12 Circles

12.1 Introduction

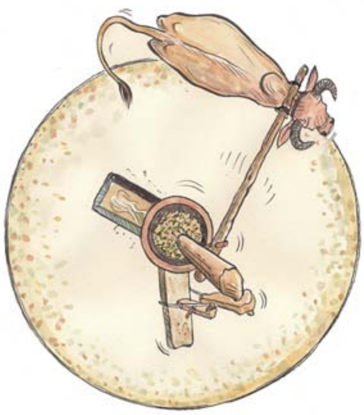
We come across many round shaped objects in our surroundings such as coins, bangles, clocks, wheels, buttons etc. All these are circular in shape.

You might have drawn an outline along the edges of a coin, a bangle, a button in your childhood to form a circle.

So, can you tell, the difference between the circular objects and the circles you have drawn with the help of these objects?

All the circular objects we have observed above have thickness and are 3-dimensional objects, where as, a circle is a 2-dimensional figure, with no thickness.

Let us take another example of a circle. You might have seen the oil press called oil mill (Spanish wheel - in Telugu known as ganuga). In the figure, a bullock is tied to fulcrum fixed at a point. Can you identify the shape of the path in which the bullock is moving? It is circular in shape.

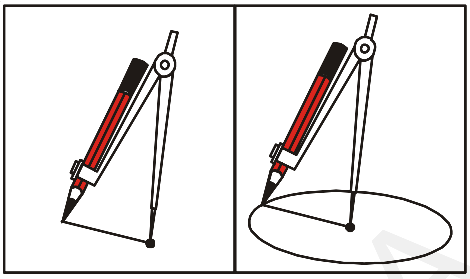


A line along the boundary made by the bullock is a circle. The oil press is attached to the ground at a fixed point, which is the centre of the circle. The length of the fulcrum with reference to the circle is radius of the circle. Think of some other examples from your daily life about circles.

In this chapter we will study circles, related terms and properties of the circle. Before this, you must know how to draw a circle with the help of a compass.

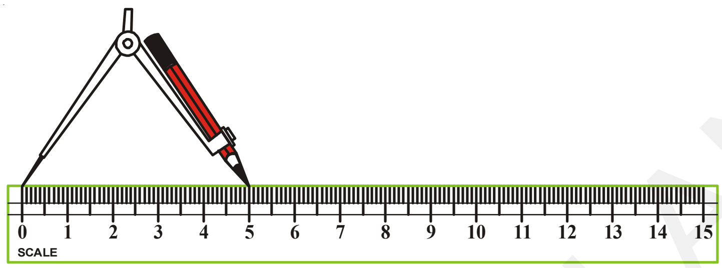
Let us do this.

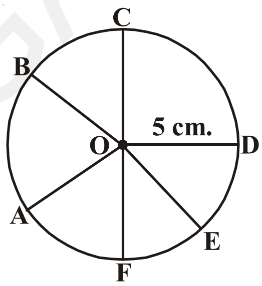
Insert a pencil in the pencil holder of the compass and tighten the screw. Mark a point 'O' on the drawing paper. Fix the sharp point of the compass on 'O'. Keeping the point of the compass firmly move the pencil round on the paper to draw the circle as shown in the figure.



If we need to draw a circle of given radius, we do this with the help of a scale.

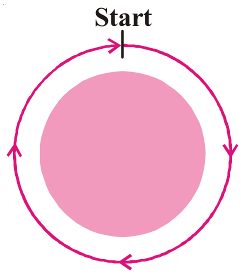
Adjust the distance between the sharp point of the compass and tip of the pencil equal to the length of the given radius, mark a point 'O'(radius of the circle in the figure is 5 cm.) and draw circle as described above.





Mark any 6 points A, B, C, D E and F on the circle. You can see that the length of each line segment OA, OB, OC, OD, OE and OF is 5 cm., which is equal to the given radius. Mark some other points on the circle and measure their distances from the point 'O'. What have you observed? We can say that a circle is a collection of all the points in a plane which are at a fixed distance from a fixed point on the plane.

The fixed point 'O' is called the centre of the circle and the fixed distance OA, is called the radius of the circle.



In a circular park Narsimha started walking from a point around the park and completed one round. What do you call the distance covered by Narsimha? It is the total length of the boundary of the circular park, and is called the circumference of the park.

So, the complete length of a circle is called its circumference.

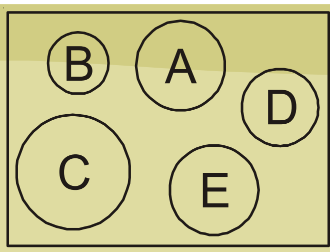
ACTIVITY

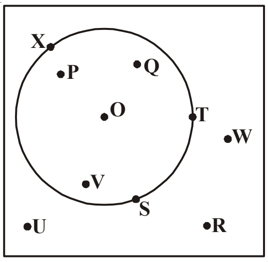
Let us now do the following activity. Mark a point on a sheet of paper. Taking this point as centre draw a circle with any radius. Now increase or decrease the radius and again draw some more circles with the same centre. What do you call the circles obtained in this activity?

Circles having same centre with different radii are called **concentric circles**.

Do This

1. In the figure, which circles are congruent to the circle A?
2. What measure of the circles make them congruent?





A circle divides the plane on which it lies into three parts. They are (i) inside the circle, which is also called interior of the circle; (ii) on the circle, this is also called the circumference and (iii) outside the circle, which is also called the exterior of the circle. From the above figure, find the points which are inside, outside and on the circle.

The circle and its interior make up the circular region.

Activity

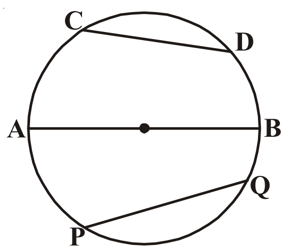
Take a thin circular sheet and fold it to half and open. Again fold it along any other half and open. Repeat this activity for several times. Finally when you open it, what do you observe?

You observe that all creases (traces of the folds) are intersecting at one point. Do you remember what do we call this point? This is the centre of the circle.

Measure the length of each crease of a circle with a divider. What do you notice ? They are all equal and each crease is dividing the circle into two equal halves. That crease is called diameter of circle. Diameter of a circle is twice its radius. A line segment joining any two points on the circle that passes through the centre is called the **diameter**.

