the relation $\mathrm{R}=\{((a_1, \mathrm{b}_1),(a_2, \mathrm{b}_2)) \in(A \times B, A \times B): a_1$ divides b_2 and a_2 divides $\mathrm{b}_1\}$ is

JEE Main 2023 (Online) 10th April Evening Shift

A 18

B 24

C 36

D 12

Question:4

Let $\mathrm{A}=\{1,2,3,4,5,6,7\}$. Then the relation $\mathrm{R}=\{(x, y) \in \mathrm{A} \times \mathrm{A}: x+y=7\}$ is

JEE Main 2023 (Online) 8th April Evening Shift

A reflexive but neither symmetric nor transitive

B transitive but neither symmetric nor reflexive

C symmetric but neither reflexive nor transitive

D an equivalence relation

Question:5

Let $P(S)$ denote the power set of $S=\{1,2,3, \ldots ., 10\}$. Define the relations R_1 and R_2 on $P(S)$ as $\mathrm{AR}_1 \mathrm{B}$ if $(\mathrm{A} \cap \mathrm{B}^{\mathrm{c}}) \cup(\mathrm{B} \cap \mathrm{A}^{\mathrm{c}})=\emptyset$ and $\mathrm{AR}_2 \mathrm{B}$ if $\mathrm{A} \cup \mathrm{B}^{\mathrm{c}}=\mathrm{B} \cup \mathrm{A}^{\mathrm{c}}, \forall \mathrm{A}, \mathrm{B} \in \mathrm{P}(\mathrm{S})$.
Then :

JEE Main 2023 (Online) 1st February Evening Shift

A only R_2 is an equivalence relation

B both R_1 and R_2 are not equivalence relations

C both R_1 and R_2 are equivalence relations

D only R_1 is an equivalence relation

Question:6

Let R be a relation on $\mathbb{R}$, given by $R = \{(a, b) : 3a - 3b + \sqrt{7}$ is an irrational number $\}$. Then R is

JEE Main 2023 (Online) 1st February Morning Shift

A an equivalence relation

B reflexive and symmetric but not transitive

C reflexive and transitive but not symmetric

D reflexive but neither symmetric nor transitive

Question:7

Among the relations

$$\mathrm{S} = \left\{(\mathrm{a}, \mathrm{b}) : \mathrm{a}, \mathrm{b} \in \mathbb{R} - \{0\}, 2 + \frac{\mathrm{a}}{\mathrm{b}} > 0\right\}$$

and $\mathrm{T} = \left\{(\mathrm{a}, \mathrm{b}) : \mathrm{a}, \mathrm{b} \in \mathbb{R}, \mathrm{a}^2 - \mathrm{b}^2 \in \mathbb{Z}\right\}$,

Question:8

Let R be a relation on $\mathrm{N} \times \mathbb{N}$ defined by $(a, b) \mathrm{R}(c, d)$ if and only if $a d(b-c)=b c(a-d)$. Then R is

JEE Main 2023 (Online) 31st January Morning Shift

A. symmetric and transitive but not reflexive

B. reflexive and symmetric but not transitive

C. transitive but neither reflexive nor symmetric

D. symmetric but neither reflexive nor transitive

Question:9

The minimum number of elements that must be added to the relation $\mathrm{R}=\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{c})\}$ on the set $\{a, b, c\}$ so that it becomes symmetric and transitive is :

JEE Main 2023 (Online) 30th January Morning Shift

A. 7

B. 3

C. 4

D. 5

Question:10

Let R be a relation defined on $\mathbb{N}$ as $a \mathrm{R} b$ if $2a+3b$ is a multiple of $5, a, b \in \mathbb{N}$. Then R is

JEE Main 2023 (Online) 29th January Evening Shift

A. an equivalence relation

B. non reflexive

C. symmetric but not transitive

D. transitive but not symmetric

Question:11

The relation $\mathrm{R}=\{(\mathrm{a}, \mathrm{b}): \operatorname{gcd}(\mathrm{a}, \mathrm{b})=1,2 \mathrm{a} \neq \mathrm{b}, \mathrm{a}, \mathrm{b} \in \mathbb{Z}\}$ is :

JEE Main 2023 (Online) 24th January Morning Shift

A. reflexive but not symmetric

B. transitive but not reflexive

C. symmetric but not transitive

D. neither symmetric nor transitive

MCQ Answers Key

1. Ans. (D)
2. Ans. (C)
3. Ans. (C)
4. Ans. (C)
5. Ans. (C)
6. Ans. (D)
7. Ans. (D)
8. Ans. (D)
9. Ans. (A)
10. Ans. (A)
11. Ans. (D)

MCQ Explanation

Ans.1

Given that the sets are $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$, for the relation R on the set $A \times B$, we need to find the combinations of pairs that satisfy the conditions $a_1 \leq b_2$ and $b_1 \leq a_2$.

