Magnetic Coupled Circuits :-

Case - (1) :-

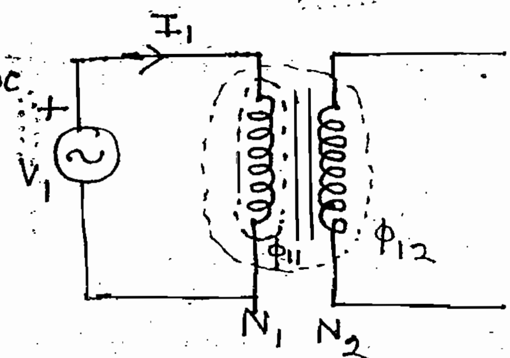
$I_1 \longrightarrow \phi_1$

$\phi_1 \nearrow \phi_{11} \rightarrow$ leakage flux

$\phi_1 \searrow \phi_{12}$

$\downarrow$

Useful flux
(or)
Mutual flux



$$e_1 \propto \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{di_1} \cdot \frac{di}{dt} \quad \left(L = \frac{N\phi}{i} \right)$$

$$\boxed{e_1 = -L_1 \frac{di_1}{dt}}$$

↓

Self induced emf

$$e_2 \propto \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} \quad \left(M_{21} = \frac{N_2 \phi_{12}}{i_1} \right)$$

↓ Mutual inductance of second inductor w.r.t. 1

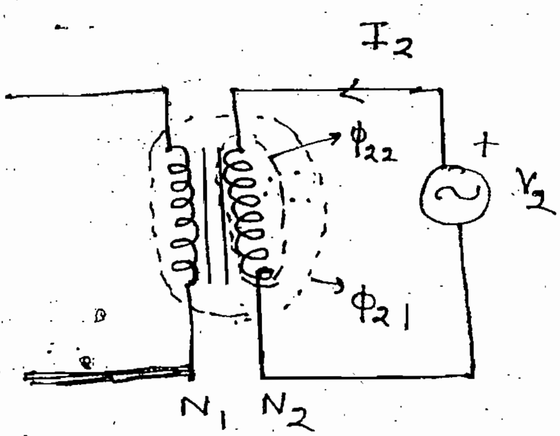
$$\boxed{e_2 = -M_{21} \frac{di_1}{dt}}$$

↓

Mutual Induced emf

Case-(II):-

$I_2 \rightarrow \phi_2$
- $\rightarrow \phi_{22} \rightarrow$ leakage flux
- $\rightarrow \phi_{21} \rightarrow$ Useful flux or Mutual flux



$$e_2 \propto \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} \quad \left(L = \frac{N\phi}{i} \right)$$

$$\boxed{e_2 = -L_2 \frac{di_2}{dt}}$$

↓

self induced emf

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$$e_1 \propto \frac{d\phi_{21}}{dt}$$

$$e_1 = -N_1 \frac{d\phi_{21}}{dt}$$

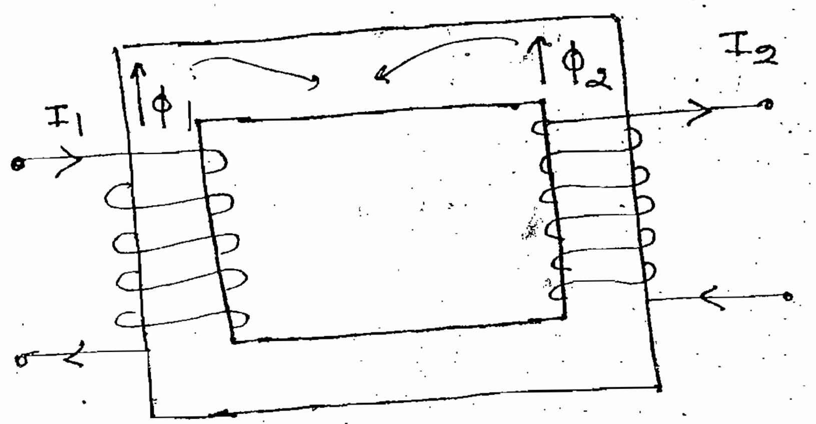
$$e_1 = -N_1 \frac{d\phi_{21}}{di_2} \cdot \frac{di_2}{dt} \qquad \left(M_{12} = \frac{N_1 \phi_{21}}{i_2} \right)$$

$$\boxed{e_1 = -M_{12} \frac{di_2}{dt}}$$

Mutual induced emf

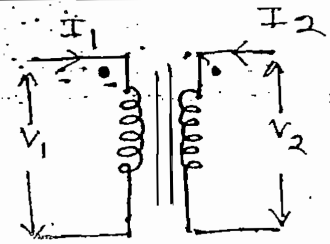
If current is flowing in any one of inductor then the sign of self & mutually induced voltage is same

Note:-

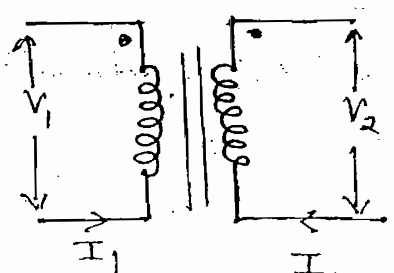


In above figure flux of the two inductors are completely closed path in opposite direction. Hence sign of mutually induced voltage is opposite to sign of self induced voltage