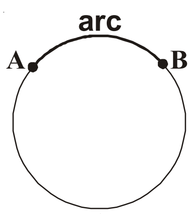
In the above activity if we fold the paper in any manner not only in half, we see that creases joining two points on circle. These creases are called chords of the circle.

So, a line segment joining any two points on the circle is called a **chord.**



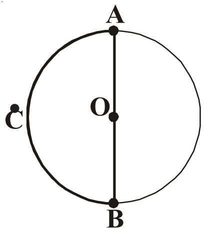
What do you call the longest chord? Is it passes through the centre?

See in the figure, CD, AB and PQ are chords of the circle.

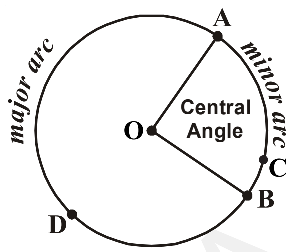


(i)

In the fig.(i), two points A and B are on the circle and they are dividing the circumference of the circle into two parts. The part of the circle between any two points on it is called an arc. In the fig.(i) AB is called an 'arc' and it is denoted by $\overset{\frown}{AB}$. If the end points of an arc become the end points of a diameter then such an arc is called a semicircular arc or a semicircle. In the fig.(ii) $\overset{\frown}{ACB}$ is a semicircle

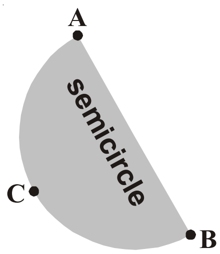


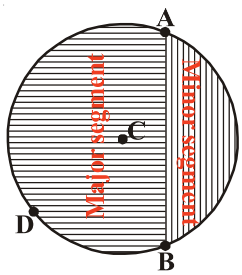
(ii)



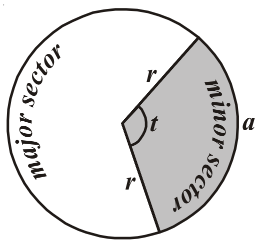
(iii)

If the arc is smaller than a semicircle, then the arc is called a minor arc and if the arc is longer than a semicircle, then the arc is called a major arc. In the fig.(iii) $\overset{\frown}{ACB}$ is a minor arc and $\overset{\frown}{ADB}$ is a major arc.





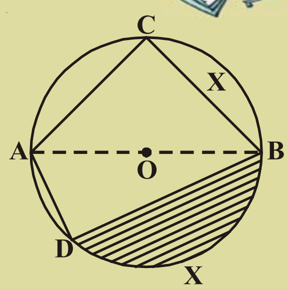
If we join the end points of an arc by a chord, the chord divides the circle into two parts. The region between the chord and the minor arc is called the **minor segment** and the region between the chord and the major arc is called the **major segment**. If the chord happens to be a diameter, then the diameter divides the circle into two equal segments.



The region enclosed by an arc and the two radii joining the centre to the end points of an arc is called a ***sector***. One is minor sector and another is major sector (see adjacent figure).

EXERCISE -12.1

1. Name the following parts from the adjacent figure where 'O' is the centre of the circle.



(i) $\overline{AO}$ (ii) $\overline{AB}$ (iii) $\overset{\frown}{BC}$

(iv) $\overline{AC}$ (v) $\overset{\frown}{DCB}$ (vi) $\overset{\frown}{ACB}$

(vii) $\overline{AD}$ (viii) shaded region

2. State true or false.

i. A circle divides the plane on which it lies into three parts. ( )

ii. The region enclosed by a chord and the minor arc is minor segment. ( )

iii. The region enclosed by a chord and the major arc is major segment. ( )

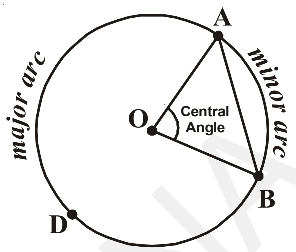
iv. A diameter divides the circle into two unequal parts. ( )

v. A sector is the area enclosed by two radii and a chord ( )

vi. The longest of all chords of a circle is called a diameter. ( )

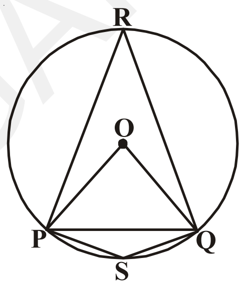
vii. The mid point of any diameter of a circle is the centre. ( )

12.2 Angle subtended by a chord at a point on the circle



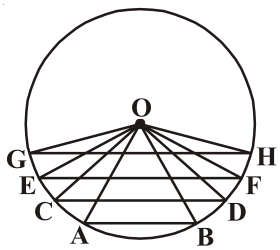
Let A, B be any two points on a circle with centre 'O'. Join A,O and B,O. Angle is made at centre 'O' by $\overline{AO}$, $\overline{BO}$ i.e. $\angle AOB$ is called the angle subtended by the chord $\overline{AB}$ at the centre 'O'.

What do you call the angles $\angle POQ$, $\angle PSQ$ and $\angle PRQ$ in the figure?



i. $\angle POQ$ is the angle subtended by the chord PQ at the centre 'O'

ii. $\angle PSQ$ and $\angle PRQ$ are respectively the angles subtended by the chord PQ at point S and point R on the minor and major arc.



In the figure, O is the centre of the circle and AB, CD, EF and GH are the chords of the circle.

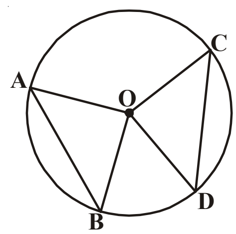
We can observe from the figure that GH > EF > CD > AB.

Now what do you say about the angles subtended by these chords at the centre?

After observing the angles, you will find that the angles subtended by the chords at the centre of the circle increases with increase in the length of chords.

So, now imagine what will happen to the angle subtended at the centre of the circle, if we take two equal chords of a circle?

Construct a circle with centre 'O' and draw equal chords AB and CD using the compass and ruler.



Join the centre 'O' with A, B and with C, D. Now measure the angles $\angle AOB$ and $\angle COD$. Are they equal to each other? Draw two or more equal chords of a circle and measure the angles subtended by them at the centre.