We find the number of combinations by considering the possible values for b_2 for each a_1 and the possible values for a_2 for each b_1 :

For each a_1 in $A = \{1, 3, 4, 6, 9\}$, the number of valid b_2 values in $B = \{2, 4, 5, 8, 10\}$ are :

- For $a_1 = 1$, there are 5 choices for b_2.

- For $a_1 = 3$, there are 4 choices for b_2.

- For $a_1 = 4$, there are 4 choices for b_2.

- For $a_1 = 6$, there are 2 choices for b_2.

- For $a_1 = 9$, there is 1 choice for b_2.

This results in a total of $5 + 4 + 4 + 2 + 1 = 16$ possible pairs (a_1, b_2).

Similarly, for each b_1 in B, the number of valid a_2 values in A are :

- For $b_1 = 2$, there are 4 choices for a_2.

- For $b_1 = 4$, there are 3 choices for a_2.

- For $b_1 = 5$, there are 2 choices for a_2.

- For $b_1 = 8$, there is 1 choice for a_2.

- For $b_1 = 10$, there are no choices for a_2.

This results in a total of $4 + 3 + 2 + 1 + 0 = 10$ possible pairs (b_1, a_2).

Therefore, the total number of elements in the relation R, which satisfies the given conditions, is $16 \times 10 = 160$.

So, the correct answer is 160.

Ans.2

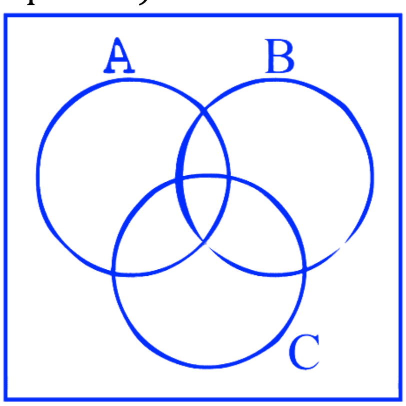
We are given the number of medals for events A, B, and C which are 48, 25, and 18 respectively. We are also given that the total number of unique medal recipients across all events is 60 and that 5 people received a medal in all three events.

Using the Principle of Inclusion and Exclusion (PIE), we know that the total number of unique medal recipients can be calculated by adding the number of medal recipients in each event, subtracting the number of people who received a medal in any two events (to correct for double counting), and then adding back the number of people who received a medal in all three events (since we subtracted these people too much).

Mathematically, this can be represented as :

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

However, we want to find the total number of people who received a medal in any two events (which is represented by $|A \cap B| + |B \cap C| + |C \cap A|$ in the equation).



To find this, we rearrange the PIE formula to solve for $|A \cap B| + |B \cap C| + |C \cap A|$:

$|A \cap B| + |B \cap C| + |C \cap A| = |A| + |B| + |C| + |A \cap B \cap C| - |A \cup B \cup C|$

Substituting the given values, we find that the total number of people who received a medal in any two events is 48 + 25 + 18 + 5 - 60 = 36.

However, this includes people who received a medal in all three events, and we want to find the number of people who received a medal in exactly two events. Therefore, we need to subtract the people who received a medal in all three events from our calculated value.

Since each person who received a medal in all three events is counted three times in |A ∩ B| + |B ∩ C| + |C ∩ A| (once for each pair of events), we subtract three times the number of people who received a medal in all three events from our calculated value:

Number of people who received a medal in exactly two events = |A ∩ B| + |B ∩ C| + |C ∩ A| - 3 × |A ∩ B ∩ C|

Substituting the values we know, we find that the number of people who received a medal in exactly two events is 36 - 3 × 5 = 21.

Therefore, 21 people received a medal in exactly two of the three events.

Ans.5

$S = \{1, 2, 3, \ldots\ldots 10\}$

$P(S) =$ power set of S

$AR_1B \Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = \phi$

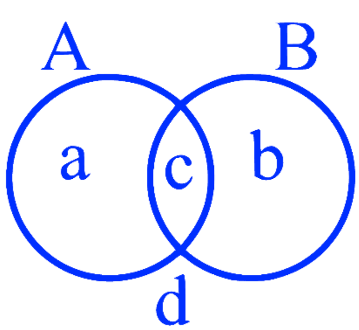
R_1 is reflexive, symmetric

For transitive

$(A \cap \overline{B}) \cup (\overline{A} \cap B) = \phi; \{a\} = \phi = \{b\} \therefore A = B$

$(B \cap \overline{C}) \cup (\overline{B} \cap C) = \phi \therefore B = C$

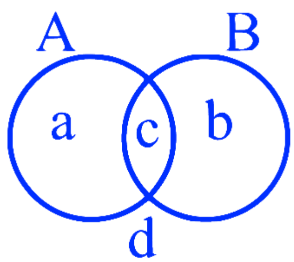
$\therefore A = C$ equivalence.