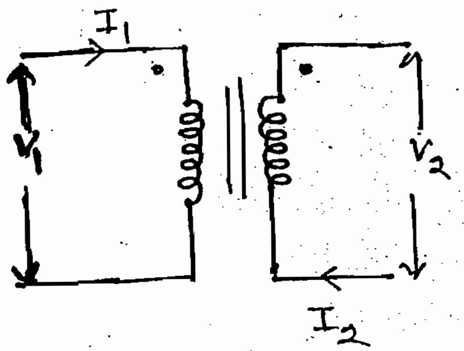
Dot convention:-



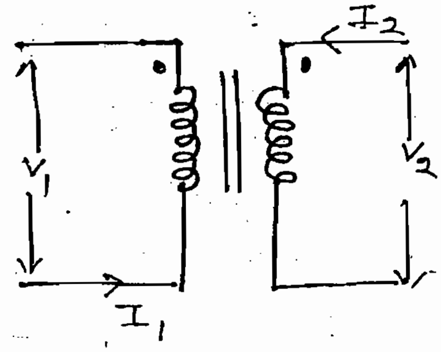
(I)



(II)



(III)



(IV)

$M_{12} = M_{21} = M$

$$\left.\begin{aligned} V_1 &= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ V_2 &= L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{aligned}\right\} \text{ Valid for (I) \& (II)}$$

$$\left.\begin{aligned} V_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ V_2 &= L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \end{aligned}\right] \text{ Valid for (III) \& (IV)}$$

Note:-

→ When either both currents are entering or both are leaving at dotted terminal sign of mutually induced voltage is same as the sign of self induced voltage.

→ When one current is entering and other current is leaving at dotted terminal sign of the mutually induced voltage is opposite to sign of self induced voltage

Coefficient of Coupling / Coupling Factor:-

$$K = \frac{\text{Useful flux}}{\text{Total flux}} \longrightarrow K_1 = K_2 \rightarrow \text{conditio}$$

$$\phi_1 \begin{cases} \rightarrow \phi_{11} \\ \rightarrow \phi_{12} \end{cases} \qquad K_1 = \frac{\phi_{12}}{\phi_1}$$

$$\phi_2 \begin{cases} \longrightarrow \phi_{22} \\ \longrightarrow \phi_{21} \end{cases} \qquad k_2 = \frac{\phi_{21}}{\phi_2}$$

$$K = \sqrt{k_1 k_2}$$

For ideal system $k = 1$ and for practical system the range of K is 0 to 1.

$$M_{21} = \frac{N_2 \phi_{12}}{i_1}$$

$$M_{12} = \frac{N_1 \phi_{21}}{i_2}$$

$$M_{12} = M_{21} = M$$

$$M^2 = M_{12} M_{21}$$

$$\Rightarrow \quad M^2 = \frac{N_1 \phi_{12}}{i_1} \cdot \frac{N_2 \phi_{12}}{i_1} \quad \text{— (1)}$$

$$\phi_{12} = k_1 \phi_1 \quad \text{— (II)}$$

$$\phi_{21} = k_2 \phi_2 \quad \text{— (III)}$$

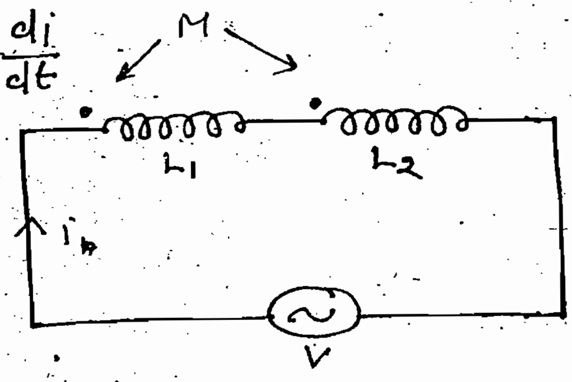
subsitute eq-(II) & (III) in eq-(1)

$$M^2 = k_1 k_2 \frac{N_1 \phi_1}{i_1} \cdot \frac{N_2 \phi_2}{i_2}$$

$$M^2 = K^2 L_1 L_2$$

$$\Rightarrow \quad \boxed{M = K\sqrt{L_1 L_2}}$$

$$V = L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$

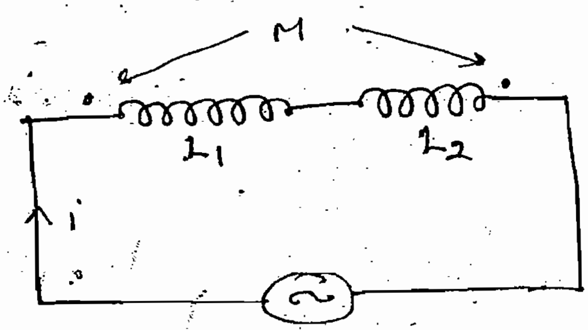


$$L_{eq}\frac{di}{dt} = (L_1+L_2+2M)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1+L_2+2M}$$