You will find that the angles subtended by them at the centre are equal.

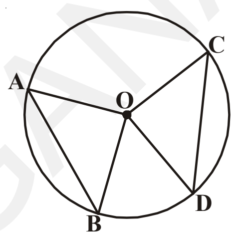
Let us try to prove this fact.

Theorem-12.1 : Equal chords of a circle subtend equal angles at the centre.

Given : Let 'O' be the centre of the circle. $\overline{AB}$ and $\overline{CD}$ are two equal chords and $\angle AOB$ and $\angle COD$ are the angles subtended by the chords at the centre.

R.T.P. : $\angle AOB \cong \angle COD$

Construction : Join the centre to the end points of each chord and you get two triangles ΔAOB and ΔCOD.



Proof: In triangles AOB and COD

AB = CD (given)

OA = OC (radii of same circle)

OB = OD (radii of same circle)

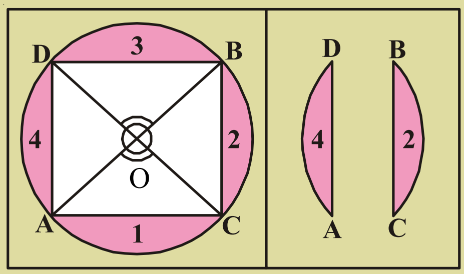
Therefore Δ AOB $\cong$ Δ COD (SSS rule)

Thus $\angle AOB \cong \angle COD$ (corresponding parts of congruent triangles)

In the above theorem, if in a circle, two chords subtend equal angles at the centre, what can you say about the chords? Let us investigate this by the following activity.

ACTIVITY

Take a circular paper. Fold it along any diameter such that the two edges coincide with each other. Now open it and again fold it into half along another diameter. On opening, we find two diameters meet at the centre 'O'. There forms two pairs of vertically opposite angles which are equal. Name the end points of the diameter as A, B, C and D



Draw the chords $\overline{AC}$, $\overline{BC}$, $\overline{BD}$ and $\overline{AD}$.

Now take cut-out of the four segments namely 1, 2, 3 and 4

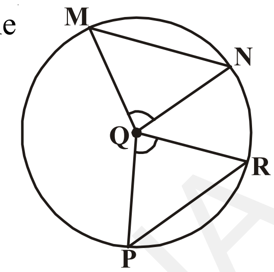
If you place these segments pair wise one above the other the edges of the pairs (1,3) and (2,4) coincide with each other.

Is $\overline{AD} = \overline{BC}$ and $\overline{AC} = \overline{BD}$?

Though you have seen it in this particular case, try it out for other equal angles too. The chords will all turn out to be equal. We will prove it as a theorem.

Can you state converse of the above theorem (12.1)?

Theorem-12.2 : If the angle subtended by the chords of a circle at the centre are equal, then the chords are equal.



This is the converse of the theorem 12.1.

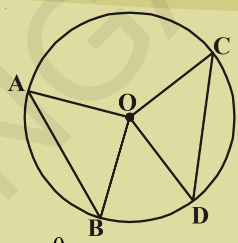
Note that in adjacent figure $\angle PQR = \angle MQN$, then

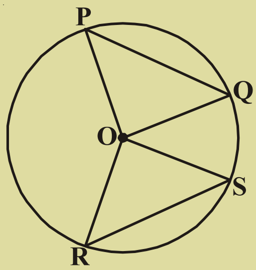
$\Delta PQR \cong \Delta MQN$ (Why?)

Is $PR = MN$? (Verify)

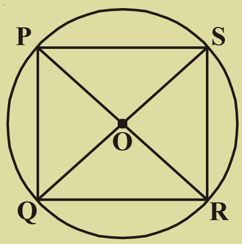
Exercise - 12.2

1. In the figure, if $AB = CD$ and $\angle AOB = 90^0$ find $\angle COD$





2. In the figure, $PQ = RS$ and $\angle ORS = 48^0$. Find $\angle OPQ$ and $\angle ROS$.

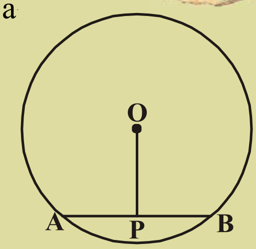


3. In the figure PR and QS are two diameters. Is $PQ = RS$?

12.3 Perpendicular from the centre to a Chord

Activity

- Construct a circle with centre O. Draw a chord $\overline{AB}$ and a perpendicular to the chord $\overline{AB}$ from the centre 'O'.
- Let the point of intersection of the perpendicular on $\overline{AB}$ be P.
- After measuring PA and PB, we will find $PA = PB$.



Theorem-12.3 : The perpendicular from the centre of a circle to a chord bisects the chord.

Write a proof by yourself by joining O to A and B and prove that $\Delta OPA \cong \Delta OPB$.

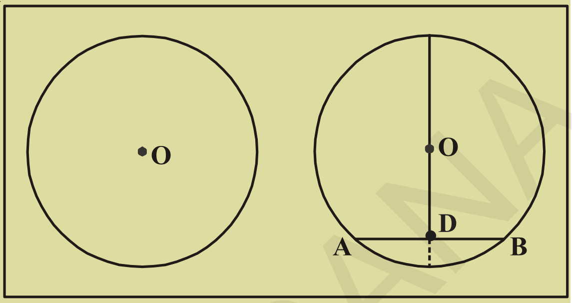
What is the converse of the theorem 12.3?

"If a line drawn from the centre of a circle bisects the chord then the line is perpendicular to that chord"

ACTIVITY

Take a circular sheet of paper and mark centre as 'O'

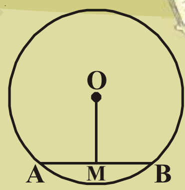
Fold it into two unequal parts and open it. Let the crease represent a chord AB, and then make a fold such that 'A' coincides with B. Mark the point of intersection of the two folds as D. Is AD=DB? and $\angle ODA = \angle ODB$? (Measure the angles between the creases). They are right angles. So, we can make a hypothesis "the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord".