$R_2 \equiv A \cup \overline{B} = \overline{A} \cup B$

$R_2 \rightarrow$ Reflexive, symmetric

For transitive :



$A \cup \overline{B} = \overline{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$

$\{a\} = \{b\} \therefore A = B$

$B \cup \overline{C} = \overline{B} \cup C \Rightarrow B = C$

$\therefore A = C \quad \therefore A \cup \overline{C} = \overline{A} \cup C \therefore$ Equivalence

Ans.6

$3a - 3a + \sqrt{7}$ is an irrational number $\forall a \in RR$ is reflexive

For symmetric :

Let $3a - 3b + \sqrt{7}$ is an irrational number

$\Rightarrow 3b - 3a + \sqrt{7}$ is an irrational number

For example, Let $3a - 3b = \sqrt{7}$

$\sqrt{7} + \sqrt{7}$ is irrational but $-\sqrt{7} + \sqrt{7}$ is not.

$\therefore R$ is not symmetric

For transitive :

Let $3a - 3b + \sqrt{7}$ is irrational and $3b - 3c + \sqrt{7}$ is irrational.

$\Rightarrow 3a - 3c + \sqrt{7}$ is irrational.

For example, take $a = 0, b = -\sqrt{7}, c = \frac{\sqrt{7}}{3}$

R is not transitive.

Ans.7

For relation $\mathrm{T} = \mathrm{a}^2 - \mathrm{b}^2 = -\mathrm{I}$

Then, (b, a) on relation R

$\Rightarrow \mathrm{b}^2 - \mathrm{a}^2 = -\mathrm{I}$

$\therefore \mathrm{T}$ is symmetric

$\mathrm{S} = \left\{(\mathrm{a}, \mathrm{b}) : \mathrm{a}, \mathrm{b} \in \mathrm{R} - \{0\}, 2 + \frac{\mathrm{a}}{\mathrm{b}} > 0\right\}$

$2 + \frac{\mathrm{a}}{\mathrm{b}} > 0 \Rightarrow \frac{\mathrm{a}}{\mathrm{b}} > -2, \Rightarrow \frac{\mathrm{b}}{\mathrm{a}} < \frac{-1}{2}$

If $(b, a) \in \mathbf{S}$ then

$2 + \frac{\mathrm{b}}{\mathrm{a}}$ not necessarily positive

$\therefore \mathrm{S}$ is not symmetric

Ans.8

Given, $(a, b) R(c, d) \Rightarrow ad(b - c) = bc(a - d)$

Symmetric :

(c, d) $R(a, b) \Rightarrow \mathrm{cb}(\mathrm{d} - \mathrm{a}) = \mathrm{da}(\mathrm{c} - \mathrm{b})$

$\Rightarrow$ Symmetric.

Reflexive :

(a, b) R (a, b) $\Rightarrow ab(b - a) \neq ba(a - b)$

$\Rightarrow$ Not reflexive.

Transitive :

$(2, 3)\mathrm{R}(3, 2)$ and $(3, 2)\mathrm{R}(5, 30)$ but

$((2, 3), (5, 30)) \notin \mathrm{R}$

$\Rightarrow$ Not transitive.

Ans.9

For symmetric $(b, a), (c, b) \in R$

For transitive $(a, c) \in R$

$\Rightarrow (c, a) \in R$

$\therefore (a, b), (b, a) \in R$

$\Rightarrow (a, a) \in R$

$(b, c), (c, b) \in R$

$\Rightarrow (b, b) \in R, (c, c) \in R$

7 elements must be added

Ans.10

a R b if 2a + 3b = 5m, m $\in l$

(1) $(a, a) \in R$ as $2a + 3a = 5a, a \in N$

Hence, R is reflexive

(2) If $(a, b) \in R$ then $2a + 3 = 5m$

Now, $5(a + b) = 5n$

$3a + 2b + 2a + 3b = 5n$

$\therefore 3a + 2b = 5(n - m)$

$\therefore (b, a) \in R$

$\therefore$ R is symmetric

(3) If $(a, b) \in R$ and $(b, c) \in R$ then

$2a + 3b = 5m, 2b + 3c = 5n$

$\Rightarrow 2a+5b+3c=5(m+n)$

$\Rightarrow 2a+3c=5(m=n-b)$

$\therefore (a,c)\in R$

$\therefore$ R is transitive

Hence, R is equivalence relation.

Option (1) is correct.

Ans.11

Given,

(a, b) belongs to relation R if $\gcd(a,b)=1, 2a\neq b$.

Here $\gcd$ means greatest common divisor. $\gcd$ of two numbers is the largest number that divides both of them.

(1) For Reflexive,

In aRa, $\gcd(a,a)=a$

$\therefore$ This relation is not reflexive.