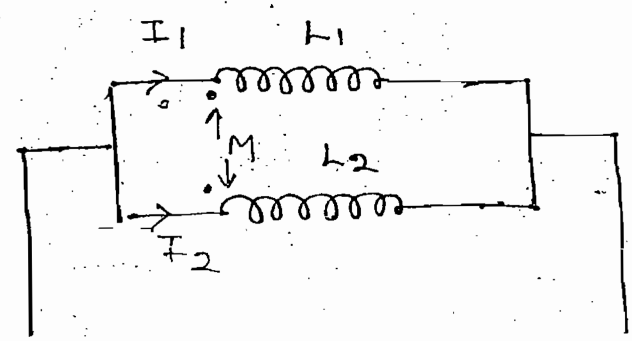
→ Series Aiding

→ **Series opposing:-**



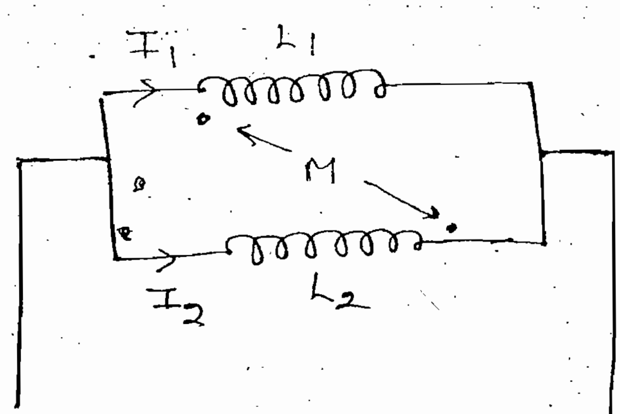
$$\boxed{L_{eq} = L_1+L_2-2M}$$

→ **Parallel Aiding:-**



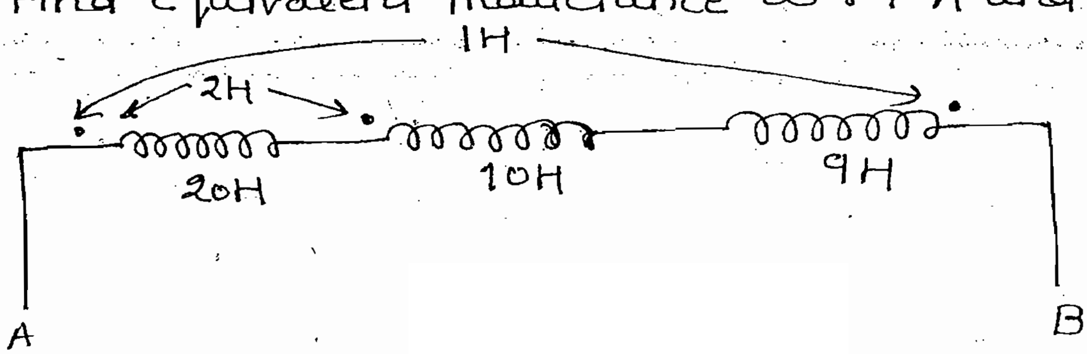
$$\boxed{L_{eq} = \frac{L_1L_2-M^2}{L_1+L_2-2M}}$$

→ **Parallel Opposing:-**



$$\boxed{L_{eq} = \frac{L_1L_2-M^2}{L_1+L_2+2M}}$$

Ques:- Find equivalent inductance w.r.t A and B :

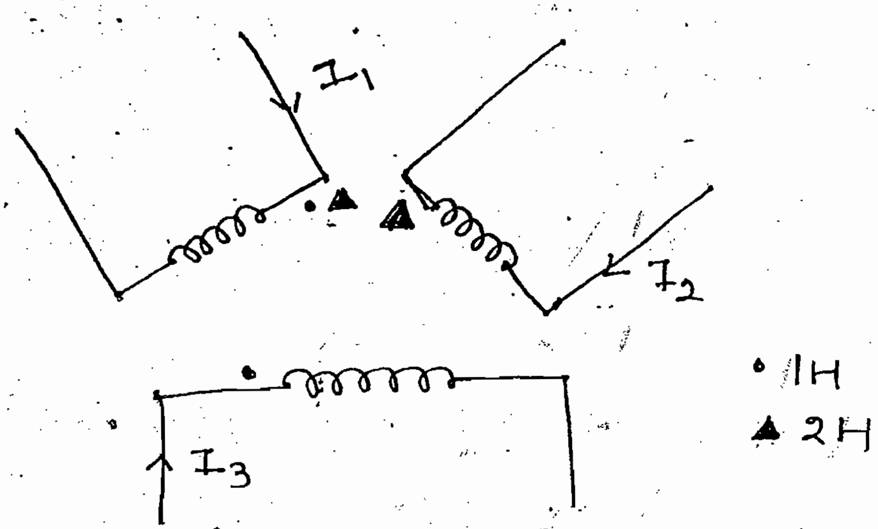


11M 44

Soln:- $L_{eq} = L_1 + L_2 + L_3 \pm 2M_1 \pm 2M_2 \pm 2M_3$

$\Rightarrow L_{eq} = 20 + 10 + 9 + 2(2) + 0 - 2(1)$

Ques:- Develop inductance matrix of the figure shown

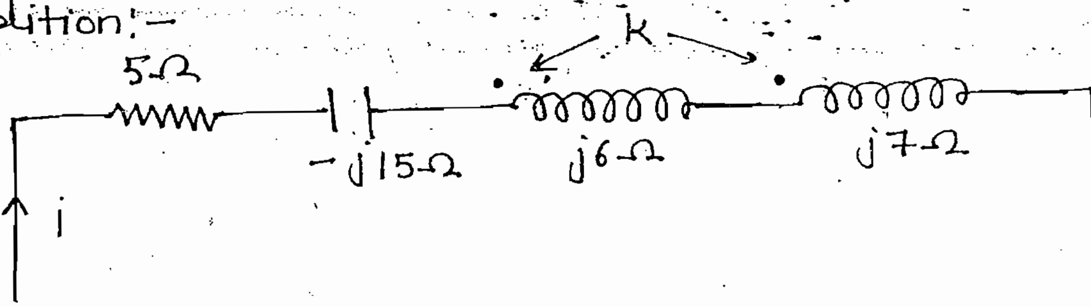


Soln:- Diagonal points denotes self inductance

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 15 & -2 & 1 \\ -2 & 20 & 0 \\ 1 & 0 & 10 \end{bmatrix}$$

