Integrals

Multiple Choice Questions :-

Concept Integrals

1.The value of $\int \frac{Cos\sqrt{x}}{\sqrt{x}}\, dx$ is

(a) $2\cos\sqrt{x} + C$ (b) $\sqrt{\frac{\cos x}{x}} + C$ (c) $\sin\sqrt{x} + C$ (d) $2\sin\sqrt{x} + C$

2. $\int \frac{x+3}{(x+4)^2}\, e^x dx$ =

(a) $\frac{e^x}{x+4} + C$ (b) $\frac{e^x}{x+3} + C$

(c) $\frac{1}{(x+4)^2} + C$ (d) $\frac{e^x}{(x+4)^2} + C$

3. $\int \frac{Cos\,2x - Cos\,2\theta}{Cos\,x - Cos\theta}\, dx$ is equal to

(a) 2 (Sin x + x Cos θ) + C (b) 2 (Sin x - x Cos θ) + C

(c) 2 (Sin x +2 x Cos θ) + C (d) 2 (Sin x - 2 x Cos θ) + C

4. $\int x^2 e^{x^3}\, dx$ equals

(a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^2} + C$

(c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$

5. $\int \sqrt{a^2 - x^2}$ dx

(a) $\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$ (b) $Sin^{-1}\frac{x}{a} + C$

(c) $\frac{1}{2}\, x\sqrt{a^2 - x^2} + \frac{1}{2}\, a^2\, Sin^{-1}\frac{x}{a}$ (d) $\frac{1}{2}\, x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 - a^2}\right| + C$

6. $\int_1^e \frac{\ln x}{x}\, dx$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{e^2}{2}$ (c) 1 (d) ∞

7. $\int_0^1 \tan(Sin^{-1}x)\ dx$

(a) 2 (b) 0 (c) -1 (d) 1

8 . The value of I = $\int_0^{\frac{\pi}{2}} \frac{(Sin\,x+Cos\,x)^2}{\sqrt{1+Sin\,2x}}\, dx$, is

(a) 3 (b) 1 (c) 2 (d) 0

9. $\int_0^{2a} f(x)\ dx$ is equal to

(a) 2 $\int_0^a f(x)\ dx$ (b) 0

(c) $\int_0^a f(x)$ + $\int_0^a f\,(2a - x)\, dx$ (d) $\int_0^a f(x)$ + $\int_0^{2a} f(x)$

10. The value of the integral $\int_{\frac{1}{3}}^{1} \frac{(x - x^3)^{\frac{1}{3}}}{x^4}$ dx is

(a) 6 (b) 0 (c) 3 (d) 4

Definite Integrsls

1. $\int_0^2 x^2 dx =$

a. 2
b. ⅔
c. 8/3
d. None of these

Answer: (c) 8/3

2. $\int_0^2 (x^2 + 3)dx$ equals

a. 24/3
b. 25/3
c. 26/3
d. None of the above.

Answer: (c) 26/3

3. $\int_1^2 dx/x^2$ equals

a. 1
b. -1
c. 2
d. ½

Answer: (d) ½

4. $\int_0^{\pi} \sin^2 x \, dx =$

a. π/2
b. π/4
c. 2π
d. 4π

Answer: (a) π/2

5. $\int_0^4 3x \, dx$ equals

a. 12
b. 24
c. 48
d. 86

Answer: (b) 24

6. Integrate $\int_0^2 (x^2+x+1)\,dx$

a. 15/2
b. 20/5
c. 20/3
d. 3/20

Answer: (c) 20/3

7. If f(a + b + 1 – x) = f(x) ∀ x, where a and b are fixed positive real numbers, then the below expression is equal to

$$\frac{1}{(a+b)}\int_a^b x(f(x)+f(x+1))\,dx$$

a. $\int_{a-1}^{b-1} f(x)\,dx$

b. $\int_{a+1}^{b+1} f(x+1)\,dx$

c. $\int_{a-1}^{b-1} f(x+1)\,dx$

d. $\int_{a+1}^{b+1} f(x)\,dx$

SOLUTION

$$2I = \int_a^b f(a+b-x+1)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$$I = \int_a^b f(x)dx$$

As, $x = t+1, dx = dt$

$$I = \int_{a-1}^{b-1} f(t+1)dt$$

$$I = \int_{a-1}^{b-1} f(x+1)dx$$

ANS: C

8. Let I= $\int_a^b \frac{IxI}{x}dx, a < b$

What is I equals to if $a < b < 0$

a) a+b b) a-b

c) b-a d) $\frac{a+b}{2}$

solution: a-b

9. Let I= $\int_a^b \frac{IxI}{x}dx, a < b$

What is I , if a$< 0 < b$

a) a+b b) a-b

c) b-a d) $\frac{a+b}{2}$

solution : a+b

10.

If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$,

then value of a is equal to
(a) 3
(b) 6
(c) 9
(d) 1
Solution: c

(c), as $\frac{dx}{dy} = \frac{1}{\sqrt{1+9y^2}}$

$\Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{18y}{2\sqrt{1+9y^2}} \cdot \frac{dy}{dx}$

11.

$\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

(a) $\frac{e^x}{1+x^2}+C$ (b) $-\frac{e^x}{1+x^2}+C$

(c) $\frac{e^x}{(1+x^2)^2}+C$ (d) $\frac{e^x}{(1+x^2)^2}+C$

SOLUTION: a

$f(x) = \frac{1}{1+x^2};$

$f'(x) = \frac{-2x}{(1+x^2)^2}$

Using $\int e^x \{f(x) + f'(x)\} dx$

$= e^x \cdot f(x) + C$

$= e^x \cdot \frac{1}{1+x^2} + C$

12.