(2) For Symmetric:

Take $a=2, b=1 \Rightarrow \gcd(2,1)=1$ Also $2a=4\neq b$

Now $\gcd(b,a)=1 \Rightarrow \gcd(1,2)=1$

and 2b should not be equal to a

But here, $2b=2=a$

$\Rightarrow \mathrm{R}$ is not Symmetric

(3) For Transitive:

Let $\mathrm{a}=14,\ \mathrm{b}=19, \mathrm{c}=21$

$\gcd(\mathrm{a},\mathrm{b})=1, 2a\neq b$

Numerical 2022

Question:1

Let $S=\{4,6,9\}$ and $T=\{9,10,11,\ldots,1000\}$. If $A=\{a_1+a_2+\ldots+a_k : k\in\mathbf{N}, a_1,a_2,a_3,\ldots,a_k \epsilon S\}$, then the sum of all the elements in the set $T-A$ is equal to _________.

JEE Main 2022 (Online) 29th July Morning Shift

Question:2

Let $A=\{1,2,3,4,5,6,7\}$ and $B=\{3,6,7,9\}$. Then the number of elements in the set $\{C\subseteq A : C\cap B\neq\phi\}$ is __________.

JEE Main 2022 (Online) 26th July Evening Shift

Question:3

Let $A=\{1,2,3,4,5,6,7\}$. Define $B=\{T\subseteq A$: either $1\notin T$ or $2\in T\}$ and $C=\{T\subseteq A : T$ the sum of all the elements of T is a prime number $\}$. Then the number of elements in the set $B\cup C$ is ______________.

JEE Main 2022 (Online) 25th July Evening Shift

Question:4

Let R_1 and R_2 be relations on the set {1, 2, ..., 50} such that

$R_1 = \{(p, p^n) : p$ is a prime and $n\geq 0$ is an integer$\}$ and

$R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or $1\}$.

Then, the number of elements in $R_1 - R_2$ is ______________.

JEE Main 2022 (Online) 28th June Morning Shift

Question:5

Let A = $\{n \in N$: H.C.F. $(n, 45) = 1\}$ and

Let B = $\{2k : k \in \{1, 2, ..., 100\}\}$. Then the sum of all the elements of $A \cap B$ is ____________.

JEE Main 2022 (Online) 26th June Morning Shift

Question:6

Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then A + B is equal to _____________.

JEE Main 2022 (Online) 26th June Morning Shift

Question:7

The sum of all the elements of the set $\{\alpha \in \{1, 2, \ldots\ldots, 100\} : HCF(\alpha, 24) = 1\}$ is _________.

JEE Main 2022 (Online) 24th June Evening Shift

Numerical Answers Key

1. Ans. (11)
2. Ans. (112)
3. Ans. (107)
4. Ans. (8)
5. Ans. (5264)
6. Ans. (1100)
7. Ans. (1633)

Numerical Explanation

Ans.1

Here $S = \{4, 6, 9\}$

And $T = \{9, 10, 11, \ \ldots\ldots, \ 1000\}$.

We have to find all numbers in the form of $4x + 6y + 9z$, where $x, y, z \in \{0, 1, 2, \ \ldots\ldots\}$.

If a and b are coprime number then the least number from which all the number more than or equal to it can be express as $ax + by$ where $x, y \in \{0, 1, 2, \ \ldots\ldots\}$ is $(a - 1)\,.\,(b - 1)$.

Then for $6y + 9z = 3(2y + 3z)$

All the number from $(2 - 1)\,.\,(3 - 1) = 2$ and above can be express as $2x + 3z$ (say t).

Now $4x + 6y + 9z = 4x + 3(t + 2)$

$= 4x + 3t + 6$

again by same rule $4x + 3t$, all the number from $(4 - 1)\,(3 - 1) = 6$ and above can be express from $4x + 3t$.

Then $4x + 6y + 9z$ express all the numbers from 12 and above.

again 9 and 10 can be express in form $4x + 6y + 9z$.

Then set $A = \{9, 10, 12, 13, \ \ldots\ldots, \ 1000\}$.

Then $T - A = \{11\}$

Only one element 11 is there.

Sum of elements of $T - A = 11$

Ans.2

As C ∩ B $\neq \phi$, c must be not be formed by {1, 2, 4, 5}

∴ Number of subsets of A = 2^7 = 128

and number of subsets formed by {1, 2, 4, 5} = 16

∴ Required no. of subsets = $2^7 - 2^4$ = 128 − 16 = 112

Ans.3

$\because (B \cup C)' = B' \cap C'$

B' is a set containing sub sets of A containing element 1 and not containing 2.

And C' is a set containing subsets of A whose sum of elements is not prime.

So, we need to calculate number of subsets of {3, 4, 5, 6, 7} whose sum of elements plus 1 is composite.