Ques:- Find the value of K under resonance condition:-



Soln:- $2\pi f$ $(L_{eq} = L_1 + L_2 + 2M)$

$$M = K\sqrt{L_1 L_2}$$

$$\Rightarrow 2\pi f M = 2\pi f K\sqrt{L_1 L_2}$$

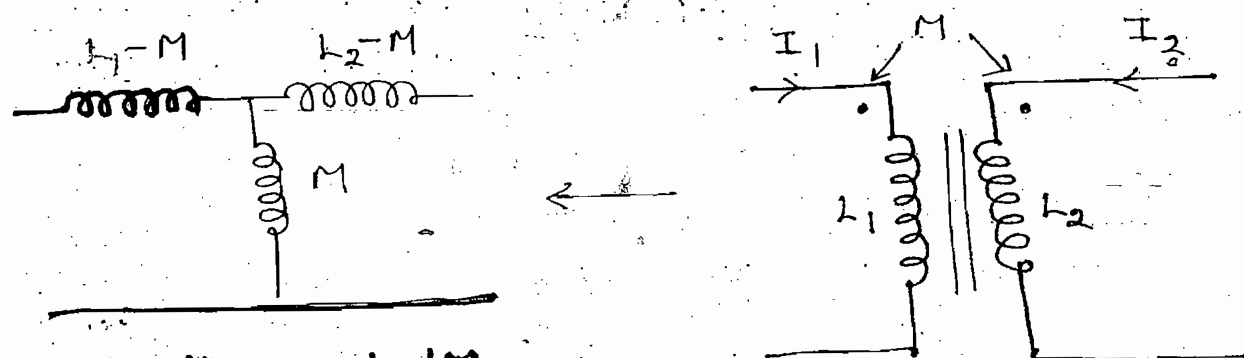
$$X_M = k\sqrt{X_1 X_2}$$

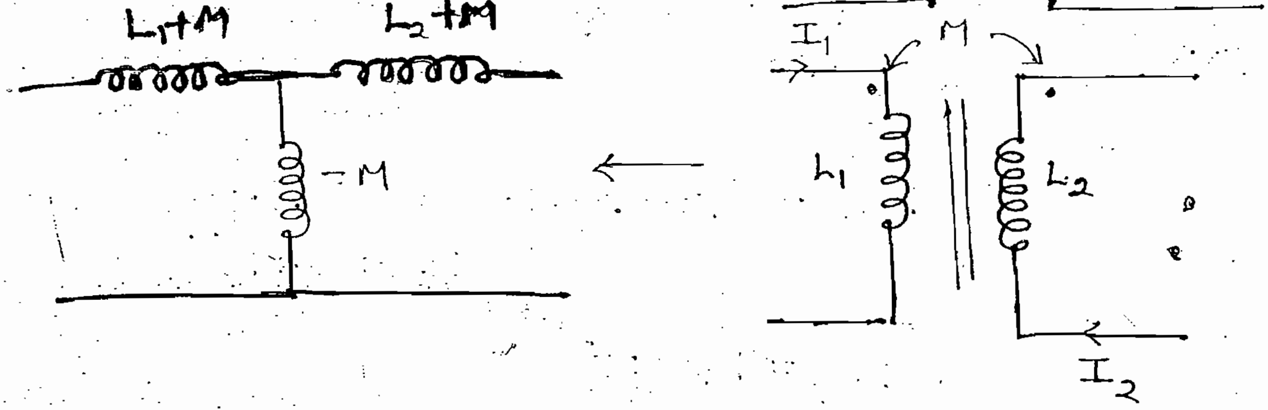
$$X_{eq} = X_1 + X_2 + 2X_M$$

$$\Rightarrow X_{eq} = X_1 + X_2 + 2k\sqrt{X_1 X_2}$$

$$\Rightarrow 15 = 6 + 7 + 2K\sqrt{6\times 7}$$

$$\Rightarrow \boxed{K = \frac{1}{\sqrt{42}}}$$ Ans



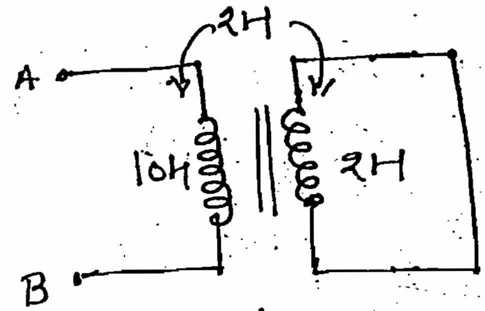


$$\boxed{W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2}$$

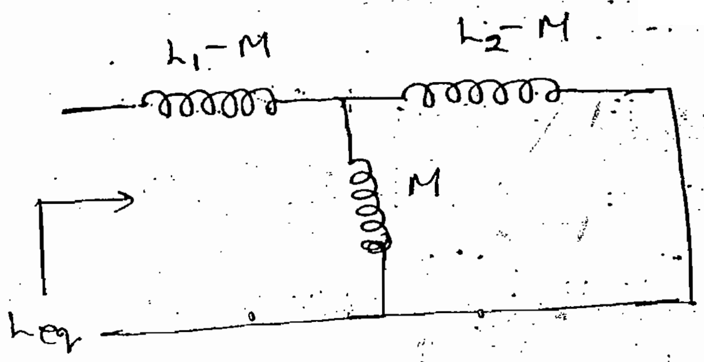
$+ \rightarrow$ fig-(1) $-$ve $\rightarrow$ fig(11)

Energy stored in the coupled coils.