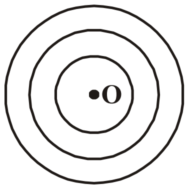
TRY THIS

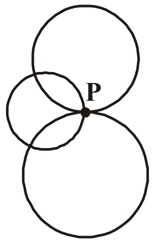
In a circle with centre 'O'. $\overline{AB}$ is a chord and 'M' is its midpoint. Now prove that $\overline{OM}$ is perpendicular to AB.

(**Hint** : Join OA and OB consider triangles OAM and OBM)



12.3.1 The three points that describe a circle



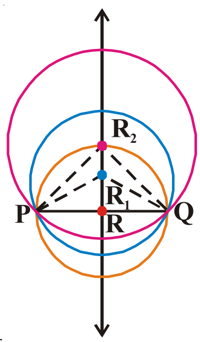


Let 'O' be a point on a plane. How many circles we can draw with centre 'O'? As many circles as we wish. We have already learnt that these circles are called concentric circles. If 'P' is a point other than the centre of the circle, then also we can draw many circles through P.

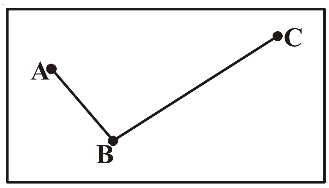
Suppose that there are two distinct points P and Q

How many circles can be drawn passing through given two points? We see that we can draw many circles passing through P and Q.

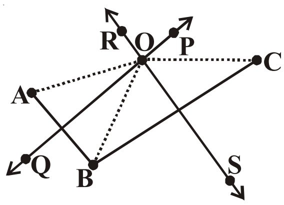
Let us join P and Q, draw the perpendicular bisector to PQ. Take any three points R, R_1 and R_2 on the perpendicular bisector and draw circles with centre R, R_1, R_2 and radii RP, R_1P and R_2P respectively. Does these circles also passes through Q (Why?)



As every point on the perpendicular bisector of a line segment is equidistant from end points of the line segment. Centre of a circle lies on the perpendicular of any chord.



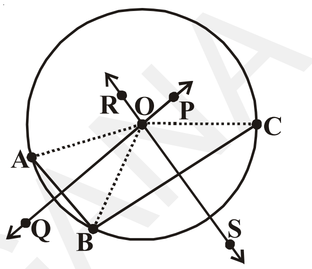
If three non-collinear points are given, then how many circles can be drawn through them? Let us examine it. Take any three non-collinear points A, B, C and join AB and BC.



Draw $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ the perpendicular bisectors to $\overline{AB}$ and $\overline{BC}$. respectively. Both of them intersect at a point 'O'(since two lines cannot have more than one point in common)

Now O lies on the perpendicular bisector of $\overline{AB}$, so OA = OB. .....(i)

As every point on $\overleftrightarrow{PQ}$ is at equidistant from A and B



Also, 'O' lies on the perpendicular bisectors of $\overline{BC}$

Therefore OB = OC ..... (ii)

From equation (i) and (ii)

We can say that OA = OB = OC (transitive law)

Therefore, 'O' is the only point which is equidistant from the points A, B and C so if we draw a circle with centre O and radius OA, it will also pass through B and C i.e. we have only one circle that passes through A, B and C.

The hypothesis based on above observation is "there is one and only one circle that passes through three non-collinear points"

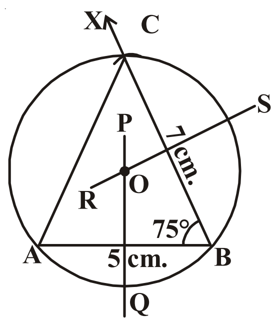
Note : If we join AC, the triangle ABC is formed. All its vertices lie on the circle. This circle is called circum circle of the triangle, the centre of the circle 'O' is circumcentre and the radius OA or OB or OC i.e. is circumradius.

TRY THIS

If three points are collinear, how many circles can be drawn through these points? Now, try to draw a circle passing through these three points.

Example-1. Construct a circumcircle of the triangle ABC where AB = 5cm; $\angle B = 75°$ and BC = 7cm

Solution : Draw a line segment AB= 5 cm. Draw BX at B such that $\angle B = 75°$. Draw an arc of radius 7cm with centre B to cut $\overrightarrow{BX}$ at C join CA to form ΔABC, Draw perpendicular bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ to $\overline{AB}$ and $\overline{BC}$ respectively. $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$ intersect at 'O'. Keeping 'O' as a centre, draw a circle with OA as radius. The circle also passes through B and C and this is the required circumcircle.

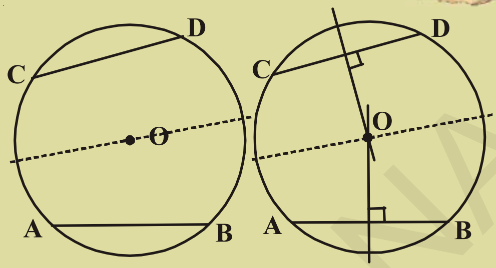


12.3.2 Chords and their distance from the centre of the circle

A circle can have infinite chords. Suppose we make many chords of equal length in a circle, then what would be the distance of these chords of equal length from the centre? Let us examine it through this activity.

ACTIVITY

Draw a big circle on a paper and take a cut-out of it. Mark its centre as 'O'. Fold it in half. Now make another fold near semi-circular edge. Now unfold it. You will get two conguent folds of chords. Name them as AB and CD. Now make perpendicular folds passing through centre 'O' for them. Using divider compare the perpendicular distances of these chords from the centre.

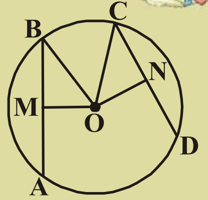


Repeat the above activity by folding congruent chords. State your observations as a hypothesis.

"The congruent chords in a circle are at equal distance from the centre of the circle"

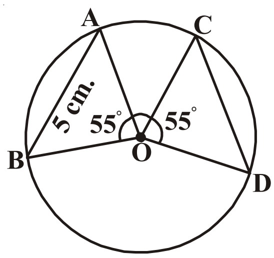
TRY THIS

In the figure, O is the centre of the circle and AB = CD. OM is perpendicular on $\overline{AB}$ and $\overline{ON}$ is perpendicular on $\overline{CD}$. Then prove that OM = ON.