Number of such 5 elements subset = 1

Number of such 4 elements subset = 3 (except selecting 3 or 7)

Number of such 3 elements subset = 6 (except selecting {3, 4, 5}, {3, 6, 7}, {4, 5, 7} or {5, 6, 7})

Number of such 2 elements subset = 7 (except selecting {3, 7}, {4, 6}, {5, 7})

Number of such 1 elements subset = 3 (except selecting {4} or {6})

Number of such 0 elements subset = 1

$n(B' \cap C') = 21 \Rightarrow n(B \cup C) = 2^7 - 21 = 107$

Ans.4

Given, $R_1 = \{(p, p^n) : p$ is a Prime and $n \geq 0$ is an integer $\}$

and, set $A = \{1, 2, 3 \ldots\ldots .50\}$

p is a Prime number which can take 15 values $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43$ and 47

$\therefore$ We can calculate no. of elements in R_1

$\mathrm{R}_1 = (2, 2^0), (2, 2^1), (2, 2^2)\ (2, 2^3) \ldots\ldots (2, 2^5) = 6$ number of ordered pairs

$(3, 3^0), (3, 3^1), (3, 3^2) \ldots\ldots\ldots (3, 3^3) = 4$ number of order paris

$(5, 5^0), (5, 5^1), (5, 5^2) \ldots\ldots\ldots\ldots = 3$ number of order paris

$(7, 7^0) \ldots\ldots\ldots (7, 7^2) \ldots\ldots\ldots\ldots = 3$ number of order paris

$(11, 11^0)$ and $(11, 11^1) = 2$ number of order paris

$\left(13,13^{0}\right)$ and $\left(13,13^{1}\right)=2$ number of order paris

$\therefore$ For the 11 prime numbers ($11,13,17,19,23,29,31,37,41,43$ and 47), n can only be 0, 1 (two pairs each).

$\therefore n\left(\mathrm{R}_{1}\right)=6+4+3+3+(2\times 11)=38$

$\mathrm{R}_{2}=\left(p,p^{n}\right)$, where n = 0 or 1

$\left(2,2^{0}\right),\left(2,2^{1}\right)\left(3,3^{0}\right)\left(3,3^{1}\right)\ldots\ldots\left(47,47^{0}\right)\left(47,47^{1}\right)$

Two ordered pairs of each element $n\left(R_{2}\right)=2\times 15=30$ elements

Hence $R_{1}-R_{2}=38-30=8$

Ans.5

Sum of all elements of A ∩ B = 2 [Sum of natural numbers upto 100 which are neither divisible by 3 nor by 5]

$=2\left[\frac{100\times 101}{2}-3\left(\frac{33\times 34}{2}\right)-5\left(\frac{20\times 21}{2}\right)+15\left(\frac{6\times 7}{2}\right)\right]$

$=10100-3366-2100+630$

$=5264$

Ans.6

$\sum_{i=1}^{10}\sum_{j=1}^{10}\{i,\ j\}$

= {1, 1} {1, 2} {1, 3} ..... {1, 10}

{2, 1} {2, 2} {2, 3} ..... {2, 10}

{3, 1} {3, 2} {3, 3} ..... {3, 10}

$\vdots$

{10, 1} {10, 2} {10, 3} ..... {10, 10}

Now, $A=\sum_{i=1}^{10}\sum_{j=1}^{10}min\{i,\ j\}$

= minimum between i and j in all sets and summation of all those values.

and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,\ j\}$

= maximum between i and j in all sets and summation of all those values.

For 1 :

1 is minimum in sets =

{1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {1, 7}, {1, 8}, {1, 9}, {1, 10}, {2, 1}, {3, 1}, {4, 1}, {5, 1}, {6, 1}, {7, 1}, {8, 1}, {9, 1}, {10, 1}

∴ 1 is minimum in 19 sets

1 is maximum in {1, 1} sets.

∴ 1 is maximum and minimum in total 20 sets.

∴ Sum of 1 in all those sets = $1 \times 20 = 20$

For 2 :

2 is minimum in sets =

{2, 2}, {2, 3}, {2, 4}, {2, 5}, {2, 6}, {2, 7}, {2, 8}, {2, 9}, {2, 10}, {3, 2}, {4, 2}, {5, 2}, {6, 2}, {7, 2}, {8, 2}, {9, 2}, {10, 2}

∴ 2 is minimum in 17 sets

2 is maximum in sets = {1, 2}, {2, 1}, {2, 2}

∴ 2 is maximum and minimum in 20 sets.

∴ Sum of 2 in all those sets = $2 \times 20 = 40$

Similarly 3 is maximum and minimum in 20 sets.

∴ Sum of 3 in all those sets = $20 \times 3 = 60$

⋮

Similarly, 10 is maximum and minimum in 20 sets.

∴ Sum of 10 in all those sets = $20 \times 10 = 200$

∴ $A + B = 20 + 20 \times 2 + 20 \times 3 + \ldots\ldots + 20 \times 10$

$= 20(1 + 2 + 3 + \ldots\ldots + 10)$

$= 20 \times \frac{10 \times 11}{2}$

$= 1100$

Ans.7

The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.

Sum of these numbers = 96

There are four such blocks and a number 97 is there upto 100.