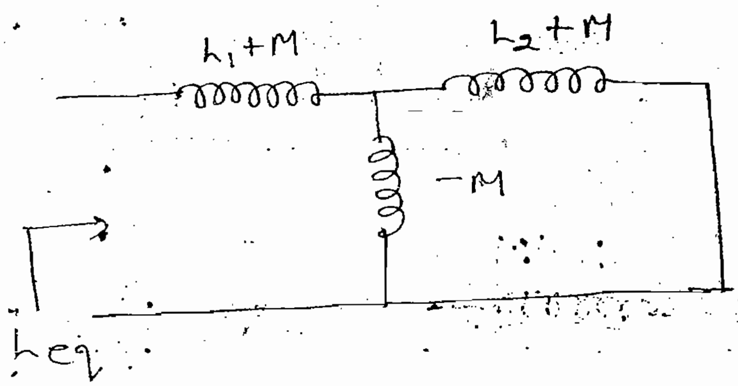
Ques:- Find L_{eq} w.r.t A and B



Soln:-



$$L_{eq} = L_1 - M + \frac{M(L_2 - M)}{L_2 - M + M} = L_1 - \frac{M^2}{L_2}$$

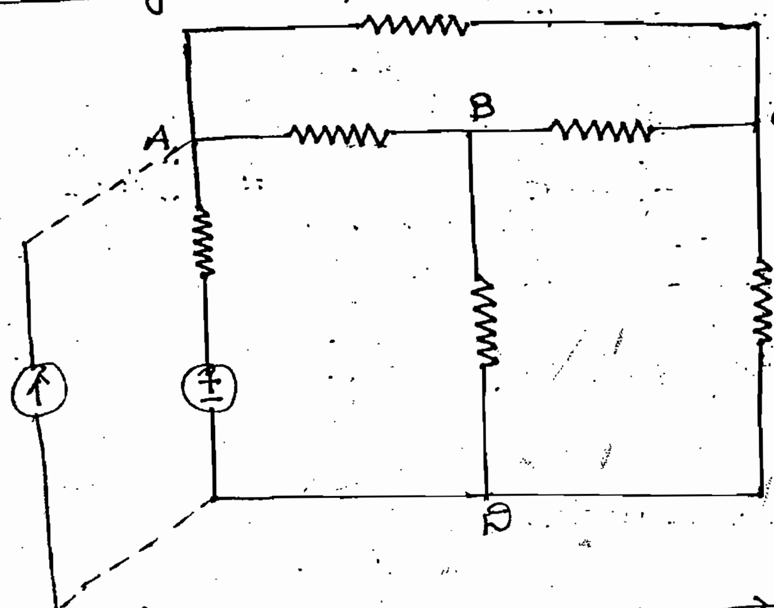


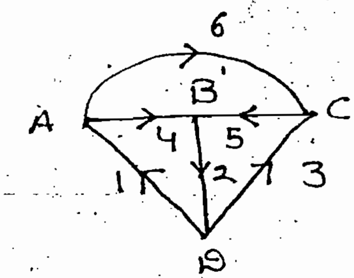
$$L_{eq} = (L_1 + M) + \frac{(-M)(L_2+M)}{L_2 + M - M}$$

$$\boxed{L_{eq} = L_1 - \frac{M^2}{L_2}}$$

$$\Rightarrow \quad L_{eq} = 10 - \frac{2^2}{2} = 8H \text{, Ans}$$

Graph Theory :-





→ N/w topology is the steady of the N/w properties by investigating interconnection b/w branches and nodes, it mainly concentrate on the geometry of the N/w

→ In the N/w topology any N/w is replace by graph. To develop graph ~~of~~ each element is replace by either straight line or arc of the semi-circle, Voltage source is replace by S.C and current source is replace by O.C and graph retains all the nodes of original N/w

→ | No. of branches of N/w $\geq$ No. of branch of graph. |

Augmented Incidence Matrix :-

	1	2	3	4	5	6
A	-1			+1		+1
B		+1		-1	-1	
C			-1		+1	-1
D	+1	-1	+1			

= $[A_a]$

Reduced Incidence Matrix :-

	1	2	3	4	5	6
A	-1			+1		+1
B		+1		-1	-1	
C			-1		+1	-1
~~D~~						

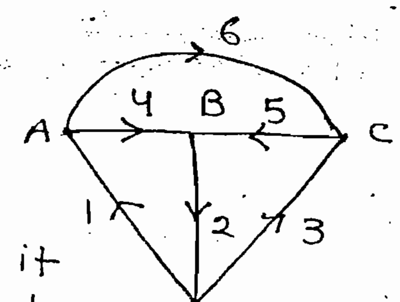
= $[A]$

D → Ref - Node or Datum Node

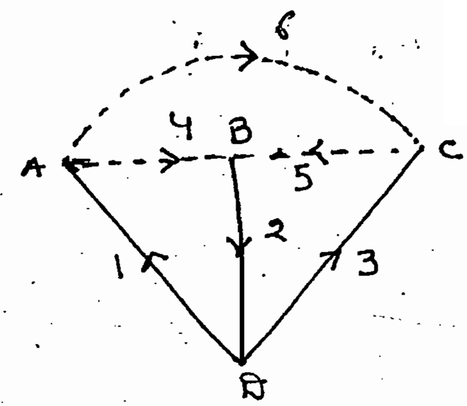
→ All the information regarding the graph can be represent mathematically inconsize form is called as Augmented incidence matrix

→ For a given graph Augmented incidence matrix is unique

Tree :-



→ Tree is a connected sub-graph. It connects the all the nodes of the N/w but it doesn't consist of any closed path



Tree {1, 2, 3}

Total Tree Branches $= N-1$

$= 4-1 = 3$

→ The set of branches which are disconnected to form a tree is called as co-tree or complementary tree

Co-Tree {4, 5, 6}

→ The branch which form a tree is called as tree branch (Twig)

→ Total no. of tree branches $= N-1$

→ The branch which is disconnected to form a tree is called as link (chord)

→ Total no. of links $(l) = b-(N-1)$

where $b =$ total No. of branches of the N/w

→ Tree is not unique

Total no. of tree $= N^{N-2} = 4^{4-2} = 16$

→ Total No of possible tree $= N^{N-2}$

<u>**Note:-**</u>

→ The above formula can be applied

(a) When connection is present b/w all the node

(b) When no repeated branch is present b/w the node

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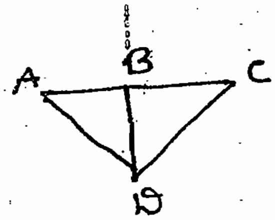
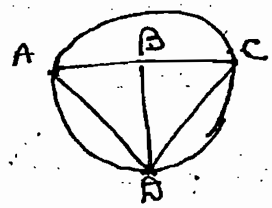