As the above hypothesis has been proved logically, it becomes the theorem 'chords of equal length are at equal distance from the centre of the circle.'

Example-2. In the figure, O is the centre of the circle. Find the length of CD, if AB = 5 cm.



Solution : In Δ AOB and Δ COD,

OA = OC (why?)

OB = OD (why?)

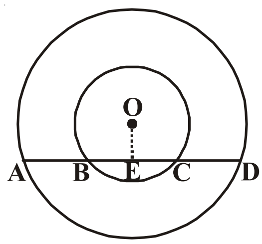
$\angle AOB = \angle COD$

$\therefore$ $\Delta AOB \cong \Delta COD$

$\because$ AB = CD (Congruent parts of congruent triangles)

$\therefore$ AB = 5 cm. then CD = 5 cm.

Example-3. In the adjacent figure, there are two concentric circles with centre 'O'. Chord AD of the bigger circle intersects the smaller circle at B and C. Show that AB = CD.



Given : In two concentric circles with centre 'O'. $\overline{AD}$ is the chord of the bigger circle. $\overline{AD}$ intersect the smaller circle at B and C.

R.T.P. : AB = CD

Construction : Draw $\overline{OE}$ perpendicular to $\overline{AD}$

Proof : AD is the chord of the bigger circle with centre 'O' and $\overline{OE}$ is perpendicular to $\overline{AD}$.

$\because$ $\overline{OE}$ bisects $\overline{AD}$ (The perpendicular from the centre of a circle to a chord bisect it)

$\therefore$ AE = ED ..... (i)

$\overline{BC}$ is the chord of the smaller circle with centre 'O' and $\overline{OE}$ is perpendicular to AD.

$\because$ $\overline{OE}$ bisects $\overline{BC}$ (from the same theorem)

$\therefore$ BE = CE ..... (ii)

Subtracting the equation (ii) from (i), we get

AE - BE = ED - EC

AB = CD

Exercise - 12.3

1. Draw the following triangles and construct circumcircles for them.

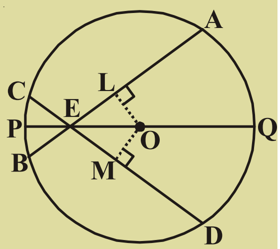
   (i) In Δ ABC, AB = 6cm, BC = 7cm and $\angle A = 60^o$

   (ii) In Δ PQR, PQ = 5cm, QR = 6cm and RP = 8.2cm

   (iii) In Δ XYZ, XY = 4.8cm, $\angle X = 60^o$ and $\angle Y = 70^o$

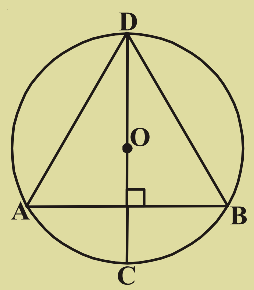
2. Draw two circles passing through A, B where AB = 5.4cm

3. If two circles intersect at two points, then prove that their centres lie on the perpendicular bisector of the common chord.

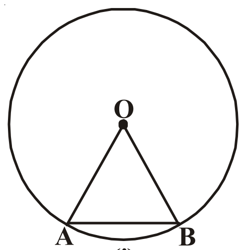


4. If two intersecting chords of a circle make equal angles with diameter passing through their point of intersection, prove that the chords are equal.

5. In the adjacent figure, AB is a chord of circle with centre O. CD is the diameter perpendicualr to AB. Show that AD = BD.



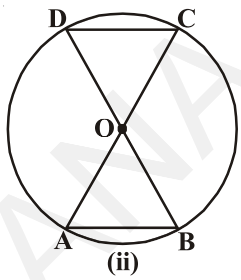
12.4 Angle subtended by an arc of a circle



(i)

In the fig.(i), $\overline{AB}$ is a chord and $\overset{\frown}{AB}$ is an arc (minor arc). The end points of the chord and arc are the same i.e. A and B.

Therefore angle subtended by the chord at the centre 'O' is the same as the angle subtended by the arc at the centre 'O'.



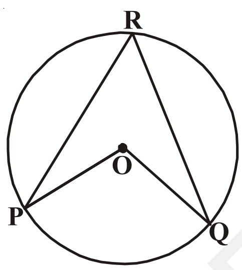
(ii)

In fig.(ii) $\overline{AB}$ and $\overline{CD}$ are two chords of a circle with centre 'O'. If AB = CD, then $\angle$AOB = $\angle$COD

Therefore we can say that the angle subtended by an arc $\overset{\frown}{AB}$ is equal to the angle subtended by the arc $\overset{\frown}{CD}$ at the centre 'O'. (Prove ΔAOB $\cong$ ΔDOC)

From the above observations we can conclude that in the same cirlce or congruent circles *"arcs of equal length subtend equal angles at the centre"* [$\therefore$ Angle subtended by an arc at the centre is called a measure of that are]

12.4.1 Angle subtended by an arc at a point on the remaining part of circle

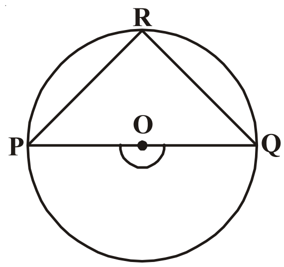


(i)

Consider the circle with centre 'O'.

Let $\overset{\frown}{PQ}$ in fig. (i) the minor arc, in fig. (ii) semicircle and in fig. (iii) major arc.

Take any point R on the circle. Join R with P and Q.

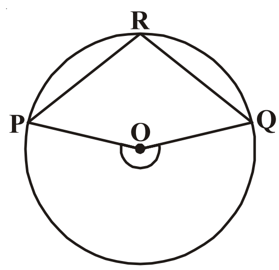


(ii)

$\angle$PRQ is the angle subtended by the arc PQ at the point R on the circle while $\angle$POQ is subtended at the centre.

Complete the following table for the given figures.