$\therefore$ Complete sum

$= 96 + (24 \times 8 + 96) + (48 \times 8 + 96) + (72 \times 8 + 96) + 97$

$= 1633$

MCQ 2022

Question:1

For $\alpha \in \mathbf{N}$, consider a relation R on $\mathbf{N}$ given by $\mathrm{R} = \{(x, y) : 3x + \alpha y$ is a multiple of $7\}$. The relation R is an equivalence relation if and only if :

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Question:2

Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by

$a\,R_1\,b \Leftrightarrow ab \geq 0$ and $a\,R_2\,b \Leftrightarrow a \geq b$

Then,

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Question:3

Let a set $A = A_1 \cup A_2 \cup ..... \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i$, $1 \leq i \leq k\}$. Then, R is :

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Question:4

Let $R_1 = \{(a, b) \in N \times N : |a - b| \leq 13\}$ and

$R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$. Then on N :

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MCQ Answers Key

1. Ans. (D)
2. Ans. (D)
3. Ans. (D)
4. Ans. (B)

MCQ Explanation

Ans.1

$R = \{(x, y) : 3x + \alpha y$ is multiple of $7\}$, now R to be an equivalence relation

(1) R should be reflexive : $(a, a) \in R \, \forall \, a \in N$

$\therefore 3a + a\alpha = 7k$

$\therefore (3 + \alpha)a = 7k$

$\therefore 3 + \alpha = 7k_1 \Rightarrow \alpha = 7k_1 - 3$

$= 7k_1 + 4$

(2) R should be symmetric : $aRb \Leftrightarrow bRa$

$aRb : 3a + (7k - 3)b = 7m$

$\Rightarrow 3(a - b) + 7kb = 7m$

$\Rightarrow 3(b - a) + 7ka = 7m$

So, $aRb \Rightarrow bRa$

$\therefore$ R will be symmetric for $a = 7k_1 - 3$

(3) Transitive : Let $(a, b) \in R, \ (b, c) \in R$

$\Rightarrow 3a + (7k - 3)b = 7k_1$ and

$3b + (7k_2 - 3)c = 7k_3$

Adding $3a + 7kb + (7k_2 - 3)\,c = 7(k_1 + k_3)$

$3a + (7k_2 - 3)\,c = 7m$

$\therefore (a, c) \in R$

$\therefore$ R is transitive

$\therefore \alpha = 7k - 3 = 7k + 4$

Ans.2

$a\,R_1\,b \Leftrightarrow ab \geq 0$

So, definitely $(a, a) \in R_1$ as $a^2 \geq 0$

If $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$

But if $(a, b) \in R_1, (b, c) \in R_1$

$\Rightarrow$ Then (a, c) may or may not belong to R_1

{Consider $a = -5, b = 0, c = 5$ so (a, b) and $(b, c) \in R_1$ but $ac < 0$}

So, R_1 is not equivalence relation

$a\,R_2\,b \Leftrightarrow a \geq b$

$(a, a) \in R_2 \Rightarrow$ so reflexive relation

If $(a, b) \in R_2$ then (b, a) may or may not belong to R_2

$\Rightarrow$ So not symmetric

Hence it is not equivalence relation

Ans.3

$R = \{(x, y) : y \in A_i,\ iff\, x \in A_i\, 1 \leq i \geq k\}$

(1) Reflexive

(a, a) $\Rightarrow a \in A_i$ iff $a \in A_i$

(2) Symmetric

(a, b) $\Rightarrow a \in A_i$ iff $b \in A_i$

(b, a) $\in$R as $b \in A_i$ iff $a \in A_i$

(3) Transitive

(a, b) $\in$R & (b, c) $\in$R.

$\Rightarrow a \in A_i$ iff $b \in A_i$ & $b \in A_i$ iff $c \in A_i$

$\Rightarrow a \in A_i$ iff $c \in A_i$

$\Rightarrow$ (a, c) $\in$ R.

$\Rightarrow$ RElation is equivalnece.

Ans.4

$R_1 = \{(a,b) \in N \times N : |a-b| \leq 13\}$ and

$R_2 = \{(a,b) \in N \times N : |a-b| \neq 13\}$

In R_1 :$\because$ $|2-11| = 9 \leq 13$

$\therefore \quad (2,11) \in R_1$ and $(11,19) \in R_1$ but $(2,19) \notin R_1$

$\therefore \quad R_1$ is not transitive

Hence R_1 is not equivalence

In R_2 : $(13,3) \in R_2$ and $(3,26) \in R_2$ but $(13,26) \notin R_2$ ($\because$ $|13-26| = 13$)

$\therefore$ R_2 is not transitive

Hence R_2 is not equivalence.