→ Total no. of possible tree for any graph

$$= \det(A A^T)$$

where A = Reduced incidence matrix

Node Pair Voltages:—

Total no. of node pair voltages $= \frac{N(N-1)}{2}$

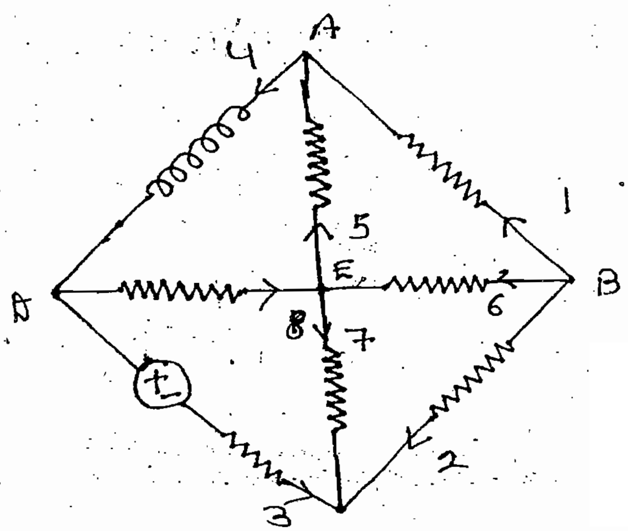
eg:- $\underbrace{V_{AB},\ V_{AC},\ V_{AD},\ V_{BC},\ V_{BD},\ V_{CD}}_{6}$

→ $\boxed{\text{Edges = Branches}}$

→ Total no. of edges (branches) $= \frac{N(N-1)}{2} = \frac{4 \times 3}{2}$

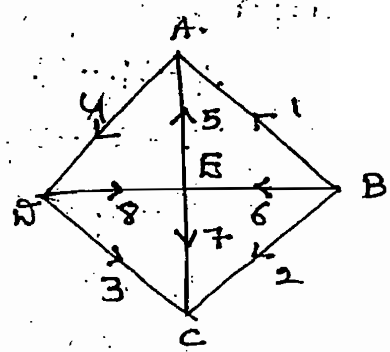
$= 6$

Ques:- Develop Tie-set matrix of the N/w shown



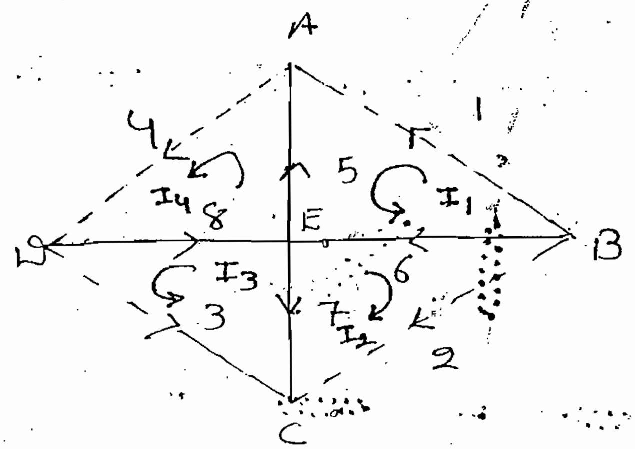
Step-1:—

Develop the graph for the given N/w



Step-(II):-

Develop a tree for the graph



Step-(III):-

Identify total no. of basic loop / fundamental loop (f-loop) or independent loops

→ **Basic** loop should consist of only one link

→ Total no. of basic loop = total no. of links

i.e $l = b - (N-1)$

→ **Basic** loop direction is same as link current direction

	V_1 1	V_2 2	V_3 3	V_4 4	V_5 5	V_6 6	V_7 7	V_8 8
I_1	+1				−1	−1		
I_2		+1				−1	−1	
I_3			+1				−1	−1
I_4				+1	+1			+1

$= [C]$

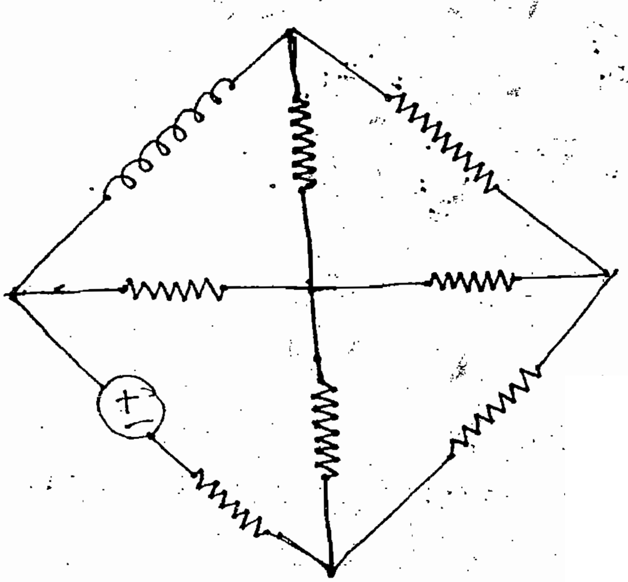
U C_b

KVL Equations :-

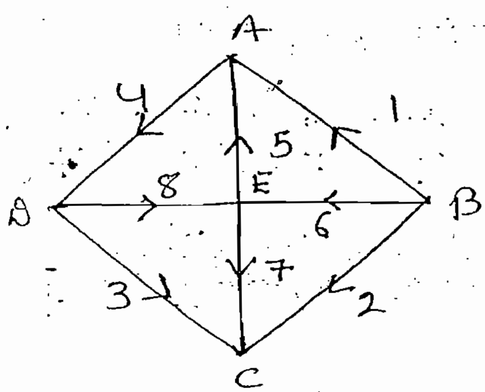
$$V_1 - V_5 - V_6 = 0$$
$$V_2 - V_6 - V_7 = 0$$
$$V_3 - V_7 - V_8 = 0$$
$$V_4 + V_5 + V_8 = 0$$