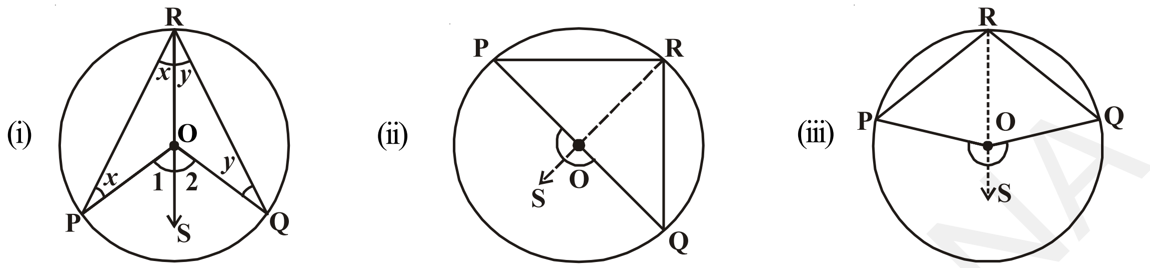
Angle	Fig. (i)	Fig. (ii)	Fig. (iii)
$\angle$PRQ			
$\angle$POQ			



(iii)

Similarly draw some circles and subtended angles on the circumference and centre of the circle by their arcs. What do you notice? Can you make a conjecture about the angle made by an arc at the centre and a point on the circle? So from the above observations, we can say that *"The angle subtended by an arc at the centre 'O' is twice the angle subtended by it on the remaining arc of the circle"*.

Theorem: The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining circle.



Given : Let O be the centre of the circle.

$\overset{\frown}{PQ}$ is an arc subtending $\angle POQ$ at the centre.

Let R be a point on the remaining part of the circle (not on $\overset{\frown}{PQ}$)

Proof: Here we have three different cases in which (i) $\overset{\frown}{PQ}$ is minor arc, (ii) $\overset{\frown}{PQ}$ is semicircle and (iii) $\overset{\frown}{PQ}$ is a major arc

Let us begin by joining the point R with the centre 'O' and extend it to a point S (in all cases)

For all the cases in Δ ROP

RO = OP (radii of the same circle)

Therefore $\angle ORP = \angle OPR$ (Angles opposite to equal sides of an isosceles triangle are equal).

$\angle POS$ is an exterior angle of Δ ROP (construction)

$\angle POS = \angle ORP + \angle OPR$ or $2\,\angle ORP$ ..... (1)

($\because$ exterior angle = sum of opp. interior angles)

Similarly for ΔROQ

$\angle SOQ = \angle ORQ + \angle OQR$ or $2\,\angle ORQ$ ... (2)

($\because$ exterior angle is equal to sum of the opposite interior angles)

From (1) and (2)

$\angle POS + \angle SOQ = 2\,(\angle ORP + \angle ORQ)$

This is same as $\angle POQ = 2\,\angle QRP$ ..... (3)

For convenience

Let $\angle ORP = \angle OPR = x$

$\angle POS = \angle 1$

$\angle 1 = x + x = 2x$

Let $\angle ORQ = \angle OQR = y$

$\angle SOQ = \angle 2$

$\angle 2 = y + y = 2y$

Now $\angle POQ = \angle 1 + \angle 2 = 2x + 2y$

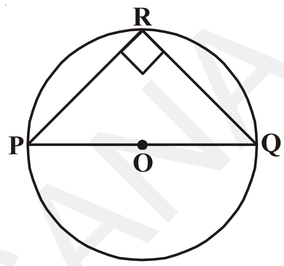
$= 2\,(x+y) = 2\,(\angle PRO + \angle ORQ)$

(i.e.) $\angle POQ = 2\,\angle PRQ$

Hence the theorem is "the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle.

Example-4. Let 'O' be the centre of a circle, PQ is a diameter, then prove that $\angle PRQ = 90^o$

(OR) Prove that angle in a semi-circle is right angle.



Solution : It is given that PQ is a diameter and 'O' is the centre of the circle.

$\therefore \angle POQ = 180^o$ [Angle on a straight line]

and $\angle POQ = 2\,\angle PRQ$ [Angle subtended by an arc at the centre is twice the angle subtended by it at any other point on circle]

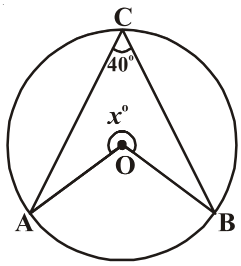
$$\therefore \angle PRQ = \frac{180^o}{2} = 90^o$$

Example-5. Find the value of x° in the adjacent figure

Solution : Given $\angle ACB = 40^\circ$

By the theorem angle made by the arc AB at the centre

$\angle AOB = 2\,\angle ACB = 2 \times 40^\circ = 80^\circ$



$\because \quad x^\circ + \angle AOB = 360^\circ$

Therefore $x^\circ = 360^\circ - 80^\circ = 280^\circ$

12.4.2 Angles in the same segment

Let us now discuss the measures of angles made by an arc in the same segment of a circle.

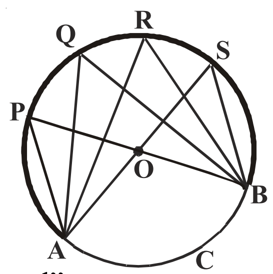
Consider a circle with centre 'O' and a minor arc AB (See figure). Let P, Q, R and S be points on the major arc AB i.e. on the remaining part of the circle. Now join the end points of the arc AB with points P, Q, R and S to form angles $\angle APB$, $\angle AQB$, $\angle ARB$ and $\angle ASB$.

$\because \quad \angle AOB = 2\angle APB$ (why?)

$\angle AOB = 2\angle AQB$ (why?)

$\angle AOB = 2\angle ARB$ (why?)

$\angle AOB = 2\angle ASB$ (why?)



Therefore $\angle APB = \angle AQB = \angle ARB = \angle ASB$

Observe that "angles subtended by an arc in the same segment are equal".

Note : In the above discussion we have seen that the point P, Q, R, S and A, B lie on the same circle. What do you call them? "Points lying on the same circle are called concyclic".

The converse of the above theorem can be stated as follows-

Theorem-12.4 : If a line segment joining two points, subtends equal angles at two other points lying on the same side of the line then these, the four points lie on a circle (i.e. they are concyclic)

Given : Two angles $\angle ACB$ and $\angle ADB$ are on the same side of a line segment $\overline{AB}$ joining two points A and B are equal.