TOPIC 1 Sets, Types of Sets, Disjoint Sets, Complementary Sets, Subsets, Power Set, Cardinal Number of Sets, Operations on Sets

1. Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is ______. **[Sep. 06, 2020 (I)]**

2. Let $S = \{1, 2, 3, \ldots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is : **[Jan. 12, 2019 (I)]**

   (a) $2^{100} - 1$ (b) $2^{50}\left(2^{50} - 1\right)$
   (c) $2^{50} - 1$ (d) $2^{50} + 1$

3. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$. Then S : **[2018]**

   (a) contains exactly one element.
   (b) contains exactly two elements.
   (c) contains exactly four elements.
   (d) is an empty set.

4. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S: **[2016]**

   (a) contains exactly two elements.
   (b) contains more than two elements.
   (c) is an empty set.
   (d) contains exactly one element.

5. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then: **[Online April 10, 2016]**

   (a) $P \subset Q$ and $Q - P \neq \phi$
   (b) $Q \not\subset P$
   (c) $P = Q$
   (d) $P \not\subset Q$

6. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is: **[Online April 12, 2014]**

   (a) 32 (b) 16 (c) 8 (d) 64

7. Let $X = \{1,2,3,4,5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is : **[2012]**

   (a) 5^2 (b) 3^5 (c) 2^5 (d) 5^3

8. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[2009]**

   (a) $A = C$ (b) $B = C$
   (c) $A \cap B = \phi$ (d) $A = B$

TOPIC 2 Venn Diagrams, Algebraic Operations on Sets, De Morgan's Law, Number of Elements in Different Sets

9. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be : **[Sep. 05, 2020 (I)]**

   (a) 63 (b) 36 (c) 54 (d) 38

10. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be : **[Sep. 04, 2020 (I)]**

    (a) 29 (b) 37 (c) 65 (d) 55

11. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to **[Sep. 04, 2020 (II)]**

    (a) 15 (b) 50 (c) 45 (d) 30

12. Let $X = \{n \in N: 1 \le n \le 50\}$. If
$A = \{n \in X: n$ is a multiple of $2\}$ and
$B = \{n \in X: n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is __________. **[Jan. 7, 2020 (II)]**

13. Let Z be the set of integers. If A = $\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and B = $\{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A × B, is : **[Jan. 12, 2019 (II)]**
(a) 2^{15} (b) 2^{18} (c) 2^{12} (d) 2^{10}

14. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **[Jan. 10, 2019 (II)]**
(a) 102 (b) 42 (c) 1 (d) 38

15. Let A, B and C be sets such that $\phi \ne A \cap B \subseteq C$. Then which of the following statements is not true ? **[April 12, 2019 (II)]**
(a) $B \cap C \ne \phi$
(b) If $(A - B) \subseteq C$, then $A \subseteq C$
(c) $(C \cup A) \cap (C \cup B) = C$
(d) If $(A - C) \subseteq B$, then $A \subseteq B$

16. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: **[April. 09, 2019 (II)]**
(a) 13.9 (b) 12.8 (c) 13 (d) 13.5

17. In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements : **[Online April 10, 2015]**
(A) 5% families own both a car and a phone
(B) 35% families own either a car or a phone
(C) 40,000 families live in the town
Then,
(a) Only (A) and (C) are correct.
(b) Only (B) and (C) are correct.
(c) All (A), (B) and (C) are correct.
(d) Only (A) and (B) are correct.

Hints & Solutions

1. **(28)** $2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 112$

$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$

$\therefore m = 7,\ n = 4 \Rightarrow mn = 28$

2. **(b)** $\because$ Product of two even number is always even and product of two odd numbers is always odd.

$\therefore$ Number of required subsets

= Total number of subsets – Total number of subsets having only odd numbers

$= 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

3. **(b)** **Case-I:** $x \in [0,9]$

$2(3 - \sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$

$\Rightarrow x = 16, 4$

Since $x \in [0,9]$

$\therefore\ x = 4$

Case-II: $x \in [9, \infty]$

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

Since $x \in [9, \infty]$

$\therefore x = 16$

Hence, $x = 4$ & 16

4. **(a)** $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ....(1)

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ ....(2)

Adding (1) and (2)

$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x}$ ...(3)

Substracting (1) from (2)

$\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x$ ...(4)

On adding (3) and (4)

$\Rightarrow f(x) = \frac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$

$x^2 = 2$ or $x = \sqrt{2}, -\sqrt{2}$

5. **(c)** $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \cos\theta + \sqrt{2}\cos\theta$

$= (\sqrt{2} + 1)\cos\theta = \left(\frac{2-1}{\sqrt{2}-1}\right)\cos\theta$

$\Rightarrow (\sqrt{2} - 1)\sin\theta = \cos\theta$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\therefore\ P = Q$

6. **(b)** $A = \{x : |x| < 3,\ x \in Z\}$

$A = \{-2, -1, 0, 1, 2\}$

$R = \{(x, y) : y = |x|,\ x \neq -1\}$

$R = \{(-2, 2), (0, 0), (1, 1), (2, 2)\}$

R has four elements

Number of elements in the power set of R

$= 2^4 = 16$

7. **(b)** Let $X = \{1, 2, 3, 4, 5\}$

$n(x) = 5$

Each element of x has 3 options. Either in set Y or set Z or none. ($\because Y \cap Z = \phi$)