$$\boxed{[C] = [U : C_b]}$$

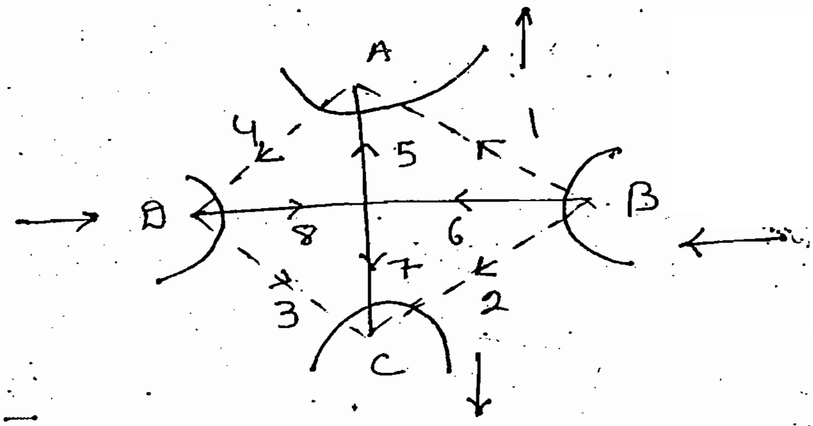
Ques:- Develop cut-set matrix of the N/w shown



Step-(I) :-
Develop a graph for the given N/w



Step-(II) :-
Develop a tree for a graph



Step-(III):-

Identify total no. of basic cut-sets or fundamental cut-sets / f-cut-sets

→ Basic cut-sets should consist of only one tree branch

→ Total No. of basic cut-sets = total No. of tree branches

i.e = N−1

→ Basic cut-set direction as same as a tree branch current direction

	I_1 1	I_2 2	I_3 3	I_4 4	I_5 5	I_6 6	I_7 7	I_8 8
A	+1			−1	+1			
B	+1	+1				+1		
C		+1	+1				+1	
D			+1	−1				+1

$= [B]$

B_l U

KCL Equations:-

$$I_1 - I_4 + I_5 = 0$$
$$I_1 + I_2 + I_6 = 0$$
$$I_2 + I_3 + I_7 = 0$$
$$I_3 - I_4 + I_8 = 0$$

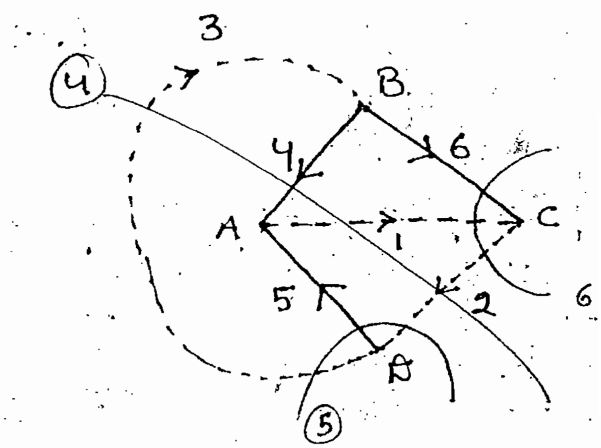
$$\boxed{[B] = [B_l \vdots U]}$$

$$[C] = [U : C_b]$$
$\downarrow$
Tie-set

$$[B] = [B_l : U]$$
$\downarrow$
cut-set

$$\boxed{\begin{aligned} [C_b] &= -[B_l]^T \\ [B_l] &= -[C_b]^T \end{aligned}}$$

Ques:- Develop cut-set matrix of the graph shown

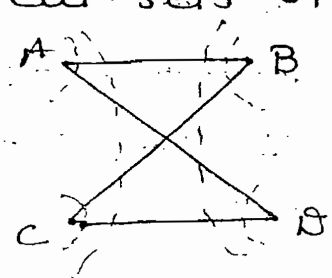


Soln:-

**

	1	2	3	4	5	6
4	-1	+1	-1	+1		
5		-1	+1		+1	
6	+1	-1				+1

Ques:- Identify total no. of cut-sets of the graph shown



(a) 3 (b) 4

(c) 5 (d) 6

Note:-

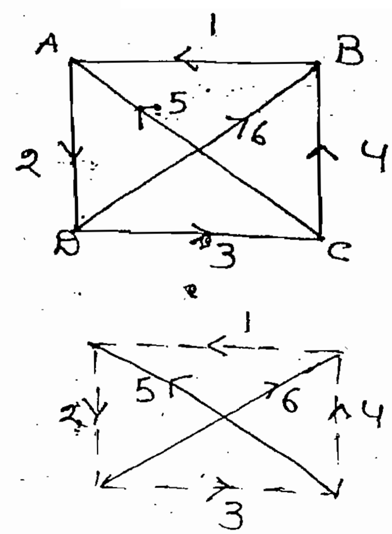
- cut-sets may consist of one tree branch or more than one tree branch
- Basic cut-sets consist of only one tree branc