R.T.P : A, B, C and D are concyclic (i.e.) they lie on the same circle.

Construction : Draw a circle passing through the three non colinear point A, B and C.

Proof: Suppose the point 'D' does not lie on the Circle.

Then there may be other point 'E' such that it will intersect AD (or extension of AD)

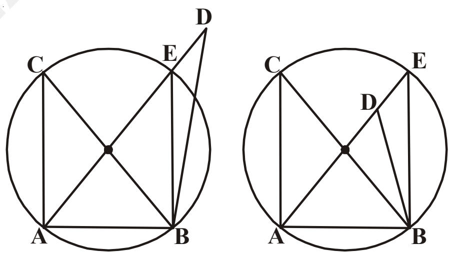
If points A, B, C and E lie on the circle then

$\angle ACB = \angle AEB$ (Why?)

But it is given that $\angle ACB = \angle ADB$.

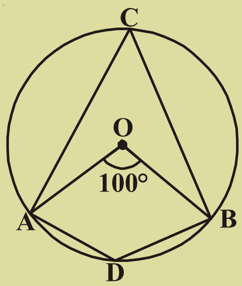
Therefore $\angle AEB = \angle ADB$

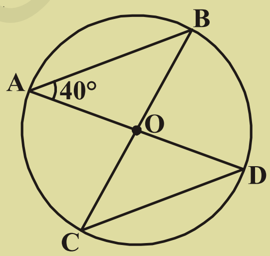
This is not possible unless E coincides with D (Why?)



EXERCISE – 12.4

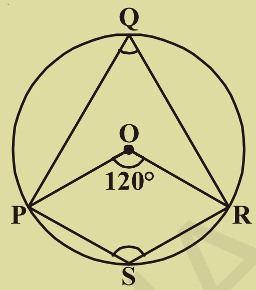
1. In the figure, 'O' is the centre of the circle. $\angle AOB = 100^{\circ}$ find $\angle ADB$.

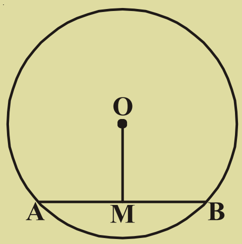




2. In the figure, $\angle BAD = 40^{\circ}$ then find $\angle BCD$.

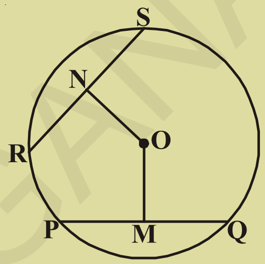
3. In the figure, O is the centre of the circle and $\angle POR = 120^\circ$. Find $\angle PQR$ and $\angle PSR$

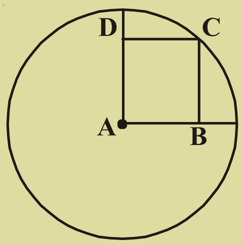




4. In the figure, 'O' is the centre of the circle. OM = 3cm and AB = 8cm. Find the radius of the circle

5. In the figure, 'O' is the centre of the circle and OM, ON are the perpendiculars from the centre to the chords PQ and RS. If OM = ON and PQ = 6cm. Find RS.

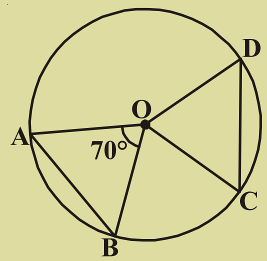




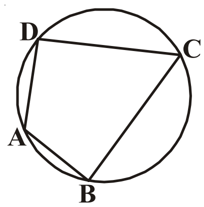
6. A is the centre of the circle and ABCD is a square. If BD = 4cm then find the radius of the circle.

7. Draw a circle with any radius and then draw two chords equidistant from the centre.

8. In the given figure 'O' is the centre of the circle and AB, CD are equal chords. If $\angle AOB = 70^\circ$. Find the angles of the ΔOCD.



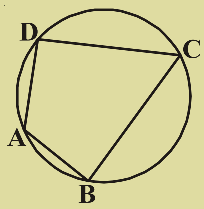
12.5 Cyclic Quadrilateral



In the figure, the vertices of the quadrilateral A, B, C and D lie on the same circle, this type of quadrilateral ABCD is called cyclic quadrilateral.

Activity

Draw a circle. Mark four points A, B, C and D on it. Draw quadrilateral ABCD. Measure its angles. Record them in the table. Repeat this activity for three more times.

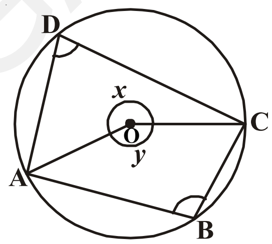


S.No	$\angle A$	$\angle B$	$\angle C$	$\angle D$	$\angle A+\angle C$	$\angle B+\angle D$
1						
2						
3						
4						

What do you infer from the table?

Theorem-12.5 : The opposite angles of a cyclic quadrilateral are supplementary.

Given : ABCD is a cyclic quadrilateral .



To Prove : $\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$

Construction : Join OA, OC

Proof: $\angle D = \frac{1}{2}\angle y$ (Why?) ..... (i)

$\angle B = \frac{1}{2}\angle x$ (Why?) ..... (ii)

By adding of (i) and (ii)

$$\angle D + \angle B = \frac{1}{2}\angle y + \frac{1}{2}\angle x$$

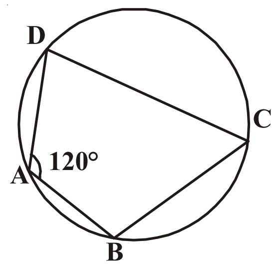
$$\angle D + \angle B = \frac{1}{2}(\angle y + \angle x)$$

$$\angle B + \angle D = \frac{1}{2} \times 360°$$

$$\angle B + \angle D = 180°$$

Similarly $\angle A + \angle C = 180°$

Example-6. In the figure, $\angle A = 120^{\circ}$ then find $\angle C$?



Solution: ABCD is a cyclic quadrilateral

Therefore $\angle A + \angle C = 180^{\circ}$

$120^0 + \angle C = 180^{\circ}$

Therefore $\angle C = 180^{\circ} - 120^{\circ} = 60^{\circ}$

What is the converse of the above theorem?