So, number of ordered pairs $= 3^5$

8. **(b)** $\because\ B = (B \cap A) \cup B$

$= (A \cap C) \cup B$

$= (A \cup B) \cap (C \cup B)$

$= (A \cup C) \cap (B \cup C)$

$= (A \cap B) \cup C$

$= (A \cap C) \cup C$

$= C$

9. **(b)** Given, $n(C) = 73,\ n(T) = 65,\ n(C \cap T) = x$

$\therefore 65 \geq n(C \cap T) \geq 65 + 73 - 100$

$\Rightarrow 65 \geq x \geq 38 \Rightarrow x \neq 36.$

10. **(d)** Let $n(U) = 100$, then $n(A) = 63, n(B) = 76$

$n(A \cap B) = x$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) \leq 100$

$= 63 + 76 - x \leq 100$

$\Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$

$\because n(A \cap B) \le n(A)$

$\Rightarrow x \le 63$

$\therefore 39 \le x \le 63$

11. **(d)** $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$

$\because \quad n(X_i) = 10,\ n(Y_i) = 5$

So, $\bigcup_{i=1}^{50} X_i = 500,\ \bigcup_{i=1}^{n} Y_i = 5n$

$\Rightarrow \frac{500}{20} = \frac{5n}{6} \Rightarrow n = 30$

12. **(29)** From the given conditions,

$n(A) = 25, n(B) = 7$ and $n(A \cap B) = 3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3 = 29$

13. **(a)** Let $x \in A$, then

$\because \quad 2^{(x+2)(x^2-5x+6)} = 1 \Rightarrow (x+2)(x-2)(x-3) = 0$

$x = -2, 2, 3$

$A = \{-2, 2, 3\}$

Then, $n(A) = 3$

Let $x \in B$, then

$-3 < 2x - 1 < 9$

$-1 < x < 5$ and $x \in Z$

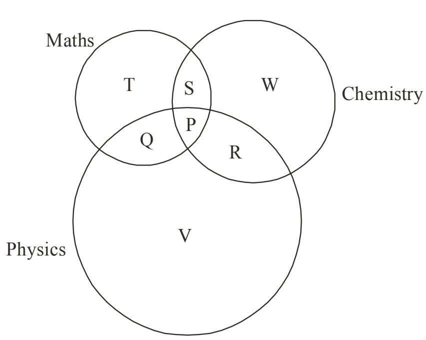
$\therefore \quad B = \{0, 1, 2, 3, 4\}$

$n(B) = 5$

$n(A \times B) = 3 \times 5 = 15$

Hence, Number of subsets of $A \times B = 2^{15}$

14. **(d)**



$P = \{30, 60, 90, 120\}$

$\Rightarrow \quad n(P) = 4$

$Q = \{6n: n \in N, 1 \le n \le 23\} - P$

$\Rightarrow \quad n(Q) = 19$

$R = \{15n: n \in N, 1 \le n \le 9\} - P$

$\Rightarrow \quad n(R) = 5$

$S = \{10n: n \in N, 1 \le n \le 14\} - P$

$\Rightarrow \quad n(S) = 10$

$n(T) = 70 - n(P) - n(Q) - n(S) = 70 - 33 = 37$

$n(V) = 46 - n(P) - n(Q) - n(R) = 46 - 28 = 18$

$n(W) = 28 - n(P) - n(R) - n(S) = 28 - 19 = 9$

$\Rightarrow$ Number of required students

$= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$

$= 140 - 102 = 38$

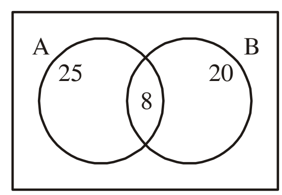
15. **(d)** (1), (2) and (4) are always correct.

In (3) option,

If $A = C$ then $A - C = \phi$

Clearly, $\phi \subseteq B$ but $A \subseteq B$ is not always true.

16. **(a)**



% of people who reads A only = 25 – 8 = 17%

% of people who read B only = 20 – 8 = 12%

% of people from A only who read advertisement

$= 17 \times 0.3 = 5.1\%$

% of people from B only who read advertisement

$= 12 \times 0.4 = 4.8\%$

% of people from A & B both who read advertisement

$= 8 \times 0.5 = 4\%$

$\therefore$ total % of people who read advertisement

$= 5.1 + 4.8 + 4 = 13.9\%$

17. **(c)** n(P) = 25%

n(C) = 15%

$n(P' \cup C') = 65\%$

$\Rightarrow n(P \cup C)' = 65\%$

$n(P \cup C) = 35\%$

$n(P \cap C) = n(P) + n(C) - n(P \cup C)$

$25 + 15 - 35 = 5\%$

$x \times 5\% = 2000$

$x = 40,000$