

TOPIC – FUNDAMENTAL THEOREM OF ARITHMETIC

- Q.1** Express each of the following integers as a product of its prime.
- (i) 420
 - (ii) 468
 - (iii) 945
 - (iv) 7325
- Q.2** Determine the prime factorisation of each of the following positive integers:
- (i) 20570
 - (ii) 58500
 - (iii) 45470971
- Q.3** Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.
- Q.4** Check whether 6^n can end with the digit 0 for any natural number n .
- Q.5** Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Sol.1 (i) 420

$$\begin{array}{r|l} 2 & 420 \\ \hline 2 & 210 \\ \hline 5 & 105 \\ \hline 7 & 21 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$420 = 2 \times 2 \times 3 \times 5 \times 7$$

$$\therefore 420 = 2^2 \times 3 \times 5 \times 7$$

(ii) 468

$$\begin{array}{r|l} 2 & 468 \\ \hline 2 & 234 \\ \hline 3 & 117 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$468 = 2 \times 2 \times 3 \times 3 \times 13$$

$$\therefore 468 = 2^2 \times 3^2 \times 13$$

(iii) 945

$$\begin{array}{r|l} 5 & 945 \\ \hline 3 & 189 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$945 = 3 \times 3 \times 3 \times 5 \times 7$$

$$\therefore 945 = 3^3 \times 5 \times 7$$

(iv) 7325

$$\begin{array}{r|l} 5 & 7325 \\ \hline 5 & 1465 \\ \hline 293 & 293 \\ \hline & 1 \end{array}$$

$$7325 = 5 \times 5 \times 293$$

$$\therefore 7325 = 5^2 \times 293$$

Sol.2 (i)

$$\begin{array}{r|l} 5 & 20570 \\ \hline 2 & 4114 \\ \hline 11 & 2057 \\ \hline 11 & 187 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$$205770 = 2 \times 5 \times 11 \times 11 \times 17$$

$$\therefore 205770 = 2 \times 5 \times 11^2 \times 17$$

(ii)

$$\begin{array}{r|l} 5 & 58500 \\ \hline 5 & 11700 \\ \hline 5 & 2340 \\ \hline 2 & 468 \\ \hline 2 & 234 \\ \hline 3 & 117 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$58500 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 13$$

$$\therefore 58500 = 2^2 \times 3^2 \times 5^3 \times 13$$

(iii)

7	45470971
7	6495853
13	927979
13	71383
17	5491
17	323
19	19
	1

$$45470971 = 7 \times 7 \times 13 \times 13 \times 17 \times 17 \times 19$$

$$\therefore 45470971 = 7^2 \times 13^2 \times 17^2 \times 19.$$

Sol.3 So, basically, there are two types of numbers, i.e., prime numbers and composite numbers.

Understanding that,

Prime numbers are those numbers having 1 and the number itself as factors.

And composite numbers are those numbers having factors other than 1 and itself.

It's seen that,

$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) \text{ [taking 13 out as common]}$$

$$= 13 \times (77 + 1)$$

$$= 13 \times 78$$

$$= 13 \times 13 \times 6$$

So, the given expression has 6 and 13 as its factors. Therefore, we can conclude that it is a composite number.

Similarly,

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \text{ [taking 5 out- common]}$$

$$= 5 \times (1008 + 1)$$

$$= 5 \times 1009$$

Since 1009 is a prime number, the given expression has 5 and 1009 as its factors other than 1 and the number itself. Hence, it is a composite number.

Sol.4 In order to check whether 6^n can end with the digit 0 for any natural number n , let us find the factors of 6.

It's seen that the factors of 6 are 2 and 3.

$$\text{So, } 6^n = (2 \times 3)^n$$

$$6^n = 2^n \times 3^n$$

Since the prime factorisation of 6 does not contain 5 and 2 as its factor together, we can conclude that 6^n can never end with the digit 0 for any natural number n .

Sol.5 Basically, there are two types of numbers, i.e., prime numbers and composite numbers.

Understanding that,

Prime numbers are those numbers having 1 and the number itself as factors.

And composite numbers are those numbers having factors other than 1 and itself.

It's seen that,

$$3 \times 5 \times 7 + 7 = 7 \times (3 \times 5 + 1) = 7 \times (15 + 1) = 7 \times 16$$

Since the given expression has 7 and 16 as its factors, we can conclude that it is a composite number.