Conclusion:-

- Total no. of possible tree $= N^{N-2}$
- Tie-set matrix is not unique, total no. of possible tie-set matrix $= N^{N-2}$
- cut-set matrix is not unique, Total no. of possible cut-set matrix ~~is~~ $= N^{N-2}$
- Rank of Tie-set matrix = total no. of links

  i.e. $l = b-(N-1)$
- Rank of cut-set matrix = total no. of tree branches

  $= N-1$
- Rank of Incidence Matrix $= N-1$
- The given figure is invalid tree. Since no connection is present b/w tree branches



Duality:-

$R \longleftrightarrow G$

$L \longleftrightarrow C$

$V \longleftrightarrow I$

Series $\longleftrightarrow$ Parallel

O.C $\longleftrightarrow$ S.C

KVL $\longleftrightarrow$ KCL

loop $\longleftrightarrow$ Node
(Mesh

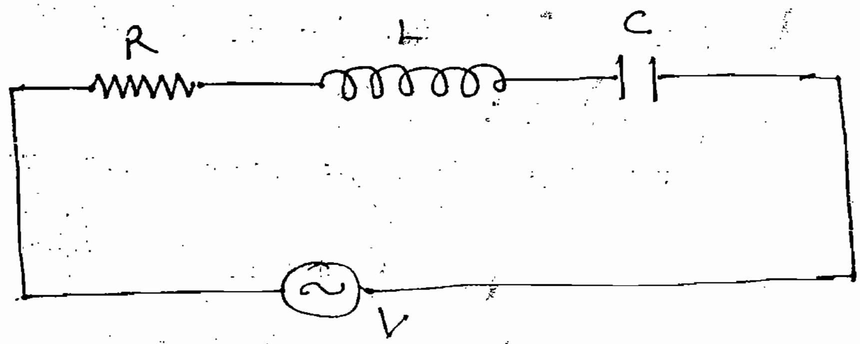
Tie-set $\longleftrightarrow$ Cut-set

Thevenin's $\longleftrightarrow$ Norton's

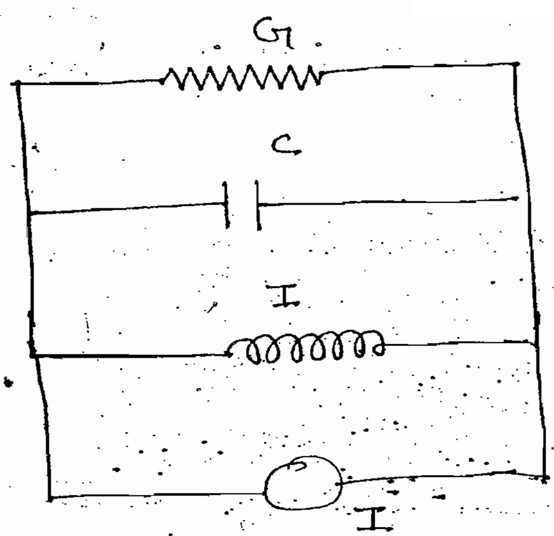
Foster-I form $\longleftrightarrow$ Foster-II form
(series) (Parallel)

$$\frac{dV}{dt} \longleftrightarrow \frac{dI}{dt}$$

$$\int V dt \longleftrightarrow \int i dt$$



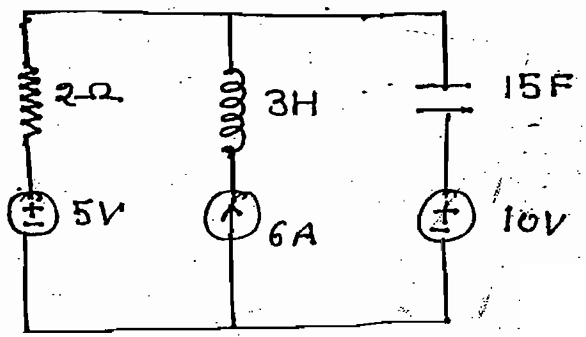
$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i dt \longrightarrow KVL$$



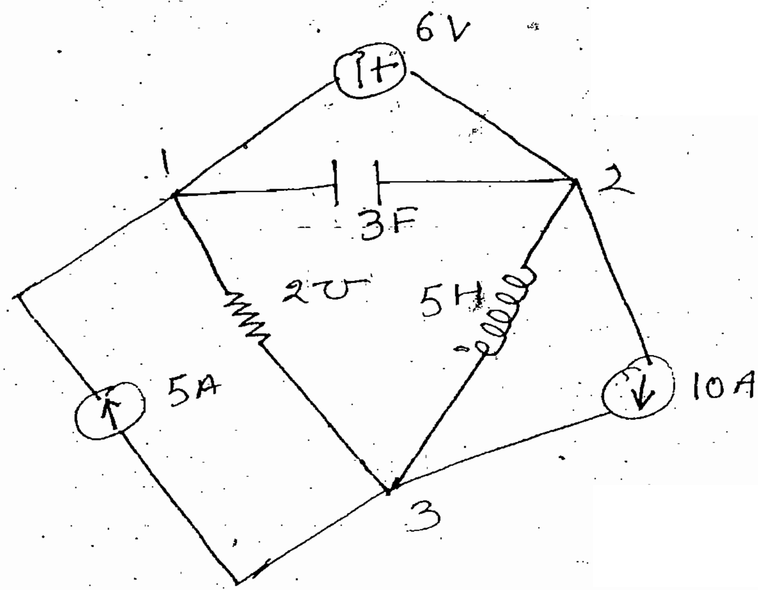
$$I = VG + C\frac{dV}{dt} + \frac{1}{L}\int V dt \longrightarrow KCL$$

Duality does not means the equivalence but it means that mathematical representation of both N/w are identicals.

Ques:- Develop dual of the N/w shown



Soln:-



Note:-

→ When voltage source circulate a current in clockwise direction arrowmark of the current source is indicated towards respective node

→ When current source circulate a current in clockwise direction +ve sign is assign to respecti node