$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$ equals to

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{3}{2}$

Solution: c

(c), as $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)}$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\cdot\left[\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$$

$$= -\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{4}-0\right) = 1.$$

13.

Let $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$, then

(a) $I_1 > I_2$
(b) $I_2 > I_1$
(c) $I_1 = I_2$
(d) $I_1 > 2I_2$
Solution: b

(b), we have $1 + x^2 > x^2$

$$\Rightarrow \sqrt{1+x^2} > x \text{ for } x \in [1, 2]$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} < \frac{1}{x}$$

$$\Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}}dx < \int_1^2 \frac{dx}{x} \Rightarrow I_1 < I_2$$

Indefinite Integrals

1) If (d/dx) f(x) is g(x), then the antiderivative of g(x) is

a) f(x)
b) f'(x)
c) g'(x)
d) None of the above

Answer: (a) f(x)

Given: (d/dx) f(x) = g(x)

We know that the integration is the inverse process of differentiation, then the antiderivative of g(x) is f(x).

Hence, option (a) f(x) is the correct answer.

2) $\int \cot^2 x \, dx$ equals to

a) cot x – x + C
b) -cot x – x + C
c) cot x + x + C
d) -cot x + x + C

Answer: (b) -cot x – x + C

Explanation:

We know that $\cot^2 x = \operatorname{cosec}^2 x - 1$

$\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1) \, dx = -\cot x - x + C$. [Since, $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$]

Hence, the correct answer is option (b) -cot x – x + C.

3) If $\int \sec^2(7 - 4x)dx = a \tan (7 - 4x) + C$, then value of a is

a. -4
b. -¼
c. 3
d. 7

Answer: (b) -¼

Explanation:

Given: $\int \sec^2(7 - 4x)dx = a \tan (7 - 4x) + C$

$\int \sec^2(7 - 4x)dx = \{[\tan (7-4x)]/-4\} + C$

$\int \sec^2(7 - 4x)dx = (-¼) \tan (7-4x) + C$

Hence, the value of a is -¼.

$\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

(a) $\frac{e^x}{1+x^2} + C$ (b) $-\frac{e^x}{1+x^2} + C$

(c) $\frac{e^x}{(1+x^2)^2} + C$ (d) $\frac{e^x}{(1+x^2)^2} + C$

Answer: (a)

Explanation:

$$f(x) = \frac{1}{1+x^2};$$

$$f'(x) = \frac{-2x}{(1+x^2)^2}$$

Using $\int e^x \{f(x) + f'(x)\} dx$

$$= e^x \cdot f(x) + C$$

$$= e^x \cdot \frac{1}{1+x^2} + C$$

5.

$\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx, \frac{3\pi}{4} < x < \frac{7\pi}{4}$ is equal to

(a) log |sin x + cos x|
(b) x
(c) log |x|
(d) -x

Answer: d
Explaination:

(d), as $\int \frac{\sin x + \cos x}{|\sin x + \cos x|} dx,$

$$\Rightarrow \quad -\int 1 \cdot dx = -x + C$$

$\{$as $\sin x + \cos x < 0$ for $\frac{3\pi}{4} < x < \frac{7\pi}{4}\}$

6. If $\int \sec^2(7 - 4x)dx = a \tan(7 - 4x) + C$, then value of a is
(a) 7
(b) -4
(c) 3
(d) −14
Answer: d
Explaination:
(d), $\int \sec^2(7 - 4x)dx = \tan(7-4x)-4 + C = -14 \tan(7 - 4x) + C$.

7.Given $\int 2^x\, dx = f(x) + C$, then f(x) is
(a) 2^x (b) $2^x \log_e 2$
(c) $\frac{2^x}{\log_e 2}$ (d) $\frac{2^{x+1}}{x+1}$
Explaination:
(c), as $\frac{d}{dx}\left(\frac{2^x}{\log_e 2}\right)$

$$= \frac{1}{\log_e 2} \cdot 2^x \cdot \log_e 2 = 2^x.$$

8. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$ is equal to
a) $2(\sin x + x \cos\theta) + C$
(b) $2(\sin x - x \cos\theta) + C$
(c) $2(\sin x + 2x \cos\theta) + C$
(d) $2(\sin x - 2x \cos\theta) + C$
Answer : (a)

Explaination:

(a), as $\int \frac{2(\cos^2 x - \cos^2\theta)}{\cos x - \cos\theta} dx,$

using $\cos 2x = 2\cos^2 x - 1$

$$= 2\int (\cos x + \cos\theta)\, dx$$

$$= 2\sin x + 2x \cdot \cos\theta + C$$

9.

$$\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx, \frac{3\pi}{4} < x < \frac{7\pi}{4} \text{ is equal to}$$

(a) log |sin x + cos x|
(b) x
(c) log |x|
(d) –x

Answer: d
Explaination:

(d), as $\int \frac{\sin x + \cos x}{|\sin x + \cos x|} dx,$

$$\Rightarrow \quad -\int 1 \cdot dx = -x + C$$

{as $\sin x + \cos x < 0$ for $\frac{3\pi}{4} < x < \frac{7\pi}{4}$}

10.

If $\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3}\sin^{-1}(ax) + C$, then value of a is

(a) 2
(b) 4
(c) $\frac{3}{2}$
(d) $\frac{2}{3}$

Answer: c
Explaination:

(c), as $\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}} dx = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$

$$\Rightarrow \quad a = \frac{3}{2}.$$