"If the sum of a pair of opposite angles of a quadrilateral is 180^0, then the quadrilateral is cyclic".

The converse is also true.

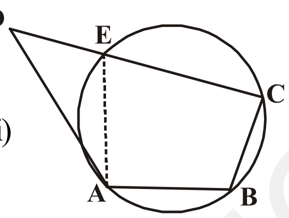
Theorem-12.6 : If the sum of any pair of opposite angles in a quadrilateral is 180^o, then it is cyclic.

Given : Let ABCD be a quadrilateral such that

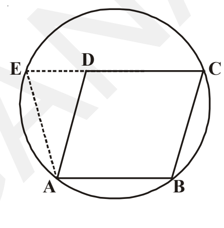
$\angle ABC + \angle ADC = 180^o$

$\angle DAB + \angle BCD = 180^o$

(i)



(ii)



R.T.P. : ABCD is a cyclic quadrilateral.

Construction : Draw a circle through three non-collinear points A, B, and C.

If it passes through D, the theorem is proved since A, B, C and D are concyclic. If the circle does not pass through D, it intersects $\overline{CD}$ [fig (i) or $\overline{CD}$ produced [fig (ii)] at E.

Draw $\overline{AE}$

Proof : ABCE is a cyclic quadrilateral (construction)

$\angle AEC + \angle ABC = 180^o$ [sum of the opposite angles of a cyclic quadrilateral]

But $\angle ABC + \angle ADC = 180^o$ Given

$\therefore \angle AEC + \angle ABC = \angle ABC + \angle ADC \Rightarrow \angle AEC = \angle ADC$

But one of these is an exterior angle of ΔADE and the other is an interior opposite angle.

We know that the exterior angle of a triangle is always greater than either of the opposite interior angles.

$\therefore \angle AEC = \angle ADC$ is a contradiction.

So our assumption that the circle passing through A, B and C does not pass through D is false.

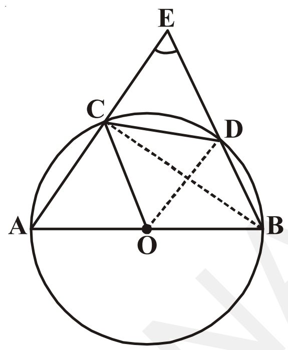
$\therefore$ The circle passing through A, B, C also passes through D.

$\therefore$ A, B, C and D are concyclic. Hence ABCD is a cyclic quadrilateral.

Example-7. In figure, $\overline{AB}$ is a diameter of the circle, $\overline{CD}$ is a chord equal to the radius of the circle. $\overrightarrow{AC}$ and $\overrightarrow{BD}$ when extended intersect at a point E. Prove that $\angle AEB = 60°$.

Solution : Join O,C, join O,D and join B,C.

Triangle ODC is equilateral (Why?)



Therefore, $\angle COD = 60°$

Now, $\angle CBD = \frac{1}{2}\angle COD$ (Why?)

This gives $\angle CBD = 30°$

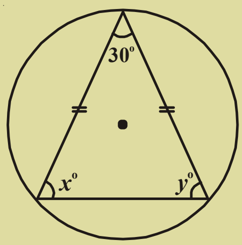
Again, $\angle ACB = 90°$ (Why?)

So, $\angle BCE = 180° - \angle ACB = 90°$

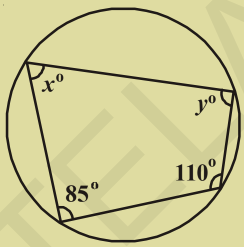
Which gives $\angle CEB = 90° - 30° = 60°$, i.e. $\angle AEB = 60°$

EXERCISE 12.5

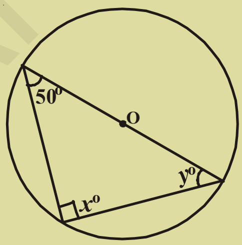
1. Find the values of x and y in the figures given below.



(i)



(ii)



(iii)

2. Given that the vertices A, B, C of a quadrilateral ABCD lie on a circle.

   Also $\angle A + \angle C = 180°$, then prove that the vertex D also lie on the same circle.

3. If a parallelogram is cyclic, then prove that it is a rectangle.
4. Prove that a cyclic rhombus is a square.
5. For each of the following, draw a circle and inscribe the figure given. If a polygon of the given type can't be inscribed, write not possible.
   (a) Rectangle
   (b) Trapezium
   (c) Obtuse triangle
   (d) Non-rectangular parallelogram
   (e) Accute isosceles triangle
   (f) A quadrilateral PQRS with $\overline{PR}$ as diameter.

WHAT WE HAVE DISCUSSED

- A collection of all points in a plane which are at a fixed distance from a fixed point in the same plane is called a circle. The fixed point is called the centre and the fixed distance is called the radius of the circle

G7P3R6

- A line segment joining any points on the circle is called a chord
- The longest of all chords which also passes through the centre is called a diameter
- Circles with same radii are called congruent circles
- Circles with same centre and different radii are called concentric circles
- Diameter of a circle divides it into two semi-circles
- The part between any two points on the circle is called an arc
- The area enclosed by a chord and an arc is called a segment. If the arc is a minor arc then it is called the minor segment and if the arc is major arc then it is called the major segment
- The area enclosed by an arc and the two radii joining the end points of the arc with centre is called a sector
- Equal chords of a circle subtend equal angles at the centre
- Angles in the same segment are equal
- An angle in a semi circle is a right angle.
- If the angles subtended by two chords at the centre are equal, then the chords are congruent
- The perpendicular from the centre of a circle to a chord bisects the chords. The converse is also true
- There is exactly one circle passes through three non-collinear points
- The circle passing through the vertices of a triangle is called a circumcircle
- Equal chords are at equal distance from the centre of the circle, conversely chords at equidistant from the centre of the circle are equal in length
- Angle subtended by an arc at the centre of the circle is twice the angle subtended by it at any other point on the circle.
- If the angle subtended by an arc at a point on the remaining part of the circle is 90^o, then the arc is a semi circle.
- If a line segment joining two points subtends same angles at two other points lying on the same side of the line segment, the four points lie on the circle.
- The pairs of opposite angles of a cyclic quadrilateral are supplementary.